Reparameterisations of the neutrino mixing matrix in long-baseline analysis

NBIA Summer School on Neutrinos Here There and Everywhere

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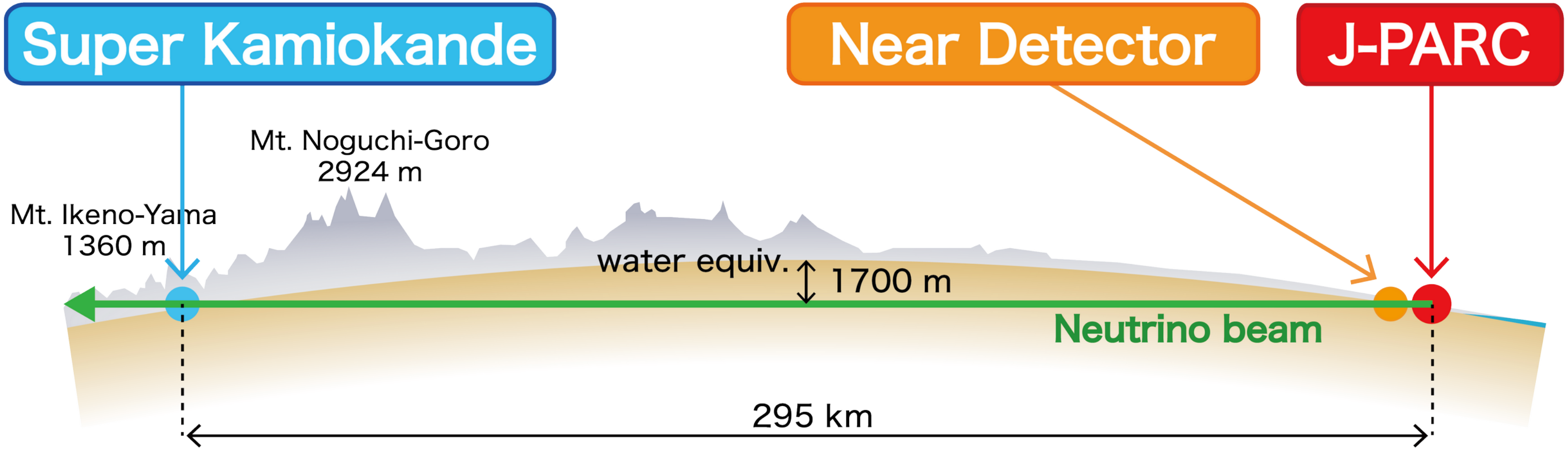
Long Baseline oscillation experiments



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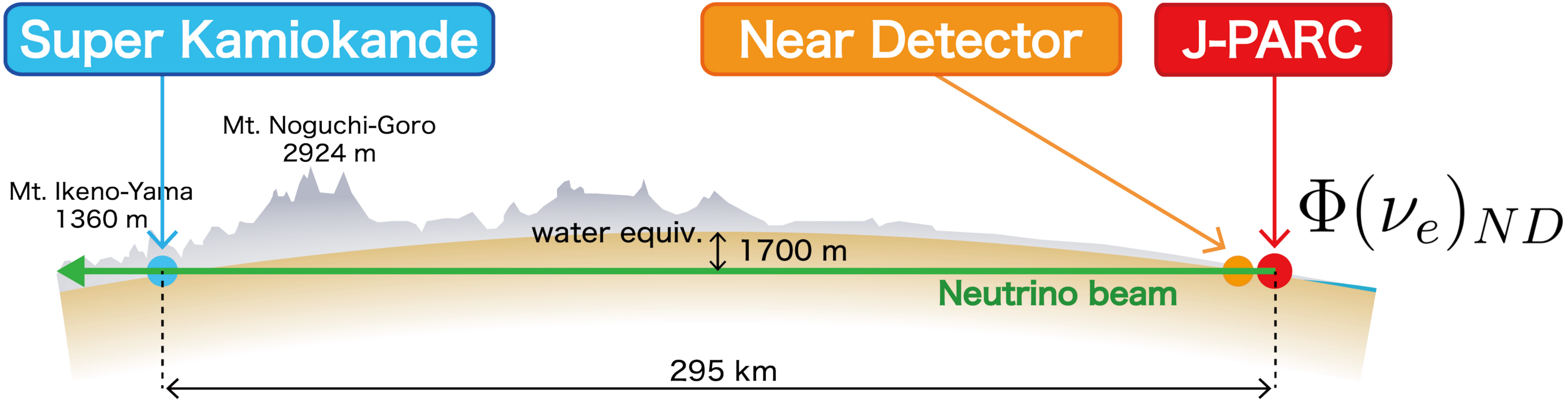

