

# A deep-learning solution to the gravitational-wave population problem

**Davide Gerosa**

University of Milano-Bicocca

**arXiv:2203.03651**

with M. Mould and S. Taylor

Jun 2, 2022

Black Hole Dynamics: From Gaseous  
Environments to Empty Space

Copenhagen, Denmark



European Research Council

Fondazione  
**CARIPLO**



LEVERHULME  
TRUST



davide.gerosa@unimib.it  
www.davidegerosa.com

# A deep-learning solution to the gravitational-wave population problem

**Davide Gerosa**

University of Milano-Bicocca

**arXiv:2203.03651**

with M. Mould and S. Taylor

↪ Soon on the job market!

Jun 2, 2022

Black Hole Dynamics: From Gaseous  
Environments to Empty Space

Copenhagen, Denmark



European Research Council

Fondazione  
**CARIPLO**



LEVERHULME  
TRUST



davide.gerosa@unimib.it  
www.davidegerosa.com

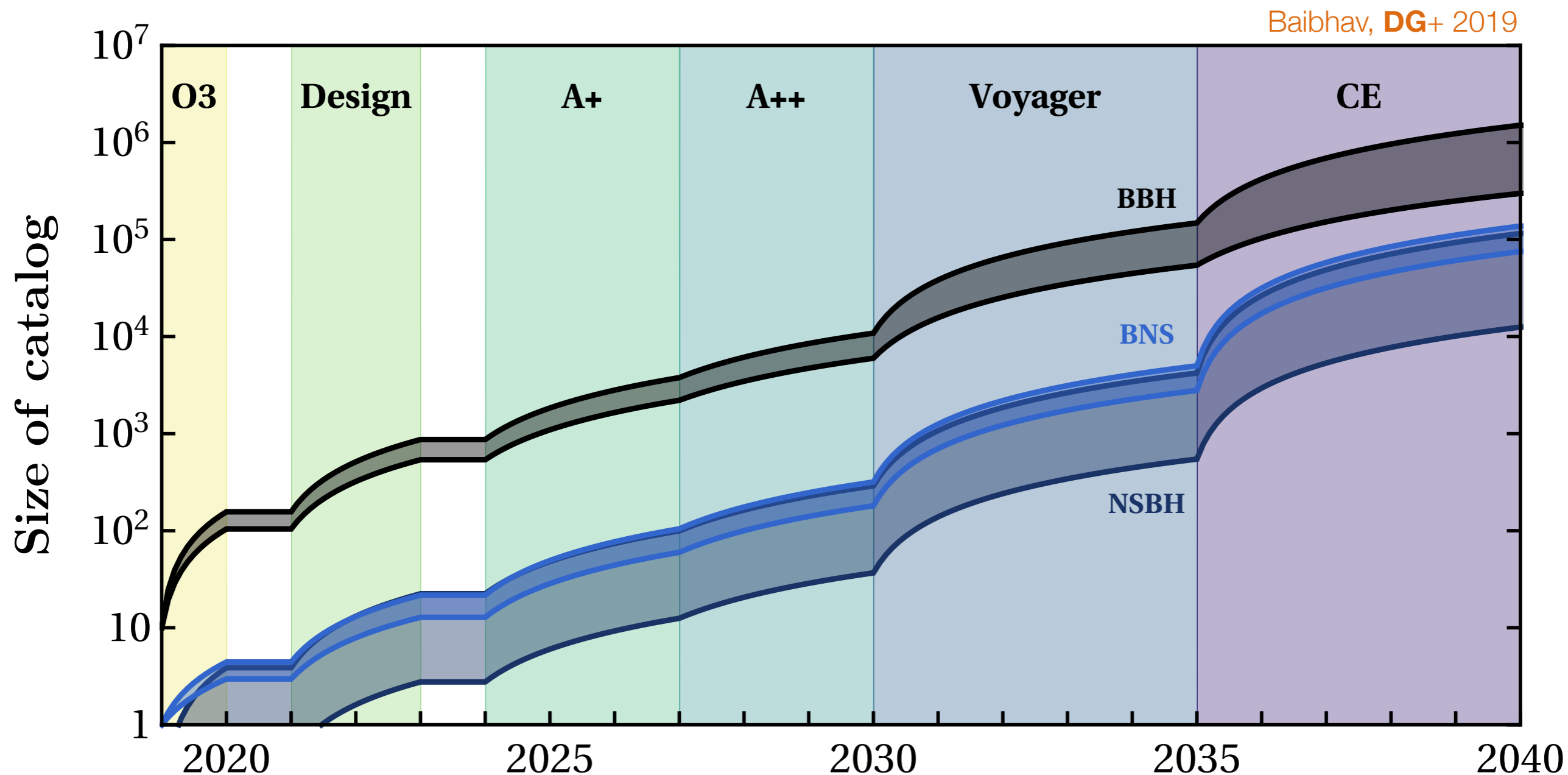
# 90 waves and counting

Discoveries are piling up!

**About 90 black-hole binary mergers detected so far.**

Will become millions in ~20 years!

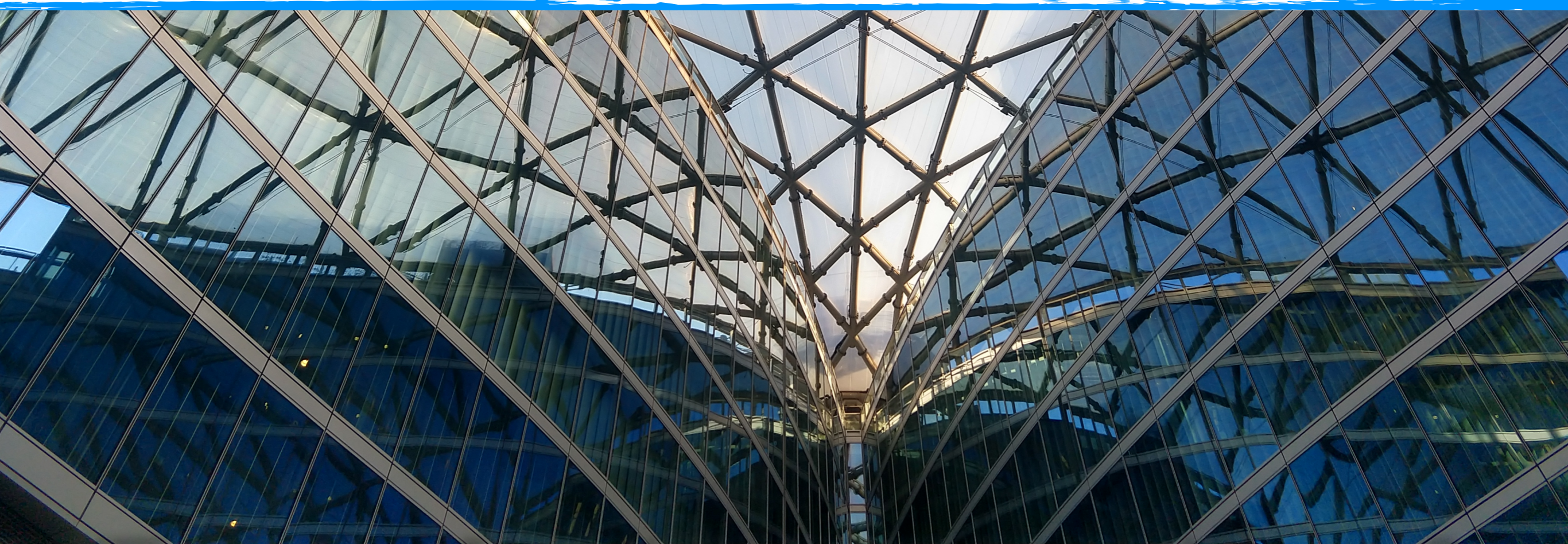
LIGO 2021





# Outline

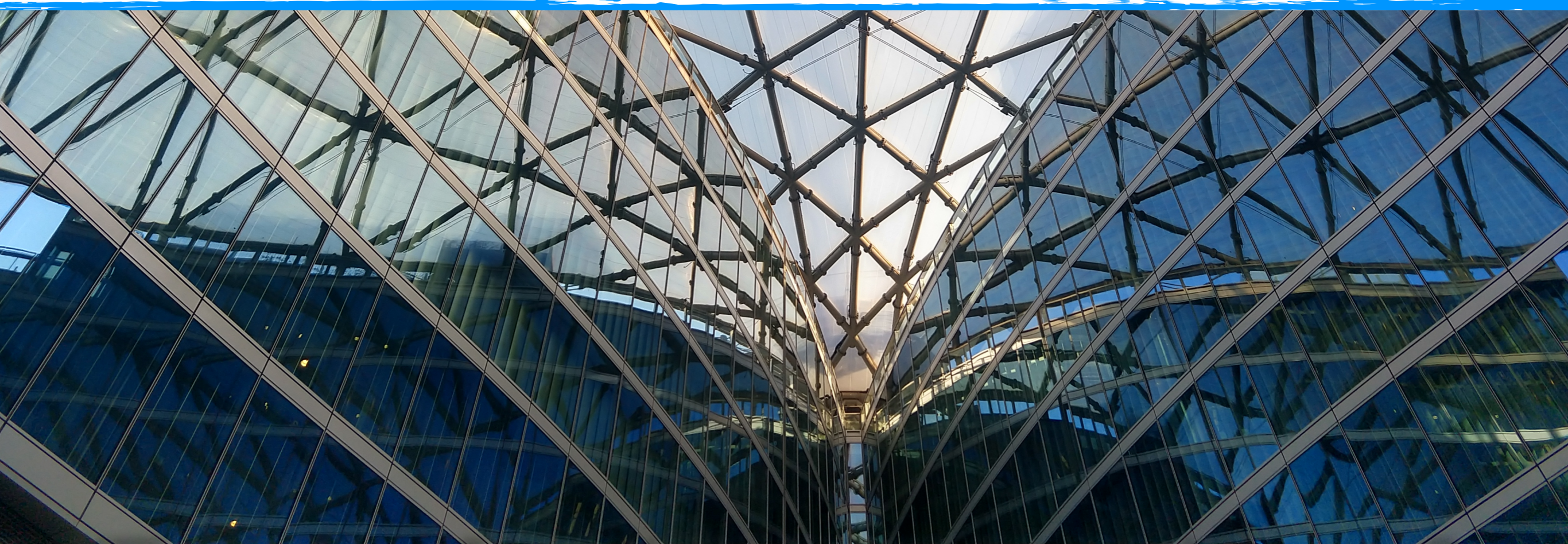
1. The population problem
2. Current pipeline: hierarchical mergers
3. Current pipeline: machine learning
4. Things we (haven't) figured out (yet)





# Outline

1. The population problem
2. Current pipeline: hierarchical mergers
3. Current pipeline: machine learning
4. Things we (haven't) figured out (yet)





# How we put things together

$\theta$  **Single-event parameters:** masses, spins, redshifts

$\lambda$  **Population parameters:** spectral index of mass distribution, cutoffs

Inhomogeneous Poisson process:

Loredo 2004, Mandel+ 2019,  
Thrane, Talbot 2019, Vitale, **DG+** 2022,

$$p(\lambda|d) \propto \pi(\lambda) \sigma^{-N}(\lambda) \prod_{i=1}^N \int p_{\text{pop}}(\theta|\lambda) \mathcal{L}(d_i|\theta) d\theta$$

Population prior

Population posterior

N events...

Population model

Single-event likelihood

Selection effects:  $\sigma(\lambda) = \int p_{\text{pop}}(\theta|\lambda) p_{\text{det}}(\theta) d\theta$

Detection probability



# What model for the Universe?

Option 1: **Simple, parametrized functional forms**

LIGO/Virgo and many others

↪ Evaluating  $p_{\text{pop}}(\theta|\lambda)$  is straightforward and can be done at each likelihood evaluation

**But:** *Astrophysicists like you guys put a lot of effort in simulating stellar evolution, clusters, AGN, and all of that!*

Option 2:

**Can we instead interpret GW data using cool astro predictions *directly*?**

↪ Evaluating  $p_{\text{pop}}(\theta|\lambda)$  now is a costly simulation...

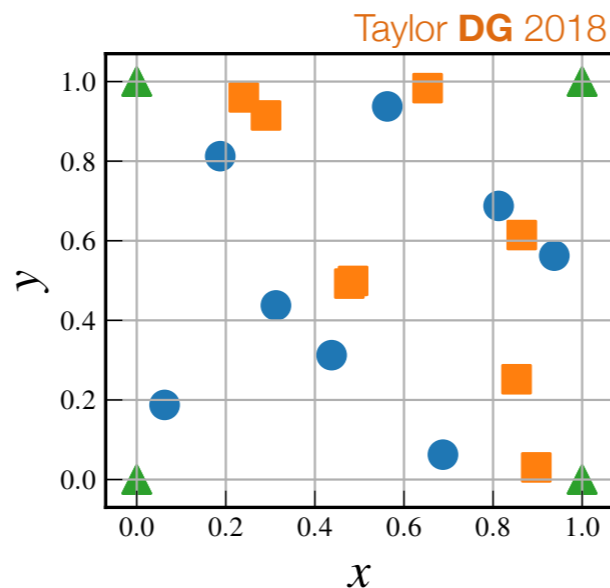
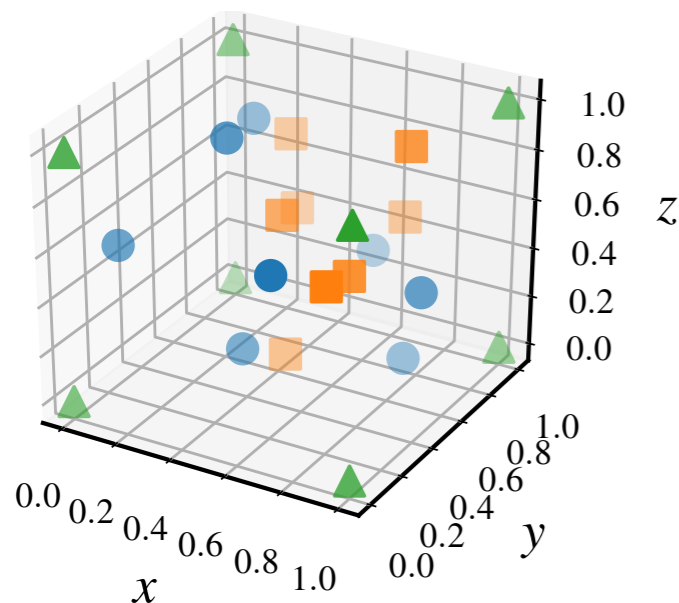
# Ingredients in the blender

## 1. A population synthesis code

I'm not going to even try citing people here! So many excellent studies

- Early prototype with limited set of COMPAS runs [Taylor DG 2018](#)
- Current application: simple hierarchical merger populations [Mould DG Taylor 2022](#)
- Hopefully soon: full isolated formation channel inference
- (But can you help us doing dynamics?)

