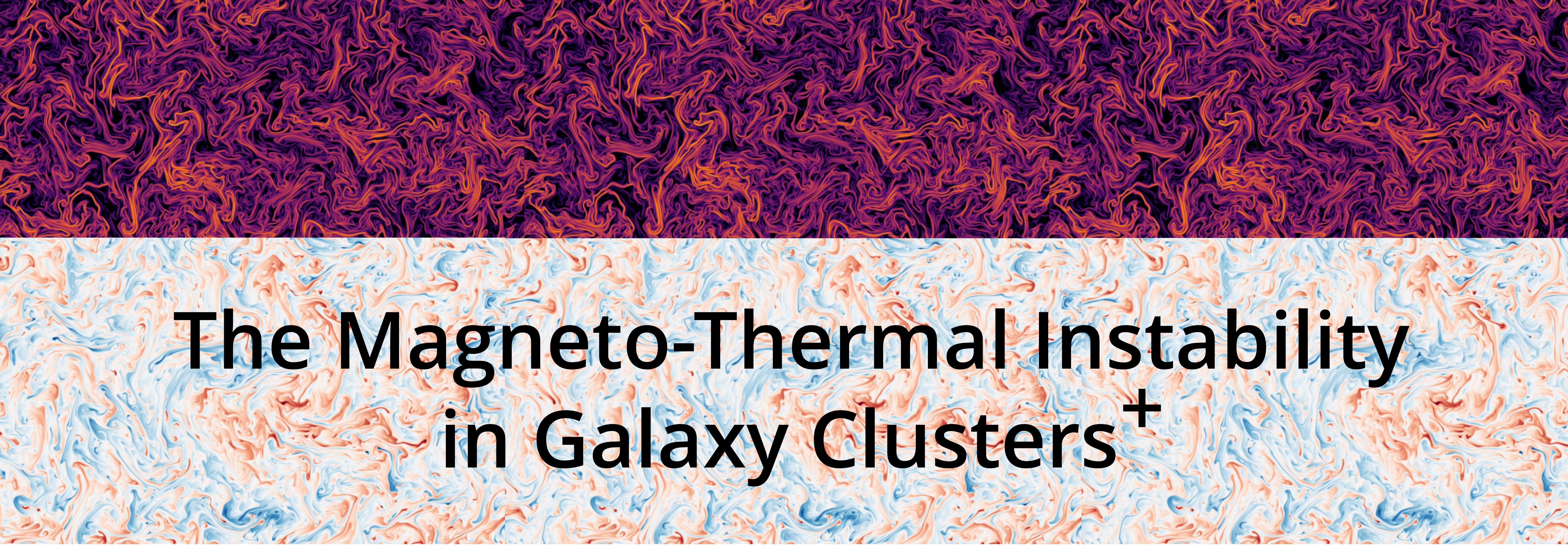


The Magneto-Thermal Instability in Galaxy Clusters[+]

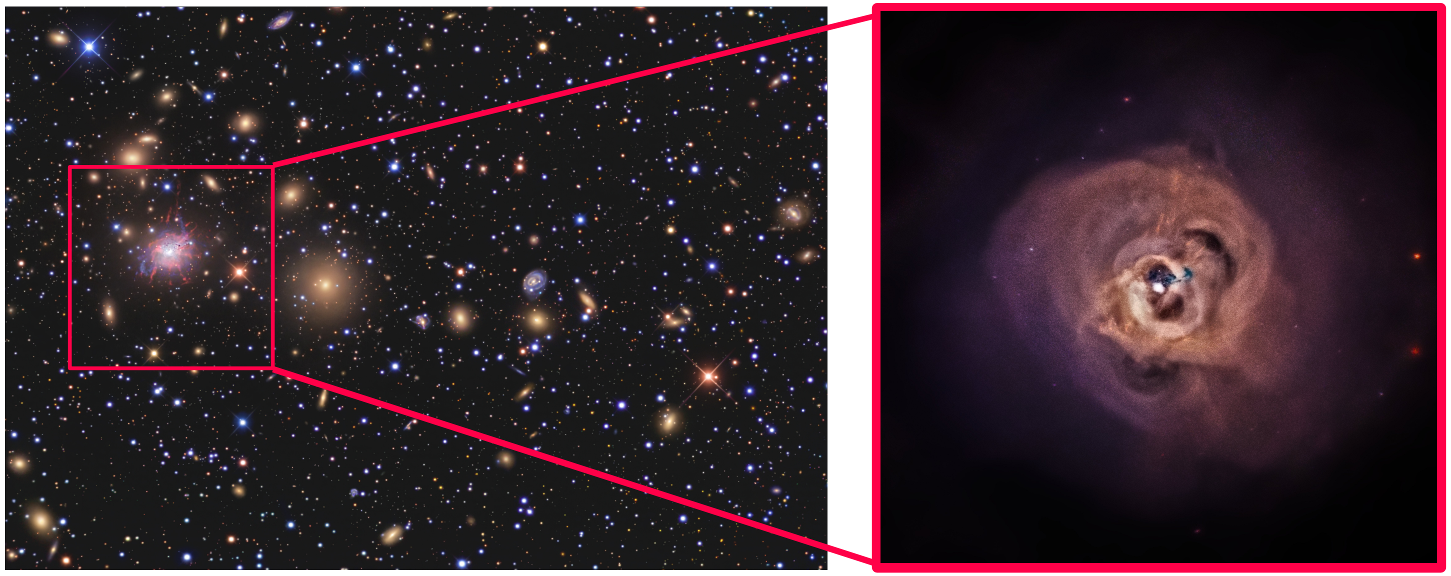
Lorenzo M. Perrone, Henrik Latter

Department of Applied Mathematics and Theoretical Physics
University of Cambridge, UK

[+]MNRAS, 513 (1 July 2022) 4605-24
MNRAS, 513 (1 July 2022) 4625-44

Introduction - Galaxy Clusters

- Largest bound systems in the Universe: typical size $\sim \mathrm{Mpc}$
- Filled with hot and dilute plasma: Intra-cluster medium (ICM) $T \lesssim 10\mathrm{keV}, n \sim 10^{-3}\mathrm{cm}^{-3}$
- Account for most of the baryonic matter in galaxy clusters



Perseus cluster: optical light (Blackbird Observatory)

Chandra: X-rays

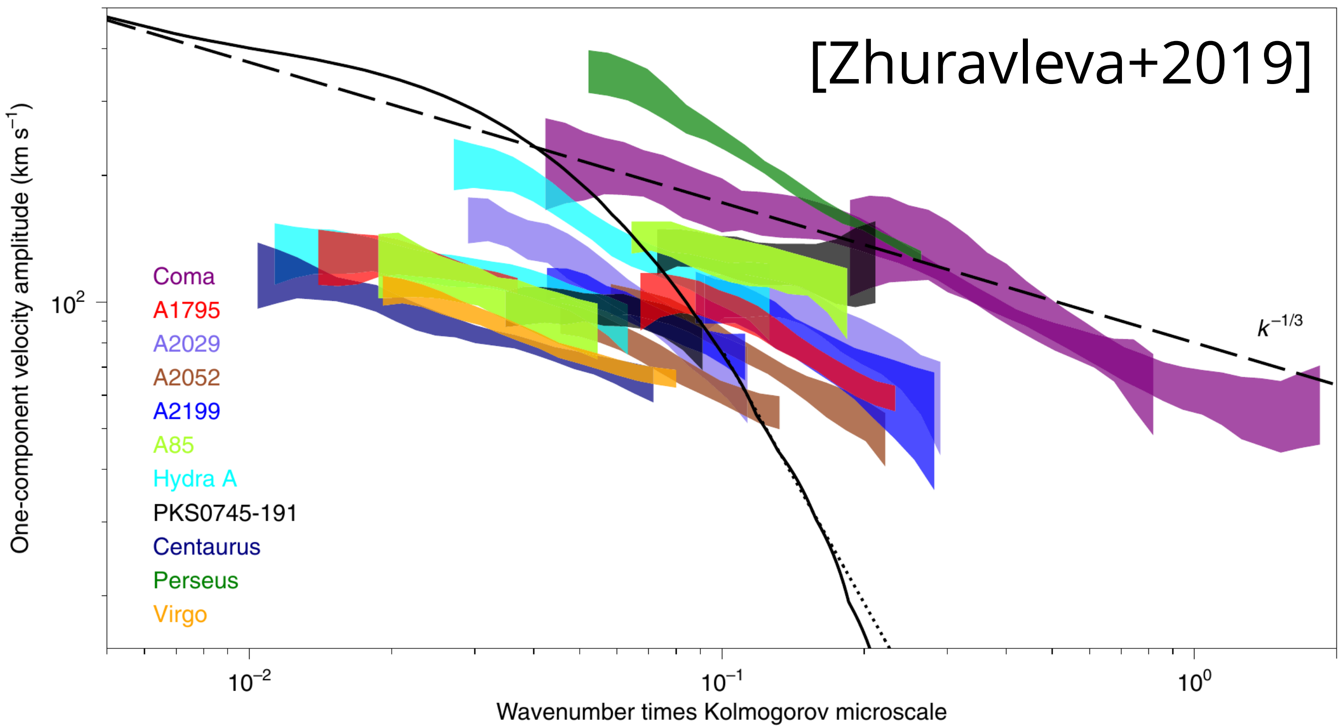
Introduction - The Intra-Cluster Medium

- ICM is turbulent

  accreting substructures/mergers, AGN feedback

  ...

$$\delta\rho/\rho \sim 5 - 10\%, \quad u_{rms} \sim \text{few } 100s\,\text{km/s}$$



