Nonlinear Propagation of Low-Energy Cosmic Rays from Supernova Remnants

arXiv:2112.09708

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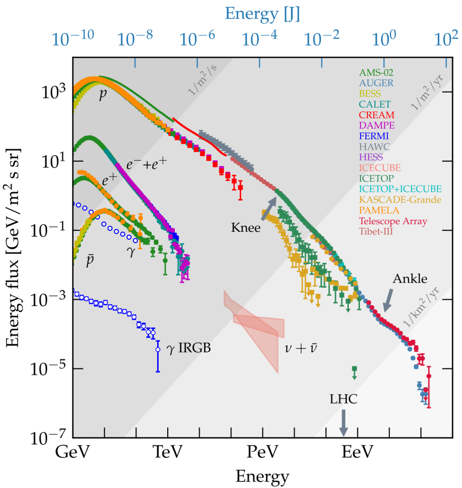
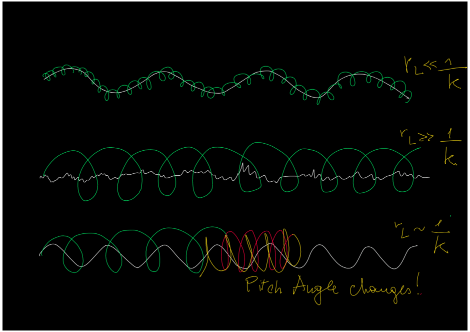
Basics

Cosmic ray spectrum



Carmelo Evoli

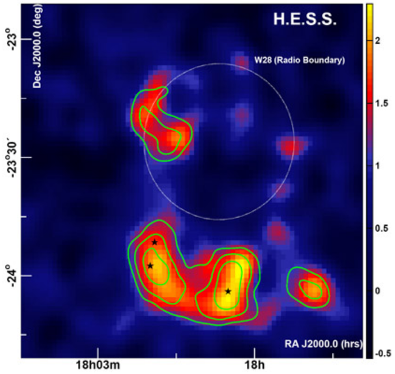
Resonant scattering



Pasquale Blasi

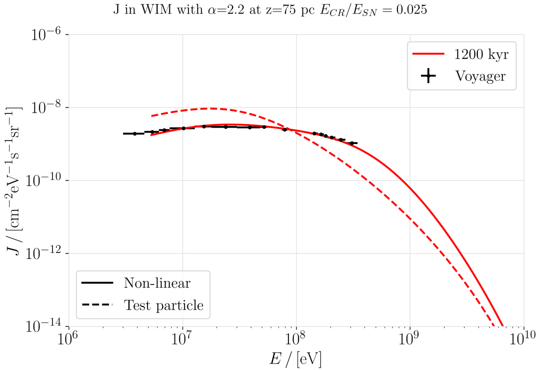
Motivation

Suppressed diffusion around SNR



H.E.S.S. collaboration (2007)

Voyager data at low energies

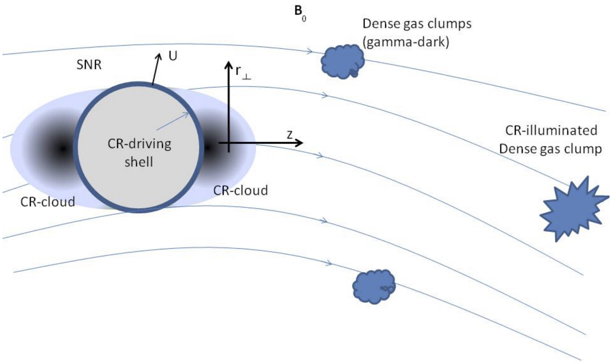


H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*

Self confinement of particles by the resonant streaming instability (SI) after the escape

Physical setup

Malkov *et al.* (2013)



Cosmic ray self-confinement

Ptuskin, Zirakashvili, Plesser (2008); Malkov *et al.* (2013); Nava *et al.* (2016/2019); Recchia *et al.* (2021)

$$\partial_t f_{\mathrm{CR}} = \partial_z \left(D_{zz}(z,p) \partial_z f_{\mathrm{CR}} \right)$$

$$D_{zz}(z,p) \sim \left. \frac{D_B(p)}{kW(k)} \right|_{k=1/r_g}$$

$$\Gamma_{\mathrm{CR}}(z,k) = -\frac{v_A}{kW} \partial_z p^4 f_{\mathrm{CR}}$$

$$\partial_t W = \left(\Gamma_{\mathrm{CR}}(z,k) - \Gamma_D \right) W$$

- Diffusion coefficient $D_{zz}(z,p)$
- Bohm value $D_{\mathrm{B}}(p)$
- Alfvén speed v_A
- Spectral power $W(k)$
- Growth rate $\Gamma_{\mathrm{CR}}(z,k)$
- Damping rate Γ_D

