Universality in the mass and eccentricity distribution in the dynamical channel

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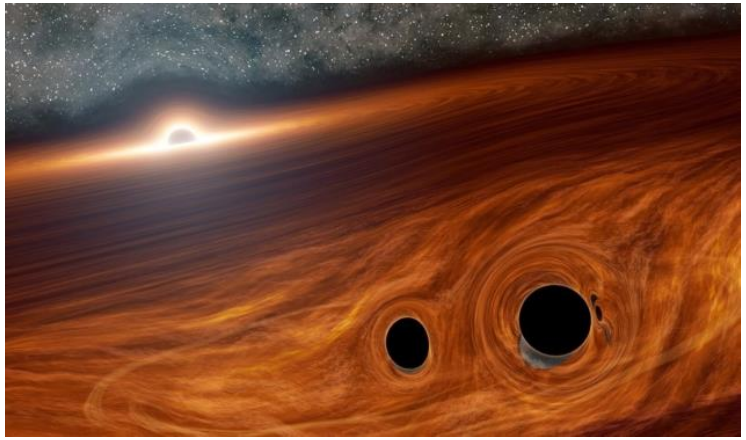
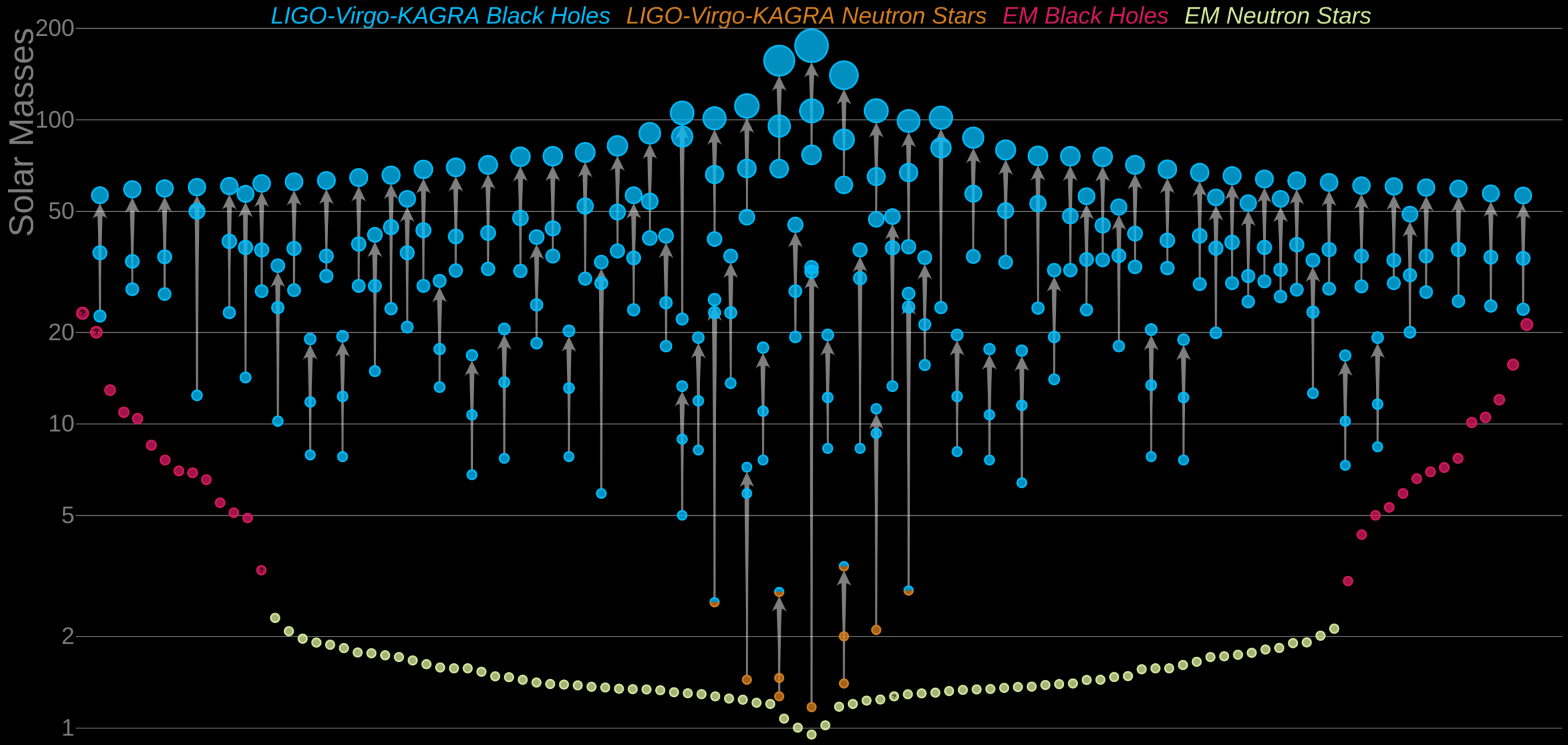
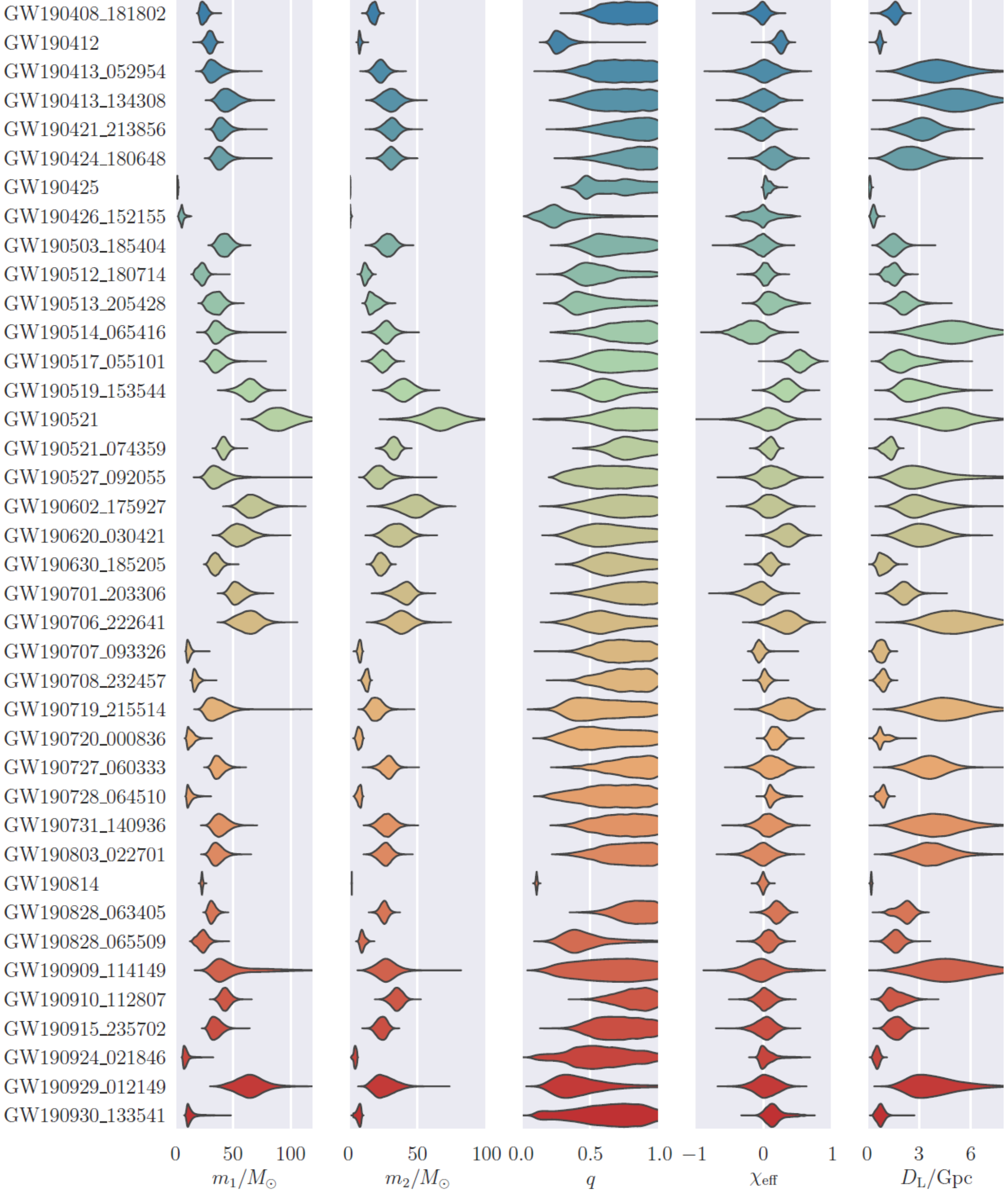
May 31, 2022 NBIA Workshop on Black Hole Dynamics: From Gaseous Environments to Empty Space