Introduction - The Intra-Cluster Medium

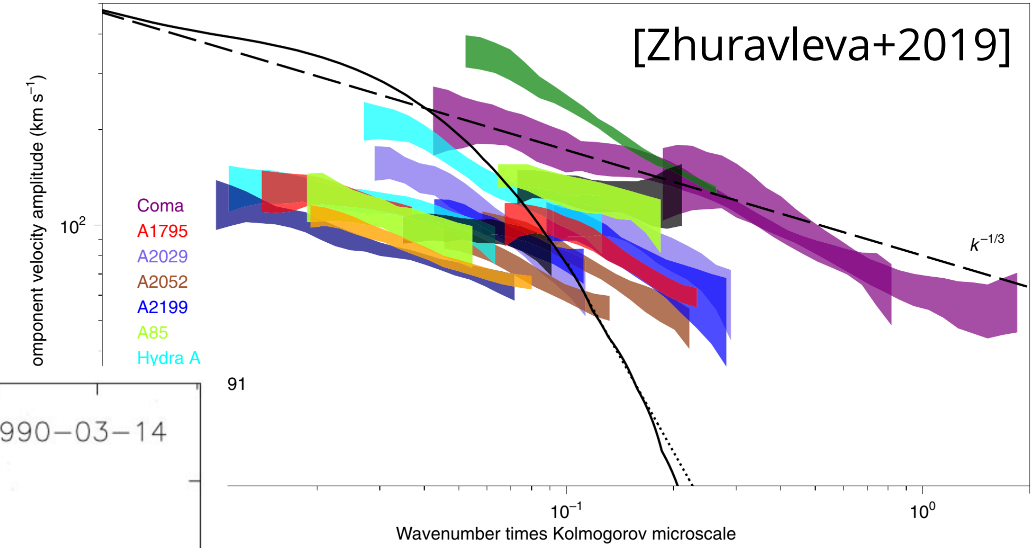
- ICM is turbulent

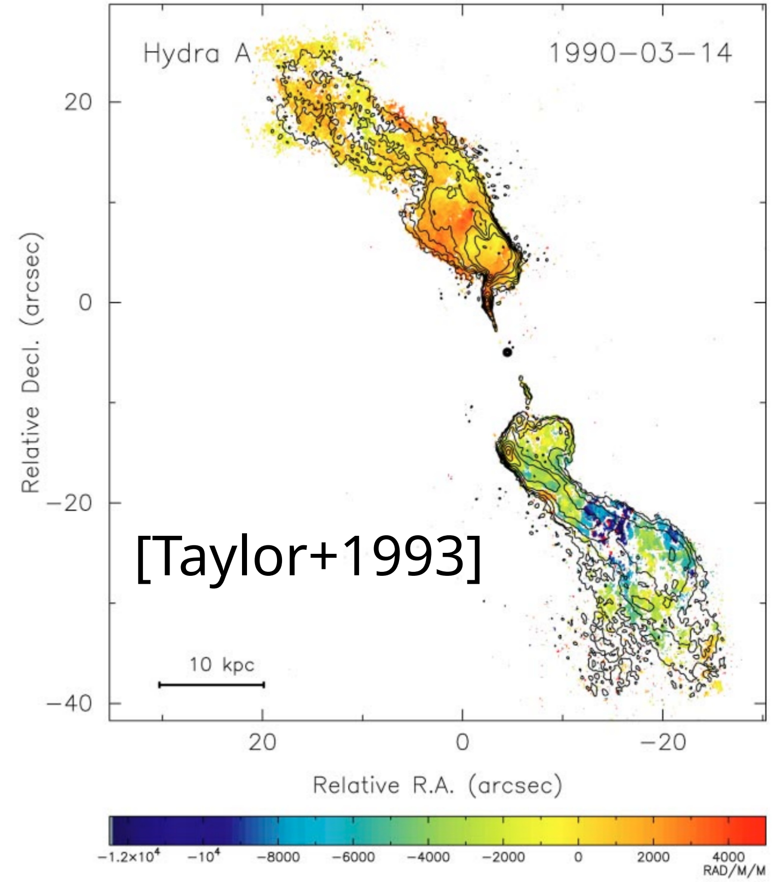
- ICM is magnetized

  Typical magnetic field strength $1 - 10\mu\mathrm{G}$

  Coherence length $\sim 10\mathrm{kpc}$

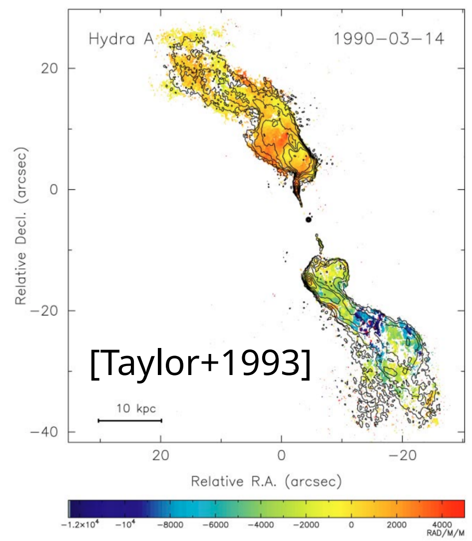
  High plasma beta $\beta \sim 100$

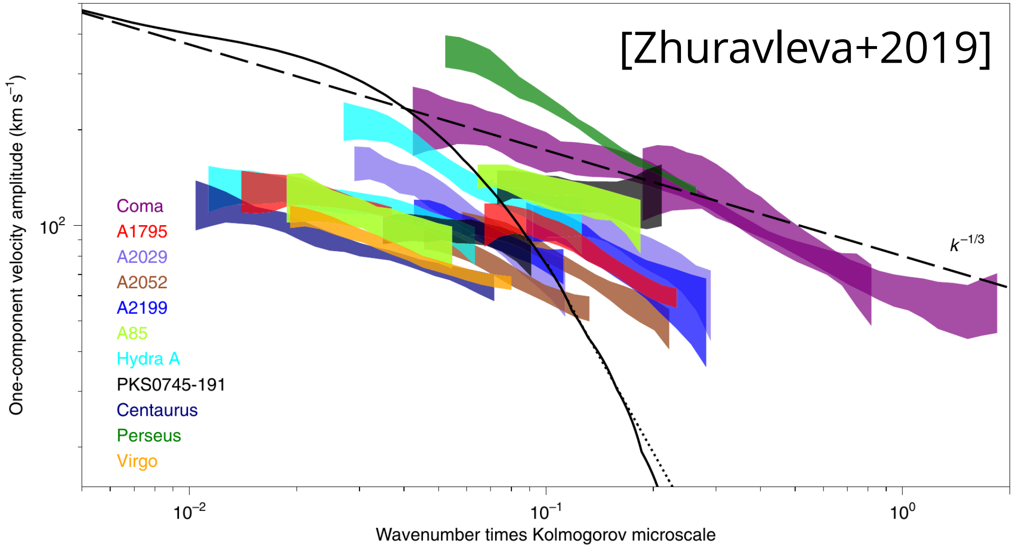




Introduction - The Intra-Cluster Medium

- ICM is turbulent

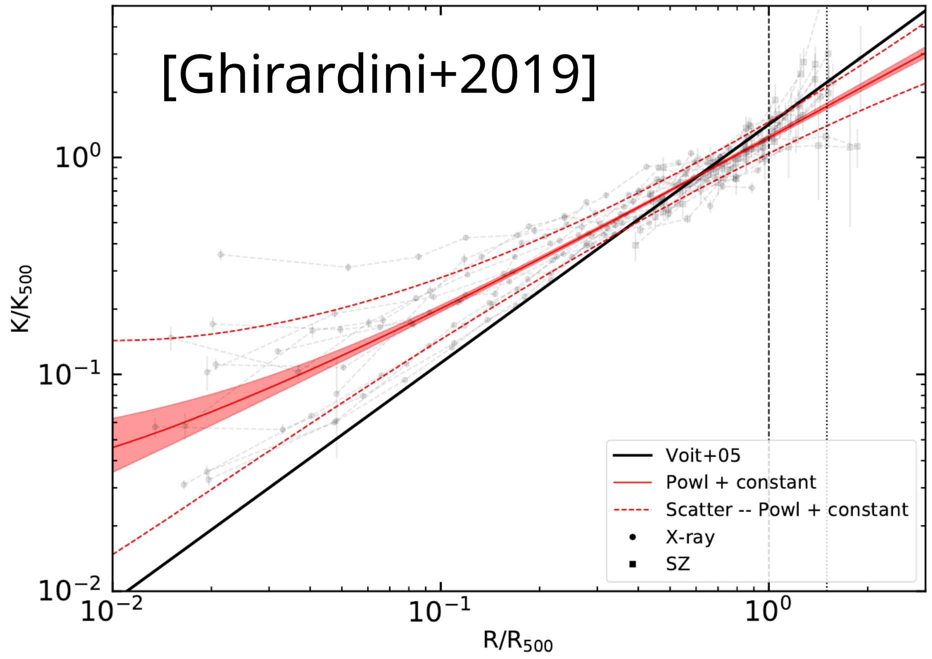




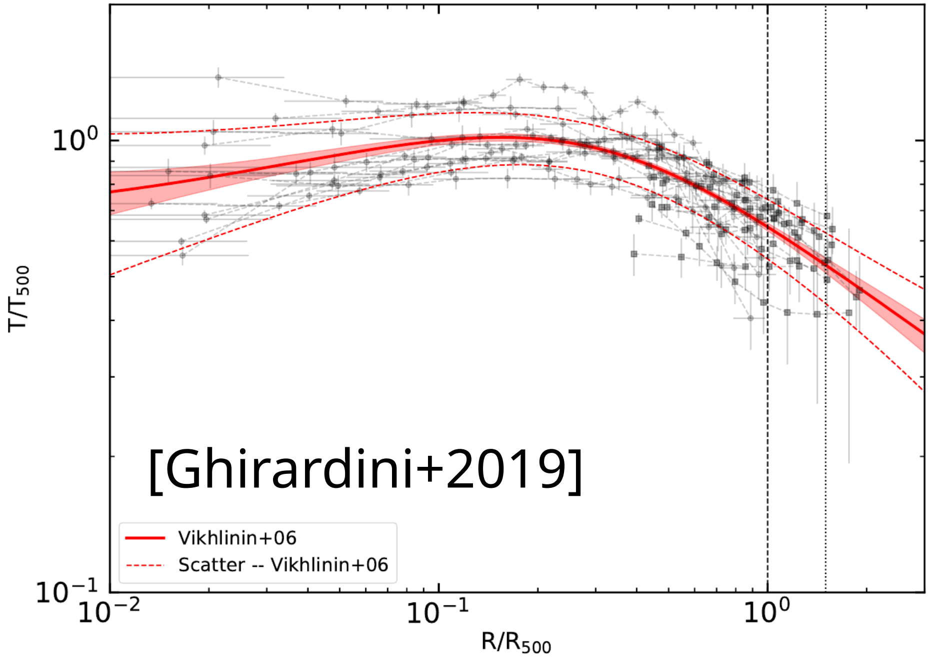
- ICM is magnetized

- ICM is stratified

Stable entropy stratification



Core: temperature increases / remains flat
Periphery: temperature decreases



Physics of ICM

Strongly magnetized

$$\frac{\rho_i}{\lambda_{\rm mfp}} \simeq 10^{-14} - 10^{-12}$$

$$\rho_i \ll \lambda_{\rm mfp} \lesssim H$$

$$\Omega_i \gg \nu_{ii} \gtrsim \omega_{\rm dyn}$$

Weakly collisional

$$\frac{\lambda_{\rm mfp}}{H} \simeq 10^{-3} - 10^{-2}$$

- Heat conduction and viscosity are anisotropic with respect to local direction of **B**:

$$\boldsymbol{q}_e \simeq -\kappa_{\|} \boldsymbol{b} \cdot \nabla T_e + \text{ h.o.t.}$$

$$\kappa_{\|} \simeq n_e v_{th,e} \lambda_{{\rm mfp},e}$$

Spitzer conductivity

$$\Pi_i \simeq -3 \frac{p_i}{\nu_{ii}} \left(\boldsymbol{bb} - \frac{\boldsymbol{I}}{3} \right) \left(\boldsymbol{bb} - \frac{\boldsymbol{I}}{3} \right) : \nabla \boldsymbol{u} + \text{h.o.t.}$$

The Magneto-Thermal Instability

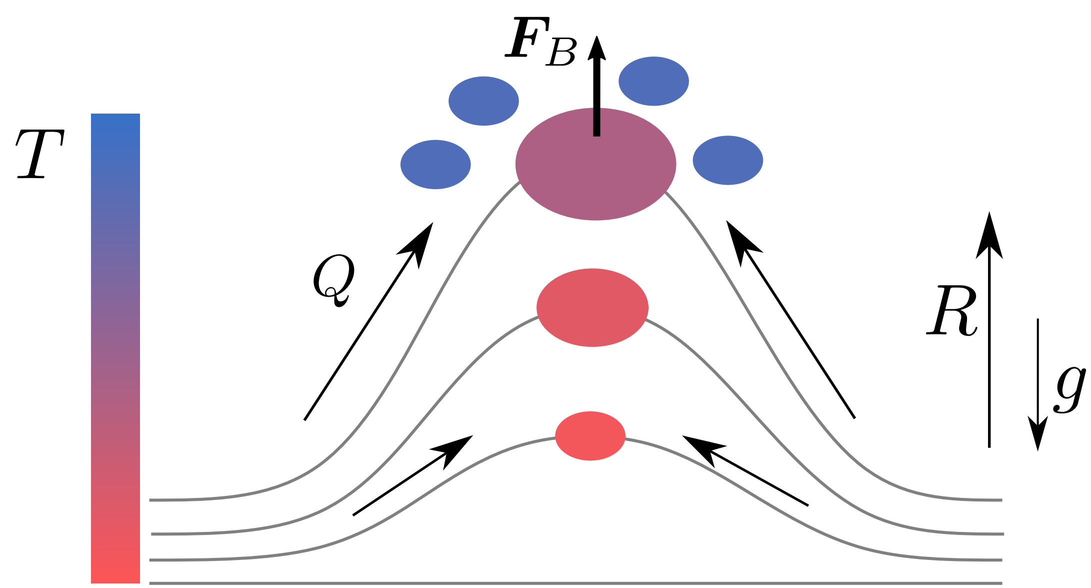
- Large-scale equilibrium affected by anisotropic heat conduction

Schwarzschild criterion ⟶ MTI instability criterion

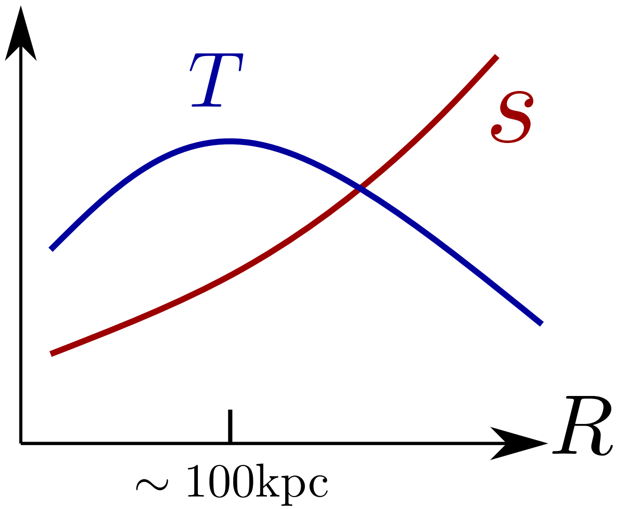
$$\partial s/\partial R < 0 \rightarrow \partial T/\partial R < 0$$ [Balbus2000]

(for weak, horizontal B)

Basic mechanism of MTI:



Anisotropic heat conduction can destabilize outskirts of ICM



- Interesting parallel with Rayleigh criterion and MRI:

$$\partial l/\partial R < 0 \rightarrow \partial \Omega/\partial R < 0$$

The Magneto-Thermal Instability

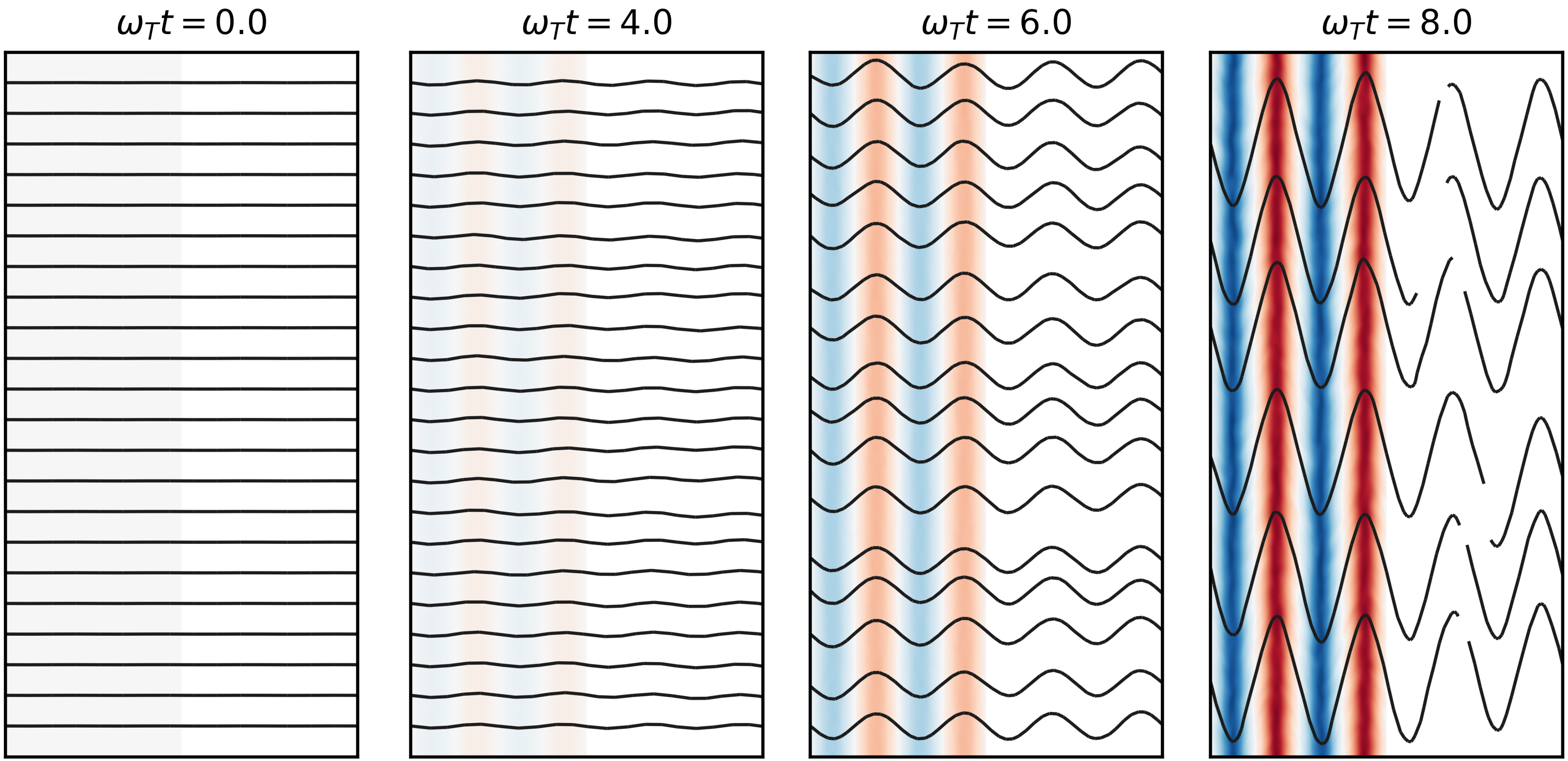
- Maximum growth rate of MTI: $\omega_T \doteq \sqrt{-g \frac{\mathrm{d} \ln T}{\mathrm{d} R}}$ independent of conductivity!
- Efficient heat conduction necessary for isothermality $t_{\text{cond}} \doteq 1/(\chi k^2)$
- Competition between different processes: buoyancy, magnetic tension, viscosity/resistivity...

timescales: $t_{\text{cond}} \ll \min\left\{N^{-1}, \omega_T^{-1}\right\} \ll (k v_A)^{-1}$ $N = \sqrt{\frac{g}{\gamma} \frac{\mathrm{d} \ln p \rho^{-\gamma}}{\mathrm{d} R}}$

lengthscales: $\frac{v_A}{\omega_T} \ll k^{-1} \ll \min\left\{\sqrt{\frac{\chi}{\omega_T}}, \sqrt{\frac{\chi}{N}}\right\}$

conduction length $l_\chi = \sqrt{\chi/\omega_T}$

Linear Evolution



Aim of this work

- Perform extensive parameter study and look at saturation of the MTI:
  how do saturated properties depend on physical parameters?