## 2. Design a training bank. Space filling algorithms



- Latin hypercubes
- Now working on implementing progressive hypercube sampling



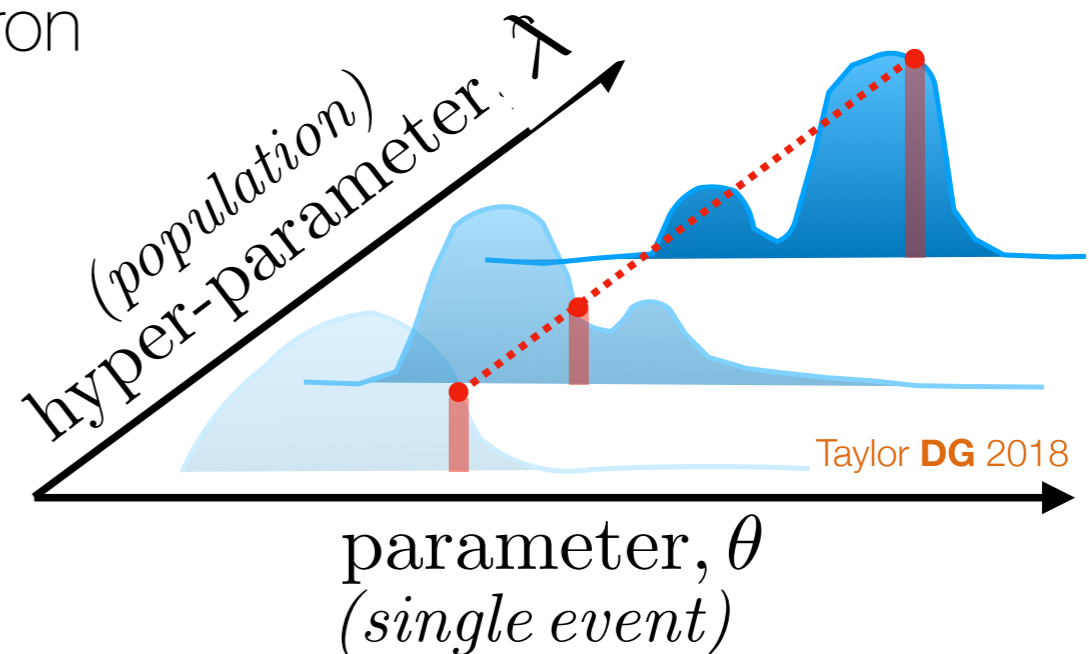
# Ingredients in the blender

## 3. Some form of data compression

- Used principal component analysis successfully Taylor, **DG** 2018
- Tucker decomposition to avoid array raveling?
- Non-linear dimensionality reduction schemes?

## 4. A powerful conditional density estimation scheme $p_{\text{pop}}(\theta|\lambda)$

- Gaussian process regression  
Taylor, **DG** 2018, Wong, **DG** 2019
- FFT-based KDE and a multilayer perceptron  
Mould **DG** Taylor 2022
- Autoregressive flows  
Wong, Contardo, Ho 2020



# Ingredients in the blender

5. A model for the detector  $p_{\text{det}}(\theta)$

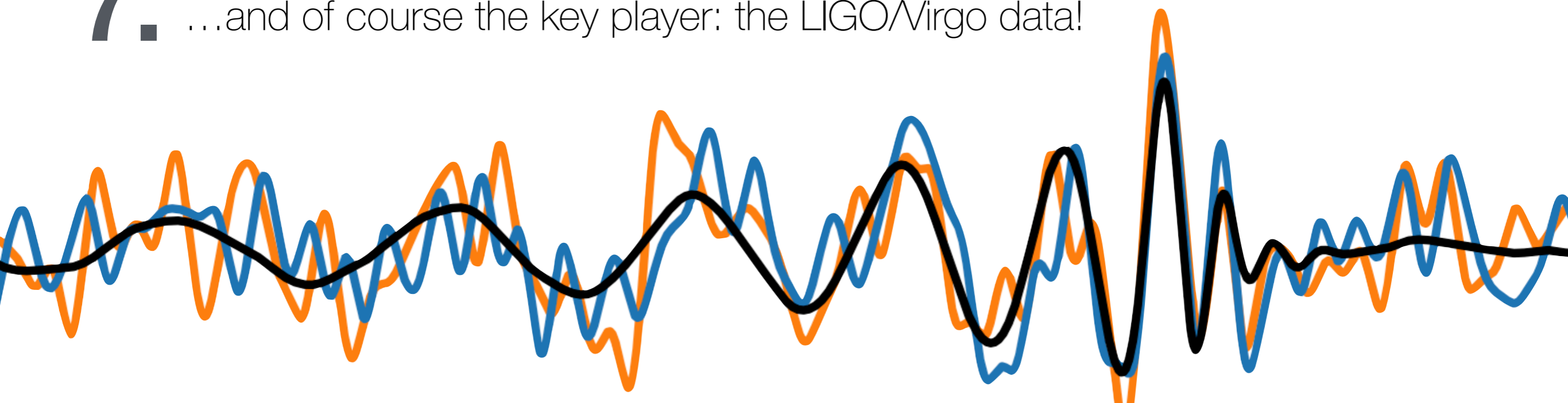
- A simple SNR cut? [Finn Chernoff 1992](#)
- Pipeline injections? [LIGO/Virgo 2019, 2021](#)
- Some attempts at machine-learn the GW detectability.

**DG** [Pratten Vecchio 2020, Talbot Thrane 2022](#)

6. A sampler for  $p(\lambda|d)$

- A vanilla nested sampling for now... but should we?

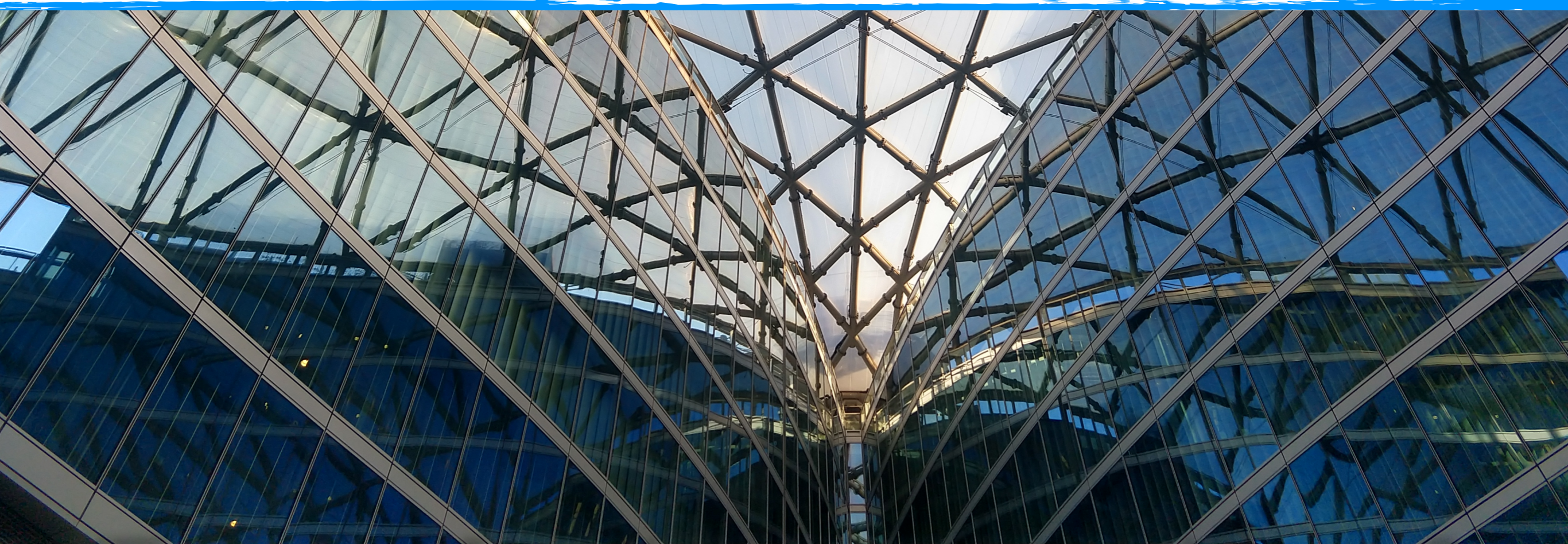
7. ...and of course the key player: the LIGO/Virgo data!





# Outline

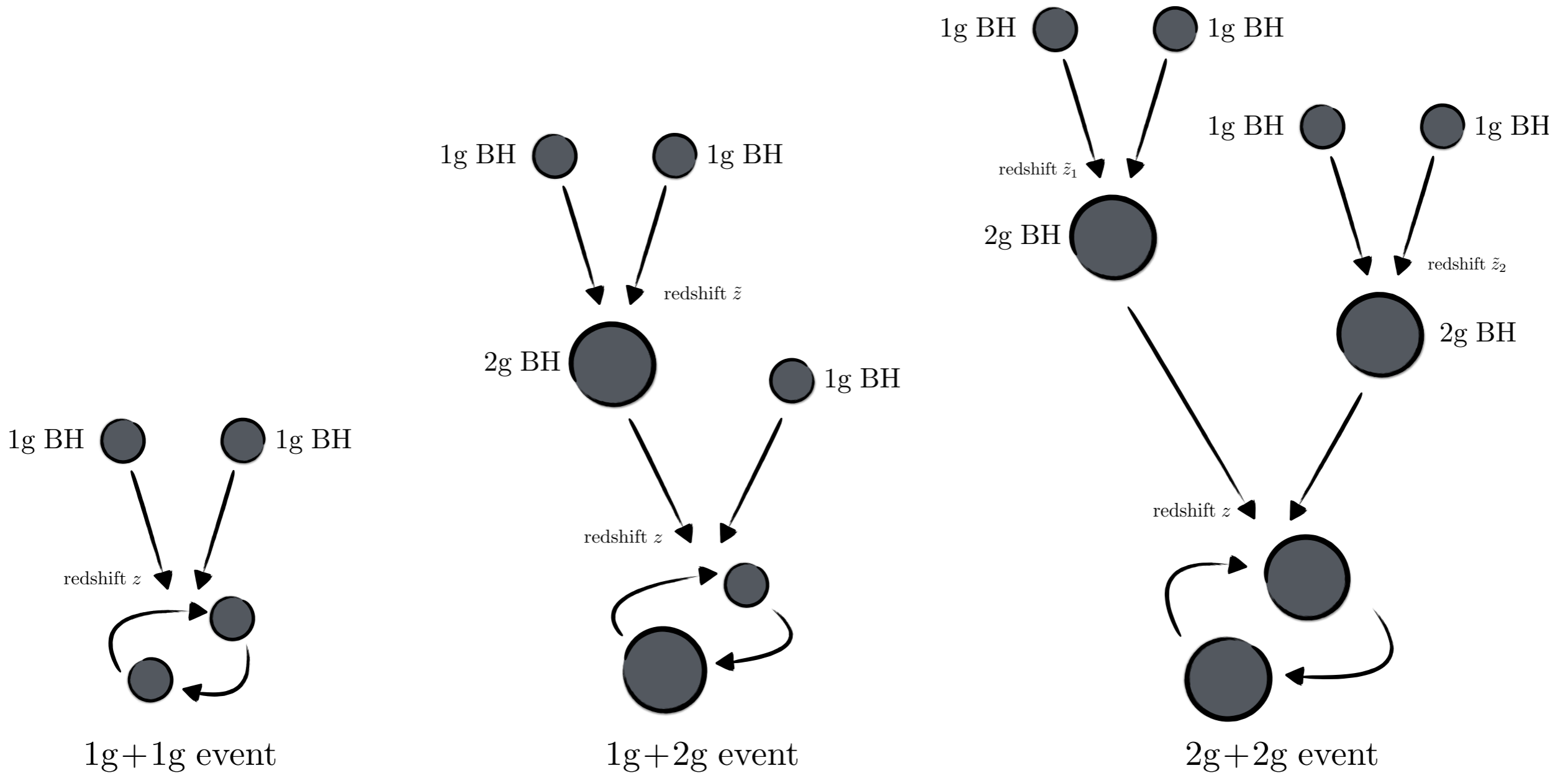
1. The population problem
2. Current pipeline: hierarchical mergers
3. Current pipeline: machine learning
4. Things we (haven't) figured out (yet)





