Alessandro Alberto Trani

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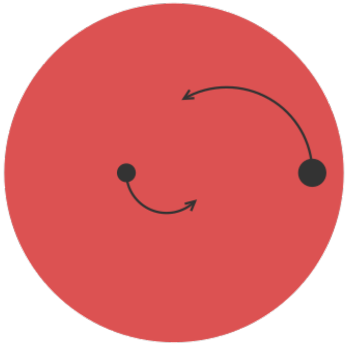
Revisiting Common Envelope Evolution

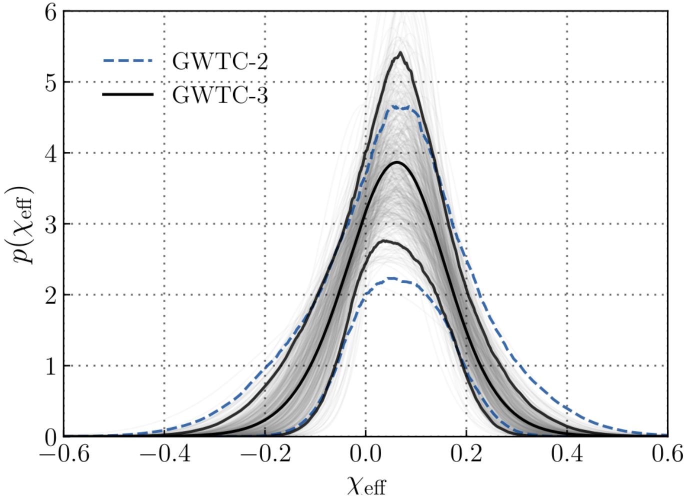
A New Semi-Analytic Model for N-body and Population Synthesis Codes



The University of Tokyo

Okinawa Institute of Science and Technology

(not) Triple interactions for gravitational waves



1. Three-body encounters in stellar clusters

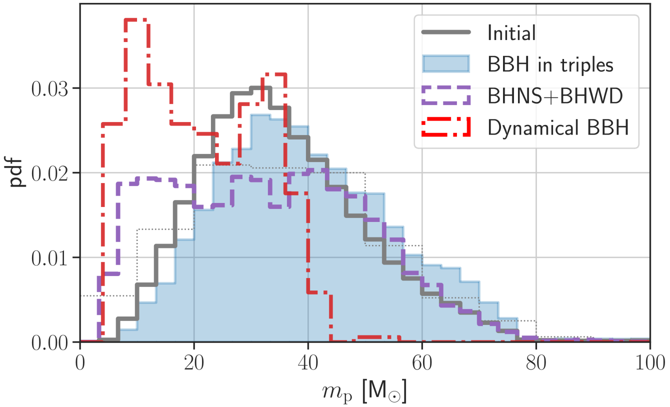
implications for spin-orbit misalignment in BH binaries

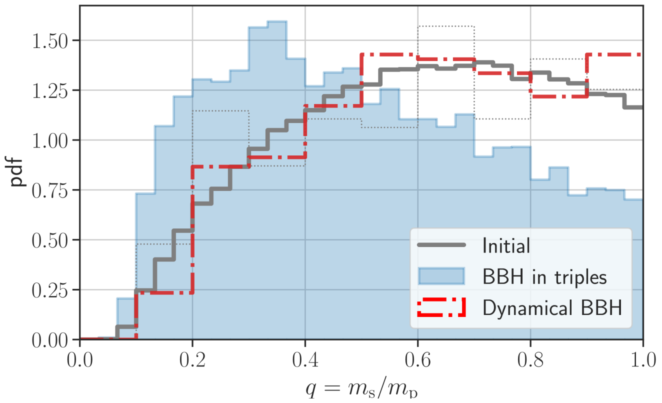
arXiv:2102.01689

2. Hierarchical triple systems from low mass clusters

properties of compact object mergers

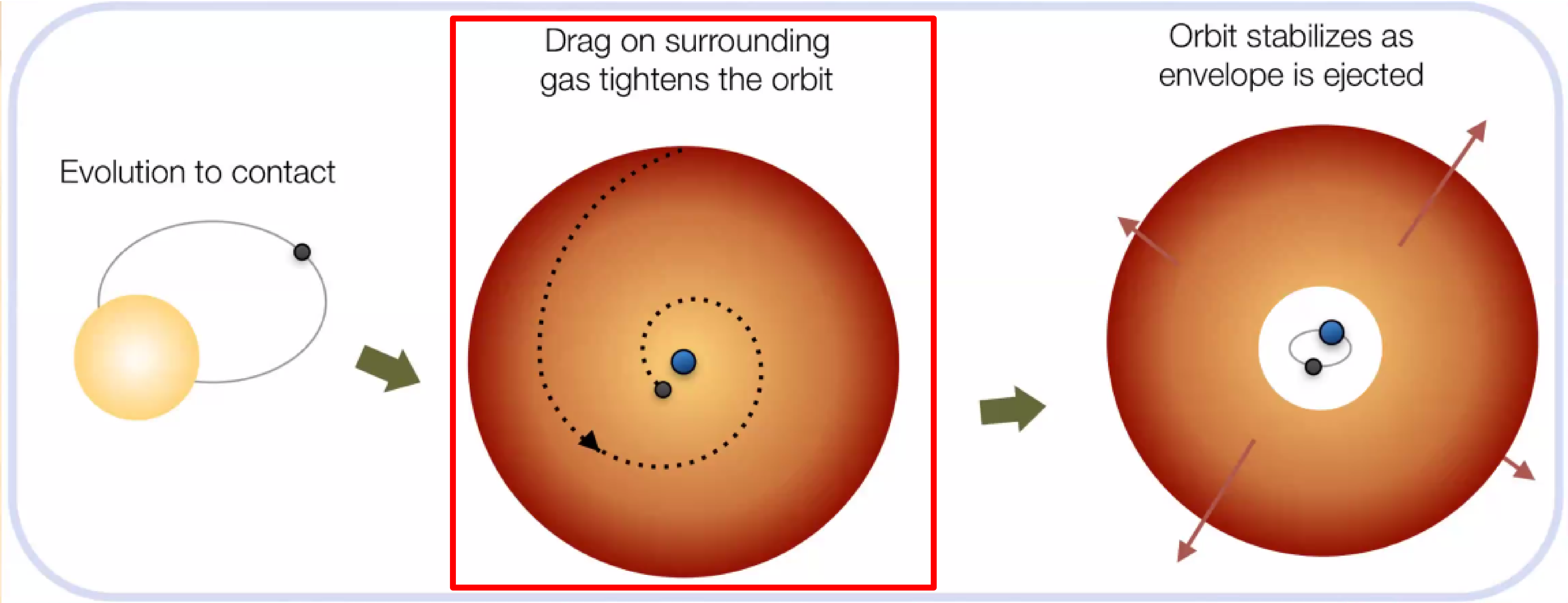
arXiv:2111.06388





WHAT IS COMMON ENVELOPE?

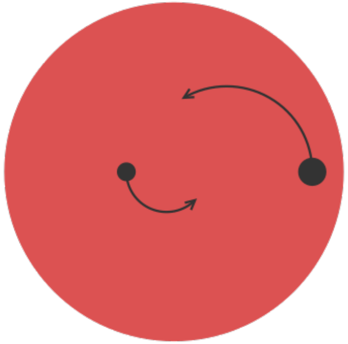
Evolutionary phase in interacting stellar binary systems



BOTH STARS ARE ENGULFED IN A COMMON STELLAR ENVELOPE

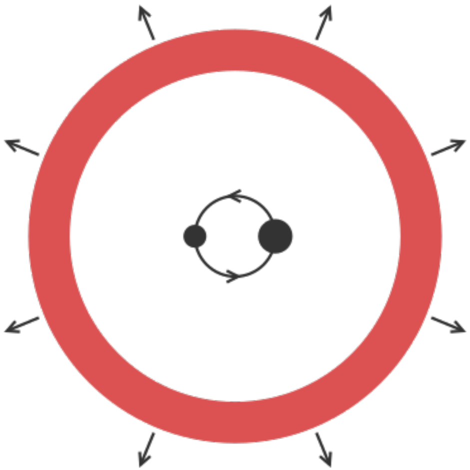
WHAT HAPPENS NEXT?

Envelope exerts a drag on both stars: spiral-in begins



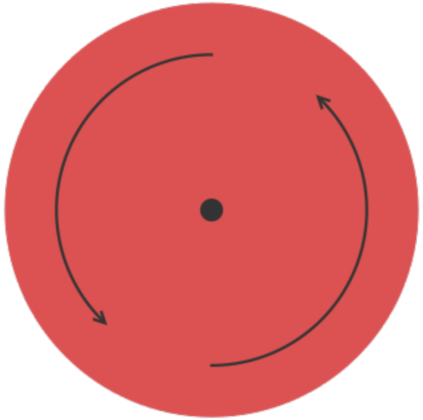
Two possible outcomes

Drag forces heat the envelope up until it unbinds, a more compact binary is formed



Energetically: orbital energy is used up to heat and unbind the envelope

The envelope does not unbind in time: spiral-in continues until the two cores merge into a single object



WHY COMMON ENVELOPE IS IMPORTANT?

IT MAKES BLACK HOLES MERGE WITHIN A HUBBLE TIME

The main process to make isolated binaries shrink so that they can coalesce via gravitational waves

Also to explain:
- Type Ia supernovae,
- X-ray binaries
- double neutron stars

+ other phenomena (e.g. optical transients)

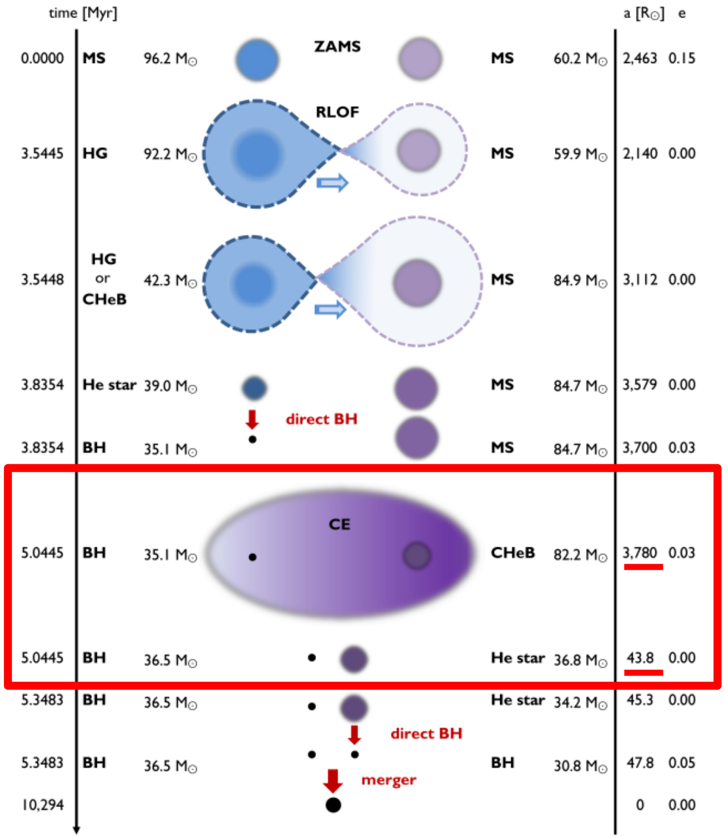
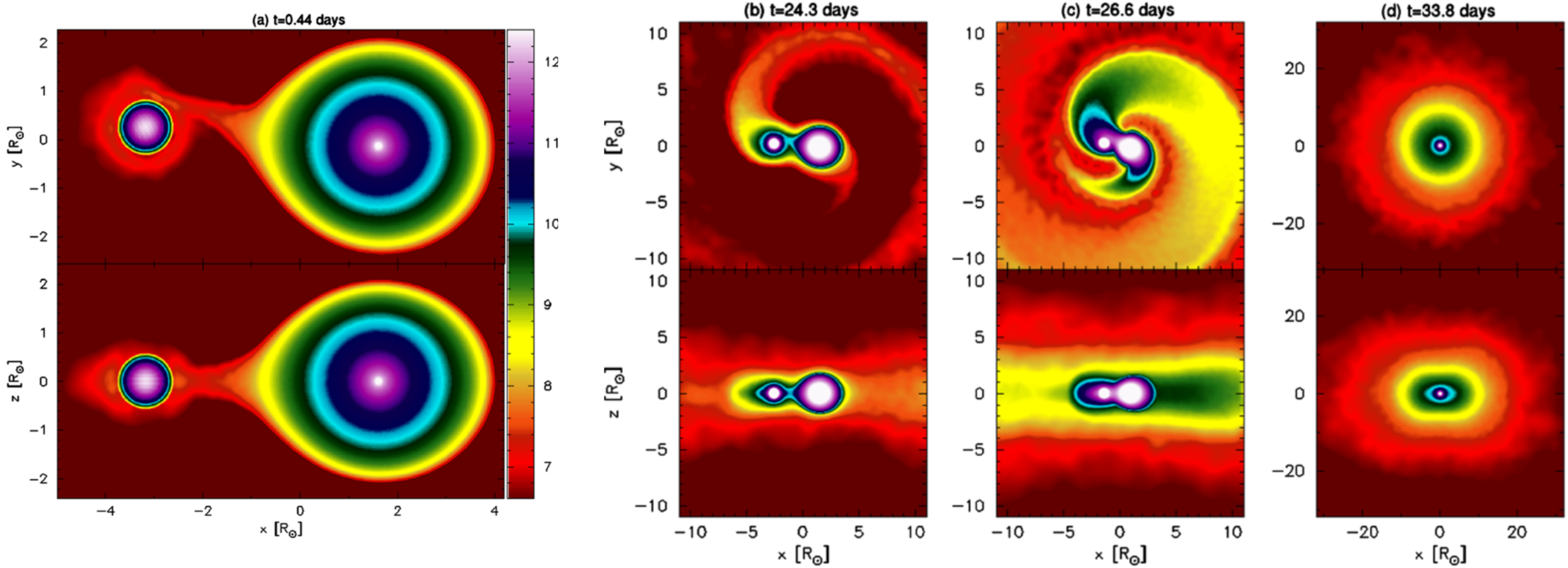


Figure 1: **Example binary evolution leading to a BH-BH merger similar to GW150914.** A

HOW IS THIS MODELED?

Hydrodynamic simulations

e.g. Lombardi et al. 2011



Very detailed BUT

- Numerically expensive
- Cannot follow the entire CE evolution
- Cannot model the stellar and orbital response properly

HOW IS COMMON ENVELOPE MODELED?