Masses in the Stellar Graveyard

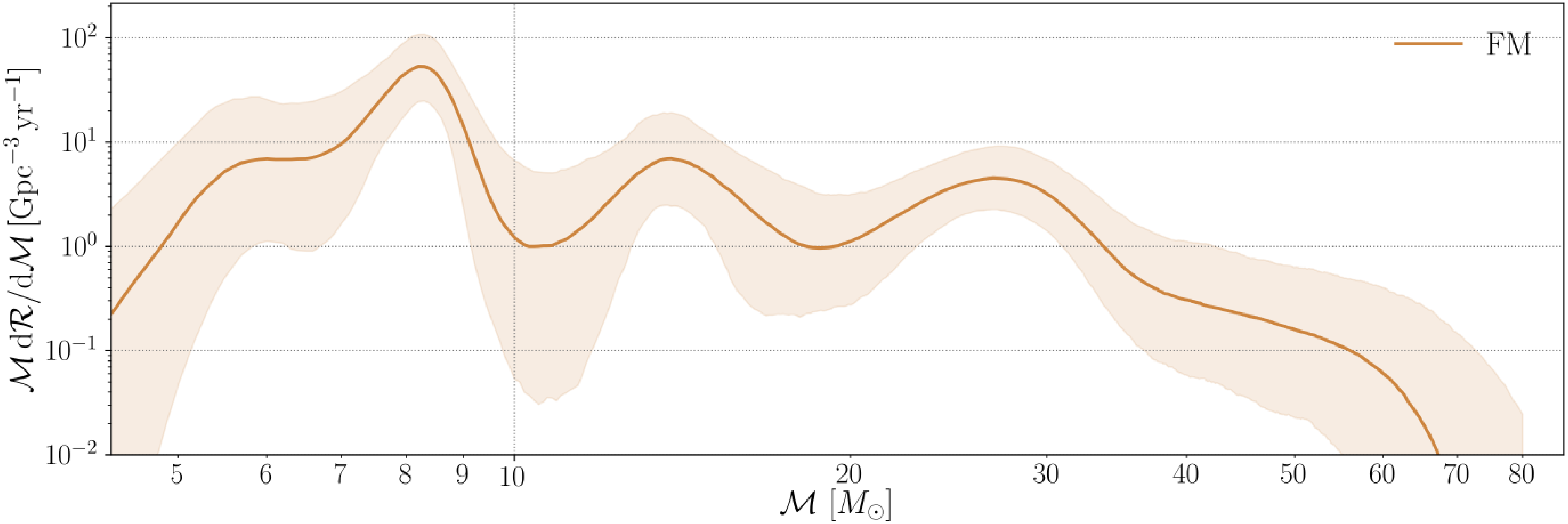


LIGO-Virgo-KAGRA | Aaron Geller | Northwestern



LIGO/VIRGO Collaboration
arxiv:2010.14527

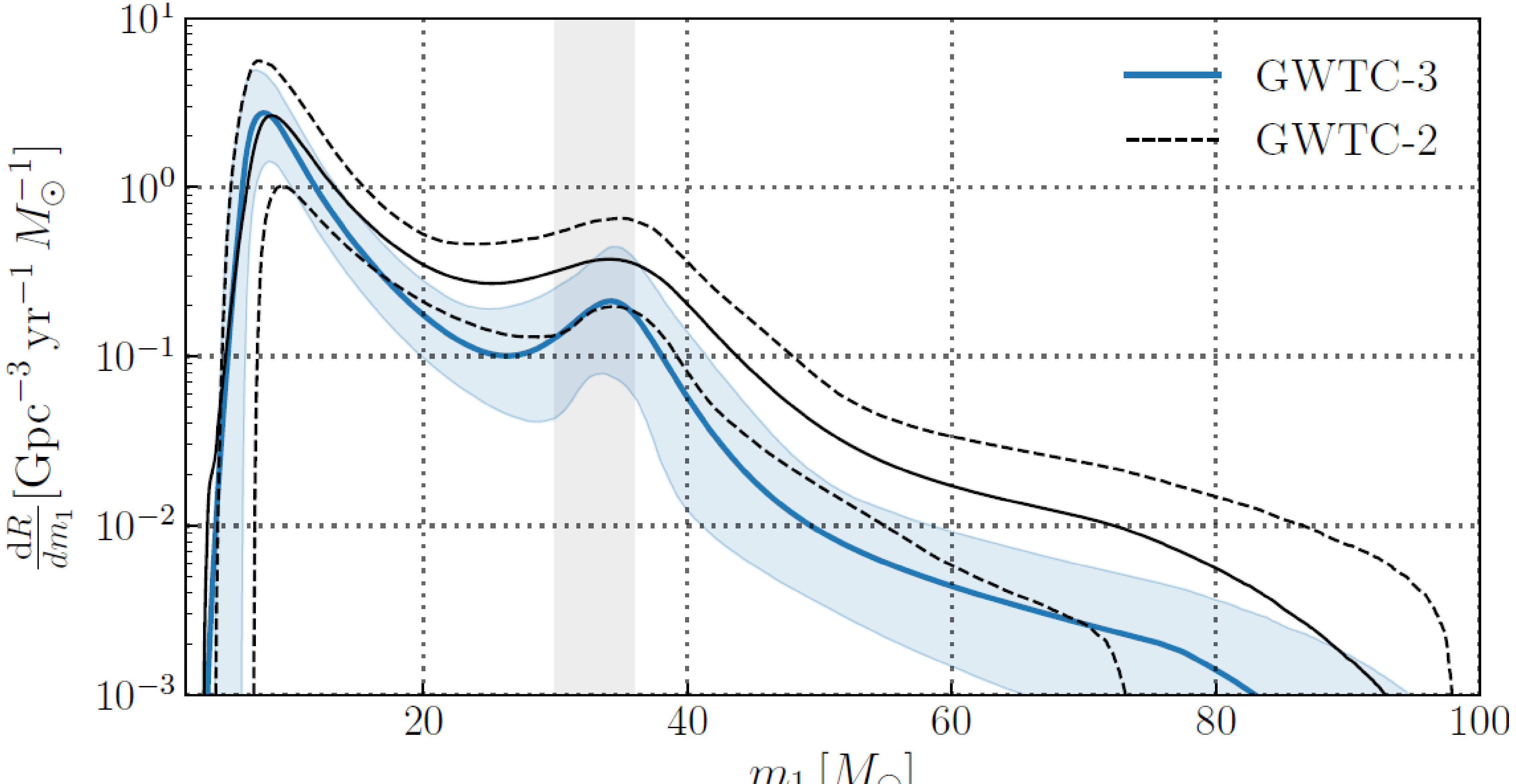
Black hole chirp mass



Very top heavy!

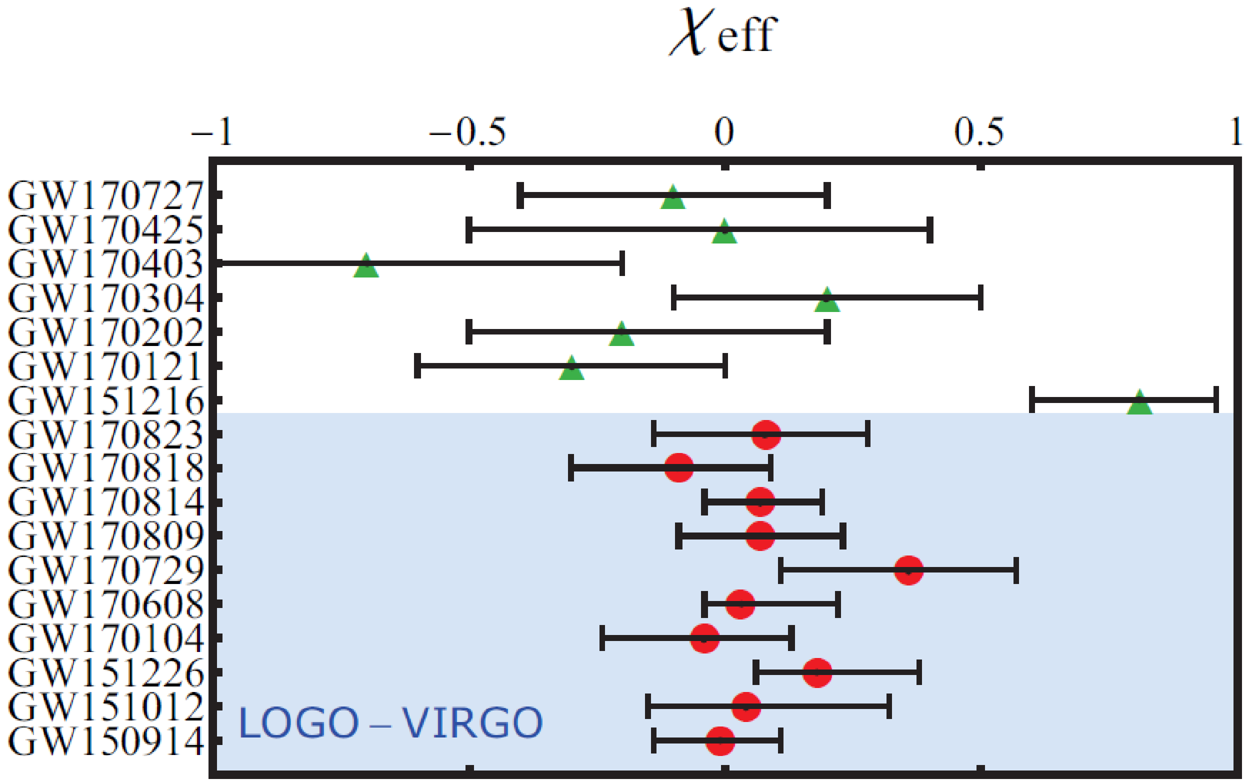
LIGO/VIRGO Collaboration
arxiv:2010.14533

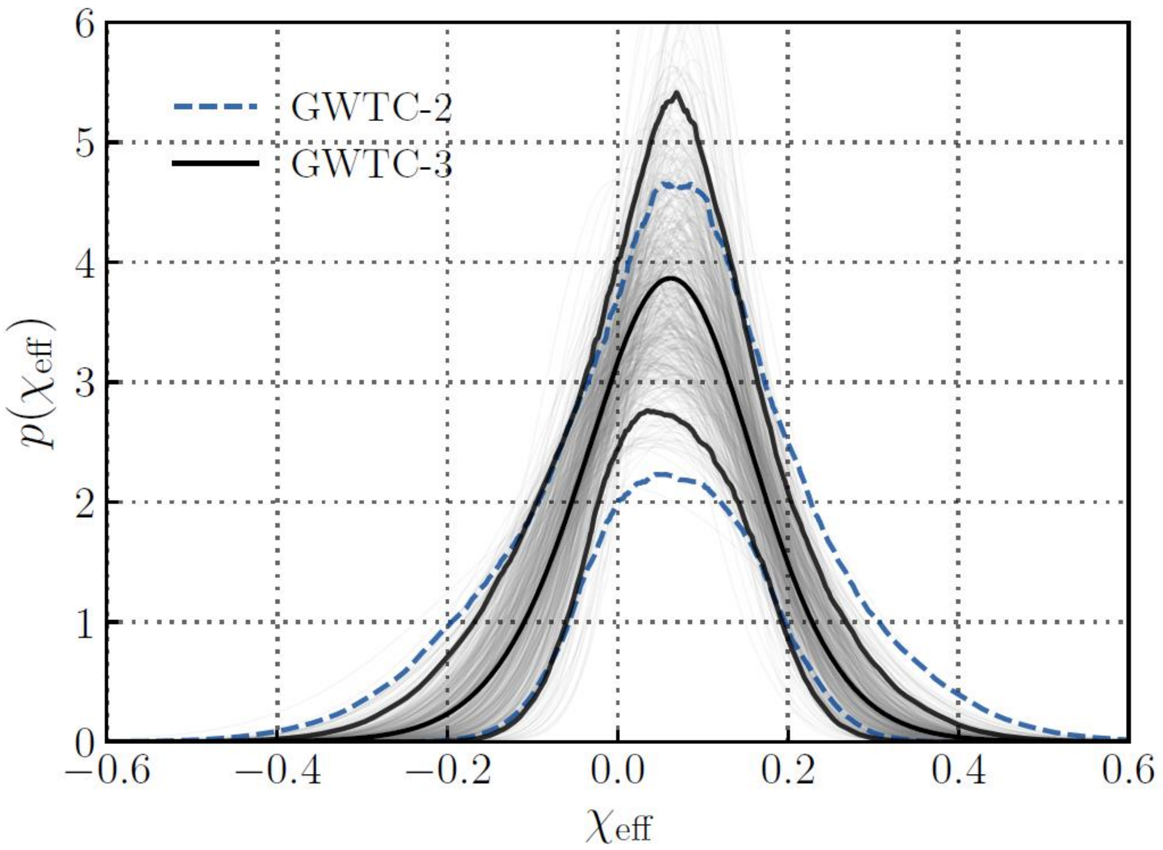
Primary black hole mass



Spins

$$\chi_{\mathrm{eff}} = \frac{(m_1 \chi_1 \cos\theta_1 + m_2 \chi_2 \cos\theta_2)}{M}$$





clustered around zero!

