Stratified & multiphase turbulence in the ICM

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based on Rajsekhar Mohapatra’s thesis work at ANU
stratified turbulence: arXiv:2001.06494, 2010.12602
multiphase turbulence: arXiv:2107.07722, 2109.01771, 2206.03602

ICM is turbulent

[Hitomi Collaboration 2016]

Hitomi spectrum of Perseus core

$v_{\mathrm{turb,3D}} \sim 3^{1/2}\sigma_v \sim 284$ km s^{-1}

$\mathcal{M}\sim0.3$, subsonic turbulence

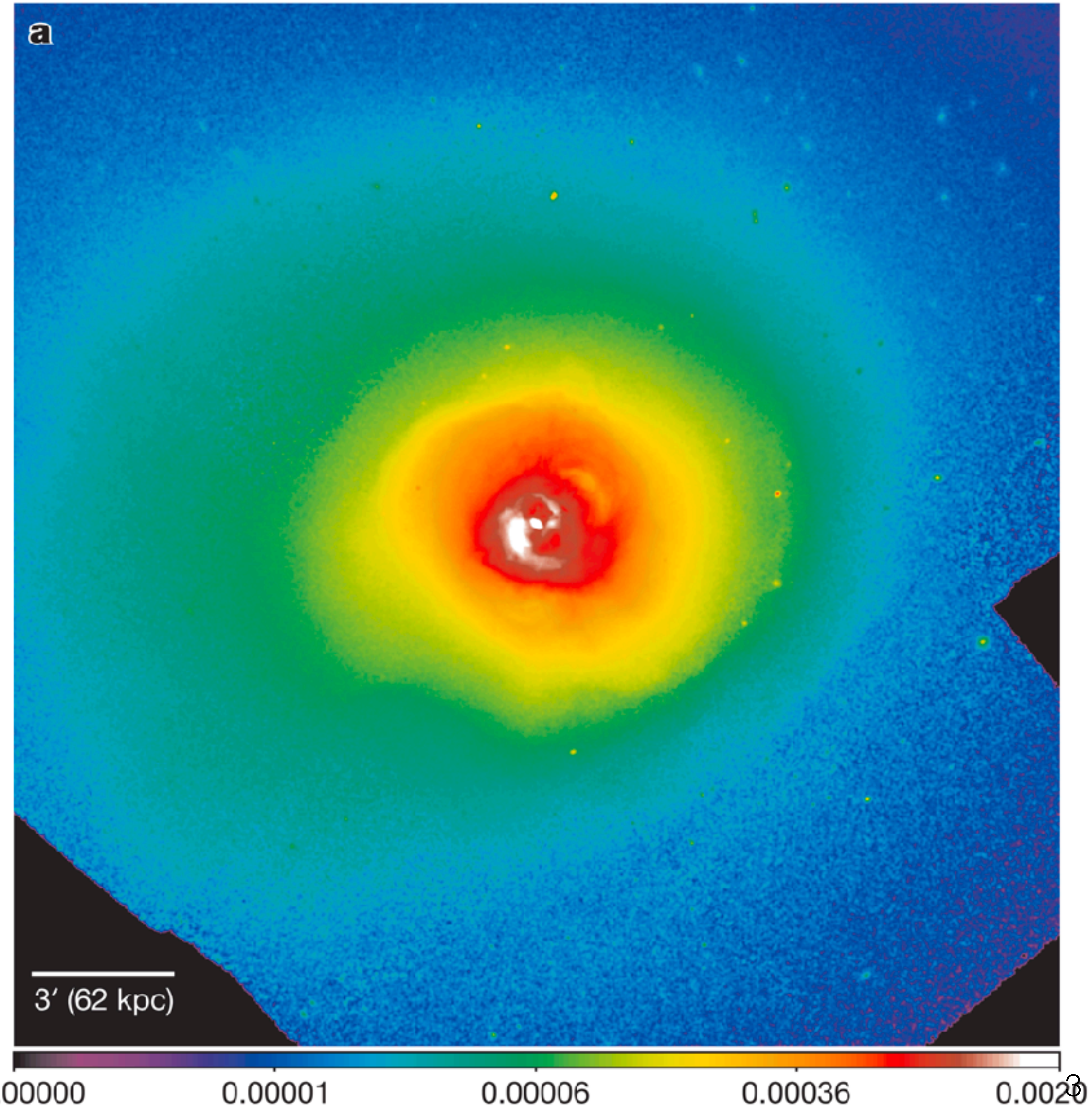
$$\sigma^2_{\mathrm{Dop}} = \sigma^2_v + \frac{2k_BT}{Am_p}$$