2. Parametrized models

e.g. Webbink 1984, de Kool 1990

Compare **envelope binding energy** vs **orbital energy** to estimate the CE energy loss

Energy final orbit = Energy initial orbit – Envelope Binding energy

$$\frac{m_1 m_{1,\mathrm{env}}}{\lambda R_1} = \alpha_{\mathrm{CE}}\left(-\frac{G m_1 m_2}{2 a_{\mathrm{i}}} + \frac{G m_{1,\mathrm{c}} m_2}{2 a_{\mathrm{f}}}\right)$$

- ✔ Fast
- ✔ Easy to fit to stellar models

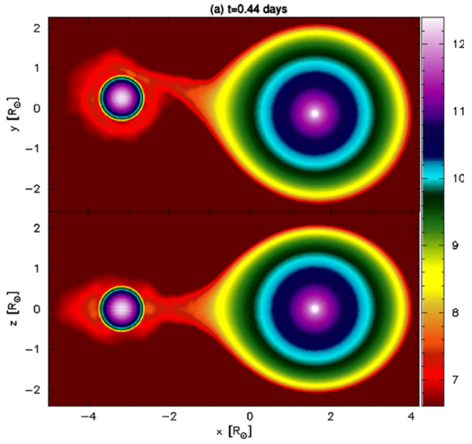
- ✗ Misses information on angular momentum:
- ✗ Instantaneous change of orbital parameters

"quantum jump"

COMMON ENVELOPE MODELS

Can model complex physics

Hydrodynamical simulations



my idea:
a new semi-analytic model for CE evolution

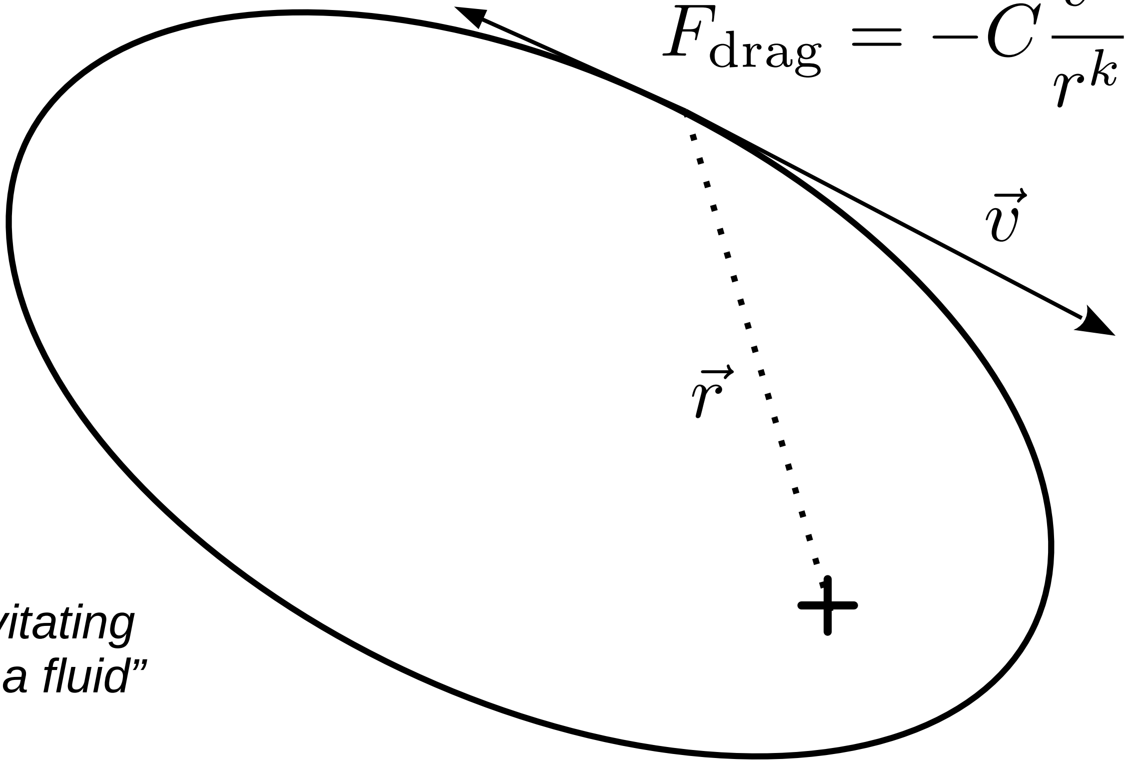
Parametrized model

Computationally inexpensive

2-BODY PROBLEM + PERTURBATIVE FORCE

Let’s imagine that the two stellar cores are orbiting in the envelope medium, which exerts a drag force

$$\vec{F}_{\mathrm{drag}} = -C\,\frac{v^l}{r^k}\,\hat{v}$$



k,l arbitrary exponents *“effective” force*

or the “two self-gravitating bodies immersed in a fluid” problem

Perturbation theory to derive the changes in orbital parameters

rate of change in semimajor axis

rate of change in eccentricity

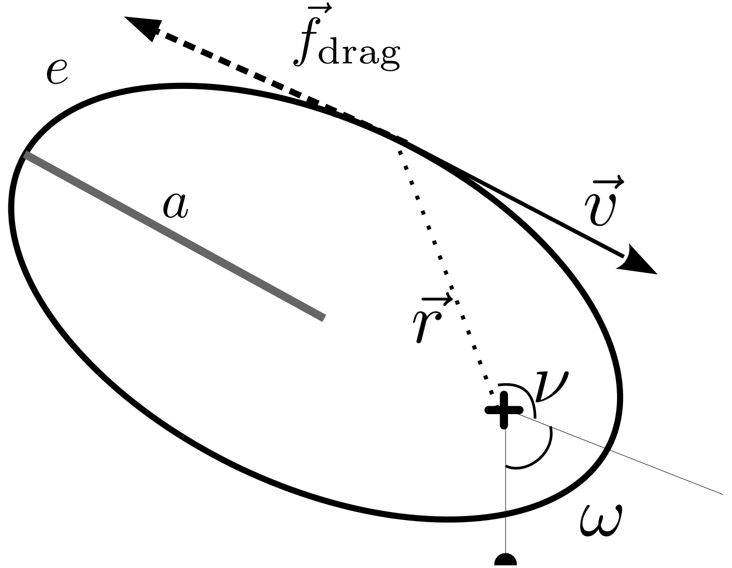
apsidal precession

$$\dot{a} = 2\,\mu^{\frac{l-1}{2}}\,a^{\frac{3-l-2k}{2}}\,(1-e^2)^{-\frac{l+1+2k}{2}}\,(1+e^2+2e\cos\nu)^{\frac{l+1}{2}}\,(1+e\cos\nu)^k \tag{1}$$

$$\dot{e} = 2(1-e^2)^{-\frac{l-1+2k}{2}}\,\mu^{\frac{l-1}{2}}\,a^{\frac{1-l-2k}{2}}\,(1+e\cos\nu)^k\,(1+e^2+2e\cos\nu)^{\frac{l-1}{2}}\,(e+\cos\nu) \tag{2}$$

$$\dot{\omega} = 2\mu^{\frac{l-1}{2}}\,a^{-\frac{l-1+2k}{2}}\,\frac{(1-e^2)^{-\frac{l-1+2k}{2}}}{e}\,(1+e\cos\nu)^k\,(1+e^2+2e\cos\nu)^{\frac{l-1}{2}}\,\sin\nu \tag{3}$$

$$\dot{\nu} = \frac{(1+e\cos\nu)^2}{(1-e^2)^{3/2}}\sqrt{\frac{\mu}{a^3}} - \dot{\omega} \tag{4}$$



Interesting forms:

$$\vec{f} = -C\,\frac{v^l}{r^k}\,\hat{v}$$

$l = 1$ drag force linear with velocity

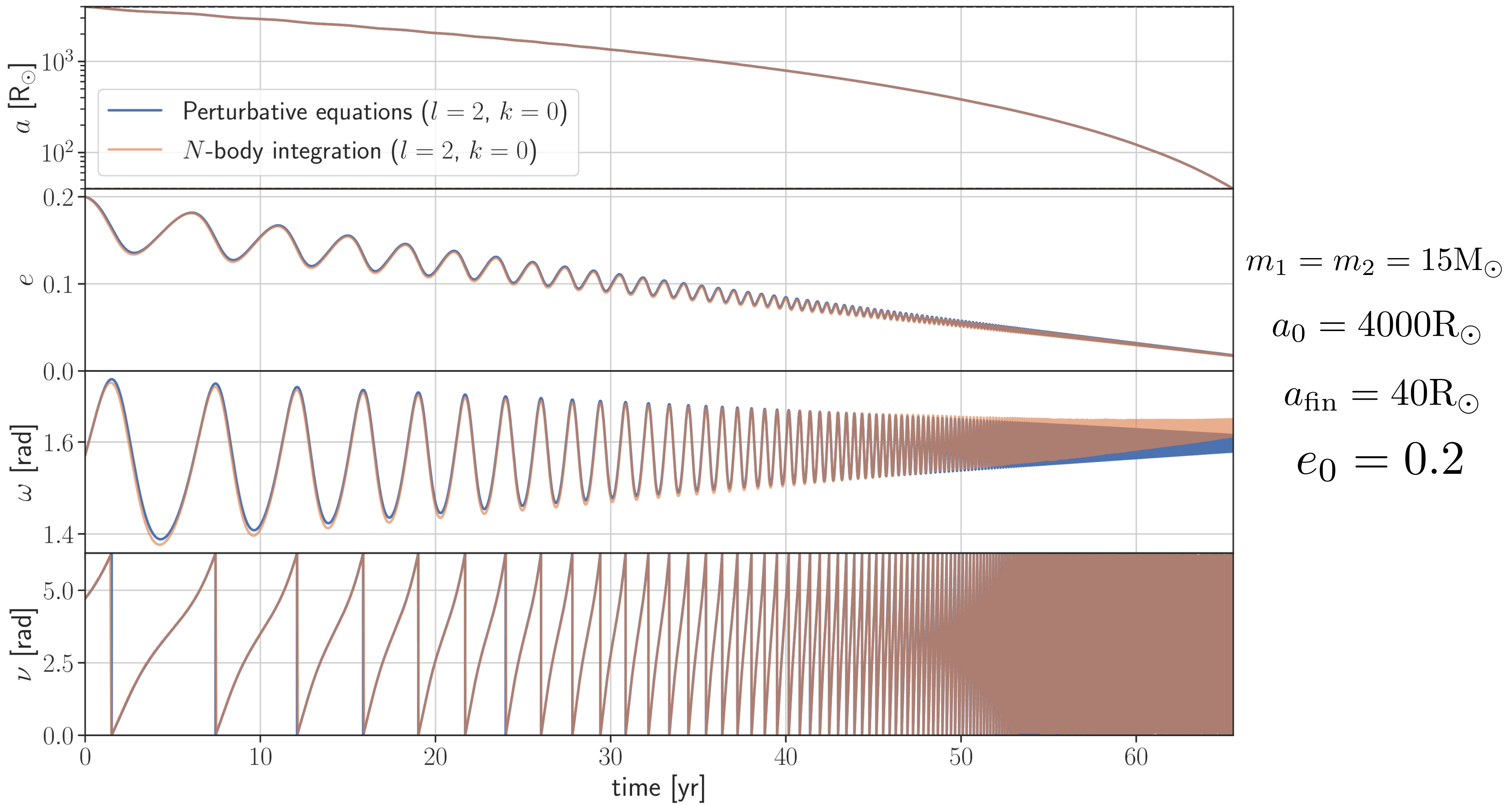
$l = 2$ drag force quadratic with velocity

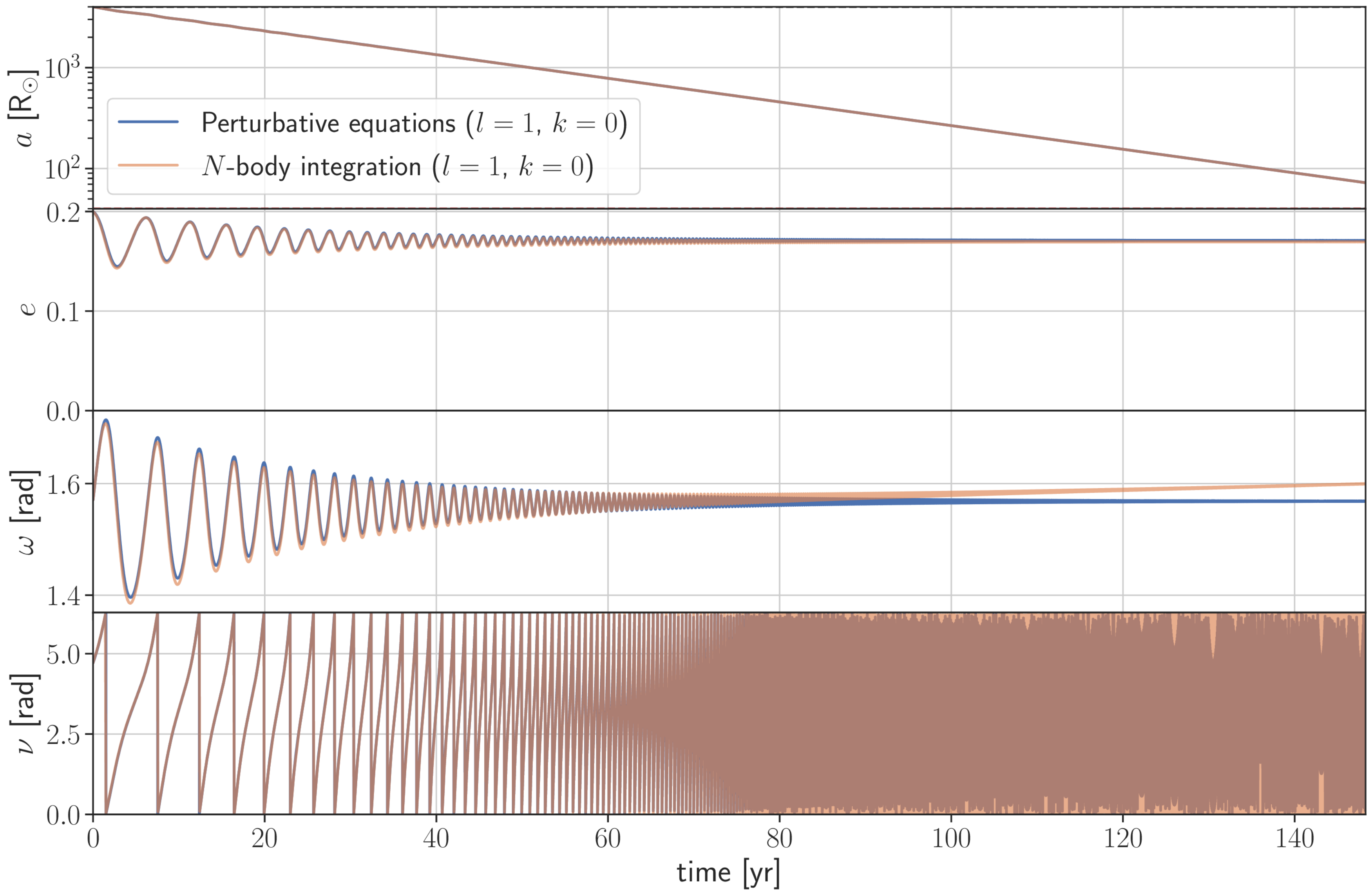
$k \geq 0$ some degree of radial dependency