LIGO/VIRGO Collaboration 2018; Zackay+ 2019, Venumadhav+ 2019

LIGO/VIRGO Collaboration arxiv:2010.14533

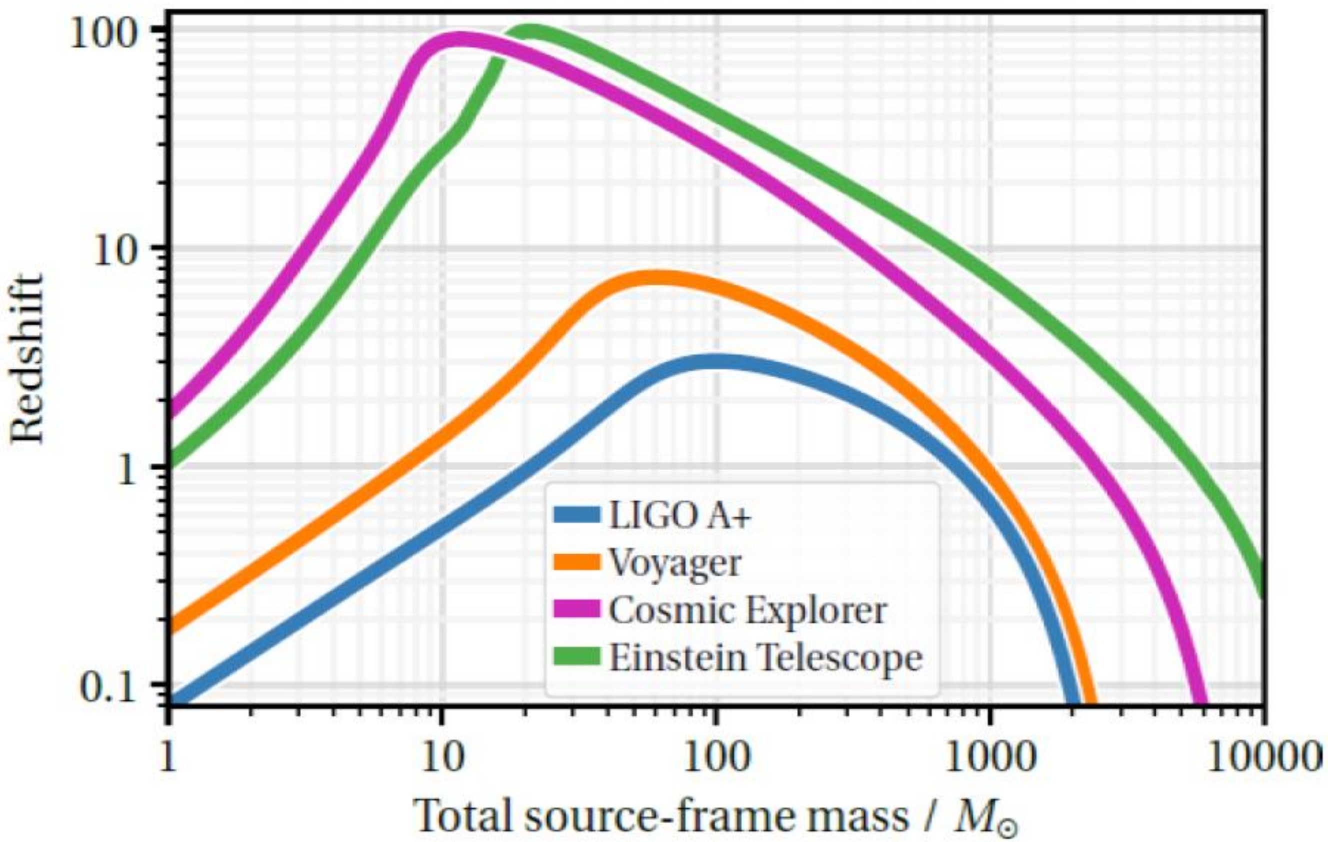
Measured merger rate

17 – 45 Gpc $^{-3}$ yr $^{-1}$ implies

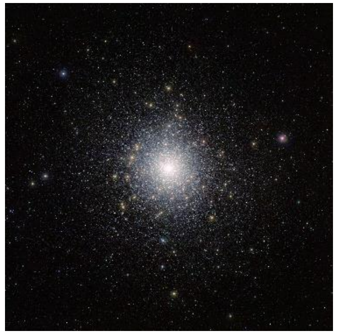
- **1-3 mergers/day** within **z=0.5**
- **1-3 mergers/hour** within **z=2**

LARGE GW SOURCE POPULATION TO BE DETECTED!

Future prospects

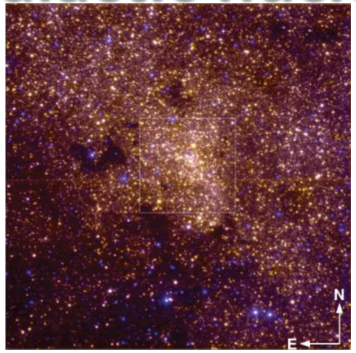


Globular clusters



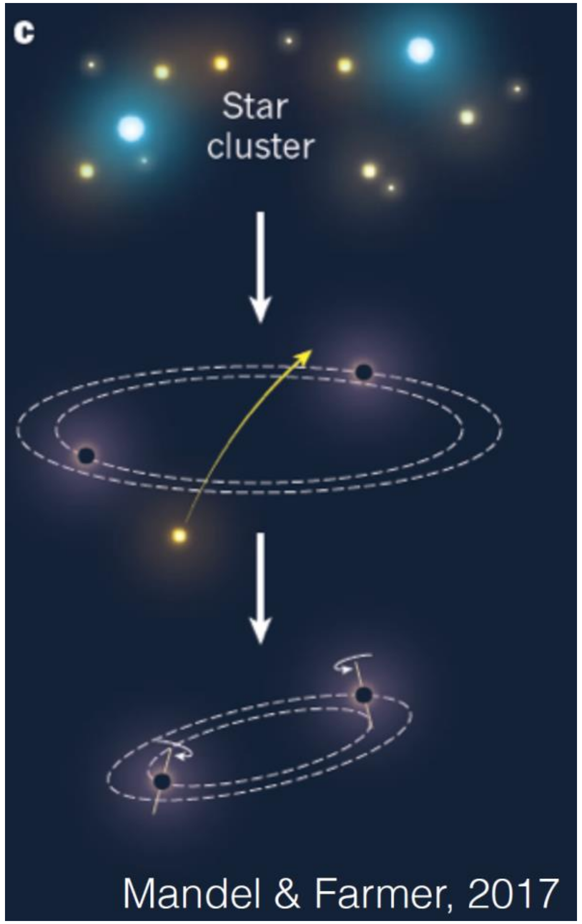
- 0.5% of stellar mass of the Universe
- 100 per galaxy
- Size: 1 pc – 10 pc
- Density 10^3—10^5 x higher

Galactic nuclei

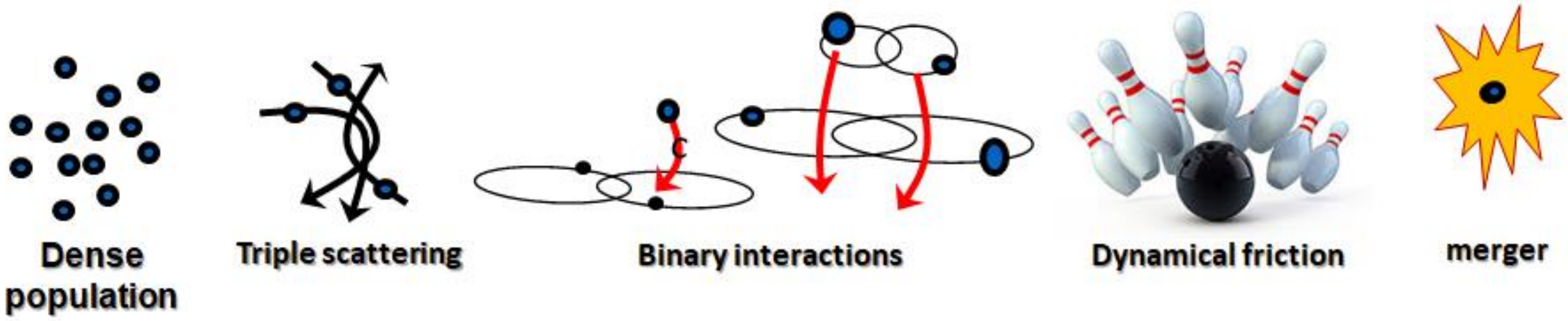


- 0.5% of stellar mass of the Universe
- 10^{6-7} M_{sun} supermassive black hole
- 10^{4-5} stellar mass black holes
- Size: 1 pc – 10pc
- Density 10^6 – 10^{10} x higher

encounter rate ~ density^2



$$\frac{d}{d\ln r}\Gamma = (4\pi r^3) n_\bullet^2 \sigma_{cs} v$$



- binary formation from singles
- exchange interactions
- mass segregation

Expectation:

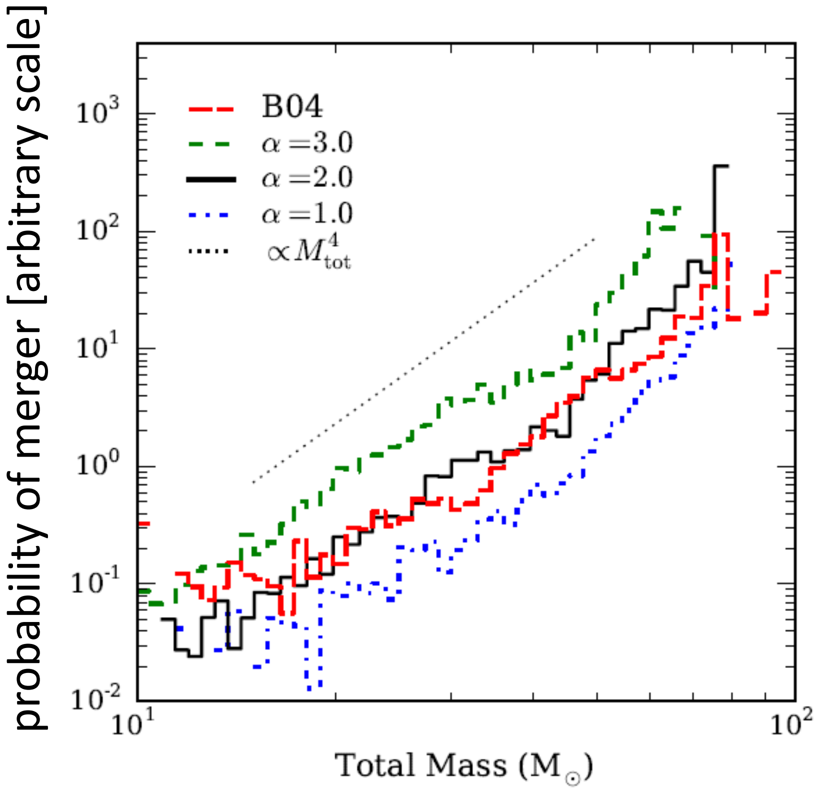
mergers more likely for heavier objects

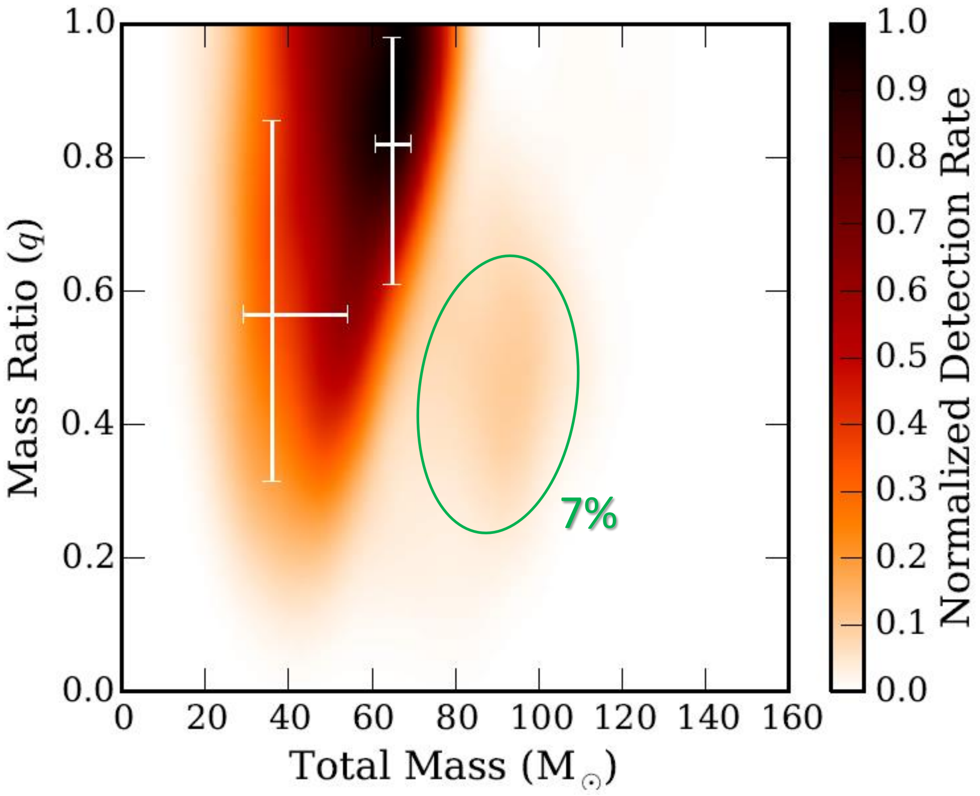
eccentric mergers possible

Mass distribution for globular clusters

Monte Carlo and Nbody simulations

O'Leary, Meiron, Kocsis (2016), Rodriguez+ '19, Askar+ '18, etc





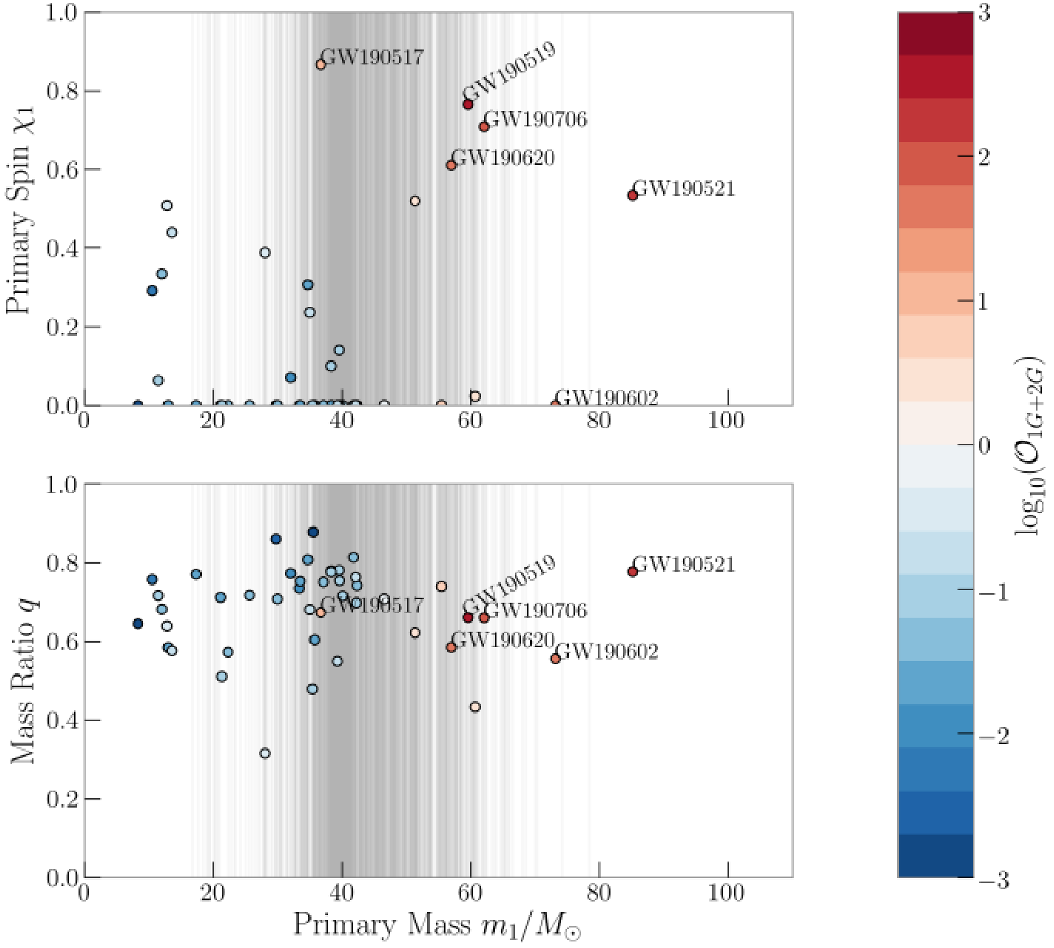
merger probability scales with $\mathbf{M^{4-5}}$

2nd generation mergers are possible: 5%-10%
3rd generation mergers are difficult to produce

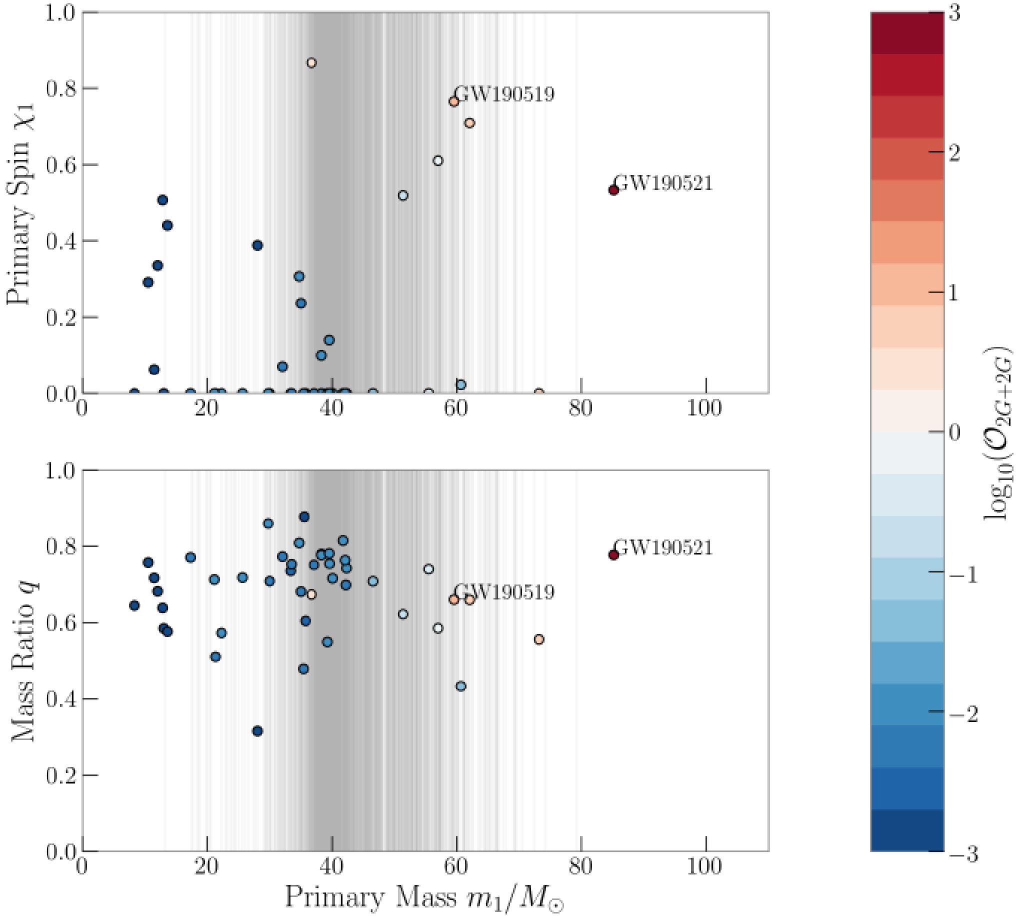
O'Leary, Meiron, Kocsis 2016

Observations show evidence of hierarchical mergers

1G+2G merger (odds ratio)



2G+2G merger (odds ratio)