instrumental+thermal broadening

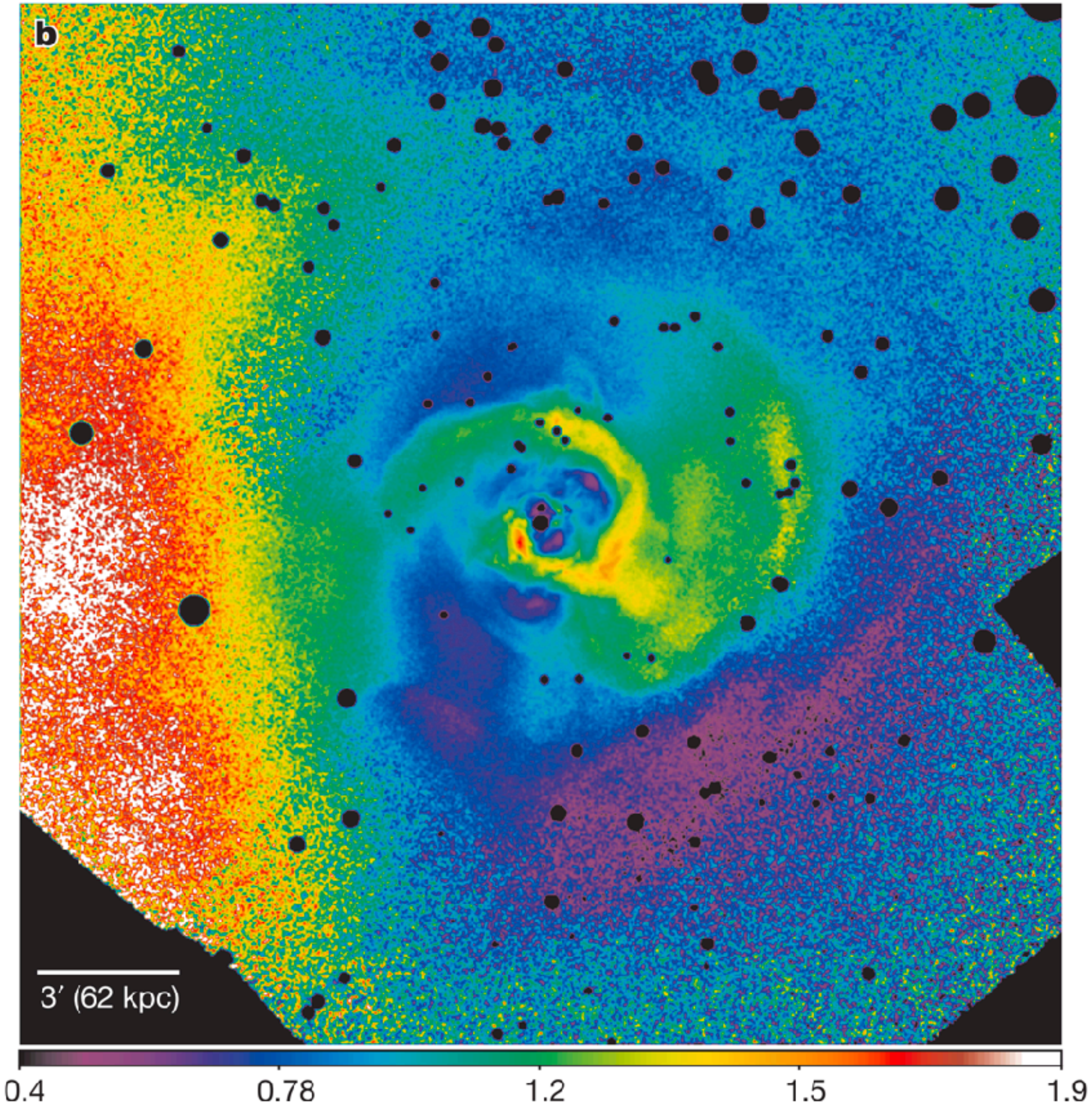
direct turbulence measurements rare
waiting for XRISM!