$l = -2$ dynamical friction in a homogeneous infinite medium

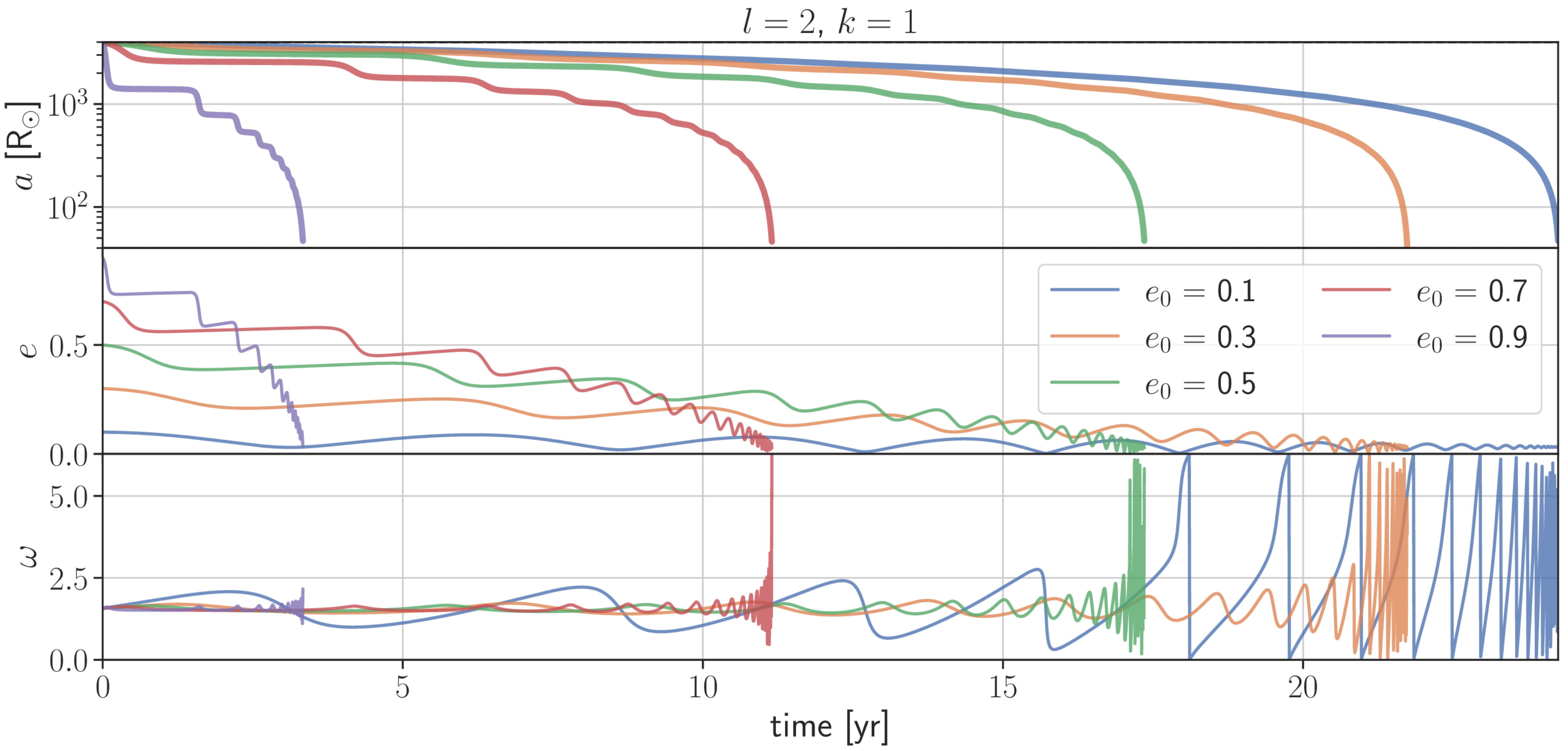
$$F_{\mathrm{drag}} = \frac{1}{2}\underline{\rho}\, v^2\, C_d\, \underline{A}$$

r^{-k} encodes the radial dependency of background density and cross-sectional area

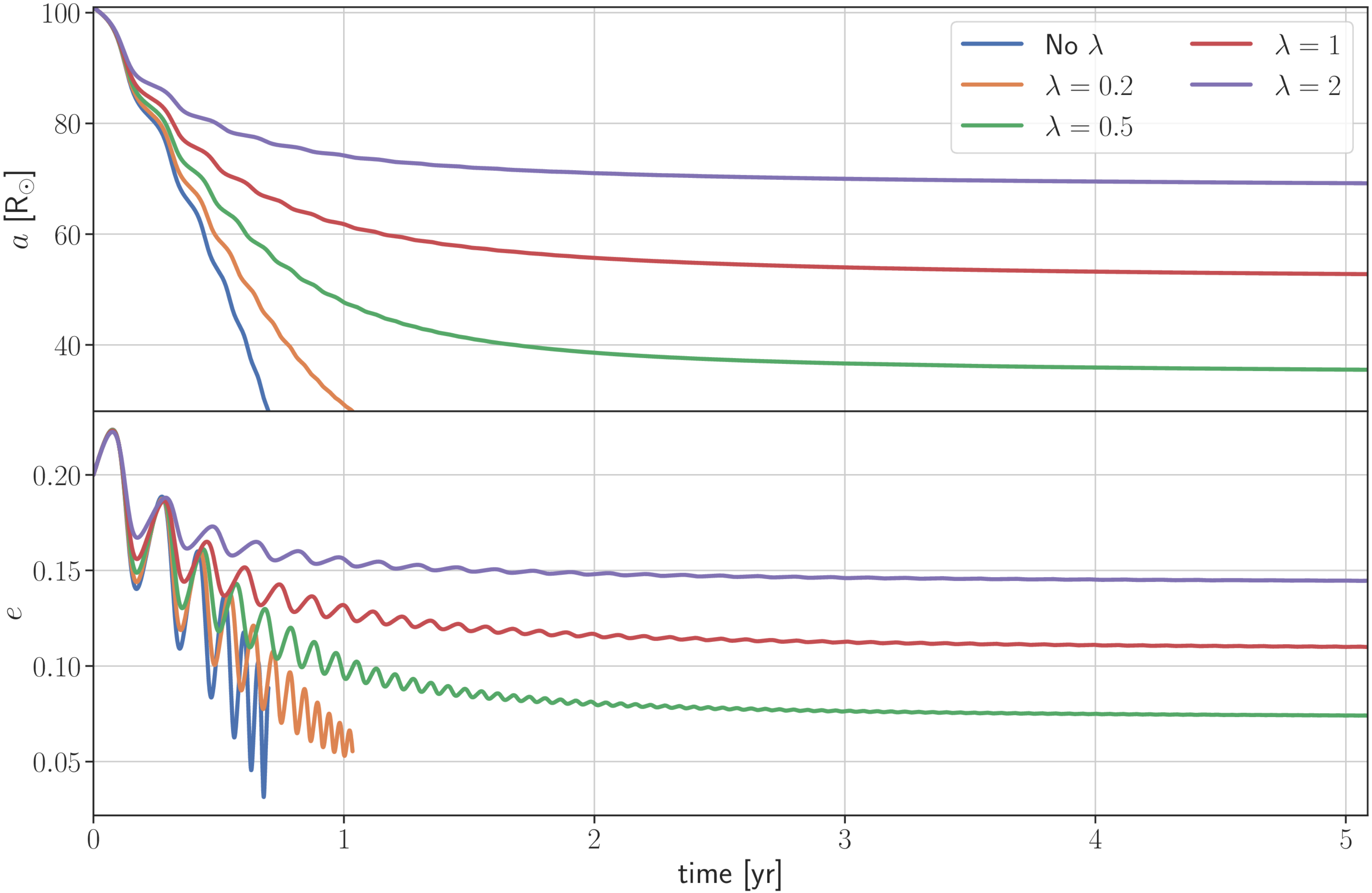

Perturbative equations ($l = 2$, $k = 0$)
N-body integration ($l = 2$, $k = 0$)
a [$R_\odot$]
10^3
10^2
e
0.2
0.1
0.0
ω [rad]
1.6
1.4
ν [rad]
5.0
2.5
0.0
0
10
20
30
40
50
60
time [yr]
$m_1 = m_2 = 15M_\odot$
$a_0 = 4000R_\odot$
$a_{\rm fin} = 40R_\odot$
$e_0 = 0.2$



a [R$_\odot$]
10^3
10^2
Perturbative equations ($l = 1$, $k = 0$)
N-body integration ($l = 1$, $k = 0$)
0.2
e 0.1
0.0
ω [rad]
1.6
1.4
ν [rad]
5.0
2.5
0.0
0
20
40
60
80
100
120
140
time [yr]



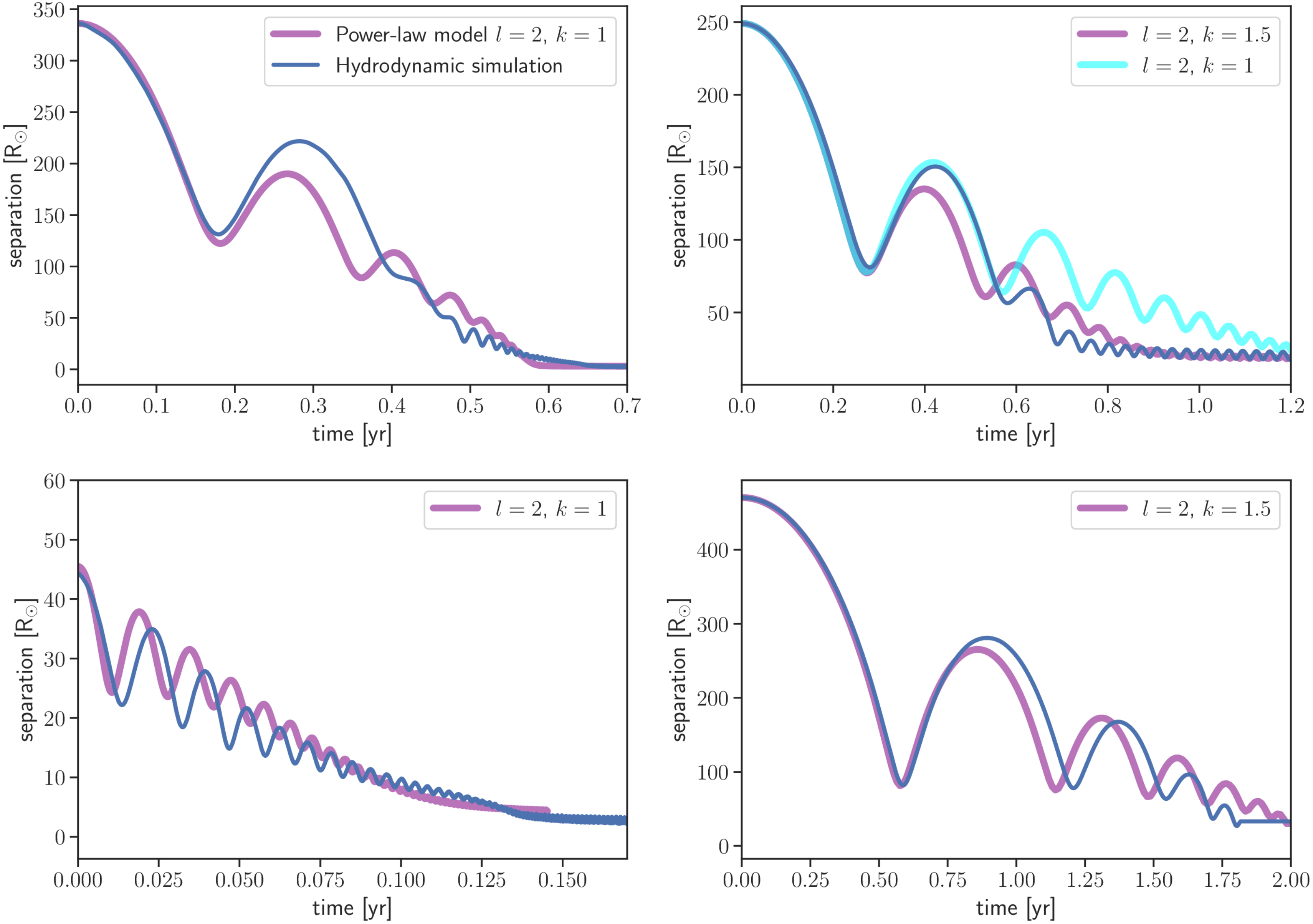
$l = 2,\ k = 1$
a [R$_\odot$]
10^3
10^2
e
0.0
0.5
$e_0 = 0.1$
$e_0 = 0.3$
$e_0 = 0.5$
$e_0 = 0.7$
$e_0 = 0.9$
ω
0.0
2.5
5.0
0
5
10
15
20
time [yr]



Self-limiting CE

Homologous expansion of the envelope

Given envelope binding energy (λ)