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Previous work

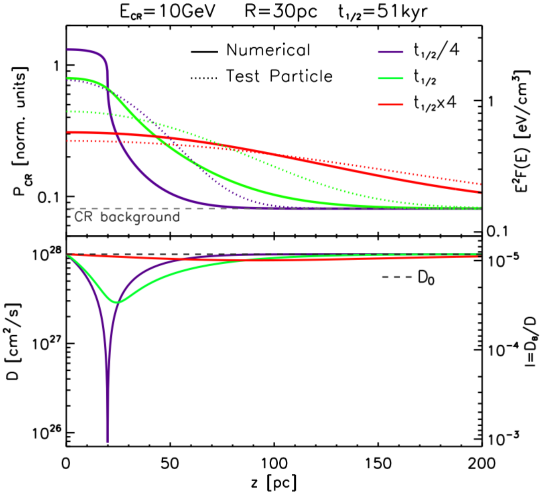
Nava *et al.* (2016)

Setup

- Supernova converts $10\,\%$ of its energy into CR
- Particles accelerated by shock to power law
- Escape at $t_{1/2}$ into ISM

Results

- Propagation slower than in test particle case
- Suppression of the diffusion coefficient up to $51\,\mathrm{kyr}$
- Recover test particle solution after $51\,\mathrm{kyr}$



The phases of the ISM

Phase	$T\,[\mathrm{K}]$	$n\,[\mathrm{cm}^{-3}]$	filling factor	ionisation fraction	neutrals	ions
HIM	10^6	10^{-2}	0.5	1	-	H^+
WIM	8000	0.35	0.25	0.6-0.9	H, He	H^+
WNM	8000	0.35	0.25	10^{-2}	H, He	H^+
CNM	80	35	~ 0	10^{-3}	H, He	C^+
DiM	50	300	~ 0	10^{-4}	H_2, He	C^+

Most of the ISM mass in molecular clouds, but filling factor tiny.

- Focus on WIM with ionisation fraction 0.9 and WNM
- Alfvén speed v_A larger in WNM due to inefficient coupling of neutrals at low energies

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Most promising at low energies (WIM, WNM)

Most of the ISM mass in molecular clouds, but filling factor tiny.

- Focus on WIM with ionisation fraction 0.9 and WNM
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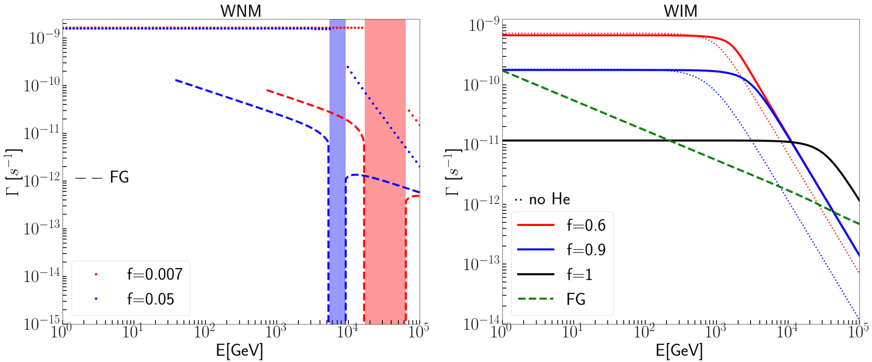
Damping Processes

- Ion-neutral damping (momentum transfer to neutrals)
- Farmer-Goldreich damping (interaction with external turbulence)
- Non-linear Landau damping (interaction of beat of waves with background plasma)

Recchia *et al.*(2021)



Propagation of low energetic protons

$$\partial_t f_{\mathrm{CR}} = \partial_z \left(D_{zz}(z,p) \partial_z f_{\mathrm{CR}} \right) - \frac{1}{p^2} \partial_p \left(\dot{p} p^2 f(z,p) \right)$$

