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Constraining very high energy diffuse gamma (and neutrino) emission with Tibet ASγ data

Presented by: Vittoria Vecchiotti

Based on a work done in collaboration with: G. Pagliaroli, F. L. Villante, F. Zuccarini

Outline:

Goal: Constrain the Galactic gamma-ray-neutrino diffuse emission using gamma-ray observations;

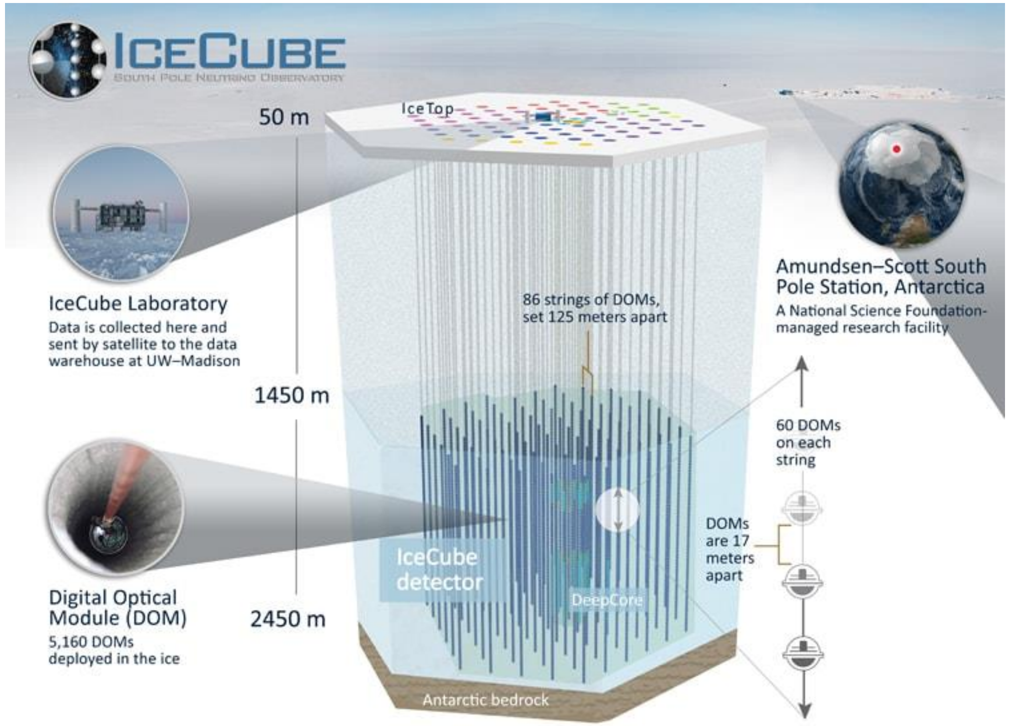
1. Galactic gamma-ray-neutrino signal;
2. Gamma-ray-neutrino Galactic diffuse emission model;
   Cataldo et al. JCAP (2019)
3. Unresolved sources *(population study of the Galactic TeV gamma-ray sources with HESS)*;
   Cataldo et al. Astrophys.J. 904 (2020)
4. Results: comparison with Tibet data and constraints on the model;
   Vecchiotti et al. Astrophys.J. (2021)

Neutrino and Gamma-ray Sky:

IceCube: Isotropic high-energy neutrino signal around 100 TeV.

The majority of the signal is expected to be of extragalactic origin.

A Galactic component cannot be excluded.



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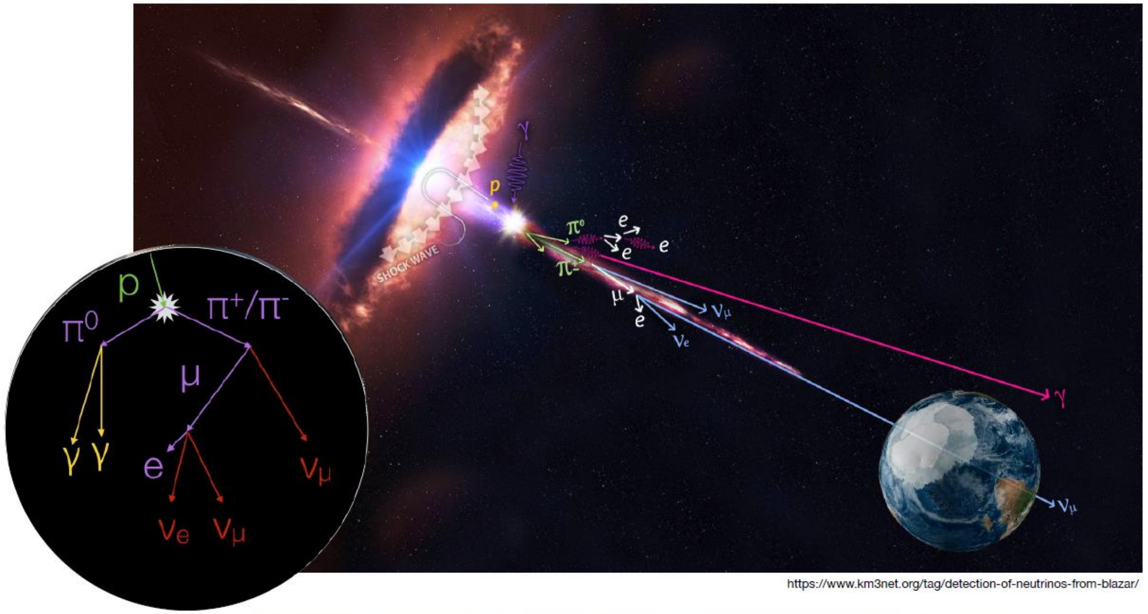
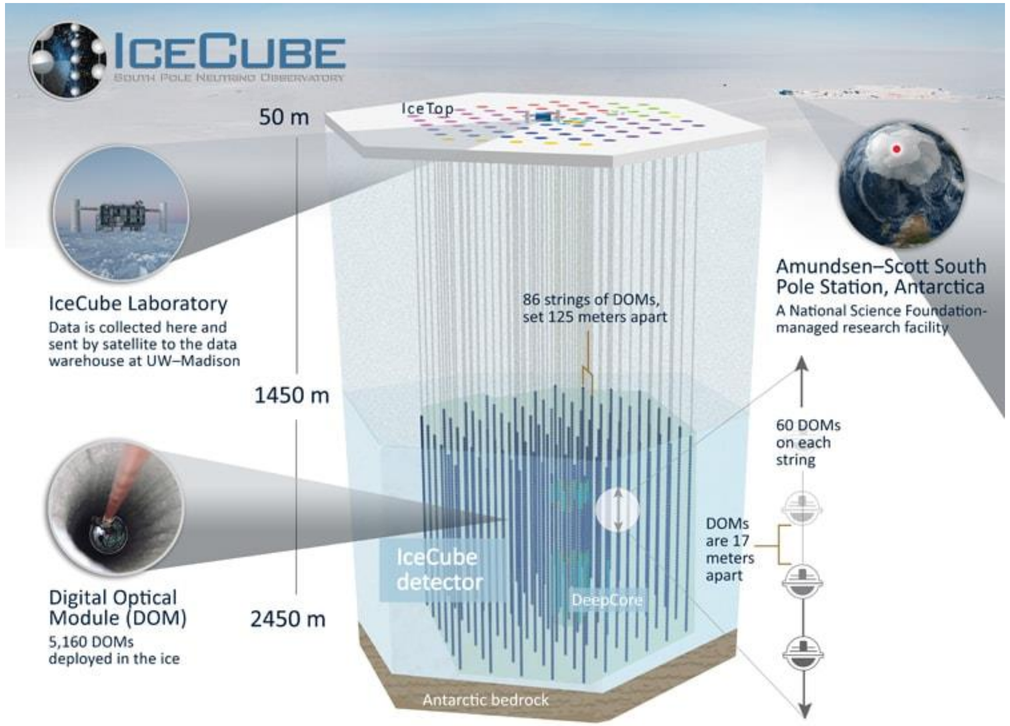


https://www.km3net.org/tag/detection-of-neutrinos-from-blazar/



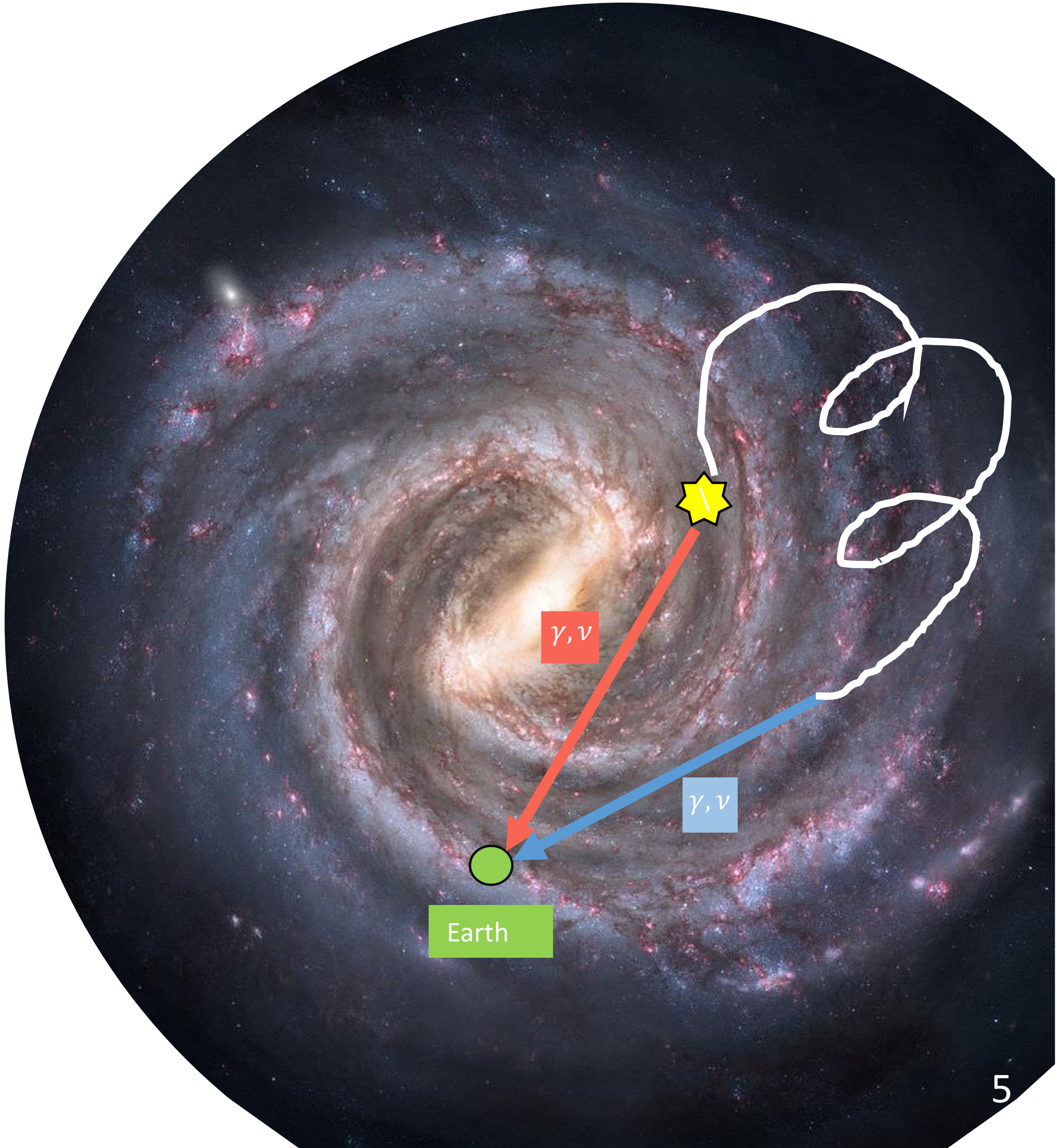
Neutrinos are produced in the same hadronic interactions like gamma-rays → gamma observations can be used to constrain the neutrino signal

Total Galactic emission at TeV-PeV:

$$\phi_{\gamma,\nu,tot} = \boxed{\phi_{\gamma,\nu,\,s}} + \boxed{\phi_{\gamma,\nu,\mathrm{diff}}}$$

Source component: due to the interaction of accelerated hadrons (*gamma* and *neutrino*) or leptons (*gamma*) with the ambient medium (ISM or CMB) within or close to an acceleration site (such as PWNe, SNRs).

Diffuse component: due to the interaction of accelerated hadrons with the interstellar medium;



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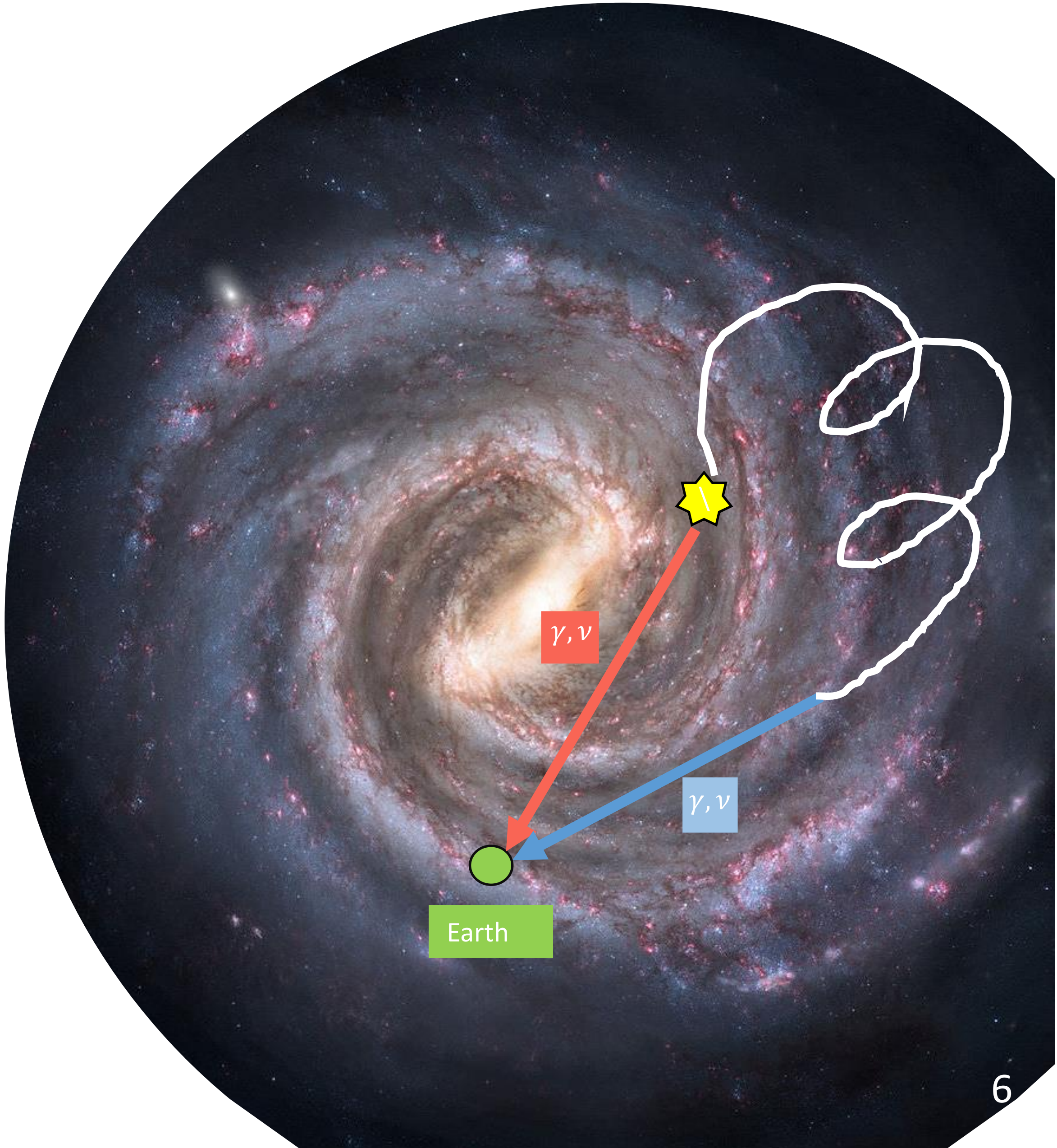
We want to constrain the diffuse emission



Tibet ASγ :

Amenomori, M., et al. 2021, Phys. Rev. Lett., 126, 141101,326

$\phi_{\gamma,\mathrm{diff}}^{Tibet}$

First measurement of the Galactic diffuse γ-ray emission in the sub-PeV energy range.