Inferred merger rate density

1G+2G: 5% -- 0.05%   2G+2G: 0.1% -- 10^-5 %

Kimball+ arxiv:2011.05332

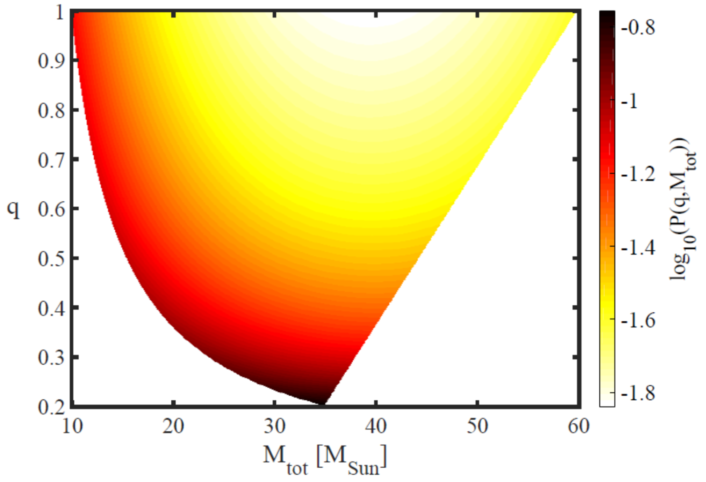
Mass distribution for different processes

universal diagnostic: independent of the mass function

Given: $\mathcal{R}(m_1, m_2) \propto \mathcal{L}(m_1, m_2) f(m_1) f(m_2)$

How can we eliminate the unknown f(m)?

$$-(m_1 + m_2)^2 \frac{\partial^2}{\partial m_1 \partial m_2} \ln \mathcal{R}(m_1, m_2, t)$$



$= \mathbf{4}$ in globular clusters

$= \mathbf{1.4 \ldots -5}$ for GW capture binaries in galactic nuclei

$= \mathbf{1.4}$ for GW capture binaries in collisionless systems

$= \mathbf{1}$ for PBH binaries formed in early universe

Kocsis, Suyama, Takahiro, Yokoyama 2018; Gondan, Kocsis, Raffai, Frei 2018

Explaining the mass exponent

Triple single scattering – binary formation

single-single encounter rate $n\sigma_{ss}v$,

cross section: $\sigma_{ss} = \pi b_{90}^2$

change in velocity $\delta v/v \sim b_{90}/b$

impact parameter for gravitational focusing: $b_{90} = 2Gm/v^2$

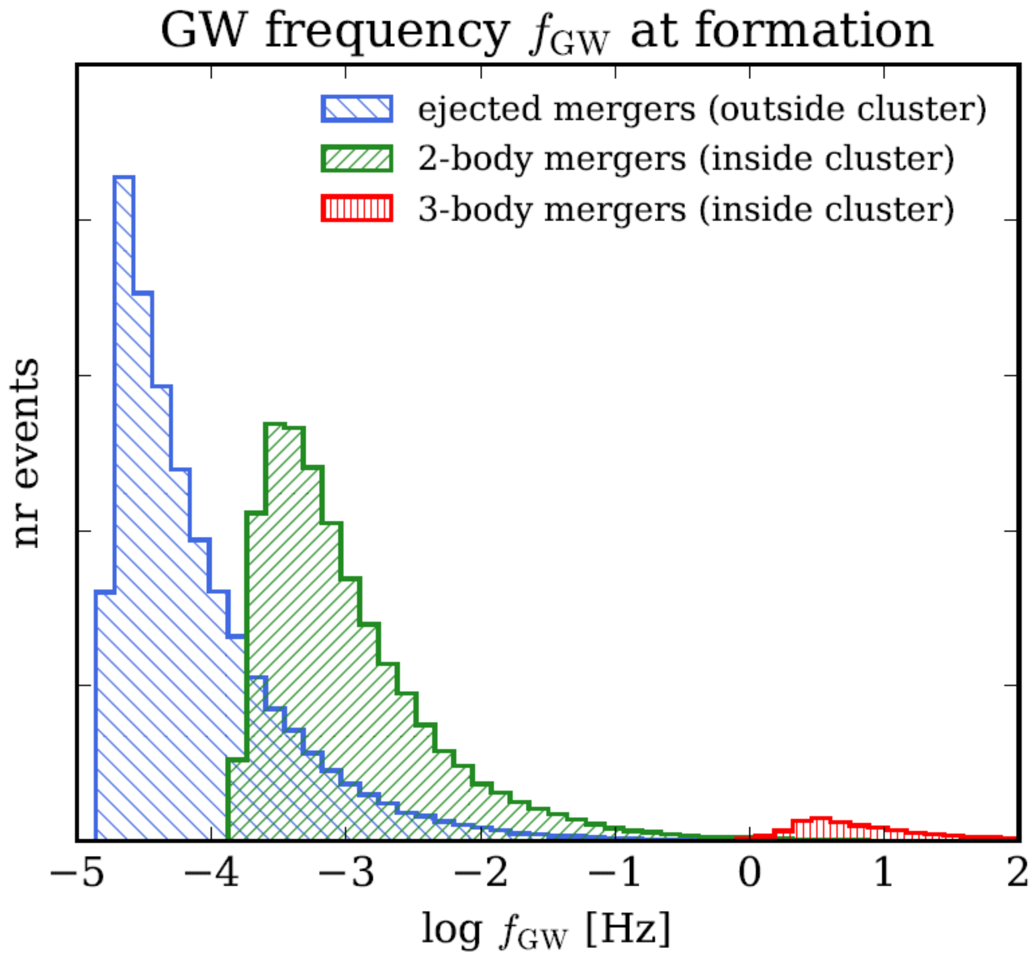
During an encounter, the probability of having a third object in the same region nb_{90}^3

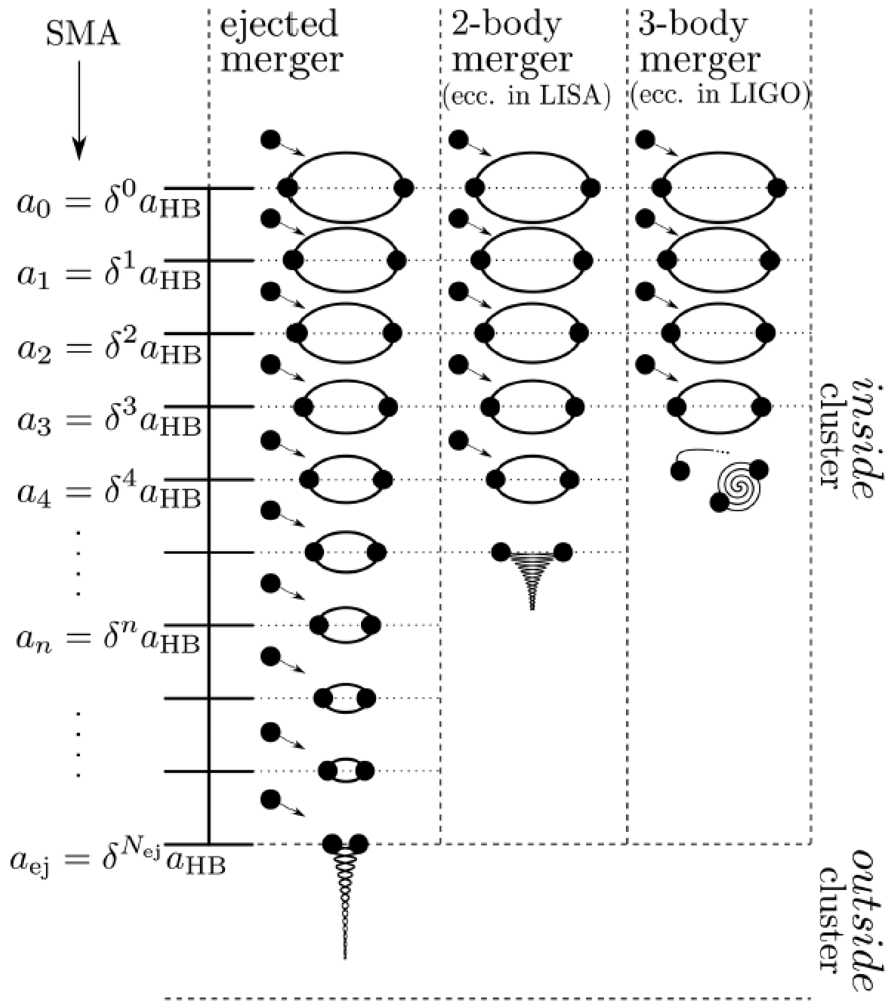
$$\Gamma_{sss} = \pi N n^2 b_{90}^5 v$$

$$\Gamma_{sss} = \frac{2^5 \pi N n^2 G^5 m^5}{v^9} = 3\times10^{-7}\mathrm{yr}^{-1}\left(\frac{v}{20\,\mathrm{km/s}}\right)^{-9}\frac{N}{10^6}\left(\frac{n}{10^5\,\mathrm{pc}^{-3}}\right)^2\left(\frac{m}{10\mathrm{M}_\odot}\right)^5$$

Is there a similar universality for the eccentricity distribution?

Eccentricity distribution
for merging binaries in globular clusters





Samsing+ (2018a, 2018b)

Binary-single scattering

Binary-single scattering rate $t_{\rm bs} = 1/(n\sigma_{\rm bs}v)$

cross section: $\sigma_{\rm bs} = 4\sqrt{\pi}(a^2 + ab_{90})$

$$t_{\rm bs} = \frac{v}{4\sqrt{\pi}n_{\rm s}Gm_{123}a} = 4\times10^7\,{\rm yr}\left(\frac{n_{\rm s}}{10^5\,{\rm pc}^{-3}}\right)^{-1}\frac{v}{20\,{\rm km/s}}\left(\frac{m_{123}}{30{\rm M}_\odot}\right)^{-1}\left(\frac{a}{1\,{\rm AU}}\right)^{-1}$$

Binary separation follows a geometrical sequence $a_n = a_{\rm h}\delta^n$

hardening factor: $\delta = 1 - \frac{2}{9}\frac{m_{\rm s}}{m}$

initial condition (hard-soft boundary)

final encounter: binary merges or it is ejected from cluster

$$\Delta E_{\rm bs} = \frac{1}{2}(2m)v_{\rm bin}^2 + \frac{1}{2}m_{\rm s}v_{\rm s}^2 = (\delta^{-1} - 1)Gm^2/a$$

Binary-single scattering

Hardening time: sum of geometrical sequence

$$t_{\rm h} = \sum_{n=0}^{n_{\rm max}} t_{\rm bs}\delta^n \approx \frac{t_{\rm bs}}{1-\delta} = \frac{v}{G H n_{\rm s} m_{\rm s} a} \qquad H = 15\text{–}20$$

