Secular Dynamics in the Triple System:

I. BH binary around a Supermassive BH
II. Star around a Massive BH binary

Bin Liu | Jun 3, 2022

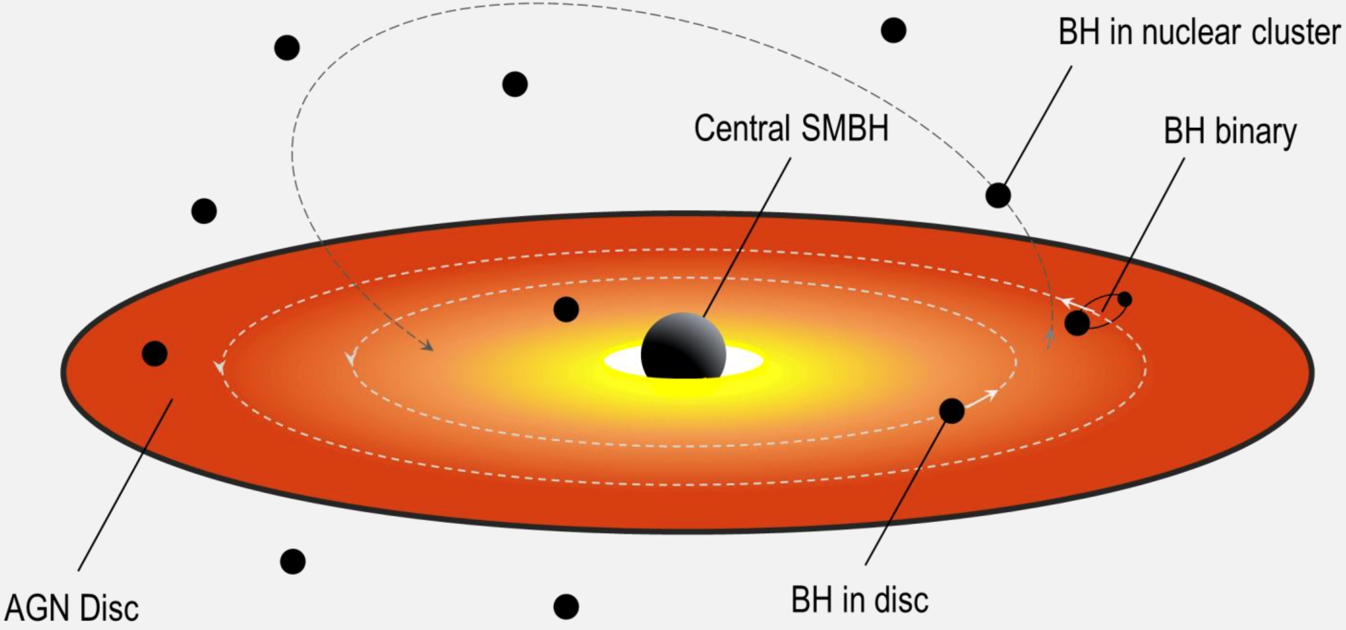
NBIA Workshop on Black Hole Dynamics

Co-financed by the Connecting Europe Facility of the European Union

The Niels Bohr International Academy

Formation Channels

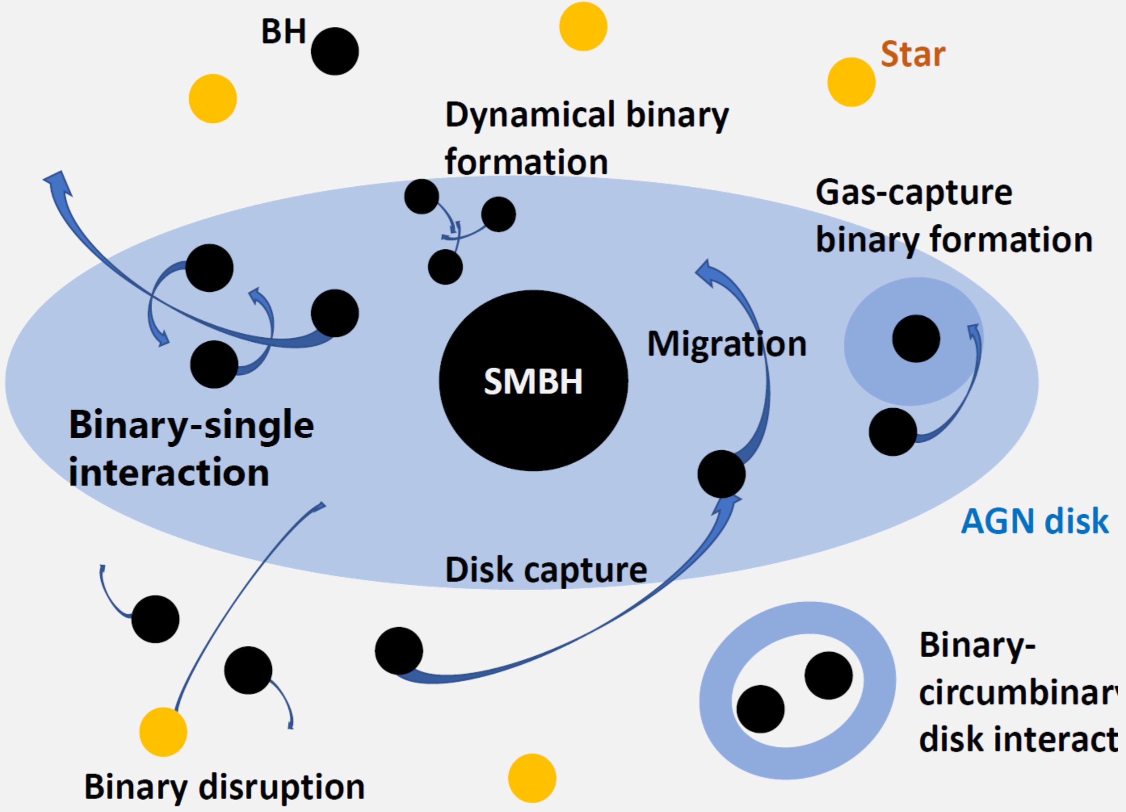
- **AGN disk Channel**



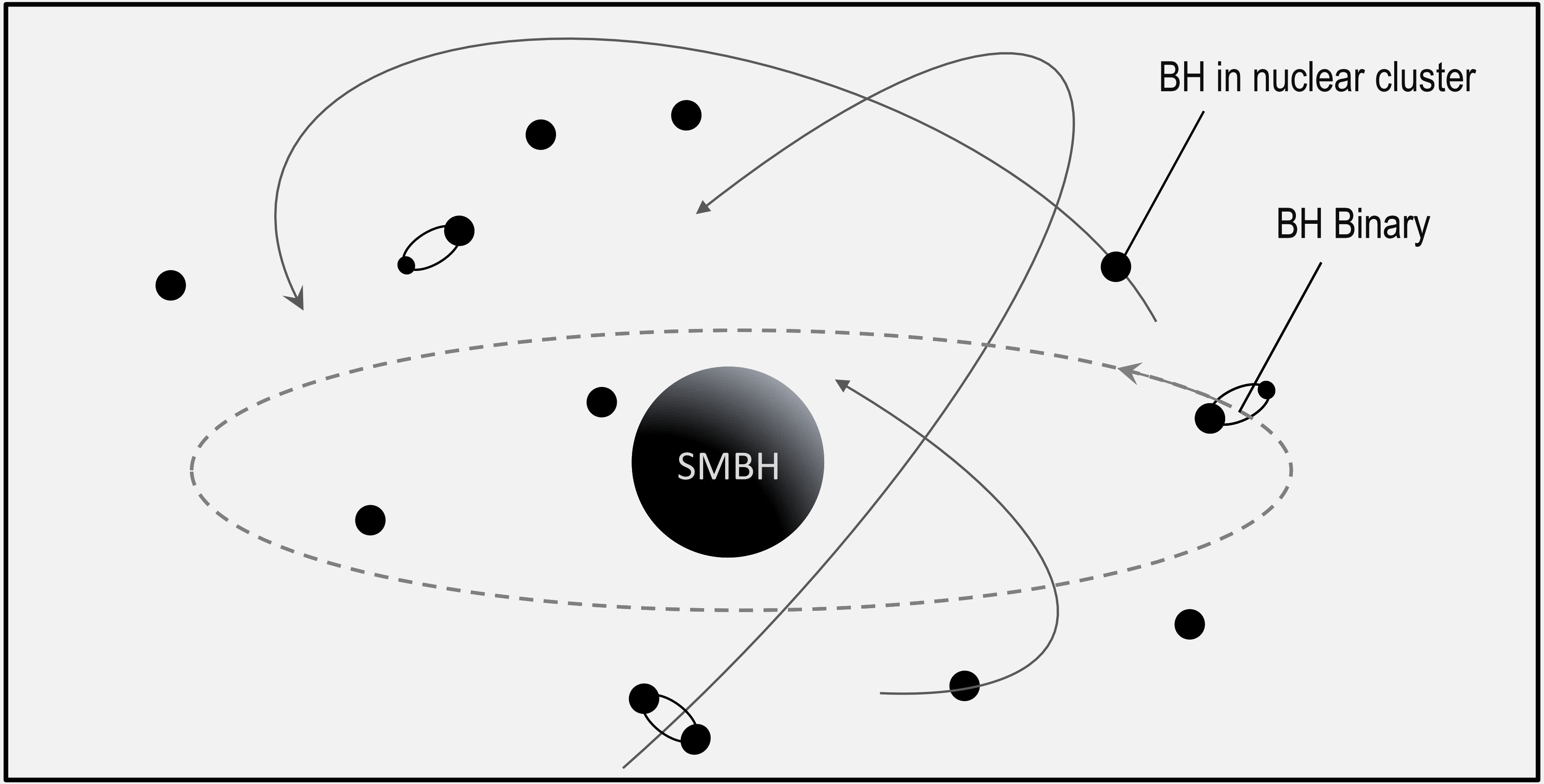
Formation Channels

- **AGN disk Channel**

Credit: Hiromichi + 2020



Binary - SMBH Interaction



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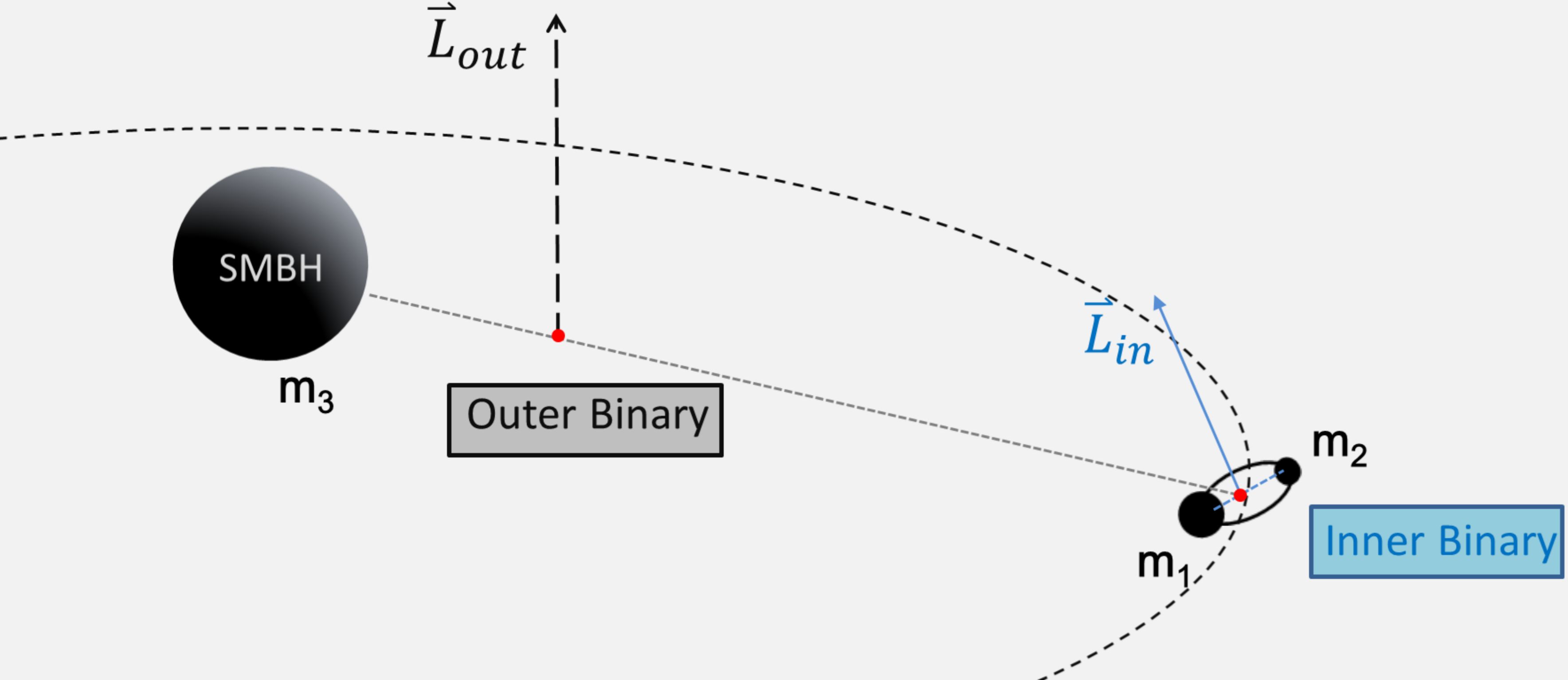