# Hierarchical black-hole mergers

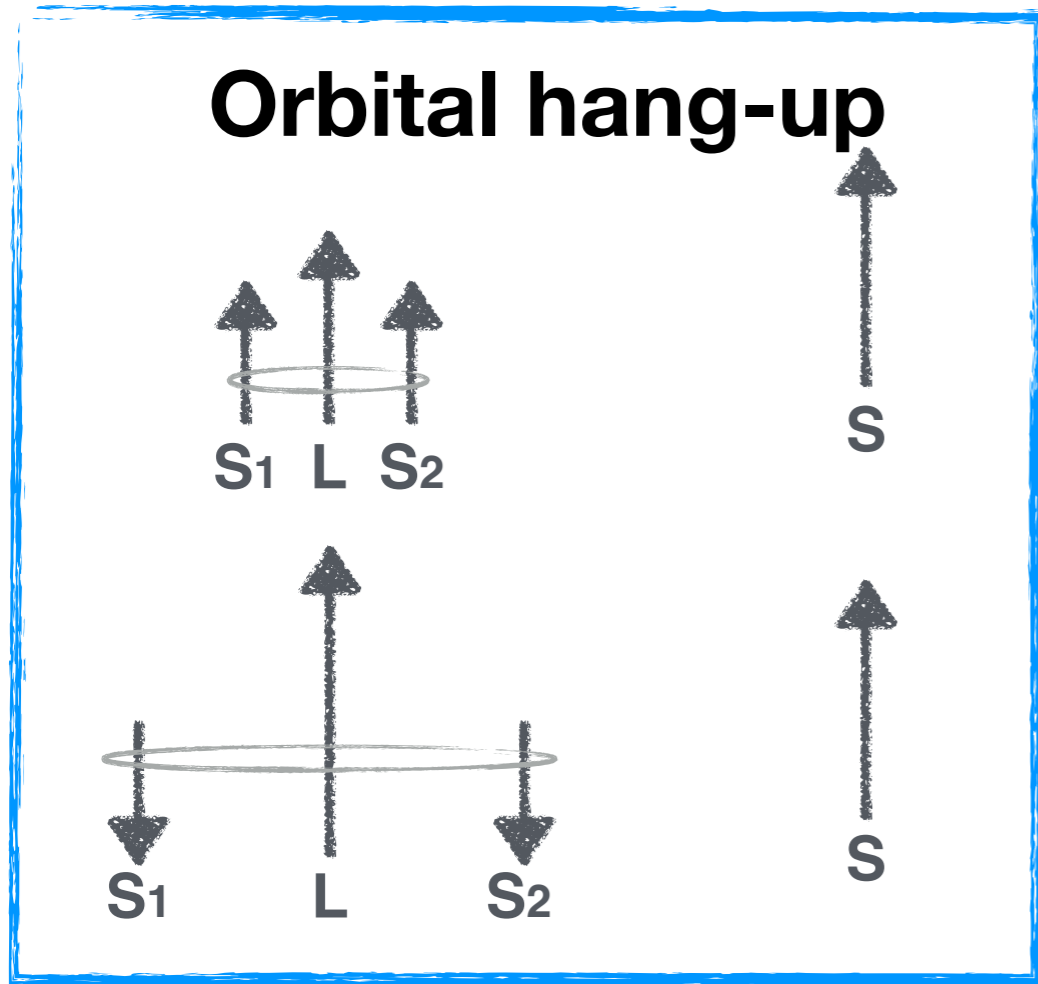
DG Berti 2017



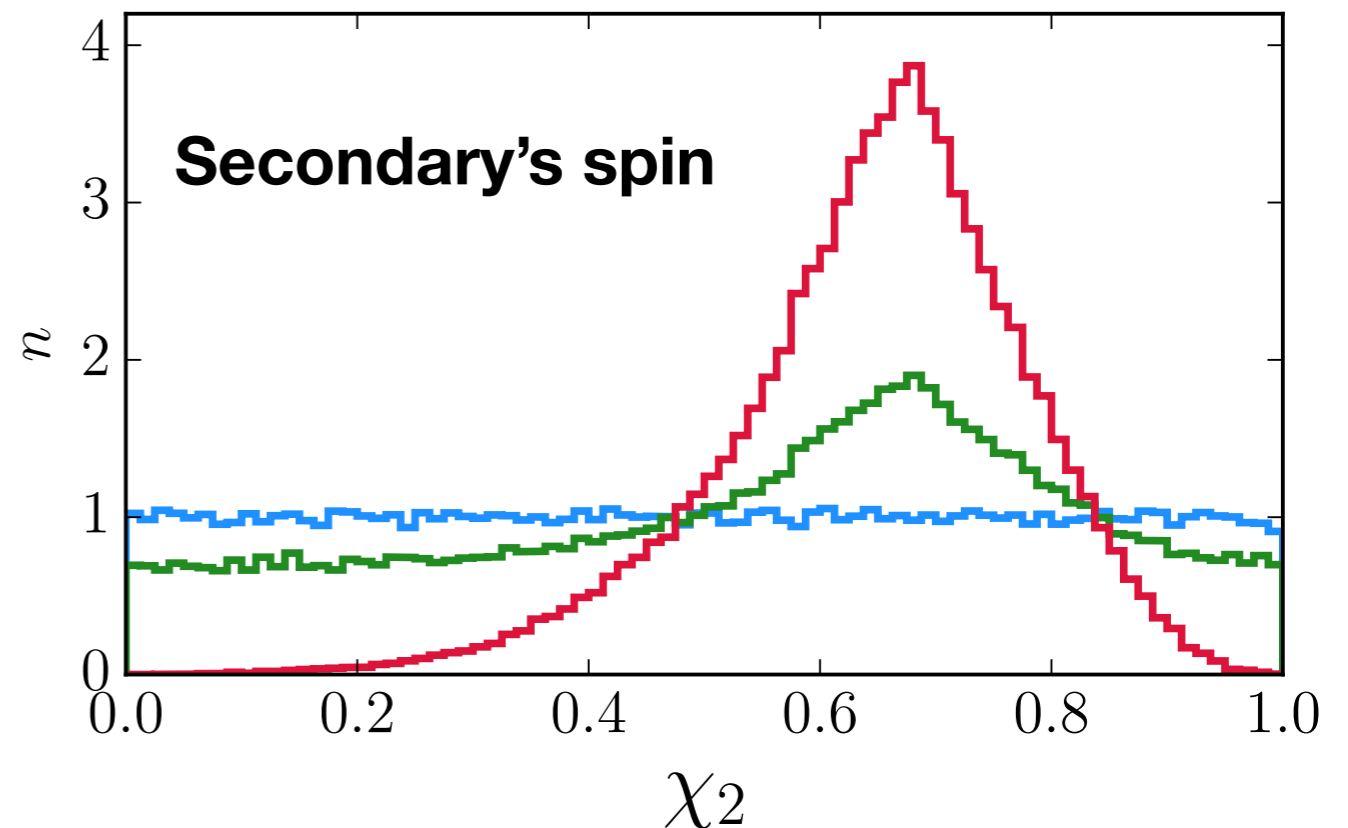
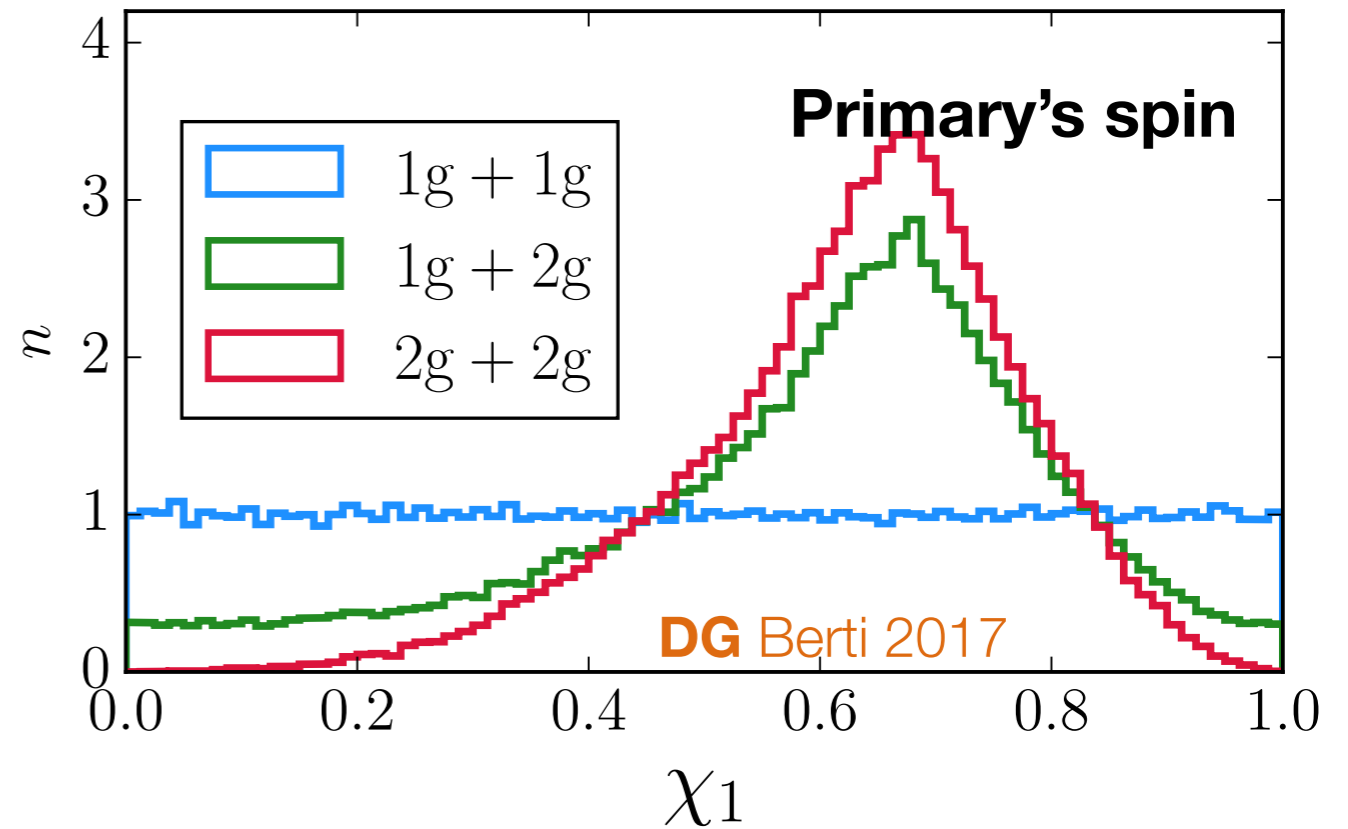
**Orthogonal**, but complementary, direction to the usual field vs. cluster debate

# Spins: the magic number

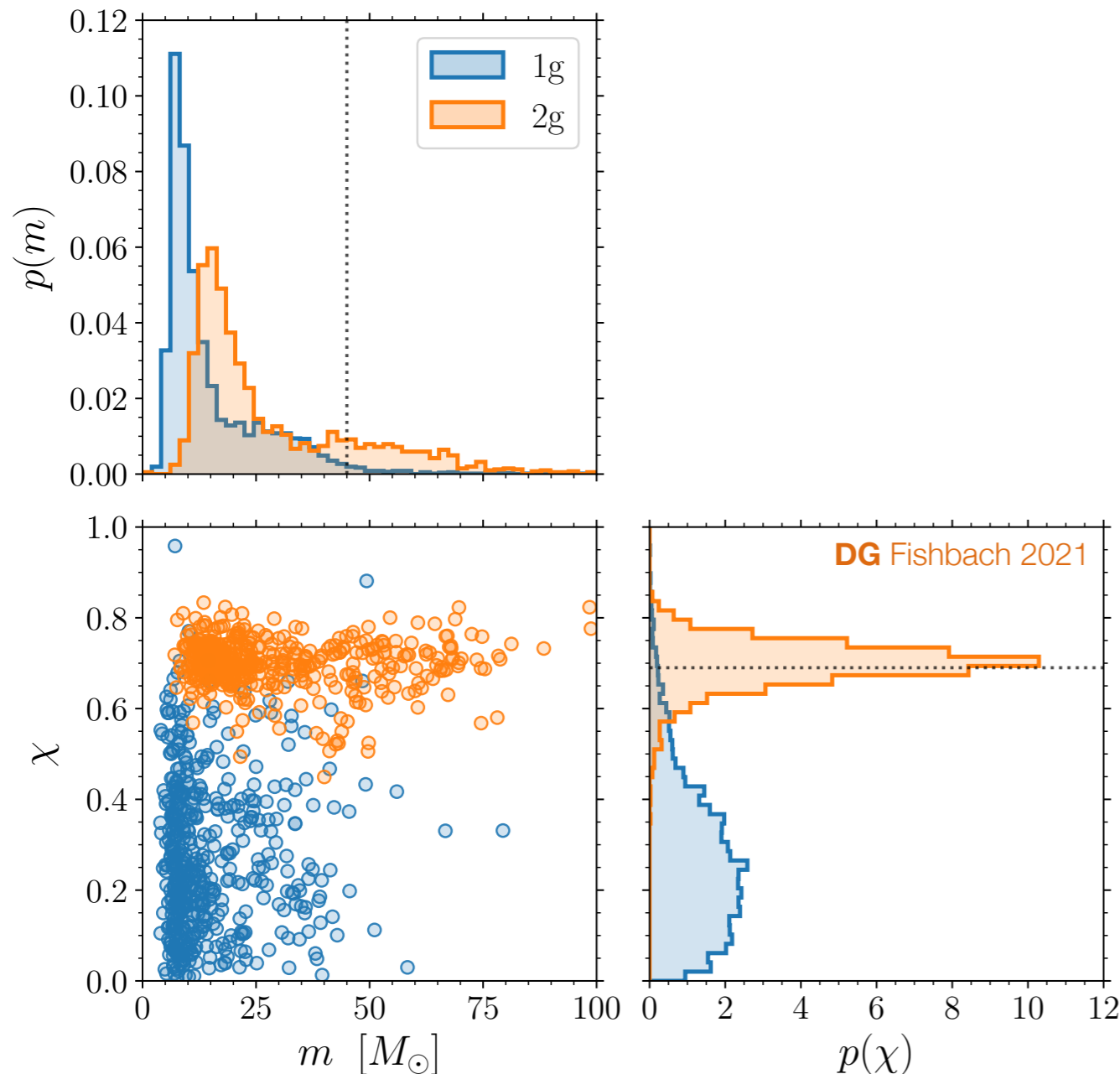
DG Berti 2017, Fishbach+ 2017, Berti Volonteri 2008



Peculiar spin distribution  
peaked at **0.7**



# An explosion of new predictions



- Masses in the pair-instability mass gap  
Heger+ 2003, Woosley+ 2007
- Peculiar spin distribution peaked at 0.7  
DG Berti 2017, Fishbach+ 2017
- But GW kicks require large escape speed  
DG Berti 2019
- Very frequent in AGNs  
Yang+ 2019, Tagawa+ 2020
- Promising for GW190412  
DG Vitale Berti 2020, Rrogriguez+ 2020
- Leading explanation for GW190521  
LIGO/Virgo 2020
- Perhaps several events in the LIGO catalog?  
Kimball+ 2021
- An exclusion region  
DG Giacobbo Vecchio 2020
- ... but don't overdo it!  
Zevin Holz 2022

And many more! Enough for a dedicated review DG Fishbach 2021



# Just balls of black holes for now....

We need a population that is easy enough for now but non-analytic...