They exclude the contribution from the known TeV sources (within 0.5 degrees) listed in the TeV source catalog.

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The Tibet measurements are contaminated by the presence of Unresolved Sources

$$\phi_{\gamma,\mathrm{diff}}^{Tibet} = \phi_{\gamma,S}^{UnRes} + \phi_{\gamma,\mathrm{diff}}$$

Population study (H.E.S.S.)→ we obtain general information on the sources

Models:
Assumptions on the CR spatial and energy distributions.

Diffuse Galactic gamma-ray emission:

$$\phi_{\gamma,S}^{UnRes} + \phi_{\gamma,\text{diff}}$$

Interstellar gas distribution in the Galaxy [*Galprop*]

$$\varphi_{\gamma}^{diff}(E_\gamma,\hat{n}_\gamma) = \int_{E_\gamma}^{\infty} dE\, \frac{d\sigma(E,E_\gamma)}{dE_\gamma} \int_0^{\infty} dl\, \varphi_{CR}(E,\bar{r}_{Sun} + l\hat{n}_\gamma)\, n_H\, (\bar{r}_{Sun} + l\hat{n}_\gamma)$$

Differential inelastic cross section of pp interaction from the SYBILL code *[Kelner,Aharonian,Bugayov (2006)]*

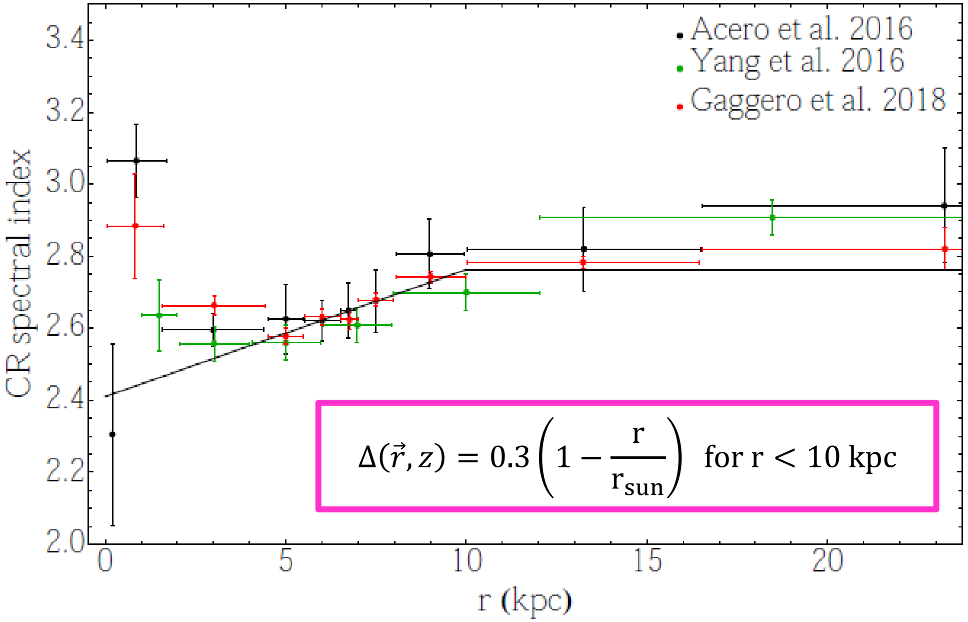
Cosmic-ray energy and spatial distribution

2 models for the diffuse fluxes **for 2 assumptions of the CR distribution** in the Galaxy.

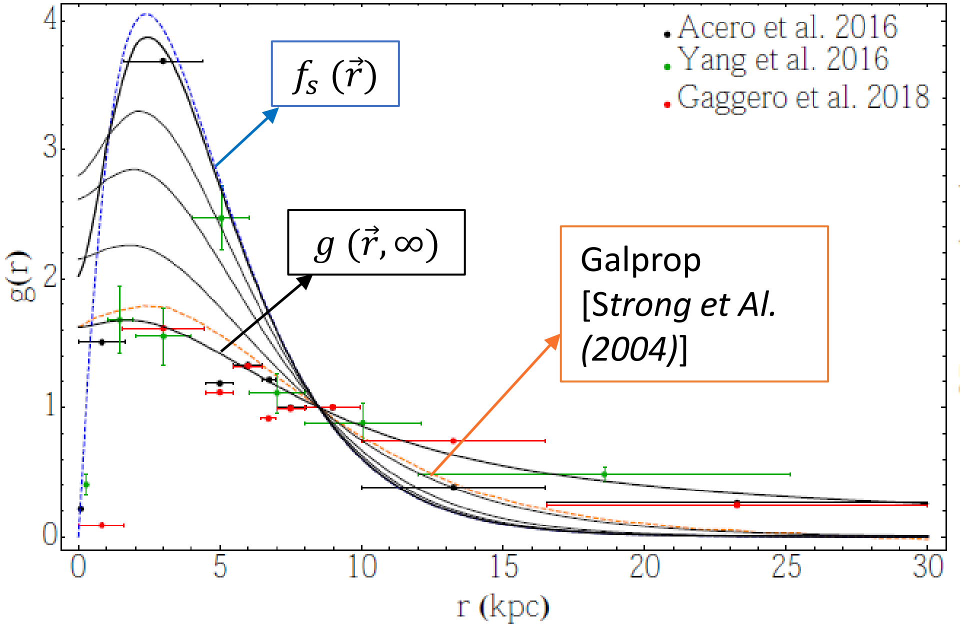
Cosmic ray distribution: $\varphi_{CR}(E,\vec{r}) = \varphi_{CR,Sun}(E\,)\,g\,(\vec{r},R)\;h(E,\vec{r})$

- Data driven local CR spectrum [*Dembinski, Engel, Fedynitch et al. (2018)*]
- g(r) is determined by the distribution of the CR sources $f_s\,(\vec{r})$(proportional to the SNR number density by Green et al. (2015), and by the propagation of CR in the Galactic magnetic field.
- **2 cases: with and without spatially dependent CR spectral index** (from the analysis of the FermiLAT data at $\sim 20\ GeV$ [*Acero et al. (2016), Yang et al. (2016), Gaggero et al. (2018)*])

$$h(E,\vec{r}) = \left(\frac{E}{20\ GeV}\right)^{\Delta(\vec{r})}$$



$$\Delta(\vec{r},z) = 0.3\left(1-\frac{r}{r_{sun}}\right) \quad \text{for r} < 10\ \text{kpc}$$



$f_s\,(\vec{r})$

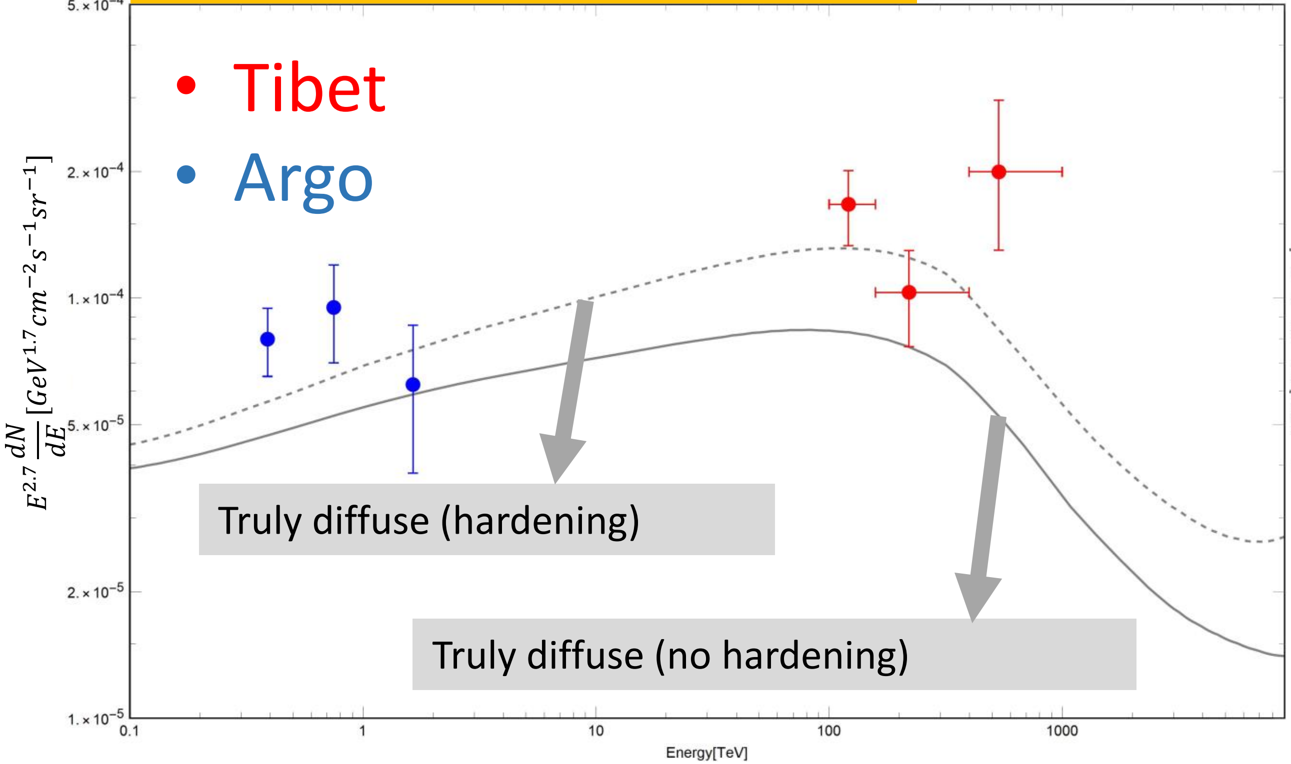
$g\,(\vec{r},\infty)$

Galprop [*Strong et Al. (2004)*]

Diffuse Galactic gamma-ray emission:

Definition: Hardening ≡ spatially dependent CR spectral index

$25° < l < 100°, |b| < 5°$



Truly diffuse (hardening)

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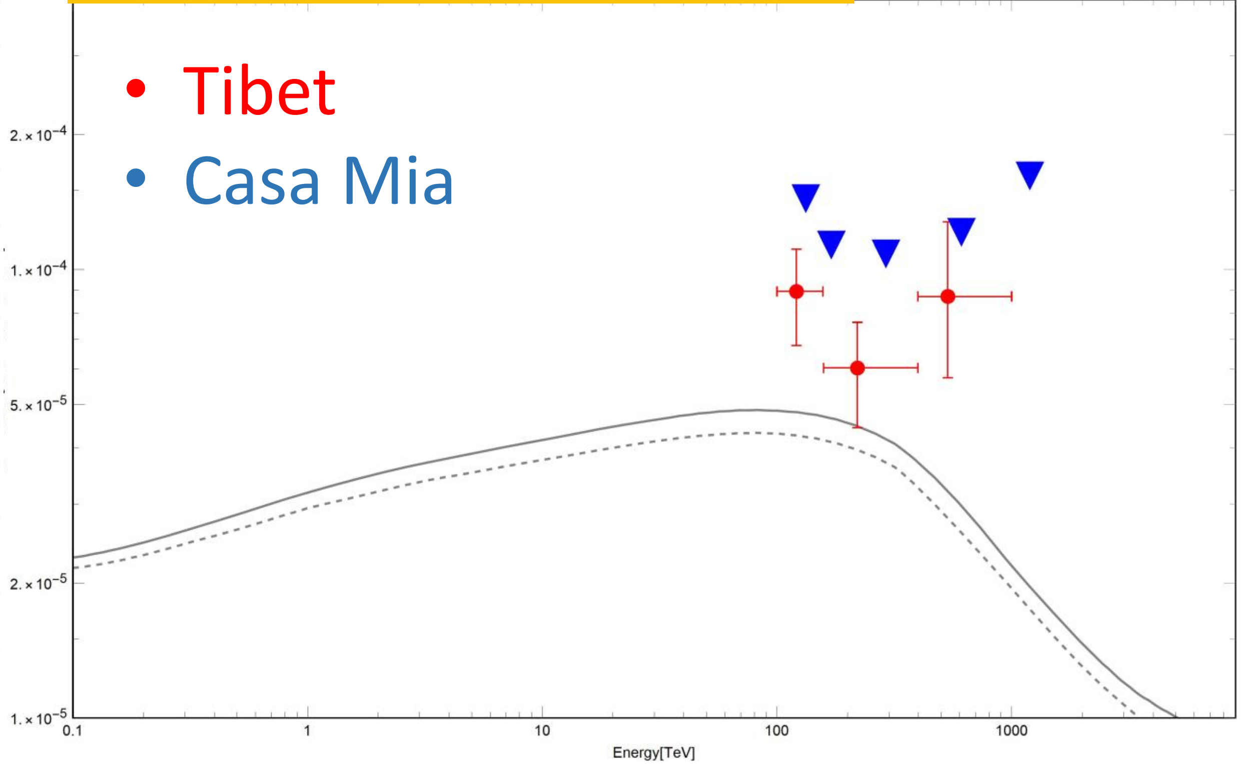