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Improvements

- Energy losses important $E < 10\,\mathrm{GeV}$
  - Ionisation
  - Cloulomb
  - Pion production
- Spatial dependent $v_A(z)$
- Non linear cascade in wave-number
- Escape at beginning of snowplow phase
- Grammage at low E

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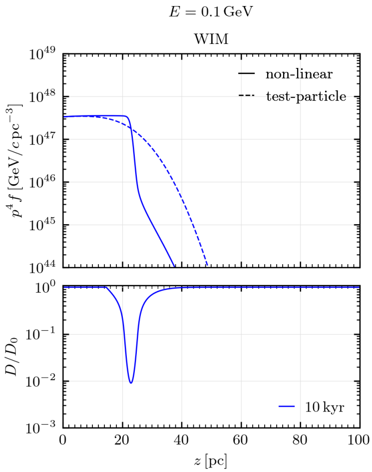
Spatial dependence

H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*

- Initially top hat profile
- Test particle solution approximately gaussian
- Particles confined longer in non-linear simulation
- Cutoff at the free escape boundary condition
- Diffusion coefficient suppressed by factor 100
- Suppression lasts $1\,\mathrm{Myr}$



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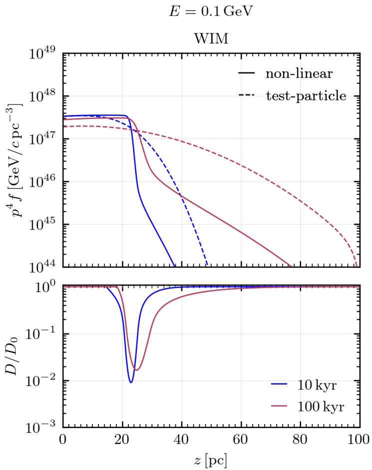

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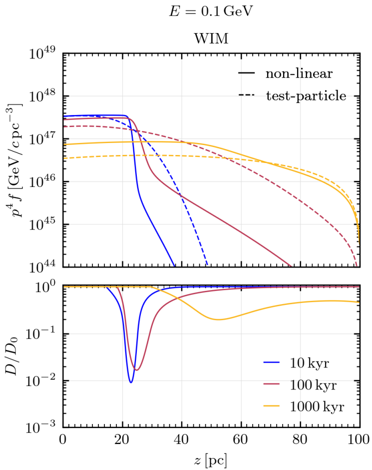
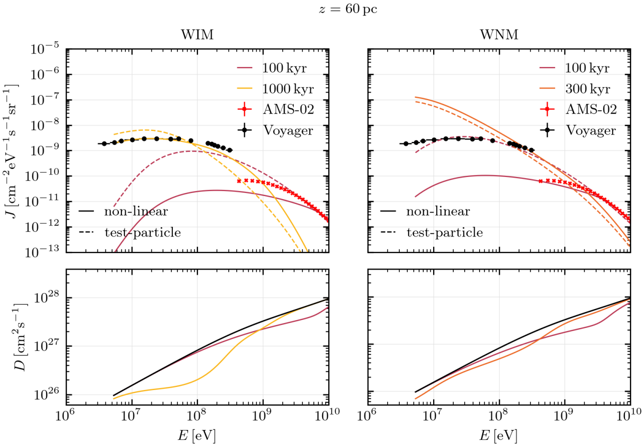


Spectral dependence: Spectral break

H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*

- Softer spectrum at later times
- More flux at later times
- Spectral break closer to Voyager than test particle solution
- Can explain Voyager1 and AMS02 data with two fine tuned sources
- Need statistical approach

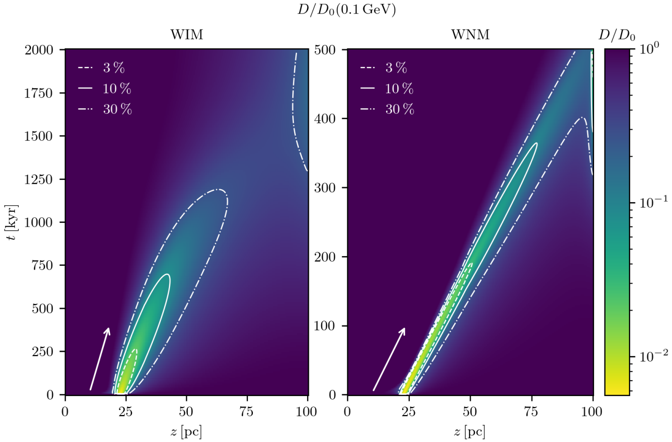
  M. Phan, F. Schulze, P. Mertsch, S. Recchia, S. Gabici (2021)



