Neutrinos in the early Universe

International PhD Summer School on Neutrinos: Here, There & Everywhere
Copenhagen, July 14, 2022

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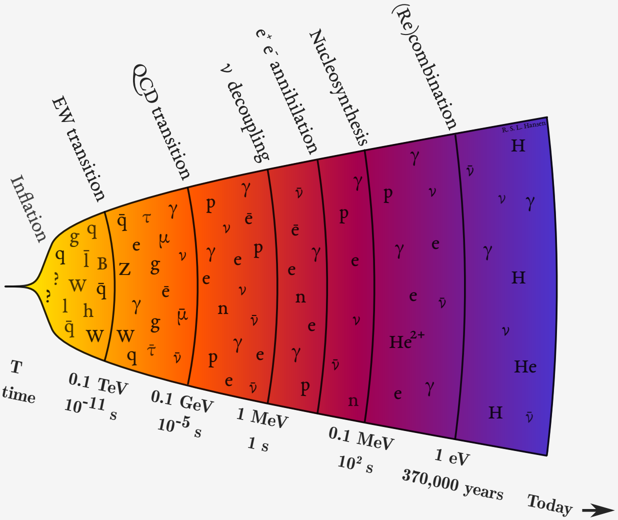
NBIA and DARK at NBI

UNIVERSITY OF COPENHAGEN

Sapere Aude

The early Universe



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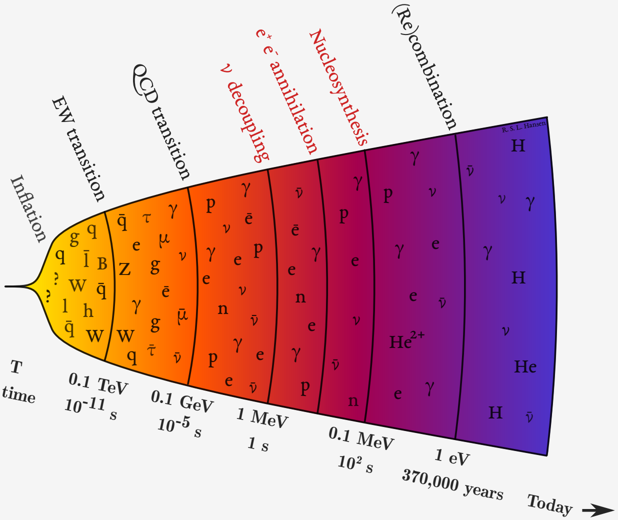
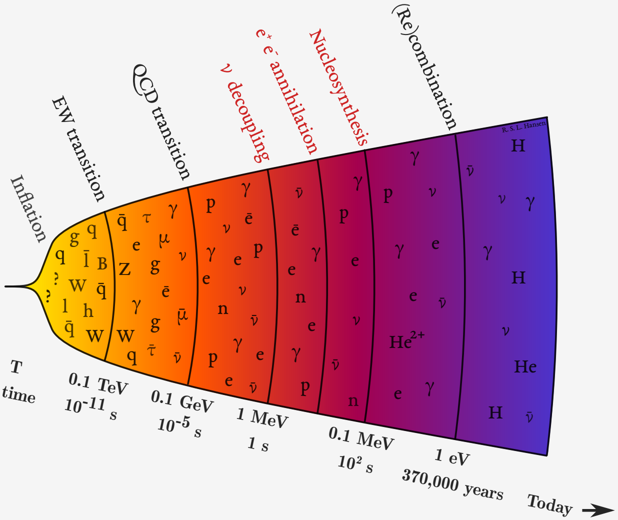


ν decoupling ($\Gamma_{\text{weak}} < H$):

$\nu + e \leftrightarrow \nu + e$

The early Universe



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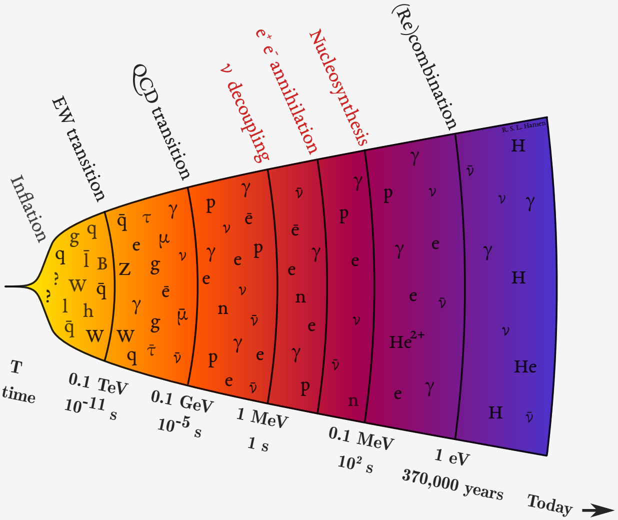