Unresolved Source component:

Study of the Pulsar wind nebulae population in the TeV range:

Cataldo et al. *Astrophys.J.* 904 (2020)

We fit the H.G.P.S. catalogue with an unbinned likelihood

$$L_{max} = L^{BF}$$
$$\tau = \tau^{BF}$$

$$\Phi_{\mathrm{PWN}}$$

$\alpha = 1.8$

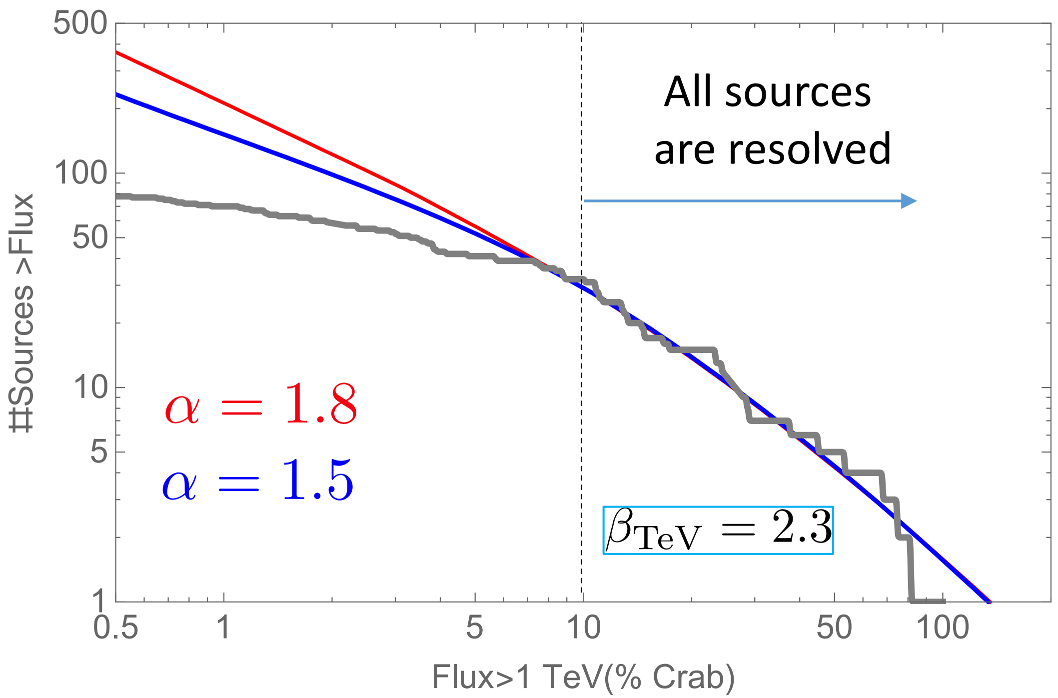
$L_{max} = 6.8 \times 10^{35}\ erg\ s^{-1}$

$\tau = 0.5\ kyr$

$\alpha = 1.5$

$L_{max} = 5.0 \times 10^{35}\ erg\ s^{-1}$

$\tau = 1.7\ kyr$



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Φ_{PWN}

Unresolved Sources: $\Phi^{\mathrm{unr}}_{1-100\,TeV}$

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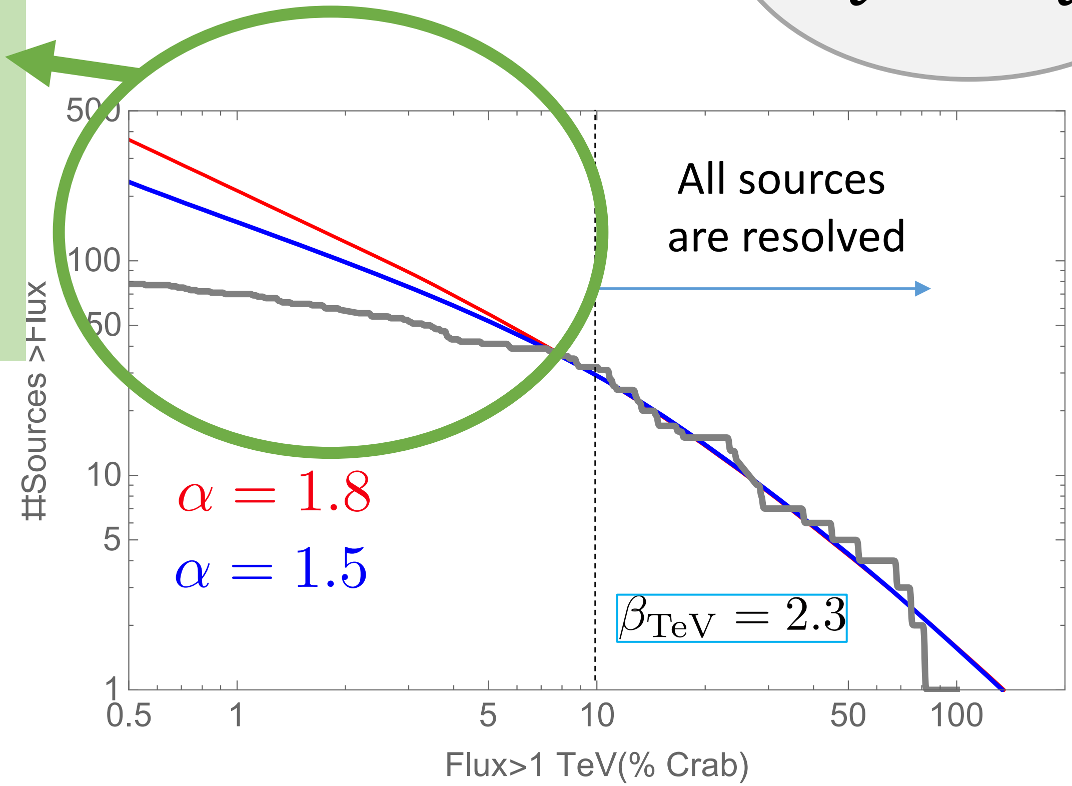

Unresolved Source component:

We have $\Phi_{1-100\ TeV}$→ we need $\phi(E)$:

- Spectral assumption: power-law with an exponential cut-off.

$$\varphi(E) = \left(\frac{E}{1\ TeV}\right)^{-\beta_{TeV}} Exp\left(-\frac{E}{E_{cut}}\right)$$

$\beta_{TeV} = 2.3$ from the HGPS catalogue;

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$E_{cut} = 500\ TeV$ still not well constrained but motivated by recent observations of Tibet, HAWC and LHAASO;

Amenomori, M., Bao, Y. W., Bi, X. J., et al. 2019, Phys.323Rev. Lett., 123, 051101

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We introduce a flux detection threshold based on the performance of H.E.S.S.

$\phi_{th} = 0.01\phi_{crab} - 0.1\phi_{crab}$

⬇

We calculate the unresolved source contribution.

Tibet ASγ : We add the contribution of unresolved sources to the truly diffuse emission without the hypothesis of CR spectral hardening.

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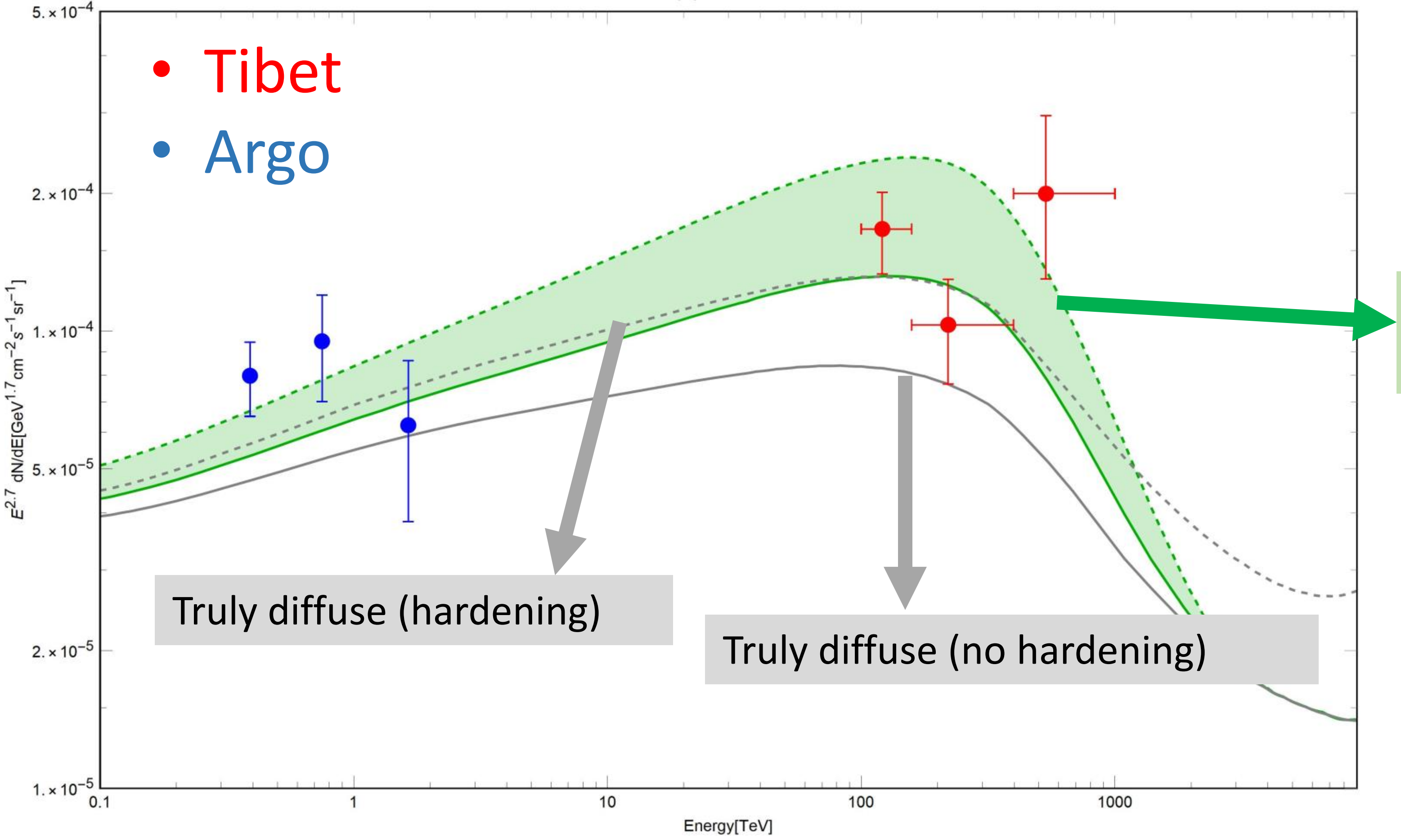
Definition: Hardening ≡ spatially dependent CR spectral index

$$25° < l < 100°, |b| < 5°, E_{cut} = 500\ TeV$$



Unresolved sources +
Truly diffuse (no hardening)

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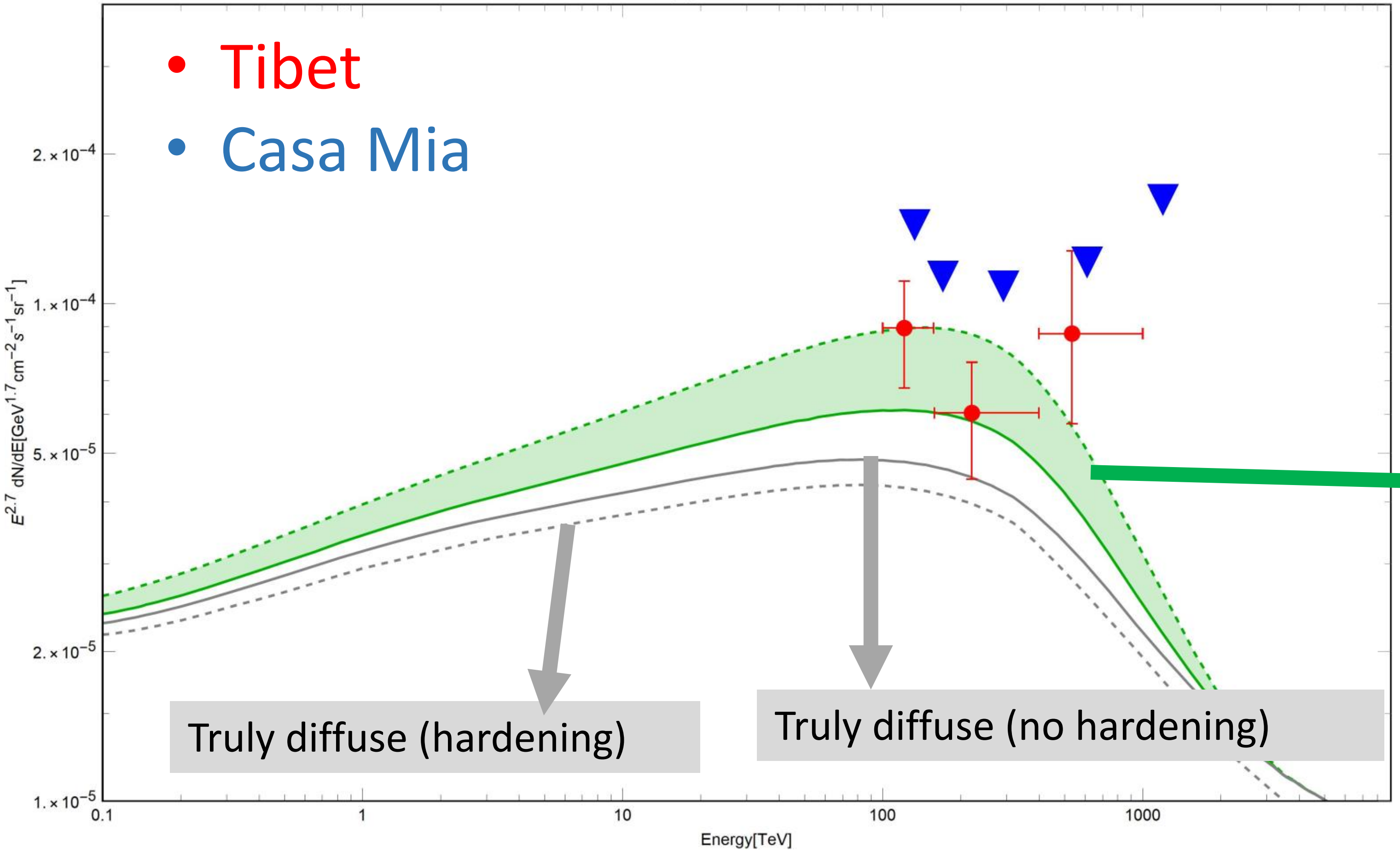

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What about the diffuse neutrinos?