- Use a Boussinesq model to study subsonic and small scale dynamics of ICM

$$u/c_s \sim \delta\rho/\rho_0 \sim \lambda/H \sim \epsilon \qquad \delta p/p_0 \sim \epsilon^2$$

- Model a plasma vertically stratified in temperature and entropy

- Compare our results with real cluster observations

- Physics not included:
  anisotropic viscosity (no micro-scale instabilities)

    - we focus on essential features of the MTI
    - anisotropic viscosity does not significantly alter MTI properties*

[Kunz+2012, Parrish+2012]

Boussinesq Equations

$$\nabla \cdot \boldsymbol{u} = \nabla \cdot \boldsymbol{B} = 0$$

$$(\partial_t + \boldsymbol{u} \cdot \nabla)\, \boldsymbol{u} = -\frac{\nabla p_{tot}}{\rho_0} - \theta \boldsymbol{e}_z + (\boldsymbol{B} \cdot \nabla)\, \boldsymbol{B} + \nu \nabla^2 \boldsymbol{u}$$

$$(\partial_t + \boldsymbol{u} \cdot \nabla)\, \boldsymbol{B} = (\boldsymbol{B} \cdot \nabla)\, \boldsymbol{u} + \eta \nabla^2 \boldsymbol{B}$$

$$(\partial_t + \boldsymbol{u} \cdot \nabla)\, \theta = N^2 u_z + \chi \nabla \cdot [\boldsymbol{b}\, (\boldsymbol{b} \cdot \nabla)\, \theta] + \chi \omega_T^2 \nabla \cdot (\boldsymbol{b} b_z)$$

$$\theta = g_0 \frac{\delta\rho}{\rho_0}, \quad N^2 = \frac{g_0}{\gamma} \left. \frac{\partial \ln p\rho^{-\gamma}}{\partial z} \right|_0, \quad \omega_T^2 = -g_0 \left. \frac{\partial \ln T}{\partial z} \right|_0, \quad \chi = \frac{\gamma - 1}{\gamma} \frac{T_0}{p_0} \kappa, \quad \boldsymbol{b} = \boldsymbol{B}/B$$

temperature fluctuation

Brunt-Vaisala frequency

MTI frequency

diffusivity

Numerical Methods

- Use pseudo-spectral code SNOOPY (Lesur2015) and implement anisotropic thermal conduction

- Look at regime of moderate conduction, weak/strong stable stratification

$$Pe = L^2 \omega_T / \chi \gg 1, \quad \tilde{N}^2 = N^2/\omega_T^2 = 10^{-2} - 1,$$
$$Pr = \nu/\chi \lesssim 1, \quad Pm = \nu/\eta \gtrsim 1$$

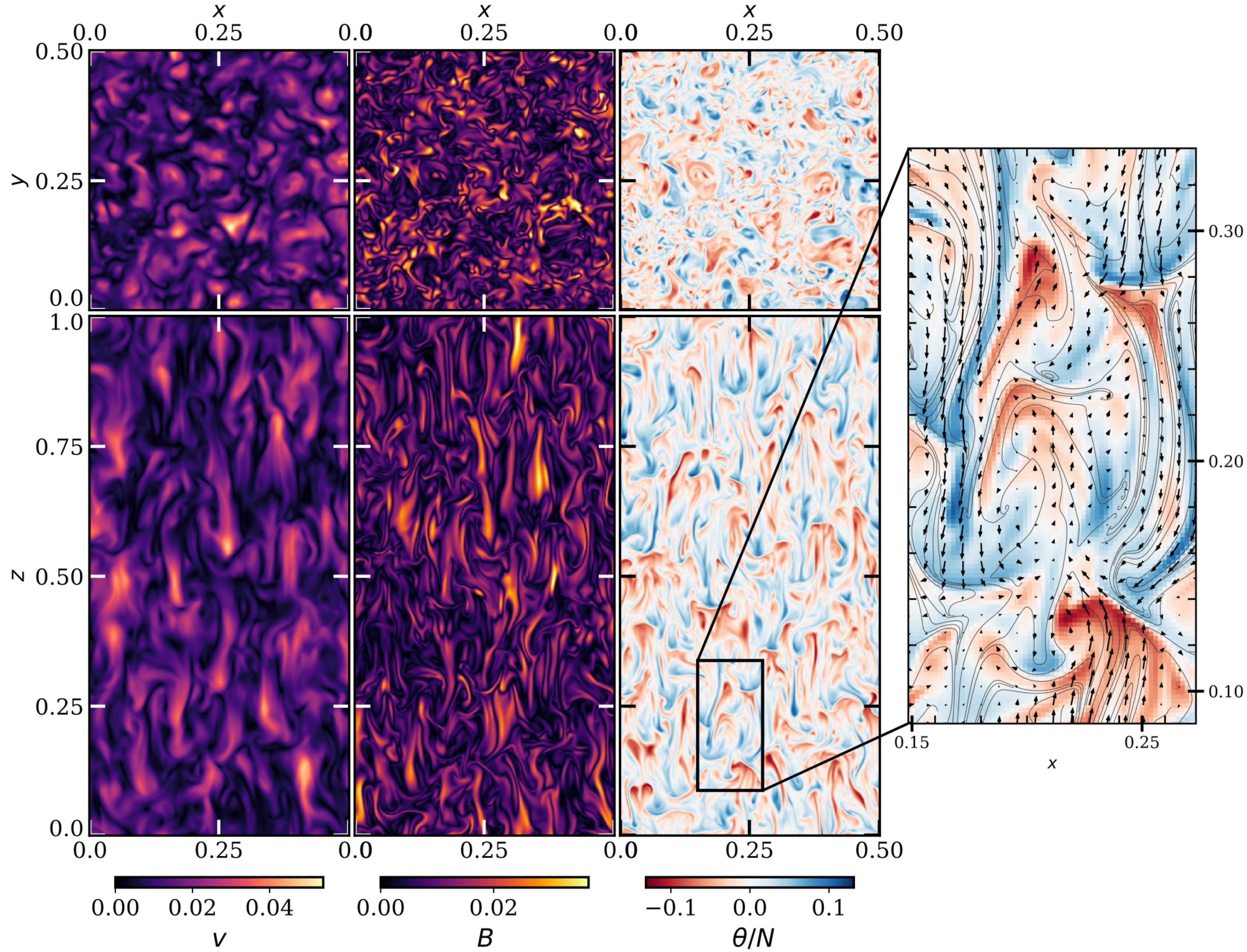
- ICM is likely* in regime of $Pr \simeq 0.02, \quad Pm \gg 1$

- Need to resolve: $L \gtrsim l_\chi \gtrsim l_\nu \gtrsim l_\eta$ computationally hard!

- Implement a super-time stepping algorithm
- Test the code and compute linear growth rates
- Run 2D/3D simulations of MTI, with triply-periodic BC and variety of ***B*** geometries

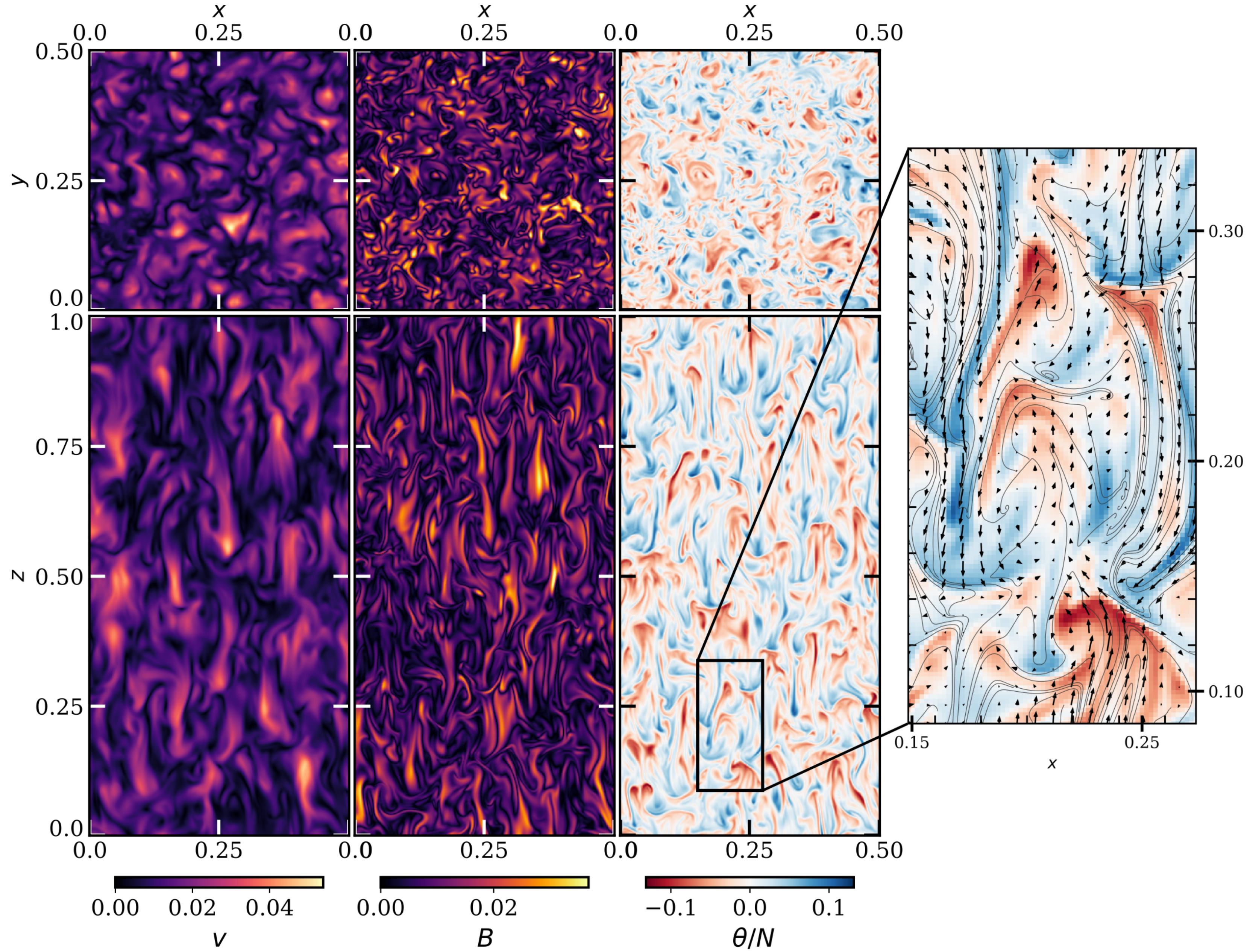
Phenomenology of MTI Turbulence

- After initial growth phase, state of sustained turbulence



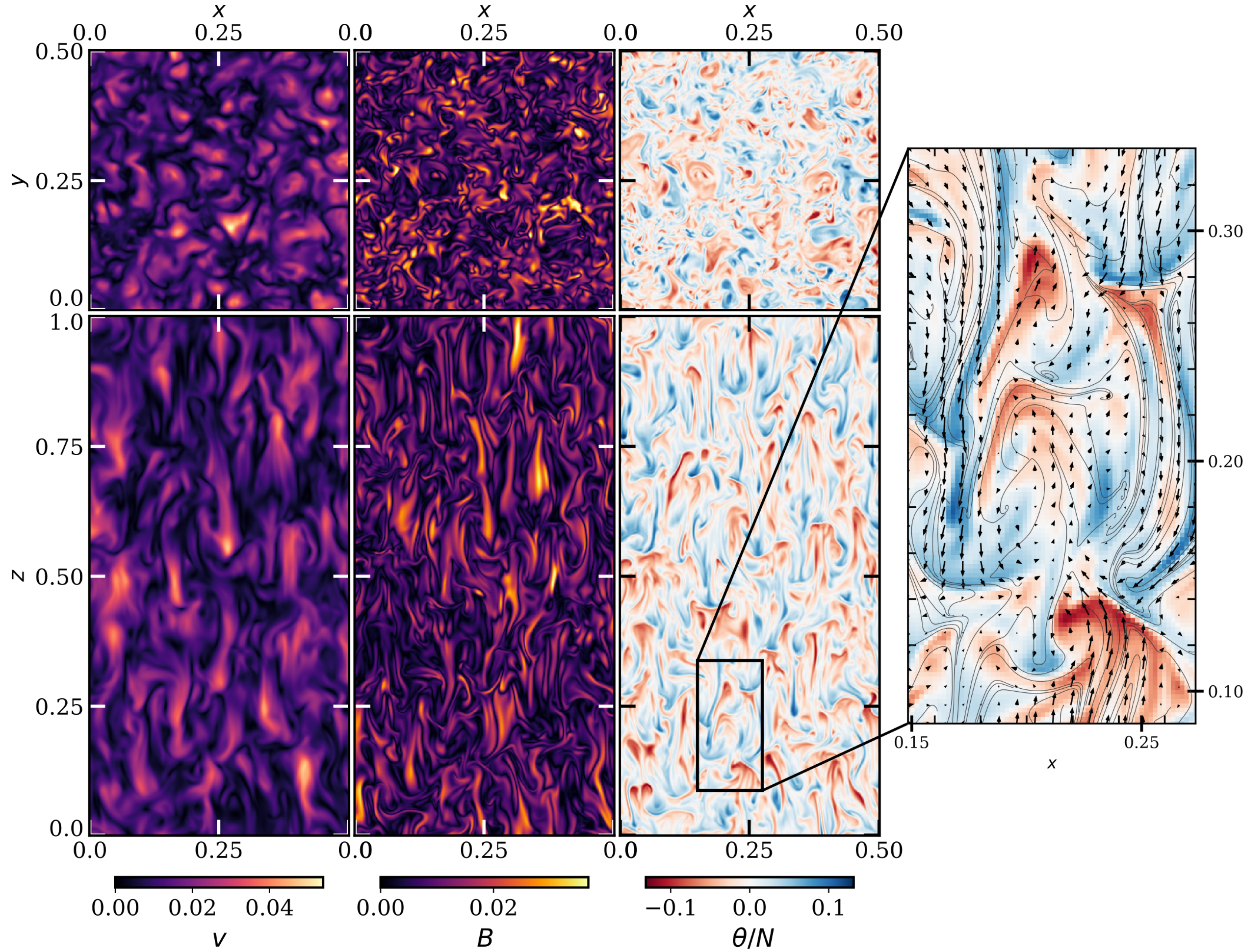
Phenomenology of MTI Turbulence

- After initial growth phase, state of sustained turbulence
- Magnetic field encapsulates the plumes: strong temperature gradients across field lines



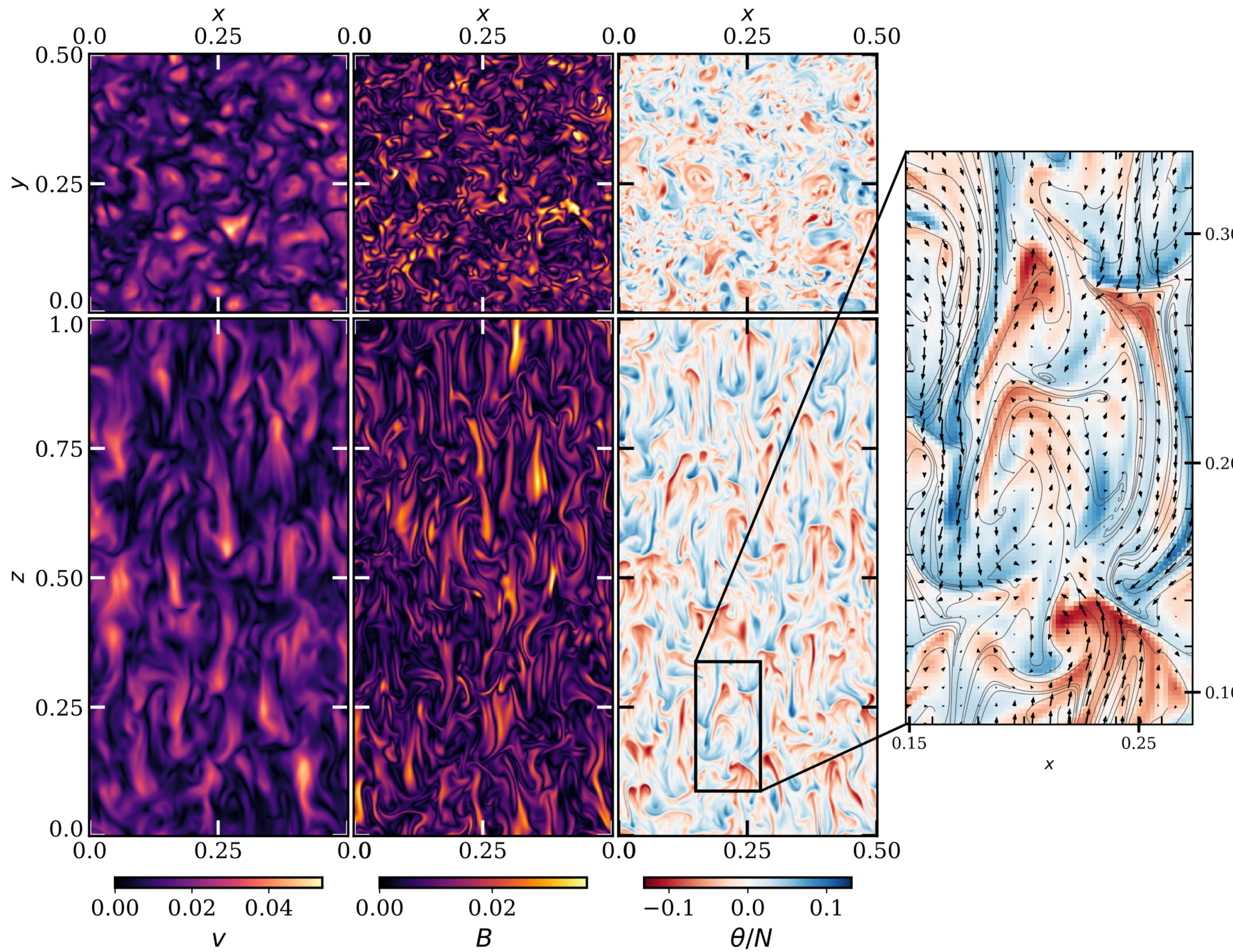
Phenomenology of MTI Turbulence

- After initial growth phase, state of sustained turbulence
- Magnetic field encapsulates the plumes: strong temperature gradients across field lines
- Convective-like behaviour: hot (lighter) plumes rise while cold (heavier) ones sink



