Collective Effects in Supernova Neutrinos

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Quick Introduction

Supernova Neutrinos

- A core-collapse supernova happens (CCSN) is a star explosion that happens when a massive star ($M \gtrsim 8 M_\odot$) ends its nuclear fuel, collapsing into itself.
- In this process, a large number of neutrinos are emitted ($\sim 10^{53}$ erg) in a time window of about 10 seconds.

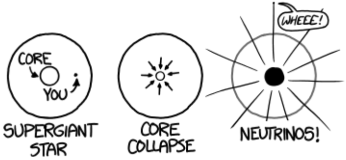


Figure: From `https://what-if.xkcd.com/73/`

- For supernovae happening in **our galaxy and its neighborhood ($\sim$ some per century)**, their neutrinos can be detected at the Earth, and these neutrinos could bring unique information about the neutrino physics and the supernova mechanism.

Forward Scattering Potentials

- Neutrino-Electron (MSW)

$$H_{\nu e} = \sqrt{2} G_F n_e$$

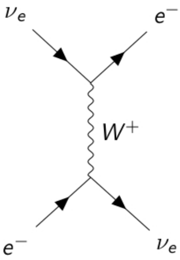


Figure: Forward scattering in electrons.

- Neutrino-Neutrino (Collective Effects) [1]

$$H_{\nu\nu,i} = \sqrt{2} G_F \sum_j (1 - \cos\theta_{ij})(n_{\nu,j}\rho_{\nu,j} - n_{\overline{\nu},j}\rho_{\overline{\nu},j})$$

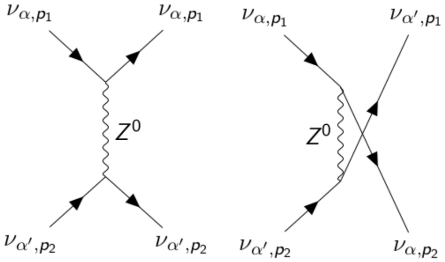


Figure: Forward scattering in neutrinos.

[1] G. Sigl and G. Raffelt, "General kinetic description of relativistic mixed neutrinos", Nucl. Phys. B **406**, 423–451 (1993), P. D. Neto and E. Kemp, "Neutrino–(anti)neutrino forward scattering potential for massive neutrinos at low energies", Mod. Phys. Lett. A **37**, 2250048 (2022)

Evolution Equation

- Considering the forward scattering potentials, the evolution equation for the i-th neutrino in the system becomes

$$i\frac{d}{dt}\rho_i = [\omega H_{vac} + \lambda H_{\nu e} + \mu H_{\nu\nu,i}, \rho_i], \qquad \rho_i \equiv |\psi_{\nu,i}(t)\rangle \langle\psi_{\nu,i}(t)|$$
$$\omega \equiv \frac{\Delta m^2}{2E_i}, \quad \lambda \equiv \sqrt{2}G_F n_e, \quad \mu \equiv \sqrt{2}G_F n_\nu \tag{1}$$

- Although it may appear simple at first glance, **there is no definitive solution to this problem in a supernova environment.**
- The main complications are:
  1. The nonlinear evolution, due to the $\nu - \nu$ interactions;
  2. The angular momenta distribution dependency in the term $\cos\theta_{ij} = \hat{p}_i \cdot \hat{p}_j$;
  3. The complicated geometry of a supernova.

Our Approach to the Problem

Polarization Vectors Formalism

- If we consider 2 families of neutrinos $\{\nu_e, \nu_x\}$, all the complex matrices in the evolution equation can be decomposed into the Pauli Matrices, such that the coefficients of expansions will work as components of a vector.

$$H_V = -\tfrac{1}{2}\vec{\sigma}\cdot\vec{B}, \quad H_{\nu e} = -\tfrac{1}{2}\vec{\sigma}\cdot\vec{L}, \quad \rho = \tfrac{1}{2}\mathbf{1} + \tfrac{1}{2}\vec{\sigma}\cdot\vec{P}, \quad \vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3) \tag{2}$$