The total isotropic neutrino flux observed by IceCube:

IceCube collaboration 2020, Phys.Rev. D., 104.022002

$$\Phi_{tot,\nu} \simeq 6.37 \times 10^{-15} \left(\frac{E}{100\ TeV}\right)^{-2.87} TeV^{-1} cm^{-2} s^{-1} sr^{-1}$$

We calculate the expected Galactic neutrino diffuse emission at 100 TeV, we integrated it over $|l| < 180°, |b| < 5°$ and we compare it with the above quantity at 100 TeV integrated over the same window.

The Galactic neutrino diffuse emission can contribute at most up to 1 % of the total neutrino flux observed by IceCube.

Summary:

- We modelled the gamma/neutrino diffuse emission;
- We calculate the unresolved component from the H.E.S.S. observations;
- In the **PeV** energy range the inclusion of the **unresolved PWNe** contribution produces a better description of the Tibet data than CR spectral hardening (*spatially dependent CR spectral index*);
- The Galactic neutrino diffuse emission can contribute at most up to 1 % of the total neutrino flux observed by IceCube.

Backup slides

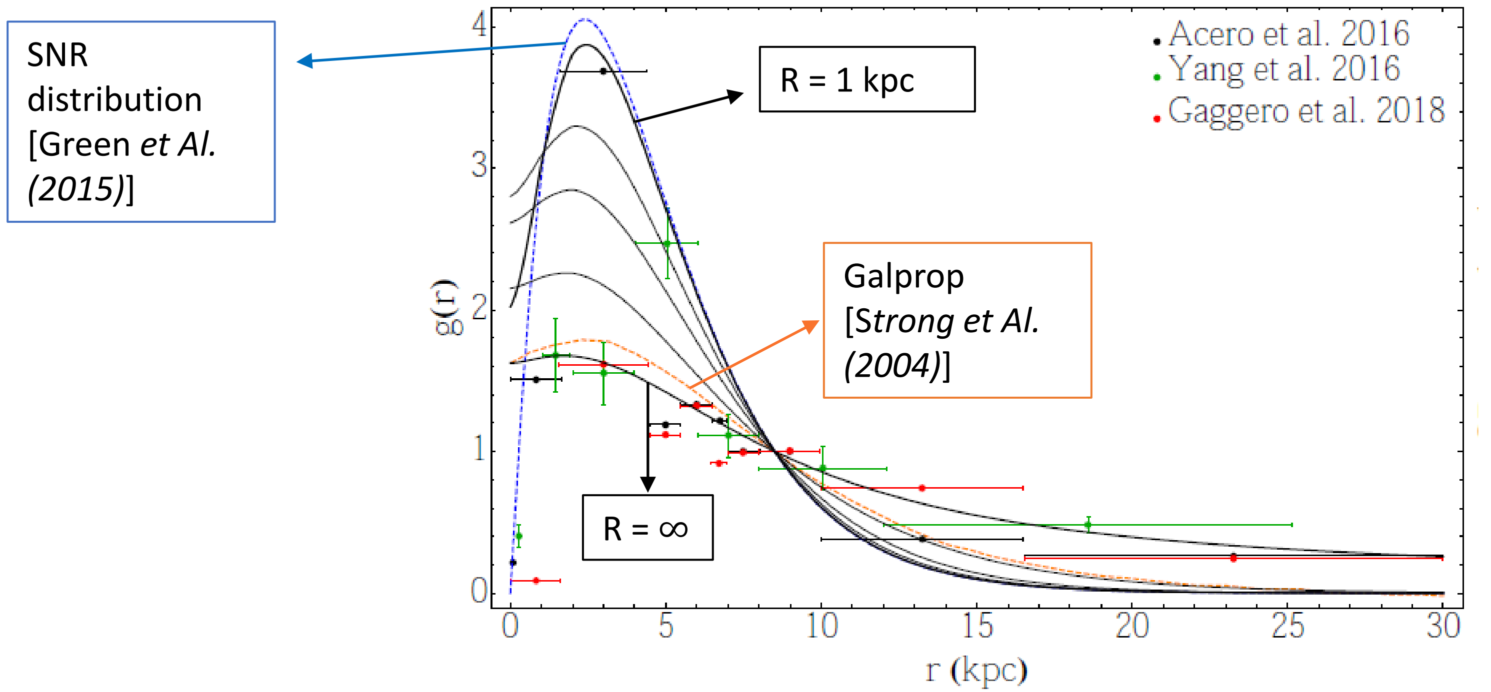
Cosmic ray distribution: 2 standard cases:

The function g(r) is determined by the distribution of the CR sources $f_s(\vec{r})$, that is assumed to follow the SNR number density parametrization given by Green et al. (2015), and by the propagation of CR in the Galactic magnetic field.

$$\varphi_{CR}(E,\vec{r}) = \varphi_{CR,Sun}(E)\, g(\vec{r},R) \qquad \left\{ \begin{array}{l} R = 1\, kpc \\ R = \infty \end{array} \right.$$

Diffusion (smearing) radius

Data driven local CR spectrum [*Dembinski, Engel, Fedynitch et al. (2018)*]



SNR distribution [Green *et Al. (2015)*]

R = 1 kpc

Galprop [*Strong et Al. (2004)*]

R = ∞

Cosmic ray distribution: 2 cases with hardening:

We consider the possibility of spatially dependent CR spectral index recenty emerged from the analysis of the FermiLAT data at $\sim 20\ GeV$ [*Acero et al. (2016), Yang et al. (2016), Gaggero et al. (2018)*]

$$\varphi_{CR}(E,\vec{r}) = \varphi_{CR,Sun}(E\ )g\ (\vec{r},R)\ h(E,\vec{r})$$

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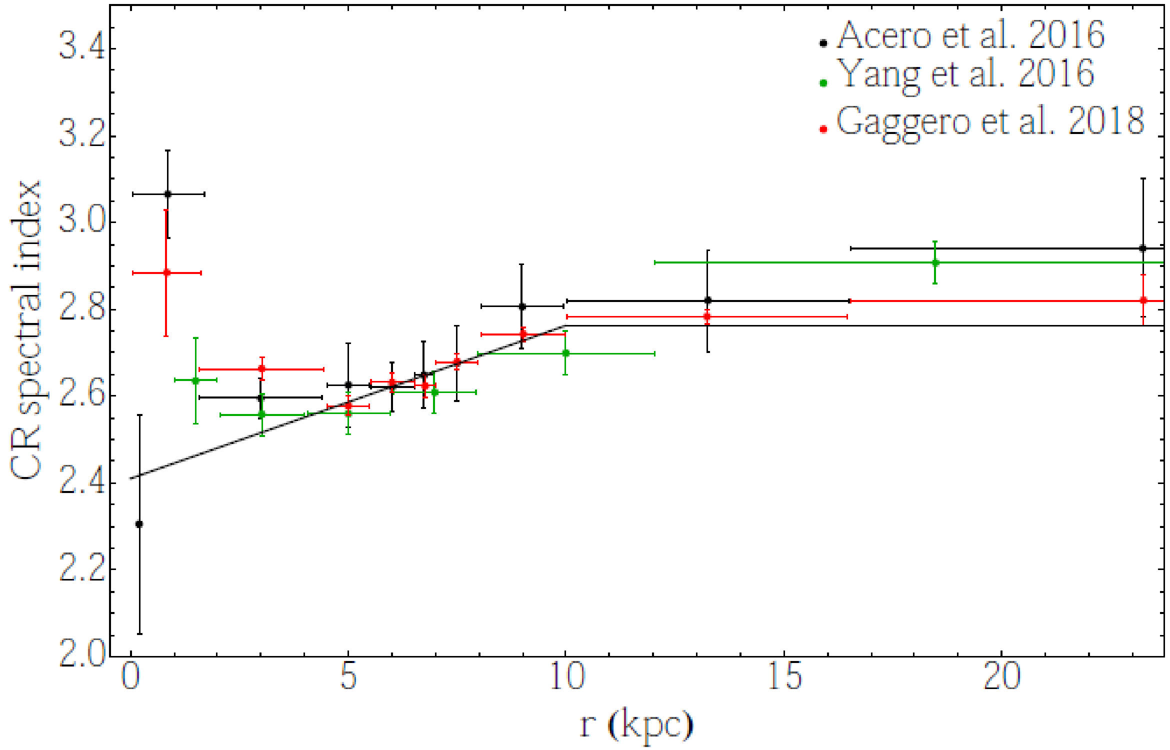

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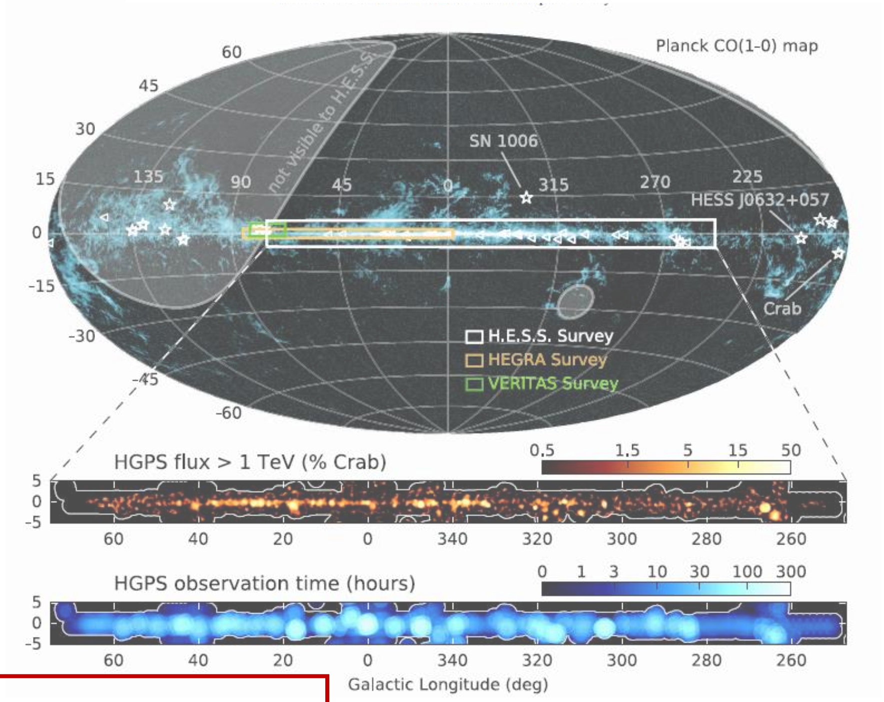
$$\Delta(\vec{r},z) = 0.3\left(1 - \frac{\mathrm{r}}{\mathrm{r}_{\mathrm{sun}}}\right)\quad \text{for r} < 10\ \text{kpc}$$

Unresolved Source component:

Study of the Pulsar wind nebulae population in the TeV range:

Cataldo et al. *Astrophys.J.* 904 (2020)

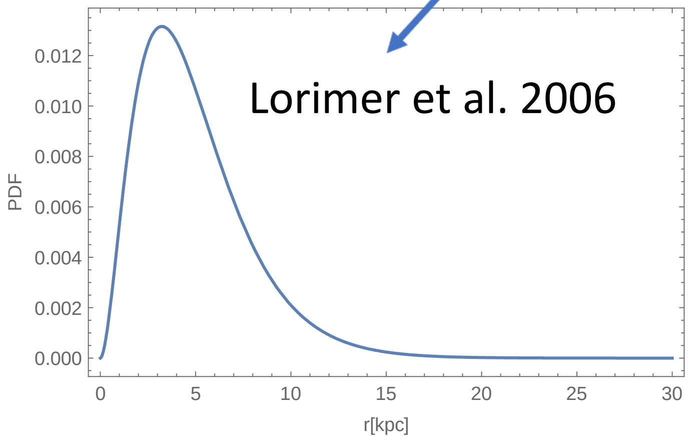
Abdalla et al, *A&A*, 612, A1 (2018)



- The HGPS catalogue ($\phi > 0.1\phi_{Crab}$);
- Model for TeV source population:

we assume the spatial distribution and the luminosity distribution of the sources;

$$\frac{dN}{d^3r\,dL} = \rho(r)\,Y(L)$$



Lorimer et al. 2006

$$Y(L) = \frac{R\,\tau\,(\alpha - 1)}{L_{\max}} \left(\frac{L}{L_{\max}}\right)^{-\alpha}$$

$$\alpha = 1/\gamma + 1$$

$$R = 0.019\,\mathrm{yr}^{-1}$$

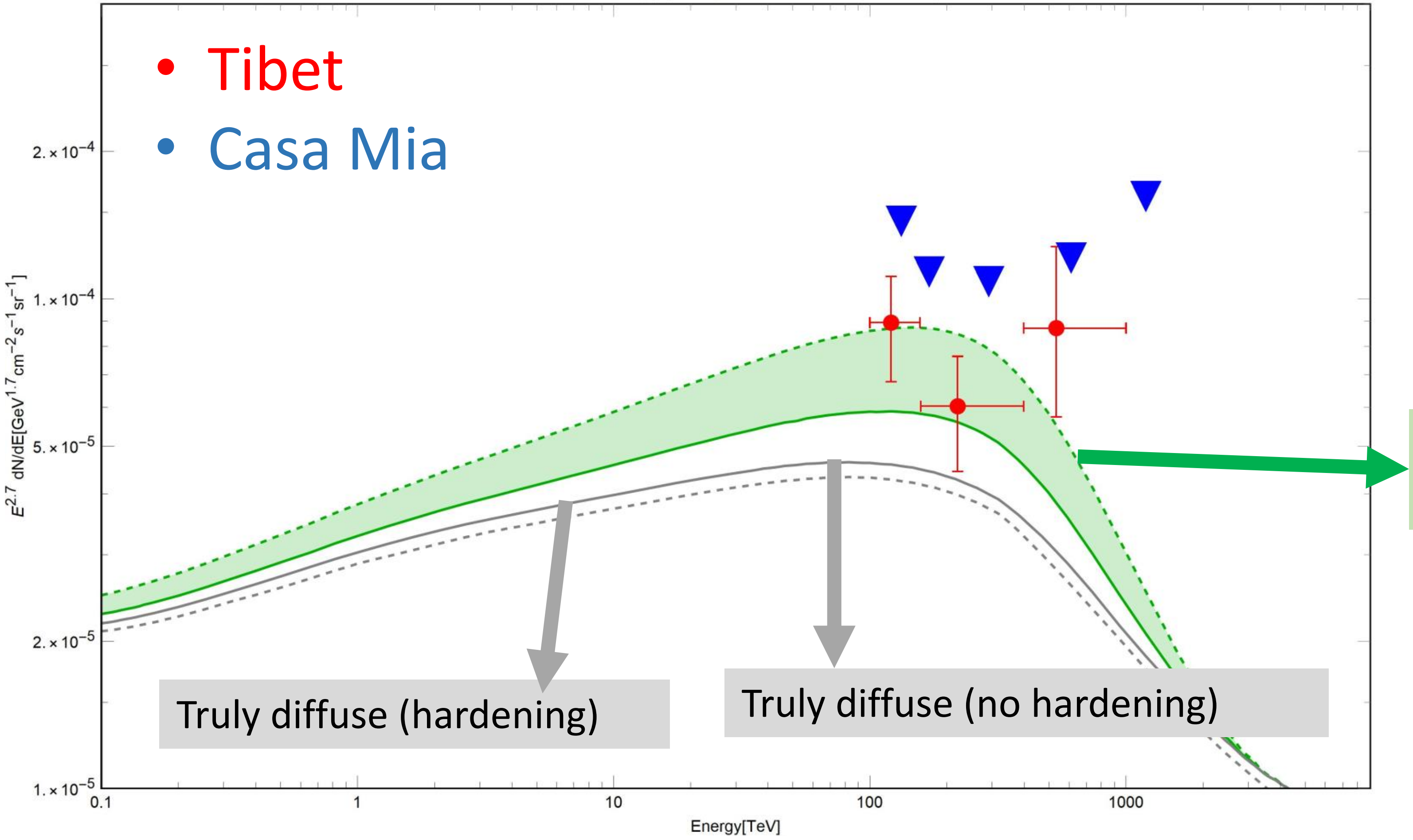
For pulsar-powered sources:

$$L(t) = L_{\max}\left(1 + \frac{t}{\tau}\right)^{-\gamma}$$

We assume a power-law energy spectrum with index $\boldsymbol{\beta_{TeV} = 2.3}$ that is the average index for all the sources in the HGPS catalogue.