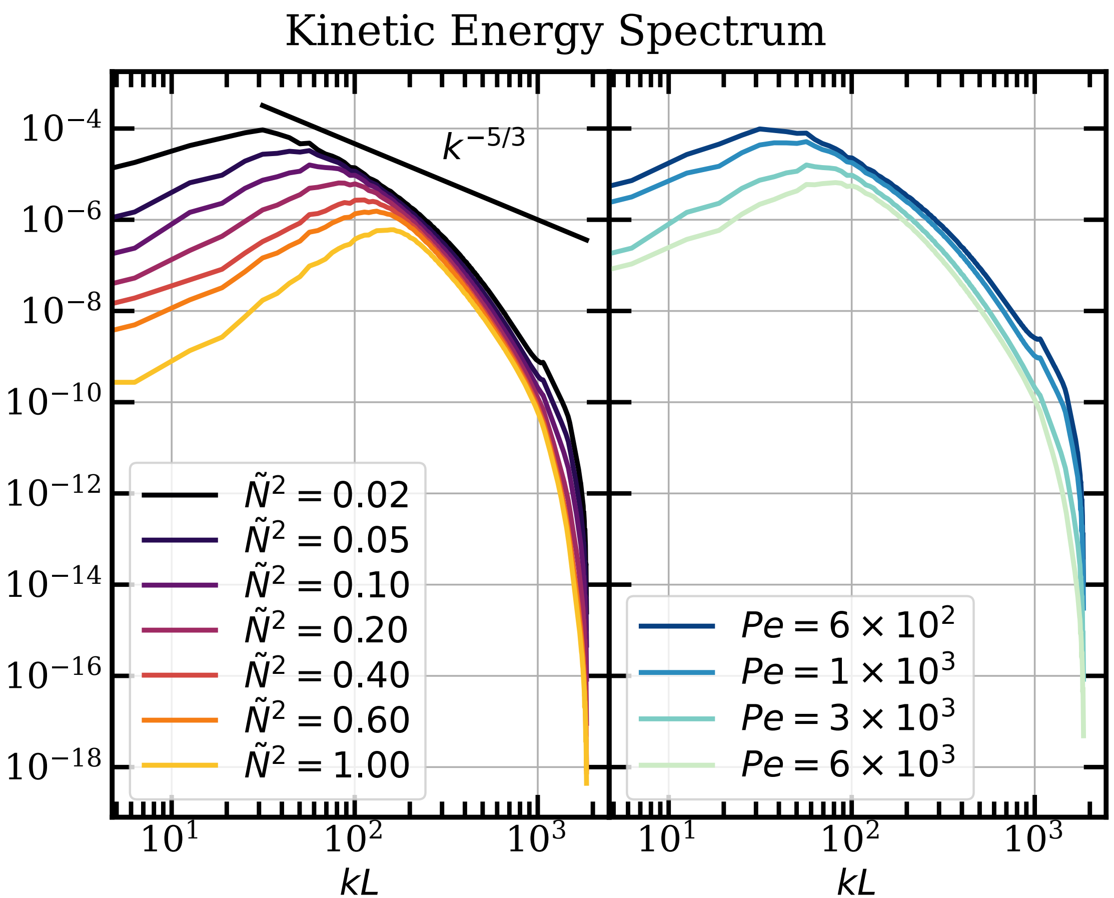
Phenomenology of MTI Turbulence

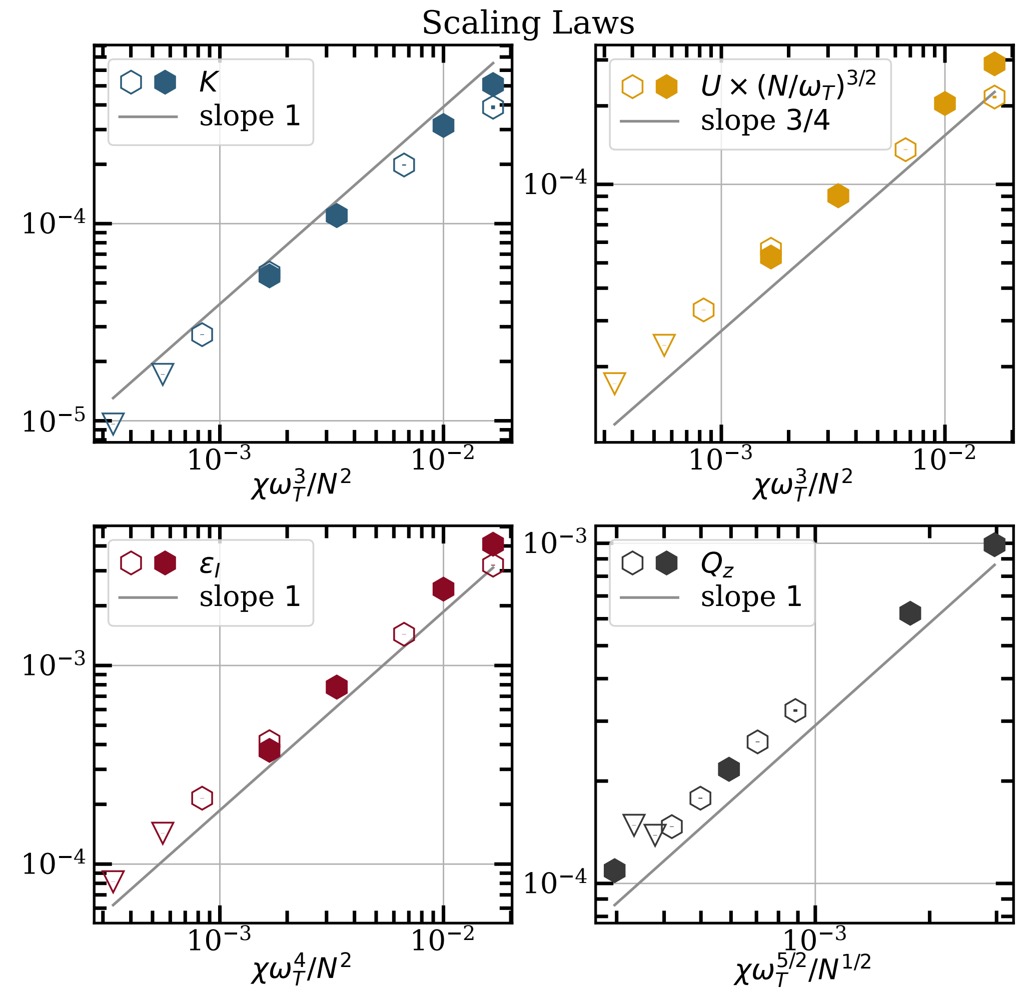
- After initial growth phase, state of sustained turbulence
- Magnetic field encapsulates the plumes: strong temperature gradients across field lines
- Convective-like behaviour: hot (lighter) plumes rise while cold (heavier) ones sink
- Effective transport of heat due to both advection and conduction



Effect of entropy stratification and thermal diffusivity

- In 2D/3D N and χ set the integral scale (~ "buoyancy scale") and the strength of turbulence
- Strong stratification / lower diffusivity → integral scale becomes shorter; turbulence is less vigorous
- No formation of structures at the box size that dominate dynamics (contrary to MRI, RB convection)





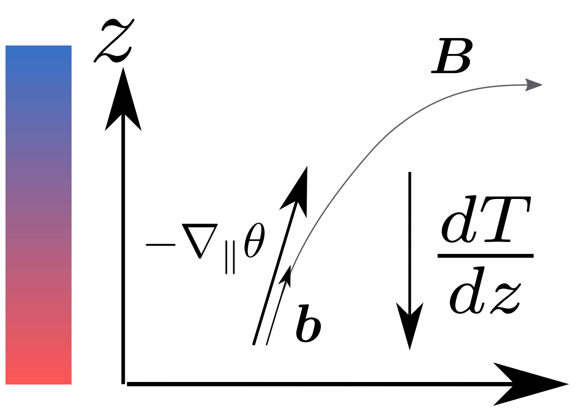
MTI Turbulence - Theoretical Scalings

- Energy budget of the MTI:

$$\frac{d}{dt}E_{tot} = -\nu\langle|\nabla \boldsymbol{u}|^2\rangle - \eta\langle|\nabla \boldsymbol{B}|^2\rangle - \frac{\chi}{N^2}\langle|\boldsymbol{b}\cdot\nabla\theta|^2\rangle - \frac{\chi\omega_T^2}{N^2}\langle b_z \boldsymbol{b}\cdot\nabla\theta\rangle$$

viscous dissipation | resistive dissipation | thermal dissipation | energy injection rate ϵ_I

- In $Pr \lesssim 1, Pm \gtrsim 1$ regime last two terms dominate. Balancing:



$$\nabla_{\parallel}\theta \approx -\omega_T^2 b_z, \quad \nabla_{\parallel} = \boldsymbol{b}\cdot\nabla$$

not just in volume averaged sense but also scale-by-scale*

$$\Rightarrow \epsilon_I \sim \chi\omega_T^4/N^2$$

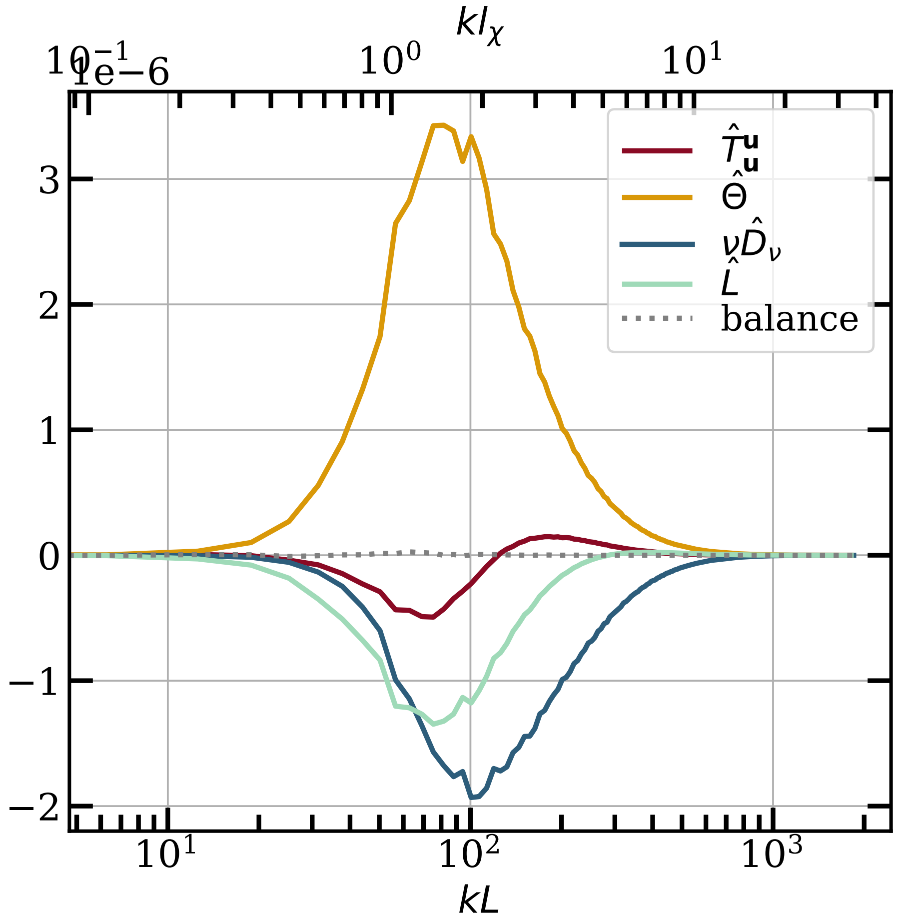
- Differences between 2D and 3D:

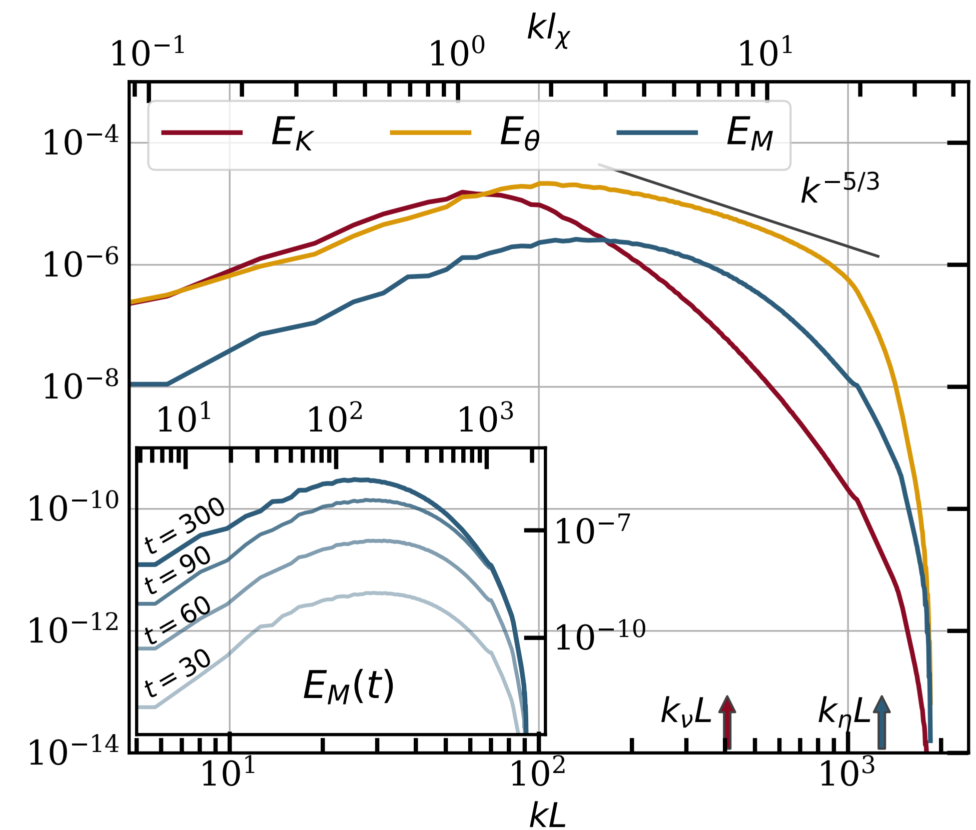
2D: inverse cascade

3D: local dissipation of energy

MTI Turbulence - 3D

- In 3D injection and dissipation are local processes in spectral space
- Buoyancy force is one-directional: transfer energy from density to kinetic





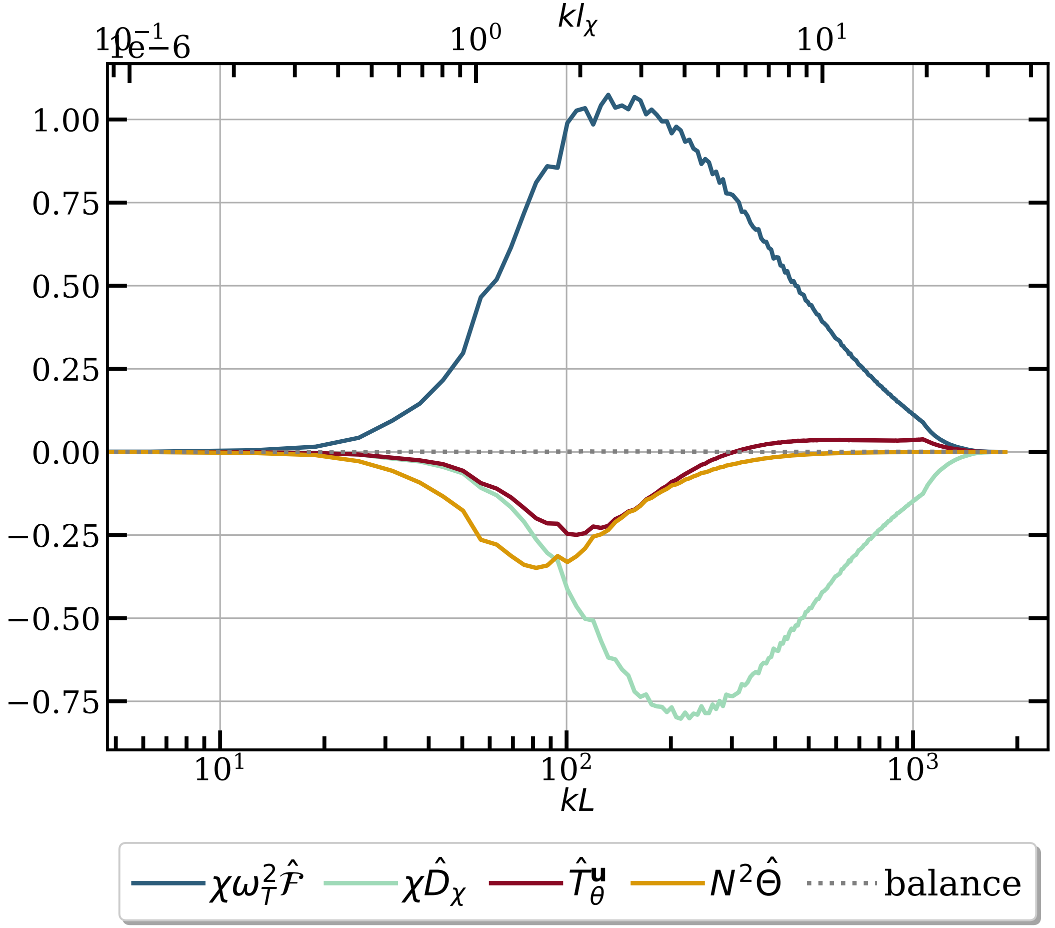
- Nonlinear advection terms subdominant

Buoyancy-driven flow

- Turbulence is anisotropic with elongated eddies in z direction and $u_z \gg u_x, u_y$