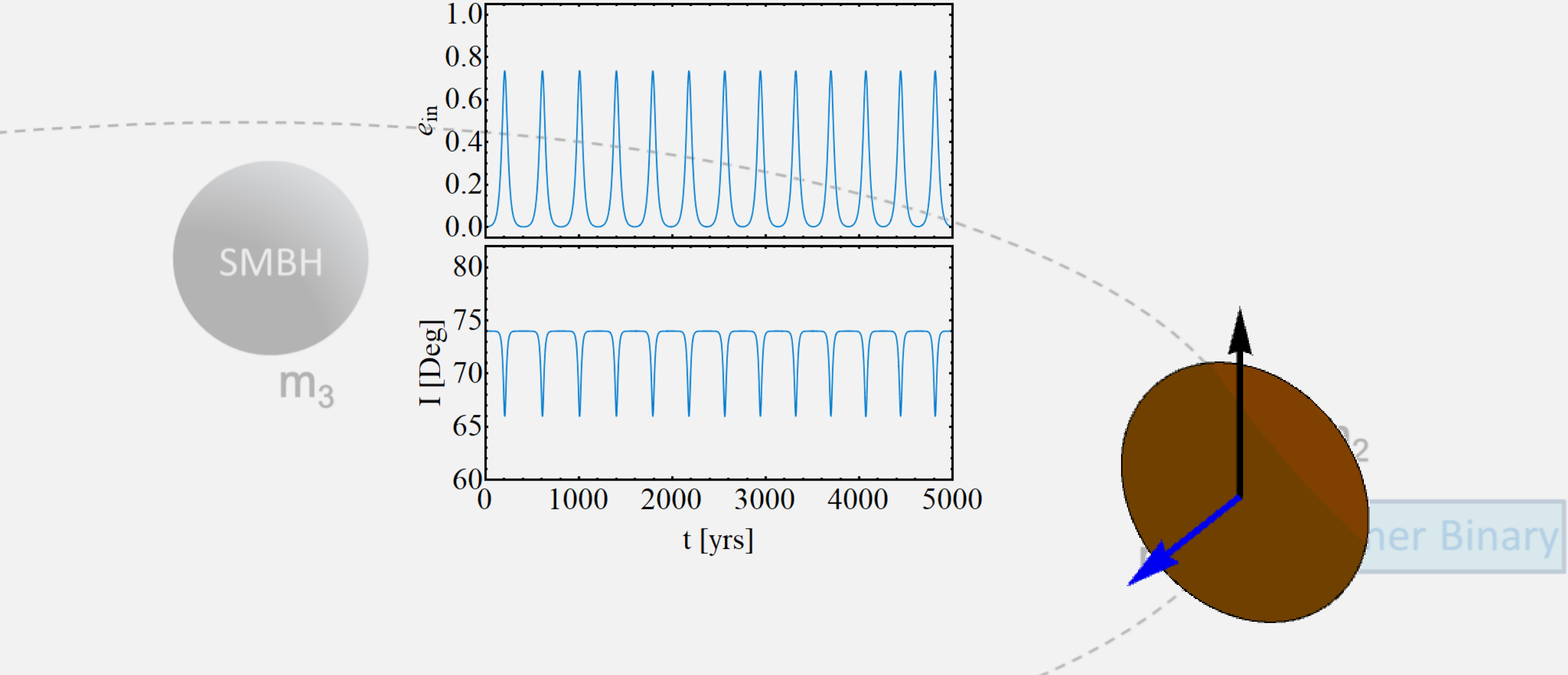


Binary - SMBH Interaction

--- Newtonian Effect: Lidov-Kozai Oscillations

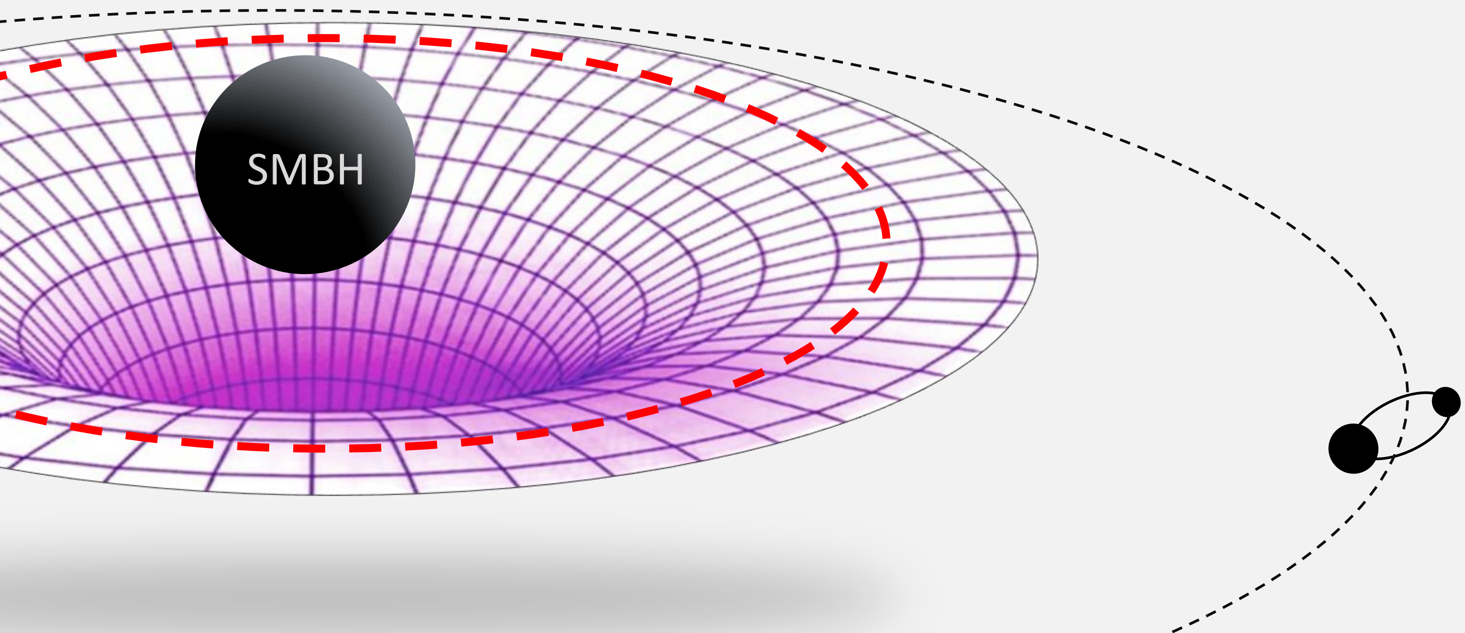


Binary Dynamics Near a Supermassive BH

--- Newtonian Effect: Lidov-Kozai Oscillations

--- GR Effect involving SMBH Tertiary

Strong GR effect induced by the SMBH

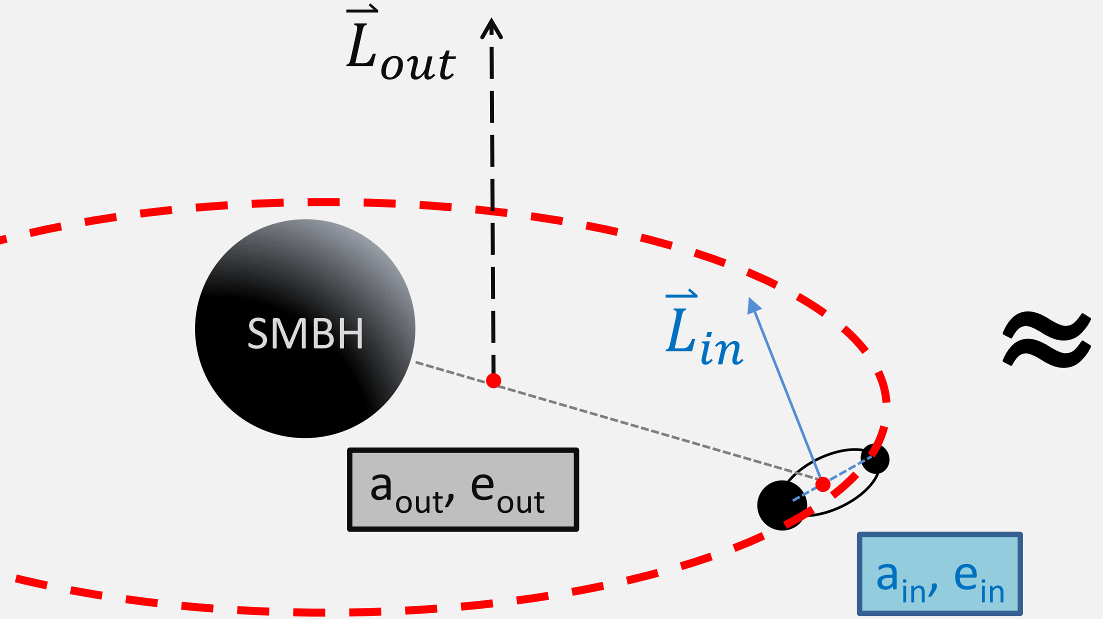


Liu & Lai ApJL 2019

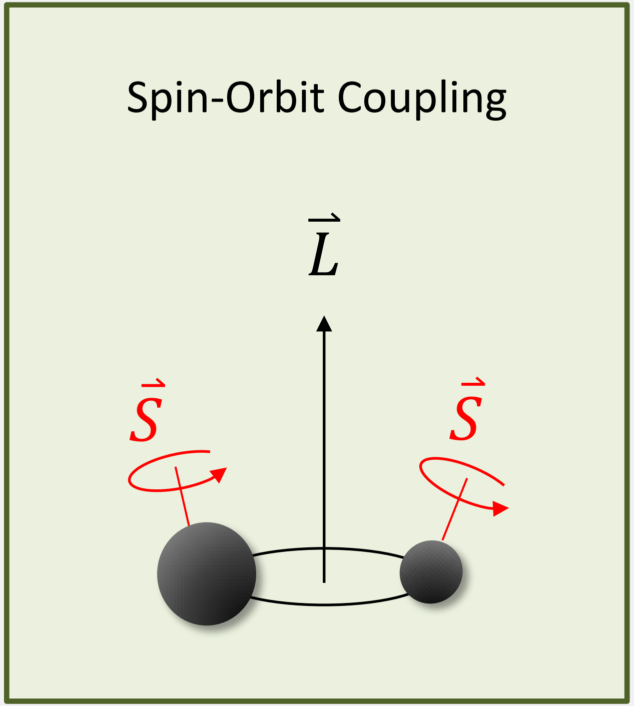
Binary Dynamics Near a Supermassive BH

--- Newtonian Effect: Lidov-Kozai Oscillations

--- GR Effect: Precession of L_{in} around L_{out} (1.5 PN)



