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# 2. Neutrino Mixing & Oscillations

## 2.1 Three Flavours of Neutrinos

- $m_{\alpha\beta}$ in general off-diagonal
- Diagonalize $\boxed{\underbrace{\nu_{\alpha L}}_{\text{flavour eigenstates}} = \underbrace{U_{\alpha j}}_{\text{unitary mixing matrix}} \underbrace{\nu_{jL}}_{\text{mass eigenstates}}}$

with $U^T m U = \text{diag}(m_1, m_2, m_3)$

This only works because $m_{\alpha\beta}$ was symmetric

Note: For Dirac $\nu$: $\mathcal{L} \supset \sum_{\alpha,\beta} m_{\alpha\beta} \bar{\nu}_{\alpha L} \nu_{\beta R}$

↳ the trafo $\nu_{\alpha L} = U_{\alpha j} \nu_{jL}$

$\nu_{\beta R} = V_{\beta j} \nu_{jR}$

$U^\dagger m V = \text{diag}(m_1, m_2, m_3)$

works for general complex $(m_{\alpha\beta})$

In the mass basis:

$$\mathcal{L} \supset \sum_j \left[\bar{\nu}_{jL} i\not{\partial} \nu_{jL}\right]$$
$$+ \sum_{\alpha,j} \frac{g}{\sqrt{2}} \left(W^\mu \bar{\nu}_{jL} U^*_{\alpha j} \gamma_\mu e_{\alpha L} + h.c.\right)$$
$$+ \frac{g}{2\cos\theta_W} \sum_j Z^\mu \bar{\nu}_{jL} \gamma_\mu \nu_{jL}$$
$$+ \sum_j \frac{1}{2} m_j \overline{(\nu_{jL})^c} \nu_{jL}$$

CC $\nu$ interaction involves superposition of all 3 mass eigenstates


## 2.2 Neutrino Oscillations

In a CC interaction, we produce

$$|\nu_\alpha\rangle = U^*_{\alpha j} |\nu_j\rangle$$

CC detection process maps the neutrino onto flavour eigenstate

$$\langle \nu_\beta| = U_{\beta k} \langle \nu_k|$$

$$\hookrightarrow \mathcal{A} = \langle \nu_\beta | \underbrace{\nu_\alpha(t, L)\rangle}_{e^{-i\hat{H}t + i\hat{p}L}|\nu_\alpha\rangle}$$

$$= \langle \nu_k | U_{\beta k} e^{-iE_j t + i p_j L} U^*_{\alpha j} |\nu_j\rangle$$

$$P(\nu_\alpha \to \nu_\beta) = |\mathcal{A}|^2$$

$$= |U_{\beta j}\, e^{-iE_j t + i p_j L} U^*_{\alpha j}|^2$$

$$= U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \cdot \underbrace{e^{-i(E_j - E_k)t + i(p_j - p_k)L}}_{\text{interference}}$$

Note: States with different $E$ and $p$ can interfere only if $E$- and $p$-uncertainties are larger than $|E_j - E_k|$, $|p_j - p_k|$
This is always the case in $\nu$ oscillation experiments

Typically, we do not know $t$ precisely because the uncertainty in the neutrino production time is much larger than $|E_j - E_k|^{-1}$

$$\hookrightarrow \bar{P}(\nu_\alpha - \nu_\beta) = \frac{1}{\mathcal{N}} \int dt\, |\mathcal{A}|^2$$

normalization constant

$$= \frac{1}{\mathcal{N}} \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \cdot 2\pi\, \delta(E_j - E_k)$$

$$\cdot \underbrace{\exp\left[i\left(\sqrt{E^2 - m_j^2} - \sqrt{E^2 - m_k^2}\right)L\right]}_{\simeq \exp\left[-i \frac{\Delta m^2_{jk} L}{2E}\right]}$$

with $\Delta m^2_{jk} \equiv m_j^2 - m_k^2$

The factor $2\pi\, \delta(E_j - E_k)$ is absorbed into $\mathcal{N}$

Consider only 2 flavours:

$$\ldots \quad \begin{pmatrix} \cos\Theta & \sin\Theta \end{pmatrix}$$

$$U = \begin{pmatrix} -\sin\theta & \cos\theta \end{pmatrix}$$

$$\hookrightarrow P(\nu_\alpha \to \nu_\beta) \overset{2\,\text{fl.}}{\simeq} \underbrace{\sin^2 2\theta}_{\text{osc. amplitude}} \; \underbrace{\sin^2 \frac{\Delta m^2 L}{4E}}_{\text{osc. term}}$$