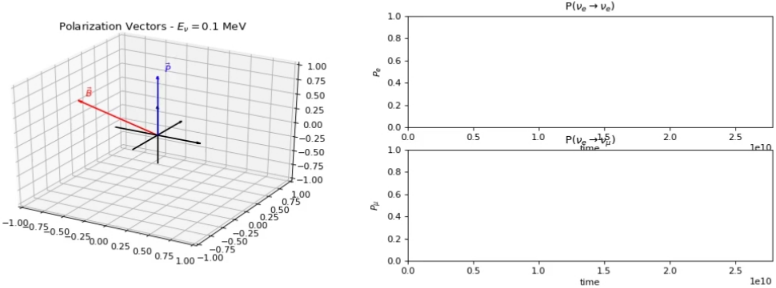
$$\tfrac{d}{dt}\vec{P}_{\nu_i} = \vec{P}_{\nu_i} \times \left[\omega\vec{B} + \lambda\vec{L} + \mu\textstyle\sum_j (1 - \cos\theta_{ij})(\vec{P}_{\nu,j} - \vec{P}_{\overline{\nu},j})\right] \tag{3}$$

- Here, the polarization vector $\vec{P}$ has information about the neutrino state, with its third component given the flavor content (ν_e or ν_x) in the flavor basis (P_3^F).

$$P_\beta(t) = \begin{cases} P_e(t) = \tfrac{1}{2}\left[1 + P_3^F(t)\right] \\ P_x(t) = \tfrac{1}{2}\left[1 - P_3^F(t)\right] \end{cases} \tag{4}$$

Example - Vacuum Oscillations ($\lambda = 0, \mu = 0$)

$$\frac{d}{dt}\vec{P}_{\nu_i} = \vec{P}_{\nu_i} \times \omega \vec{B} \tag{5}$$



Vacuum oscillation of a neutrino created as $|\nu(0)\rangle = |\nu_e\rangle$

Isotropic and Mono-energetic Neutrino Gas

Isotropic and Mono-energetic Neutrino Gas

- As a first approach in trying to solve neutrino evolution in a high-density neutrino environment, we consider a **mono-energetic and isotropic** ($\langle \cos\theta_{ij} \rangle = 0$) neutrino gas composed of electron neutrinos and antineutrinos $\{\nu_e, \overline{\nu}_e\}$.
- For simplicity, let us consider no matter potential (λ). For the relevant cases, the matter potential is almost constant over the collective regime, changing only the mixing angle to the one effective in matter.

$$\frac{d}{dt}\vec{P}_{\nu_i} = \vec{P}_{\nu_i} \times \left[\omega\vec{B} + +\mu(\vec{P}_\nu - \vec{P}_{\overline{\nu}})\right] \tag{6}$$

$$\frac{d}{dt}\vec{P}_{\overline{\nu}_i} = \vec{P}_{\overline{\nu}_i} \times \left[-\omega\vec{B} + \mu(\vec{P}_\nu - \vec{P}_{\overline{\nu}})\right] \tag{7}$$

- Here, $\vec{P}_\nu \equiv \sum_i \vec{P}_{\nu_i}$ and $\vec{P}_{\overline{\nu}} \equiv \sum_i \vec{P}_{\overline{\nu}_i}$ represent the entire ensemble of neutrinos and antineutrinos, respectively.

The Pendulum Analogy

- It remarkable that the equations for this system is **equivalent to a pendulum**.
- To see this, let us define the vectors $\vec{D} = \vec{P}_\nu - \vec{P}_{\overline{\nu}}$ (difference), $\vec{S} = \vec{P}_\nu + \vec{P}_{\overline{\nu}}$ (sum) and $\vec{Q} = \vec{S} - \frac{\omega}{\mu}\vec{B}$, so that $\vec{Q} \approx \vec{S}$ for $\mu \gg \omega$ and we can rewrite the equations as[2]:

Neutrino Equations

$$\dot{\vec{Q}} = \mu\vec{D} \times \vec{Q} \tag{8a}$$

$$\dot{\vec{D}} = \omega\vec{Q} \times \vec{B} \tag{8b}$$

Pendulum Equations

$$I\dot{\vec{r}} = \vec{L} \times \vec{r} \tag{9a}$$

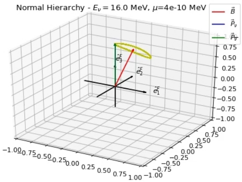
$$\dot{\vec{L}} = \vec{\tau} = \vec{r} \times \vec{F} \tag{9b}$$