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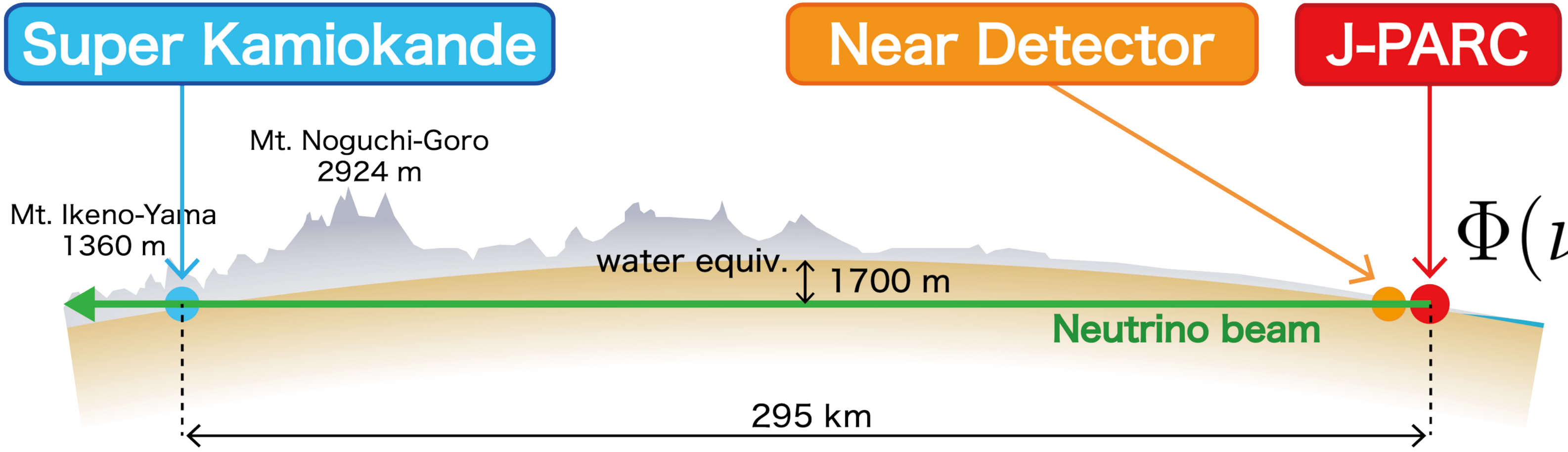
Long Baseline oscillation experiments