Isolated Binary



Binary Dynamics Near a Supermassive BH

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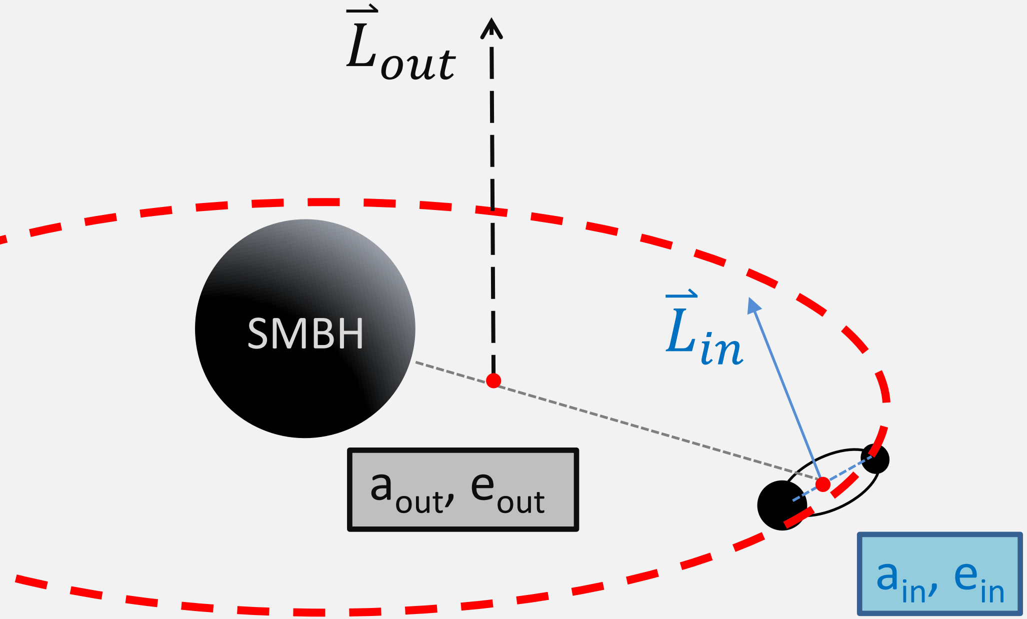
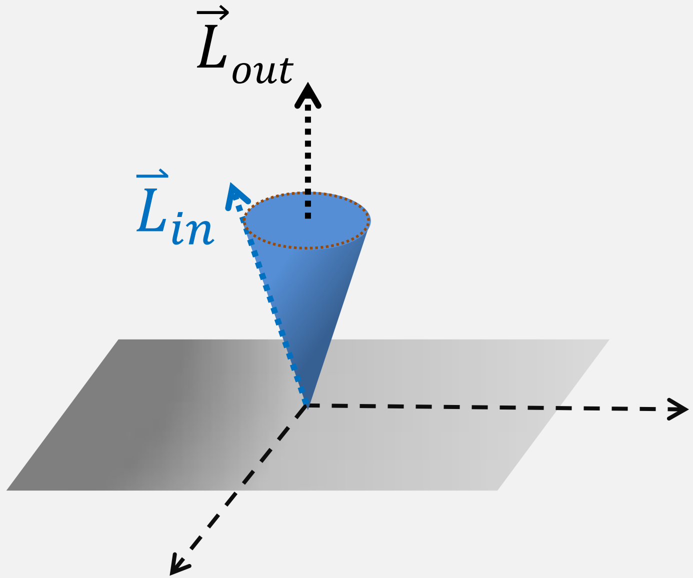
--- GR Effect: Precession of L_{in} around L_{out} **(1.5 PN)**

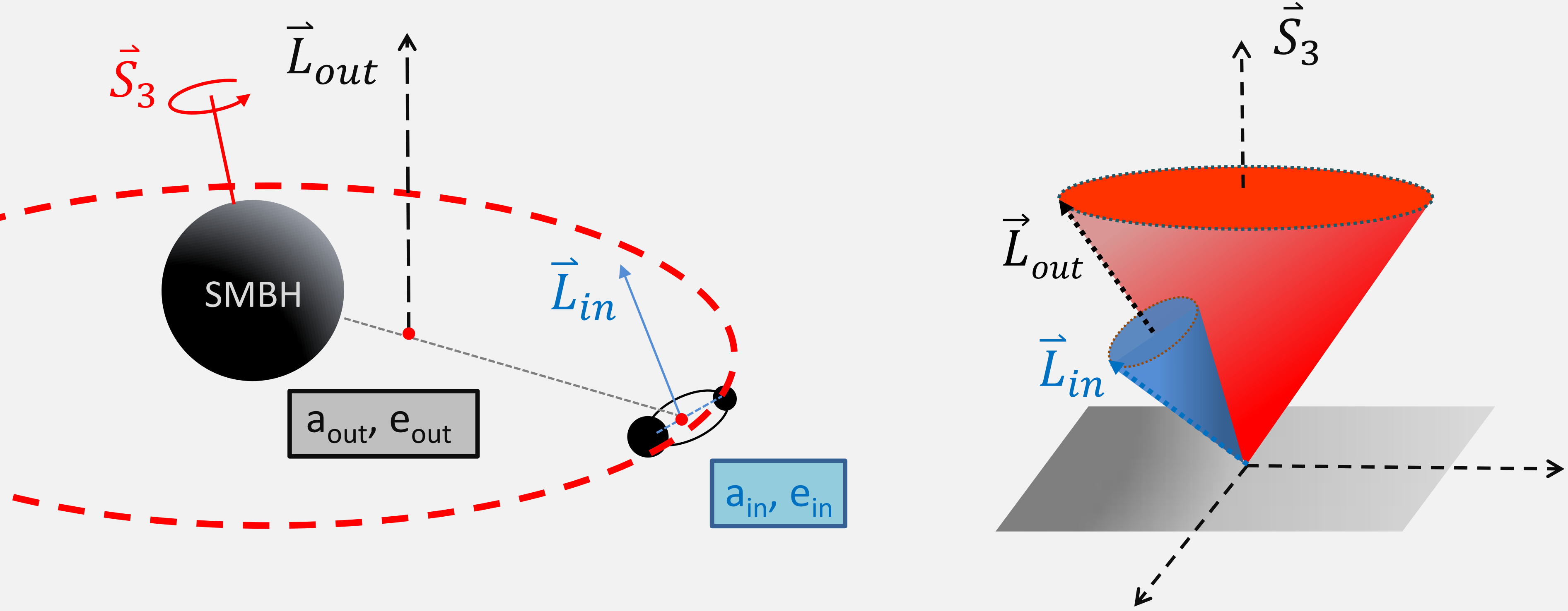




Binary Dynamics Near a Supermassive BH

--- Newtonian Effect: Lidov-Kozai Oscillations

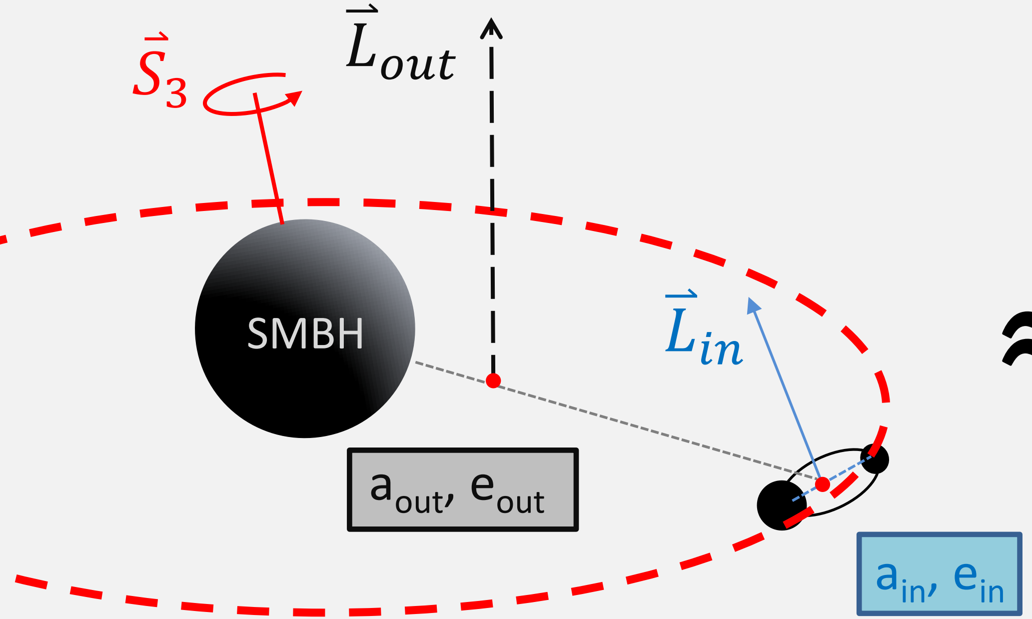
--- GR Effect: Precession of $\boldsymbol{L_{out}}$ around $\boldsymbol{S_3}$ **(1.5 PN)**



Binary Dynamics Near a Supermassive BH

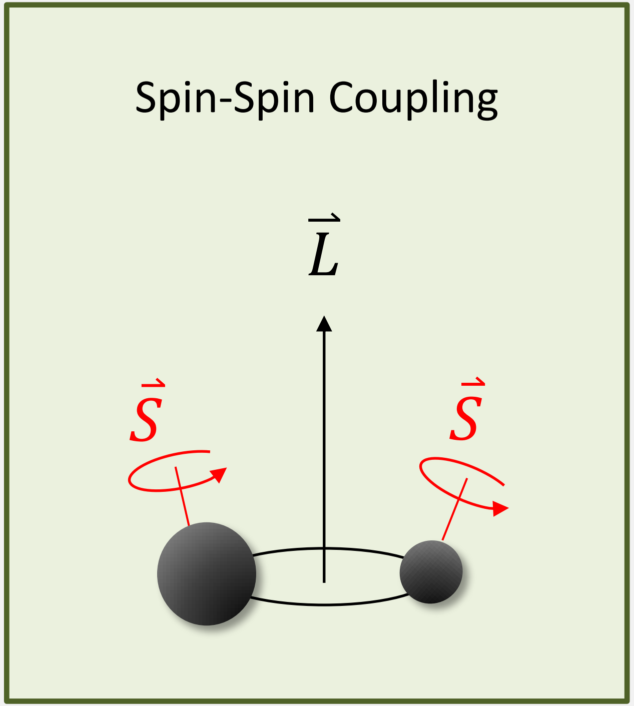
--- Newtonian Effect: Lidov-Kozai Oscillations

--- GR Effect: Precession of L_{in} around S_3 (2 PN)



$\approx$

Isolated Binary



Binary near the Spinning Supermassive Black Hole

The leading-order effects (recognizing L_{in} behaves like a "spin")

- **Effect I:** *de-Sitter-like Precession of* $\boldsymbol{L}_{in}$ *around* $\boldsymbol{L}_{out}$

  --- 1.5 PN

$$\begin{cases} \left.\frac{d\mathbf{L}}{dt}\right|_{\mathrm{L_{in}L_{out}}} = \Omega^{(\mathrm{GR})}_{\mathrm{L_{in}L_{out}}} \hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{L}, \\ \left.\frac{d\mathbf{e}}{dt}\right|_{\mathrm{L_{in}L_{out}}} = \Omega^{(\mathrm{GR})}_{\mathrm{L_{in}L_{out}}} \hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{e}, \end{cases}$$

- **Effect II:** *Precession of* $\boldsymbol{L}_{out}$ *around* $\boldsymbol{S}_3$