Final encounter at ejection

$$a_{\rm ej} = \left(\frac{1}{\delta} - 1\right) \frac{m m_{\rm s}}{2(2m + m_{\rm s})} \frac{G}{v_{\rm esc}^2}$$

mean orbital velocity

$$\overline{v}_{\rm orb,ej} = \sqrt{\frac{8m + 4m_{\rm s}}{m_{\rm s}} \frac{\delta}{1-\delta}}\, v_{\rm esc} = \sqrt{2\left(1 + 2\frac{m}{m_{\rm s}}\right)\left(9\frac{m}{m_{\rm s}} - 2\right)}\, v_{\rm esc} = \sqrt{42} v_{\rm esc} \frac{m}{\langle m_{\rm BH}\rangle}$$

$$80\,\frac{\rm km}{\rm s}\frac{m}{\langle m_{\rm BH}\rangle} \leq \overline{v}_{\rm orb,ej} \leq 370\,\frac{\rm km}{\rm s}\frac{m}{\langle m_{\rm BH}\rangle}$$

Scattering outcomes

1. Binary merger outside cluster (ejected)
2. Binary merger inside cluster between encounter episodes (2body)
3. Binary merger inside cluster during an encounter episode during intermediate binary state (IMS)
4. Merger inside cluster during a three body scramble (3body)
5. Merger during single single encounters (SS)

Eccentricity estimate

Eccentricity draws a random value from the thermal distribution during encounters

e^2 is uniformly distributed

Merger condition

GW merger timescale $\lessgtr$ lifetime of a given episode

$$t_{\rm GW} \approx t_{\rm GWc}(1-e^2)^{7/2} \qquad t_{\rm life}$$

Probability of merger

$$p_{\rm merger} = \left(\frac{t_{\rm life}}{t_{\rm GWc}}\right)^{2/7} \qquad t_{\rm GWc} = \frac{5}{128\pi}\left(\frac{\overline{v}_{\rm orb}}{c}\right)^{-5} t_{\rm orb}$$

Merger if eccentricity is larger than

$$e_{\rm crit} = 1 - \frac{1}{2}\left(\frac{t_{\rm life}}{t_{\rm GWc}}\right)^{2/7}$$

Eccentricity after GW-driven inspiral (Peters 1964)

$$e_{10\rm Hz} \approx \left(\frac{\sqrt{8}\,\overline{v}^3_{\rm orb}}{(1.7\,f_{\rm det})(2\pi Gm)}\right)^{19/18}\left(\frac{t_{\rm life}}{t_{\rm GWc}}\right)^{-19/42}$$

Eccentricity for different channels

"lifetime of a given episode"

1. ejected binary mergers: $t_{life} = t_{Hubble}$
2. 2body mergers: $t_{life} = t_{binary\text{-}single\ encounter}$
3. intermediate state binary $t_{life} = t_{ejected\ scatterer\ orb\ period}$
4. 3-body merger $t_{life} = t_{orb}$

Eccentricity for different channels

increases exponentially until

$$\sqrt{42}\, v_{\rm esc} \frac{m}{\langle m_{\rm BH}\rangle}$$

for 3body mergers

$$p_{\rm merger,3b} = \left(\frac{t_{\rm life}}{t_{\rm GWc}}\right)^{2/7} = 4\frac{\pi^{2/7}}{5^{2/7}}\left(\frac{\overline{v}_{\rm orb}}{c}\right)^{10/7}$$

$$e_{\rm crit,3b} \approx 1 - \frac{1}{2}\left(\frac{t_{\rm orb}}{t_{\rm GWc}}\right)^{2/7} = 1 - \frac{2\,\pi^{2/7}}{5^{2/7}}\left(\frac{\overline{v}_{\rm orb}}{c}\right)^{10/7} = 1 - 1.8\left(\frac{\overline{v}_{\rm orb}}{c}\right)^{10/7}$$

$$e_{\rm 10Hz,3b} = 0.14\left(\frac{m}{10{\rm M}_\odot}\right)^{-19/18}\left(\frac{\overline{v}_{\rm orb}}{200\,{\rm km/s}}\right)^{19/21} \longrightarrow 0.14\left(\frac{\langle m_{\rm BH}\rangle}{10{\rm M}_\odot}\right)^{-1}\frac{v_{\rm esc}}{30\,{\rm km/s}}$$

- event rate dominated by conditions close to ejection
- eccentricity of GW sources measures orbital velocity → infer v_{esc}
- it is independent of GW source mass and cluster parameters

Merger probability

Probability of merger during each encounter: $p_{\rm merger} = \left(\frac{t_{\rm life}}{t_{\rm GWc}}\right)^{2/7}$

Probability of merger = 1 – probability of no mergers

- during all binary-single interactions from hard-soft boundary to ejection
- during all intermediate state binaries during each binary-single interaction

$$p_{\rm 2b} = 1 - e^2_{\rm crit,2b} = \left(\frac{t_{\rm bs}}{t_{\rm GWc}}\right)^{2/7} = \left(\frac{128\, G^2 m^2 v}{15\sqrt{\pi}\, c^5 n a^5}\right)^{2/7} = \left(\frac{4\, v \overline{v}^{10}_{\rm orb}}{15\sqrt{\pi}\, G^3 c^5 n m^3}\right)^{2/}$$

$$= 0.048 \left(\frac{m}{10\,{\rm M}_\odot}\right)^{-6/7} \left(\frac{n_{\rm BH}}{10^5 {\rm pc}^{-3}}\right)^{-2/7} \left(\frac{v}{30{\rm km/s}}\right)^{2/7} \left(\frac{\overline{v}_{\rm orb}}{200{\rm km/s}}\right)^{20/7} .$$

increases exponentially as a geometric sequence

$$P_{\rm 2b} = 1 - \prod_{n=0}^{n_{\rm tot}} [1 - p_{{\rm 2b},n}] = 1 - \exp\left[\sum_{n=0}^{n_{\rm tot}} \ln[1 - p_{{\rm 2b},n}]\right] \qquad (20)$$

$$\approx 1 - \exp\left[-\sum_{n=0}^{n_{\rm tot}}\left[p_{{\rm 2b},n} + \frac{p^2_{{\rm 2b},n}}{2}\right]\right] \approx 1 - \exp\left[\frac{-p_{\rm 2b,ej}}{1-\delta^{10/7}}\right] \exp\left[\frac{-p^2_{\rm 2b,ej}}{2(1-\delta^{20/7})}\right] \approx \frac{p_{\rm 2b,ej}}{1-\delta^{10/7}}$$

Similarly for IMS mergers

$$p_{\mathrm{IMS},n} = 1 - (e^2_{\mathrm{crit,IMS}})^{N_{\mathrm{IMS}}} = 1 - \left[1 - \left(\frac{t_{\mathrm{IMS}}}{t_{\mathrm{GWc}}}\right)^{2/7}\right]^{N_{\mathrm{IMS}}}$$

$$\approx 1 - \exp\left[-N_{\mathrm{IMS}}\left(\frac{t_{\mathrm{IMS}}}{t_{\mathrm{GWc}}}\right)^{2/7}\right] \approx N_{\mathrm{IMS}}\left(\frac{t_{\mathrm{IMS}}}{t_{\mathrm{GWc}}}\right)^{2/7} = N_{\mathrm{IMS}}\, p_{\mathrm{IMS}}$$

1 – (no mergers during IMS states)

$$P_{\mathrm{IMS}} = 1 - \prod_{n=0}^{n_{\mathrm{tot}}}[1 - p_{\mathrm{IMS},n}] = 1 - \exp\left[\sum_{n=0}^{n_{\mathrm{tot}}} \ln[1 - p_{\mathrm{IMS},n}]\right]$$

$$\approx 1 - \exp\left[-\sum_{n=0}^{n_{\mathrm{tot}}}\left[p_{\mathrm{IMS},n} + \frac{p^2_{\mathrm{IMS},n}}{2}\right]\right]$$

$$\approx 1 - \exp\left[\frac{-p_{\mathrm{IMS,ej}}}{1-\delta^{5/7}}\right]\exp\left[\frac{-p^2_{\mathrm{IMS,ej}}}{2(1-\delta^{10/7})}\right]$$

$$\approx \frac{p_{\mathrm{IMS,ej}}}{1-\delta^{5/7}} \approx 0.08\,\frac{N_{\mathrm{IMS}}}{20}\, q_{\mathrm{out}}^{1/7}\,\frac{m}{m_s}\left(\frac{a_{\mathrm{out}}}{100a}\right)^{3/7}\left(\frac{\overline{v}_{\mathrm{orb,ej}}}{200\,\mathrm{km/s}}\right)^{10/7}$$

1 – (no mergers during binary-single interactions)

and for 3-body mergers

$$p_{3b} \approx 1 - e^2_{\mathrm{crit,3b}} = \frac{4\,\pi^{2/7}}{5^{2/7}} \left(\frac{\overline{v}_{\mathrm{orb}}}{c}\right)^{10/7}$$

$$p_{3b,n} \approx N_{\mathrm{IMS}}\, p_{3b}$$

$$P_{3b} \approx \frac{p_{3b,\mathrm{ej}}}{1-\delta^{5/7}} = \frac{\pi^{2/7}}{5^{2/7}} \frac{4N_{\mathrm{IMS}}}{1-\delta^{5/7}} \left(\frac{\overline{v}_{\mathrm{orb}}}{c}\right)^{10/7} \approx 0.01 \frac{N_{\mathrm{IMS}}}{20} \frac{m}{m_s} \left(\frac{\overline{v}_{\mathrm{orb,ej}}}{200\,\mathrm{km/s}}\right)^{10/7}$$

Merger probability – summary

In cluster mergers:

$$P_{\rm in} = 1-(1-P_{\rm 2b})(1-P_{\rm IMS})(1-P_{\rm 3b}) \approx 1-e^{-(P_{\rm 2b}+P_{\rm IMS}+P_{\rm 3b})} \approx P_{\rm 2b}+P_{\rm IMS}+P_{\rm 3b}\,,$$

For typical **smallish** black holes:

For $M_{\rm gc} = 10^5\,{\rm M}_\odot$, $P_{\rm in} = 4\%$ for $m = m_{\rm s} = 10{\rm M}_\odot$

$M_{\rm gc} = 10^6\,{\rm M}_\odot$ $P_{\rm in} = 59\%$

$(P_{\rm 2b}, P_{\rm IMS}, P_{\rm 3b})/(P_{\rm 2b} + P_{\rm IMS} + P_{\rm 3b}) = (70\%, 26\%, 4\%)$ eccentric

For **heavier** black holes:

For $M_{\rm gc} = 10^5\,{\rm M}_\odot$, $P_{\rm in} = 10\%$ for $m = 3m_{\rm s} = 30{\rm M}_\odot$

$M_{\rm gc} = 10^6\,{\rm M}_\odot$ $P_{\rm in} = 78\%$

$(P_{\rm 2b}, P_{\rm IMS}, P_{\rm 3b})/(P_{\rm 2b} + P_{\rm IMS} + P_{\rm 3b}) = (50\%, 42\%, 8\%)$ eccentric