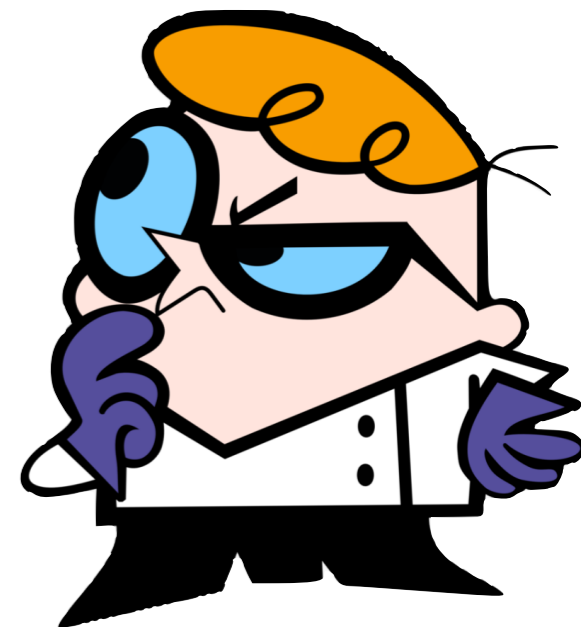
Key idea: **take a parametrized model but allow for hierarchical mergers**

In this talk a cluster is... a “thing” with a given escape speed  $v_{\text{esc}}$

DG, Berti 2019, DG Giacobbo Vecchio 2021, Zevin Holz 2022

- Masses:  $p(m) \propto m^\gamma$   $m \in [5M_\odot, m_{\text{max}}]$
- Spins:  $p(\chi) = \text{const}$   $\chi \in [0, \chi_{\text{max}}]$
- Pairing:  $p_{\text{pair}}(m_1) \propto m_1^\alpha$   
 $p_{\text{pair}}(m_2|m_1) \propto m_2^\beta$
- Clusters:  $p(v_{\text{esc}}) \propto v_{\text{esc}}^\delta$

Here is a real expert on  
BH dynamics when  
hearing about my model



# Just balls of black holes for now....

We need a population that is easy enough for now but non-analytic...

Key idea: **take a parametrized model but allow for hierarchical mergers**

In this talk a cluster is... a “thing” with a given escape speed  $v_{\text{esc}}$

DG, Berti 2019, DG Giacobbo Vecchio 2021, Zevin Holz 2022

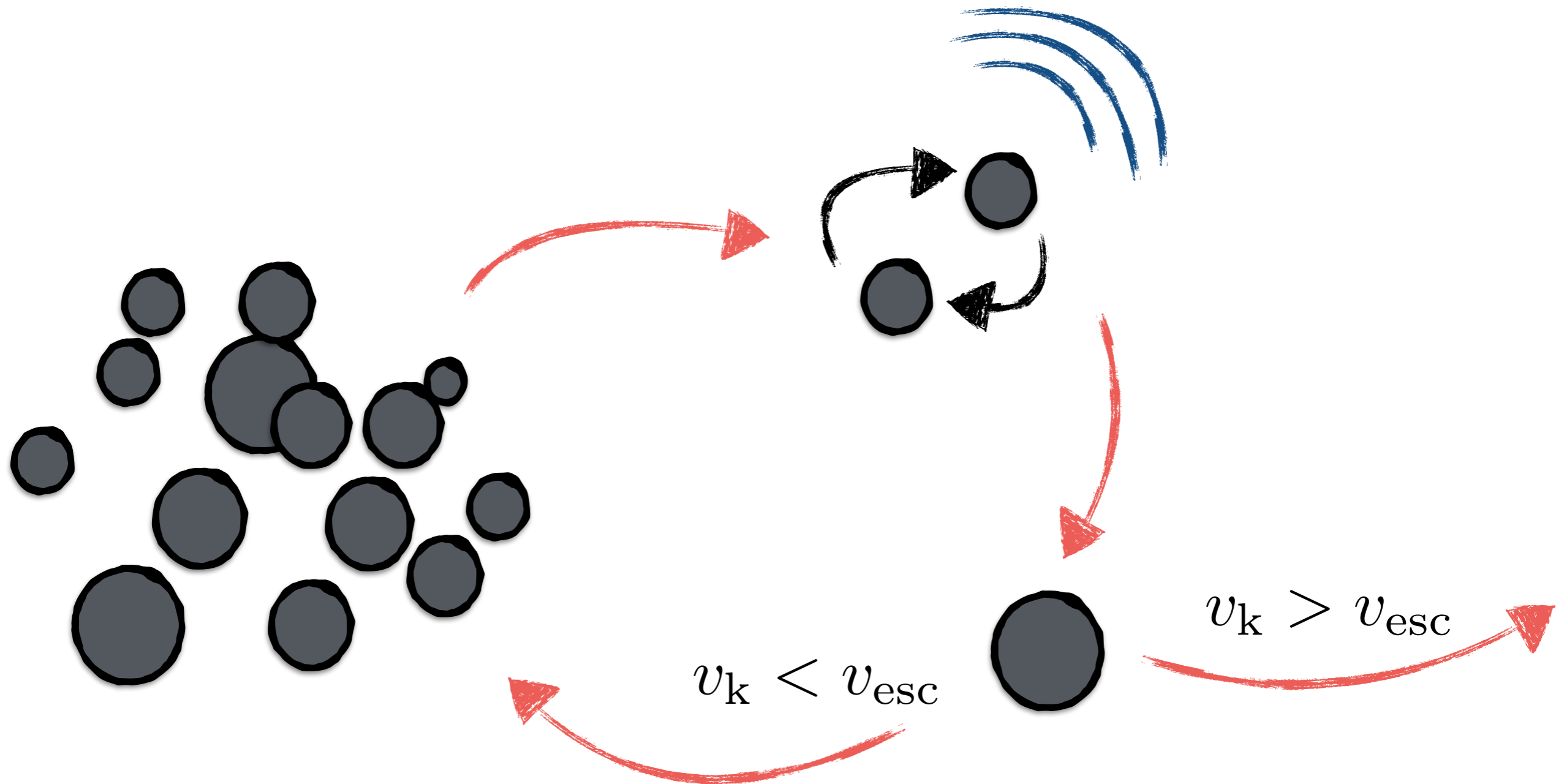
- Masses:  $p(m) \propto m^{\gamma}$   $m \in [5M_{\odot}, m_{\text{max}}]$
- Spins:  $p(\chi) = \text{const}$   $\chi \in [0, \chi_{\text{max}}]$
- Pairing:  $p_{\text{pair}}(m_1) \propto m_1^{\alpha}$   
 $p_{\text{pair}}(m_2|m_1) \propto m_2^{\beta}$
- Clusters:  $p(v_{\text{esc}}) \propto v_{\text{esc}}^{\delta}$

**Six population parameters**

Here is a real expert on  
BH dynamics when  
hearing about my model



# Out of the cluster, one kick at the time

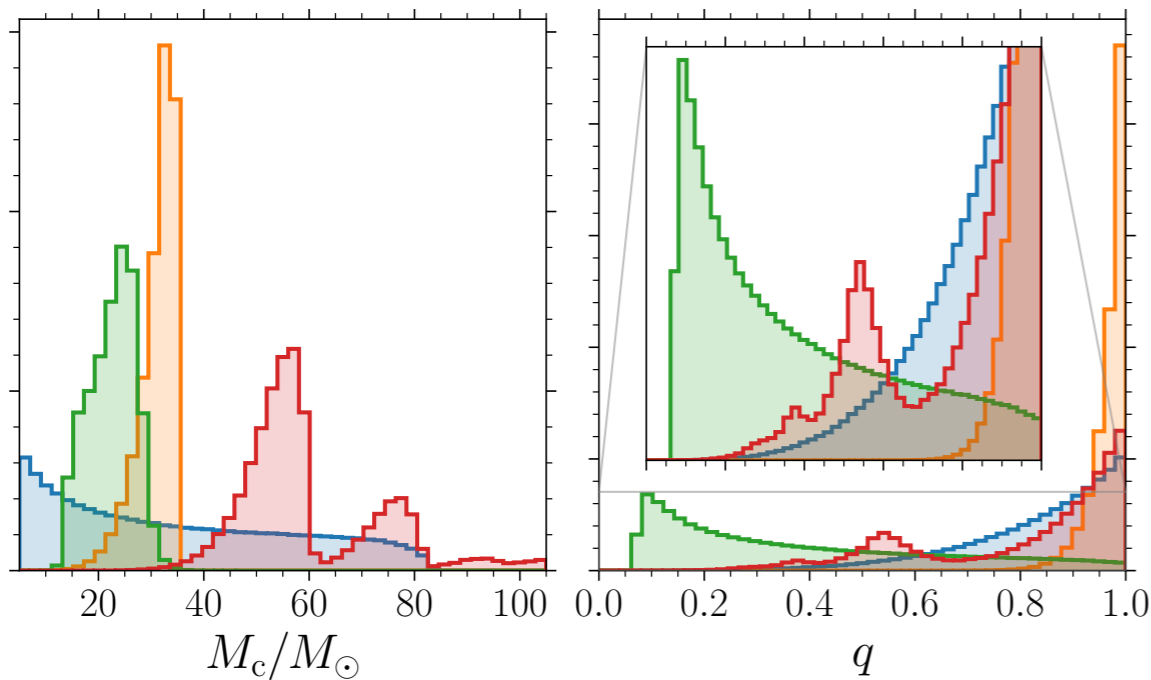




# Targeted populations

Tackling inference on four event parameters  $\theta = \{M_c, q, \chi_{\text{eff}}, \chi_p\}$

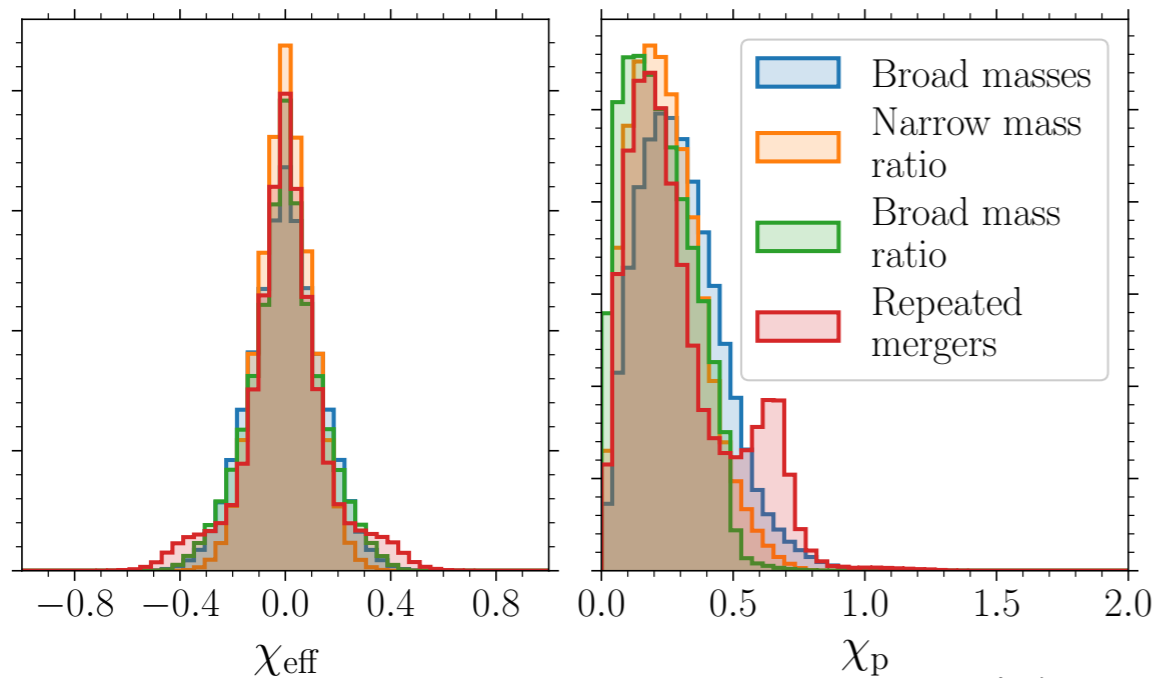
Mould, **DG**, Taylor 2022



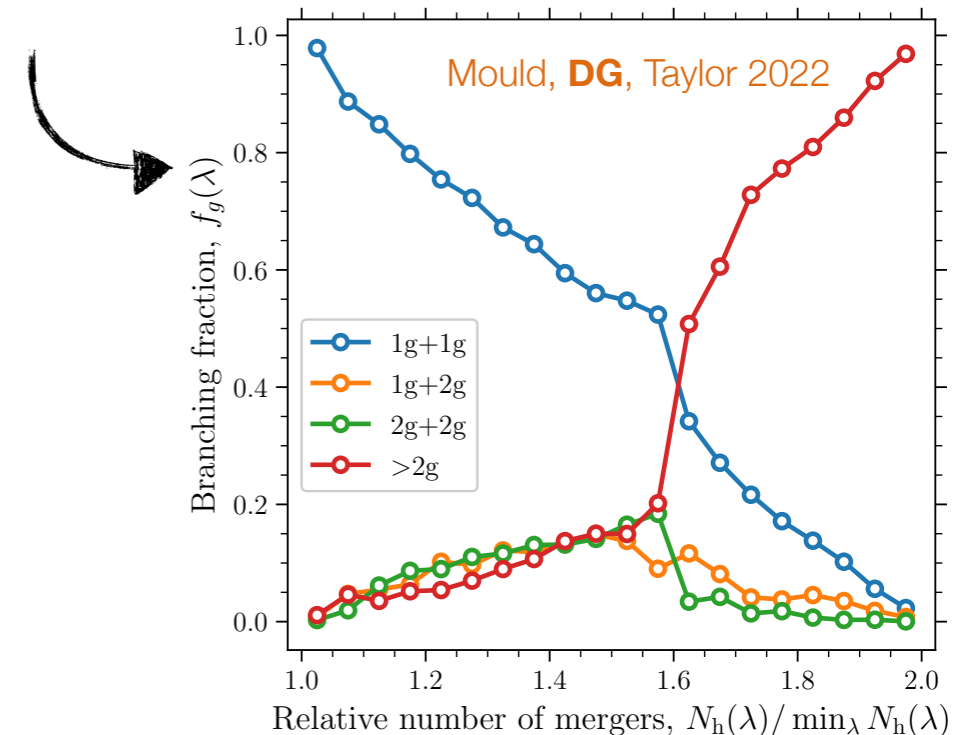
Four representative hyperparameter locations