  --- 1.5 PN

$$\begin{cases} \left.\frac{d\mathbf{L}_{\mathrm{out}}}{dt}\right|_{\mathrm{L_{out}S_3}} = \Omega_{\mathrm{L_{out}S_3}} \hat{\mathbf{S}}_3 \times \mathbf{L}_{\mathrm{out}}, \\ \left.\frac{d\mathbf{e}_{\mathrm{out}}}{dt}\right|_{\mathrm{L_{out}S_3}} = \Omega_{\mathrm{L_{out}S_3}} \hat{\mathbf{S}}_3 \times \mathbf{e}_{\mathrm{out}} \\ \qquad -3\Omega_{\mathrm{L_{out}S_3}} (\hat{\mathbf{L}}_{\mathrm{out}} \cdot \hat{\mathbf{S}}_3) \hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{e}_{\mathrm{out}} \end{cases}$$

- **Effect III:** *Lense-Thirring Precession of* $\boldsymbol{L}_{in}$ *around* $\boldsymbol{S}_3$

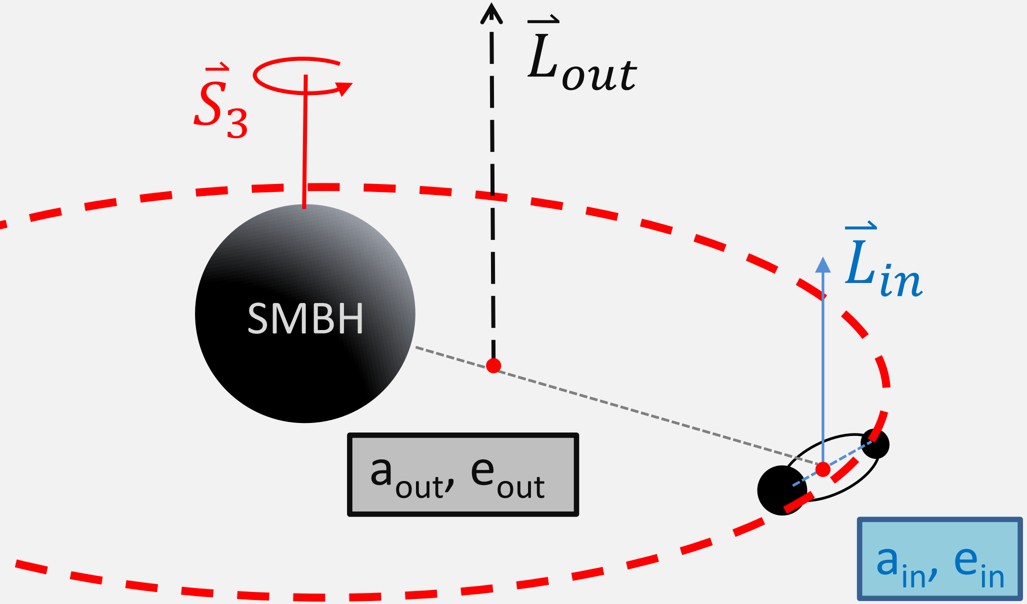
  --- 2 PN

$$\begin{cases} \left.\frac{d\mathbf{L}}{dt}\right|_{\mathrm{L_{in}S_3}} = \Omega_{\mathrm{L_{in}S_3}} \hat{\mathbf{S}}_3 \times \mathbf{L} \\ \qquad -3\Omega_{\mathrm{L_{in}S_3}} (\hat{\mathbf{L}}_{\mathrm{out}} \cdot \hat{\mathbf{S}}_3) \hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{L}, \\ \left.\frac{d\mathbf{e}}{dt}\right|_{\mathrm{L_{in}S_3}} = \Omega_{\mathrm{L_{in}S_3}} \hat{\mathbf{S}}_3 \times \mathbf{e} \\ \qquad -3\Omega_{\mathrm{L_{in}S_3}} (\hat{\mathbf{L}}_{\mathrm{out}} \cdot \hat{\mathbf{S}}_3) \hat{\mathbf{L}}_{\mathrm{out}} \times \mathbf{e}. \end{cases}$$

See more details in Lim & Rodriguez 2020; Fang & Huang 2020

Binary Dynamics Near a Supermassive BH

--- Coplanar + GR Effect

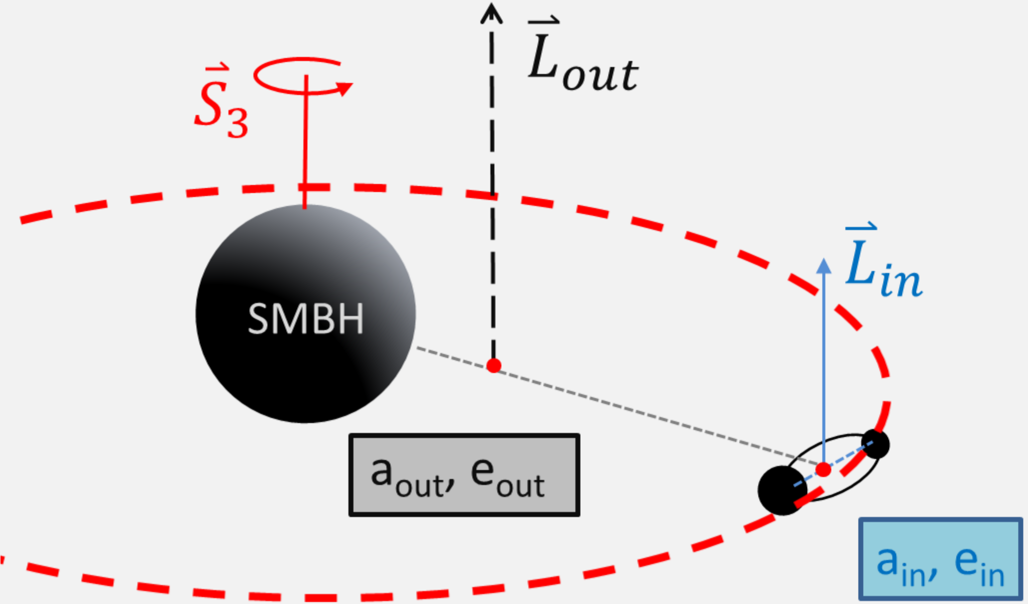


$$\begin{cases} \dfrac{d\hat{L}}{dt} = 0 \\ \dfrac{d\vec{e}}{dt} = \omega\, \hat{L} \times \vec{e} \end{cases}$$

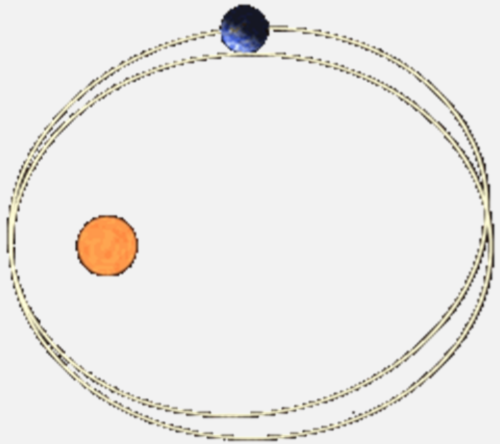
Liu & Lai PRD 2020

Binary Dynamics Near a Supermassive BH

--- Coplanar + GR Effect



Apsidal Precession



Apsidal Precession Resonance

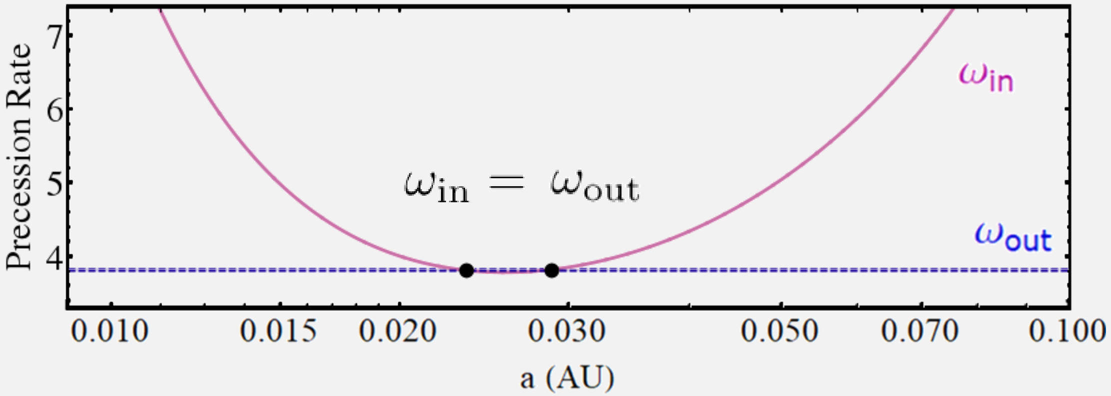
- **Precession Rate**

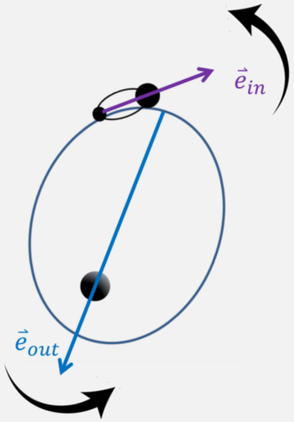
Newtonian 1PN 1.5+2 PN

$$\omega_{\rm in} = \omega_{\rm LK,in} + \omega_{\rm GR,in} + \omega^{\rm (GR)}_{\rm L_{in}L_{out}} \mp 2\omega_{\rm L_{in}S_3},$$

$$\omega_{\rm out} = \omega_{\rm GR,out} \mp 2\omega_{\rm L_{out}S_3}$$

- **Consider one case:**

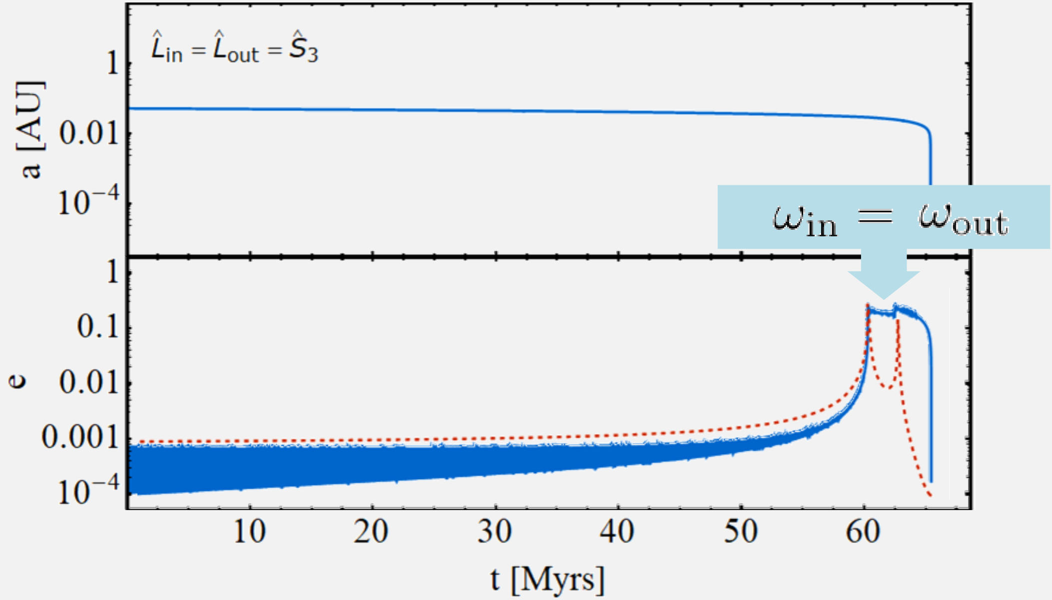


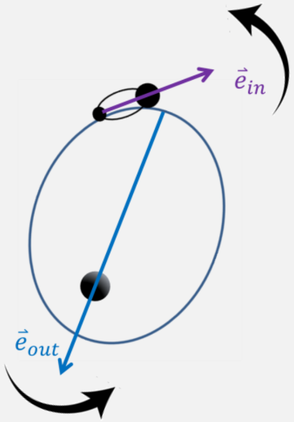


Apsidal Precession Resonance

- **Orbital Evolution:**

$(m_1, m_2, m_3) = (30, 20, 1\times10^8)\ M_\odot$
$(a_{in,0}, a_{out}) = (0.05, 90)$ AU
$(e_{in,0}, e_{out,0}) = (0.0001, 0.7)$