Comparison

orbital separation as function of time

semi-analytic model

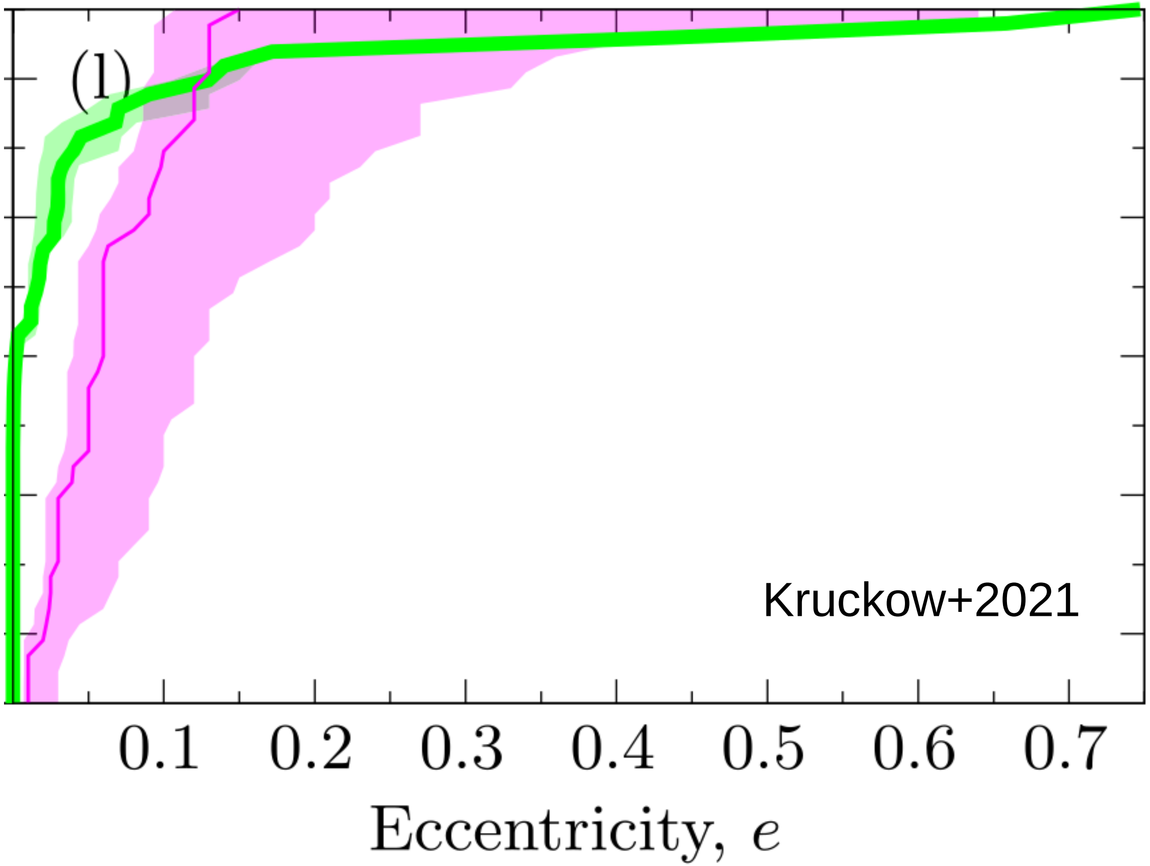
vs

hydrodynamic simulations

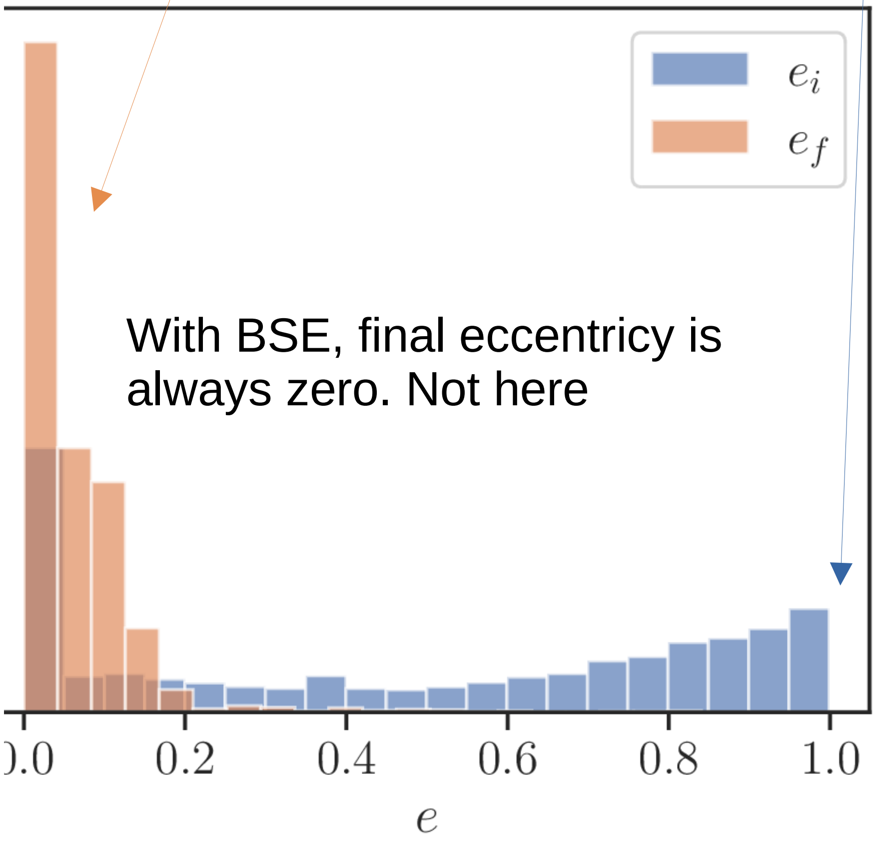
(Glanz & Perets 2021)

2600 CE events from **binary population synthesis** (Tanikawa+2020)

Final semimajor axis   Initial semimajor axis

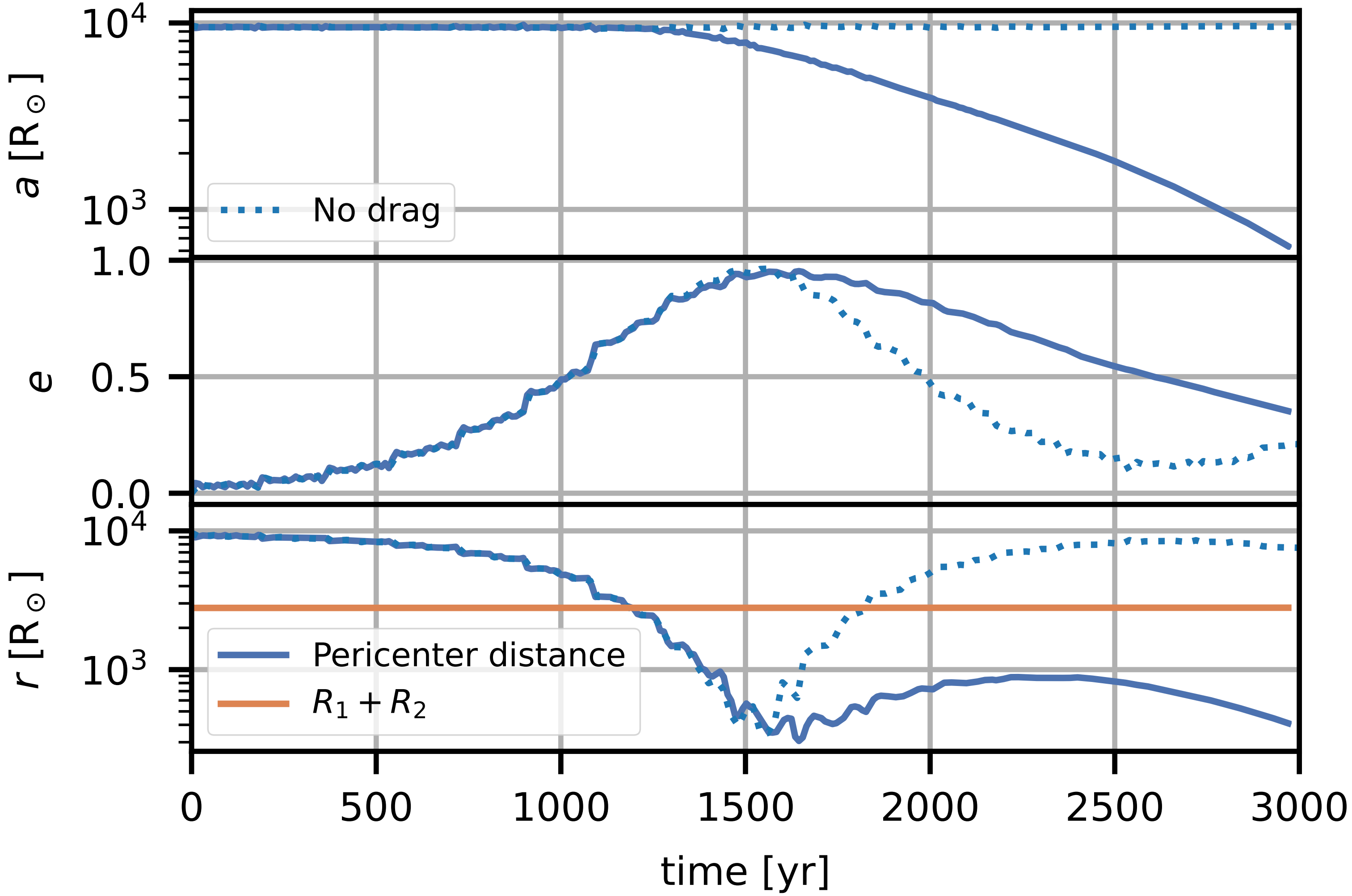


Final eccentricity   Initial eccentricity



With BSE, final eccentricity is always zero. Not here

CE inspiral triggered by von Zeipel-Kozai-Lidov evolution in triple systems



New semi-analytic, descriptive model for common envelope evolution

✔ Avoids “quantum” orbit jumps

✔ Gives information about the final eccentricity

✔ Can be made consistent with the α-λ model…

✔ …or can be alternative to the α-~~λ~~ model

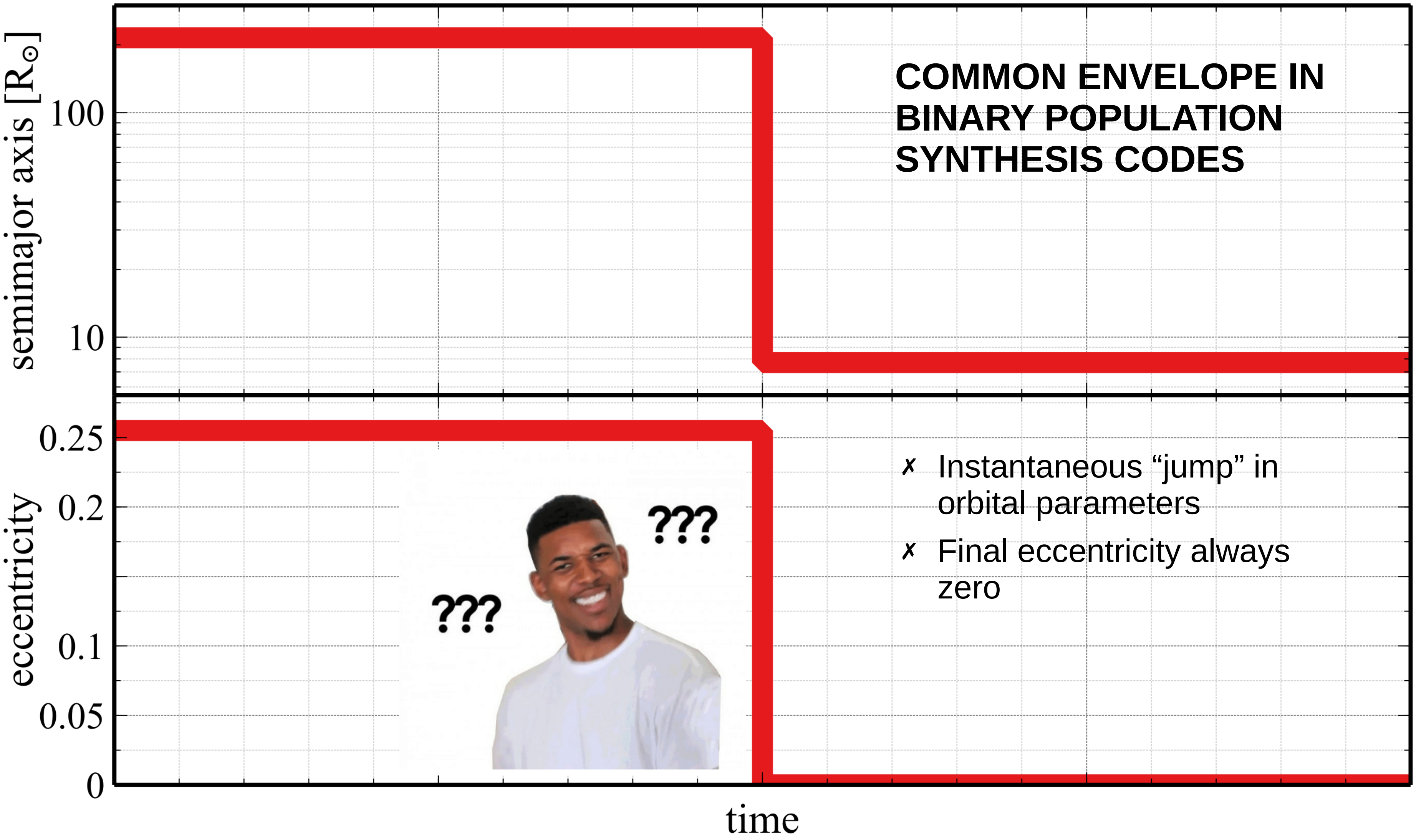
Can be used both in

- Binary population synthesis codes (perturbation theory approach)
- N-body codes, with direct integration (current implementation in AMUSE)

arXiv:2205.13537

Revisiting Common Envelope Evolution -- A New Semi-Analytic Model for N-body and Population Synthesis Codes

Alessandro Alberto Trani, Steven Rieder, Ataru Tanikawa, Giuliano Iorio, Riccardo Martini, Georgii Karelin, Hila Glanz, Simon Portegies Zwart


COMMON ENVELOPE IN BINARY POPULATION SYNTHESIS CODES
✗ Instantaneous “jump” in orbital parameters
✗ Final eccentricity always zero
???
???
semimajor axis [$R_\odot$]
100
10
eccentricity
0.25
0.2
0.1
0.05
0
time

- ✗ Misses information on angular momentum:
  - ➢ what about eccentricity evolution?
- ✗ Instantaneous change of orbital parameters
  - ➢ creates problems when combined with continuous derivatives / when extending BSPS to multiple systems

CAN WE MAKE A MODEL THAT CAN OVERCOME THESE ISSUES?