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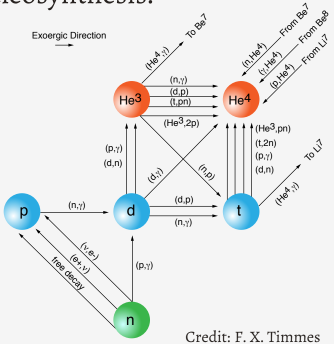
Nucleosynthesis:



Credit: F. X. Timmes

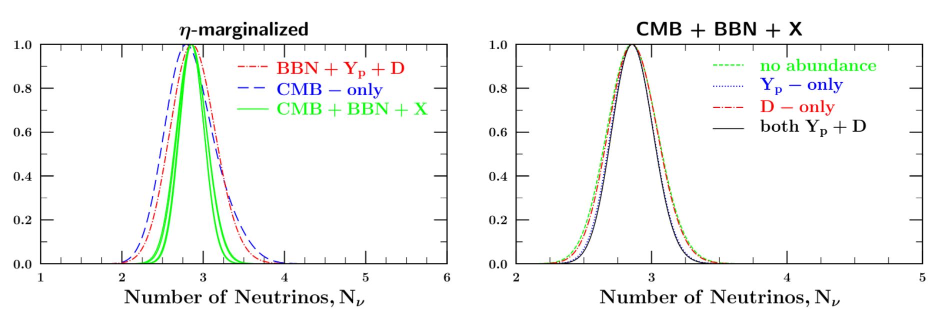
Observables - N_{eff}

Energy density of radiation: $\rho_{\text{rad}} = \rho_\gamma + \rho_\nu + \rho_{\text{BSM}} = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right)$.

N_{eff} - the effective number of relativistic neutrino species.

It affects both BBN and CMB.

Instantaneous decoupling approximation: $N_{\text{eff}} = 3$. Detailed calculation: $N_{\text{eff}} \approx 3.044$.



Fields, Olive, Yeh and Young (1912.01132)

Standard Model corrections to N_{eff}

As Olga introduced, the corrections come from different effects:

Standard-model corrections to $N_{\text{eff}}^{\text{SM}}$	Leading-digit contribution
m_e/T_d correction	$+0.04$
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	$+0.01$
Non-instantaneous decoupling+spectral distortion	-0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	$+0.0005$
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

Bennett et al. (2012.02726)

Final result: $N_{\text{eff}} = 3.0440 \pm 0.0002$.

From Olga:

Particle decoupling in the early universe: Neutrinos

- The entropy density is: $s \equiv \frac{\rho + p}{T}$

¿How are related the photon and the neutrino temperatures?

- Electron positron annihilation takes place AFTER neutrino decoupling.

- In an expanding universe the entropy density per comoving volume is conserved:

- Boson's entropy contribution: $2\pi^2 T^3/45$
- Fermion's entropy contribution: $7/8 \times 2\pi^2 T^3/45$

- Before electron/positron annihilation= electrons (g=2), positrons (g=2), neutrinos (3), antineutrinos (3) and photons (g=2) therefore:

$$s(a_1) = 2\pi^2 T_1^3/45(2 + 7/8(2+2+3+3))$$

- After, only neutrinos, antineutrinos and photons but at different temperature!

$$s(a_2) = 2\pi^2/45(2T_\gamma^3 + 7/8(3+3)T_\nu^3)$$

$$s(a_1)a_1^3 = s(a_2)a_2^3 \qquad a_1 T_1 = a_2 T_\nu \longrightarrow \left(\frac{T_\nu}{T_\gamma}\right) = \left(\frac{4}{11}\right)^{1/3}$$