Diffusion coefficient

H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*

- Low diffusion zone advected with gradient of CR (arrow)
- WIM: suppression lasting over $1\,\mathrm{Myr}$
- WNM: suppression advected to boundary at $500\,\mathrm{kyr}$
- Instantaneous transition from 1D to 3D at boundary overestimation



Conclusion

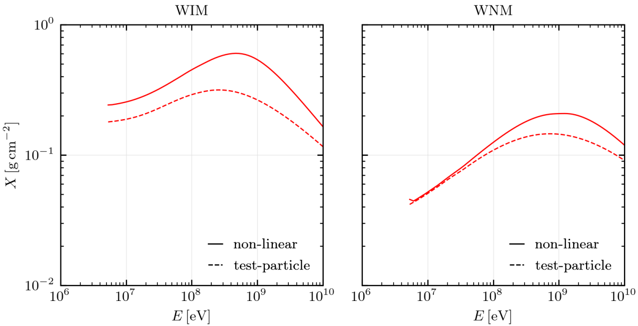
- Diffusion coefficient suppressed for more than $1\,\mathrm{Myr}$ at $E = 100\,\mathrm{MeV}$ in WIM and $500\,\mathrm{kyr}$ in WNM
- Spectral break at $100\,\mathrm{MeV}$ as required by Voyager1
- Grammage in near source region increased by factor 3

Outlook

- Propagation into a molecular cloud
- Compare to Ionisation rate measured around $W28$

Grammage

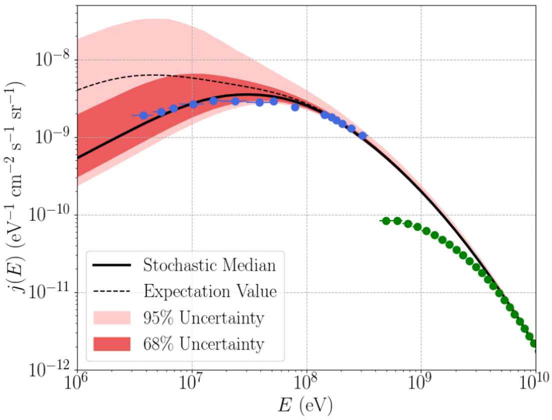
H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*



- Increased by factor 3
- Similar results to Recchia *et al.* (2021) (fig. 4) at $10\,\mathrm{GeV}$
- WIM: Constant at lowest energies
- WNM: Advection dominated at lowest energies

Stochasticity: Voyager spectrum

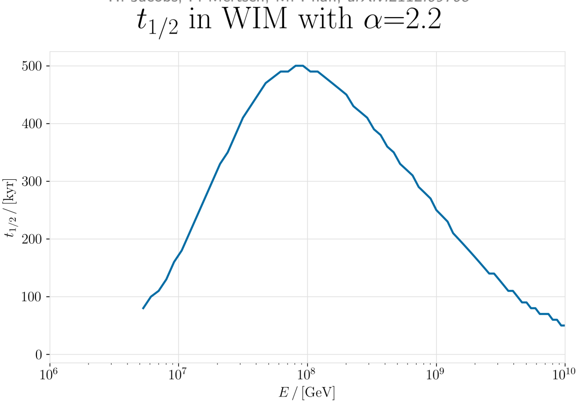
M. Phan, F. Schulze, P. Mertsch, S. Recchia, S. Gabici (2021)



- Combine stochasticity and non-linear approach

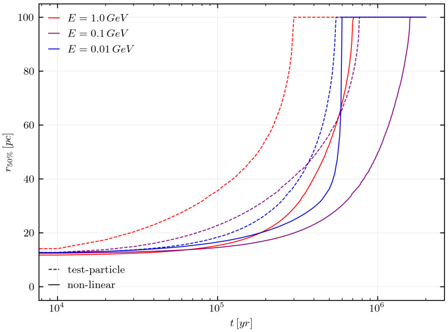
Half time of the cloud

H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*



- Energy loss dominant at low E
- Diffusion dominant at high E
- Comparable to Nava *et al.*2016 (fig. 4) at high E