**This is a hard problem!**  
 Strong correlations, multimodalities,  
 spikes, gaps, degeneracies

... and we keep track of the merger generation



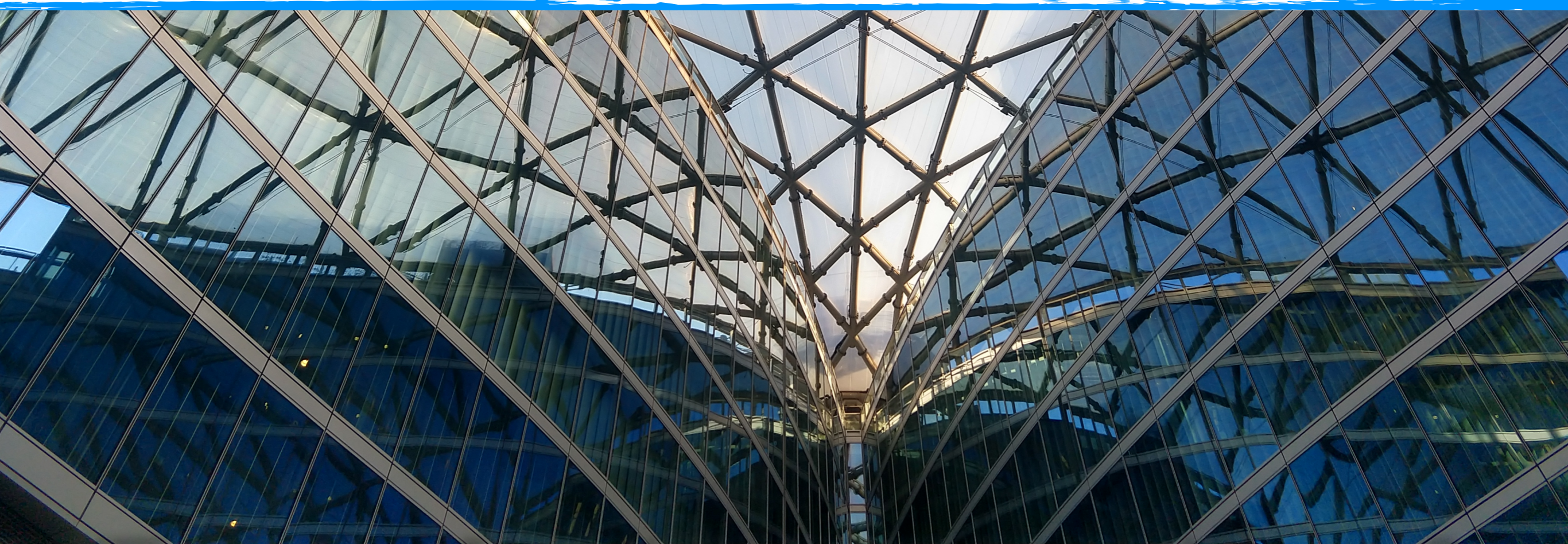
Ask me at coffee  
 why  $\chi_p$  goes up to 2... **DG+** 2021





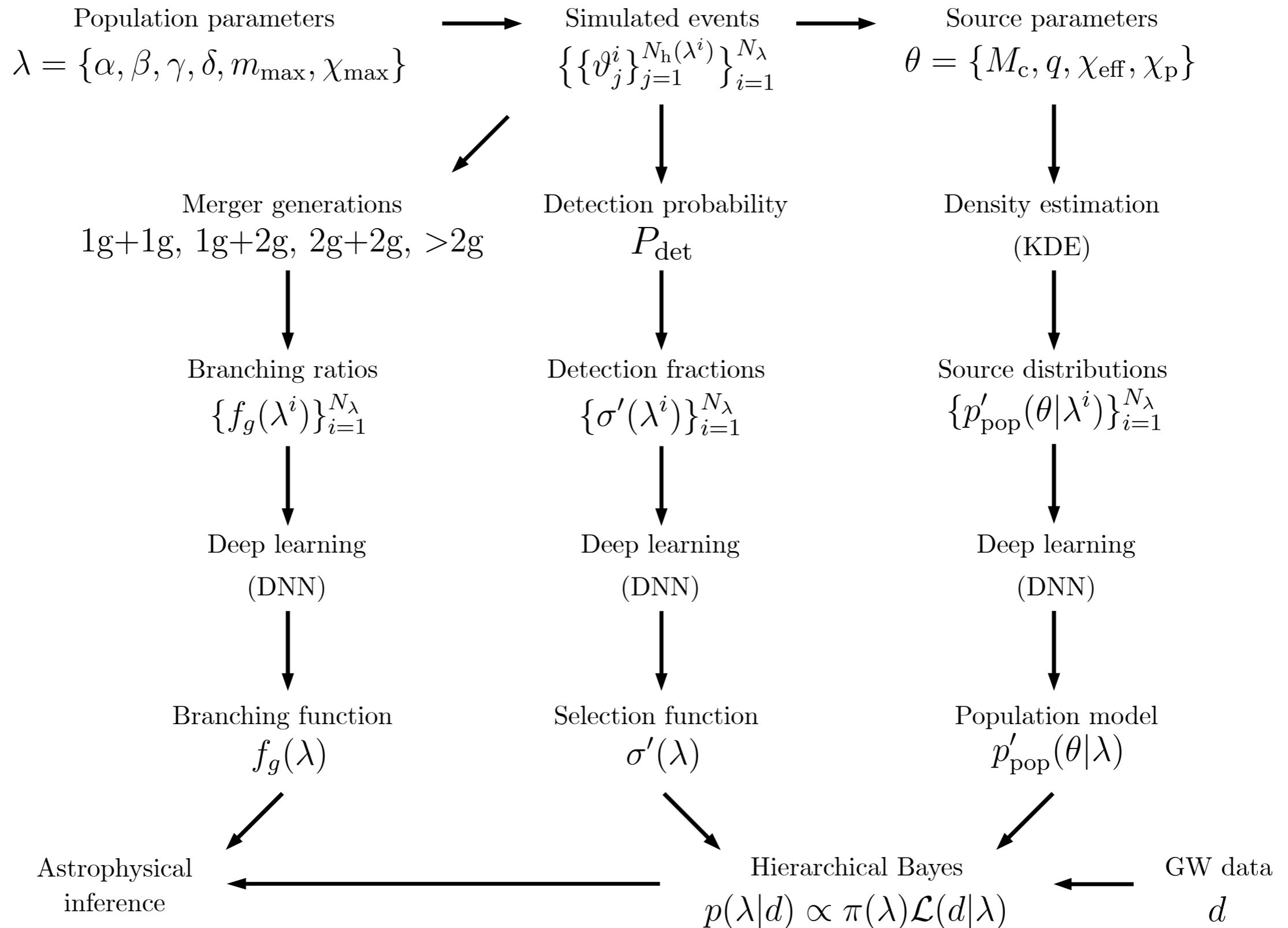
# Outline

1. The population problem
2. Current pipeline: hierarchical mergers
3. Current pipeline: machine learning
4. Things we (haven't) figured out (yet)