A model that can:

- Follow the inspiral phase as a function of time – no orbital quantum jumps
- Easy to incorporate in binary stellar evolution codes (as derivative of orbital parameters) BUT also in N-body codes (as a perturbative force)
- Can still reproduce the outcome of the α-λ model

LET'S START WITH A SIMPLE DRAG-FORCE MODEL

$$\vec{f} = -C\,\frac{v^l}{r^k}\hat{v}$$

C has physical dimensions $[C] = L^{1-l+k}T^{l-2}$

and l, k are real numbers that set the drag force dependence on (relative) velocity and distance

Physically, what is a drag force?

$$F_{\rm drag} = \frac{1}{2}\rho\, v^2\, C_d\, A$$

Cross section area $\equiv 4\pi R_*^2$

Drag coefficient.
Depends on the Reynolds coefficient $R_e = \frac{v R_*}{\nu}$

kinematic viscosity

Fluid density

Relative velocity with the fluid

1) $R_e < 1 \;\Rightarrow\; C_d \propto R_e^{-1} \propto v^{-1} \;\Rightarrow\; F_{\rm drag} \propto v$

2) $R_e \gg 1 \;\Rightarrow\; C_d \approx {\rm const} \;\Rightarrow\; F_{\rm drag} \propto v^2$

During common envelope, we are most likely in case 2)

Cautionary note: part of the drag during common envelope can be from a form of "dynamical friction" due to gravitationally focused fluid, rather than viscosity

Physically, what is a drag force?

$$F_{\rm drag} = \frac{1}{2}\rho\, v^2\, C_d\, A$$

Cross section area $\equiv 4\pi R_*^2$

Fluid density

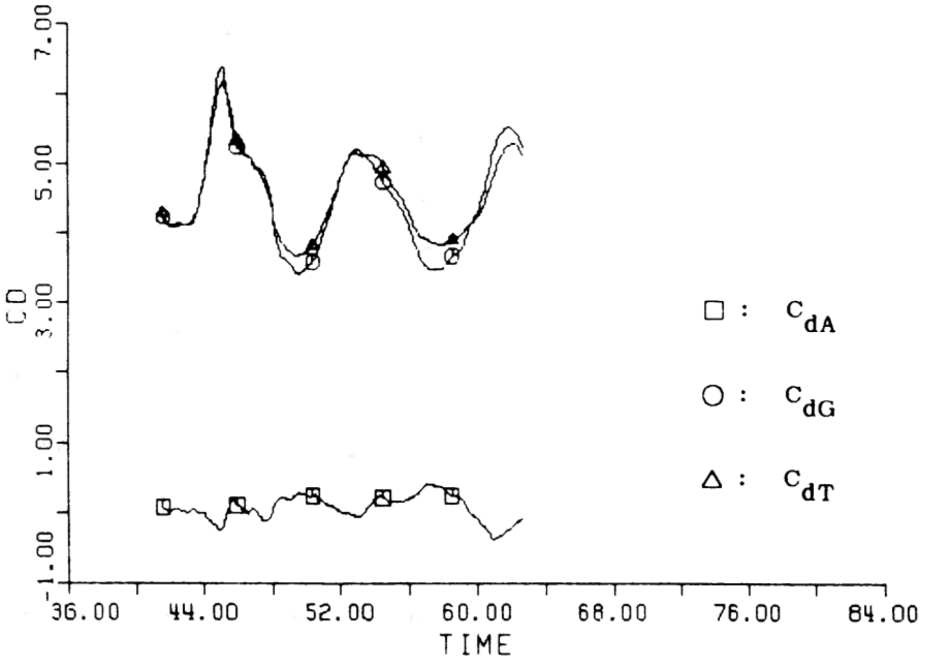
Relative velocity with the fluid

$$R_* = \frac{2GM}{c_s^2 + v^2}$$

The radius is the **Bondi accretion radius**:

The radius within which the fluid gets gravitationally focused

Hydro simulations showed that this drag force form reproduces the "gravitational drag" (Shima et al. 1985; McLeod et al. 2017; Reichardt et al. 2019)



Let's derive the changes in orbital parameters using perturbation theory

$$\vec{f} = -C\,\frac{v^l}{r^k}\,\hat{v}$$

Interesting forms:

$l = 1$ drag force linear with velocity

$l = 2$ drag force quadratic with velocity

$k \geq 0$ some degree of radial dependency

$$F_{\text{drag}} = \frac{1}{2}\underline{\rho}\, v^2\, C_d\, \underline{A}$$

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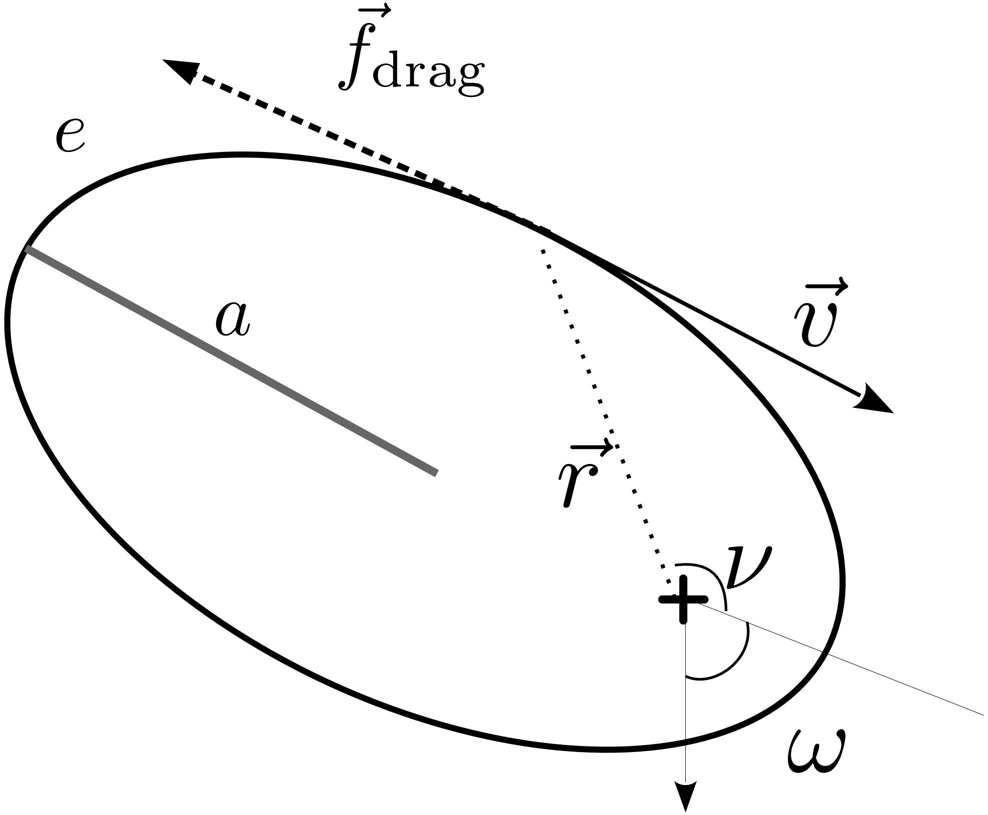


$\dot{e}$ rate of change in eccentricity

$\dot{a}$ rate of change in semimajor axis

$\dot{\omega}$ apsidal precession

Let's derive the changes in orbital parameters using perturbation theory

Instantaneous change in semimajor axis a and specific angular momentum h

$$\dot{a} = \frac{2a^2}{\mu}\left(\dot{r}\, f_r + r\,\dot{\nu}\, f_\nu\right) \qquad \dot{h} = r\, F_\nu$$

where ν is the true anomaly

$\mu = G(m_1 + m_2)$ standard gravitational parameter

+ argument of pericenter precession

$$\dot{\omega} = \left(\frac{1}{r} - \frac{\varepsilon}{e\mu}\cos\nu\right)\left(\frac{2h\dot{h}}{e\mu\sin\nu}\right) - \frac{h^2}{e^2\mu^2}\dot{\varepsilon}\cot\nu$$

After some pages of calculations...

$$\dot{a} = 2\,\mu^{\frac{l-1}{2}}\,a^{\frac{3-l-2k}{2}}\,(1-e^2)^{-\frac{l+1+2k}{2}}\,(1+e^2+2e\cos\nu)^{\frac{l+1}{2}}\,(1+e\cos\nu)^k \tag{1}$$

$$\dot{e} = 2(1-e^2)^{-\frac{l-1+2k}{2}}\,\mu^{\frac{l-1}{2}}\,a^{\frac{1-l-2k}{2}}\,(1+e\cos\nu)^k\,(1+e^2+2e\cos\nu)^{\frac{l-1}{2}}\,(e+\cos\nu) \tag{2}$$

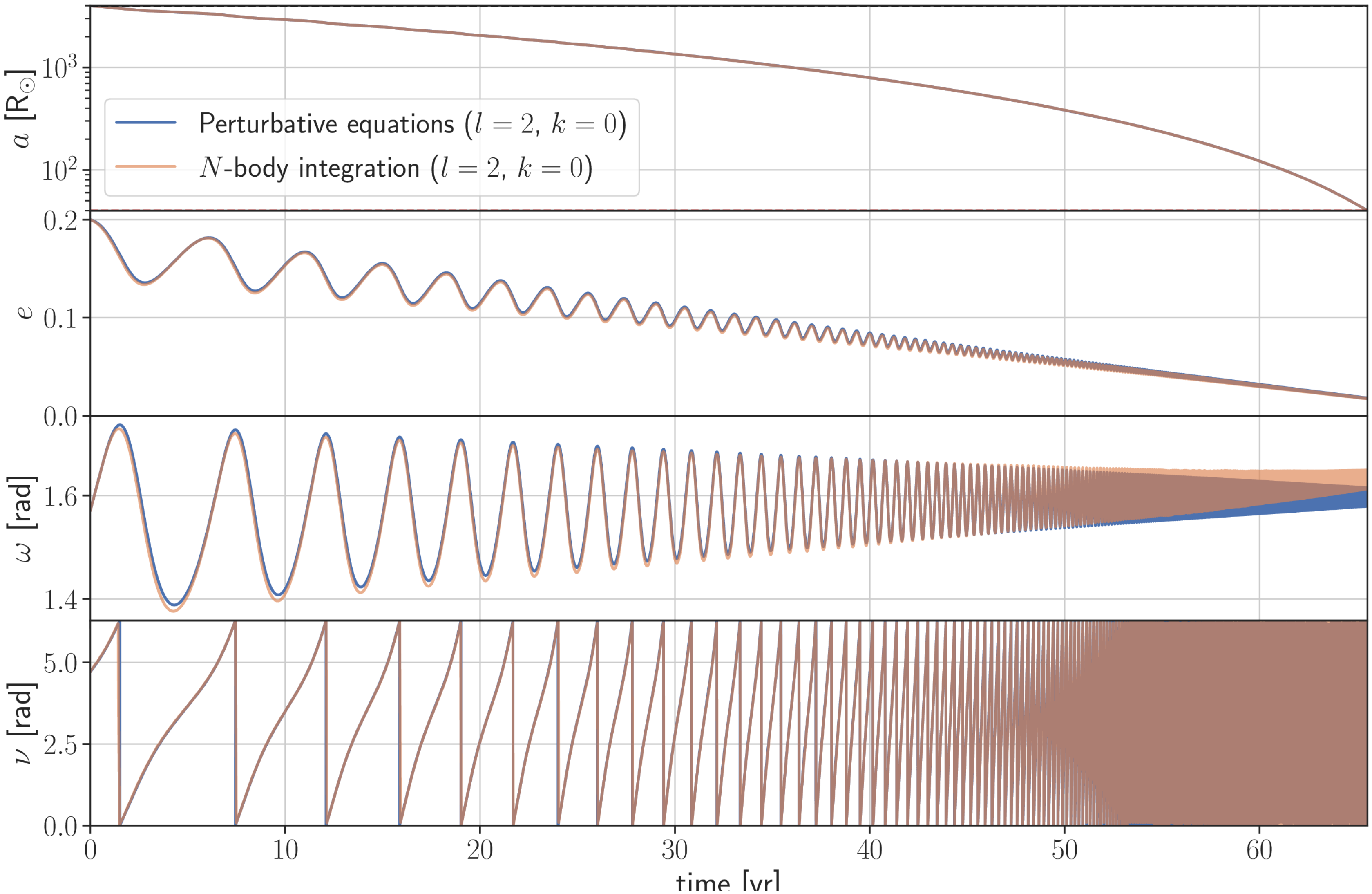
$$\dot{\omega} = 2\mu^{\frac{l-1}{2}}\,a^{-\frac{l-1+2k}{2}}\,\frac{(1-e^2)^{-\frac{l-1+2k}{2}}}{e}\,(1+e\cos\nu)^k\,(1+e^2+2e\cos\nu)^{\frac{l-1}{2}}\,\sin\nu \tag{3}$$

$$\dot{\nu} = \frac{(1+e\cos\nu)^2}{(1-e^2)^{3/2}}\sqrt{\frac{\mu}{a^3}} - \dot{\omega} \tag{4}$$

These 4 equations constitute a closed set of ODEs

$$l = 2 \qquad \vec{f} = -C\, v^2\, \hat{v}$$
$$k = 0$$

$$
\begin{aligned}
\frac{da}{dt} &= -2C\sqrt{a\,\mu}\,\left(\frac{1+e^2+2e\cos\nu}{1-e^2}\right)^{3/2} \\
\frac{de}{dt} &= -2C\sqrt{\frac{\mu}{a}}\,\sqrt{1-e^2}\,\sqrt{1+e^2+2e\cos\nu}\,(e+\cos\nu) \\
\frac{d\omega}{dt} &= -2C\sqrt{\frac{\mu}{a}}\,\frac{\sqrt{1+e^2+2e\cos\nu}}{e\sqrt{1-e^2}}\,\sin\nu \\
\frac{d\nu}{dt} &= \frac{(1+e\cos\nu)^2}{(1-e^2)^{3/2}}\sqrt{\frac{\mu}{a^3}} - \frac{d\omega}{dt}
\end{aligned}
\qquad (1)
$$