Tibet ASγ : We add the contribution of unresolved sources to the truly diffuse emission without the hypothesis of CR spectral hardening.

$50° < l < 200°, |b| < 5°, E_{cut} = 500\ TeV, \mathrm{R} = 1\ kpc$



- Tibet
- Casa Mia

Truly diffuse (hardening)

Truly diffuse (no hardening)

Unresolved sources +
Truly diffuse $\mathrm{R} = 1\ kpc$ (no hardening)

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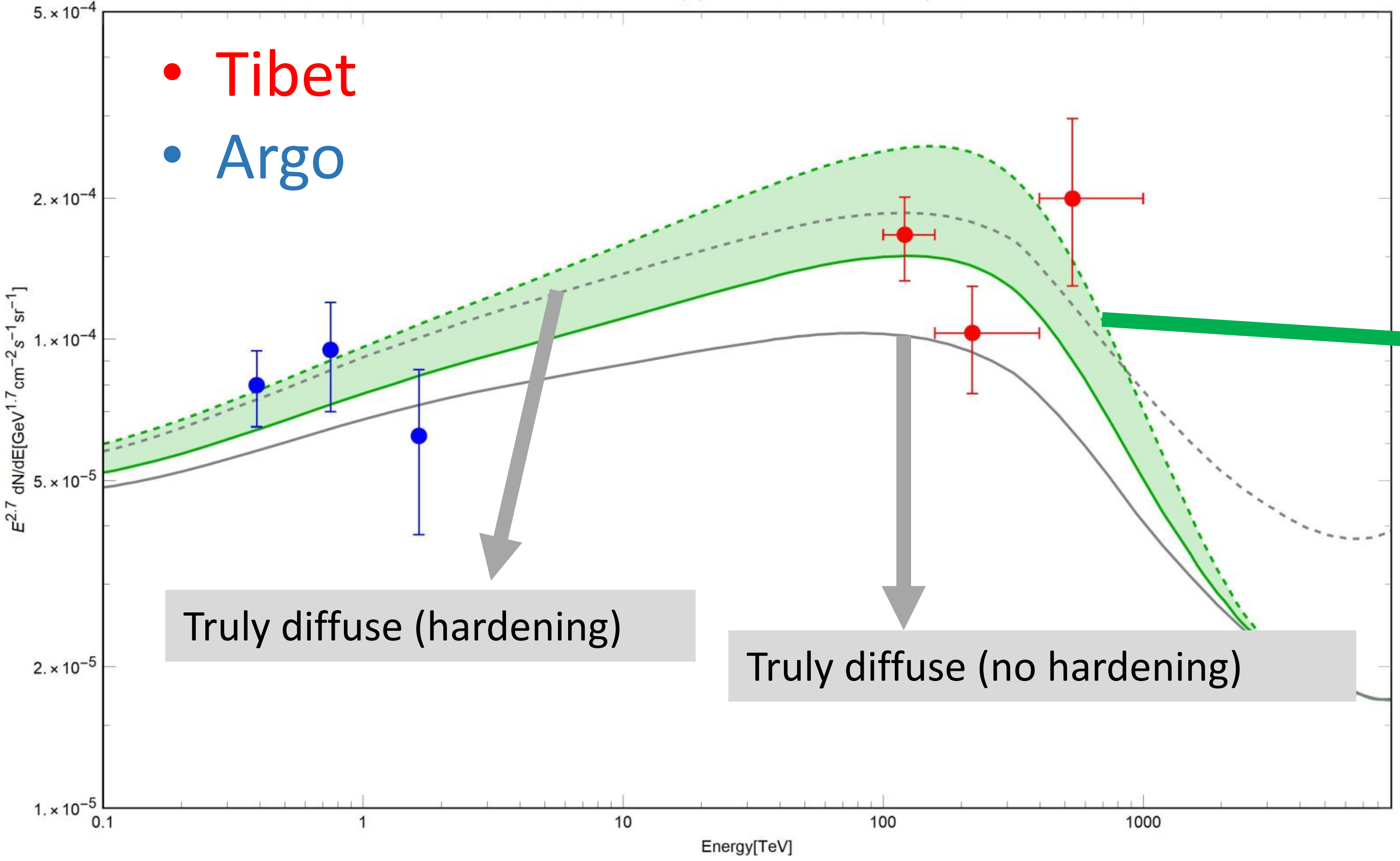

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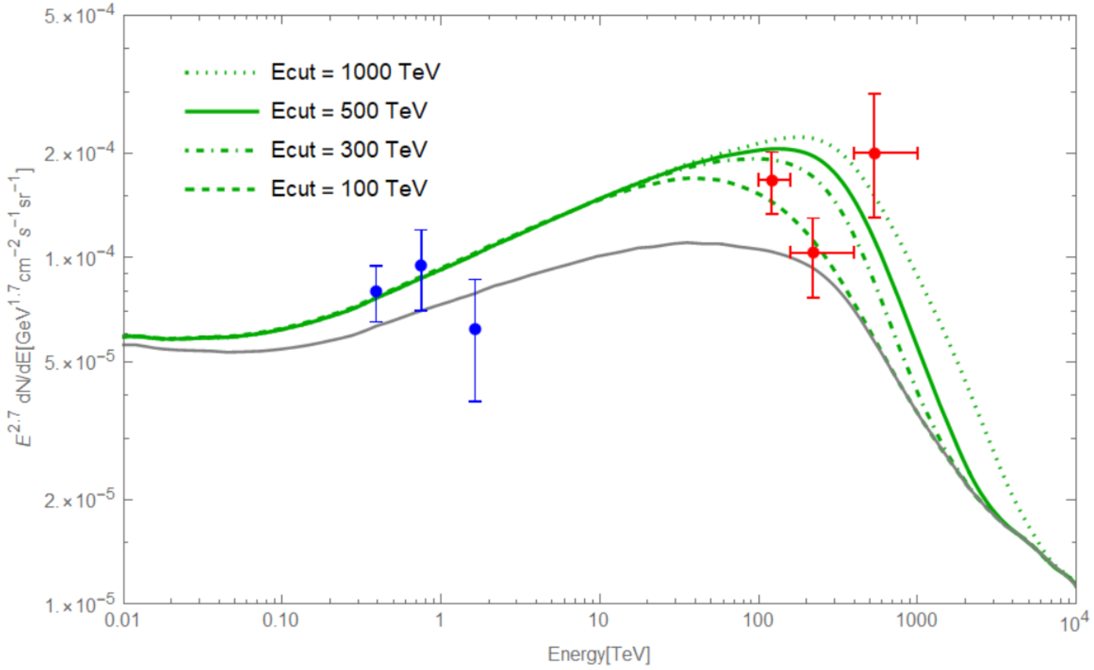
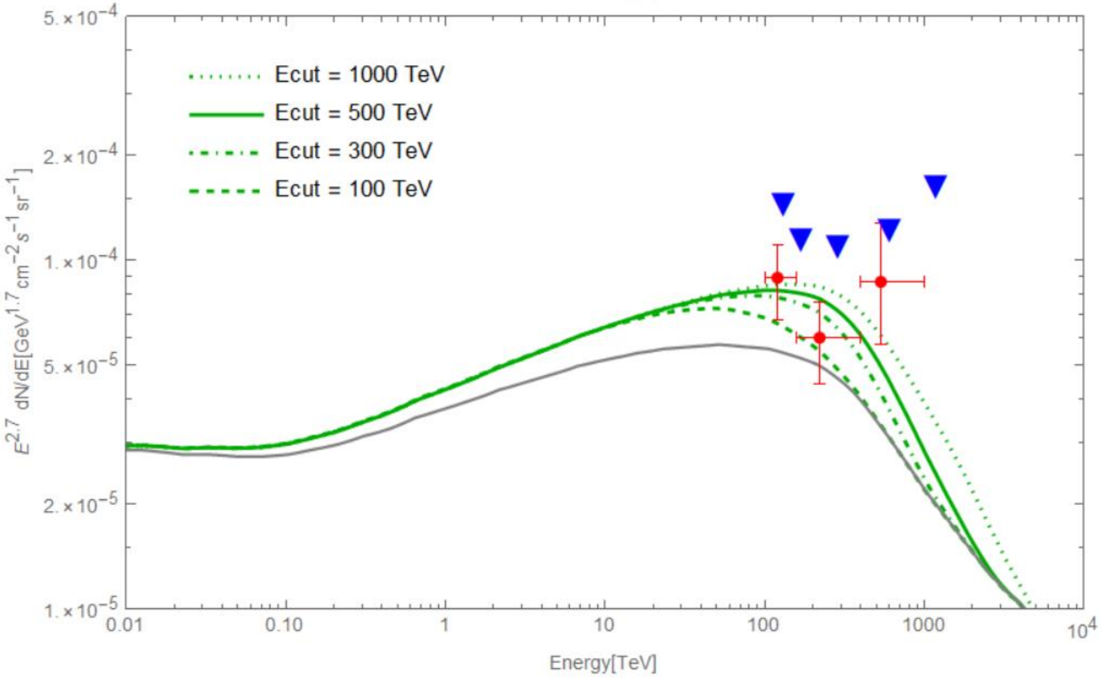
Energy cut effect:

$25^\circ < l < 100^\circ, |b| < 5^\circ$

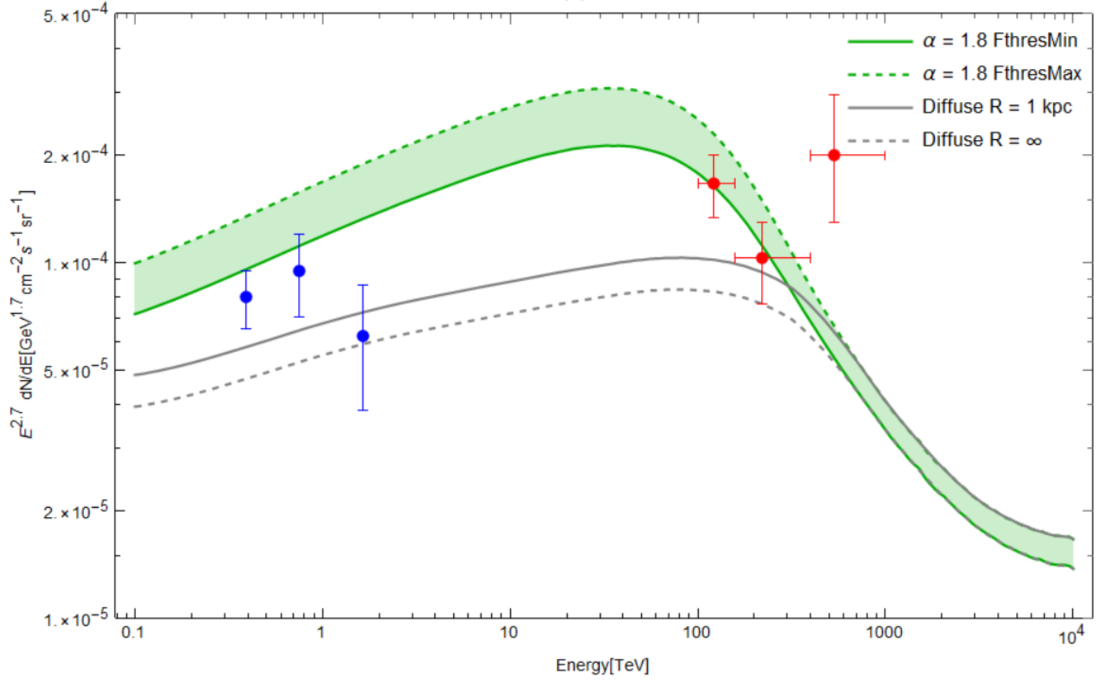


$50^\circ < l < 200^\circ, |b| < 5^\circ$



Luminosity index 1.8 and Ecut = 100 TeV:

$25^\circ < l < 100^\circ, |b| < 5^\circ$



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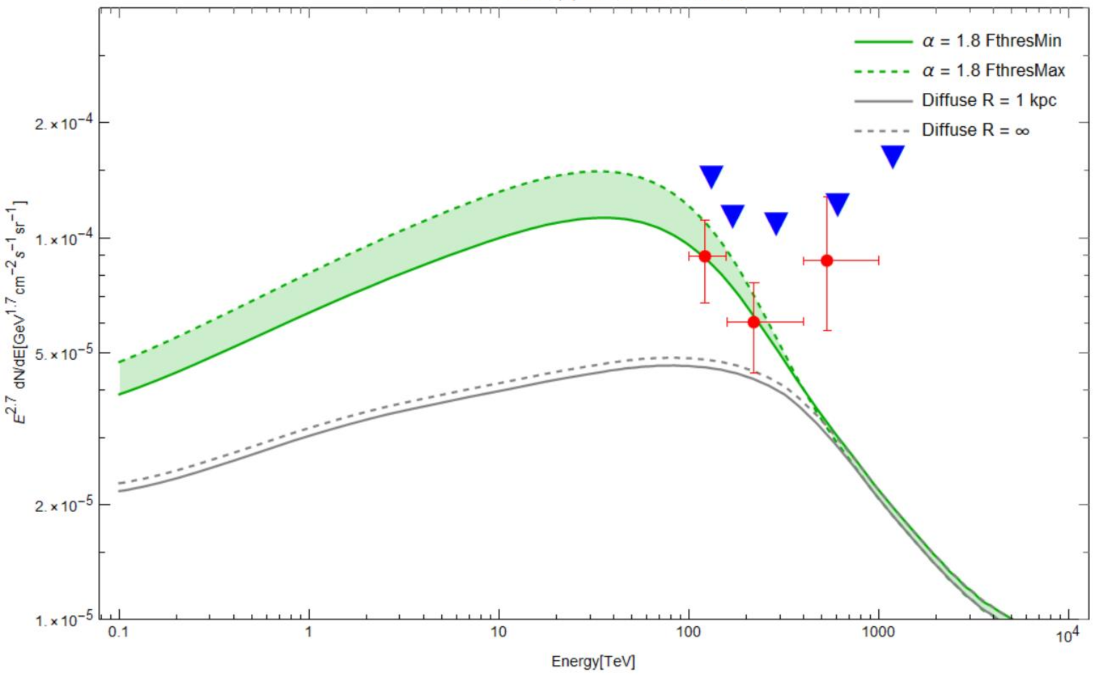