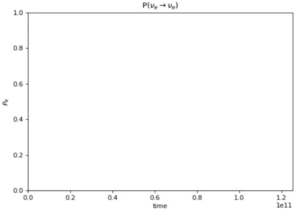
- The neutrino system works as a pendulum attracted by a **force field** $\vec{F} = \omega\vec{B}$, with **angular momentum** $\vec{L} = \vec{D}$, **length** $\vec{r} = \vec{Q}$, and **moment of inertia** $I = m|\vec{r}|^2 = \mu^{-1}$

[2] S. Hannestad et al., "Self-induced conversion in dense neutrino gases: Pendulum in flavour space", Phys. Rev. D **74**, [Erratum: Phys.Rev.D 76, 029901 (2007)], 105010 (2006).

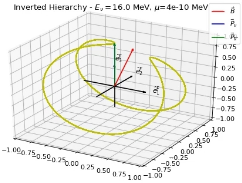
1^{st} Scenario - Symmetric and Constant μ

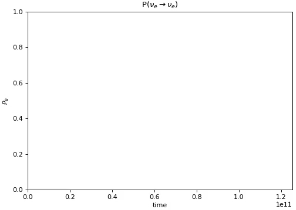
- **Normal Hierarchy ($\Delta m^2 > 0$):** The system is attracted by $\vec{F} = |\omega|\vec{B}$





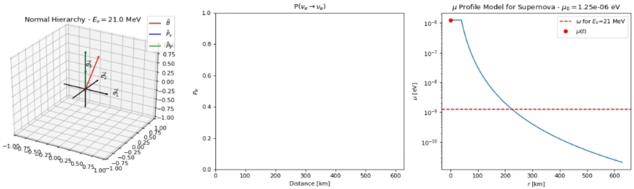
- **Inverted Hierarchy ($\Delta m^2 < 0$):** The system is attracted by $\vec{F} = -|\omega|\vec{B}$



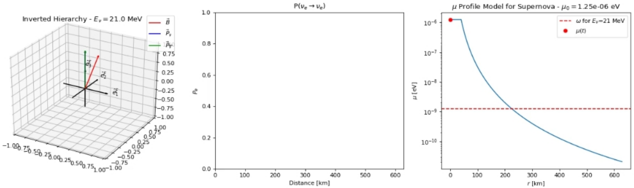


2^{nd} Scenario - Symmetric and Decreasing μ

- **Normal Hierarchy:** As $I = \mu^{-1}$ increases, the oscillation is damped towards $\vec{F} = |\omega|\vec{B}$.

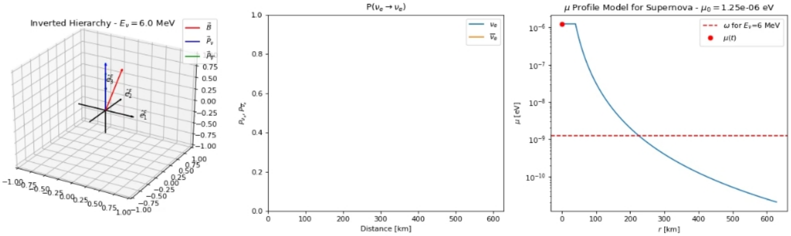


- **Inverted Hierarchy:** As $I = \mu^{-1}$ increase, the oscillation is damped towards $\vec{F} = -|\omega|\vec{B}$



3rd Scenario - Asymmetric and Decreasing μ

- From the equations of motion, $\vec{D} \cdot \vec{B}$ is conserved, which means that **the initial difference of eigenstates is conserved**. If the mixing angle is small $\theta << 1$, which is the case for the supernova environment due to the high density of matter, this is equivalent to **conservation of net lepton number**.[3]
- **Inverted Hierarchy:** Asymmetry of $N_{\overline{\nu}_e}/N_{\nu_e} = 0.8$.



[3] A. Mirizzi et al., "Supernova Neutrinos: Production, Oscillations and Detection", Riv. Nuovo Cim. **39**, 1–112 (2016).