Correction from $m_e/T_d > 0$ $(\delta N_{\rm eff} \sim +0.04)$

Assume entropy conservation:

$$s(a_1)a_1^3 = s(a_2)a_2^3 \,, \qquad s_\nu(a_1)a_1^3 = s_\nu(a_2)a_2^3 \,.$$

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a_1 at neutrino decoupling (T_d), a_2 after $e^\pm$ annihilation. Entropies:

$$s(a_1) = \frac{2\pi^2}{45}\left(g_\gamma + \frac{7}{8}g_e\right)T_d^3 + \delta s\,, \qquad s(a_2) = \frac{2\pi^2}{45}g_\gamma T(a_2)^3\,,$$

$$s_\nu(a_1) = 3\times\frac{7}{8}\,\frac{2\pi^2}{45}g_\nu T_d^3\,, \qquad s_\nu(a_2) = 3\times\frac{7}{8}\,\frac{2\pi^2}{45}g_\nu T_\nu(a_2)^3$$

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The change in entropy when relaxing the $T_d/m_e \to \infty$ approximation: $(E_e = \sqrt{p^2+m_e^2})$

$$\delta s = \frac{g_e}{2\pi^2 T_d}\int_0^\infty dp p^2\left(E_e + \frac{p^2}{3E_e}\right)\frac{1}{\exp(E_e/T)+1} - s_{\rm Rel} \approx -0.009859\, s_{\rm Rel}\ .$$

Correction from $m_e/T_d > 0$ $(\delta N_{\text{eff}} \sim +0.04)$

Assume entropy conservation:

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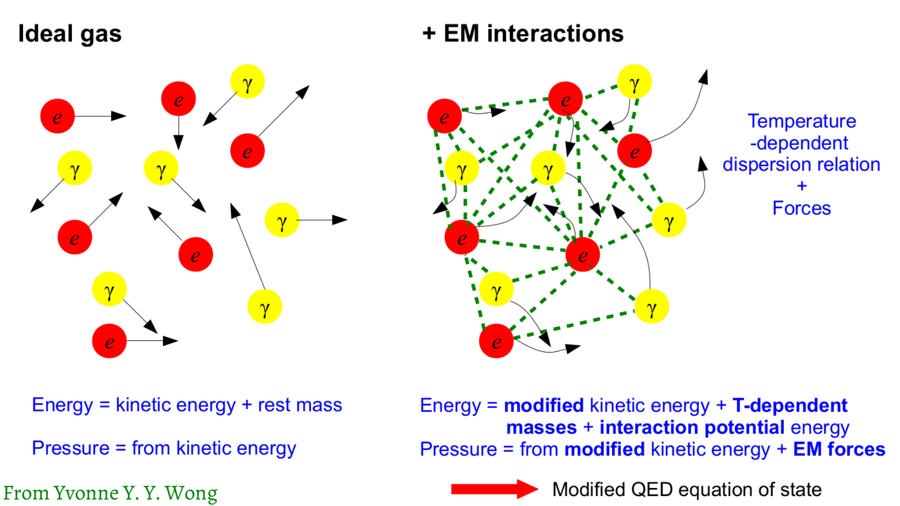
The change in N_{eff} is:

$$\delta N_{\text{eff}} = 3\left(\left[1 + \frac{\delta s}{s_{\text{Rel}}}\right]^{-4/3} - 1\right) \approx 0.039895 \,.$$

Finite-temperature QED corrections ($\delta N_{\rm eff} \sim +0.01$)



From Yvonne Y. Y. Wong

Finite-temperature QED corrections ($\delta N_{\rm eff} \sim +0.01$)

see also Bennett et al. 1911.04504

More formally: QED equation of state can be computed from the grand canonical partition function.
Expanded in powers of e:

$$\ln Z = \ln Z^{(0)} + \ln Z^{(2)} + \ln Z^{(3)} + \dots \,.$$

Pressure, energy density and entropy can be calculated as

$$P^{(n)} = \frac{T}{V} \ln Z^{(n)} \,,$$

$$\rho^{(n)} = \frac{T^2}{V} \frac{\partial \ln Z^{(n)}}{\partial T} = -P^{(n)} + T \frac{\partial P^{(n)}}{\partial T} \,,$$

$$s^{(n)} = \frac{1}{V} \frac{\partial \left[T \ln Z^{(n)} \right]}{\partial T} = \frac{\rho^{(n)} + P^{(n)}}{T} \,.$$

Finite-temperature QED corrections ($\delta N_{\rm eff} \sim +0.01$)

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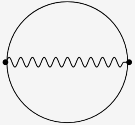
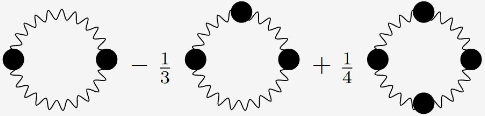
Zeroth order term gives the equations for an ideal gas.