$\Phi(\nu_e)_{FD}$



$\Phi(\nu_e)_{ND}$

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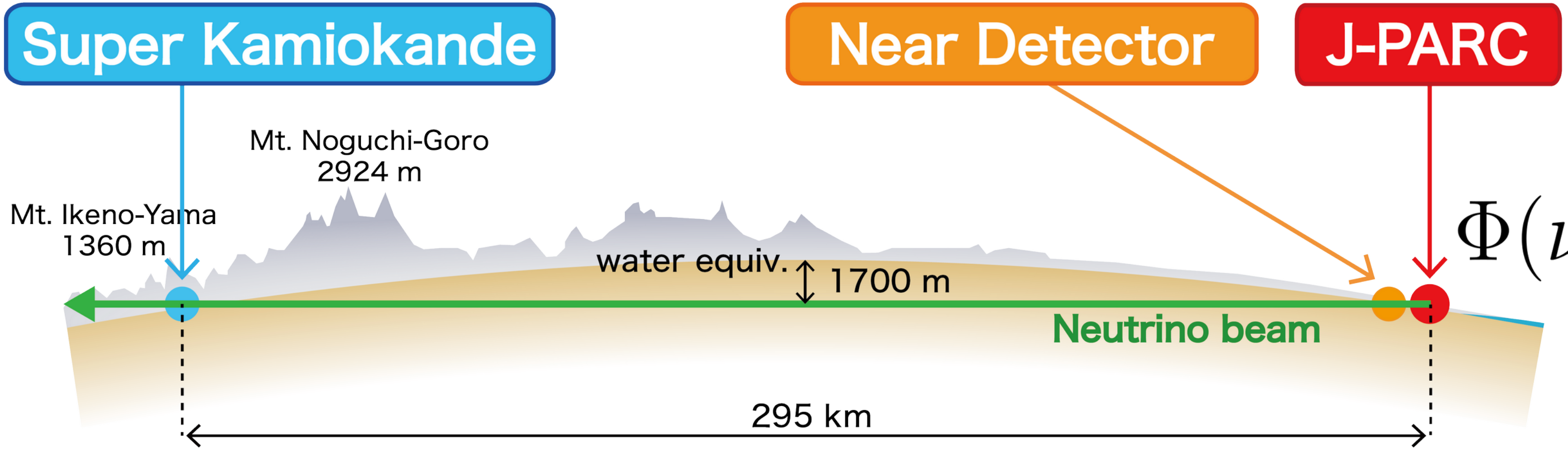
Long Baseline oscillation experiments

$\Phi(\nu_e)_{FD}$ $\Phi(\nu_\mu)_{FD}$



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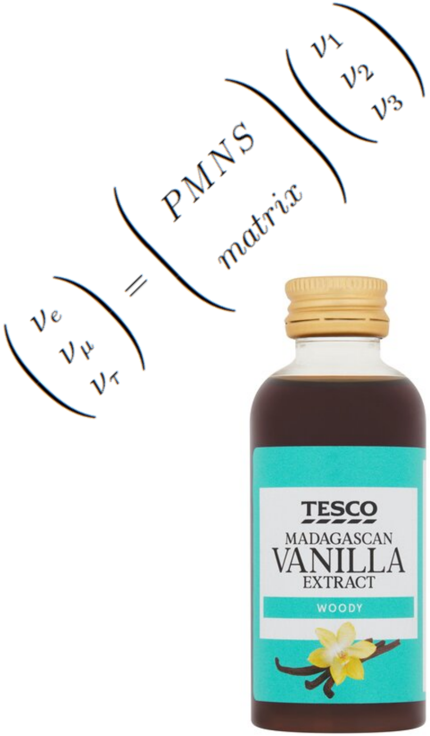
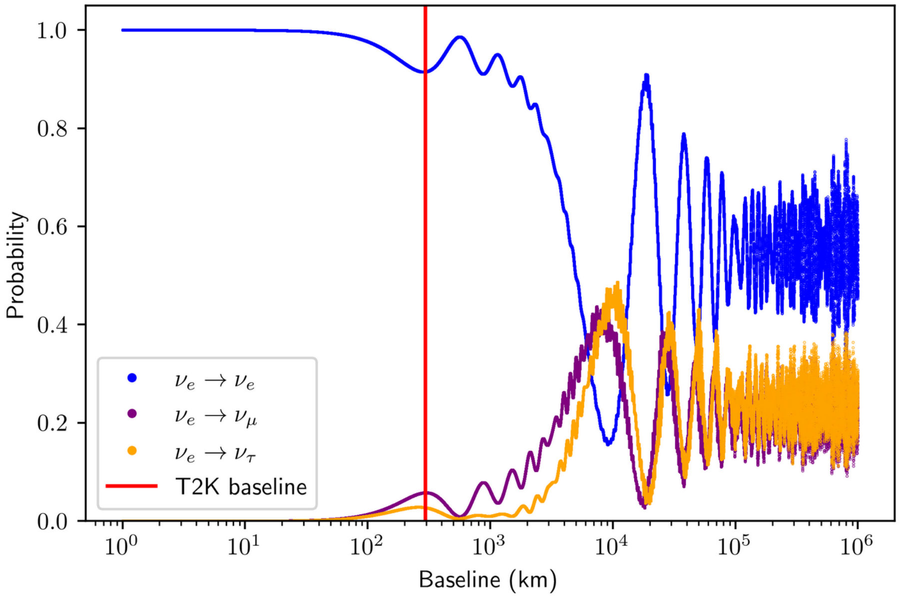
Long Baseline oscillation experiments