50% containment radius



- Radius which contains $50\,\%$ of the particles as a function of time for the WIM with an initial spectral index of 2.2 and Kraichnan turbulence.
- Diffusion dominant at high E
- Energy loss dominated at low E

Grammage

Single particle grammage

- grammage:
  - $X_{1p}(E,t) = \int_0^t \rho v_p(E,t')dt'$
- $v_p(E,t')$ given by:
  - $t = -\int_{E_0}^{E} \frac{\mathrm{d}E'}{b(E')}$

Escape flux

- particles in simulation domain:
  - $N_{in}(E,t) = \int_0^L f(z,E,t)\mathrm{d}z$
- integrate TPE and use BC:
  - $\Phi(E,t) = -\frac{D_B}{kW(z,E,t)} \frac{\partial f(z,E,t)}{\partial z}\Big|_{z=L}$

Average grammage

- Fraction of particles escaping at t:
  - $dF(E,t) = \frac{\Phi(E,t)\mathrm{d}t}{\int_o^\infty \Phi(E,t)\mathrm{d}t}$
- Average grammage
  - $\langle X(E)\rangle = \frac{\int_0^\infty X_{1p}(E,t)\Phi(E,t)\mathrm{d}t}{\int_o^\infty \Phi(E,t)\mathrm{d}t}$

Open questions

- What is a good approximation for $t = \infty$?
- Where exactly is L?

Spatial dependence

H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*

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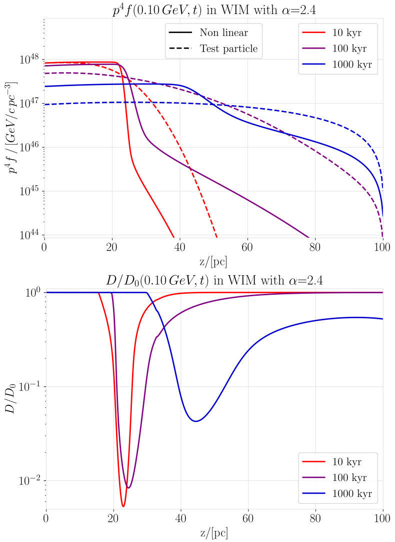
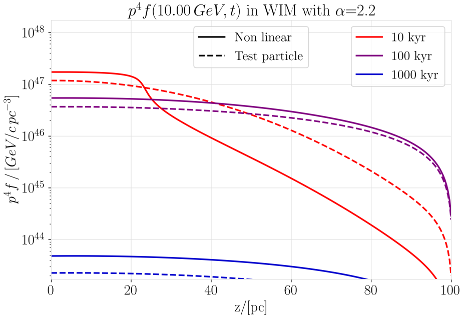
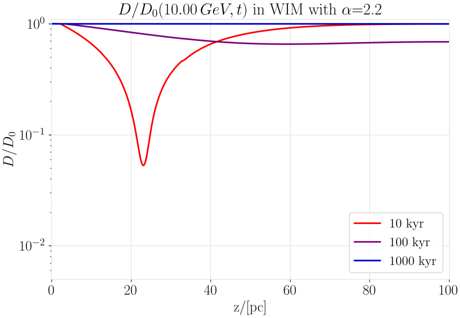


Spatial dependence high energies

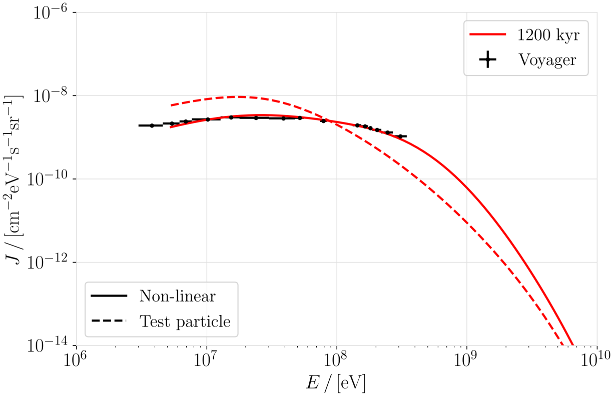
H. Jacobs, P. Mertsch, M. Phan, *in prep.*