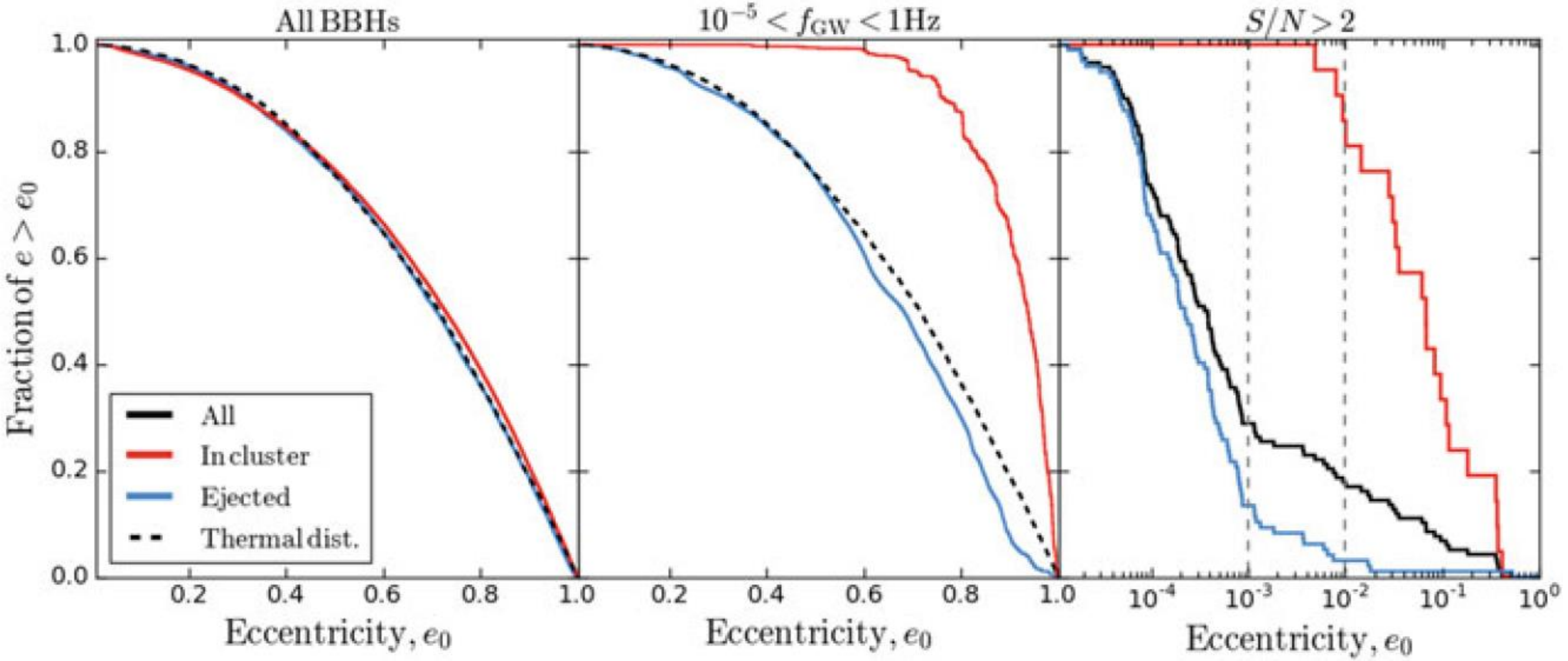
cf.: currently one eccentric source: GW190521 e=0.7 Gayathri+ (2022), e=0.1 Romero-Shaw+ (2020)

Eccentricity distribution function

A superposition of truncated thermal distributions. As v_{orb} grows, merger probability grows, leads to superthermal distribution.

$$\ln P_{\rm IMS}(e_0 < e) \approx N_{\rm IMS}(1-e^2)\frac{\ln(1-e^2)-\ln(p_{\rm IMS,ej})}{\frac{5}{7}\ln\delta} \quad \text{if } e^2 > 1 - p_{\rm IMS,ej}$$

Explains simulation results of Kimball+ arxiv:2011.05332



Universality in eccentricity distribution

Consider the following measurable quantity

$$\kappa(e) = -\frac{(1-e^2)}{2e}\frac{d}{de}\left[\frac{\ln P(e_0 < e)}{1-e^2}\right]$$

This is independent of binary parameters and cluster parameters

$$\kappa_{\rm 2b} = \frac{10}{7|\ln\delta|} = \frac{10}{7|\ln\frac{7}{9}|} = 2.785$$

$$\kappa_{\rm IMS} = \kappa_{\rm 3b} = \frac{5}{7|\ln\delta|} = \frac{5}{7|\ln\frac{7}{9}|} = 5.571$$

$$\kappa_{\rm ss} = 0\,.$$

Universality broken for single-single GW captures in galactic nuclei

Specifically for single-single captures

→ velocity dispersion determines the eccentricity

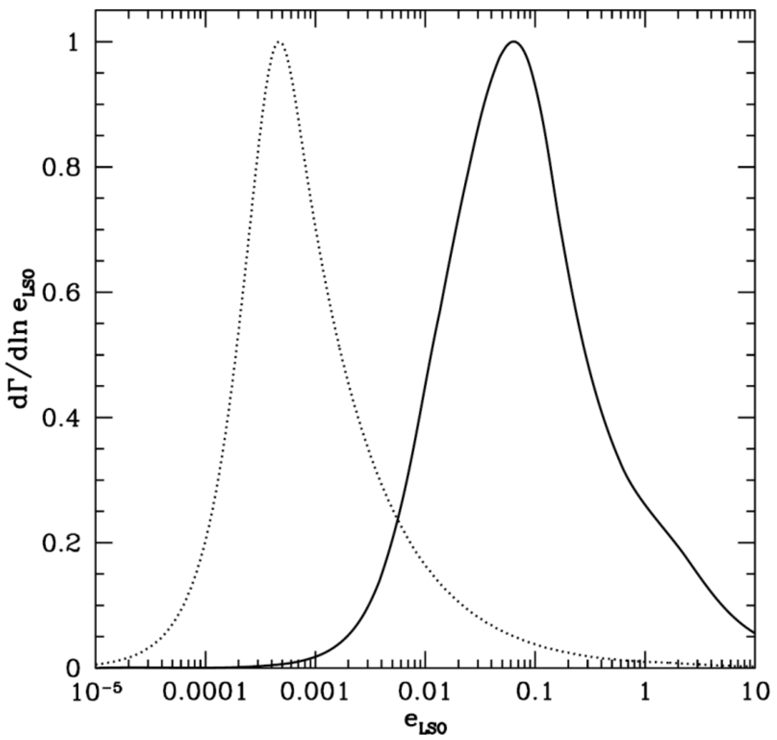
→ mass segregation

→ correlation between mass – velocity dispersion

Eccentricity distribution
for GW capture binaries following single-single encounters

Velocity dispersion → maximum initial pericenter distance r_p/M → eccentricity at merger

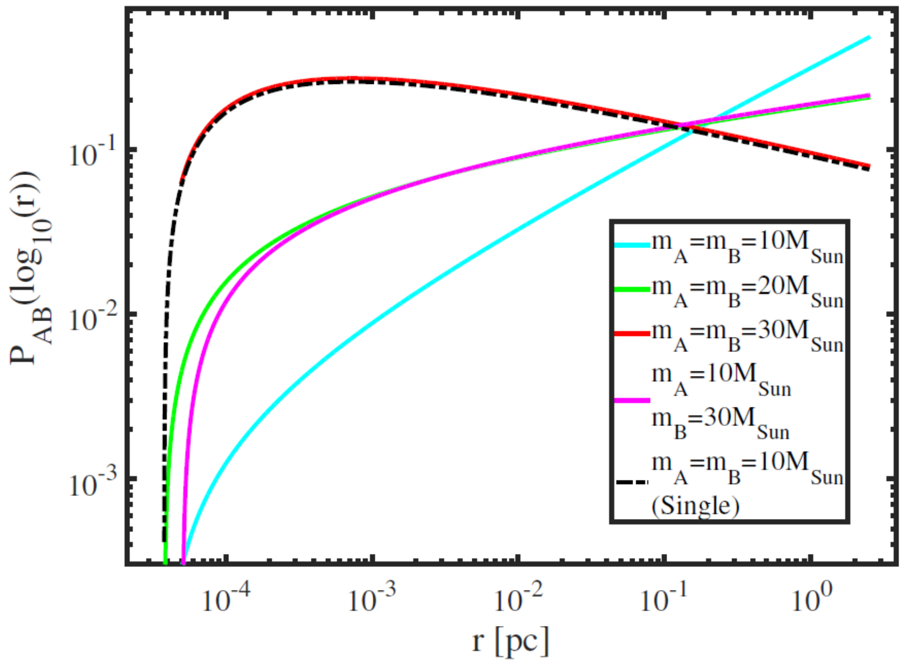
$$\sigma \sim 258\,\frac{\mathrm{km}}{\mathrm{s}}\,(4\eta)^{1/2}\left(\frac{e_{\mathrm{LSO,peak}}}{0.01}\right)^{35/32}$$



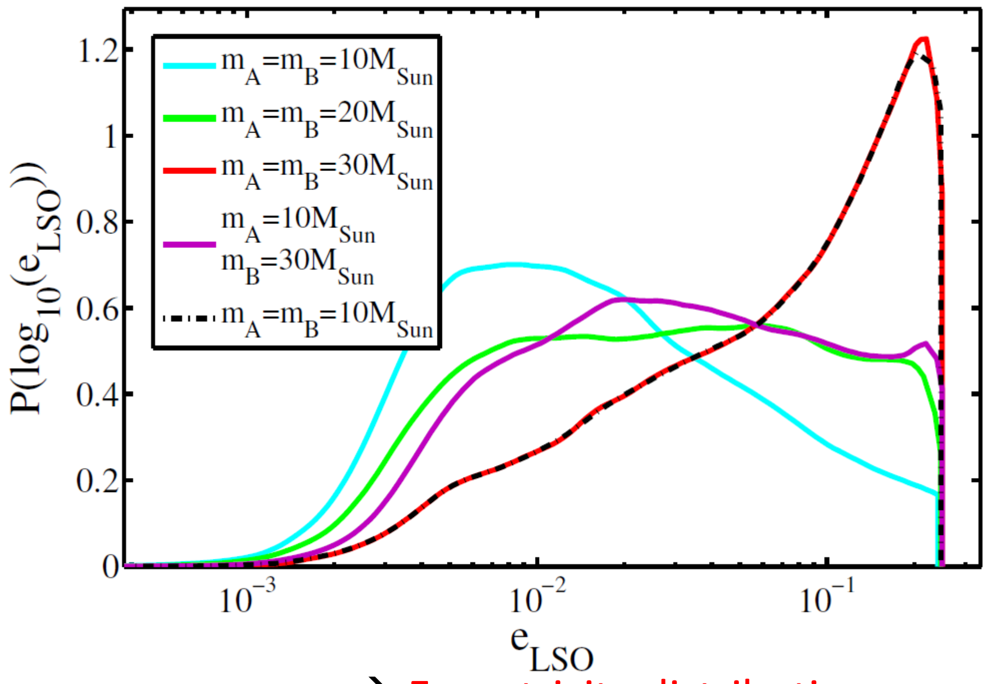
O'Leary, Kocsis, Loeb (2009); see also Rodriguez+ 2016, Gondan+ 2018, Samsing 2017, Gondan & Kocsis 2021