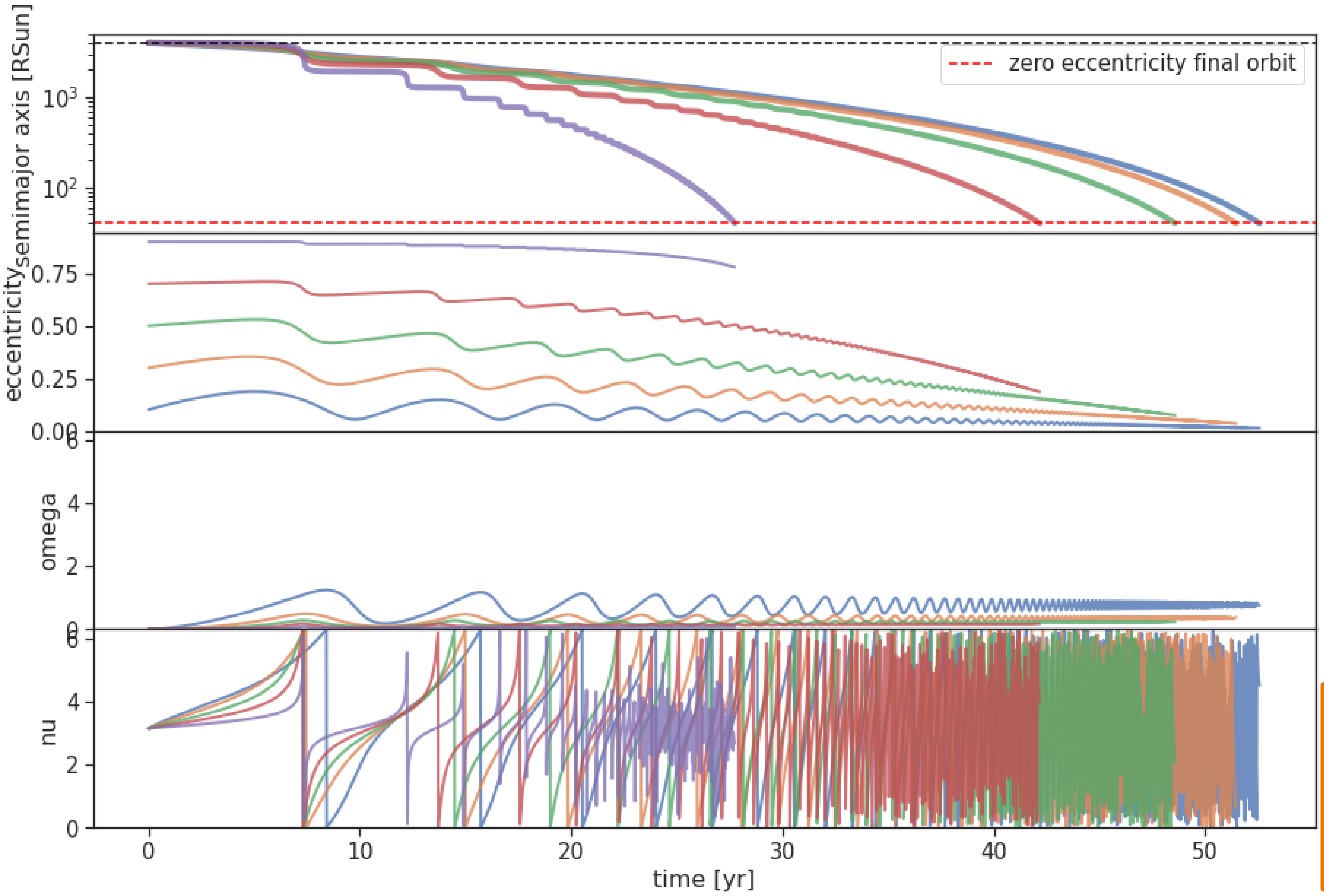
comparison with direct N-body: Hermite integrator + velocity kick

$$m_1 = m_2 = 15\mathrm{M}_\odot$$

$$a_0 = 4000\mathrm{R}_\odot$$

$$a_{\mathrm{fin}} = 40\mathrm{R}_\odot$$

$$e_0 = 0.2$$


semimajor axis [RSun]
zero eccentricity final orbit
10^3
10^2
eccentricity
0.75
0.50
0.25
0.00
omega
6
4
2
0
nu
6
4
2
0
0
10
20
30
40
50
time [yr]

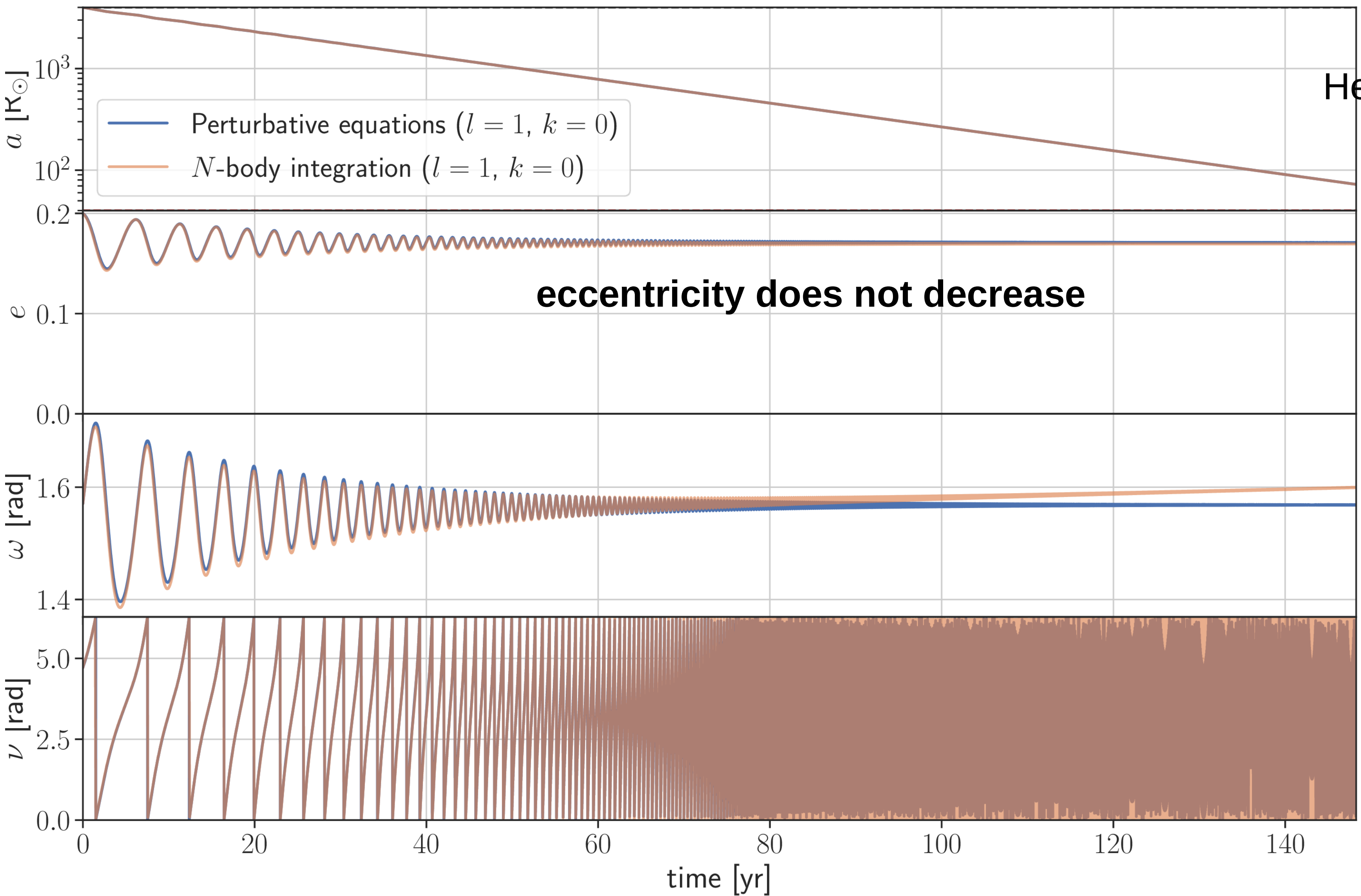
$l = 2$

$k = 0$

$$l = 1 \qquad k = 0 \qquad \vec{f} = -C\, v\, \hat{v}$$

$$\begin{aligned}
\frac{da}{dt} &= -2C\, a\, \frac{1 + e^2 + 2e\cos\nu}{1 - e^2} \\
\frac{de}{dt} &= -2C(1 - e^2)\,(e^2 + e\cos\nu)(e + \cos\nu) \\
\frac{d\omega}{dt} &= -\frac{2C}{e}\sin\nu \\
\frac{d\nu}{dt} &= \frac{(1 + e\cos\nu)^2}{(1 - e^2)^{3/2}}\sqrt{\frac{\mu}{a^3}} - \frac{d\omega}{dt}
\end{aligned} \tag{1}$$

comparison with direct N-body:
Hermite integrator
+ velocity kick



$m_1 = m_2 = 15\mathrm{M}$

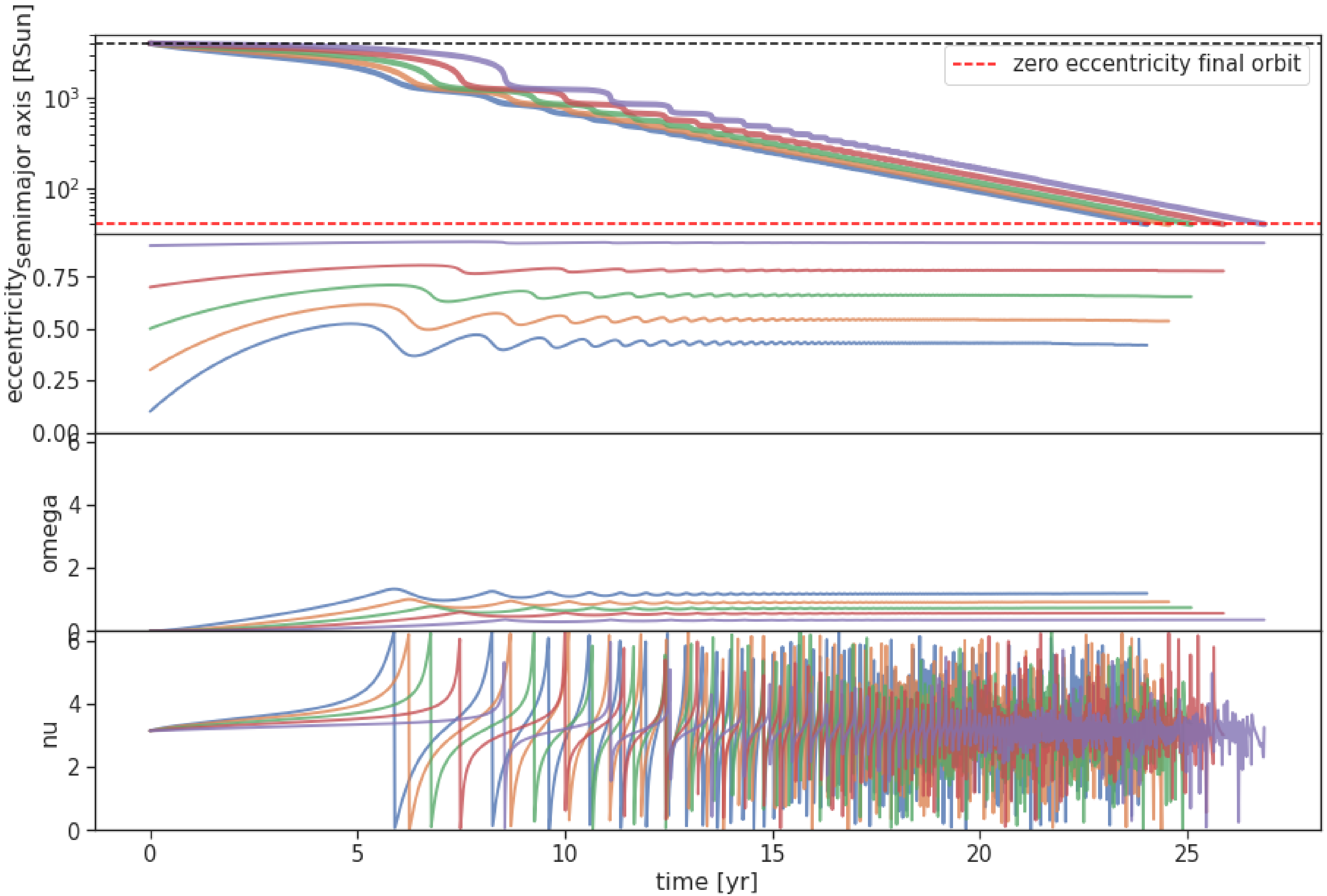
$a_0 = 4000\mathrm{R}_\odot$

$a_{\mathrm{fin}} = 40\mathrm{R}_\odot$

$e_0 = 0.2$

$l = 1$

$k = 0$



semimajor axis [RSun]
zero eccentricity final orbit
eccentricity
omega
nu
time [yr]

$l = 1$

$k = 0$

With no eccentricity, the equations have analytic solution

$$\dot{a} = -2C\mu^{\frac{l-1}{2}} a^{\frac{3-l-2k}{2}}$$

$$\frac{3-l-2k}{2} = m$$

$m \neq 1$ $$a = \sqrt[1-m]{a_0^{1-m} - 2(1-m)Ct\mu^{\frac{l-1}{2}}}$$ *power-law decay*

$m = 1$ $$a = a_0 e^{-2C\,t\,\mu^{(l-1)/2}}$$ *exponential decay*

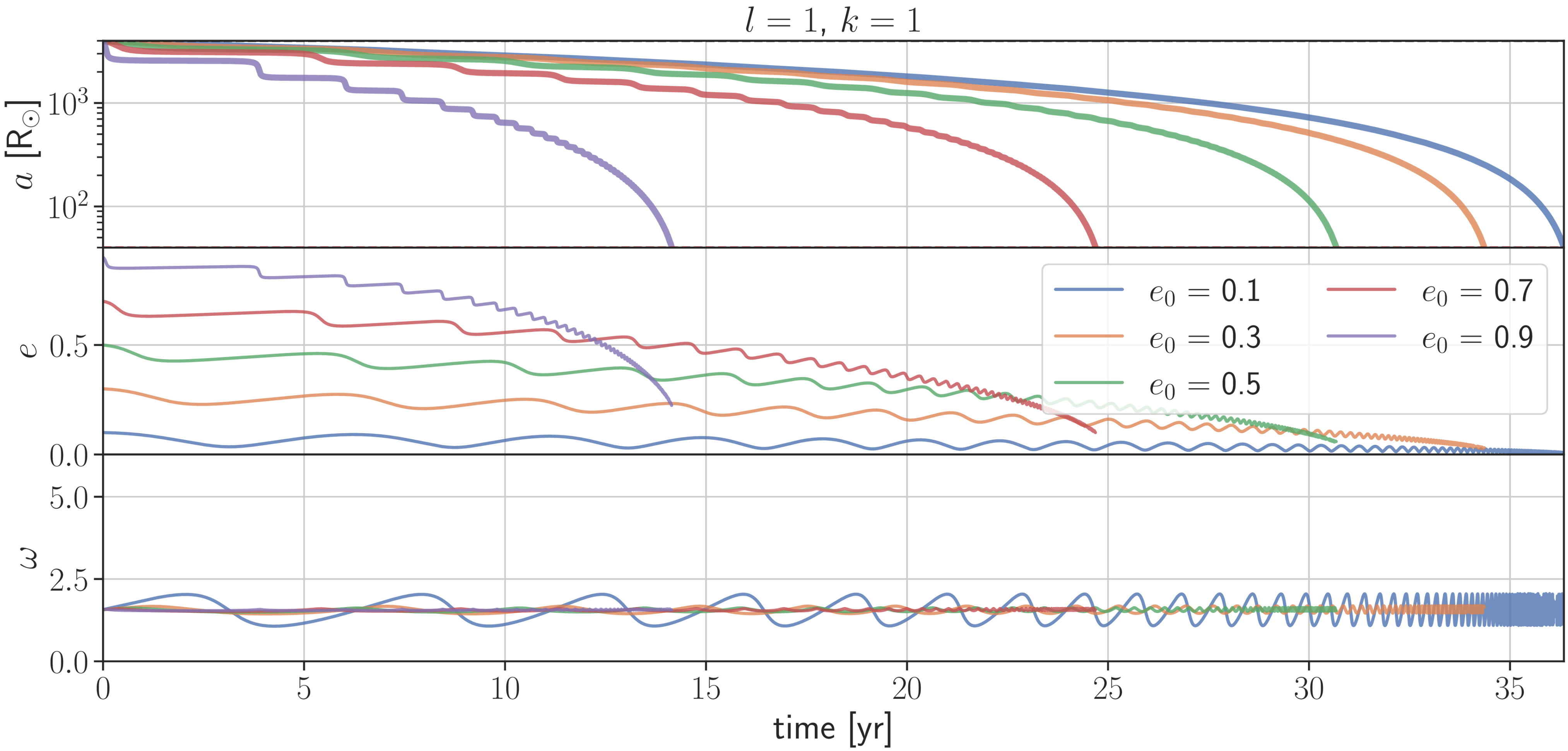
decay timescale: $\tau_a = \frac{a}{\dot{a}}$