Higher order terms come from:

$$\ln Z^{(2)} = -\tfrac{1}{2}$$



$$\ln Z^{(3)} = \tfrac{1}{2}\Big[\ \tfrac{1}{2}\ (\ldots) - \tfrac{1}{3}\ (\ldots) + \tfrac{1}{4}\ (\ldots) + \cdots\ \Big]$$



E.g. for pressure

$$P^{(2)} = \frac{T}{V}\ln Z^{(2)} = -\frac{e^2T^2}{12\pi^2}\int_0^\infty \mathrm{d}p\,\frac{p^2}{E_p}n_D - \frac{e^2}{8\pi^4}\left(\int_0^\infty \mathrm{d}p\,\frac{p^2}{E_p}n_D\right)^2 + \frac{e^2m_e^2}{16\pi^4}\iint_0^\infty \mathrm{d}p\,\mathrm{d}\tilde{p}\,\frac{p\tilde{p}}{E_pE_{\tilde{p}}}\ln\left|\frac{p+\tilde{p}}{p-\tilde{p}}\right| n_D\tilde{n}_D$$

Finite-temperature QED corrections ($\delta N_{\rm eff} \sim +0.01$)

see also Bennett et al. 1911.04504

Alternative: Quasiparticle picture

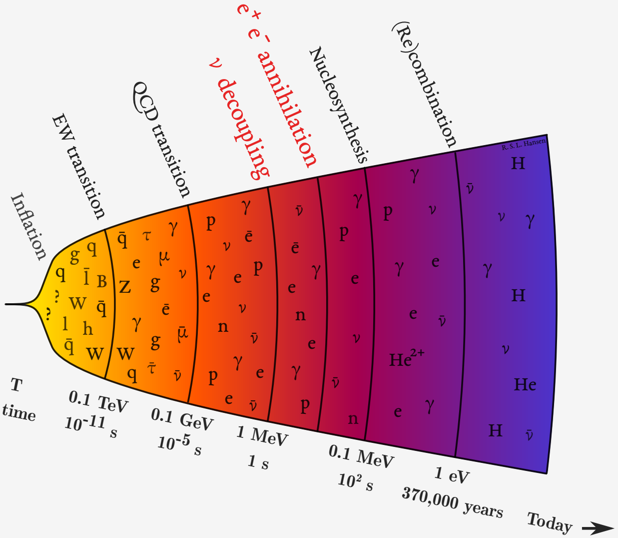
$$E_\gamma^2(p) \to E_\gamma^2(p,T) = p^2 + \delta m_\gamma^2(T),$$
$$E_e^2(p) \to E_e^2(p,T) = p^2 + m_e^2 + \delta m_e^2(p,T)$$

Works well for transport equations, but "double counts" for bulk properties.

Cannot account for collective exitations such as plasmons.

- Hence does not work beyond second order correction.

Non-instantaneous decoupling ($\delta N_{\rm eff} \sim -0.005$)



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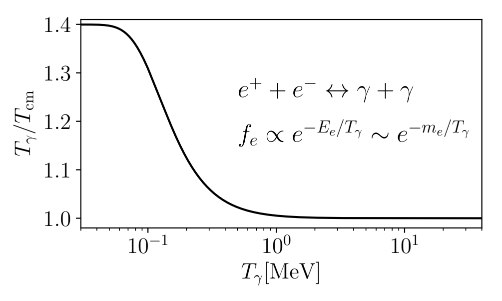

Friedmann equation + continuity equation give T_γ.

Boltzmann equation:

$$\frac{\partial f_\nu(t,p)}{\partial t} - Hp\frac{\partial f_\nu(t,p)}{\partial p} = \mathcal{C}[f_\nu, p]$$

Collision term for $1 + 2 \to 3 + 4$:

$$\mathcal{C}[f_1, p] = \frac{1}{2E_1}\int\prod_{i=2}^{4}\frac{d^3p_i}{(2\pi)^3 2E_i}(2\pi)^4\delta^4(P_1 + P_2 - P_3 - P_4)|M|^2\left[f_3f_4(1-f_1)(1-f_2) - f_1f_2(1-f_3)(1-f_4)\right]$$

Phase space E and p cons.   Matrix element

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Bennett et al. (2012.02726)

Final result: $N_{\text{eff}} = 3.0440 \pm 0.0002$.