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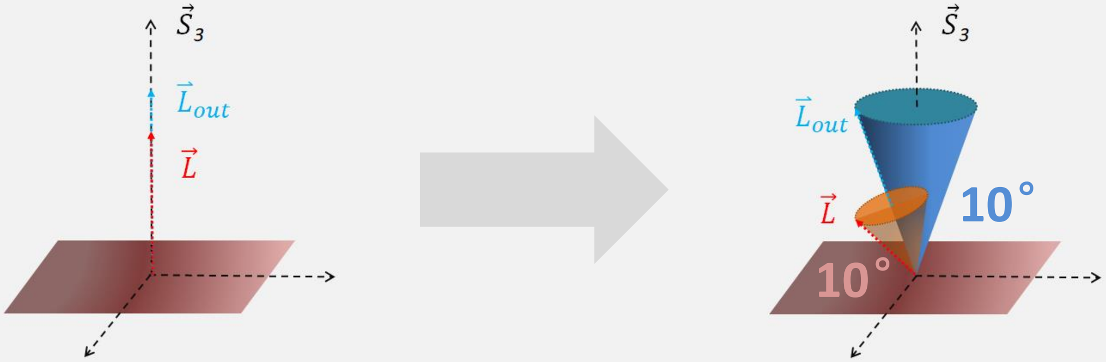
Apsidal Precession Resonance

- **Inclined System?**



- **Orbital Evolution:**

$(m_1, m_2, m_3) = (30, 20, 1\times10^8)\ M_\odot$
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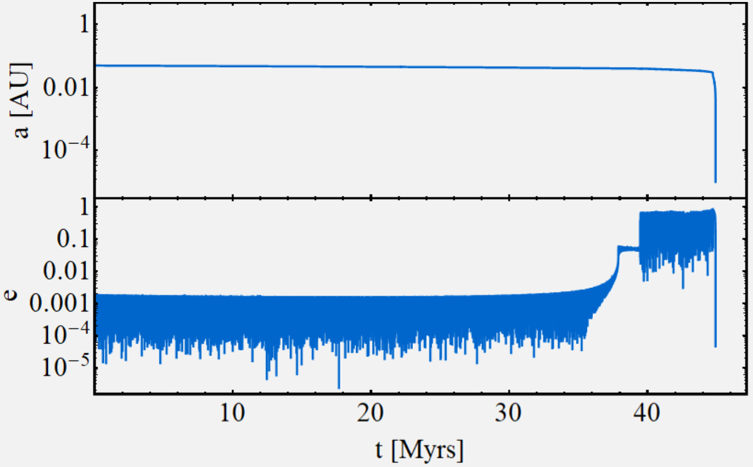
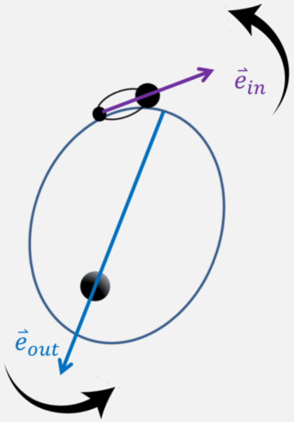
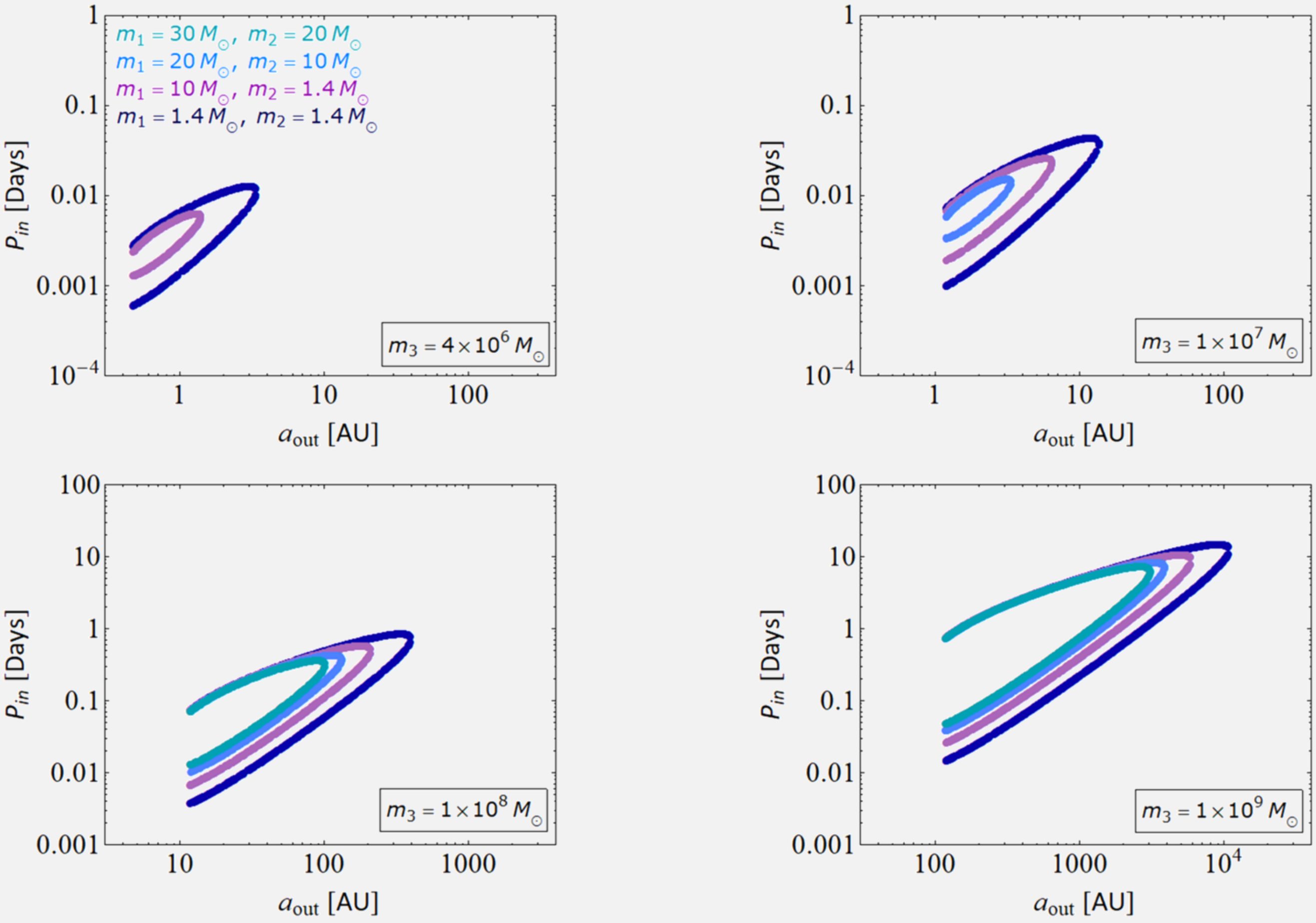
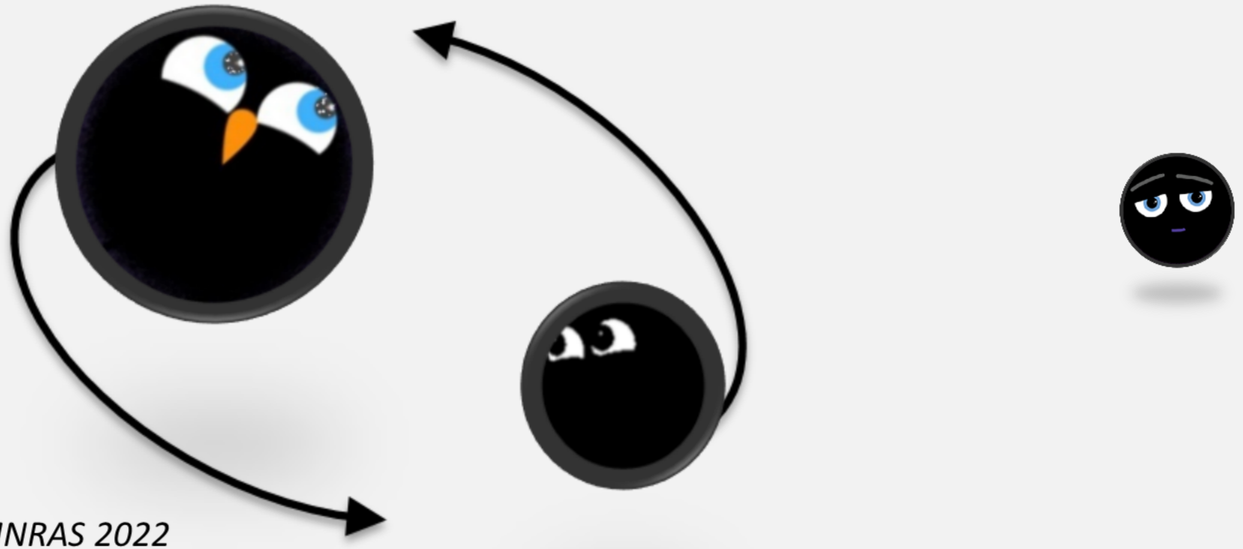




Parameter Space

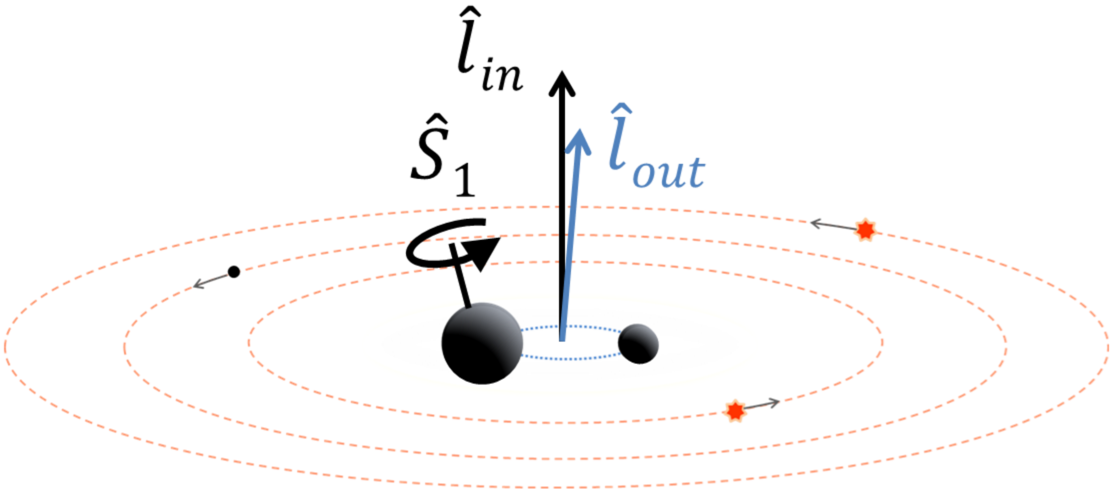


II. Inverse Secular Problem



Liu & Lai MNRAS 2022

Massive BH binary + surrounding stars

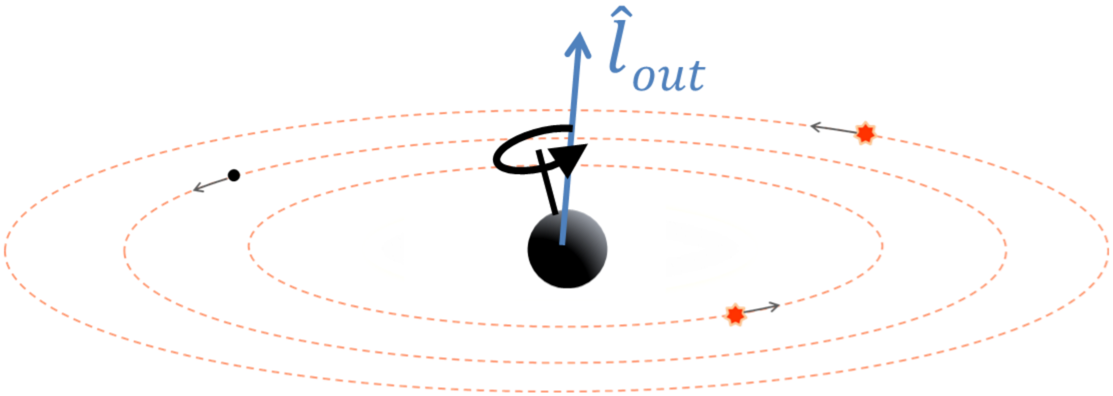


- **Effect I:**
  Precession of $\boldsymbol{L}_{out}$ around $\boldsymbol{L}_{in}$