Eccentricity distribution for single-single GW capture binaries

$$\sigma \sim 258\,\frac{\mathrm{km}}{\mathrm{s}}\,(4\eta)^{1/2}\left(\frac{e_{\mathrm{LSO,peak}}}{0.01}\right)^{35/32}$$

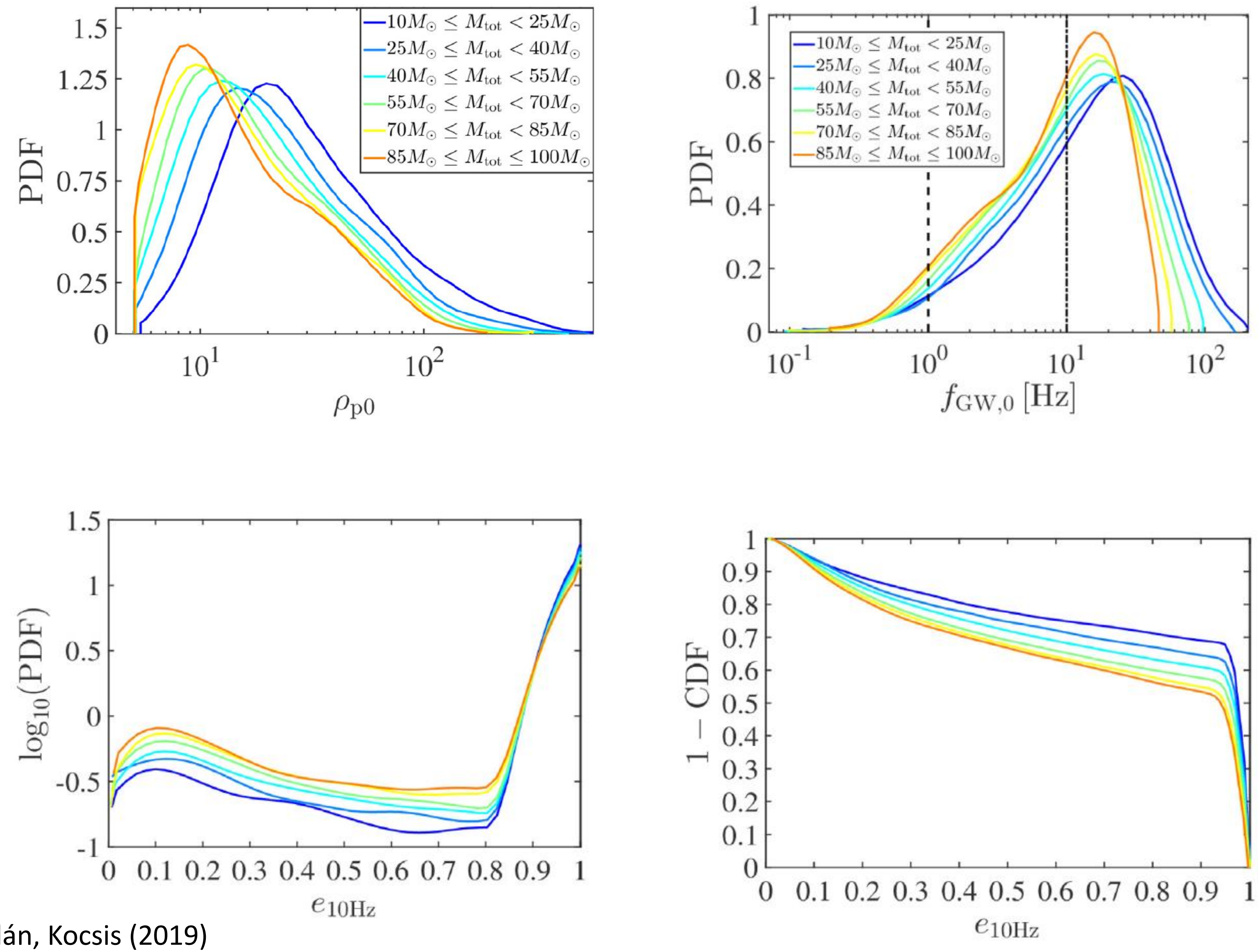


radial distribution of mergers shows mass segregation



→ Eccentricity distribution reveals mass segregation

Gondán, Kocsis, Raffai, Frei (2018b)



Gondán, Kocsis (2019)

>90% at least mildly eccentric

>50% very highly eccentric

Summary

- There is some hope to make sense of the GW distributions to infer the dominant astrophysical mechanism leading to merger
- **Universal exponents** in globular cluster mergers in **mass** and **eccentricity** distribution

- Things to do:
  - Eccentricity models neglected mass segregation, radial dependence (Samsing, D'Orazio+ 2020 for single-single), binary-binary scatterings (see Zevin, Samsing+ 2019), cluster anisotropy, Kozai-Lidov type effects (Antonini & Perets 2012, Tremaine & Silsbee 2018, Antonini+ 2016, Hoang+ 2018), AGN channel (e.g. Tagawa+ 2021, Samsing+ 2022), evolution within the host galaxy (Fragione & Kocsis 2018, Arca-Sedda+ 2021), etc.
  - similar toy model may be constructed to explain the Kozai-Lidov channel eccentricity distribution

    → look for universality there

Extra slides

$$\delta E \approx -\frac{85\pi}{12\sqrt{2}} \frac{\eta^2 M_{\mathrm{tot}}^{9/2}}{r_{\mathrm{p}}^{7/2}} \qquad \delta L \approx -\frac{6\pi M_{\mathrm{tot}}^4 \eta^2}{r_{\mathrm{p}}^2} \qquad \eta = (mM)/(m+M)^2$$

$$E_{\mathrm{final}} = M_{\mathrm{tot}}\eta w^2/2 + \delta E \qquad L_{\mathrm{final}} = M_{\mathrm{tot}}\eta b w + \delta L,$$

$$r_{\mathrm{p}} = \left(\sqrt{\frac{1}{b^2} + \frac{M_{\mathrm{tot}}^2}{b^4 w^4}} + \frac{M_{\mathrm{tot}}}{b^2 w^2}\right)^{-1} \approx \frac{b^2 w^2}{2M_{\mathrm{tot}}}\left(1 - \frac{b^2 w^4}{4M_{\mathrm{tot}}^2}\right)$$

$$a_0 = -\frac{M_{\mathrm{tot}}^2 \eta}{2E_{\mathrm{final}}} \qquad e_0 = \sqrt{1 + 2\frac{E_{\mathrm{final}} b^2 w^2}{M_{\mathrm{tot}}^3 \eta}} \qquad r_{\mathrm{p}0} = a_0(1 - e_0)$$

$$|\delta L| \ll M_{\mathrm{tot}}\eta b w$$

$$E_{\mathrm{final}} < 0 \qquad r_{\mathrm{p,max}} = \left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7} M_{\mathrm{tot}} \frac{\eta^{2/7}}{w^{4/7}} \qquad b_{\mathrm{max}} = \left(\frac{340\pi}{3}\right)^{1/7} M_{\mathrm{tot}} \frac{\eta^{1/7}}{w^{9/7}} \times \left[1 - \frac{1}{4}\left(\frac{85\pi}{3}\right)^{2/7} (4\eta)^{2/7} w^{10/7}\right]$$

$$\frac{\mathrm{d}^3\Gamma_{\mathrm{1GN}}}{\mathrm{d}r\,\mathrm{d}m\,\mathrm{d}M} = 4\pi^2 b_{\mathrm{max}}^2 v_{\mathrm{c}}(r) n_m(r) n_M(r) r^2 \qquad v_{\mathrm{c}}(r) = \sqrt{G M_{\mathrm{SMBH}}/r}$$

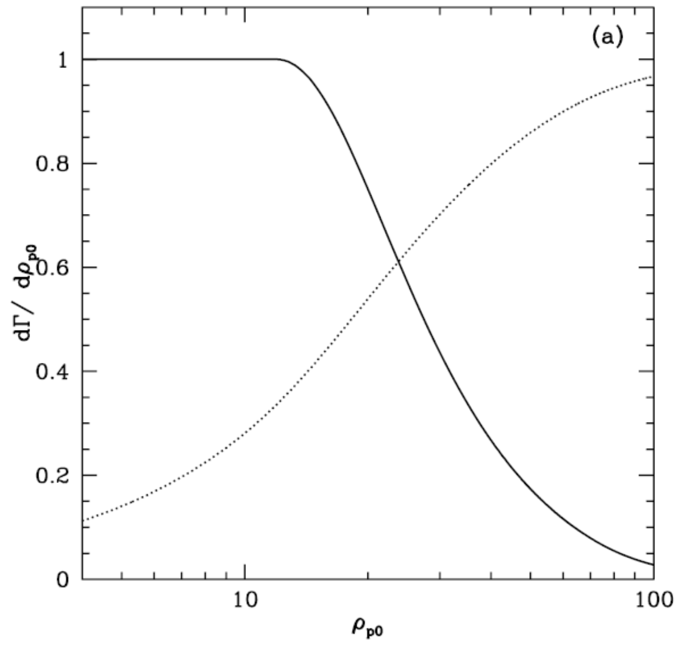
$$\mathrm{d}r_{\mathrm{p}} \approx \frac{w^2 b\,\mathrm{d}b}{M_{\mathrm{tot}}}$$

$$r_{\rm p} = \left(\sqrt{\frac{1}{b^2} + \frac{M_{\rm tot}^2}{b^4 w^4}} + \frac{M_{\rm tot}}{b^2 w^2}\right)^{-1} \approx \frac{b^2 w^2}{2M_{\rm tot}}\left(1 - \frac{b^2 w^4}{4M_{\rm tot}^2}\right)$$

$$\frac{d^3\Gamma_{\rm 1GN}}{dr\, dm\, dM} = 4\pi^2 b_{\rm max}^2 v_{\rm c}(r) n_m(r) n_M(r) r^2,$$

$$b_{\rm max} = \left(\frac{340\pi}{3}\right)^{1/7} M_{\rm tot} \frac{\eta^{1/7}}{w^{9/7}}$$

Galactic nuclei



Globular clusters