Spectral dependence: Voyager spectrum

H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*

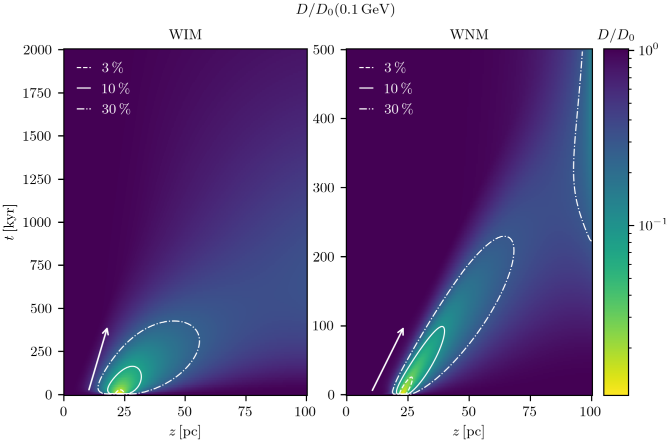


- Can reproduce Voyager spectrum for specific case.
- Need stochastic approach

Diffusion coefficient in Kolmogorov turbulence

H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*

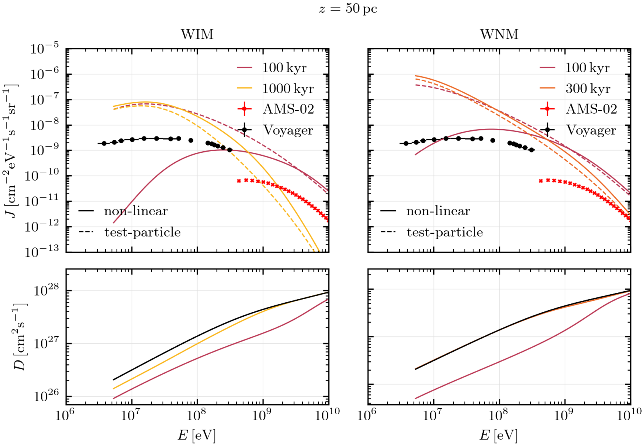
- Initial diffusion coefficient larger than Kraichnan
- WIM: suppression lasting less than $500\,\mathrm{kyr}$
- WNM: suppression lasting less than $300\,\mathrm{kyr}$
- Less effects on spectra and grammage



Spectral dependence Kolmogorov: Spectral break

H. Jacobs, P. Mertsch, M. Phan, *arXiv:2112.09708*

- Overpredict Voyager and AMS02 data
- Faster convergence to test-particle case
- Same spectral break at early times
- Need statistical approach



Non-linear transport equations

$$\partial_t f(z,p) + \partial_z \left(D_{zz}(z,p)\partial_z f(z,p)\right) + v_A \partial_z f(z,p) - \frac{p}{3}\frac{\mathrm{d}v_A}{\mathrm{d}z}\partial_p f(z,p) + \frac{1}{p^2}\partial_p\left(\dot{p}p^2 f(z,p)\right) = q_{\mathrm{CR}}(p)\theta(z - z_{\mathrm{min}})$$

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Improvements

1. Coulomb, ionisation, pion production losses
2. Non-linear cascade in wave-number
3. Adiabatic gains in turbulent power

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$$D_{zz}(z,p) \sim \frac{v r_g}{3} \left.\frac{B_0^2}{kW(k)}\right|_{k=1/r_g}$$

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$$\partial_t W(z,k) + \partial_k \left(D_{kk}\partial_k W(z,k)\right) + \partial_z (v_A W(z,k)) = q_W \delta(z)\delta(k - k_0) + \left(\Gamma_{\mathrm{CR}}(z,k) - \Gamma_{\mathrm{D}}\right) W(z,k)$$

Improvements

1. Coulomb, ionisation, pion production losses
2. Non-linear cascade in wave-number
3. Adiabatic gains in turbulent power

Alfvénic Diffusion

- Particles scatter on Alfvén waves

→ Diffusion $D(W, p) \propto 1/W(k_{res}(p))$

Resonant Streaming Instability

- Momentum transfer CR to waves

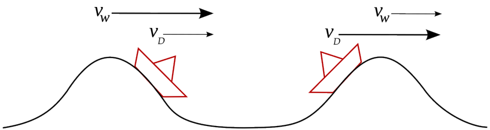
$$P_{CR} = -n_{CR} m_{CR} \gamma (v_D - v_A)$$