- **Effect II:**
  Precession of $\boldsymbol{L}_{in}$ around $\boldsymbol{S}_1$

- **Effect III:**
  Precession of $\boldsymbol{L}_{out}$ around $\boldsymbol{S}_1$

Massive BH binary + surrounding stars



- **Effect I:**
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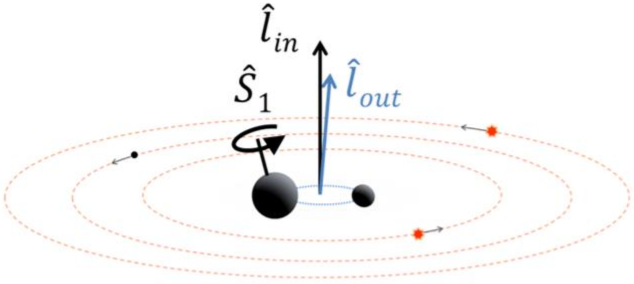
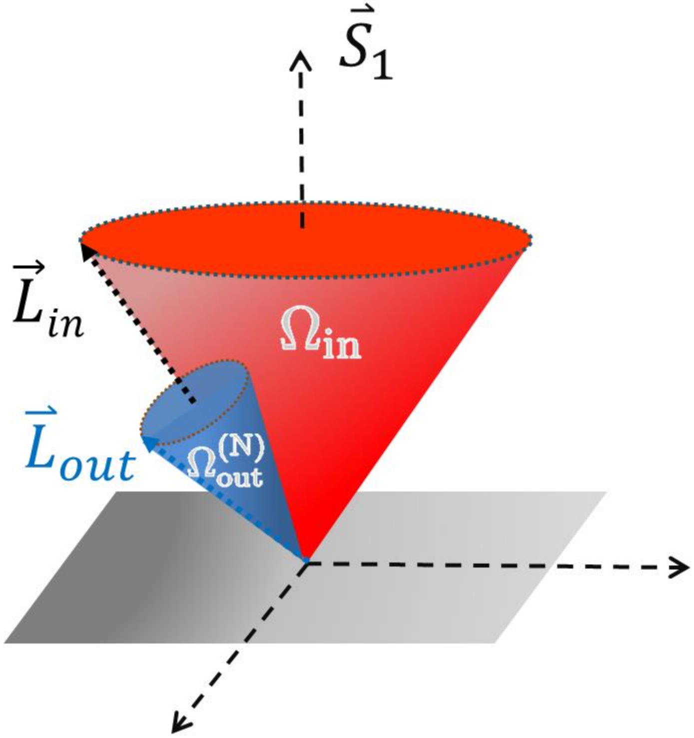
- **Effect II:**
  Precession of $\boldsymbol{L}_{in}$ around $\boldsymbol{S}_1$

- **Effect III:**
  Precession of $\boldsymbol{L}_{out}$ around $\boldsymbol{S}_1$

What happen to $\hat{l}_{out}$?

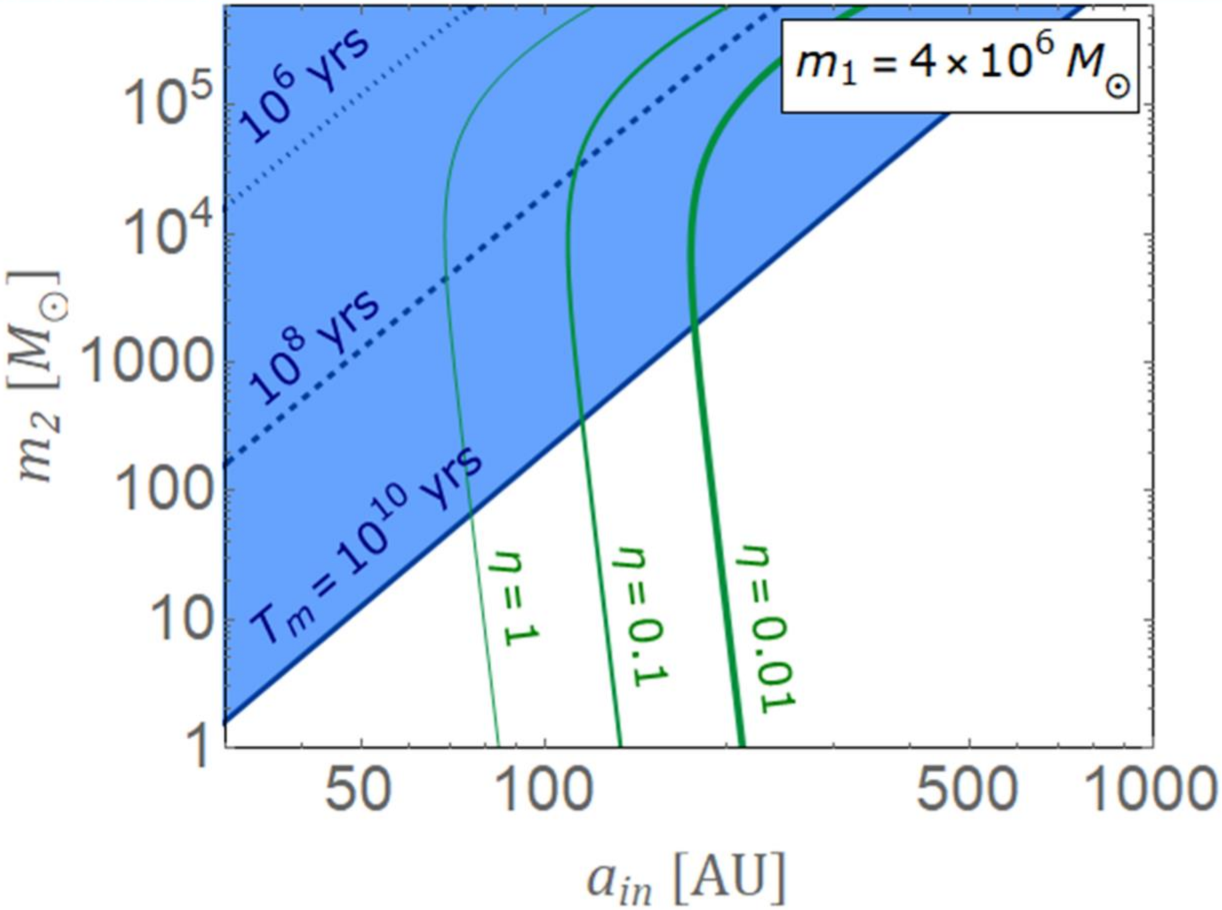
Dynamics of the System





$$\left\{ \eta = \frac{\Omega_{\text{in}}}{\Omega_{\text{out}}^{(\text{N})}} \right\}$$

Dynamics of the System



$$\left\{ \eta = \frac{\Omega_{\rm in}}{\Omega_{\rm out}^{(\rm N)}} \right\}$$

Different types of $\hat{l}_{out}$ behaviors

- **During the orbital decay of the inner BH binary**

$$\frac{\Omega_{\rm in}}{\Omega_{\rm out}^{\rm (N)}} < 1$$

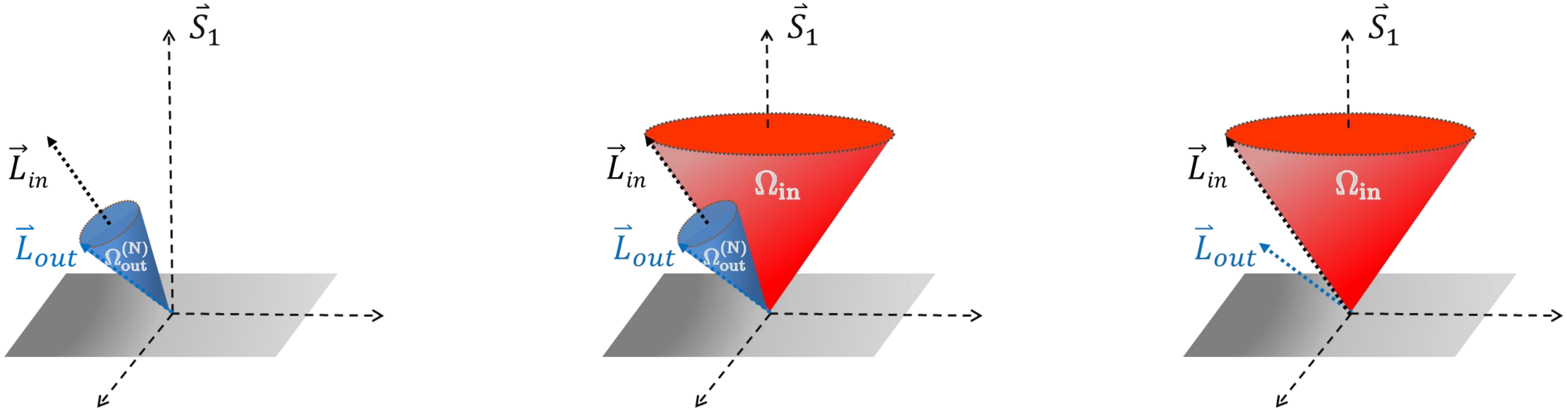
"nonadiabatic"

$$\frac{\Omega_{\rm in}}{\Omega_{\rm out}^{\rm (N)}} \sim 1$$

"trans-adiabatic"

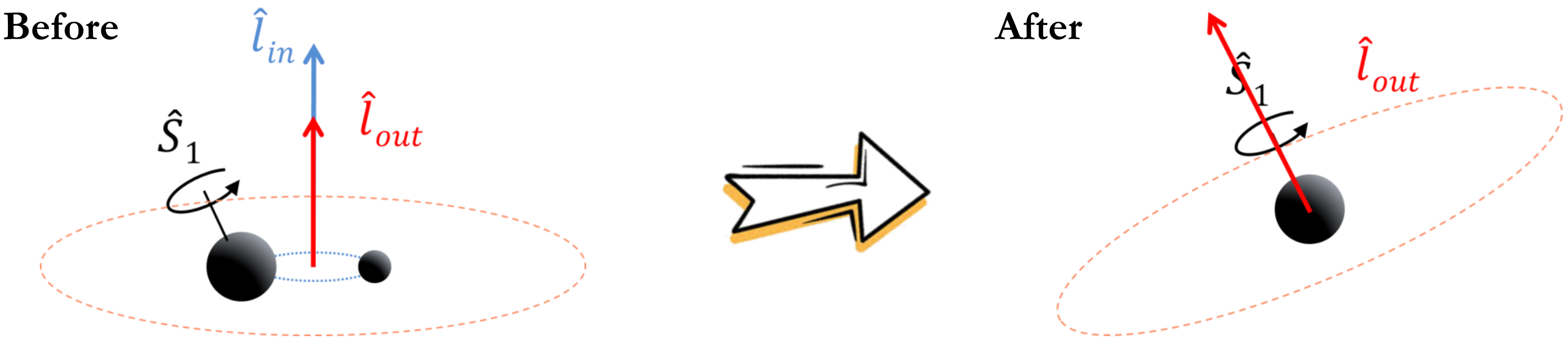
$$\frac{\Omega_{\rm in}}{\Omega_{\rm out}^{\rm (N)}} > 1$$