Isotropic Neutrino Gas with Spectral Distribution

Isotropic Neutrino Gas with Spectral Distribution

- With a spectral distribution, we can define the polarization vector $\vec{P}_{\nu,\vec{p}_1}$ for each momentum $\vec{p}_1$ (or momentum interval). Defining $\vec{D} = \sum_{\vec{p}_2} \vec{D}_{\vec{p}_2} = \sum_{\vec{p}_2}(\vec{P}_{\nu,\vec{p}_2} - \vec{P}_{\overline{\nu},\vec{p}_2})$ we may write[4]:

$$\dot{\vec{P}}_{\nu,\vec{p}_1} = \vec{P}_{\nu,\vec{p}_1} \times \left[\omega_{\vec{p}_1}\vec{B} + \mu\vec{D}\right] \tag{10a}$$

$$\dot{\vec{P}}_{\overline{\nu},\vec{p}_1} = \vec{P}_{\overline{\nu},\vec{p}_1} \times \left[-\omega_{\vec{p}_1}\vec{B} + \mu\vec{D}\right] \tag{10b}$$

$$\dot{\vec{S}}_{\vec{p}_1} = \omega\vec{D}_{\vec{p}_1} \times \vec{B} + \mu\vec{D} \times \vec{S}_{\vec{p}_1} \tag{11a}$$

$$\dot{\vec{D}}_{\vec{p}_1} = \omega\vec{S}_{\vec{p}_1} \times \vec{B} \tag{11b}$$

- If $\mu \gg \omega_{\vec{p}_1}$ for all modes, all $\vec{S}_{\vec{p}_1}$ evolve in a similar way, resulting in an evolution similar to the mono-energetic case for each mode.

[4] S. Hannestad et al., "Self-induced conversion in dense neutrino gases: Pendulum in flavour space", Phys. Rev. D **74**, [Erratum: Phys.Rev.D 76, 029901 (2007)], 105010 (2006).

Isotropic Neutrino Gas with Spectral Distribution - Spectral Split

- When considering a decreasing μ, as in a supernova, due to the **net lepton number conservation**, only a fraction of the spectrum can convert its flavor, as given by the following equation:

$$\int_{E_{split}}^{\infty} dE\,[\phi_{\nu_e}(E) - \phi_{\nu_x}(E)] = \int_0^{\infty} dE\,[\phi_{\overline{\nu}_e}(E) - \phi_{\overline{\nu}_x}(E)]\,, \tag{12}$$

- This leads to the phenomenon of **spectral split**, in which there is conversion above certain energy E_{split}, but not bellow it.

Isotropic Neutrino Gas with Spectral Distribution - Numerical Results

- Considering a supernova spectrum, we had the following results:

Normal Hierarchy

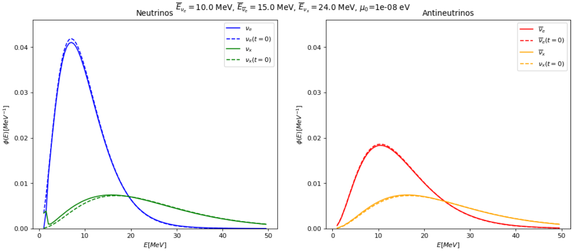


Figure: Initial and final spectrum for the isotropic neutrino gas (NH).

Inverted Hierarchy

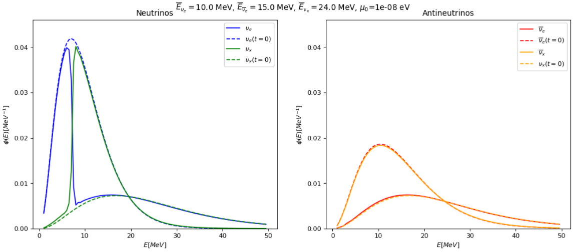


Figure: Initial and final spectrum for the isotropic neutrino gas (IH).

Bulb Model Single Angle Approximation

Bulb Model

- The Bulb model consists of considering that the neutrinos of a supernova are emitted from the surface of a sphere, as light from a bulb.