Neutrino flavor oscillations

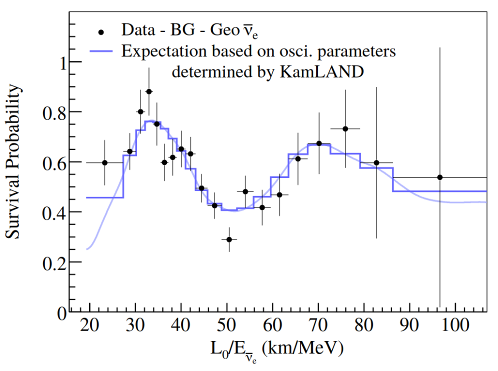
Described by the von Neumann equation (for pure states)

$$\frac{d\rho}{dt} = -i\,[\mathcal{H}, \rho]\ , \qquad \mathcal{H}_{\rm vac} = \frac{\mathcal{U}\mathcal{M}^2\mathcal{U}^\dagger}{2E}$$

ρ is the density matrix.
Equivalent to a Schrödinger-like equation:

$$\frac{d\,|\psi\rangle}{dt} = -i\mathcal{H}\,|\psi\rangle$$

$|\psi\rangle$ - wave function.
$\rho = |\psi\rangle\,\langle\psi|$ for pure states.



KamLAND (0801.4589)

Quantum Kinetic Equations - include oscillations ($\delta N_{\rm eff} \sim +0.0005$)

For three flavors of neutrinos, $\rho = \begin{pmatrix} f_{\nu_e} & \rho_{e\mu} & \rho_{e\tau} \\ \rho_{\mu e} & f_{\nu_\mu} & \rho_{\mu\tau} \\ \rho_{\tau e} & \rho_{\tau\mu} & f_{\nu_\tau} \end{pmatrix}$.

Combine the von Neumann and Boltzmann equations: Sigl and Raffelt, 1993

$$\frac{\partial\rho(t,p)}{\partial t} - Hp\frac{\partial\rho(t,p)}{\partial p} = -i\,[\mathcal{H},\rho] + \mathcal{C}[\rho,p]$$

Hamiltonian:

$$\mathcal{H}(\rho,\mathbf{p},\mathbf{x}) = \frac{\mathcal{U}\mathcal{M}^2\mathcal{U}^\dagger}{2E} + \sqrt{2}G_F\int\frac{d^3\mathbf{p}'}{(2\pi)^3}(\rho(\mathbf{p}',\mathbf{x}) - \bar\rho(\mathbf{p}',\mathbf{x}))(1-\mathbf{v}'\cdot\mathbf{v})$$

vacuum term — asymmetric neutrino — neutrino term

$$-\frac{8\sqrt{2}G_F p}{4}\,\frac{\mathcal{E}_l + \mathcal{P}_l}{m_W^2}$$

symmetric matter term

Quantum Kinetic Equations - include oscillations ($\delta N_{\rm eff} \sim +0.0005$)

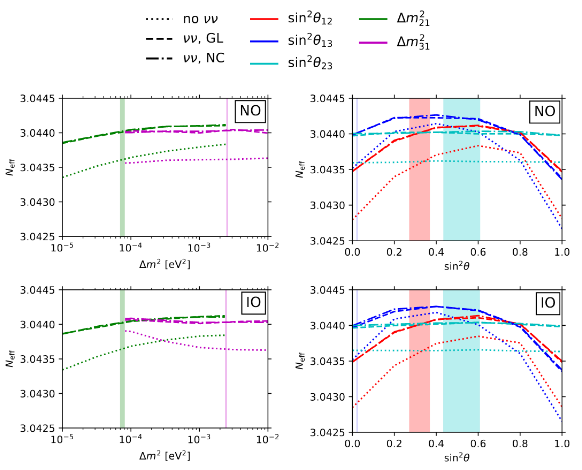
Without neutrino-neutrino term:

Oscillations tend to equilibrate the flavors.

ν_μ and ν_τ interact via NC.

ν_e interact via both NC and CC.

Hence, oscillations lead to higher $N_{\rm eff}$.