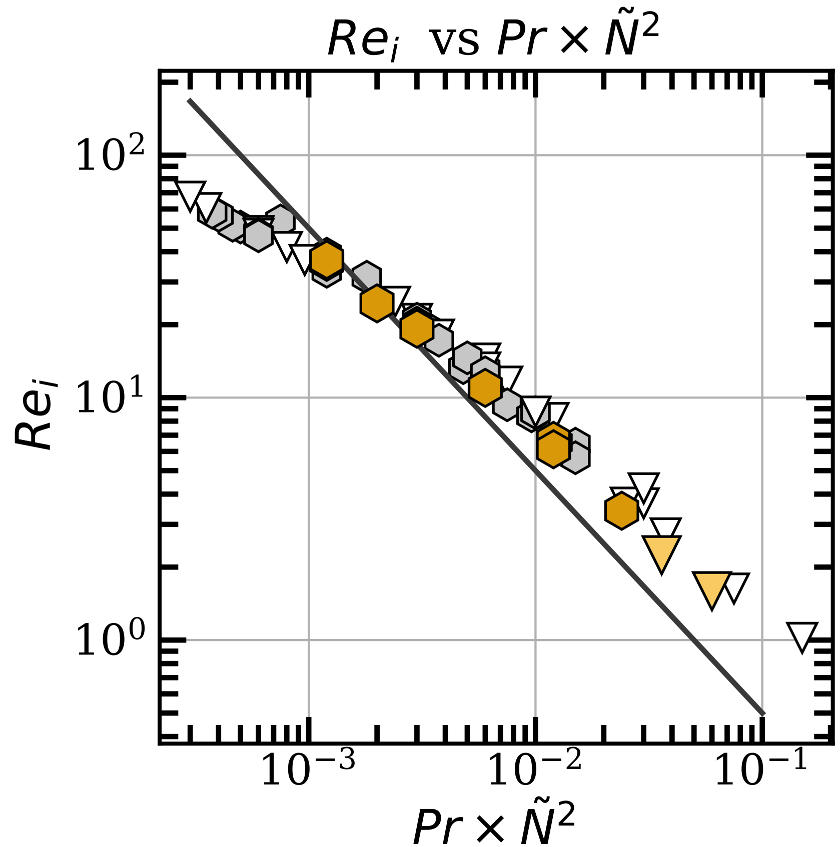
MTI Turbulence - Balancing the Intermediate Scales

- Look at the detailed thermal energy balance



- MTI forcing and buoyancy force balance each other at large scales

$$K \sim \frac{\chi\omega_T^3}{N^2} \qquad l_i \sim \frac{(\chi\omega_T)^{1/2}}{N} \qquad Re_i \sim \frac{\chi\omega_T^2}{\nu N^2}$$



- very different saturation mechanism, but same scalings as 2D!

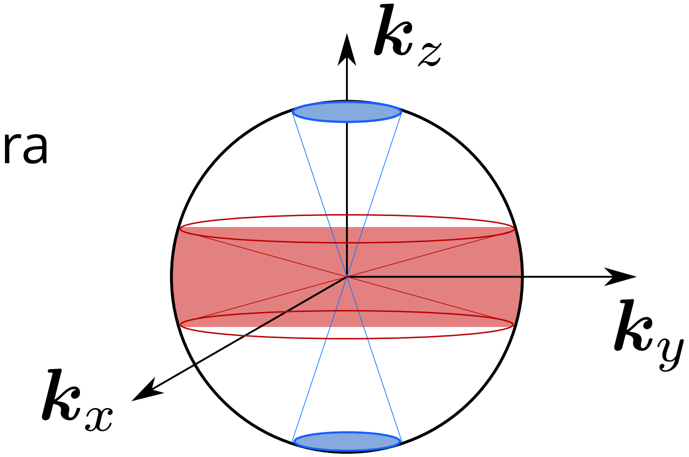
Strongly stratified MTI

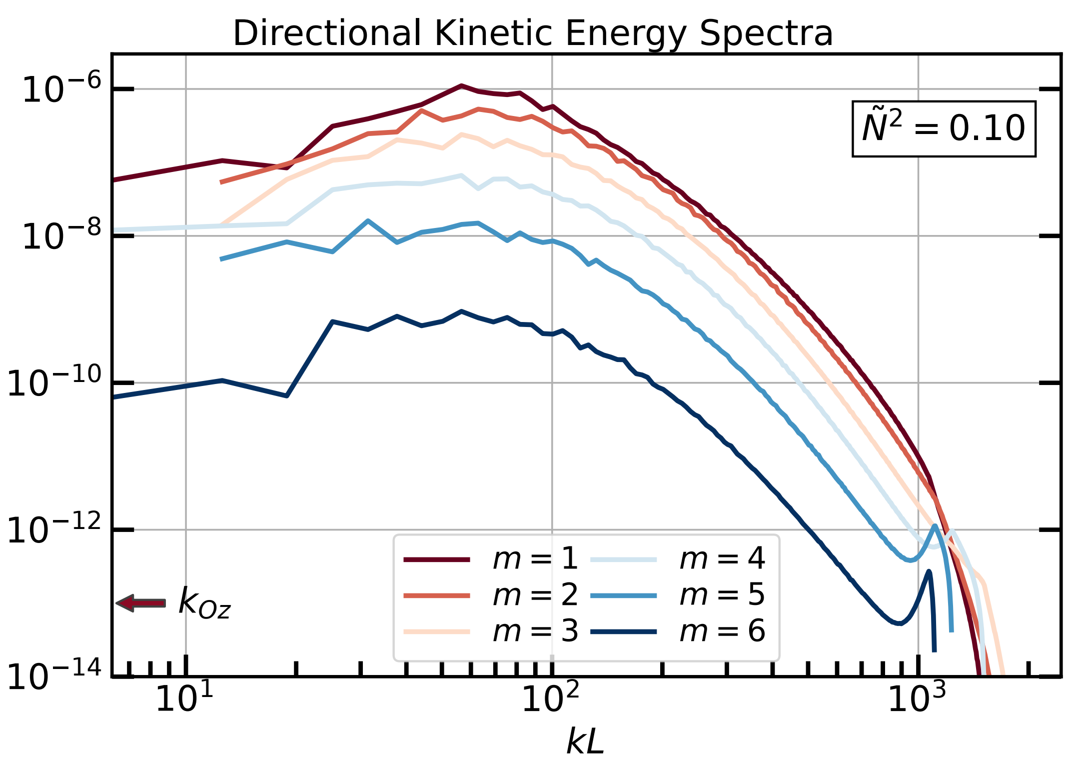
- Effect of strong entropy stratification evident from directional spectra
- Divide the spherical shell of radius *k* in latitudinal bands

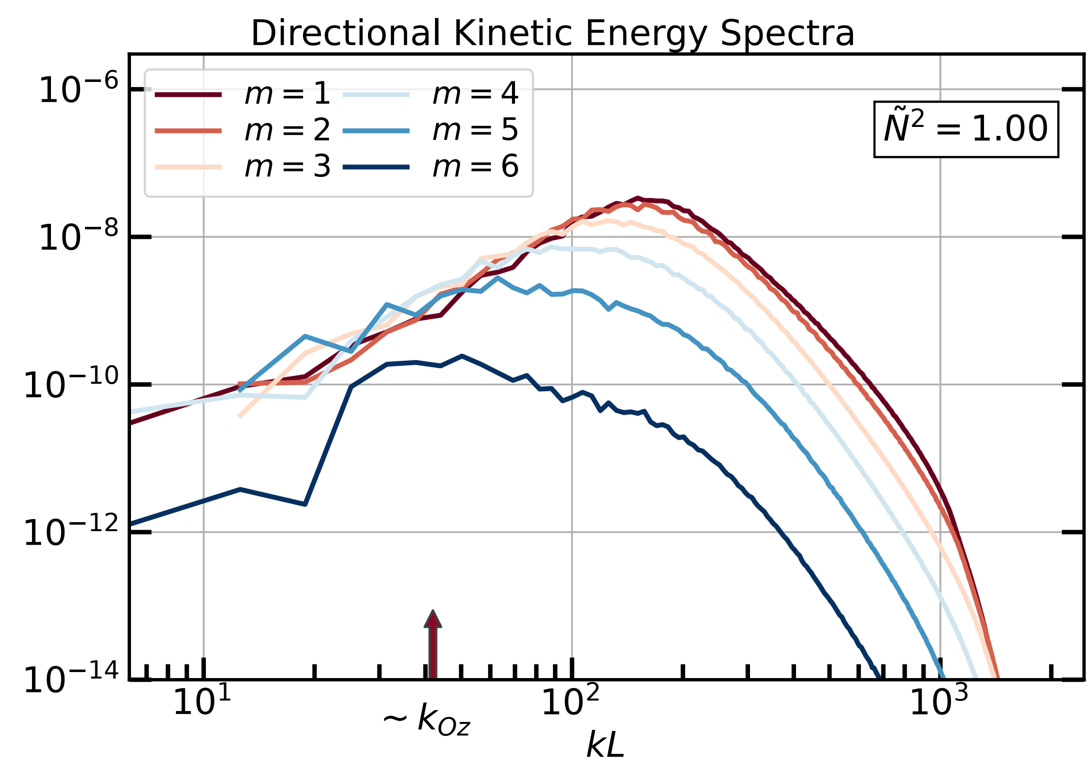
$m = 1$ near equatorial: "vertical pancakes"

$m = 6$ near polar: "horizontal pancakes"

- small scales largely unaffected by increased N
- large scales tend to get isotropized near the Ozmidov scale $k_{Oz} = (N^3/\epsilon_\nu)^{1/2}$



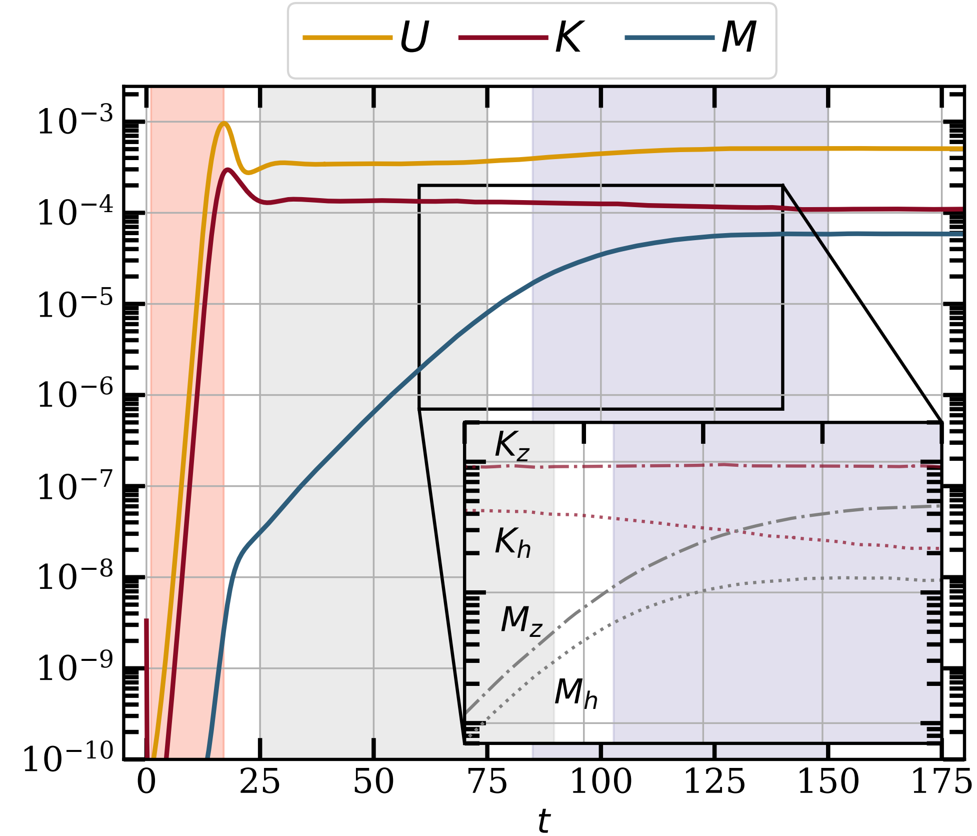


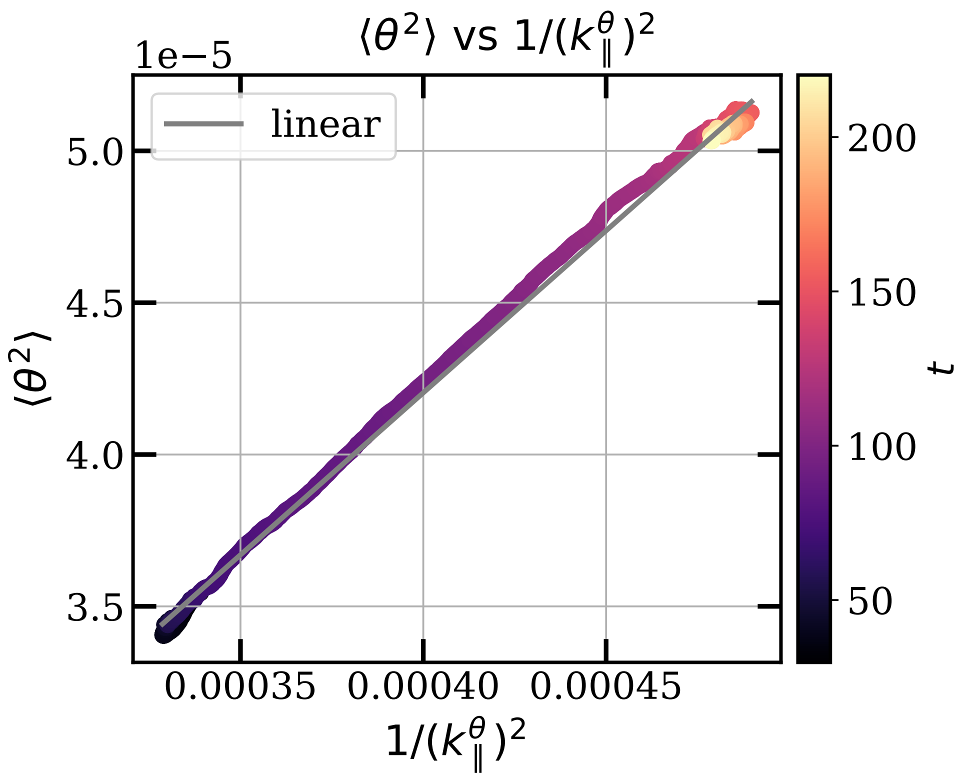


MTI-driven dynamo

- MTI turbulence can sustain a fluctuation dynamo

- The dynamo acts back on the turbulent flow
  -> becomes more biased in vertical direction
  -> increase in potential energy