$$P_{\alpha\beta}(L/E) = \frac{\Phi(\nu_\beta)_{FD}}{\Phi(\nu_\alpha)_{ND}}$$

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Long Baseline oscillation experiments

$$P_{\alpha\beta}(L/E) \equiv f(L/E, M_\nu, \Delta m^2_{ij})$$

- We (usually) require a change of basis matrix between eigenstates of H and flavour states

- If we assume there are precisely 3 neutrino states, this matrix will be in a subset of U(3)   (or SO(3), in the absence of CP-violation)

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Long Baseline oscillation experiments

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \quad B = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & -0 & c_{13} \end{pmatrix} \quad C = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{ABC} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{ABC}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{ABC}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{ABC}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{ABC}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{ABC}} & c_{23}c_{13} \end{pmatrix}$$

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Long Baseline oscillation experiments

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Small angle approximations give us our usual oscillation paradigm

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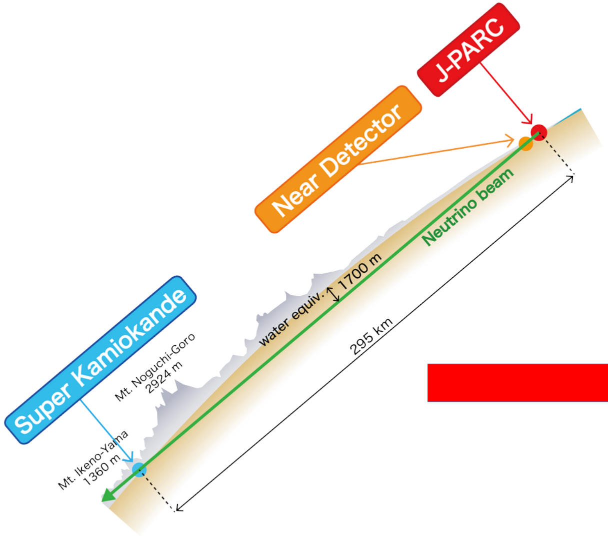
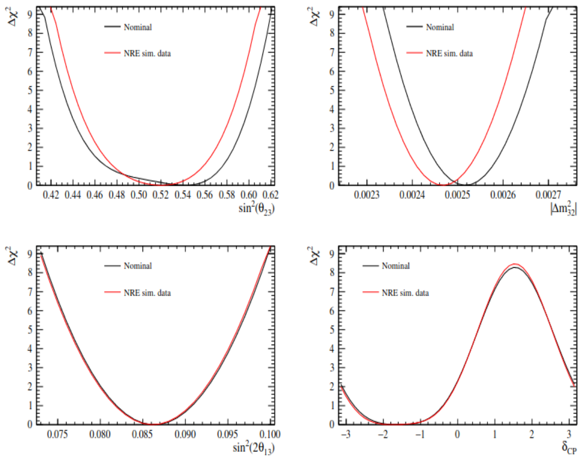
Long Baseline oscillation experiments





The T2K collab. arXiv:2101.03779

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Reparameterisations, assumptions and samplings

What are we actually measuring, and how should we sample our phase space?