Need for indirect probes of turbulence

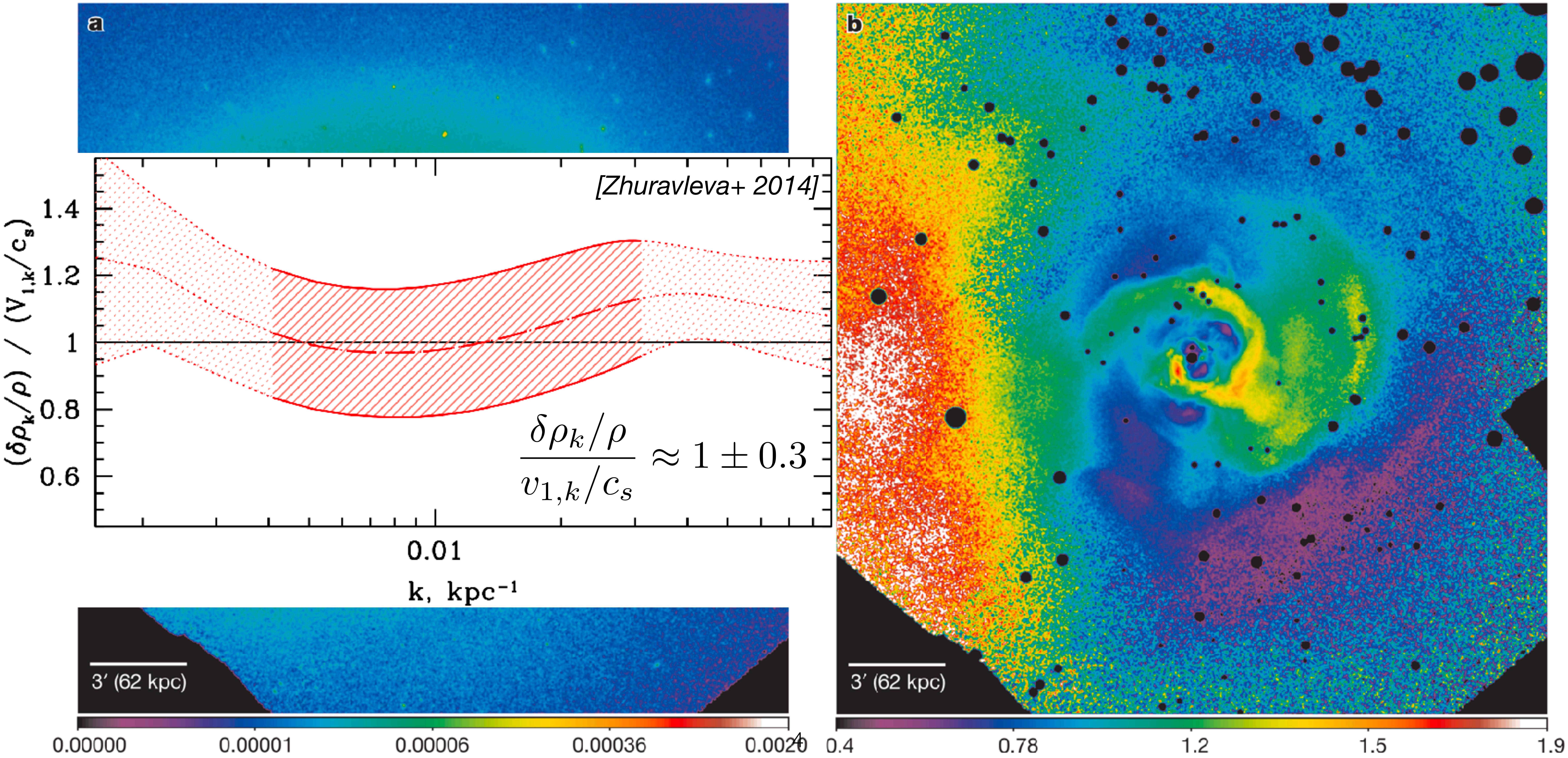
assuming density & SB fluctuations are due to turbulence