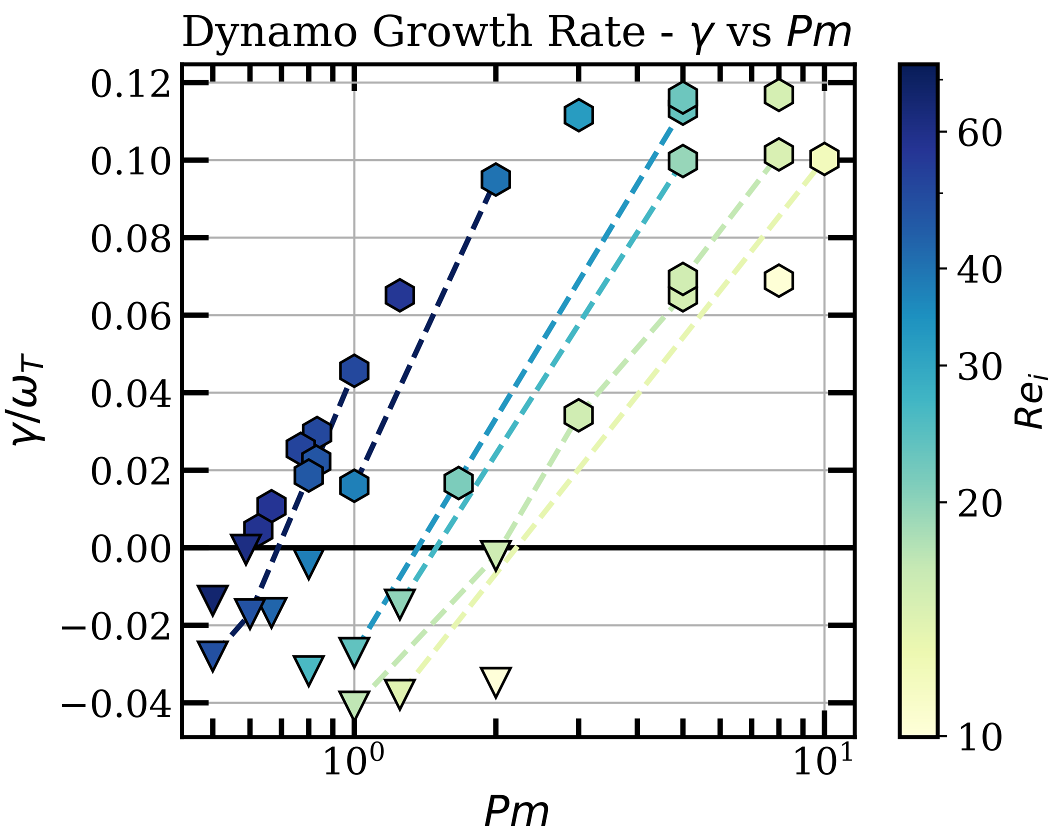


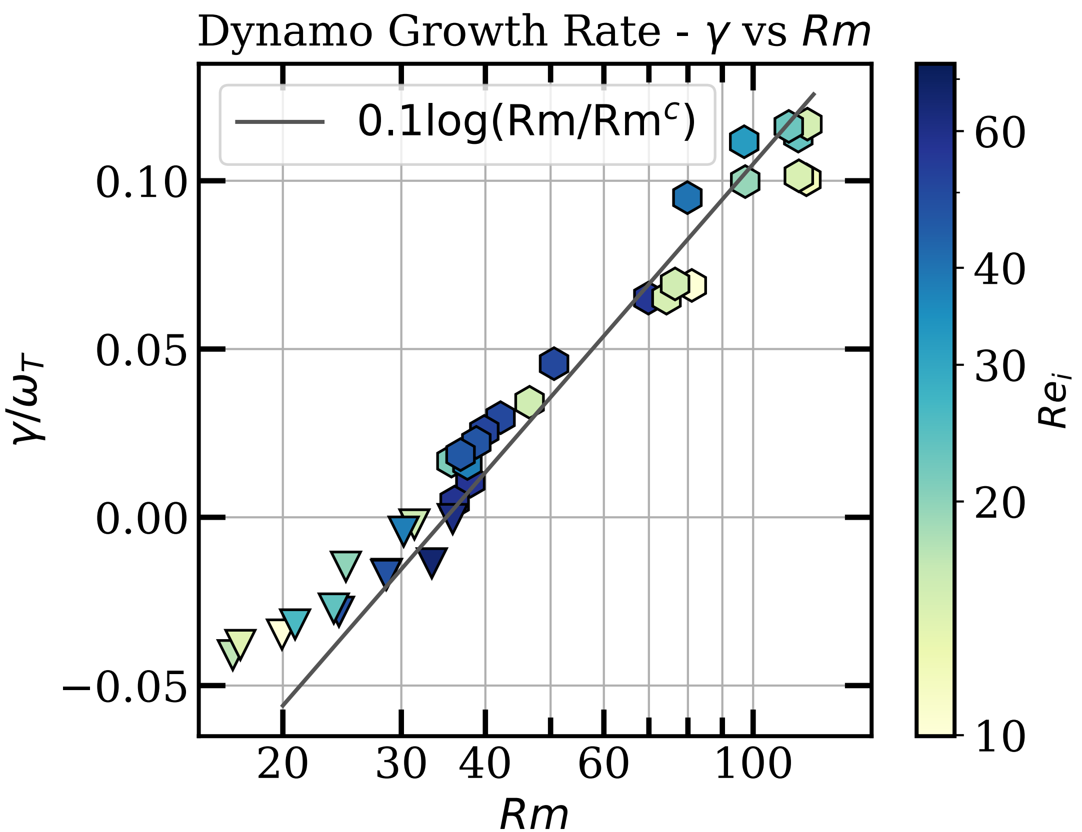
$$k_\parallel^\theta = \left(\frac{\langle |\boldsymbol{b} \cdot \nabla\theta|^2 \rangle}{\langle \theta^2 \rangle} \right)^{1/2}$$

- parallel correlation of density fluctuations increases: potential energy increases

Criterion for small-scale dynamo

- Perform large suite of simulations with no-net flux





- Growth is possible if $Rm > Rm^c \approx 35$
- At fixed Pm criterion for small-scale dynamo: $l_i > l_\nu$

Application of MTI to Galaxy Clusters

- Main source of uncertainty is limited understanding of the ICM's microphysics
  - kinetic micro-instabilities can modify viscosity and thermal conductivity

    introduce a suppression factor: $\chi = f\chi_{\rm S}$

- Use MTI scaling laws to obtain:

$$u_{rms} \approx 400 f^{1/2} \mathrm{km\,s^{-1}}, \quad \beta \approx 10 - 20 f^{-1}$$

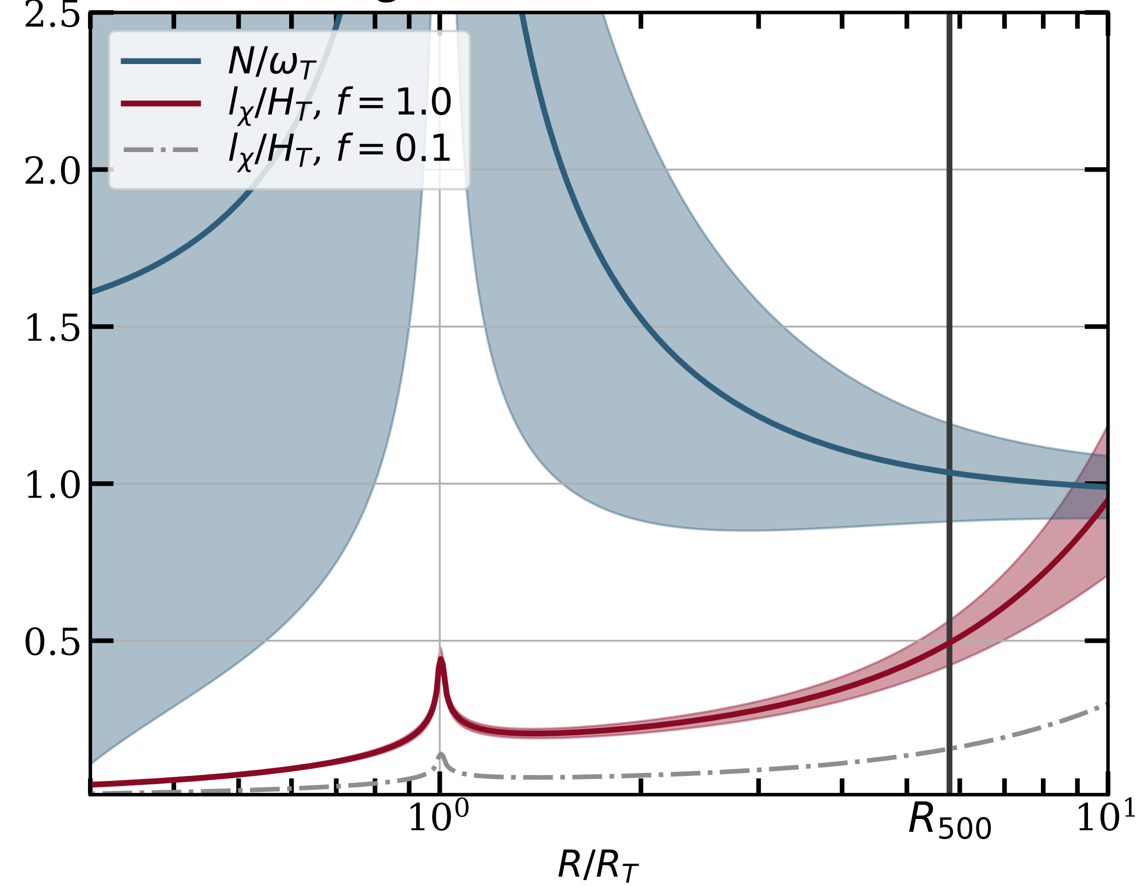
- Key MTI scales: $l_\chi \sim 100\, f^{1/2} \mathrm{kpc}$ $\quad l_{Oz} \gtrsim l_\chi$ $\quad l_\nu \sim 10\, f^{1/2} \mathrm{kpc}$

  - Observational estimates:

$$u_{rms} \sim 100 - 500 \mathrm{km\,s^{-1}}$$

MTI levels consistent with $f \simeq 0.1$

[using data from Ghirardini+2019]



MTI in Galaxy Clusters - Challenges

- **Large-scale challenges:**
  - MTI only one of several competing sources of turbulence
  - these can actively suppress the action of the MTI: unlikely to happen at all scales!
  - not much evidence of MTI in the outskirts of galaxy clusters from cosmological simulations with anisotropic conduction
    - are key MTI scales resolved?
    - numerical diffusion is not well quantified
    - the usual diagnostics (i.e. radial bias etc) not appropriate on large scales

MTI in Galaxy Clusters - Challenges

- **Large-scale challenges:**
  - MTI only one of several competing sources of turbulence
  - these can actively suppress the action of the MTI: unlikely to happen at all scales!
  - not much evidence of MTI in the outskirts of galaxy clusters from cosmological simulations with anisotropic conduction
    - are key MTI scales resolved?
    - numerical diffusion is not well quantified
    - the usual diagnostics (i.e. radial bias etc) not appropriate on large scales

- **Small-scale challenges:**
  - In the ICM thermal conduction may be partially suppressed
    - tangled magnetic fields
    - kinetic micro-instabilities, e.g. mirror, firehose, whistler, gyrothermal, eMTI?

Summary and Future Work

- MTI interesting convective/diffusive instability!
  - Scale and strength of turbulence set by N, χ
  - MTI can be a player to explain turbulence in ICM
  - MTI effective at transporting heat: efficiency $\gtrsim 1/3$ of Spitzer flux

- To study impact of microinstabilities on MTI we need more sophisticated models
  - inclusion of anisotropic viscosity
  - suppression of heat conduction by whistler
  - other kinetic closures, FLR-Landau?

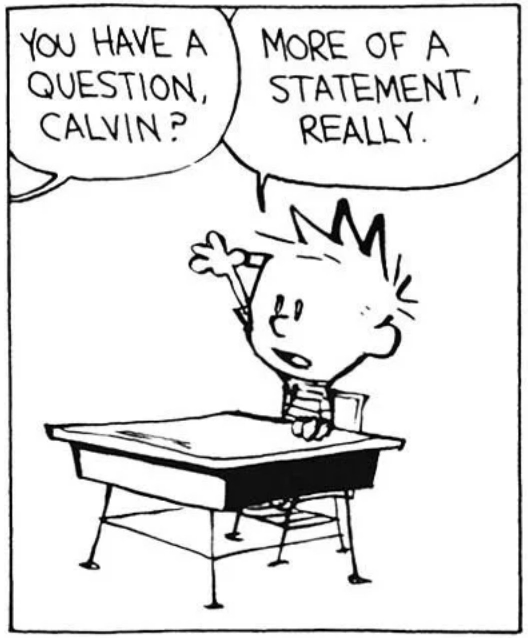
- With Boussinesq approximation cannot capture dynamics on scales $\sim H$
  - need global simulations in spherical geometry with anelastic or fully compressible

Summary and Future Work

- MTI interesting convective/diffusive instability!
  - Scale and strength of turbulence set by N, χ
  - MTI can be a player to explain turbulence in ICM
  - MTI effective at transporting heat: efficiency $\gtrsim 1/3$ of Spitzer flux

- To study impact of microinstabilities on MTI we need more sophisticated models
  - inclusion of anisotropic viscosity
  - suppression of heat conduction by whistler
  - other kinetic closures, FLR-Landau?

- With Boussinesq approximation cannot capture dynamics on scales $\sim H$
  - need global simulations in spherical geometry with anelastic or fully compressible



Thank you for your attention!

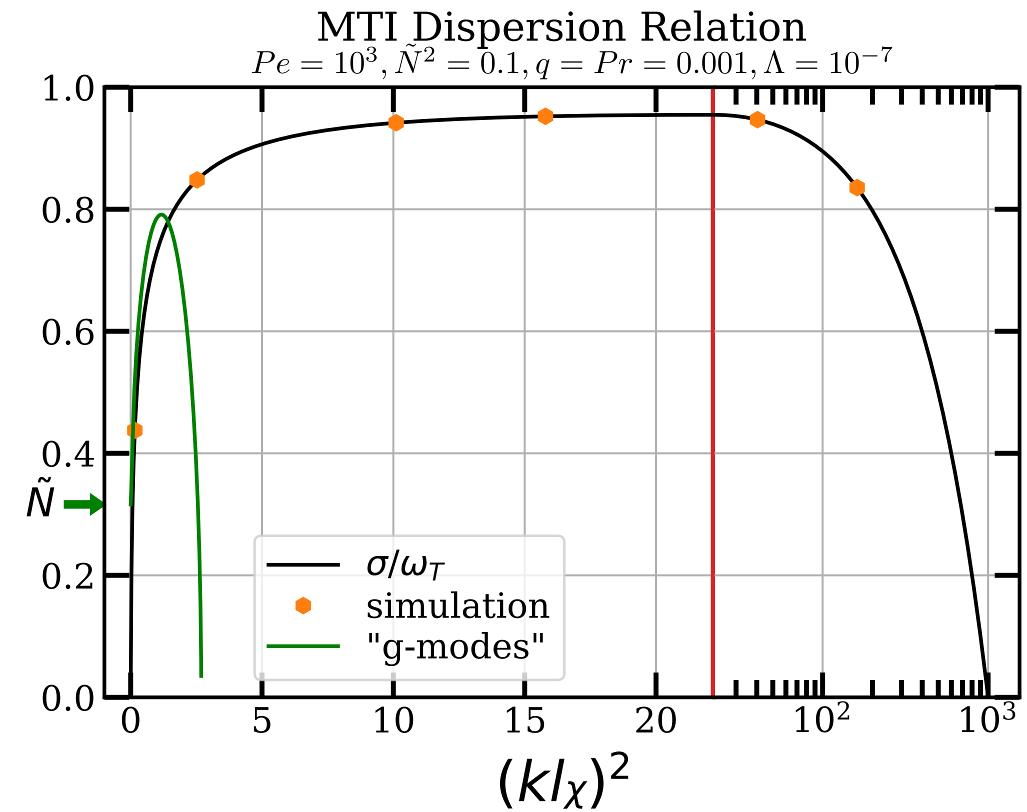
Linear Theory

- Instability criterion:

$$\frac{k^2}{k_x^2}\frac{\eta}{\chi}N^2 + v_A^2 k^2 + \frac{k^2}{k_x^2}\eta\nu k^4 < \omega_T^2$$