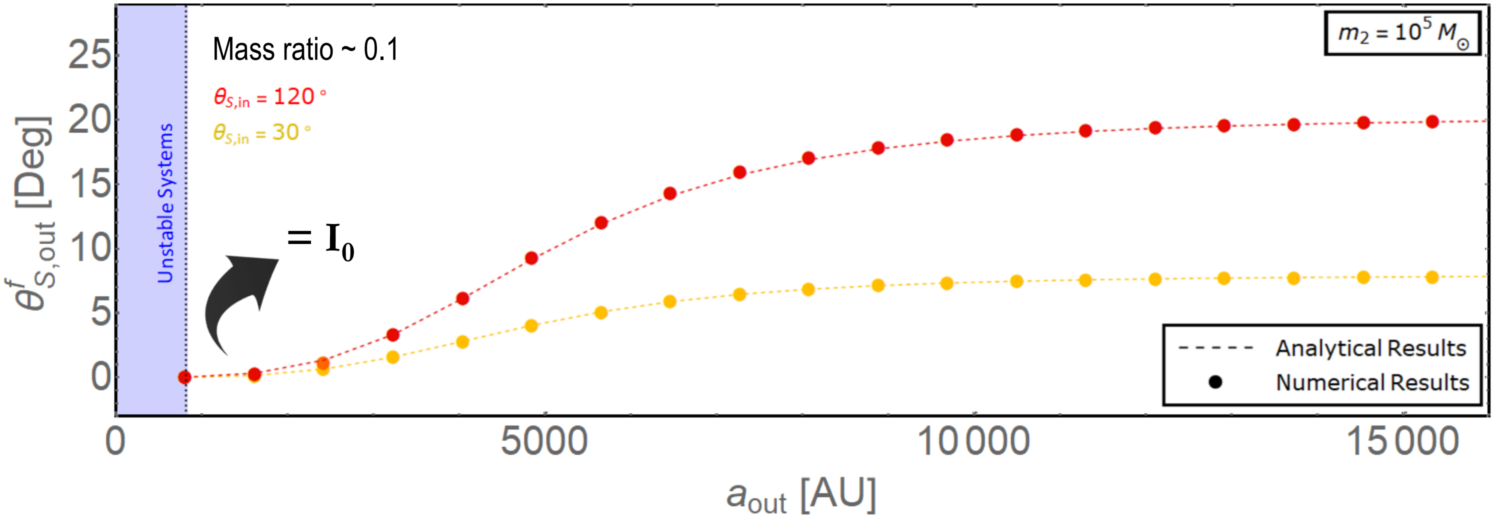
"adiabatic"



Final orientation of $\hat{l}_{out}$

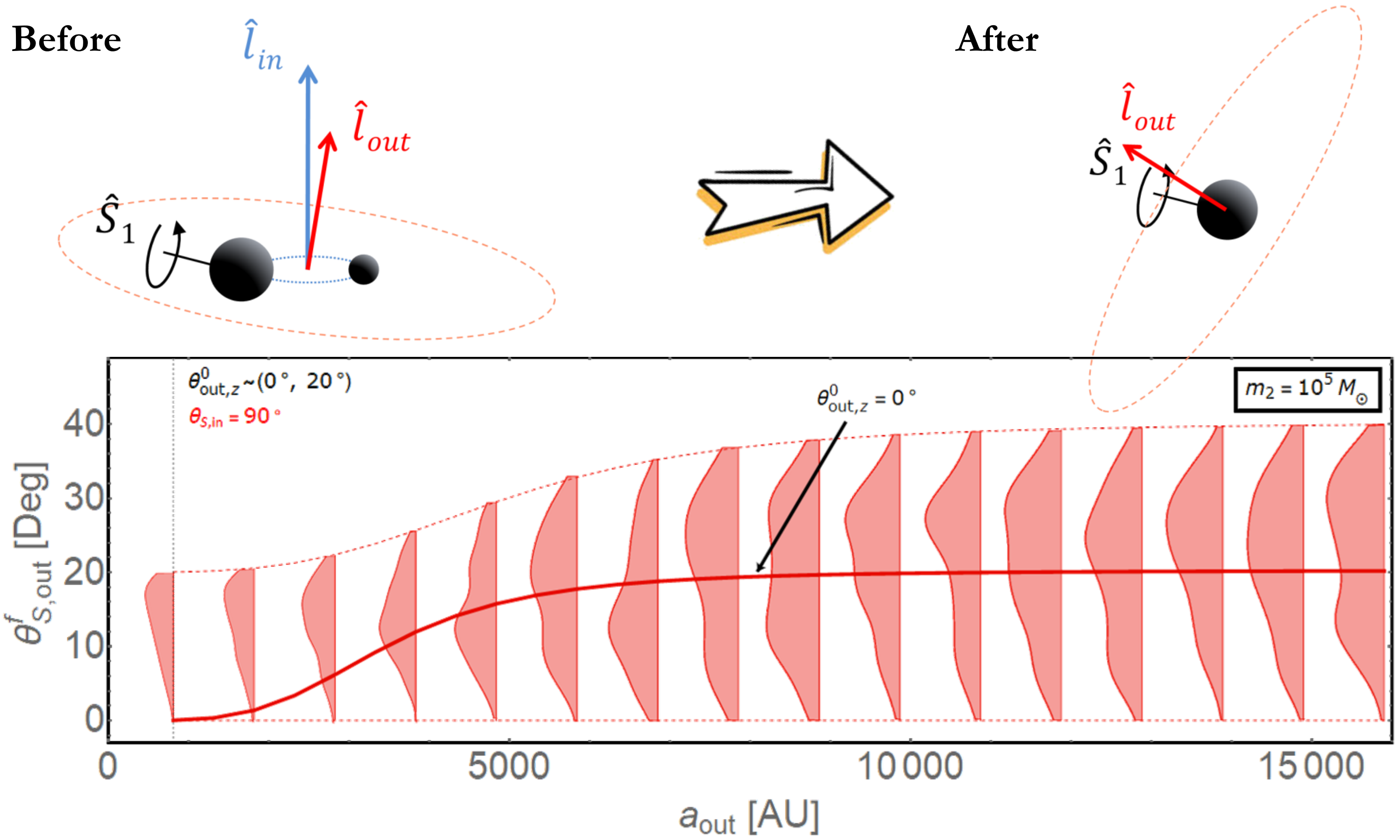
Parameters: $4\times10^6\, M_\odot + 10^5\, M_\odot$ a_{in} =475AU $e_{in} = e_{out} = 0$





Initially inclined $\hat{l}_{out}$

Parameters: $4\times10^6\ M_\odot + 10^5\ M_\odot$ a_{in} =475AU $e_{in}=e_{out}=0$



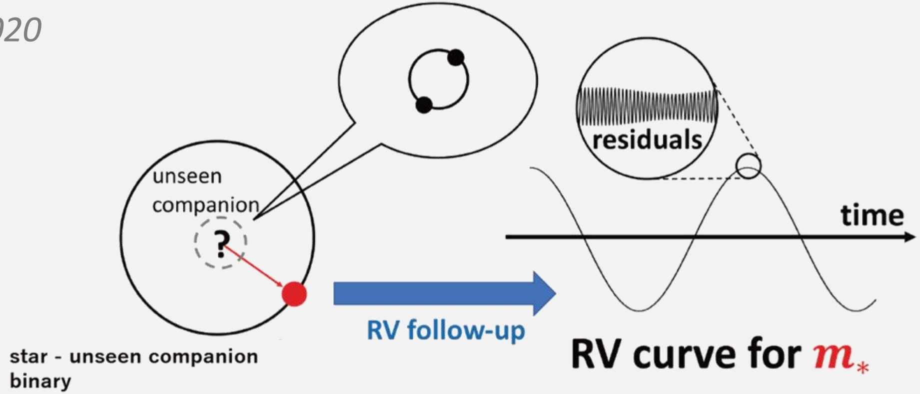
III. On-going Study:

Probing the BHB with the Change of Eccentricity of Outer Stellar Orbit

Star Near a Massive BH Binary

- **Short-term Evolution**

Hayashi + 2019, 2020



short-term RV variations due to BHB period: $\sim P_{in}/2$

- **Long-term Evolution**

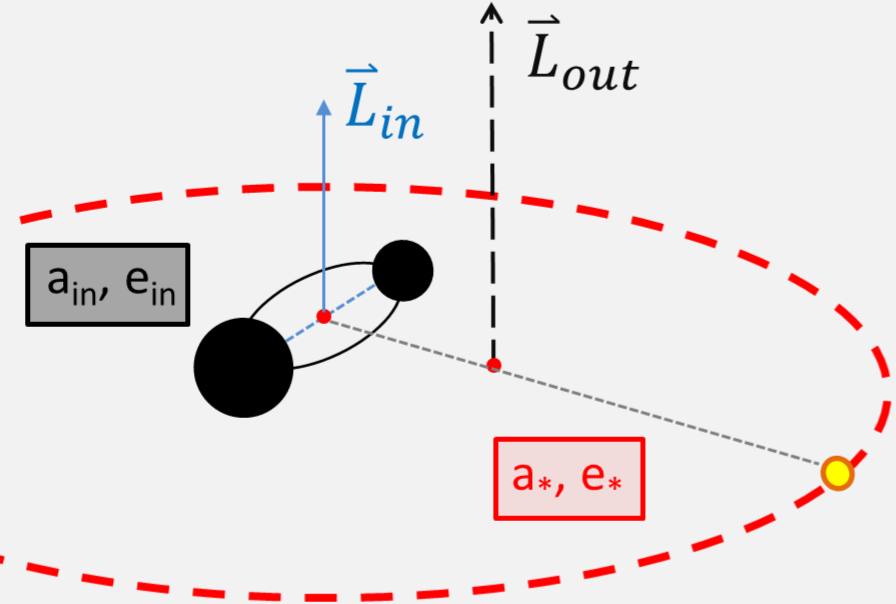
Naoz + 2017, 2020
Vinson & Chiang 2018

Δe_*

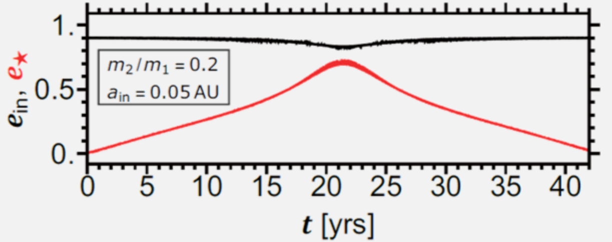
ΔI_*

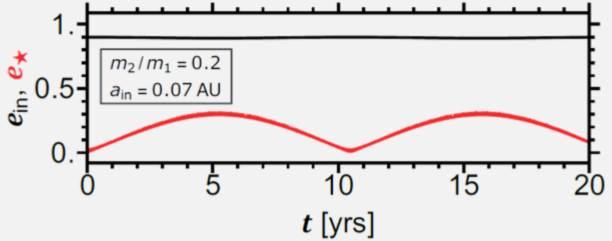
Star Near a Massive BH Binary

--- Coplanar + GR Effect



$m_{12} = 50\ M_\odot$; $e_{in,0} = 0.9$, $e_{*,0} = 0.0001$, $P_* = 15$ days

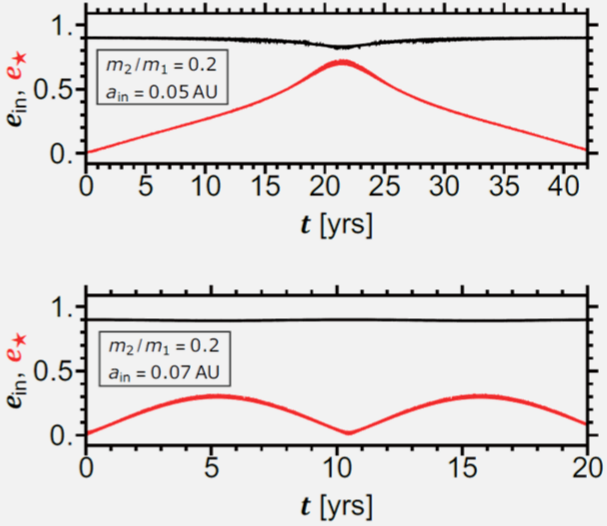


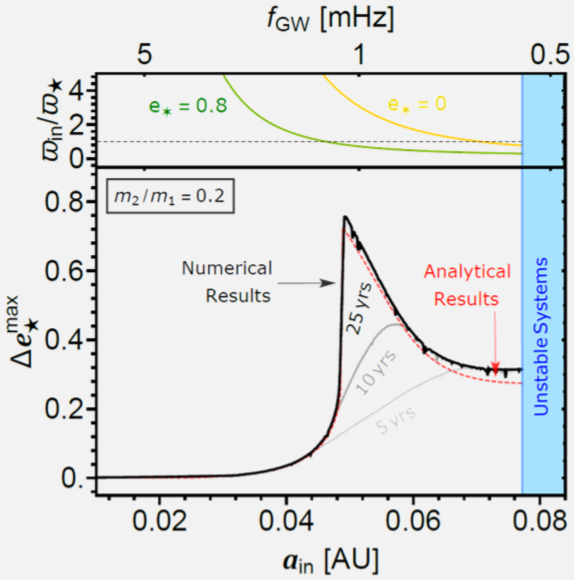


Star Near a Massive BH Binary

--- Coplanar + GR Effect

$$\frac{\omega_{\rm in} = \omega_{\rm N,in} + \omega_{\rm GR,in}}{\omega_\star = \omega_{\rm N,\star} + \omega_{\rm GR,\star}}$$