X-ray *surface* brightness fluctuations *[Zhuravleva+ 2014]*



Need for indirect probes of turbulence

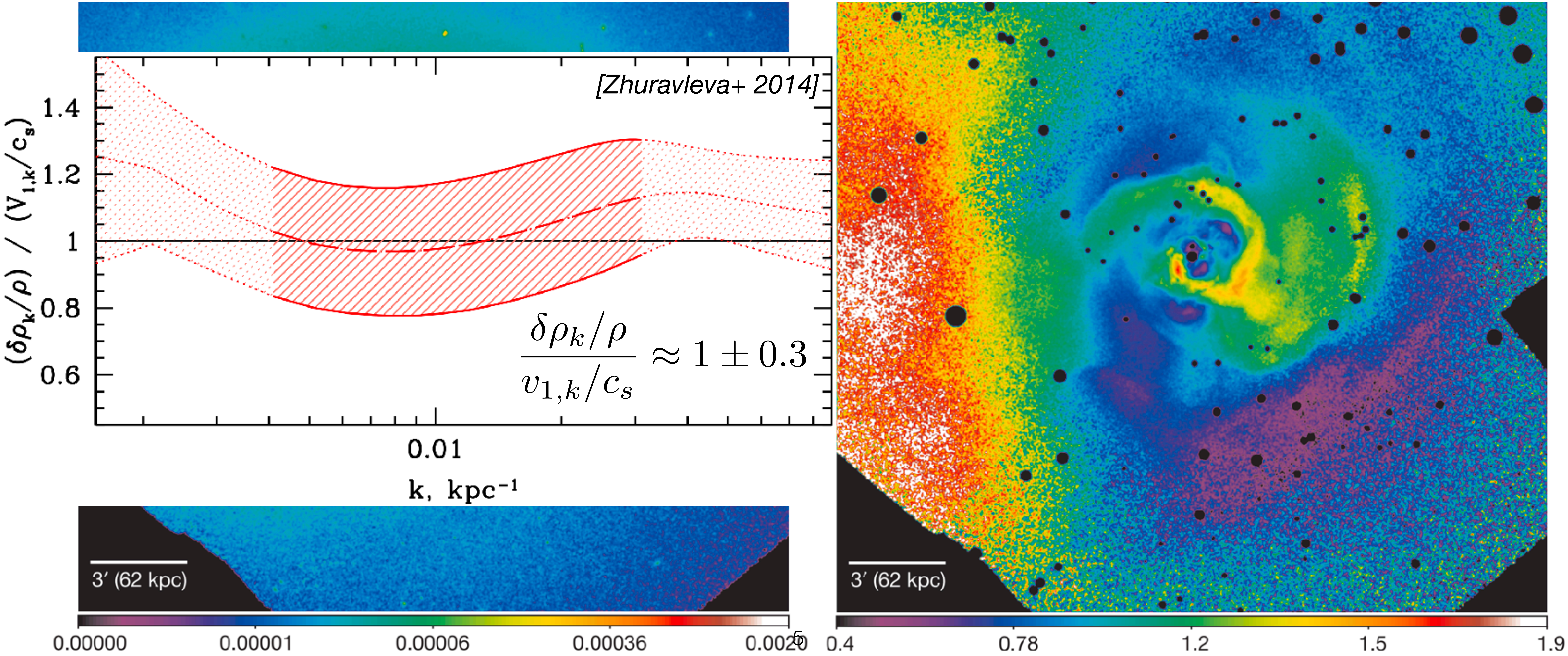


[Zhuravleva+ 2014]

$$\frac{\delta\rho_k/\rho}{v_{1,k}/c_s} \approx 1 \pm 0.3$$

Need for indirect probes of turbulence

other indirect probe is tSZ effect that measures LOS pressure through the ICM, again pressure fluctuations assumed to be mainly caused by turbulence