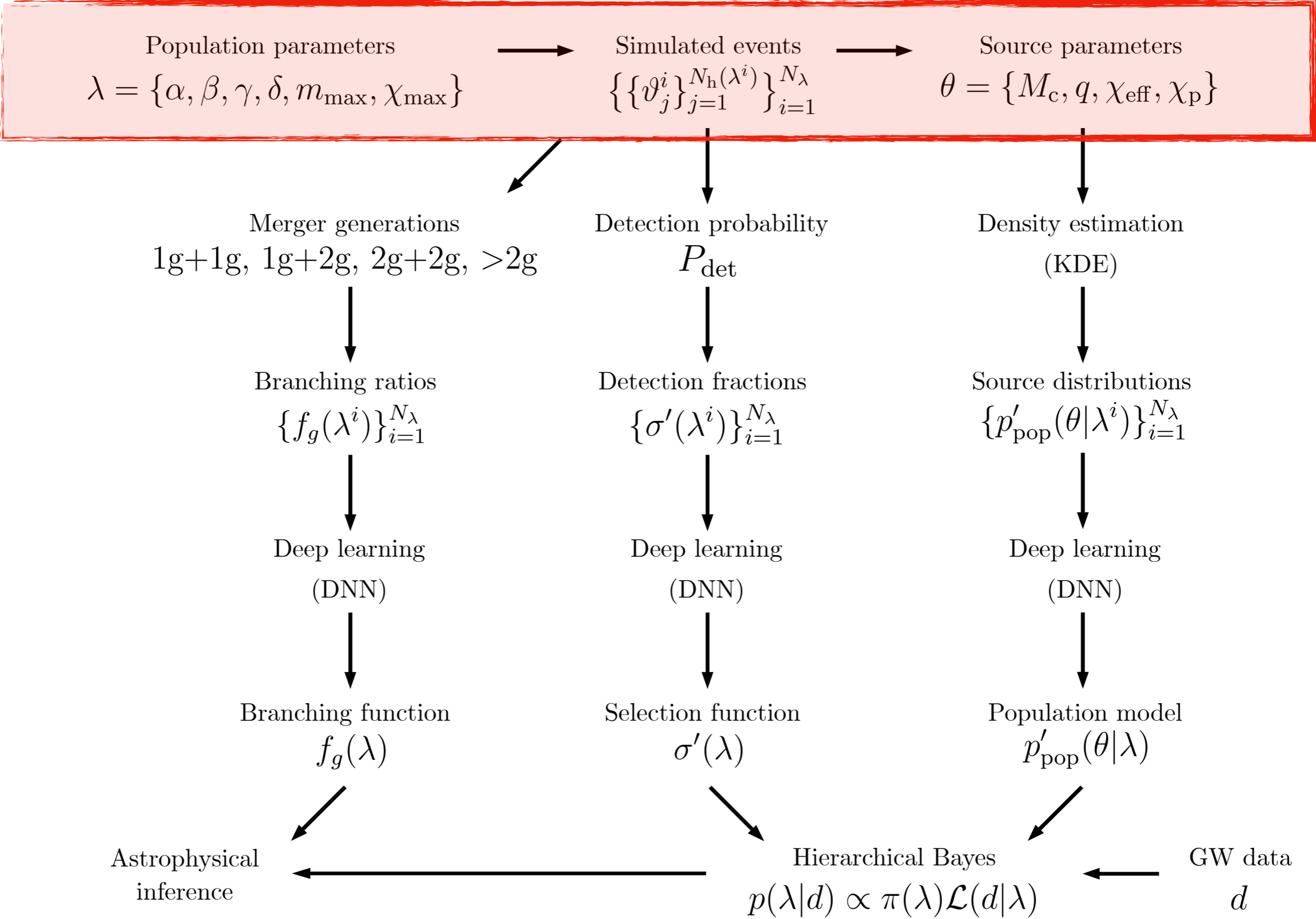
# Full pipeline



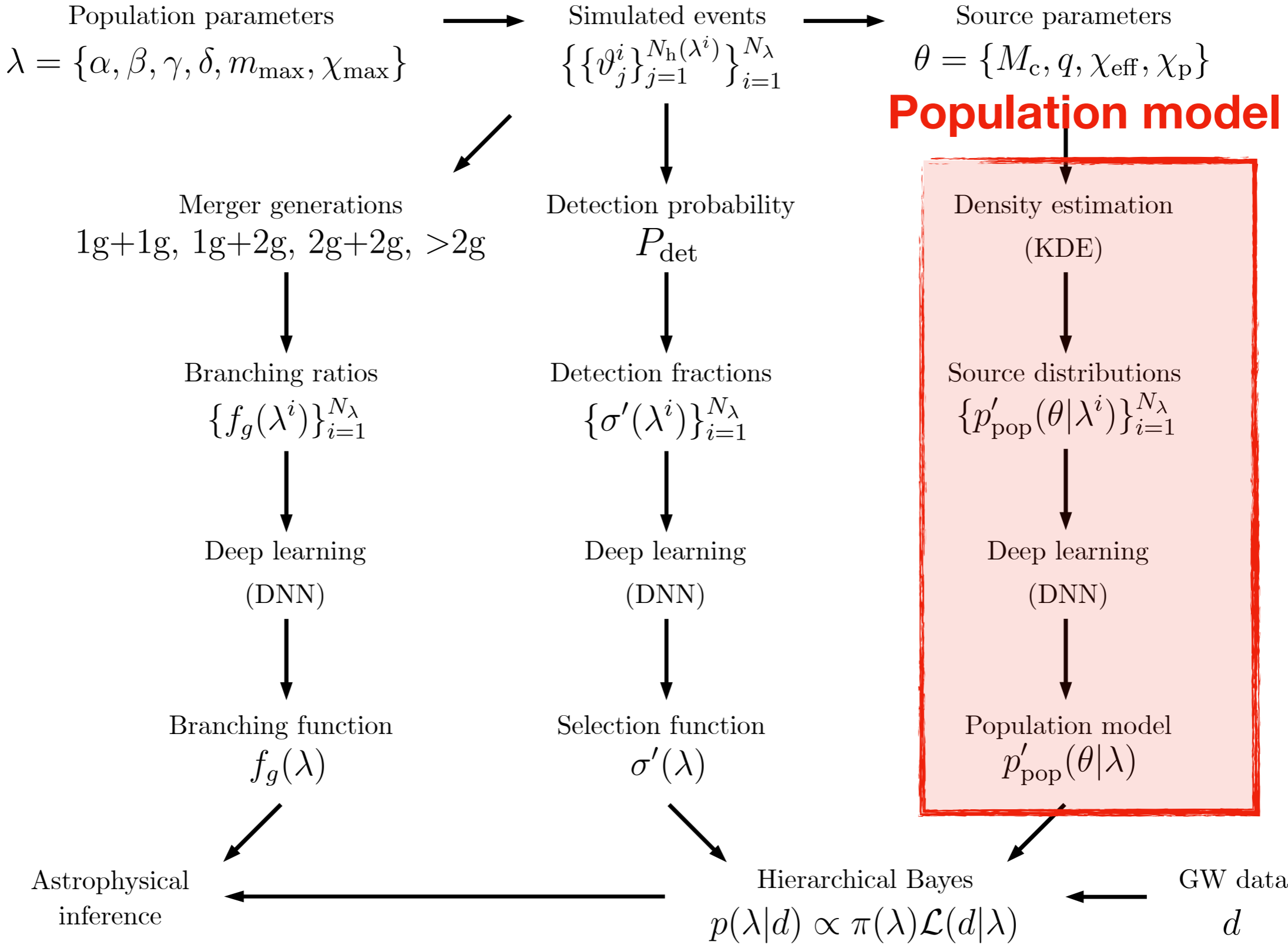


# Full pipeline

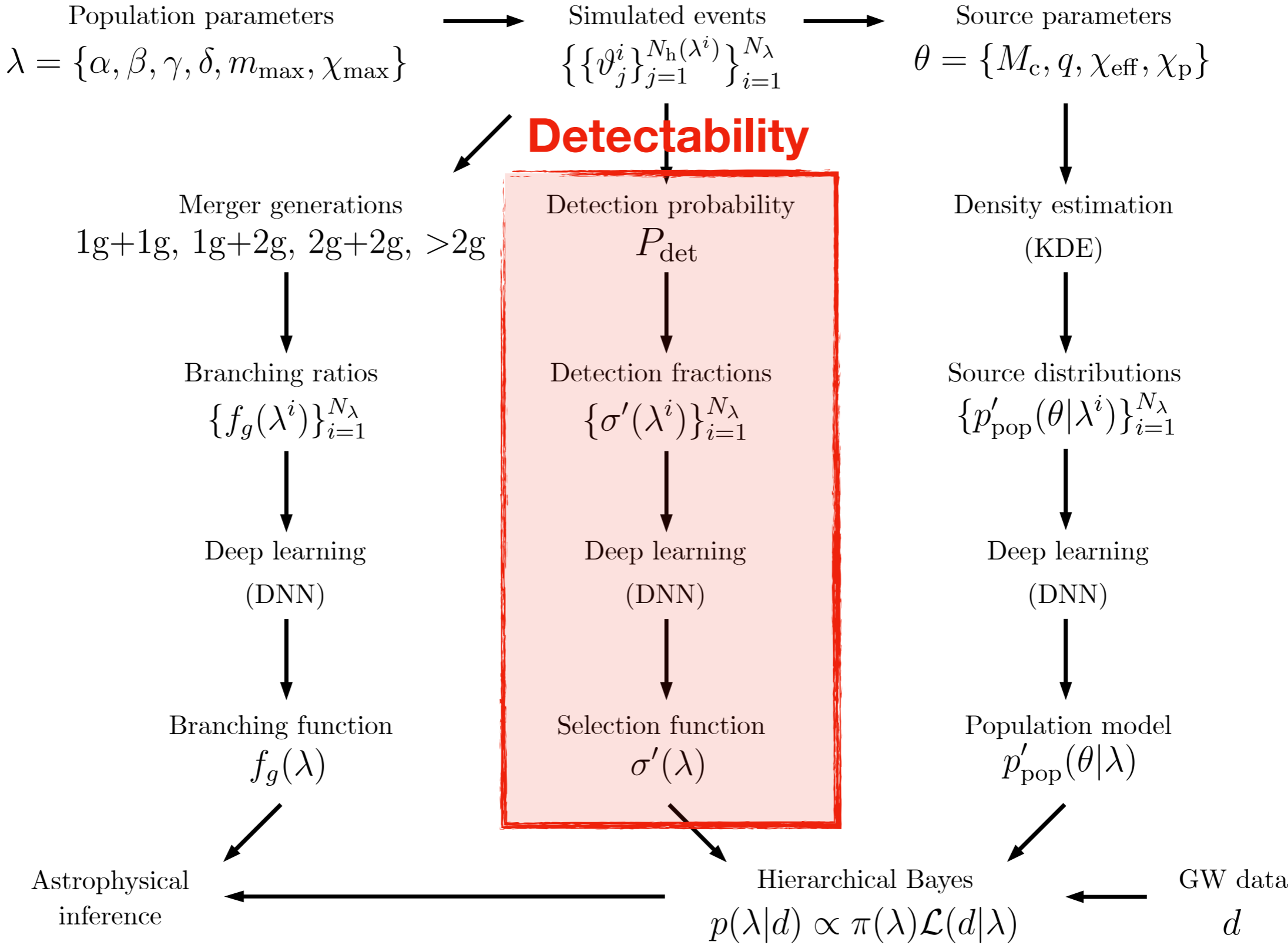
## Simulations



# Full pipeline

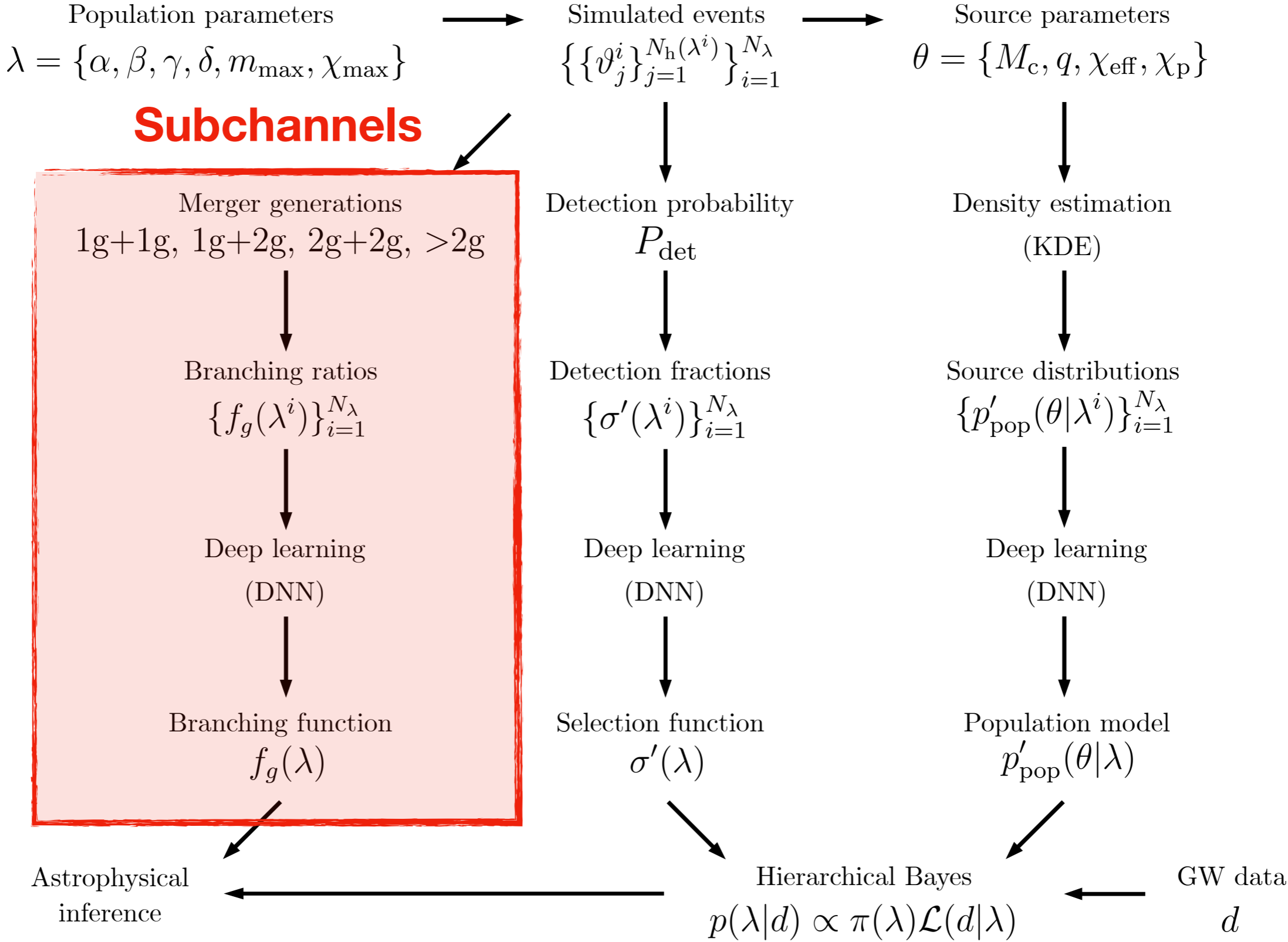


# Full pipeline

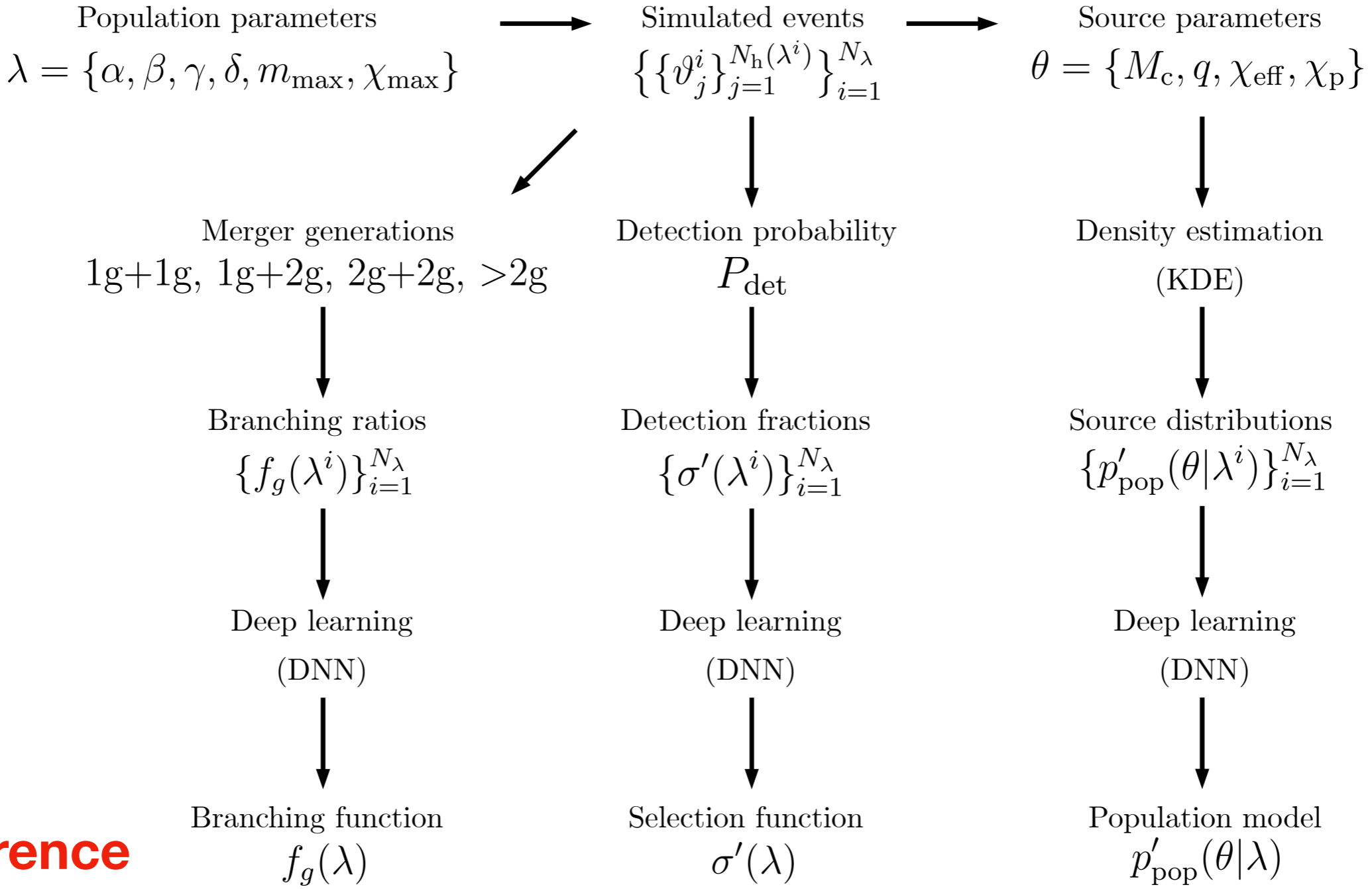




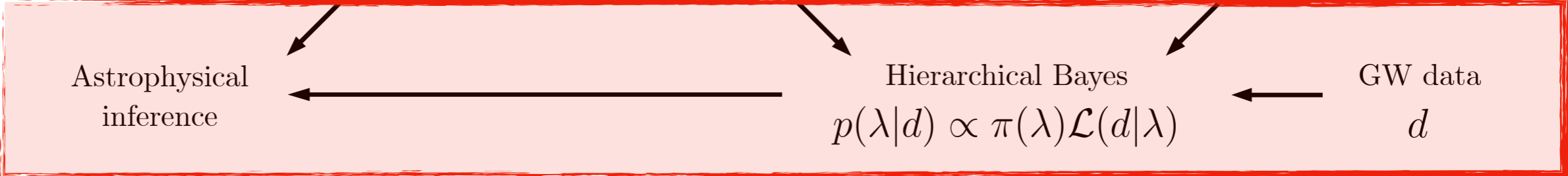
# Full pipeline



# Full pipeline



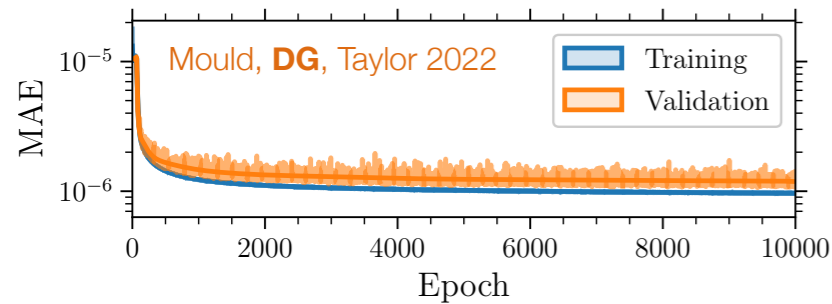
**Inference**



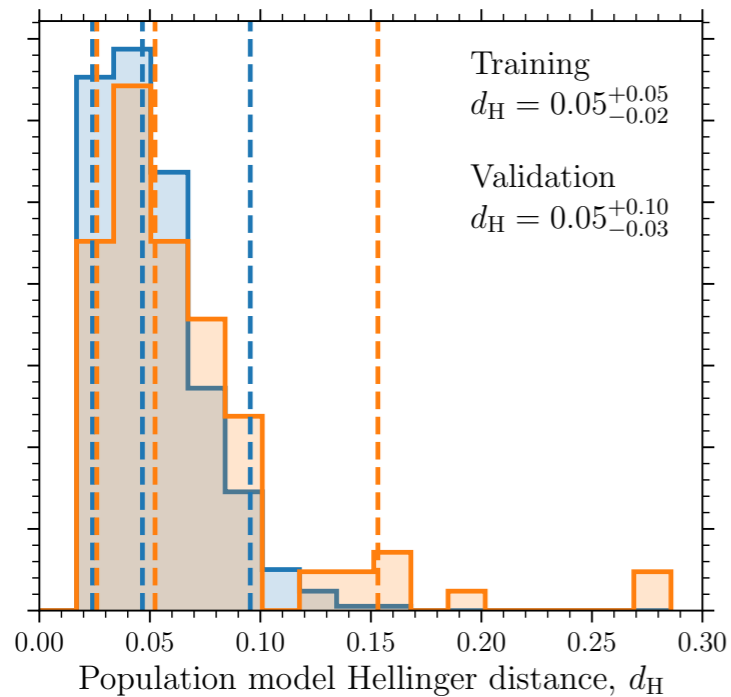
# Population neural network

- A fully connected network
- A total of **~70k parameters!**
- Implemented in Google's Tensorflow
- Fast (~days) training on GPU

| Layer   | Neurons | Activation     | Parameters |
|---------|---------|----------------|------------|
| Input   | 10      | –              | 0          |
| Dense 1 | 128     | RReLU          | 1408       |
| Dense 2 | 128     | RReLU          | 16,512     |
| Dense 3 | 128     | RReLU          | 16,512     |
| Dense 4 | 128     | RReLU          | 16,512     |
| Dense 5 | 128     | RReLU          | 16,512     |
| Output  | 1       | Absolute value | 129        |
| Total   |         |                | 67,585     |