Can we explore different phase spaces? How would we parameterise them?

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Reparameterisations, assumptions and samplings

What are we actually measuring, and how should we sample our phase space?

- Tait-Bryan parameterisations
- Haar “anarchy” samplings

Can we explore different phase spaces? How would we parameterise them?

- Non-unitary parameterisations (i.e. lower-triangular prefactors)
- T-symmetric parameterisations
- Haar SO(3) samplings

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Tait-Bryan parameterisations

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \quad B = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & -0 & c_{13} \end{pmatrix} \quad C = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- We can write down an SO(3) + dcp matrix in 6 (9) different ways, in terms of 3 angles

- The usual parameterisation takes advantage of the smallness of the U_e3 element of the PMNS matrix to allow for approximations

- Why should we assume an underlying prior which is flat in the usual parameterisation?

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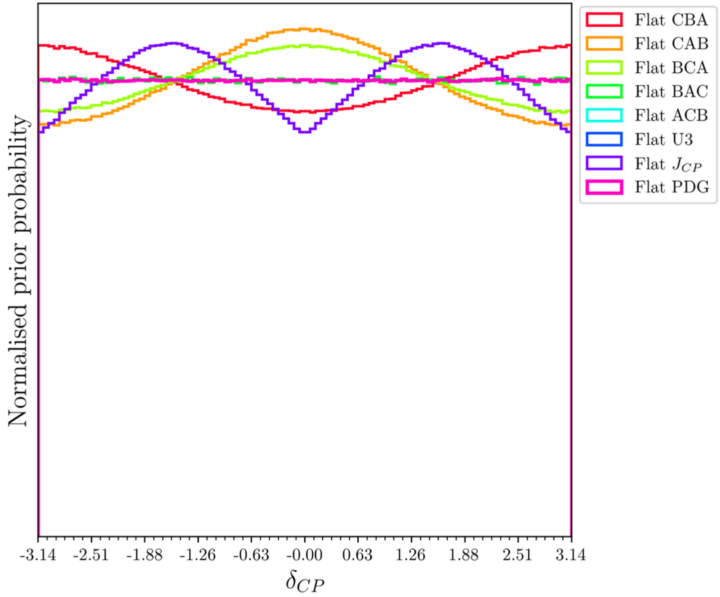
Tait-Bryan parameterisations



The underlying distributions of the spaces generated in the different parameterisations are not the same!

Sampling the phase space of neutrino oscillations in a uniform way is not a trivial problem!!

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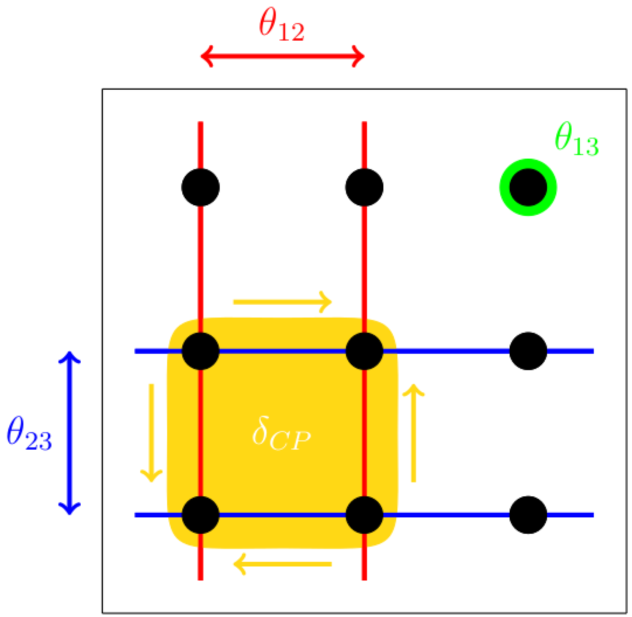
Tait-Bryan parameterisations