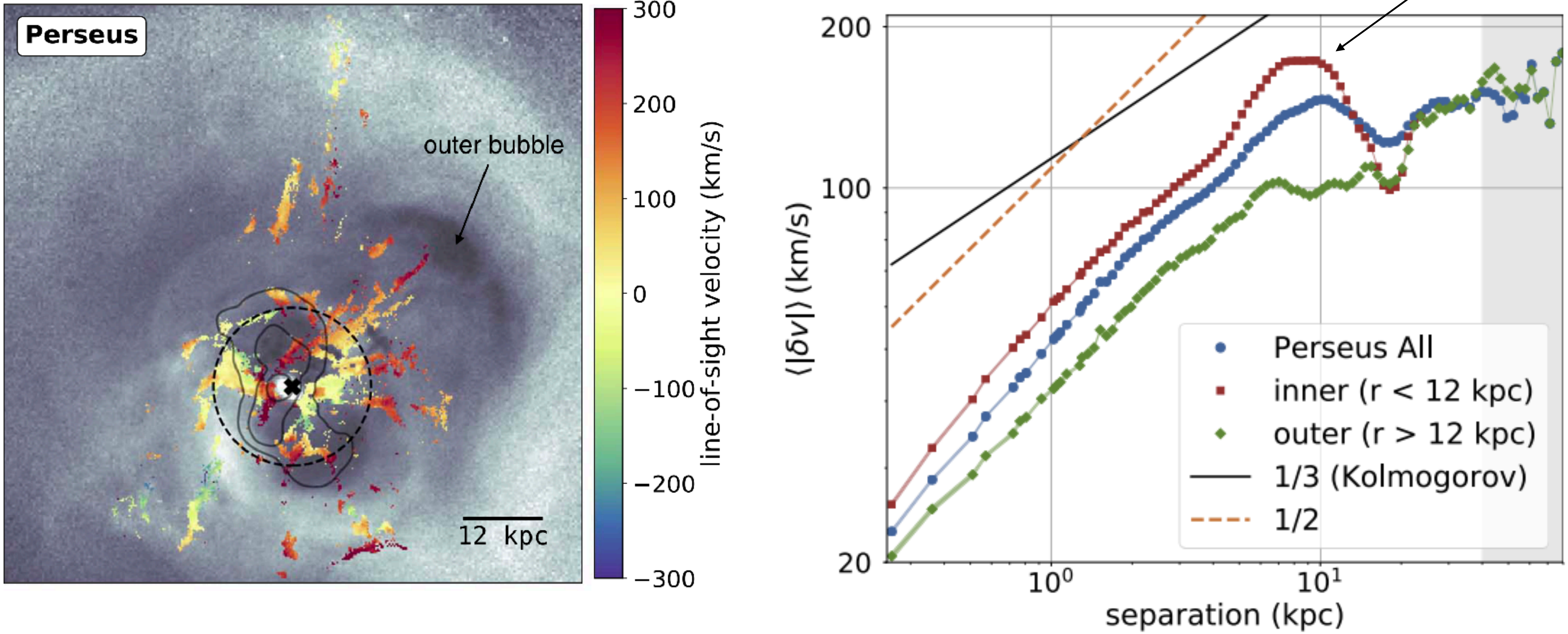
[Zhuravleva+ 2014]

$$\frac{\delta\rho_k/\rho}{v_{1,k}/c_s} \approx 1 \pm 0.3$$

Turbulence in cold filaments

projected VSFs of cold gas *[Li+ 2020]*

flattening of VSF at driving scale



v_{LOS} much easier to measure for cooler 10^4 K gas than hot X-ray emitting plasma; cold gas a tracer of background hot gas?

Need for idealized turbulent simulations

- ICM is **stratified** & cool cores are **multiphase**, substantially different from K41 (isotropic, single-phase)

- idealized boxes are simpler to understand, with a large body of work

- stirring due to jets/mergers/substructure may be mapped to large-scale driving???

Stratified turbulence

finite volume hydrodynamic code FLASH; without explicit viscosity

hydro equations with external gravity (mimicking DM gravity):