No significant overfitting

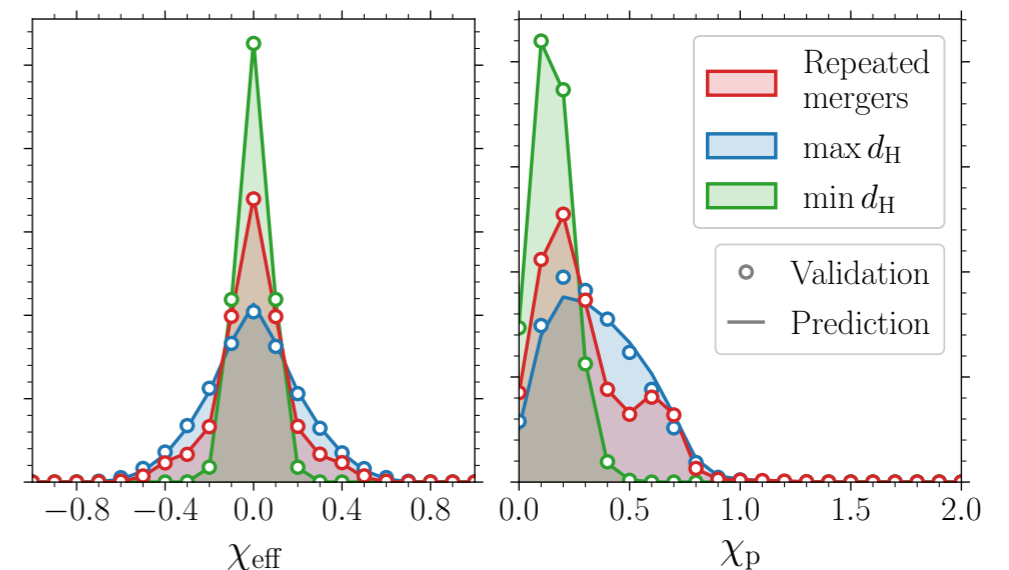
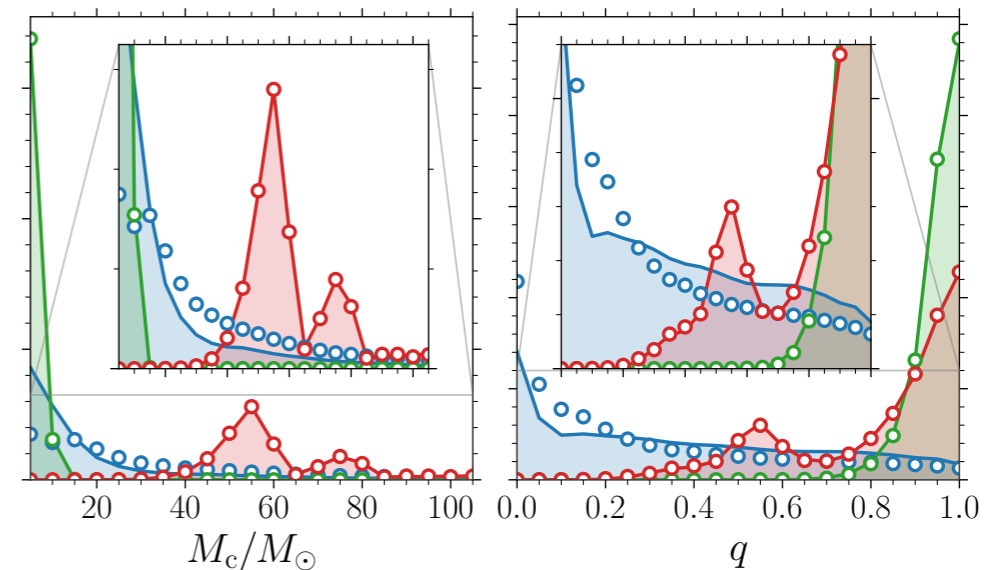


Interpolated distributions are statistically the same

Able to capture spikes and almost-discontinuous features

$$d_H(p, q)^2 = 1 - \int \sqrt{p(x)q(x)} dx.$$

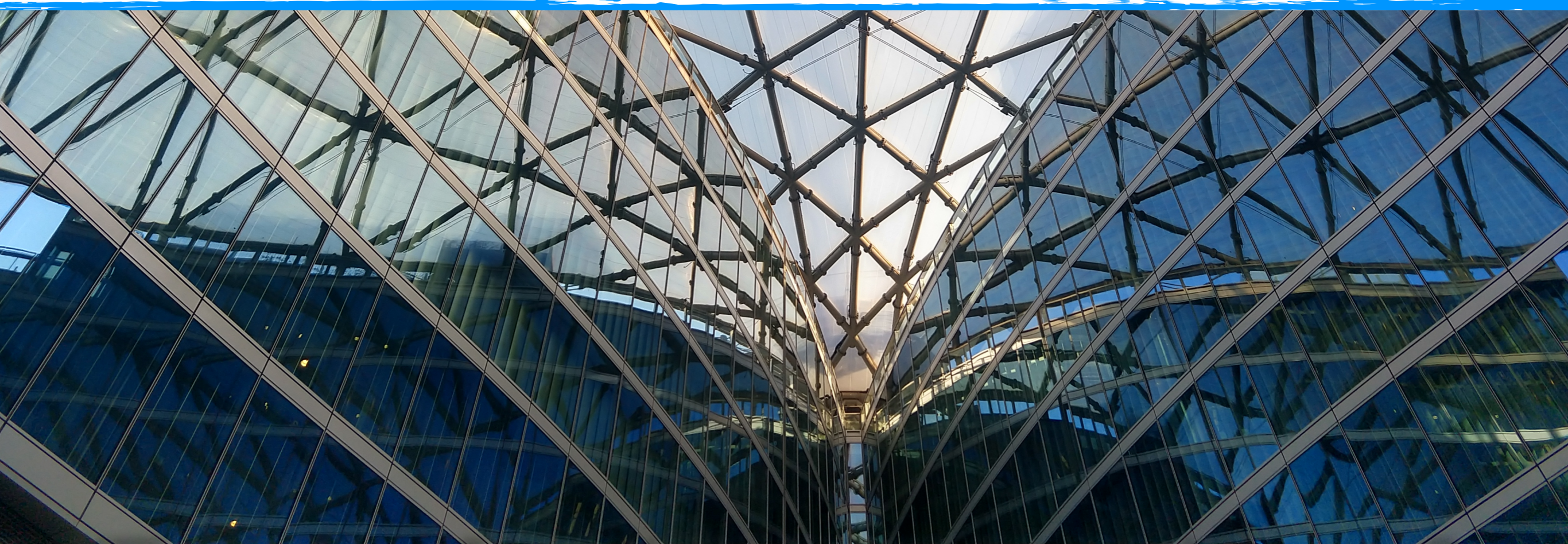
Mould, **DG**, Taylor 2022





# Outline

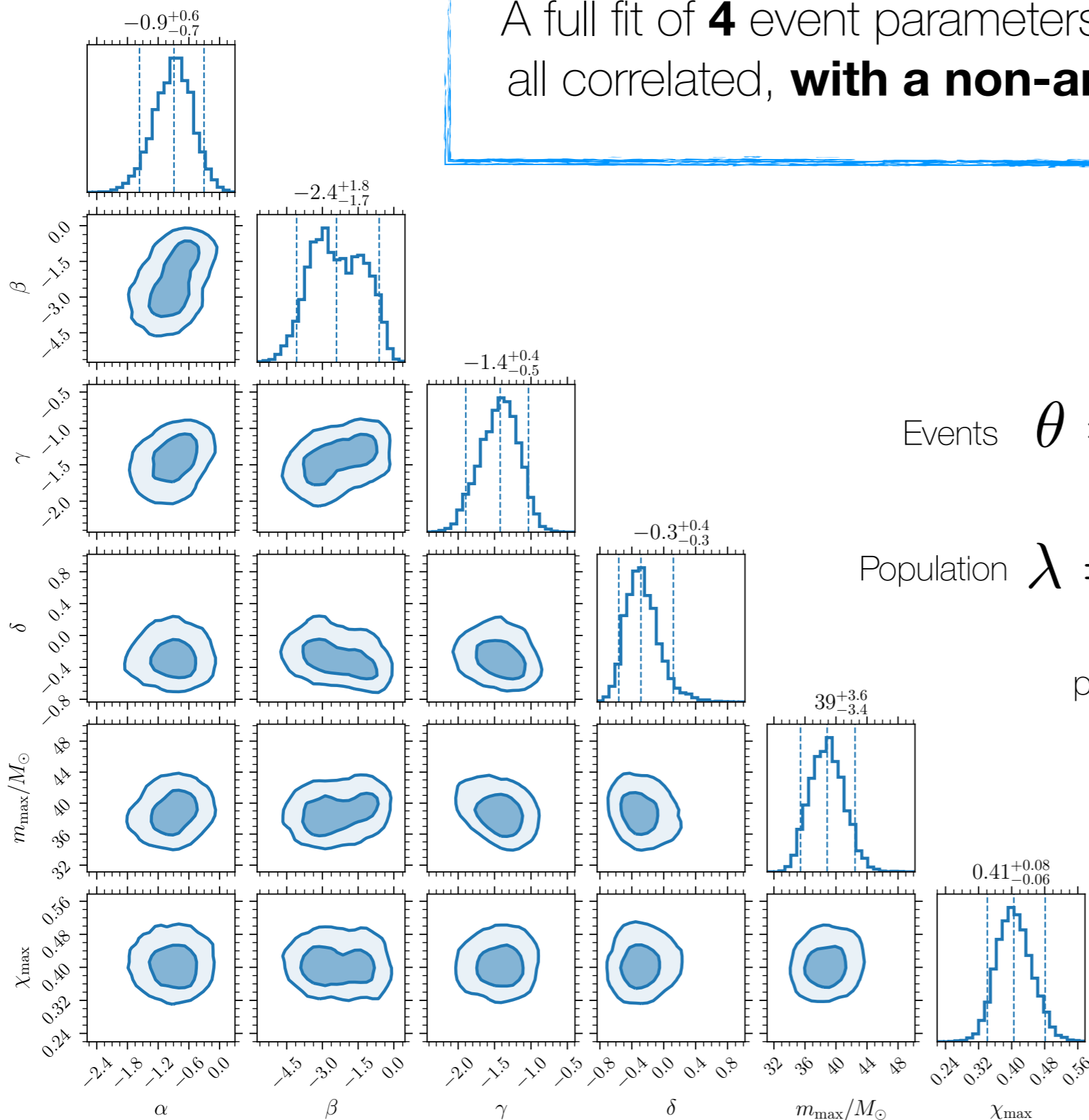
1. The population problem
2. Current pipeline: hierarchical mergers
3. Current pipeline: machine learning
4. Things we (haven't) figured out (yet)





# Full GWTC3 results

A full fit of **4** event parameters and **6** population parameters, all correlated, **with a non-analytical population model!**



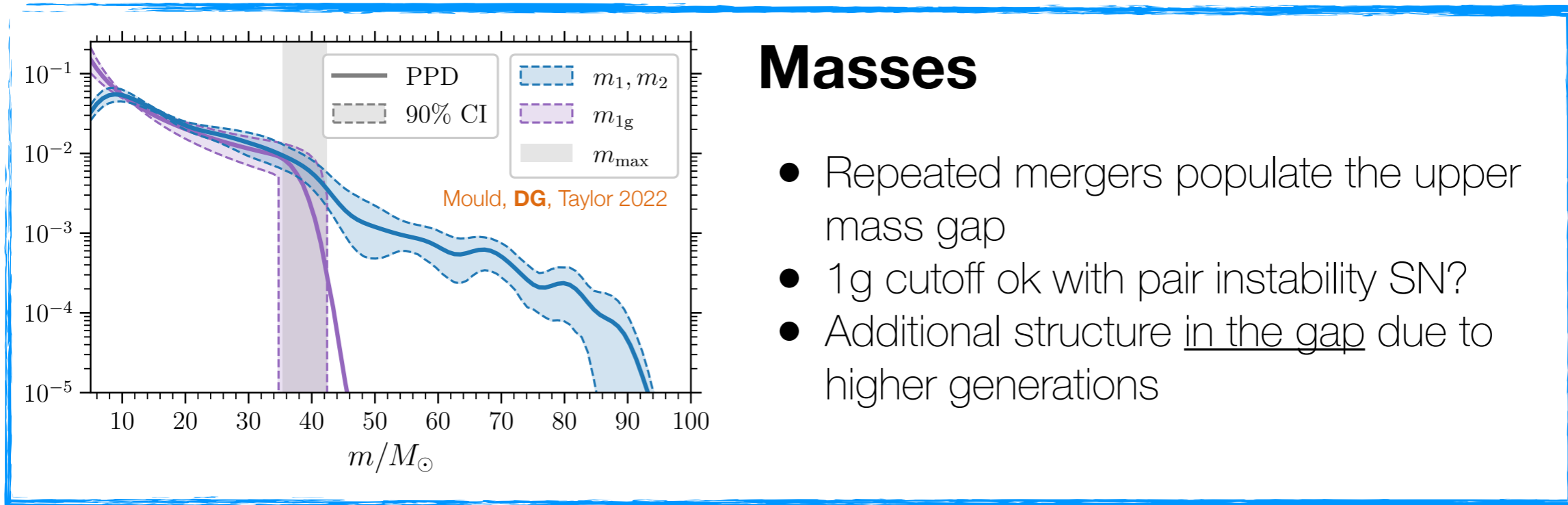
Events  $\theta = \{M_c, q, \chi_{\text{eff}}, \chi_p\}$

chirp mass      effective spin  
 mass ratio      precession

Population  $\lambda = \{\alpha, \beta, \gamma, \delta, M_{\text{max}}, \chi_{\text{max}}\}$