- Fast growth over wide range of k

- Oscillatory solutions at large scales (hybrid g-modes)



- In the limit of $Pr \sim q \sim \Lambda \ll 1$ asymptotic solution:

$$\frac{\sigma_{max}}{\omega_T} = 1 - \left[(1+\tilde{N}^2)(Pr+q+\Lambda)\right]^{1/2}$$

$$l_\chi k_{max} = \left[\frac{(1+\tilde{N}^2)}{(Pr+q+\Lambda)}\right]^{1/4}$$

$$l_\chi = \sqrt{\chi/\omega_T}$$

$$\tilde{N}^2 = N^2/\omega_T^2$$

$$Pr = \nu/\chi$$

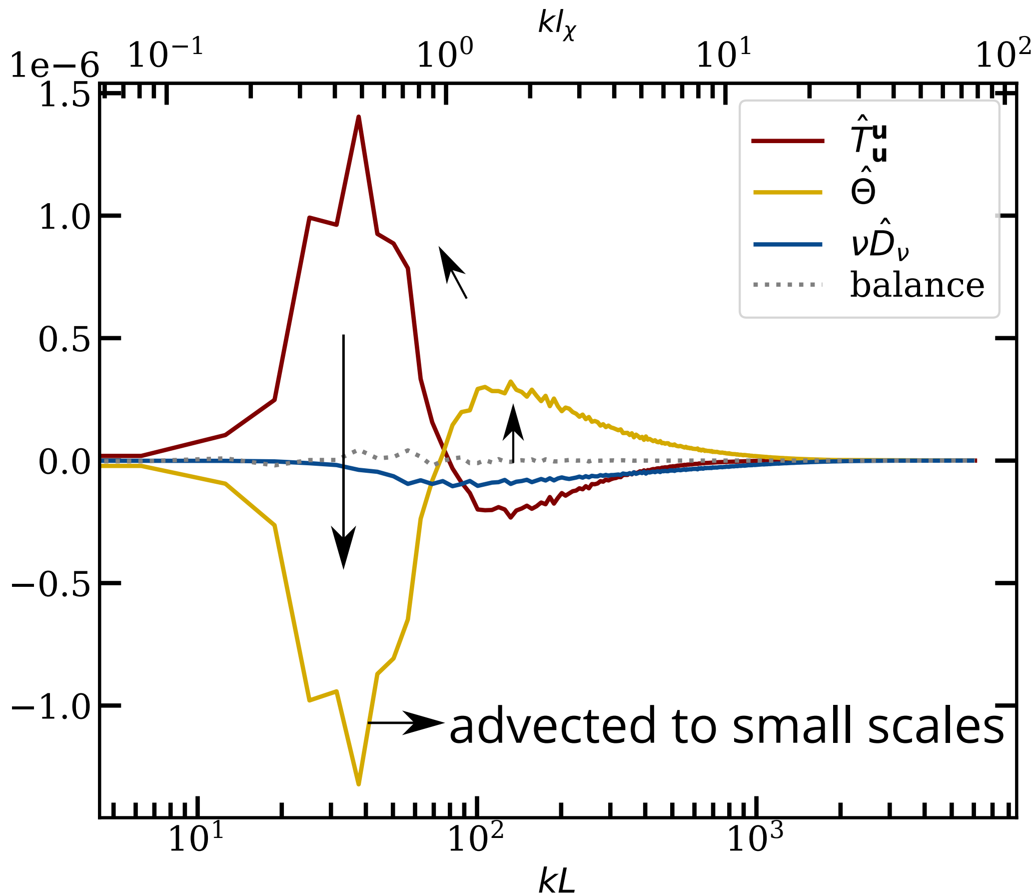
$$q = \eta/\chi$$

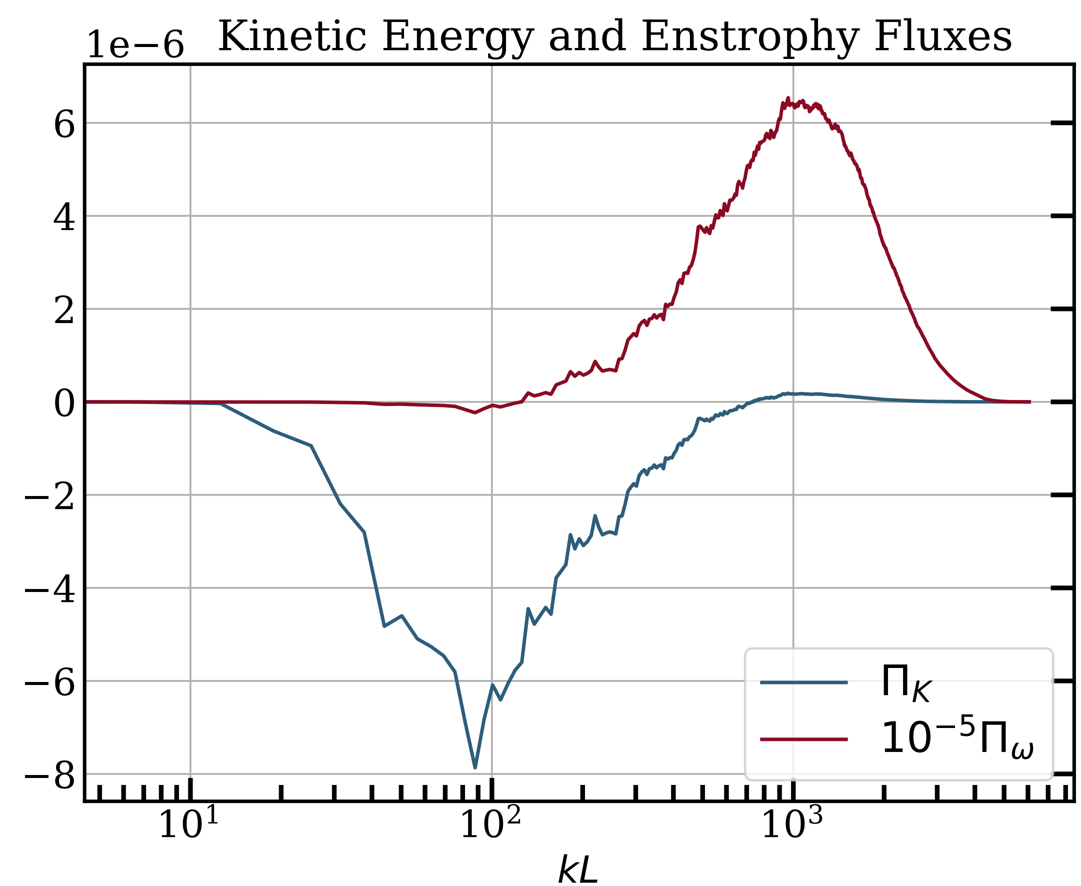
$$\Lambda = v_A^2/(\chi\omega_T)$$

MTI Turbulence - Theoretical Scalings 2D

- 2D dynamics characterized by inverse cascade:
  - MTI injects energy at small scales, then carried to large scales
  - At large scales, g-modes are excited and act as a sink of kinetic energy

Flux-loop mechanism



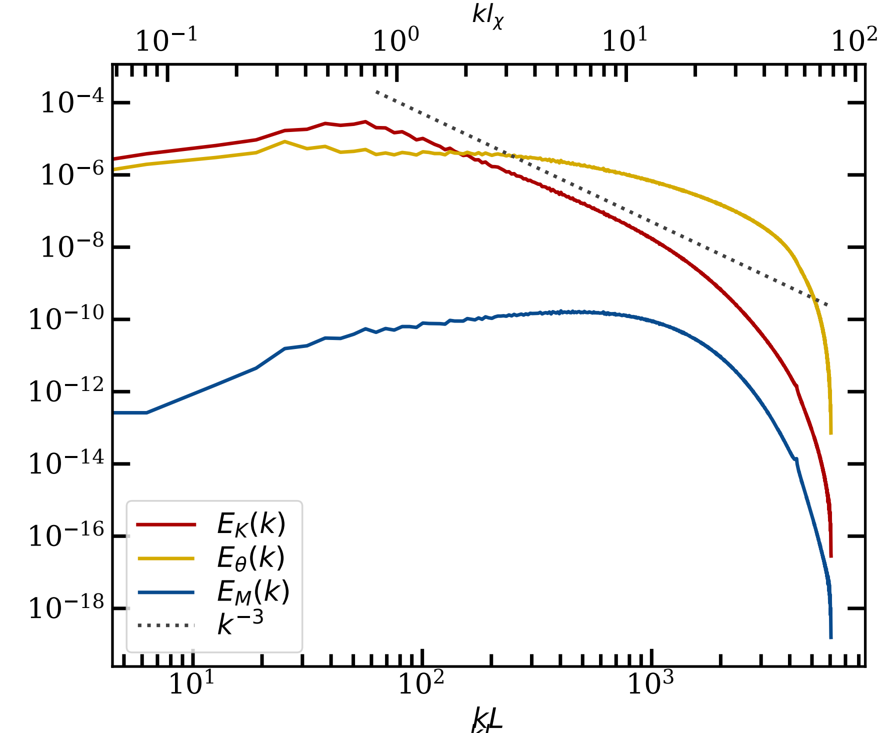


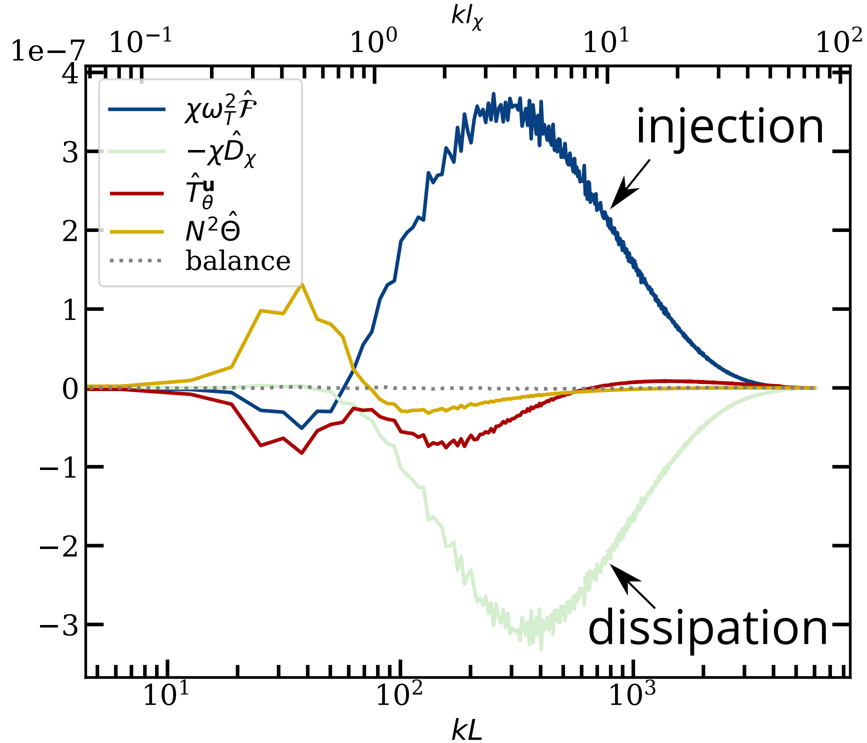
MTI Turbulence - Theoretical Scalings 2D

- Model energy removal at large scales via "Epstein drag" on timescale of $\sim 1/\omega_T$

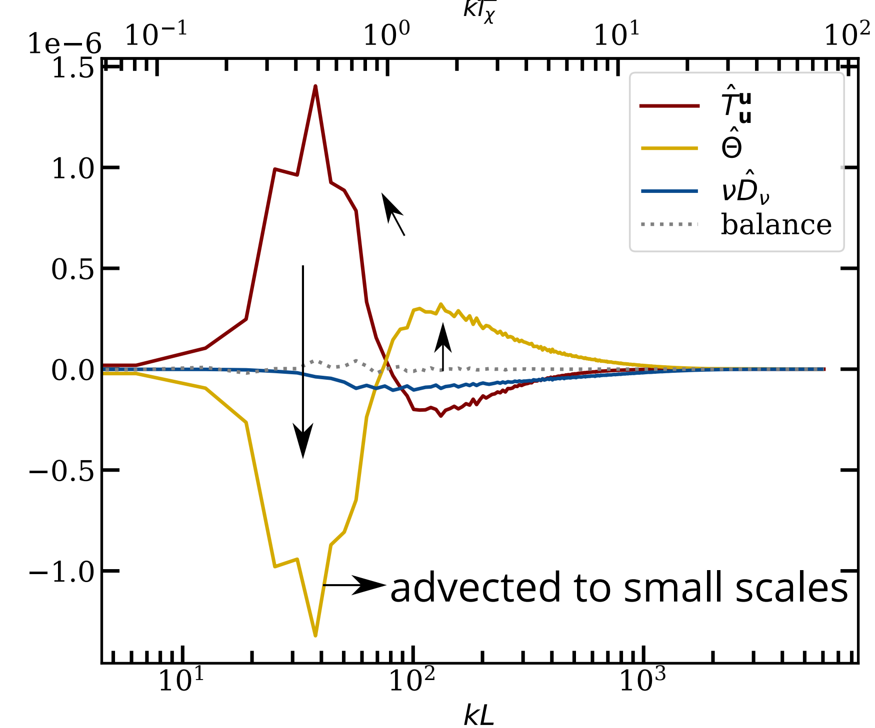
- If we do so, we obtain:

$$l_B \sim (\chi\omega_T)^{1/2}/N, \quad u_{rms}^2 \sim \chi\omega_T^3/N^2$$





Flux-loop mechanism



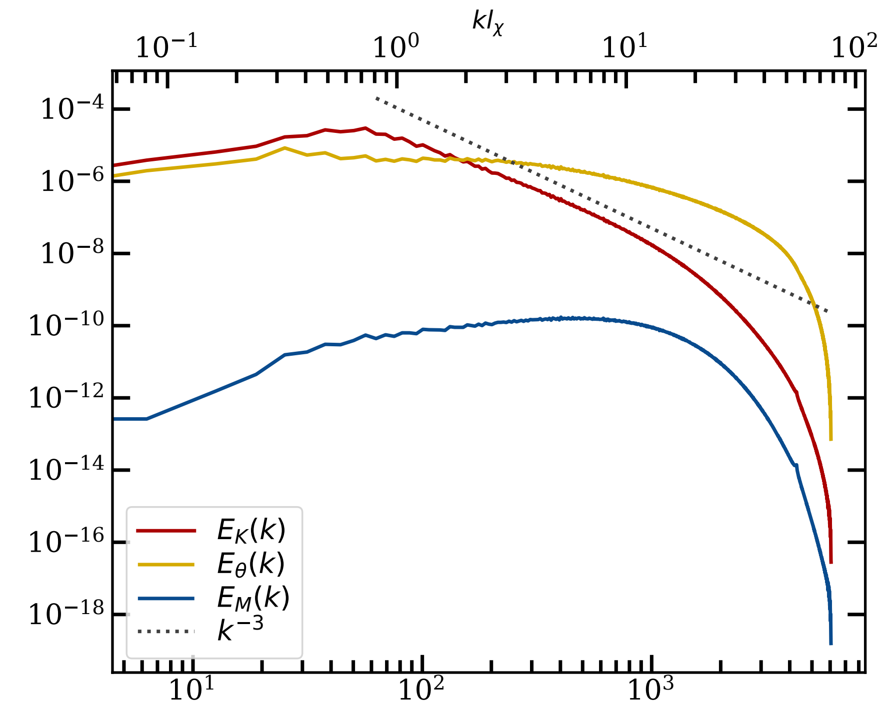
MTI Turbulence - Theoretical Scalings 2D