random forcing at ~ L/2

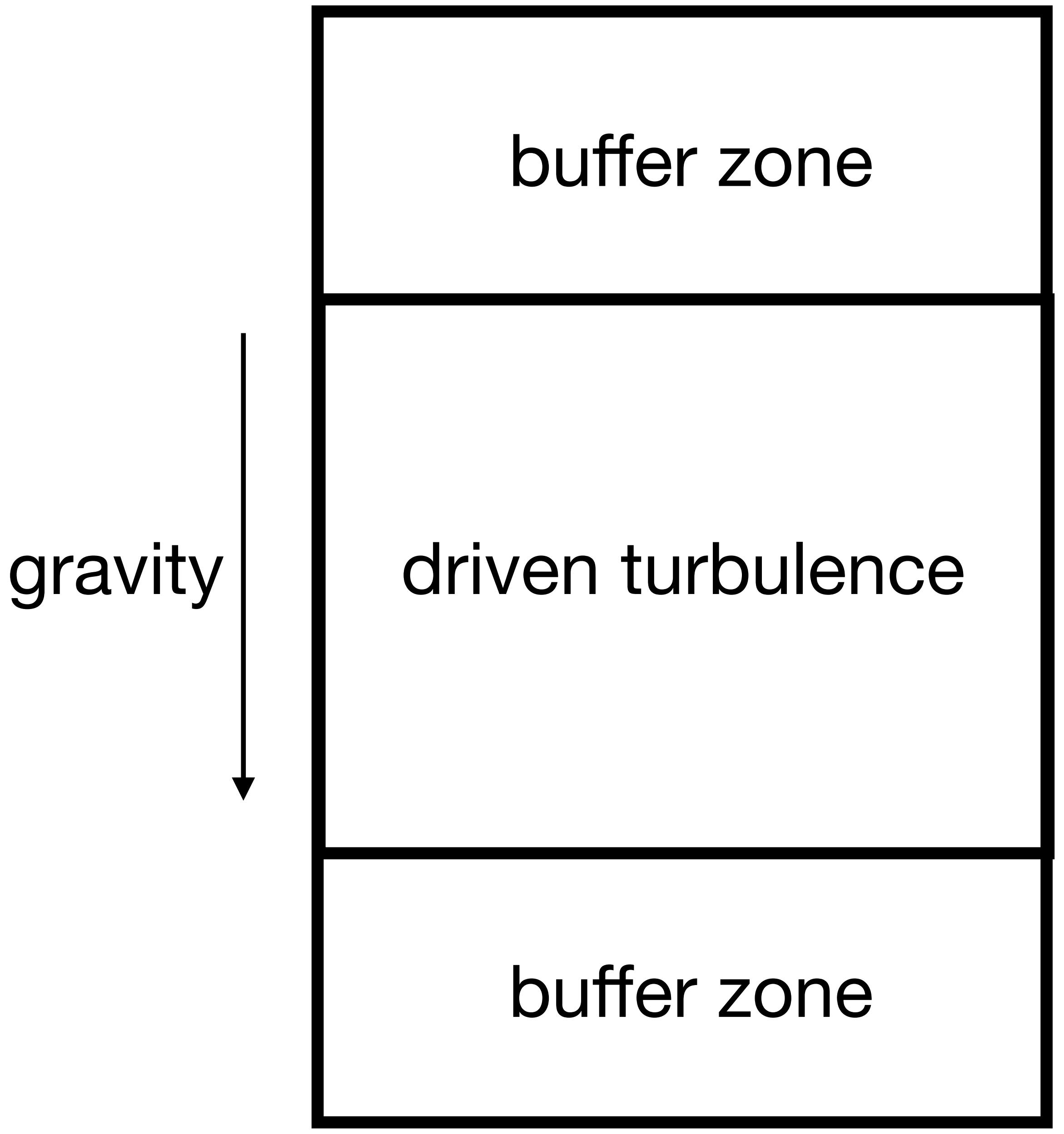
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) + \nabla P = \rho \mathbf{F} + \rho \mathbf{g},$$

$$\frac{\partial E}{\partial t} + \nabla \cdot ((E + P)\mathbf{v}) = \rho \mathbf{F} \cdot \mathbf{v} + \rho (\mathbf{v} \cdot \nabla)\Phi,$$

$\boldsymbol{g} = -\nabla\phi$, $E = \rho v^2/2 + P/(\gamma - 1)$, $\gamma = 5/3$

resolutions up to ~2048^3



[Mohapatra, Federrath & Sharma 2020, 2021]