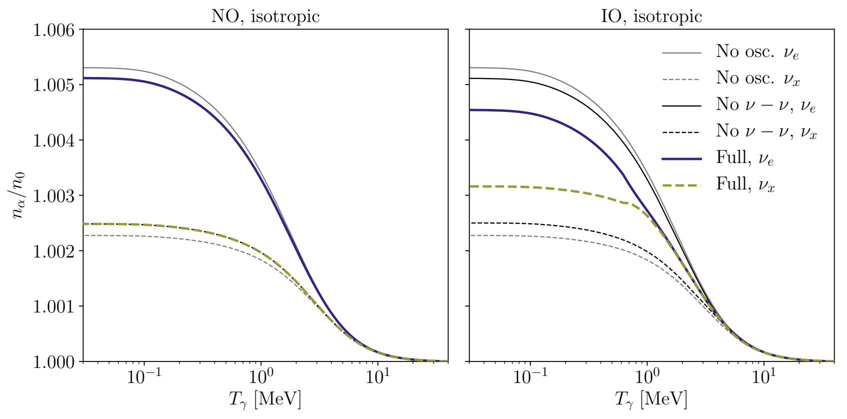


Bennett et al. (2012.02726)

Accounting for the neutrino-neutrino term

RSLH, Shalgar and Tamborra (2012.03948)

$$\mathcal{H}_{\nu\nu}(\rho, \mathbf{p}, \mathbf{x}) = \sqrt{2} G_F \int \frac{d^3\mathbf{p}'}{(2\pi)^3} (\rho(\mathbf{p}', \mathbf{x}) - \bar{\rho}(\mathbf{p}', \mathbf{x}))(1 - \mathbf{v}' \cdot \mathbf{v})$$



Breaking isotropy can change these results.

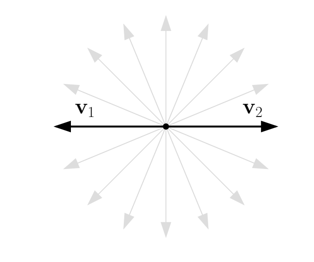
Let me try to illustrate with a bit of gymnastics.

Anisotropic neutrino oscillations

RSLH, Shalgar and Tamborra (2012.03948)

Homogeneous universe model with two angle bins:

- Universe is homogeneous.
- Angular dependence approximated with two angle bins. (left moving *L* and right moving *R* not to be confused with chirality of the particles)
- Two neutrino oscillation framework.
- Relaxation-time-like approximation for the collision term.



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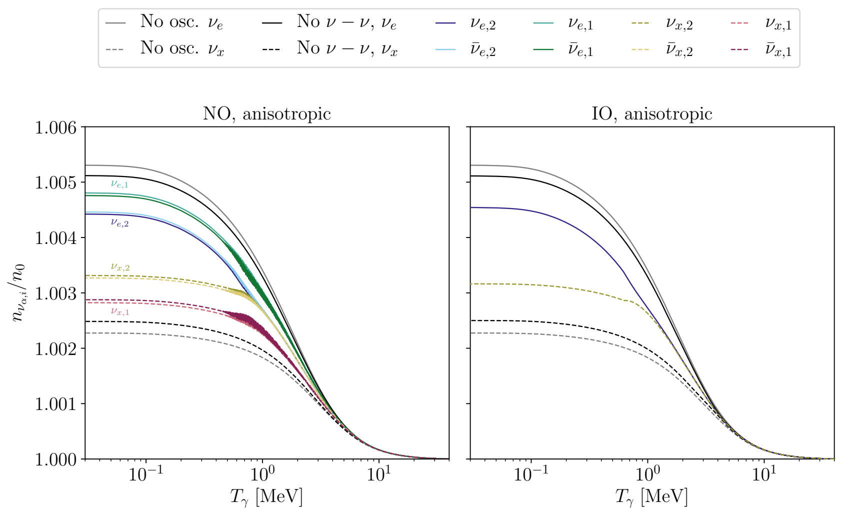

Change in N_{eff} in the two angle bin model

RSLH, Shalgar and Tamborra (2012.03948)

For no neutrino oscillations, $N_{\text{eff}} = 3.0460$ (not very accurate).

The cases with oscillations give:

	NO, isotropic	NO, anisotropic	NO, anisotropic ($\mu_{\text{ini}} = 10^{-9}$)	IO, both
δN_{eff}	$+0.0001$	$+0.0005$	$+0.0005$	$+0.0006$

These are only indications from a simple model with an approximated collision term!

From Bennett et al. (2012.02726), $N_{\text{eff}} = 3.0440 \pm 0.0002$.