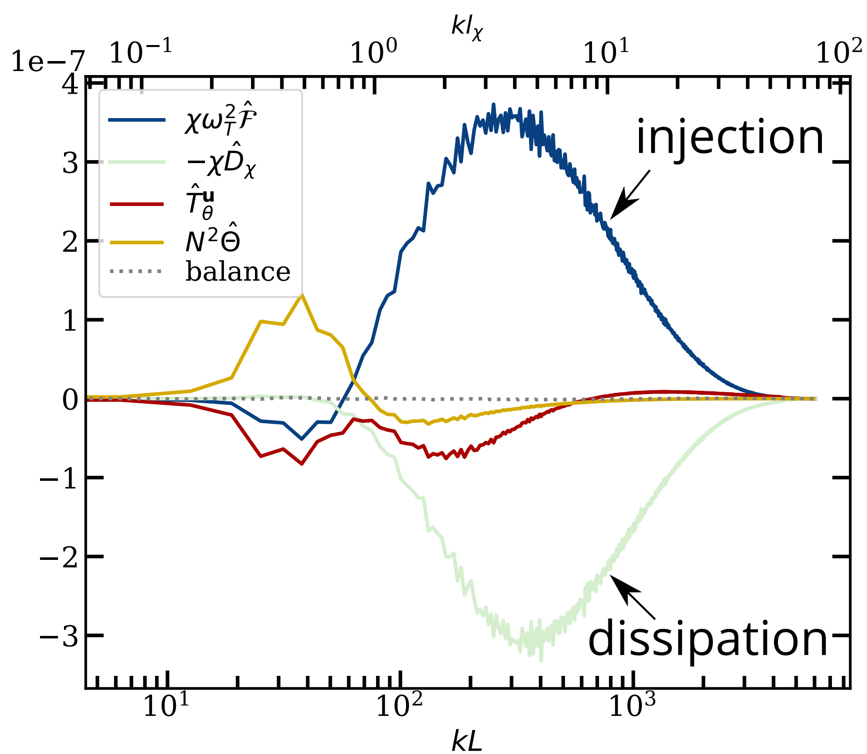
- Model energy removal at large scales via "Epstein drag" on timescale of $\sim 1/\omega_T$

- If we do so, we obtain:

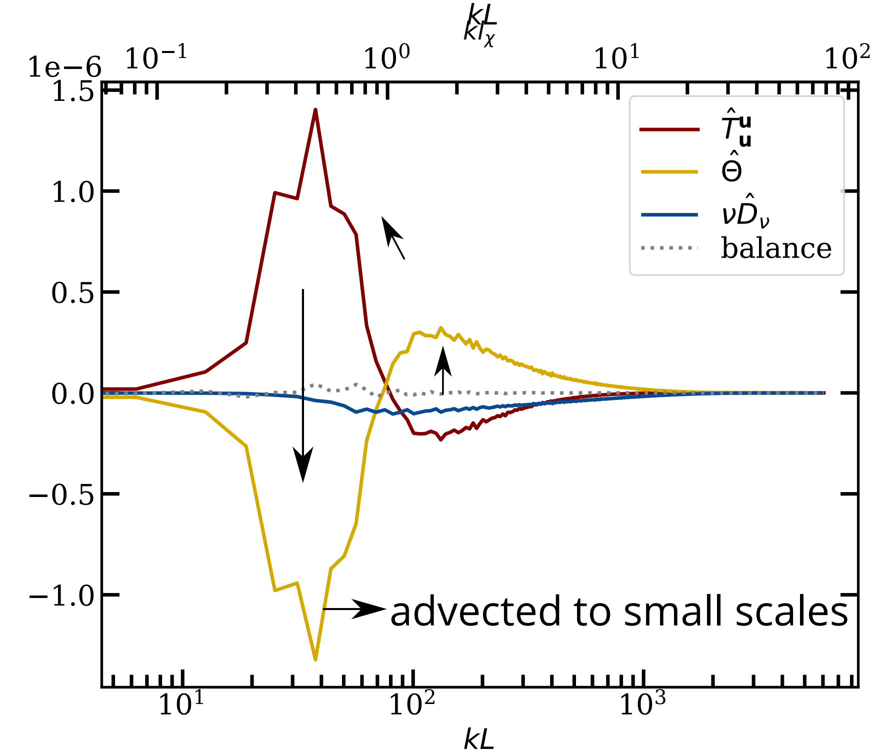
$$l_B \sim (\chi\omega_T)^{1/2}/N, \quad u_{rms}^2 \sim \chi\omega_T^3/N^2$$

In 2D $l_B \sim \chi^{0.37}/N^{1.08}$



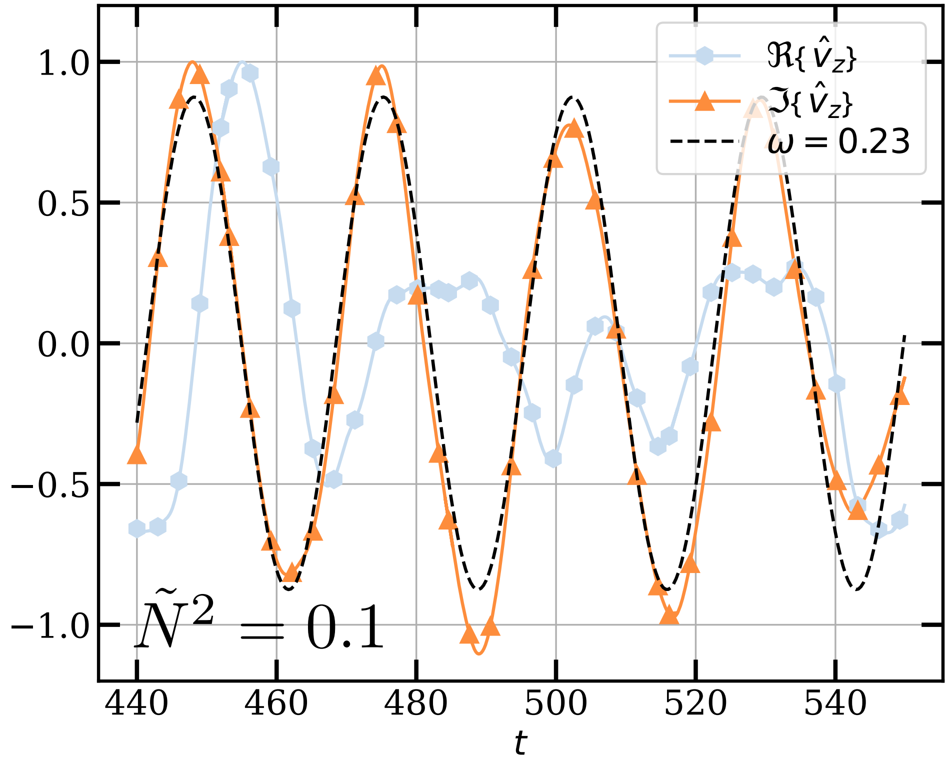


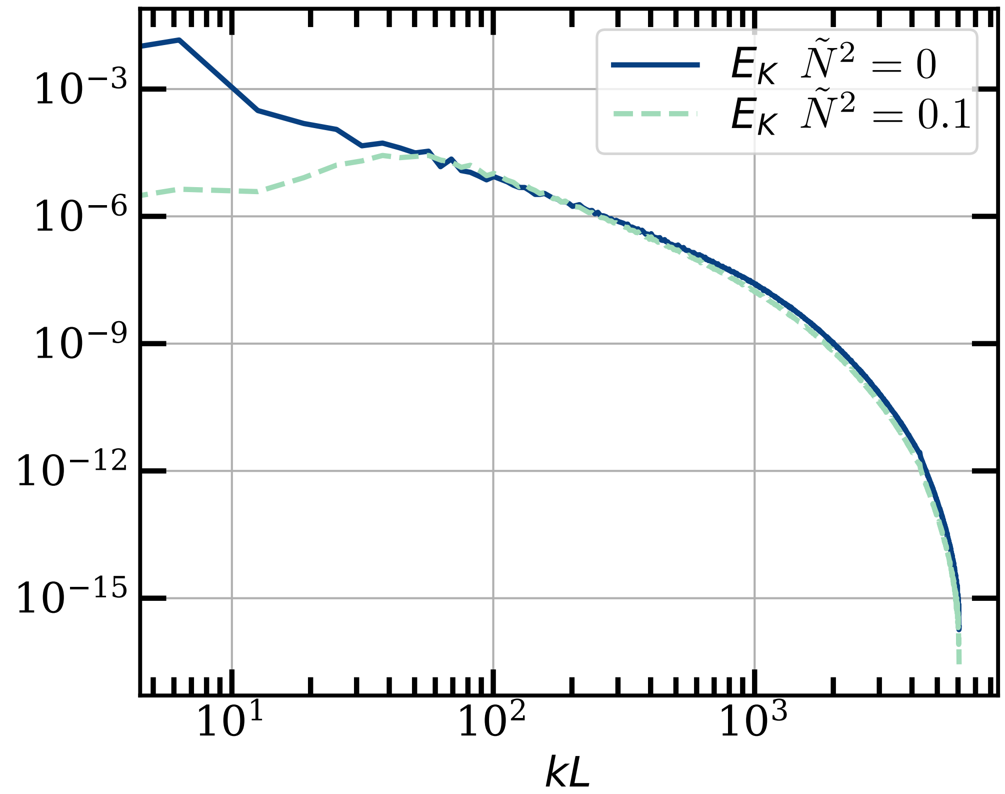
Flux-loop mechanism



MTI Turbulence - g-modes excitation

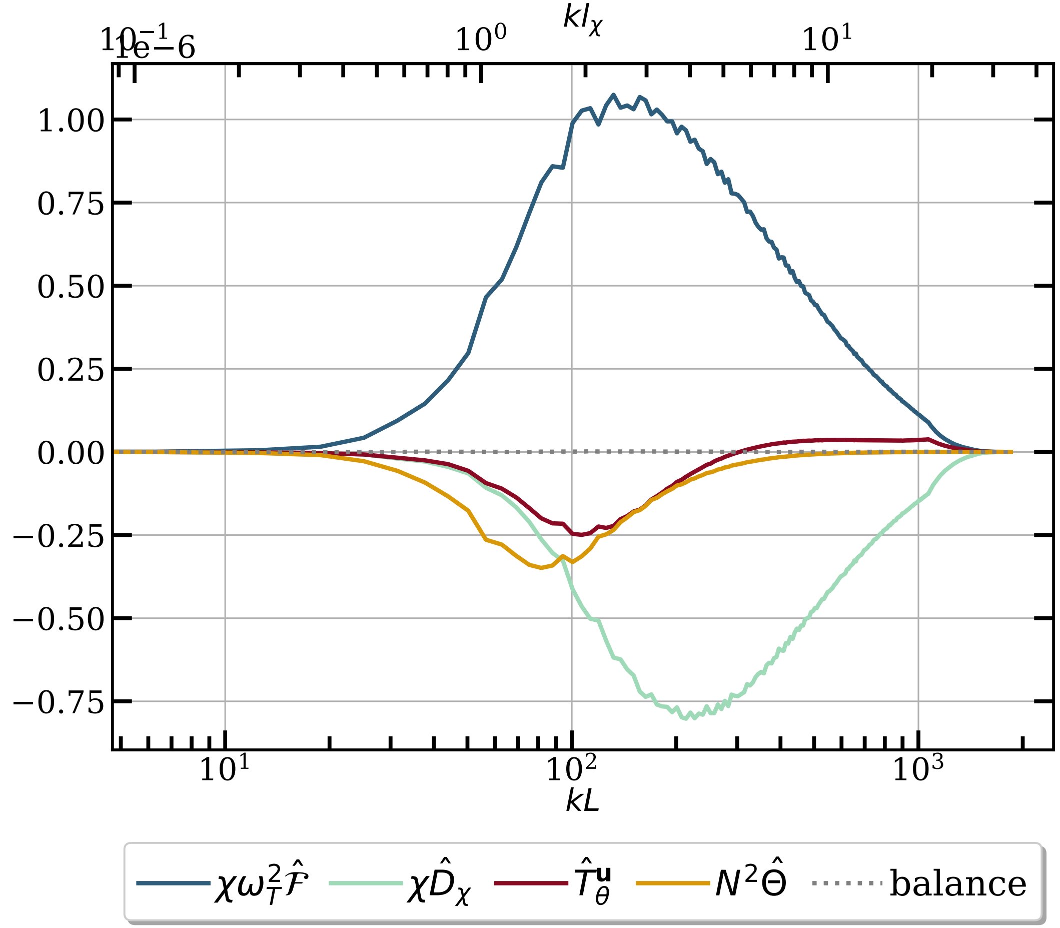
- g-modes impact on 2D turbulence in a fundamental way
  - They arrest the inverse cascade of energy to large scales
  - The excitation of g-modes isotropizes turbulence between vertical and horizontal directions





MTI Turbulence - Balancing the Small Scales

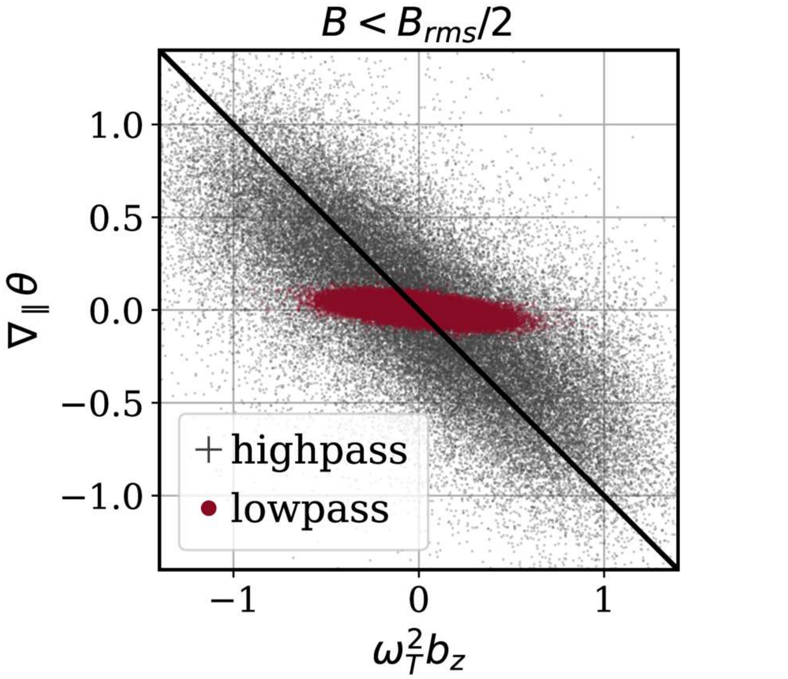
- Look at the detailed thermal energy balance



MTI forcing and anisotropic dissipation balance each other at small scales:

$$\nabla_{\parallel}\theta \approx -\omega_T^2 b_z$$

Scatter plot of $\nabla_{\parallel}\theta$ and $\omega_T^2 b_z$



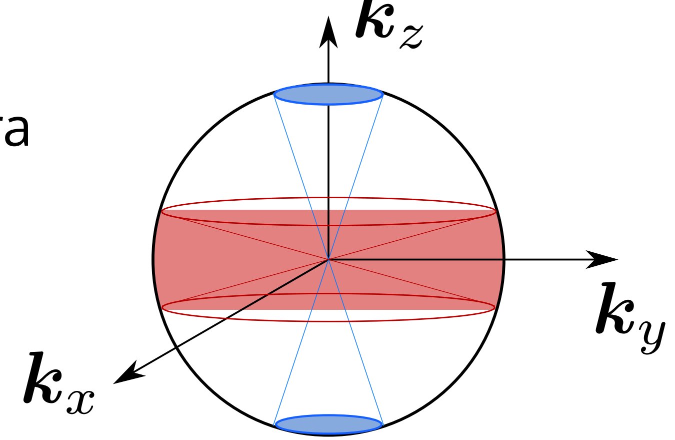
- gradient of temperature fluctuations arranges itself to counterbalance background gradient
- isothermality at small scales

Strongly stratified MTI

- Effect of strong entropy stratification evident from directional spectra
- Divide the spherical shell of radius *k* in latitudinal bands

$m = 1$ near equatorial: "vertical pancakes"

$m = 6$ near polar: "horizontal pancakes"



- small scales largely unaffected by increased N
- large scales tend to get isotropized near the Ozmidov scale $k_{Oz} = (N^3/\epsilon_\nu)^{1/2}$