Firstly, we cannot think of the different angles in the same way

$$U_{ABC} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{ABC}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{ABC}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{ABC}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{ABC}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{ABC}} & c_{23}c_{13} \end{pmatrix}$$



-θ12 is a mass ratio (onto flavour states) angle
-θ23 is a flavour ratio (onto mass states) angle
-θ13 is a projection angle
-δcp is a measure of symmetry within the adjugate

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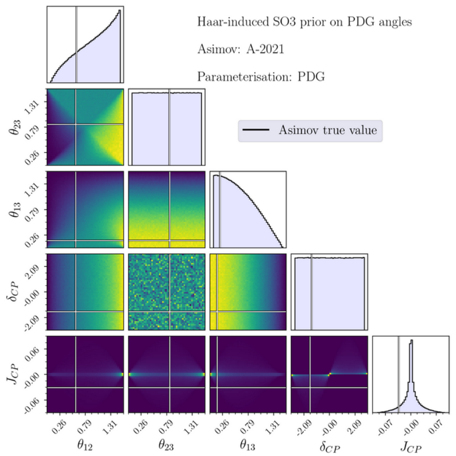
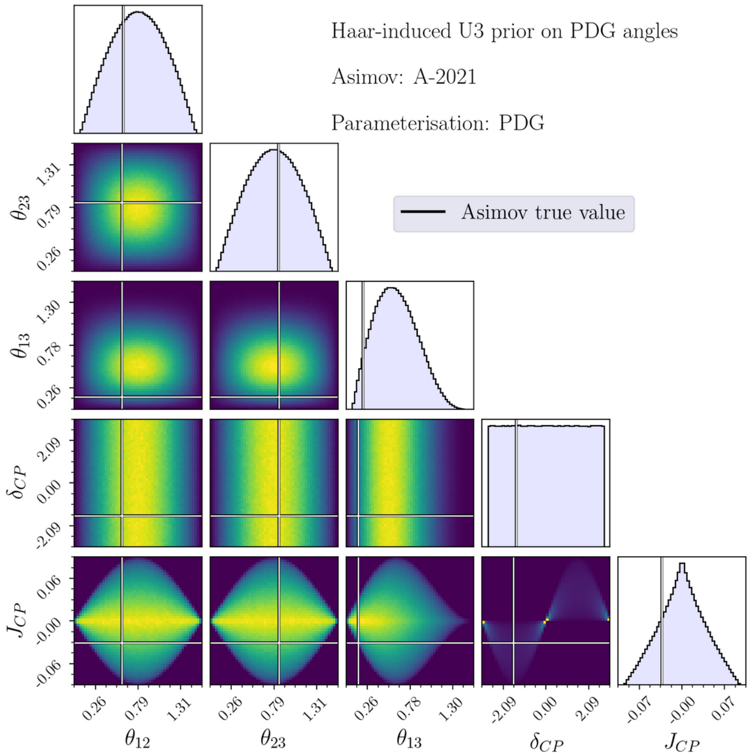
True U(3) uniform sampling

Haar (anarchy) sampling to the rescue!