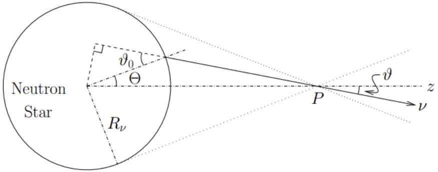


Figure: Bulb model of emission.[5]

- In this case, there is an **azimuthal symmetry** around the z axis, so that the neutrino-neutrino potential can be integrated in this angle.

$$H_{\nu\nu} = \sqrt{2} G_F 2\pi \sum_\alpha \int (1 - \cos\vartheta \cos\vartheta')[\rho_\alpha(p', \vartheta) - \bar{\rho}_\alpha(p', \vartheta)] dp' d\cos\vartheta' \qquad (13)$$

[5] H. Duan et al., "Simulation of Coherent Non-Linear Neutrino Flavor Transformation in the Supernova Environment. 1. Correlated Neutrino Trajectories", Phys. Rev. D **74**, 105014 (2006)

Bulb Model - Single-Angle Approximation

- A further approximation can be made by considering that a single angle ϑ is a good representative of all the other possible trajectories.

$$H_{\nu\nu} = \sqrt{2} G_F 2\pi D(r) \sum_{\alpha} \int [\rho_\alpha(p') - \overline{\rho}_\alpha(p')] dp' \tag{14}$$

- In this case, **the potential is identical to the isotropic case**, with the following geometric factor for $\vartheta = 0$:

$$D(r) = \frac{1}{2}(1 - \cos\vartheta_{\text{max}})^2 = \frac{1}{2}\left[1 - \sqrt{1 - \left(\frac{R_\nu}{r}\right)^2}\right]^2 \tag{15}$$

Bulb Model - Single-Angle Approximation - Numerical Results

- As expected, we have the same results of the isotropic scenario, with the phenomenon of spectral split.

Inverted Hierarchy

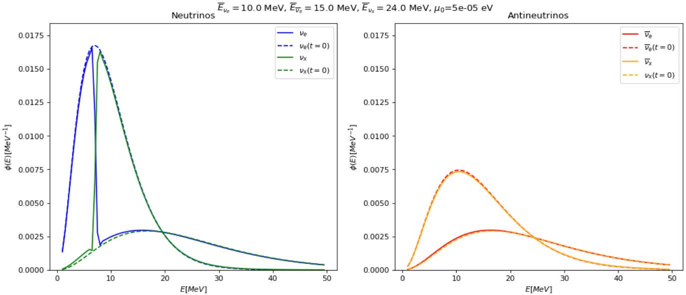


Figure: Initial and final spectrum for the Bulb model with the Single-Angle approximation (IH).

Open-Source Code

Open-Source Code

- When working with **neutrino collective effects**, it is **difficult (almost impossible) to find papers with open-source code** to verify and **reproduce the results**.
- With that in mind, our code was made available and can be found in the following repository: `https://github.com/pedrodedin/Neutrino-Collective-Effects.git`
- We hope that this may help newcomers to understand and reproduce our results without the need to ”reinvent the well“.

Conclusions

Conclusions

- The neutrino flavor evolution is still an open problem in a supernova environment due to neutrino collective effects.
- However, solutions can be found for simpler systems, such as an isotropic gas and the Bulb model.
- We saw that the polarization vector formalism is extremely powerful, with a geometric visualization and classical analogs (Pendulum).
- As next steps, we intend to explore the following scenarios:
    - Multi-Angle emission in the Bulb model;
    - Non-uniform emission in the Bulb model;
    - Fast oscillations due to electron number crossing in the angular distribution.

Thank you!

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References I

[1] G. Sigl and G. Raffelt, "General kinetic description of relativistic mixed neutrinos", Nucl. Phys. B **406**, 423–451 (1993).

[2] P. D. Neto and E. Kemp, "Neutrino–(anti)neutrino forward scattering potential for massive neutrinos at low energies", Mod. Phys. Lett. A **37**, 2250048 (2022).

[3] S. Hannestad, G. G. Raffelt, G. Sigl, and Y. Y. Y. Wong, "Self-induced conversion in dense neutrino gases: Pendulum in flavour space", Phys. Rev. D **74**, [Erratum: Phys.Rev.D 76, 029901 (2007)], 105010 (2006).

[4] A. Mirizzi, I. Tamborra, H.-T. Janka, N. Saviano, K. Scholberg, R. Bollig, L. Hudepohl, and S. Chakraborty, "Supernova Neutrinos: Production, Oscillations and Detection", Riv. Nuovo Cim. **39**, 1–112 (2016).

[5] H. Duan, G. M. Fuller, J. Carlson, and Y.-Z. Qian, "Simulation of Coherent Non-Linear Neutrino Flavor Transformation in the Supernova Environment. 1. Correlated Neutrino Trajectories", Phys. Rev. D **74**, 105014 (2006).