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Beyond Standard Model scenarios

- **Sterile neutrinos**: (see also Joachims lectures)

  If the SM and sterile neutrinos oscillate during neutrino decoupling, anisotropies and inhomogeneities could arise.

- **Low temperature reheating**:

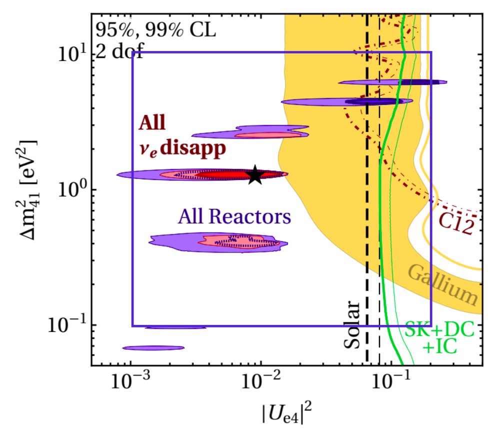
  A reheating temperature around $T = 1\text{MeV}$ would lead to non-equilibrium densities of neutrinos which could allow anisotropic an inhomogeneous oscillations to take place.

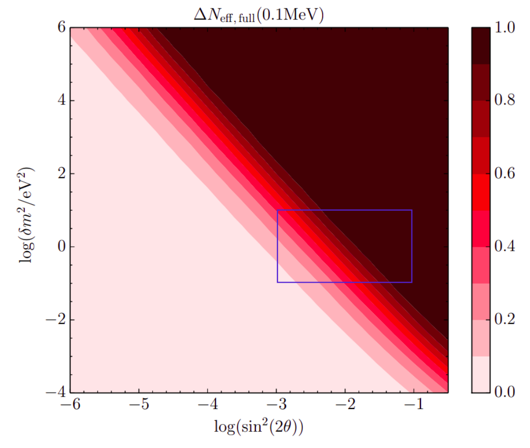
- **Large lepton asymmetries**:

  A large asymmetry between neutrinos and antineutrinos also provide good conditions for flavor oscillations.

  The linear stability analysis suggests that collective modes are suppressed.

Sterile neutrinos mixing with electron neutrinos ($\Delta N_{\rm eff} \lesssim 0.3$)





Dentler et al. (1803.10661)

Hannestad, RSLH et al. (1506.05266)

Similar for mixing with muon neutrinos

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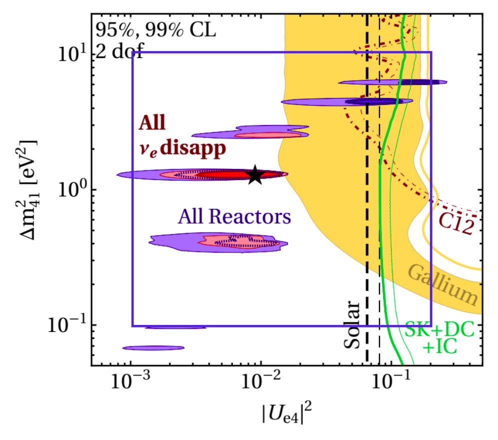

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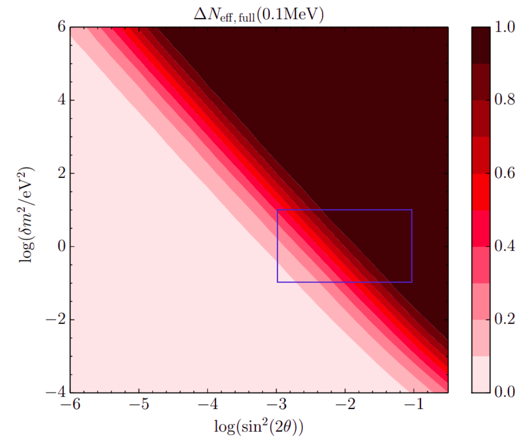

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Similar for mixing with muon neutrinos

+ constraints on $\sum m_i$

Summary

$N_{\rm eff}$ is a powerful probe of both Standard Model and Beyond Standard Model physics.

The Standard Model value is $N_{\rm eff} = 3.044$.

The corrections come from:
- m_e/T_d being finite.
- Finite temperature QED.
- Non-instantaneous decoupling
- Neutrino oscillations.

Neutrino oscillations in the early Universe can be anisotropic and inhomogeneous.

Thanks for your attention