Dimensionless decay timescale

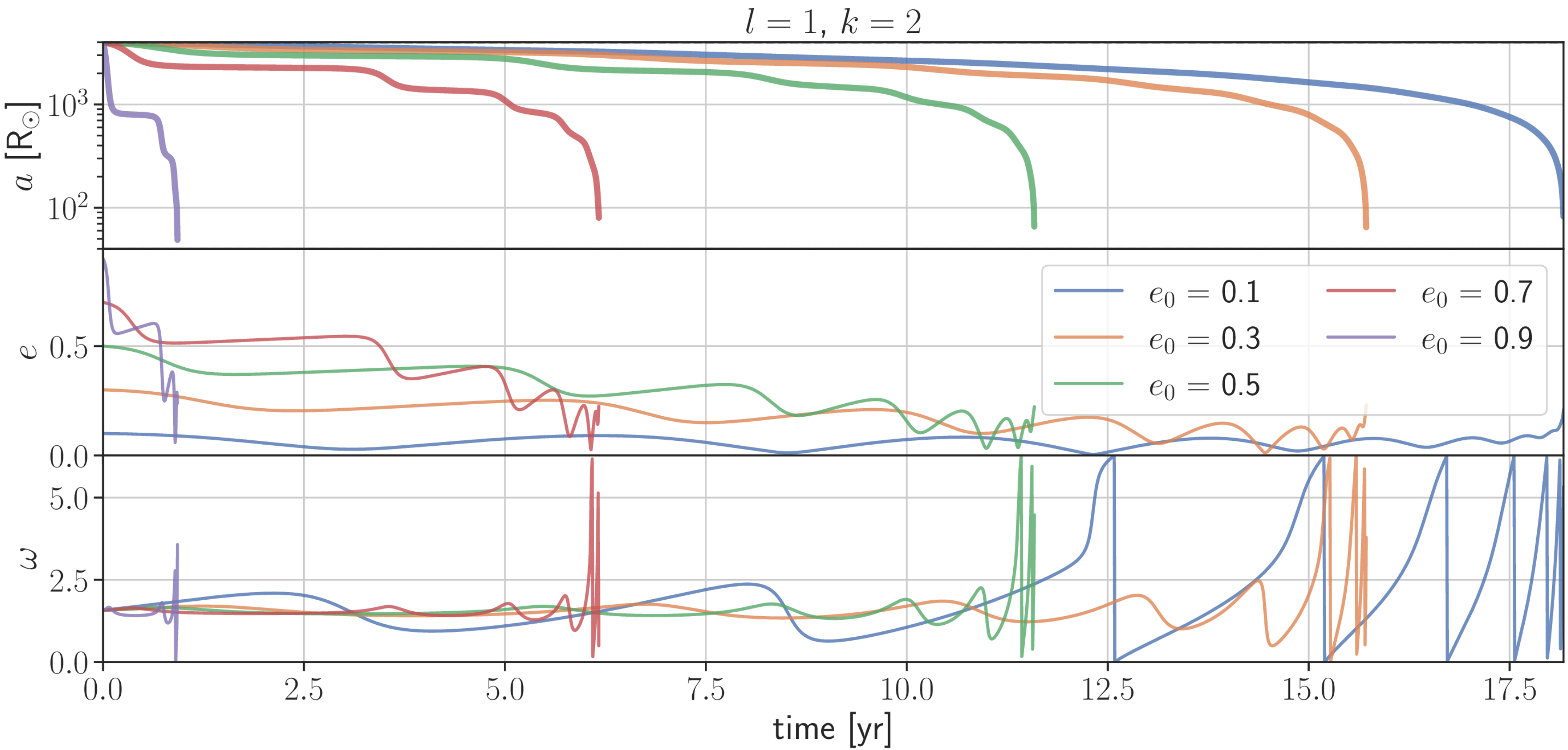
$$\chi_a = \frac{P}{\tau_a} = -\pi C\,\mu^{l/2-1}\,a^{2-l/2-k}$$

the perturbative approximation requires that

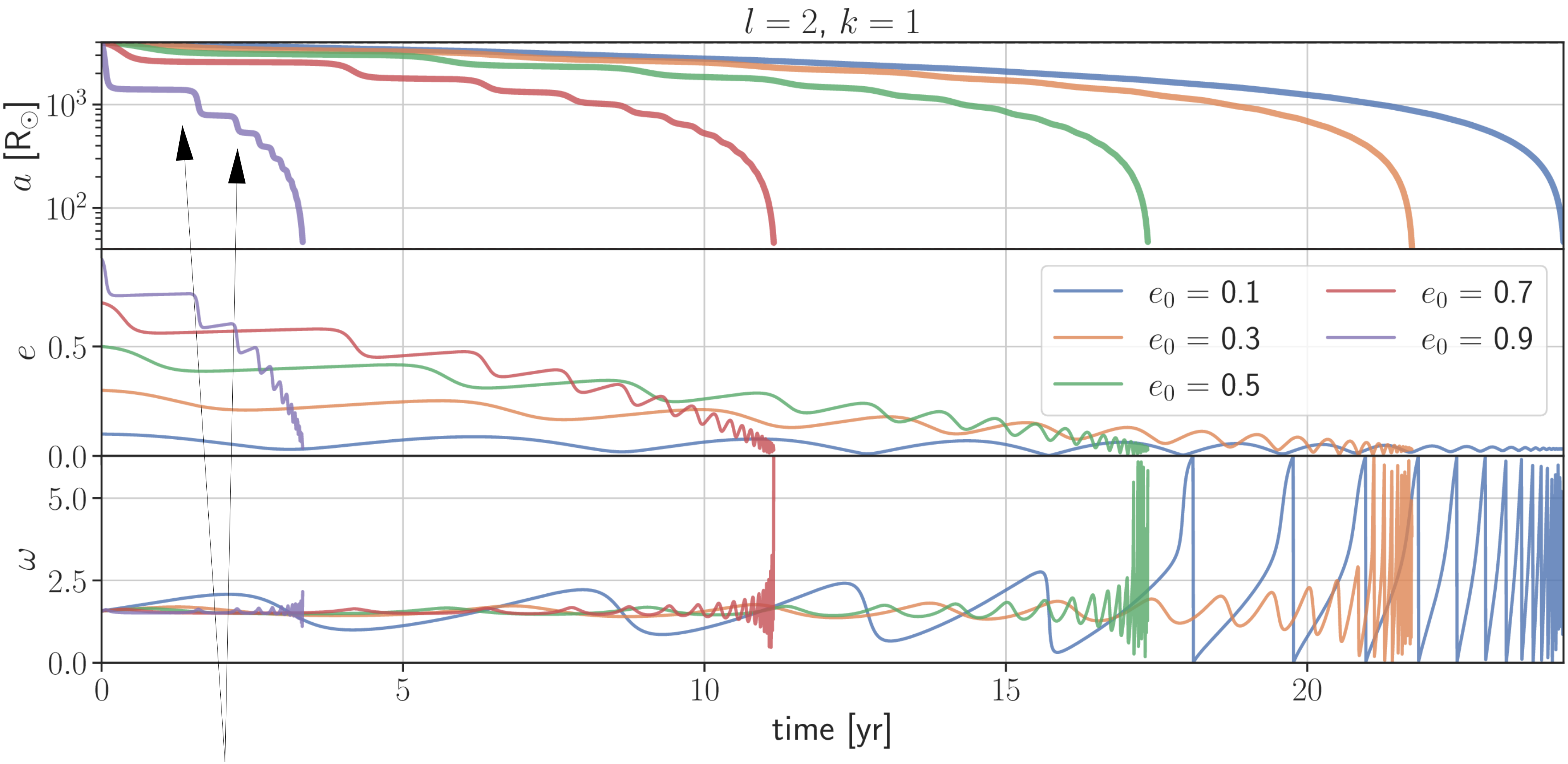
$$\chi_a < 1$$



$l = 1,\ k = 1$
a [R$_\odot$]
10^3
10^2
e
0.5
0.0
$e_0 = 0.1$
$e_0 = 0.3$
$e_0 = 0.5$
$e_0 = 0.7$
$e_0 = 0.9$
ω
5.0
2.5
0.0
0
5
10
15
20
25
30
35
time [yr]



$l = 1,\, k = 2$
a [$R_\odot$]
10^3
10^2
e
0.5
0.0
ω
5.0
2.5
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
time [yr]
$e_0 = 0.1$
$e_0 = 0.3$
$e_0 = 0.5$
$e_0 = 0.7$
$e_0 = 0.9$



the drop is at pericenter passage

Halting the inspiral

$$f = -C \frac{v^l}{r^k} \longrightarrow f_{\text{drag}} \propto \frac{\rho(r)}{m} v^2$$

$l = 2$ to agree with the above gravitational drag force

k encodes information about the radial density profile of the envelope

Let's consider evolving background density

$$\rho := \rho(t, r)$$

Halting the inspiral

Self-similar (homologous) expansion: radial profile remains the same

$$\rho := \rho(t, r) = \rho_0(r) f(t)$$

We need a way to map drag-force energy losses $(\dot{a})$

to decreasing density $(\dot{f})$

Halting the inspiral

Self-similar expansion + conservation of mass: $$\rho(t,r) = \frac{1}{g(t)^3}\rho_0\left(\frac{r}{g(t)}\right)$$

Radius expands as: $$R(t) = R_0 g(t)$$

Orbital energy losses: $$\dot{E}_{\rm orb} = \frac{Gm_1m_2}{2a^2}\dot{a}$$

Energy losses go into unbinding the envelope

Binding energy for a polytropic sphere: $$B_0 \propto \frac{GM^2}{R}$$

Hence: $$B(t) = \frac{B_0}{g(t)} \qquad \dot{B} = -\dot{g}\frac{B_0}{g^2}$$

Setting the change in binding energy $\dot{B}$ equal to the orbital energy losses $\dot{E}_{\rm orb}$

we obtain the differential equation for the expansion factor $g(t)$

$$\dot{g} = -\frac{m_{\rm red}\mu}{2a^2}\frac{\dot{a}}{B_0}g^2$$

We can solve this equation along the others, and calculate the new drag-force coefficient as

$$C(t) = C_0/g(t)^{3-k}$$

Setting the change in binding energy $\dot{B}$ equal to the orbital energy losses $\dot{E}_{\rm orb}$

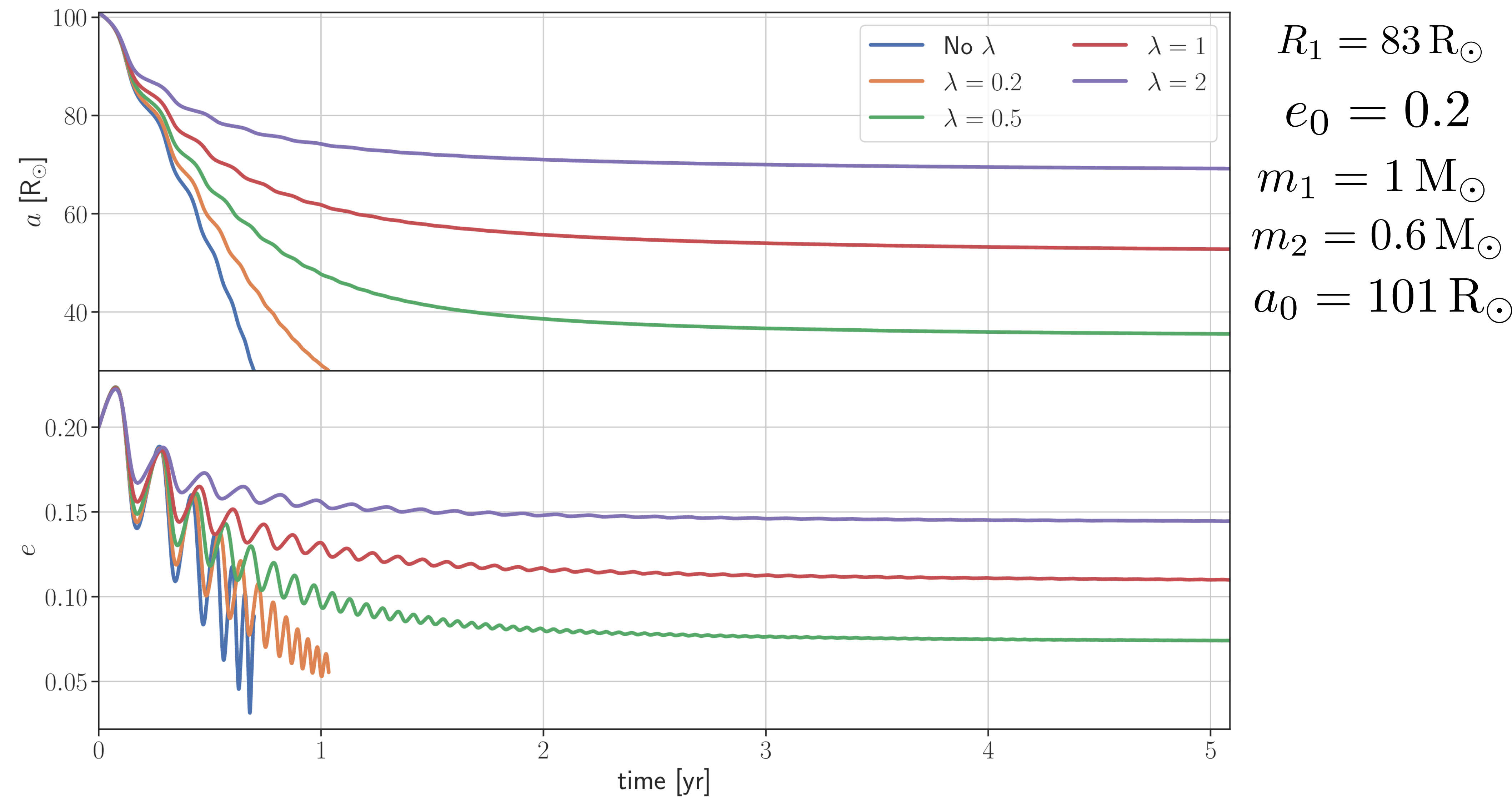
we obtain the differential equation for the expansion factor $g(t)$

$$\dot{g} = -\frac{m_{\rm red}\mu}{2a^2}\frac{\dot{a}}{B_0}g^2$$

Missing ingredient: initial value of the binding energy B_0

We can use the usual Lambda parametrization

$$B_0 = \frac{Gm_{1,\rm c}m_{1,\rm env}}{\lambda R} \qquad \lambda = 0.1\text{–}2$$