Absorption in the Sub PeV energy range:

Vernetto and Lipari, Phys. Rev. D 94, 063009 – Published 19 September 2016

The pair production cross section:

$$\sigma_{\gamma\gamma} = \sigma_T \left(\frac{3}{16}\right)(1-\beta^2)\left[2c(\beta^2-2)+(3-\beta^4)\ln\left(\frac{1+\beta}{1-\beta}\right)\right]$$

Where: $\beta = \sqrt{1-\frac{1}{x}}$ and $x = \frac{2E_\gamma \epsilon(1-\cos\theta)}{4\,m_e^2}$, $x > 1$

For a fixed values of ϵ the energy threshold is:

$$E_\gamma^{th} = \frac{2\,m_e}{\epsilon(1-cos\theta)} \simeq \frac{0.52}{\epsilon_{eV}(1-cos\theta)} TeV$$

The absorption probability per unit path length (for CMB) is:

$\mathrm{K}(E_\gamma) = \int \epsilon \int d\Omega(1-\cos(\theta))n_{\gamma,CMB}(\epsilon)\sigma_{\gamma\gamma}(x(E_\gamma,\epsilon,\theta))$

The optical depth is:

$$\tau(E_\gamma, r) = \int_0^r \mathrm{d}r' \; \mathrm{K}(E_\gamma)$$

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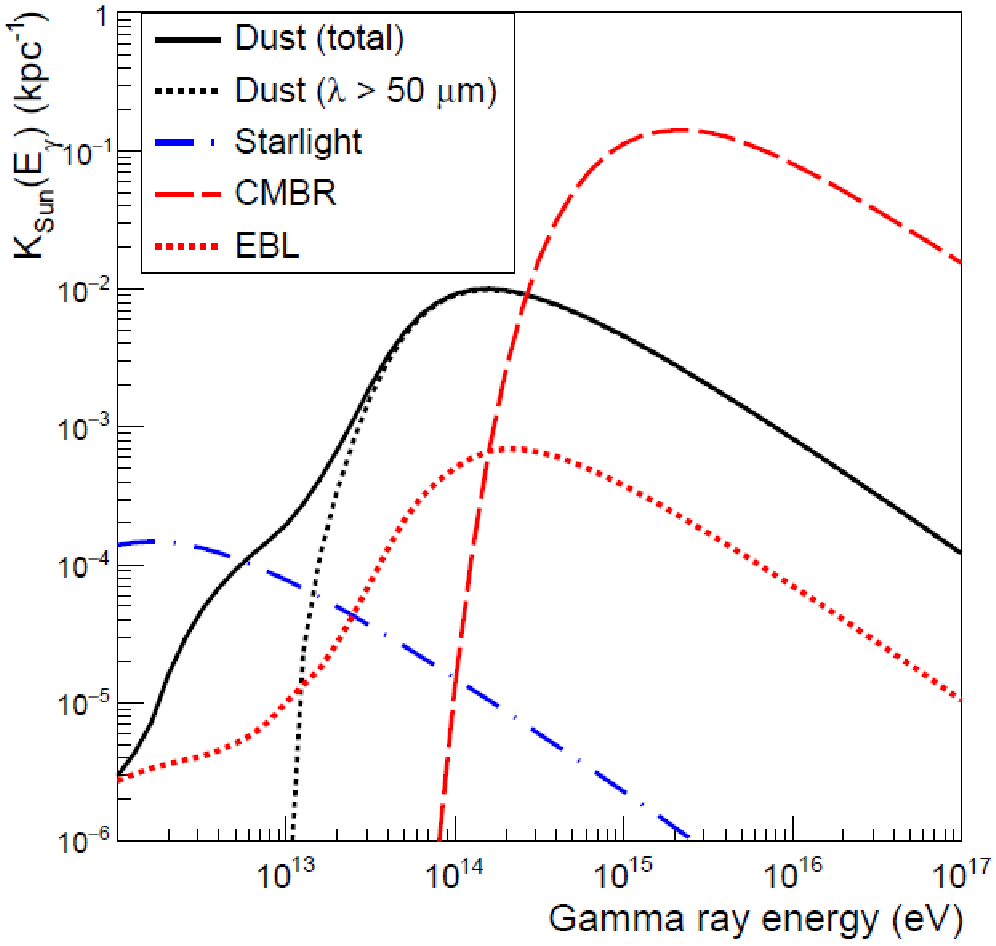

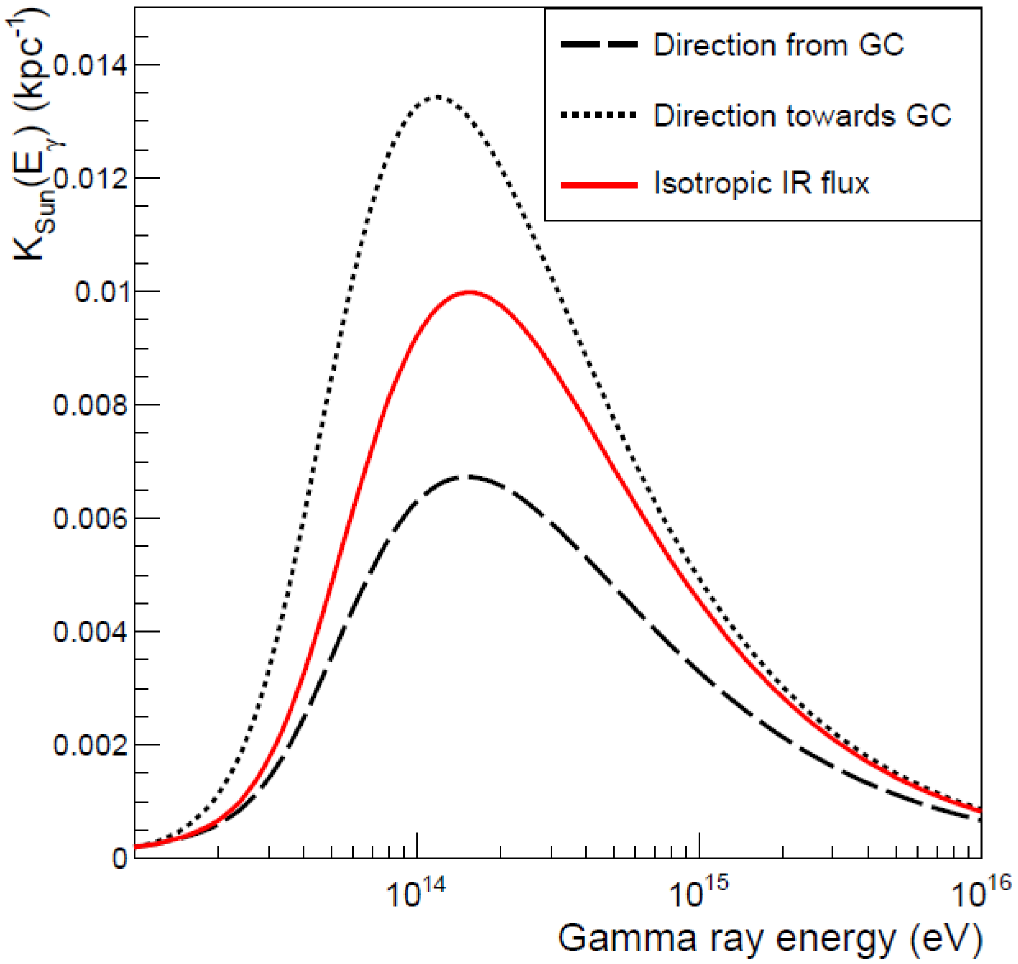

Vernetto and Lipari, Phys. Rev. D 94, 063009
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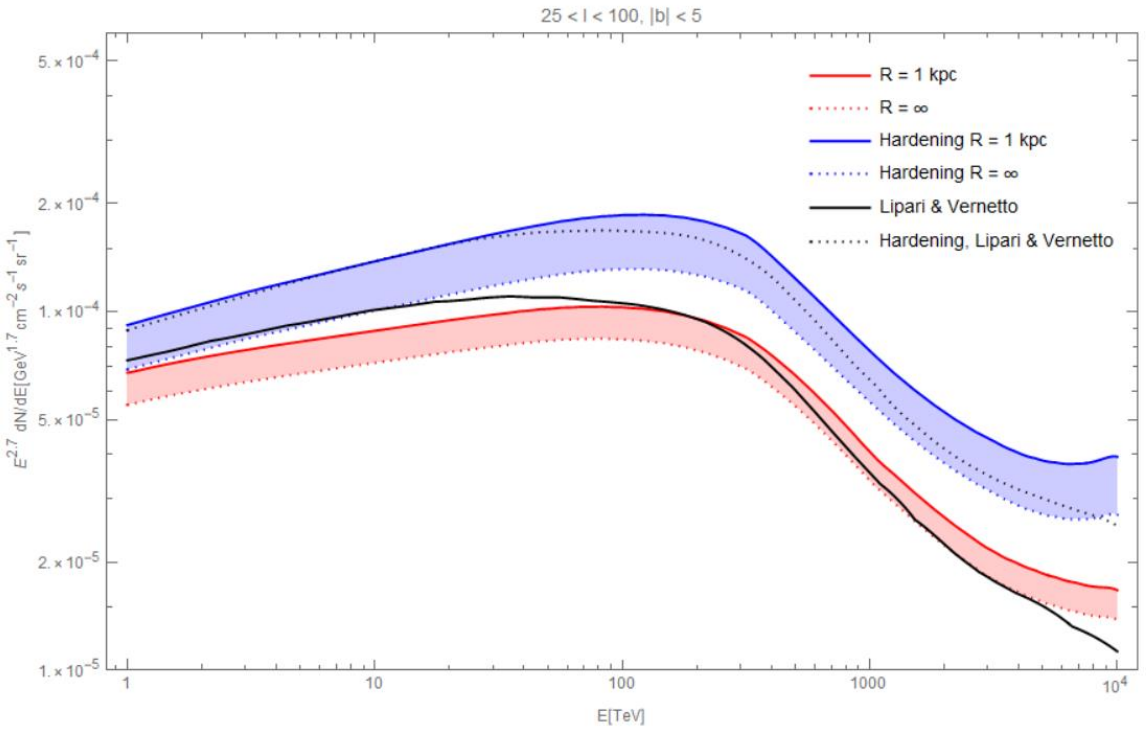
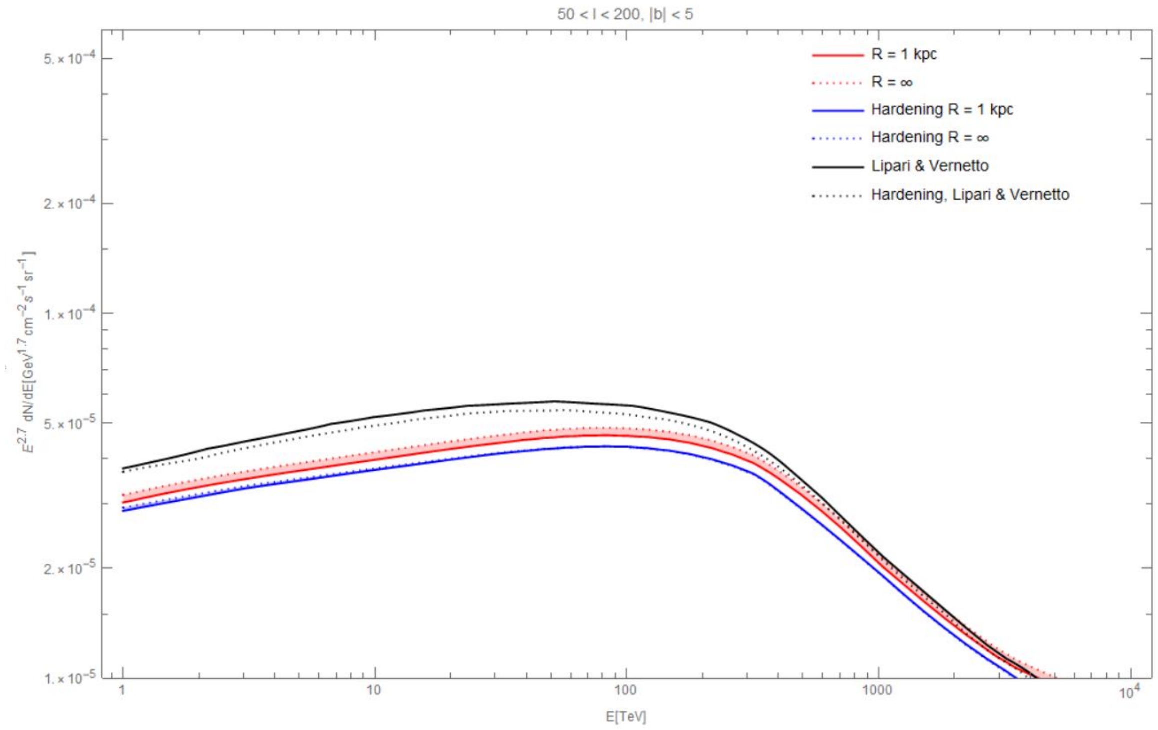
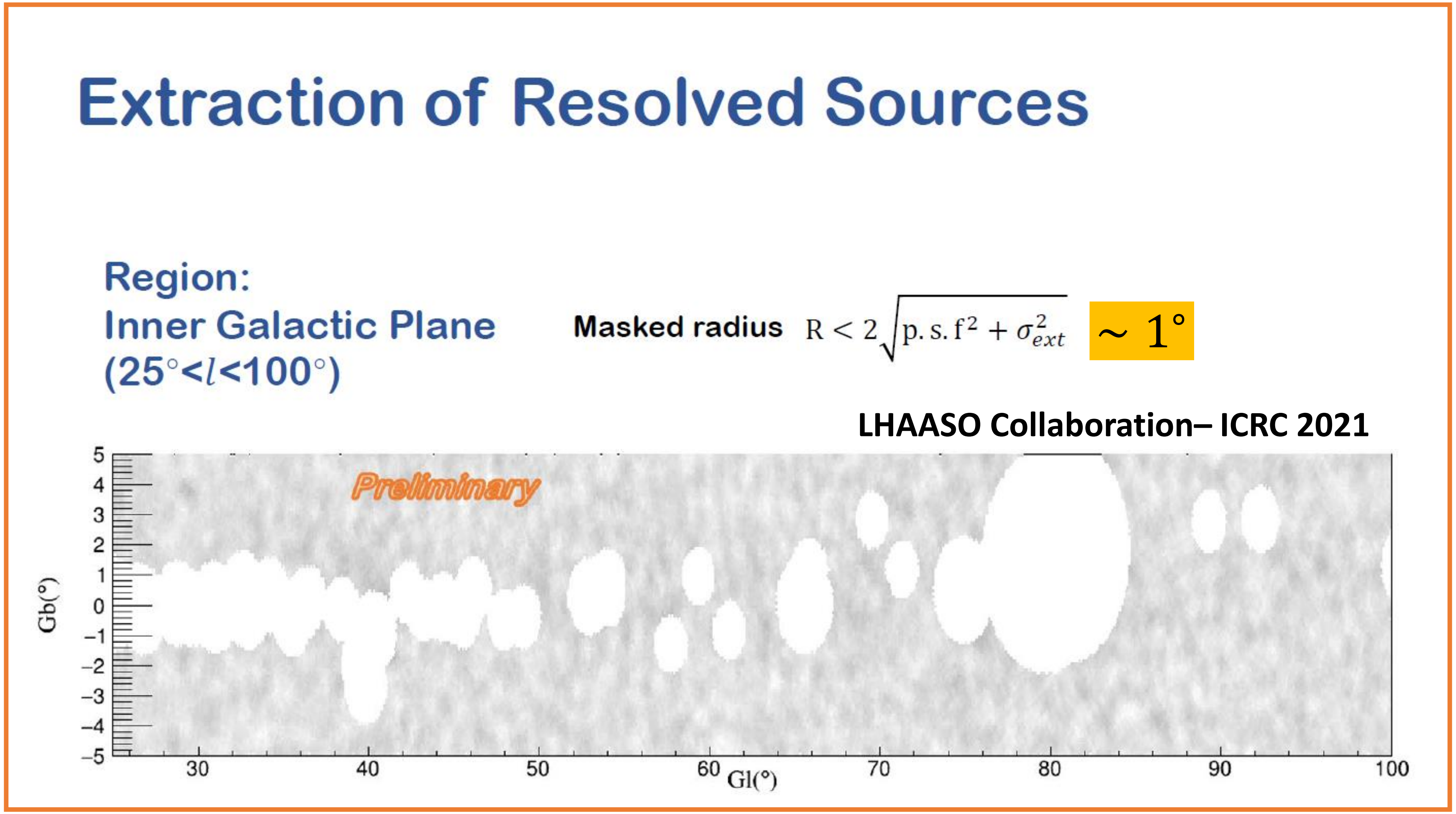
Comparison Diffuse models:





Why not LHAASO?



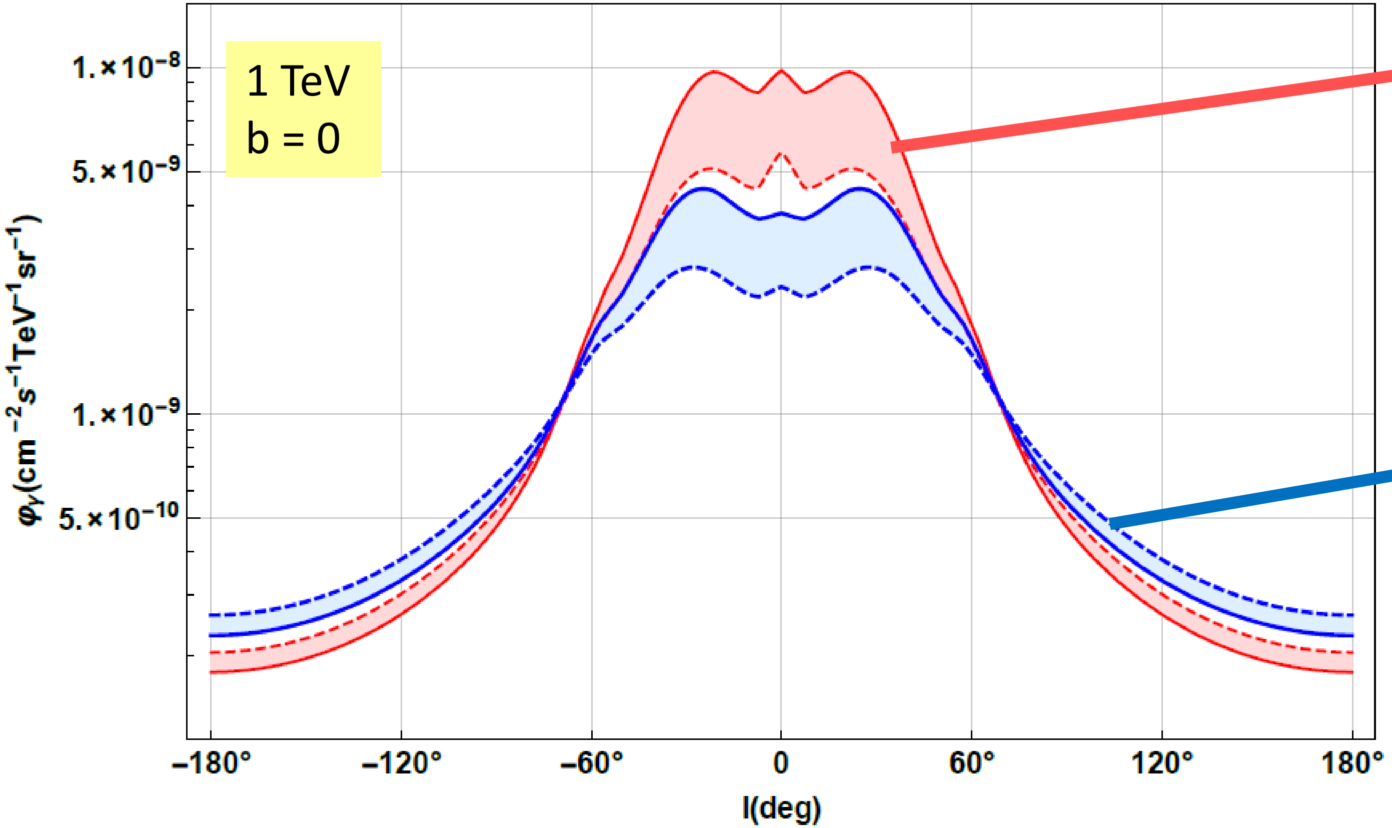
Extraction of Resolved Sources

Region:
Inner Galactic Plane
(25°<l<100°)

Masked radius $R < 2\sqrt{\mathrm{p.s.f}^2 + \sigma_{ext}^2}$ $\sim 1^\circ$

LHAASO Collaboration– ICRC 2021

Diffuse Galactic gamma-ray emission:



Model with hardening

— $R = 1\ kpc$

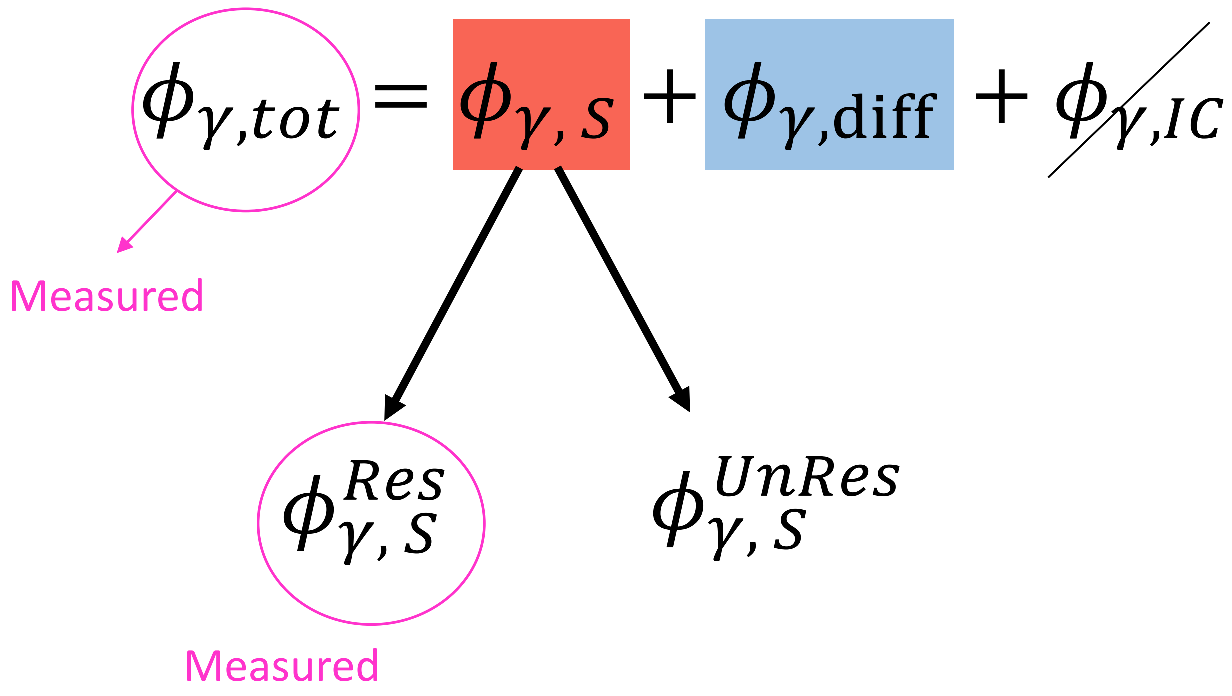
- - - $R = \infty$

Model without hardening

— $R = 1\ kpc$

- - - $R = \infty$

Unresolved sources:



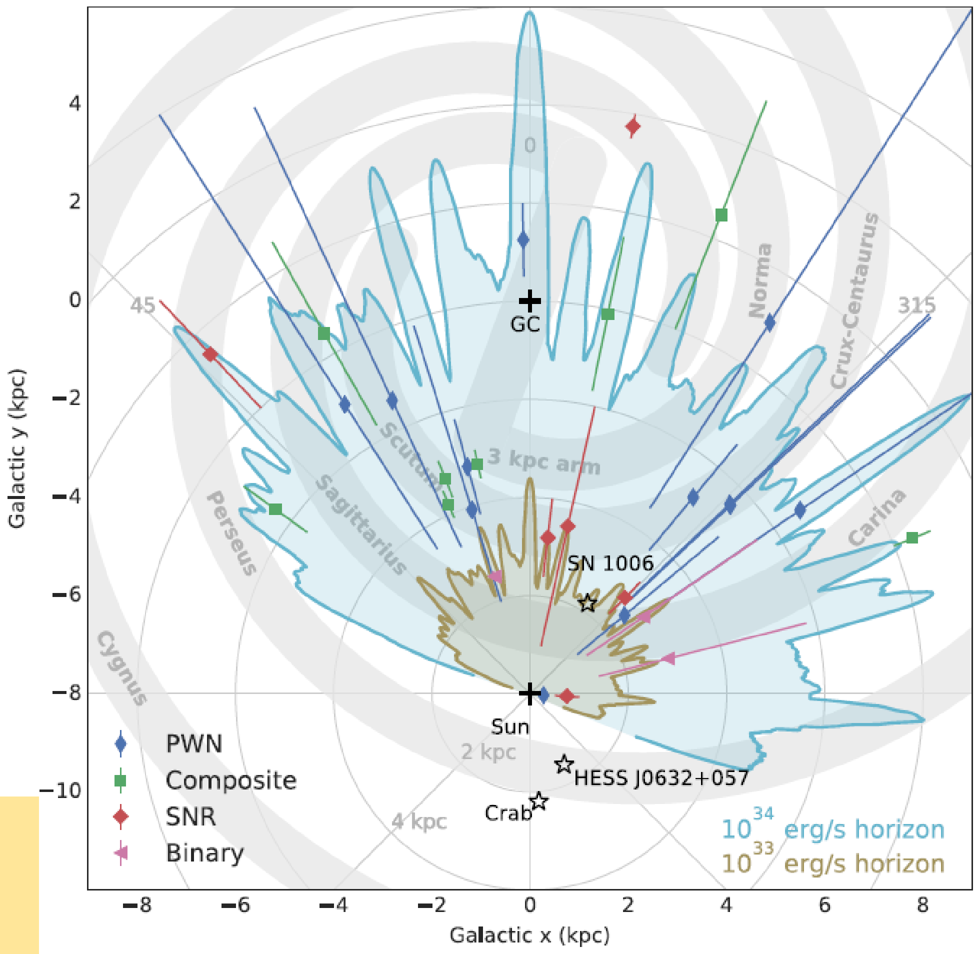
$$\phi_{\gamma,tot} = \phi_{\gamma,s} + \phi_{\gamma,\text{diff}} + \cancel{\phi_{\gamma,IC}}$$