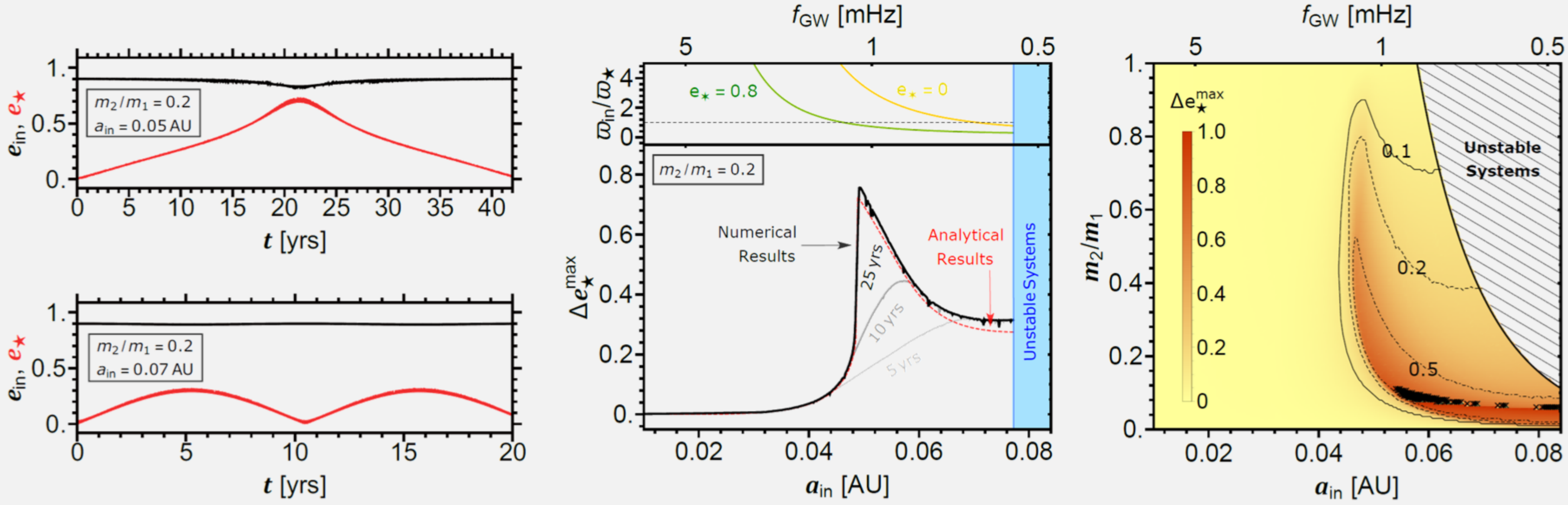




For a given stellar orbit: P_*= 15 days

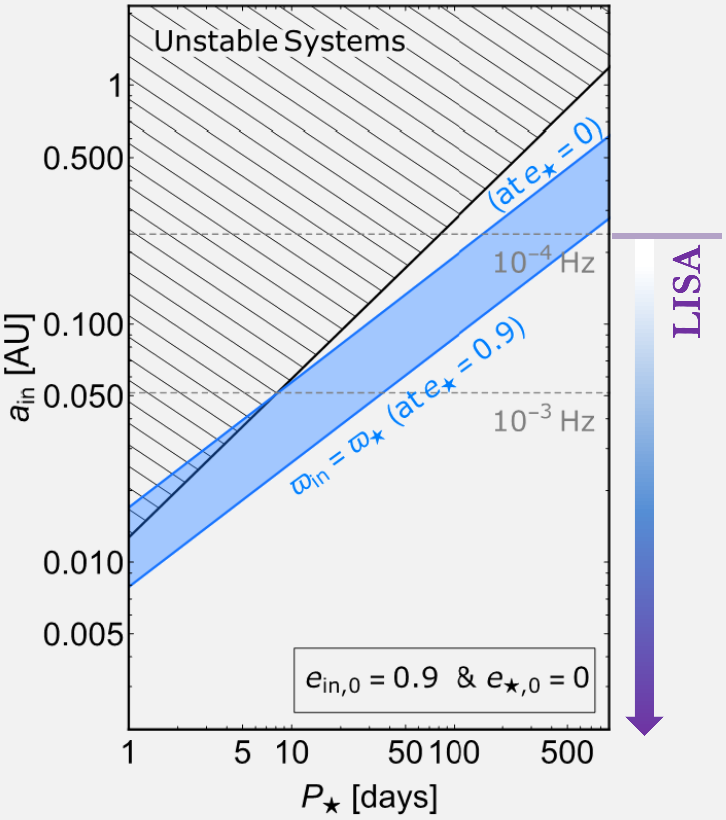
Star Near a Massive BH Binary

--- Coplanar + GR Effect

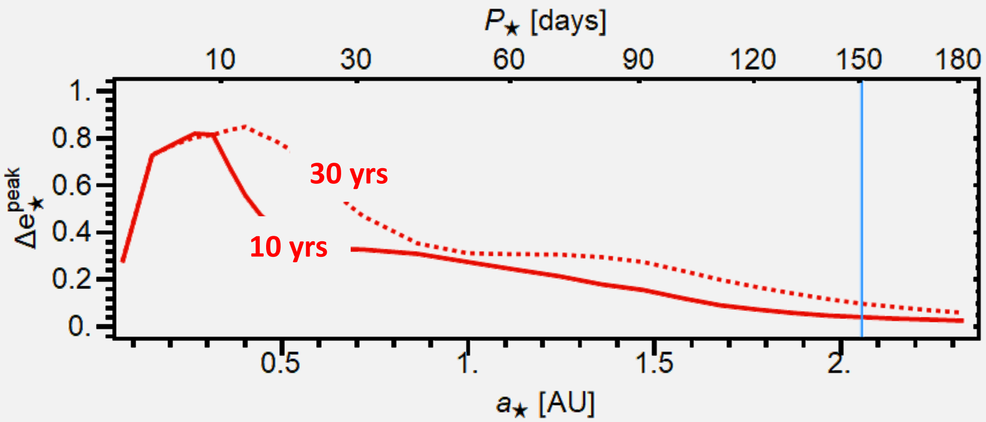


For a given stellar orbit: $P_* = 15$ days

Parameter Space



- m_{12} = 50 $M_\odot$; $e_{in,0}$ = 0.9, $e_{*,0}$= 0.0001
- LISA sources
  --- P_{in} ~ 1 day
- A range of stellar orbits



Detectability

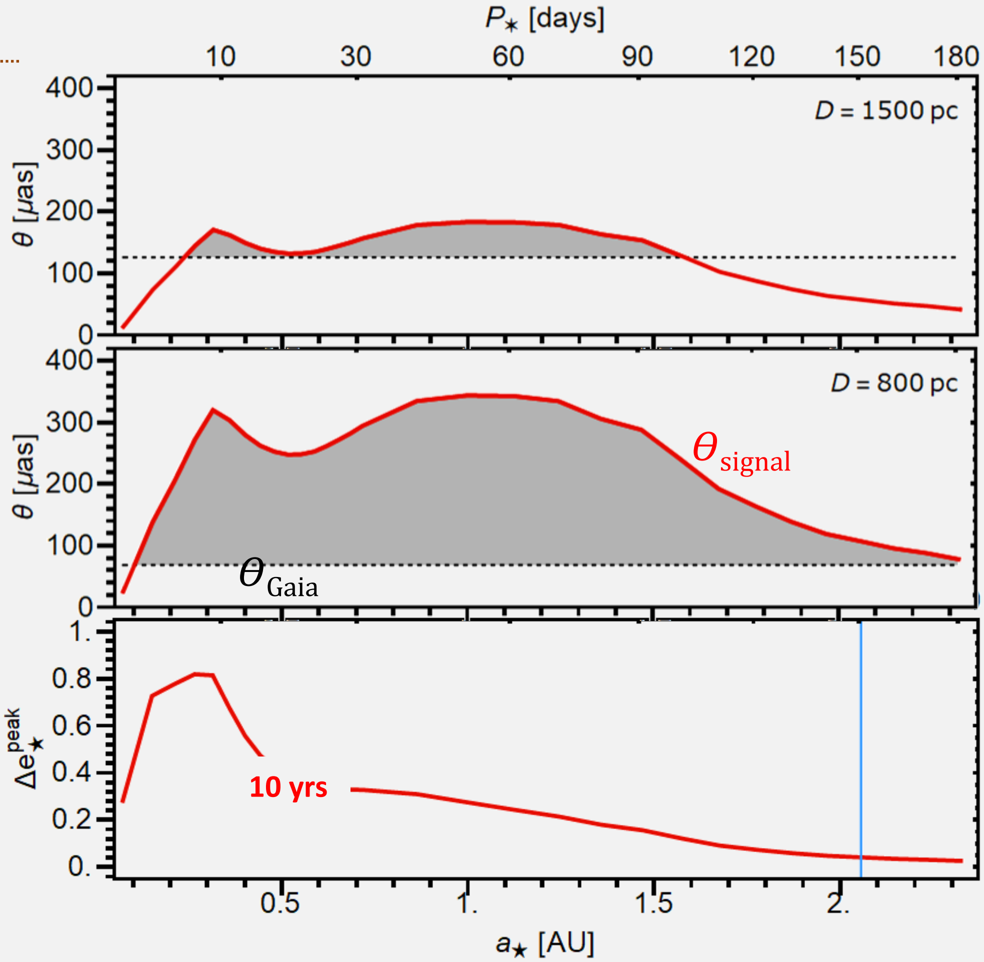
- Whether the change of e_* can be **POTENTIALLY** detected?

- *Gaia* (spacecraft)

Astrometric precision:

$$\theta_{\rm Gaia} \equiv \theta(\rm SNR; D)$$

$$\theta_{\rm signal} \simeq \frac{a_\star \Delta e_\star^{\rm peak}}{\rm D}$$



Summary

- **BH Binary near the SMBH**

  --- Eccentricity-Excitation due to Apsidal Precession Resonance

- **Star around SMBH Binary**

  --- Stellar orbit can be tilted after the merger of BH binary
  (Potential signature of the merger history of the SMBH)

- **Star around BH Binary**

  --- Signature: Change of the eccentricity of the stellar orbit
  (Detectable)