100
80
60
40
a [R$_\odot$]
0.20
0.15
0.10
0.05
e
0
1
2
3
4
5
time [yr]
No λ
$\lambda = 0.2$
$\lambda = 0.5$
$\lambda = 1$
$\lambda = 2$
$R_1 = 83\,\mathrm{R}_\odot$
$e_0 = 0.2$
$m_1 = 1\,\mathrm{M}_\odot$
$m_2 = 0.6\,\mathrm{M}_\odot$
$a_0 = 101\,\mathrm{R}_\odot$

Alternatively, we can use the α-λ model to stop the inspiral

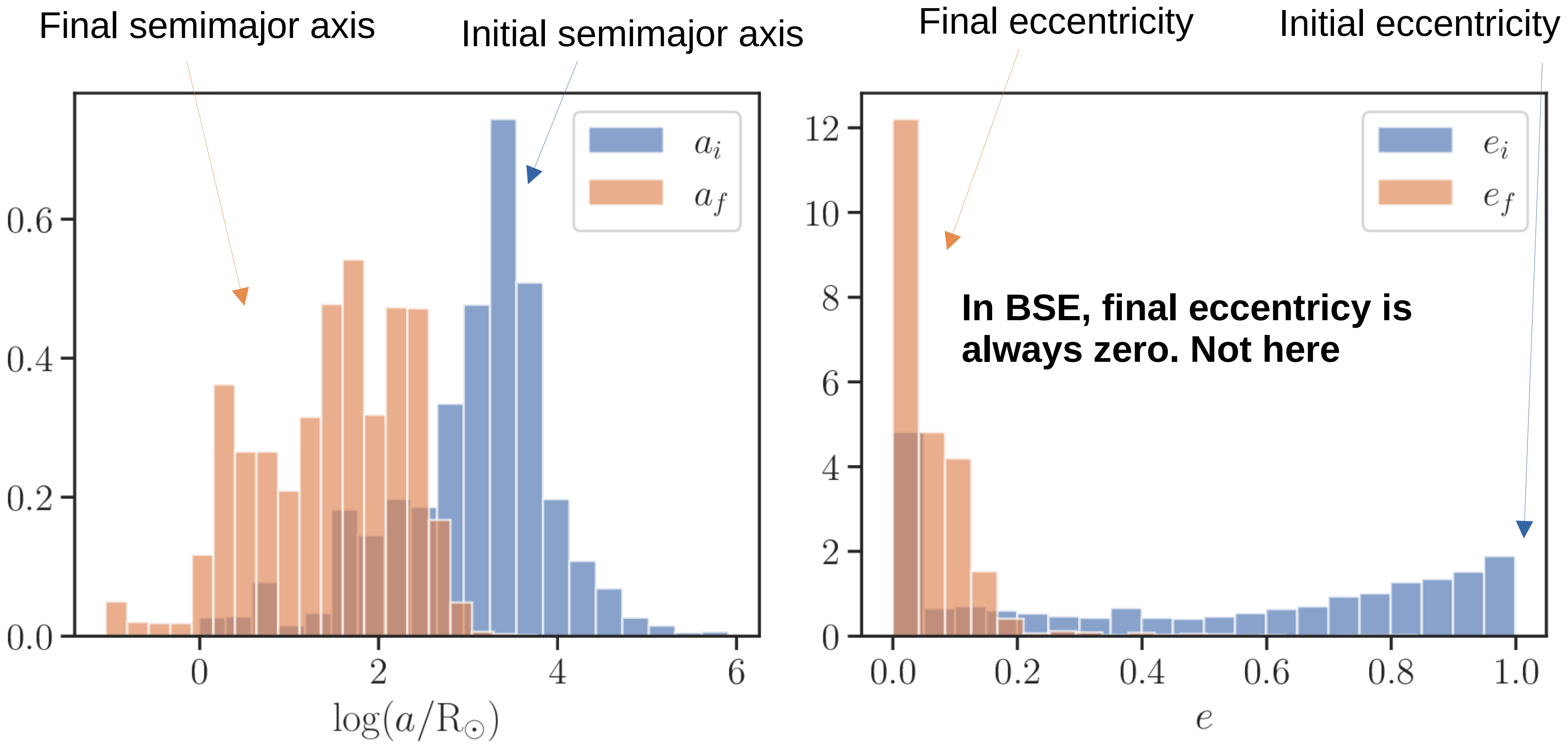
1. Obtain the final semimajor axis, or total energy loss, from the α-λ model

$$\Delta E_{\rm orb} = \frac{1}{\alpha\lambda}\frac{G m_{1,\rm c} m_{1,\rm env}}{R}$$

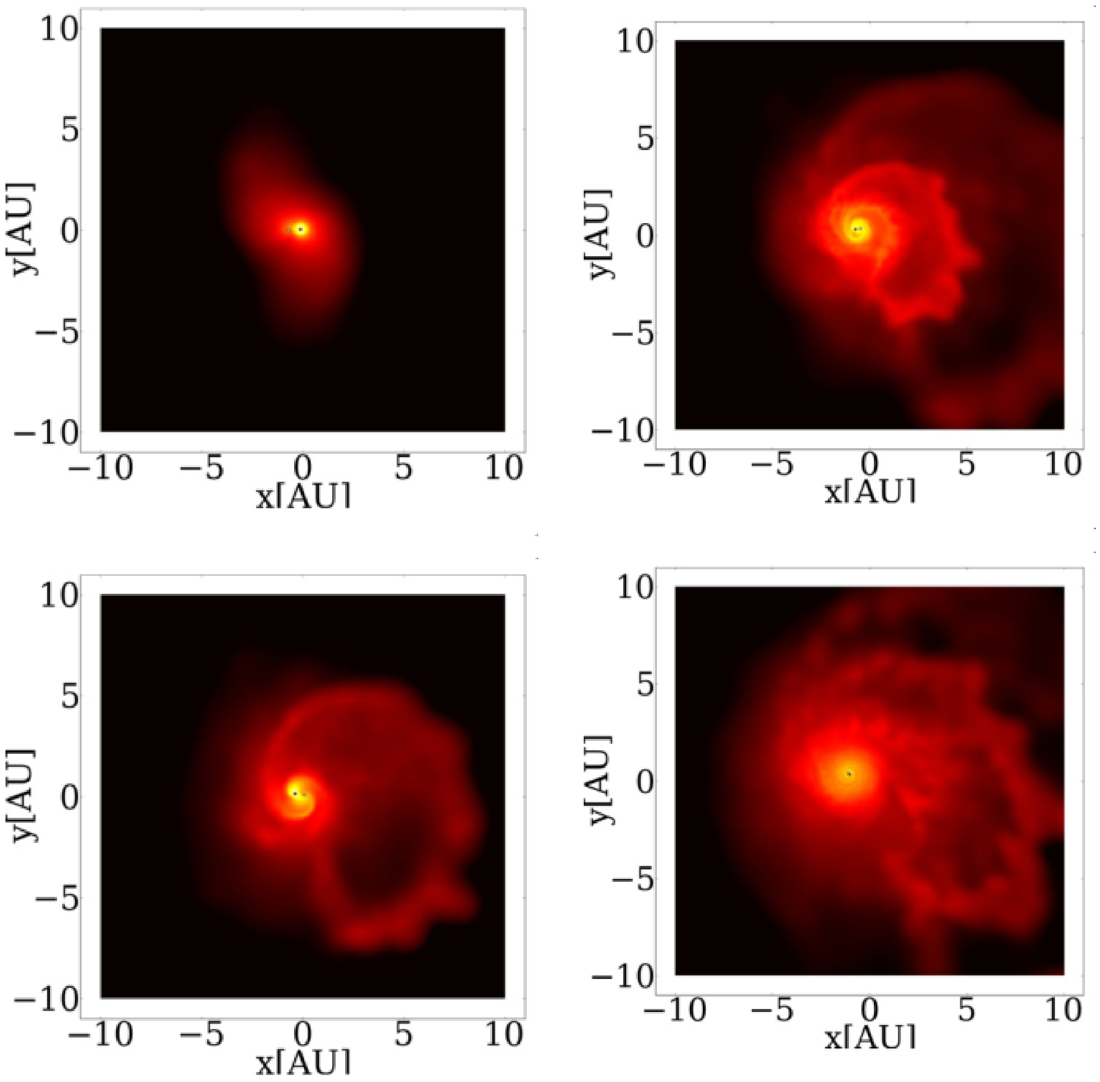
2. Stop the integration when final semimajor axis is reached, or when total integrated energy loss $\dot{E}_{\rm orb}$ reaches $\Delta E_{\rm orb}$

3. Profit!

2659 common envelope events from Tanikawa et al. 2020 (modified BSE)



Comparison with SPH simulations of common envelope, Glanz & Perets 2021

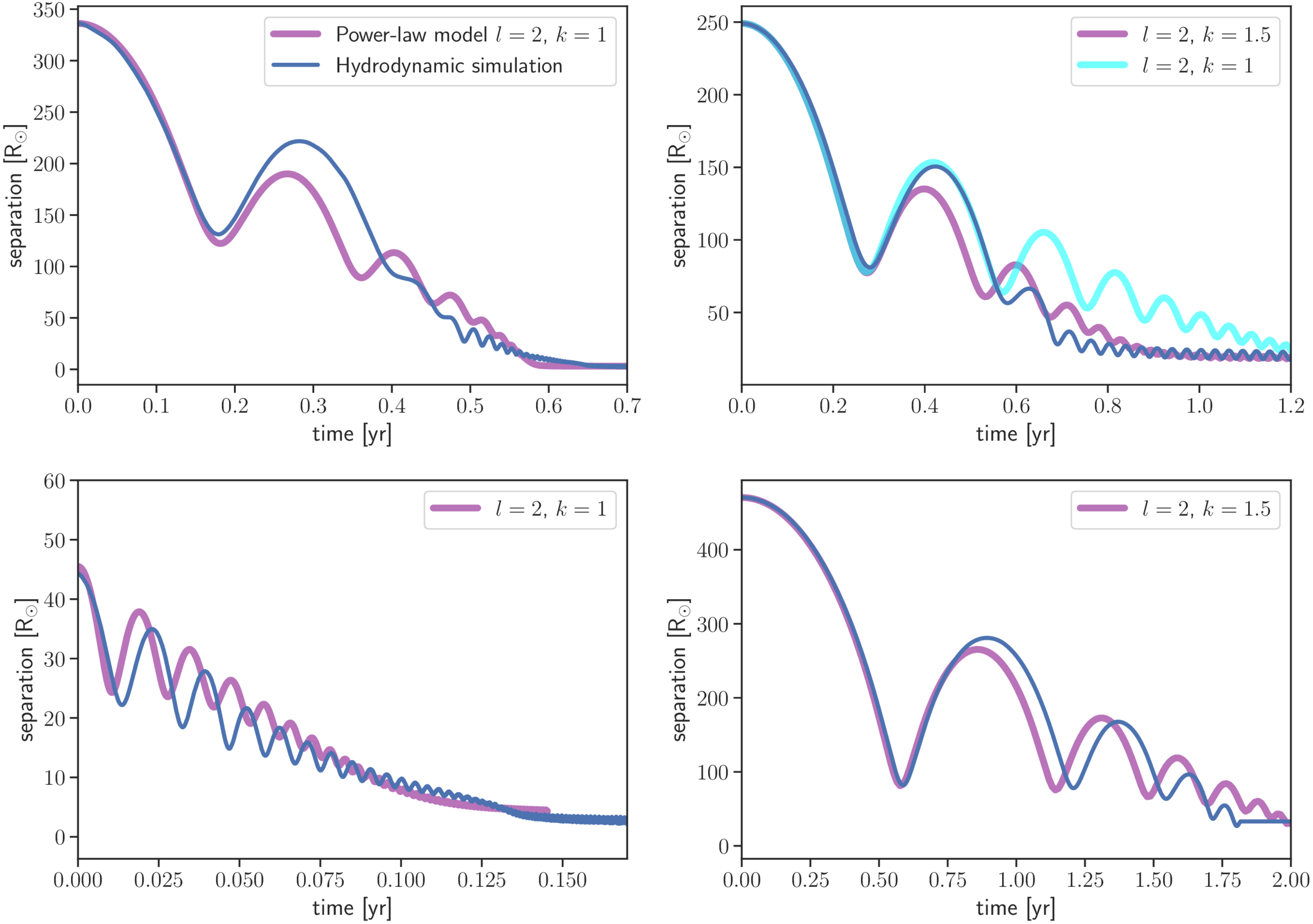


$1\,\mathrm{M}_{\odot}, 83\,\mathrm{R}_{\odot}$

Red Giant Branch

With 0.6 MSun inspiralling White Dwarf point mass

Comparison with hydrodynamic simulations from Glanz & Perets 2021



Summary

New semi-analytic, descriptive model for common envelope evolution

- ✔ Avoids “quantum” orbit jumps
- ✔ Gives information about the final eccentricity
- ✔ Can be made consistent with the α-λ model…
- ✔ …or can be alternative to the α-~~λ~~ model (still needs a “λ” parameter)

Can be used both in

- Binary population synthesis codes (perturbation theory approach)
- N-body code, with direct integration (currently implemented in AMUSE)

+ model is geometry agnostic (so far): can be also used to describe planetary migration in gas disks

Future

- Better treatment of mass loss (drag force–mass loss coupling)
- More consistent envelope expansion model
- Introduce angular momentum? Rotating polytropes?

velocity cross-terms in the force: $\vec{f} \propto |\vec{v}_{\mathrm{orb}} - \vec{v}_{\mathrm{env}}|^2$

Final goal: a self-consistent, **predictive** model for common envelope evolution