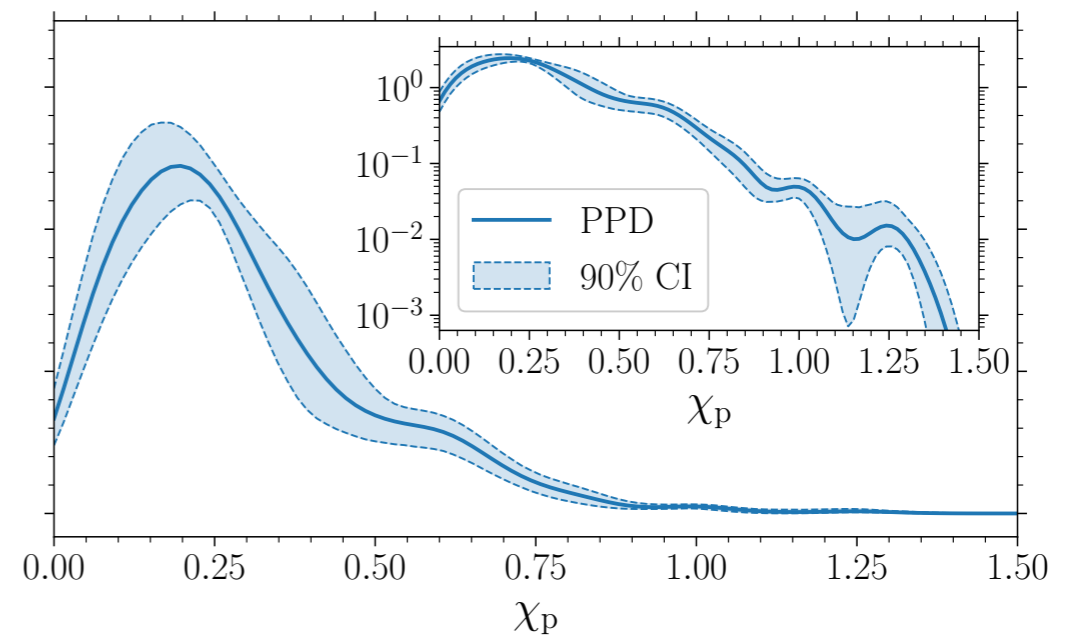
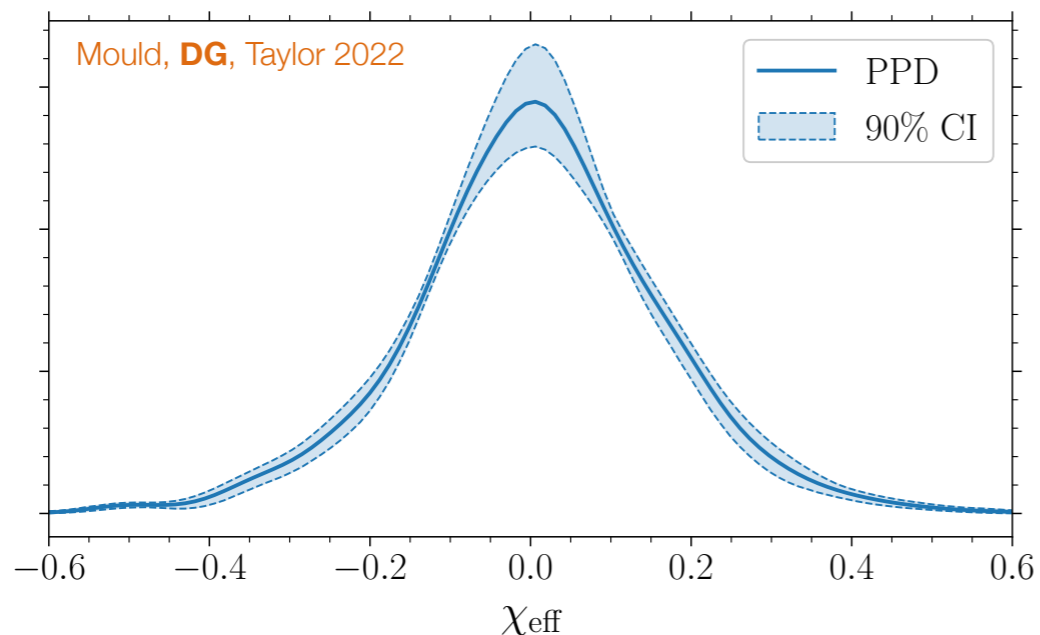
pairing      1g mass slope      max 1g mass  
 pairing      escape speeds      max 1g spin

# Inference and predictions



## Spins

- Fat tails in the effective spin
- Fine structures in spin precession

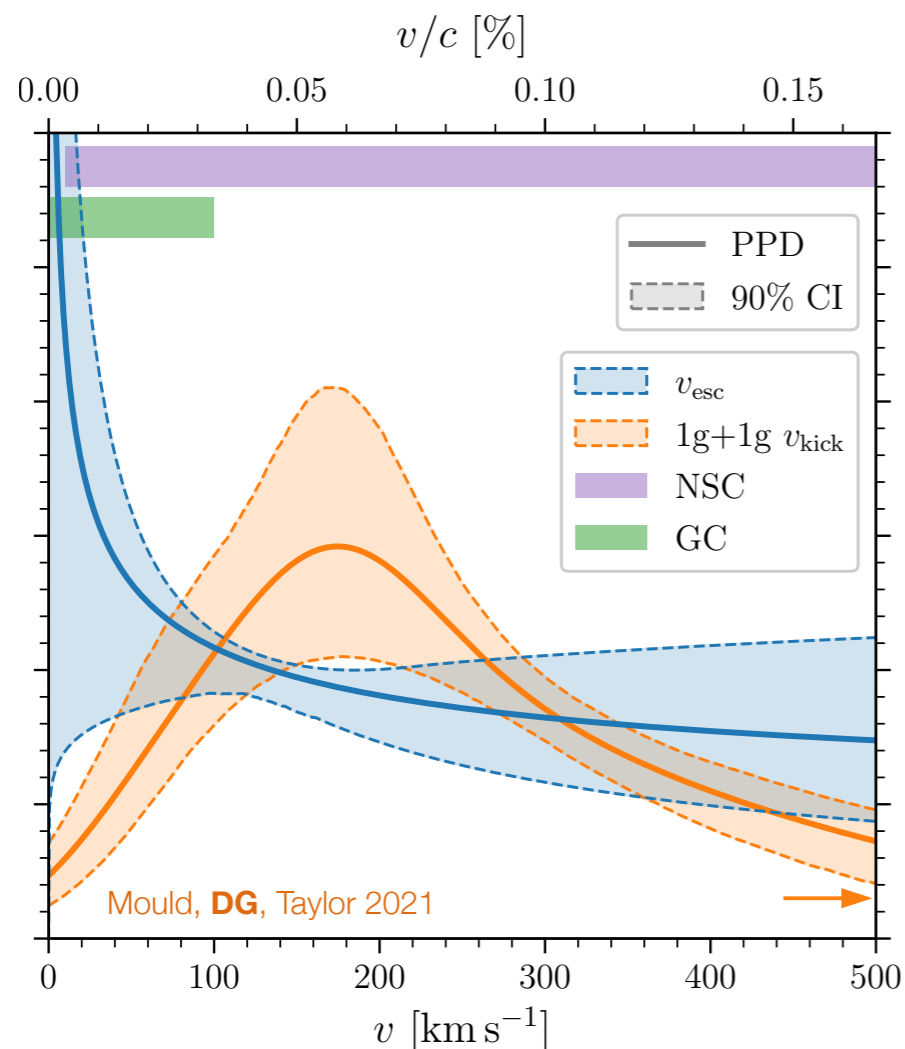




# Inference and predictions

## Escape speeds

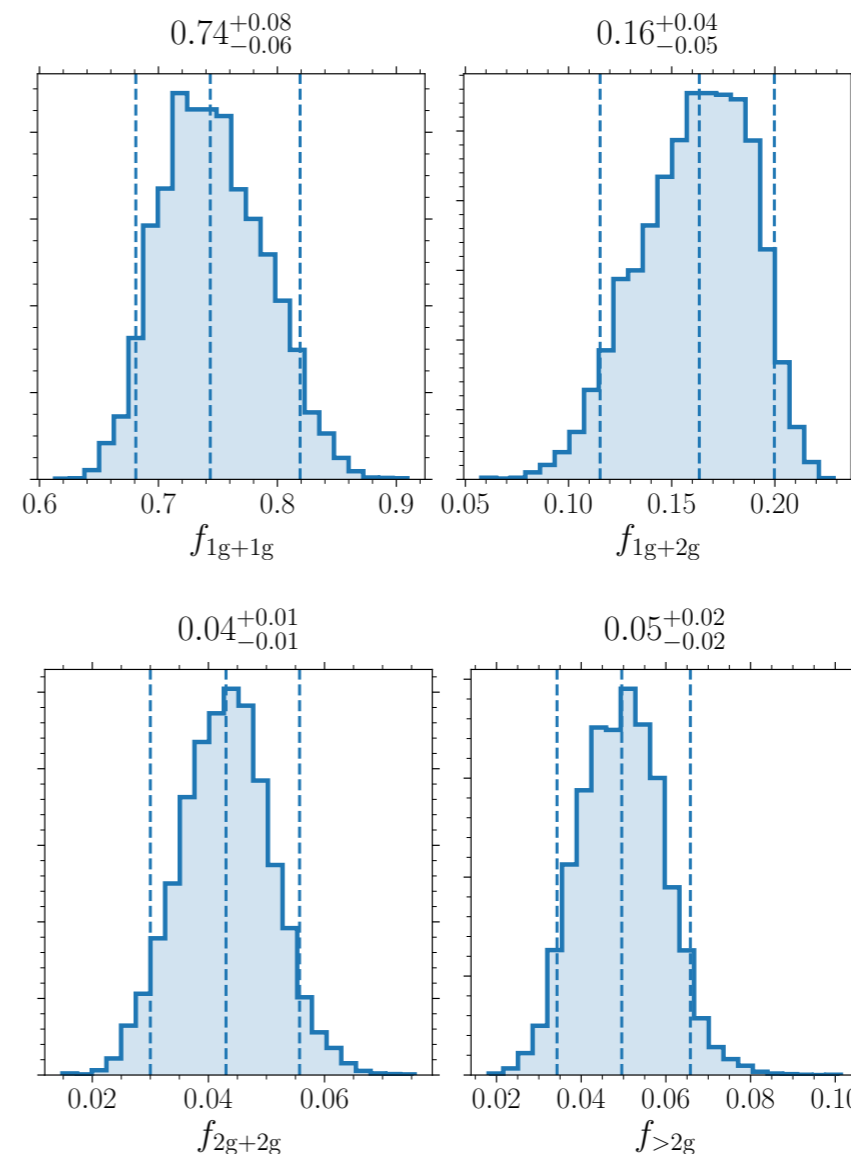
- Easy to infer secondary population parameters (here the escape speed)
- But can go crazy! Metallicity, environments, etc



## Generations

- If we allow for hierarchical mergers, the fit wants to go there! cf e.g. [Kimball+ 2021](#)
- Easy to infer subchannels (here the generation)
- But can go crazy! Any label in the population...

Mould, **DG**, Moore, in prep

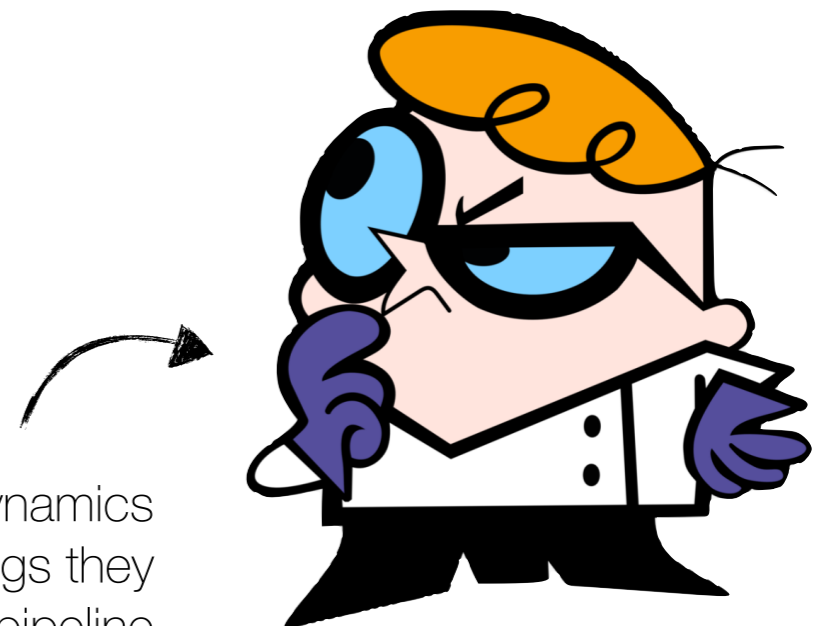


Mould, **DG**, Taylor 2022

# Ready to go!

- A complete, highly optimized population inference pipeline designed to digest outputs of astrophysical simulations and GW data
- **Deep learning is crucial here** (no other way, I think)
- Current astrophysics is admittedly too simple...
- ...but we're ready to use this beast on state-of-the-art models!

Here is a real expert on BH dynamics thinking of all the amazing things they could do with this pipeline



# A deep-learning solution to the gravitational-wave population problem

**Davide Gerosa**

University of Milano-Bicocca

**arXiv:2203.03651**

with M. Mould and S. Taylor

Jun 2, 2022

Black Hole Dynamics: From Gaseous  
Environments to Empty Space

Copenhagen, Denmark



European Research Council

Fondazione  
**CARIPLO**



LEVERHULME  
TRUST



davide.gerosa@unimib.it  
www.davidegerosa.com