Oscillation length: $\frac{\Delta m^2 L_{osc}}{4E} = \pi$

$$\Leftrightarrow \quad L_{osc} = \frac{4\pi E}{\Delta m^2}$$

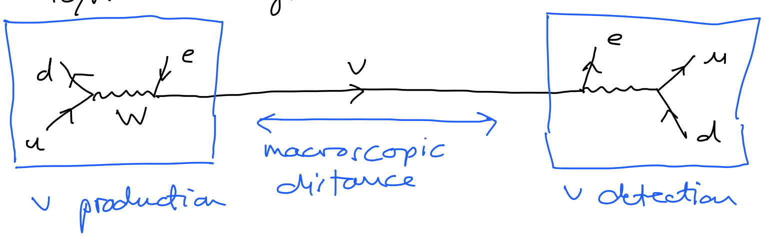
In nature:

$\Delta m^2_{21} \sim 8 \cdot 10^{-5}\,\mathrm{eV}^2 \Rightarrow L_{osc} \sim 60\,\mathrm{km}$ @ $\mathcal{O}(1\,\mathrm{MeV})$

$\Delta m^2_{31} \sim 2 \cdot 10^{-3}\,\mathrm{eV}^2 \Rightarrow L_{osc} \sim 1\,\mathrm{km}$ @ $\mathcal{O}(\mathrm{MeV})$

mixing angles control osc. amplitudes
$\Delta m^2$ control osc. lengths

Note: in QFT, $\nu$ osc. can be described as a Feynman diagram




treat external particles as wave packets

## 2.3 3-Flavor Oscillations

Mixing matrix is unitary $3\times 3$

General parameterization:

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

$s_{23} = \sin\theta_{23}$, $c_{23} = \cos\theta_{23}$

Here, phase factors have been absorbed into redefinitions of fields

## 2.4 Neutrino Oscillations in Matter

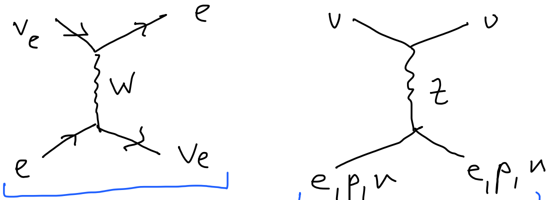
Coherent forward scattering

↑ background nuclei

Exchange of W or Z boson without momentum change
↳ all background particles contribute coherently

$$|\mathcal{M}|^2 \sim n^2 G_F^2$$

↑ number density of background matter




for $\nu_e$ only

for $\nu_{e,\mu,\tau}$

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left[ \bar{e}\, \gamma^\mu (1-\gamma^5)\, \nu_e \right] \left[ \bar{\nu}_e\, \gamma_\mu (1-\gamma^5)\, e \right]$$

$$\overset{\text{Fierz}}{=} \frac{G_F}{\sqrt{2}} \left[ \bar{e}\, \gamma^\mu (1-\gamma^5)\, e \right] \left[ \bar{\nu}_e\, \gamma_\mu (1-\gamma^5 \nu_e \right]$$

Treat e as classical background
↳ take expectation value

$$\langle H_{eff} \rangle = \frac{G_F}{\sqrt{2}} \underbrace{\langle \bar{e}\, \gamma^\mu (1-\gamma^5)\, e \rangle}_{= \begin{cases} n_e & \text{for } \mu = 0 \\ 0 & \text{for } \mu = 1,2,3 \end{cases}} \left[ \bar{\nu}_e\, \gamma_\mu (1-\gamma^5) \nu_e \right]$$

$$= \underbrace{\sqrt{2}\, G_F n_e}_{= V_{CC}} \quad \bar{\nu}_{eL} \gamma^0 \nu_{eL}$$

CC MSW potential

In the derivation of $P(\nu_\alpha \to \nu_\beta)$, we had factors of the form

$$\phi = p \cdot L = \sqrt{(\hat{H} - \hat{V})^2 - \hat{M}^2} \cdot L$$

← mass matrix

↑ $2 \times 2$ matrix in 2-flavour approx.

↑ MSW potential $2 \times 2$ matrix $\begin{pmatrix} V_{CC} & 0 \\ 0 & 0 \end{pmatrix}$

$$\simeq \left( \hat{H} - \frac{\hat{M}^2}{2\hat{H}} - \hat{V} \right) \cdot L$$

↑ $\hat{V}, \hat{M}$ small

In matrix notation

$$\hat{H} - \frac{\hat{M}^2}{2\hat{H}} - \hat{V} = E \cdot \mathbb{1} - U \begin{pmatrix} \frac{m_1^2}{2E} & \\ & \frac{m_2^2}{2E} \end{pmatrix} U^+$$

$$- \begin{pmatrix} V_{cc} & \\ & 0 \end{pmatrix}$$