- Growth rate

$$\Gamma \propto \frac{v_D}{v_A}, \qquad v_D \propto D$$

$$\Gamma \propto \frac{1}{W} \left| p^4 \frac{\partial f}{\partial z} \right|_{p_{res}(k)}$$

Simplified picture



[1]

Wave Particle Interaction - Self Confinement

Alfvénic Diffusion

- Particles scatter on Alfvén waves

→ Diffusion $D(W, p) \propto 1/W(k_{res}(p))$

Resonant Streaming Instability

- Momentum transfer CR to waves

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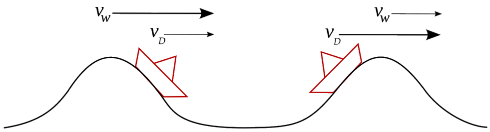
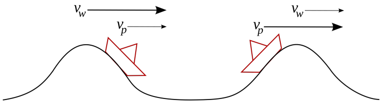


[1]

Non-Linear Landau Damping

- Linear Landau damping
  - $v_p < v_W$:
    $\rightarrow \; E_W \rightarrow E_p$
  - $v_p > v_W$:
    $\rightarrow \; E_p \rightarrow E_W$
  - Thermal background: $f(v_p < v_W) > f(v_p > v_W)$
    → Damping
- Non-Linear Landau damping
  - Beat of two waves
  - Same principles apply



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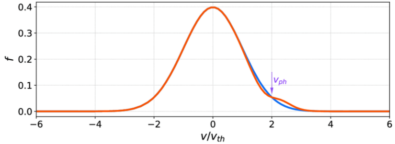

[2]

Damping rate

$$\Gamma_{NLLD}(z,k,t) = \sqrt{\frac{\pi}{2}} v_{th} k^2 W(z,k,t)$$

Non-Linear Landau Damping

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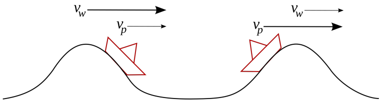

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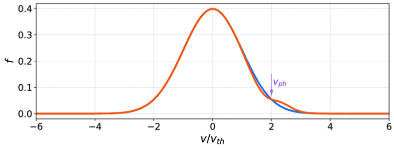

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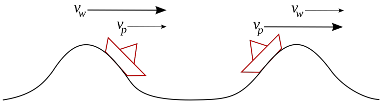

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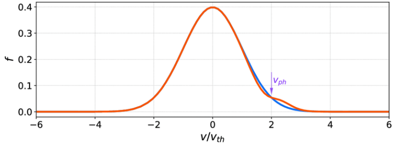

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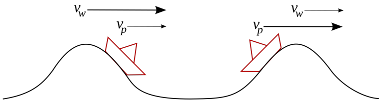

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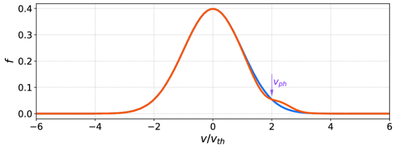

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Damping rate

$$\Gamma_{NLLD}(z,k,t) = \sqrt{\frac{\pi}{2}} v_{th} k^2 W(z,k,t)$$

Initial and boundary conditions

Boundary conditions

- symmetric around $z = 0$
- no advection at $z = 0$
- at $z = L$ 3D diffusion, fast

→ Reflecting BC at $z = 0$

→ $v_A = 0$ at $z = 0$

→ Free escape BC at $z = L$

Initial conditions

- SN with $E_{SN} = 10^{51}\,\mathrm{erg}$ and $M_{ej} = 1.4\,\mathrm{M}_{\odot}$
- $10\,\%$ into CR
- power law $\propto p^{\alpha}$ in momentum
- particles released at the beginning of the snowplow phase

References I

[1] W. Commons, *File:phys interp landau damp.png — wikimedia commons, the free media repository*, [Online; accessed 8-November-2020], 2005.

[2] P. Cagas, "Continuum kinetic simulations of plasma sheaths and instabilities", PhD thesis (Virginia Polytechnic Institute and State University, Sept. 2018).