Measured

$$\phi^{Res}_{\gamma,S} \qquad \phi^{UnRes}_{\gamma,S}$$

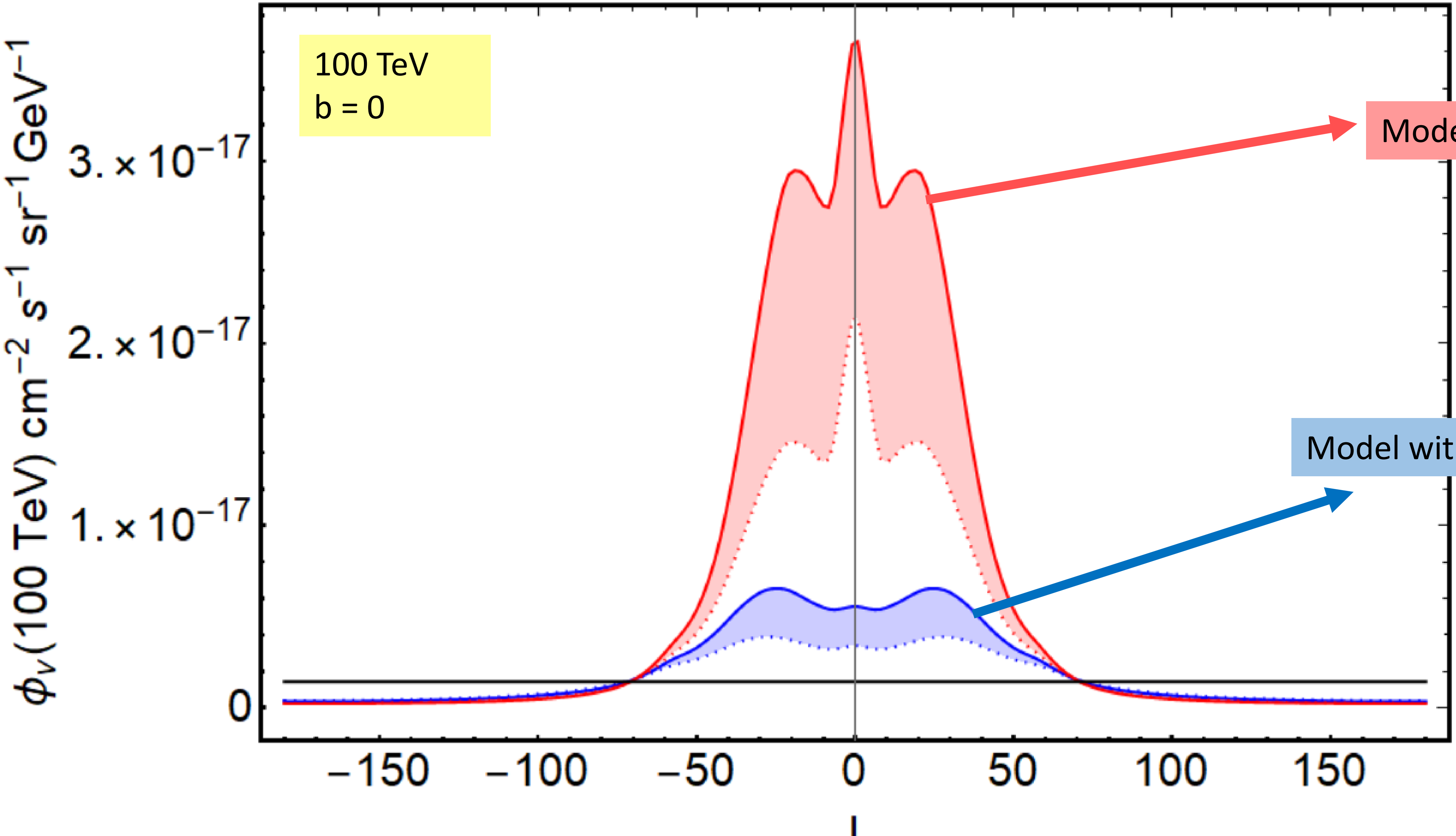
Measured

H.E.S.S. Collaboration: The H.E.S.S. Galactic plane survey



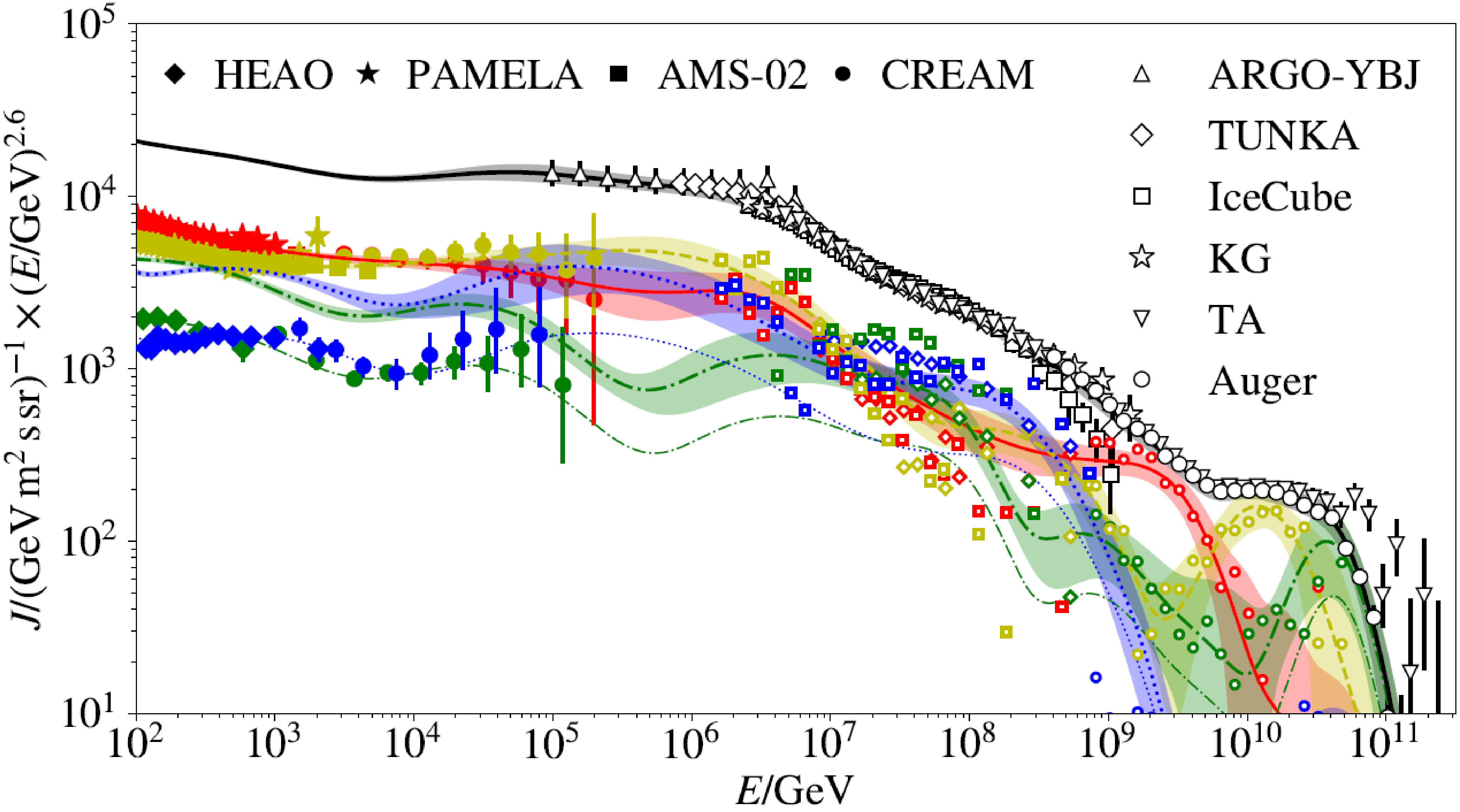
Unresolved sources contribute to the total gamma-ray signal.
Are they negligible?

Diffuse Galactic neutrino emission:



Model with hardening

Model without hardening



◆ HEAO
★ PAMELA
■ AMS-02
● CREAM
△ ARGO-YBJ
◇ TUNKA
□ IceCube
☆ KG
▽ TA
○ Auger
$J/(\mathrm{GeV\,m^2\,s\,sr})^{-1} \times (E/\mathrm{GeV})^{2.6}$
10^1
10^2
10^3
10^4
10^5
E/GeV
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
10^{11}

GAMMA–NEUTRINOS RELATION (KAPPES ET AL.)

Convert Gamma

Assumptions: total hadronic production,
no gamma-ray absorption

Using the relations:

$$\frac{dN_{\nu,\gamma}}{dE_{\nu,\gamma}} = k_{\nu,\gamma}\left(\frac{E_{\nu,\gamma}}{TeV}\right)^{-\alpha_{\nu,\gamma}} \exp\left(-\sqrt{\frac{E_{\nu,\gamma}}{E_{cut,\nu,\gamma}}}\right)$$

$$\begin{aligned} k_\nu &\sim (0.694 - 0.16\alpha_\gamma)k_\gamma, \\ \alpha_\nu &\sim \alpha_\gamma \sim \alpha_p - 0.1, \\ E_{cut,\nu} &\sim 0.59 E_{cut,\gamma} \sim \frac{E_{cut,p}}{40}. \end{aligned}$$

In Neutrinos

Model:

The power-law for the luminosity distribution can be automatically obtained assuming a fading source population (like PWNe, TeV Halos) create at a constant rate $\bar{r}$.

The spin-down power is described by: $\dot{E}\,(t) = \dot{E}_0\left(1+\frac{t}{\tau}\right)^{-2}$

Considering that a fraction $\boldsymbol{\lambda(t)}$ of the spin-down power is converted into gamma-rays then the intrinsic luminosity decreases according to:

$$L(t) = \lambda(t)\dot{E}(t) = \overset{\boldsymbol{L_{max}}}{\lambda\dot{E}_0}\left(1+\frac{t}{\tau}\right)^{-\gamma} \text{ where } \gamma = 2(\delta+1);$$

$$\lambda(t) = \lambda\left(\frac{\dot{E}(t)}{\dot{E}_0}\right)^{\delta}$$

Abdalla et al, *A&A*, 612, A2 (2018)

Then:

$$\boxed{Y(L) = \frac{\bar{r}\,\tau\,(\alpha\text{ -}1)}{L_{\max}}\left(\frac{L}{L_{\max}}\right)^{-\alpha}}$$

Where $\bar{r} = 0.019\,yr^{-1}$ is the SN's rate and $\alpha = \left(\frac{1}{\gamma}+1\right)$ therefore for $\gamma = 2$ we have $\alpha = 1.5$.

And instead of the parameter $\mathcal{N}$ we have the spin-down timescale of the Pulsar τ.

GSSI

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Abdalla et al, *A&A*, 612, A2 (2018)

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Results:

- The total TeV luminosity (1-100 TeV) of the Galaxy:

$$L_{MW} = \frac{N\, L_{max}}{(2-\alpha)}\left[1-\left(\frac{L_{min}}{L_{max}}\right)^{\alpha-2}\right] = \boxed{1.7^{+0.5}_{-0.4} \times 10^{37}\ erg\ s^{-1}}$$

0

- The flux at Earth produced by all sources (1-100 TeV) (resolved and unresolved) in the H.E.S.S. OW:

$$\phi_{tot} = \frac{L_{MW}}{4\Pi\langle E\rangle}\int_{OW} d^3r\, \rho(r) r^{-2} = \boxed{3.8^{+1.0}_{-1.0} \times 10^{-10} cm^{-2} s^{-1}}$$

3.25 TeV

- By subtraction we can obtain the contribution of unresolved sources in the H.E.S.S. observational window knowing that: $\phi_{S,res} = 2.3 \times 10^{-10} cm^{-2} s^{-1}$ (cumulative flux due to all 78 sources):

$$\boxed{\phi_{S,unres} = \phi_{tot} - \phi_{S,res} = 1.4^{+1.0}_{-0.8} \times 10^{-10}\ cm^{-2} s^{-1}} \sim 60\%\ \phi_{S,res} \sim 30\%\ \phi_{tot}$$

Likelihood:

$$\log L = -\mu_{tot} + \sum_i \log(\mu_i)$$

- μ_{tot} represents the number of expected sources;
- μ_i is the probability to observe an object with coordinates (b_i, l_i) and measured flux ϕ_i.

The source distribution per unit of flux is:

$$\mu(b, l, \phi) = \int dr\ 4\pi r^4 \rho(r, l, b) Y(4\pi r^2 \langle E \rangle \phi)$$

While is given by:

$$\mu_i = \int d\phi \mu(b_i, l_i, \phi_i) P(\phi_i, \phi, \delta\phi_i)$$

Where $P(\phi_i, \phi, \delta\phi_i)$ respresents the probability that the measured flux ϕ_i is obtained for the real flux ϕ.

We assume a Gaussian.

The $\chi^2 = -2logL$ was used for obtaining the best fit values and the allowed regions for the parameters.

Cumulative distribution:

The flux distribution can be calculated as:

$$\frac{dN}{d\Phi} = \int dr\ 4\pi r^4 \langle E \rangle\ Y(4\pi r^2 \langle E \rangle \Phi)\ \overline{\rho}(r)$$

- $\bar{\rho}(r)$ is the sources spatial distribution integrated over the longitude and latitude intervals probed by H.E.S.S.;
- The above integral is performed in the range $\mathrm{d}/\theta_{max} \leq \mathrm{r} \leq D(L,\phi) =$ where $\theta_{max} = 0.7^{\mathrm{o}}$ is the maximal angular dimension that can be probed by H.E.S.S. and the d is the physical dimention of the source. While $D(L,\phi) = (L/4\,\pi\,\langle E \rangle\,\phi)^{\frac{1}{2}}$;
- We calculate analytically the flux distribution for the 2 limits cases $L_{max} \rightarrow \infty$ and $L_{max} \rightarrow 0$:

$$\frac{dN}{d\Phi} = R\,\tau\,(\alpha - 1)\,L_{\max}^{\alpha-1}\,\boxed{\Phi^{-\alpha}} \int_0^{\infty} dr\,(4\pi\langle E \rangle)^{1-\alpha}\,r^{4-2\alpha}\,\overline{\rho}(r)$$

$$\frac{dN}{d\Phi} \simeq (4\pi\langle E \rangle)^{1-\alpha}\,\overline{\rho}(0)\,R\,\tau\,(\alpha-1)\,L_{\max}^{\alpha-1}\,\Phi^{-\alpha} \int_0^{D(L_{\max},\Phi)} dr\,r^{4-2\alpha} = \overline{\rho}(0)\,R\,\tau\left(\frac{\alpha-1}{5-2\alpha}\right)\left(\frac{L_{\max}}{4\pi\langle E \rangle}\right)^{\frac{3}{2}} \boxed{\Phi^{-\frac{5}{2}}}$$

Resolved and Unresolved sources:

The resolved flux can be calculated from:

$$\phi_{res} = \int dr\, r^2 \bar{\rho}(r) \int d\phi Y(4\pi r^2 \langle E \rangle \phi)$$

$$\phi_{res} = \int dr\, r^2 \bar{\rho}(r) \int dL\, 4\pi r^2 \langle E \rangle Y(L)$$

$$\phi_{res} = \phi_{th} \int dr \bar{\rho}(r) \int dL \overline{D}(L)^2 Y(L)$$

- $\bar{\rho}(r)$ is the sources spatial distribution integrated over the longitude and latitude intervals probed by H.E.S.S.;
- The above integral is performed in the range $\mathrm{d}/\theta_{max} \leq \mathrm{r} \leq D(L,\phi)$ where $\theta_{max} = 0.7^{\mathrm{o}}$ is the maximal angular dimension that can be probed by H.E.S.S. and the d is the physical dimention of the source. While $D(L,\phi) = (L/4\pi\langle E \rangle \phi)^{\frac{1}{2}}$;
- The luminosity integral is performed in the range $L_{min}(r) \leq \mathrm{L} \leq L_{max}$ where $L_{min} = 4\pi r^2 \langle E \rangle \phi_{th}$

$=$