But even with the usual assumptions, the PMNS matrix is not a random U3 element





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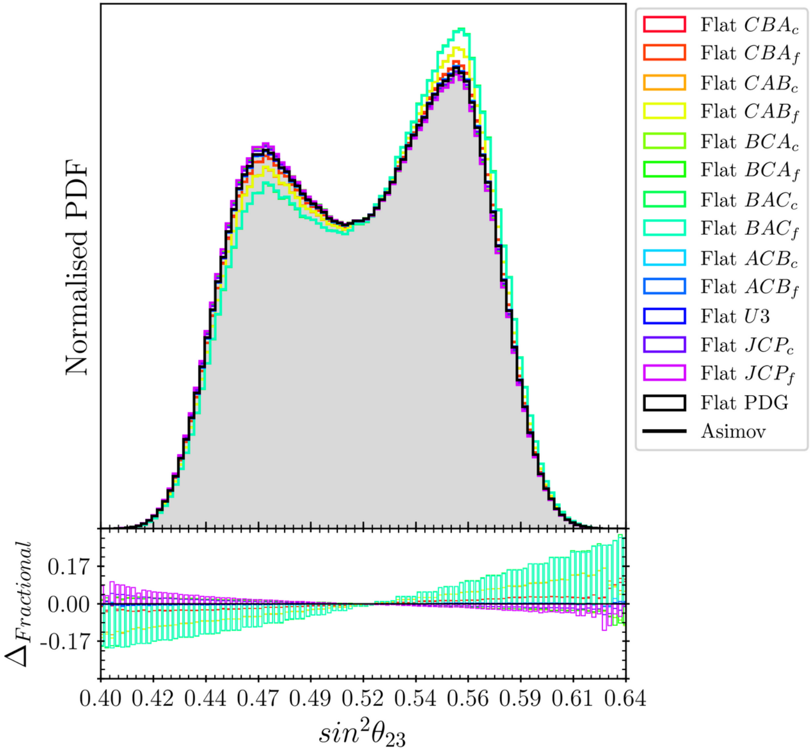
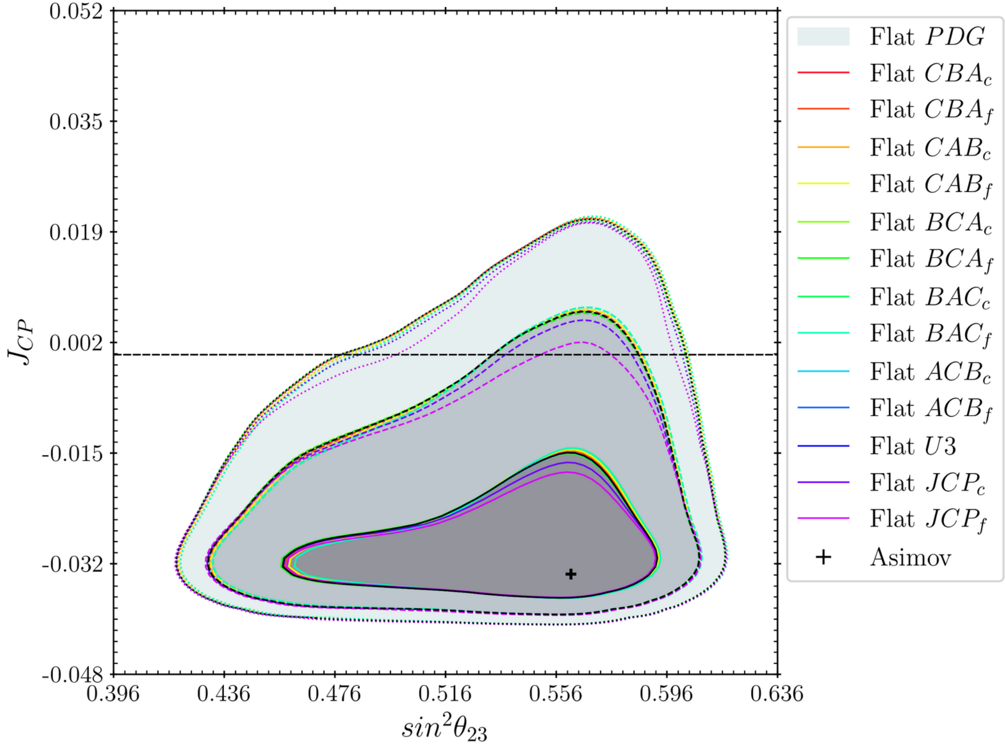
Checking the robustness of the T2K analysis





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Takeaways

- Study your underlying phase space for intrinsic structure
- Do not attach value to purely mathematically-motivated parameters
- In neutrino analysis, where we cannot directly observe what we are attempting to measure, make sure your results are robust enough to survive alternate paradigms
- Make sure you understand the assumptions behind your choice of parameterisation

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