Richardson or Froude number

$$\mathrm{Ri}_\ell = \frac{N^2}{(v_\ell/\ell)^2}$$

$$\mathrm{Fr}_\perp = \frac{v_\perp}{N\ell_\perp}$$

$$\mathrm{Fr}_\parallel = \frac{v_\perp}{Nl_\parallel}$$

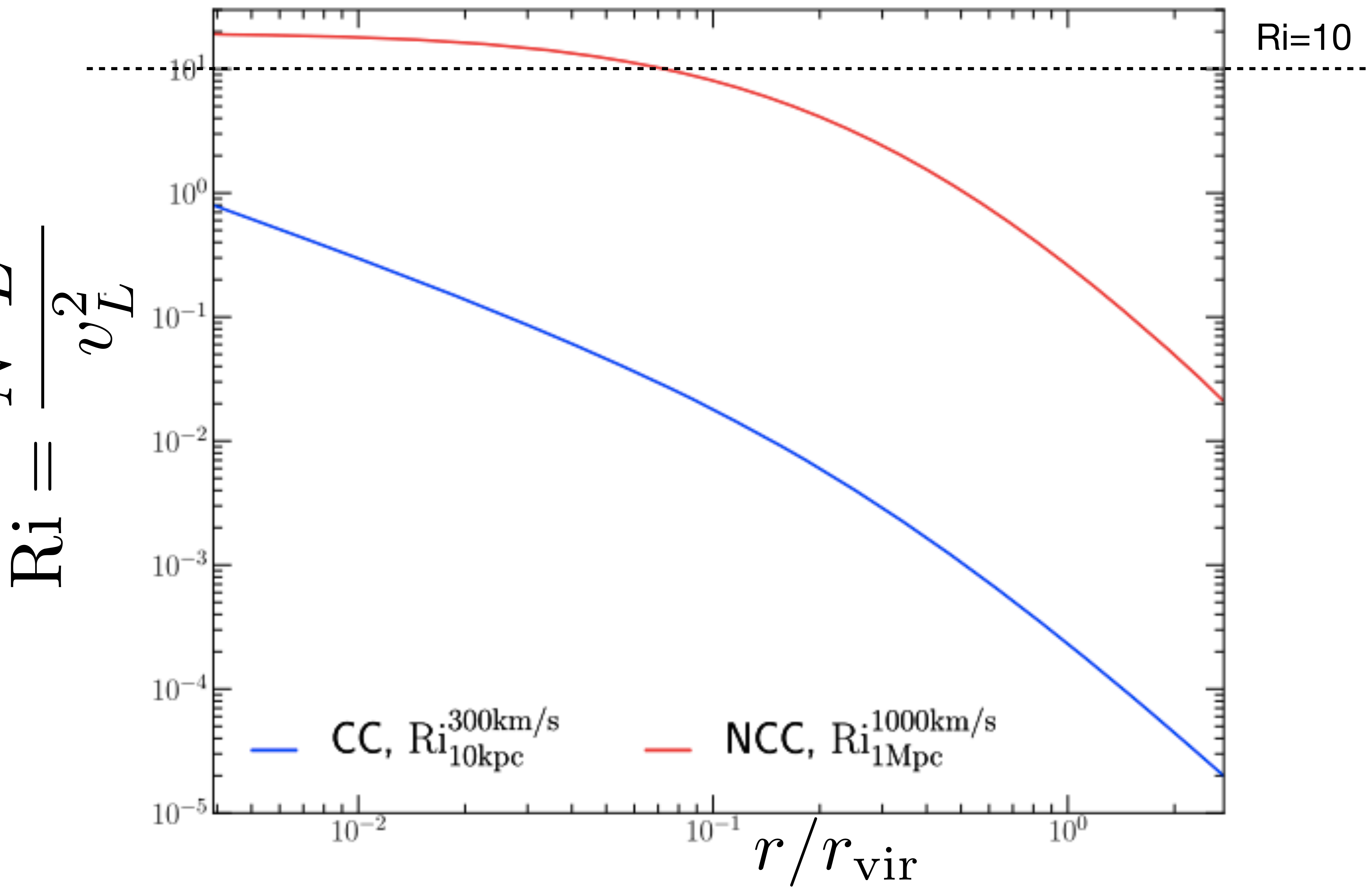
N: Brunt-Vaisala frequency

v_l : turbulent velocity at scale l

for strong stratification eddies are flattened

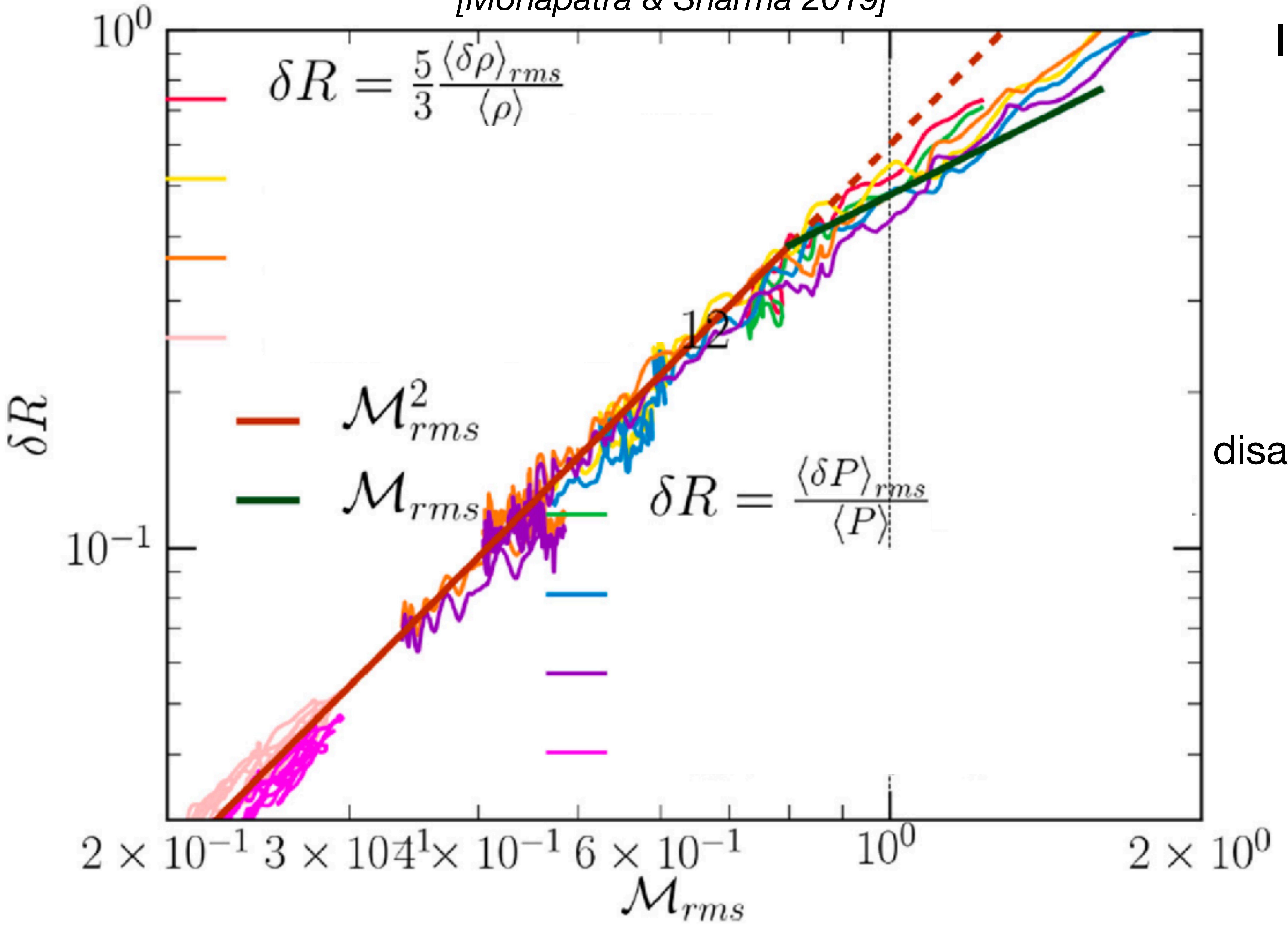
$$l_\perp > l_\parallel$$

Clusters are moderately stratified



Unstratified turbulence

[Mohapatra & Sharma 2019]



In subsonic regime relevant for ICM,

$$\left(\frac{\delta\rho}{\rho}\right)_{\rm rms} \approx \frac{3}{5}\left(\frac{\delta p}{p}\right)_{\rm rms} \approx \frac{1}{3}\mathcal{M}_{\rm rms}^2$$

disagrees with rho-v relation from cosmological sims.

$$\frac{\delta\rho_k/\rho}{v_{1,k}/c_s} \approx 1 \pm 0.3$$

is it because of stratification?

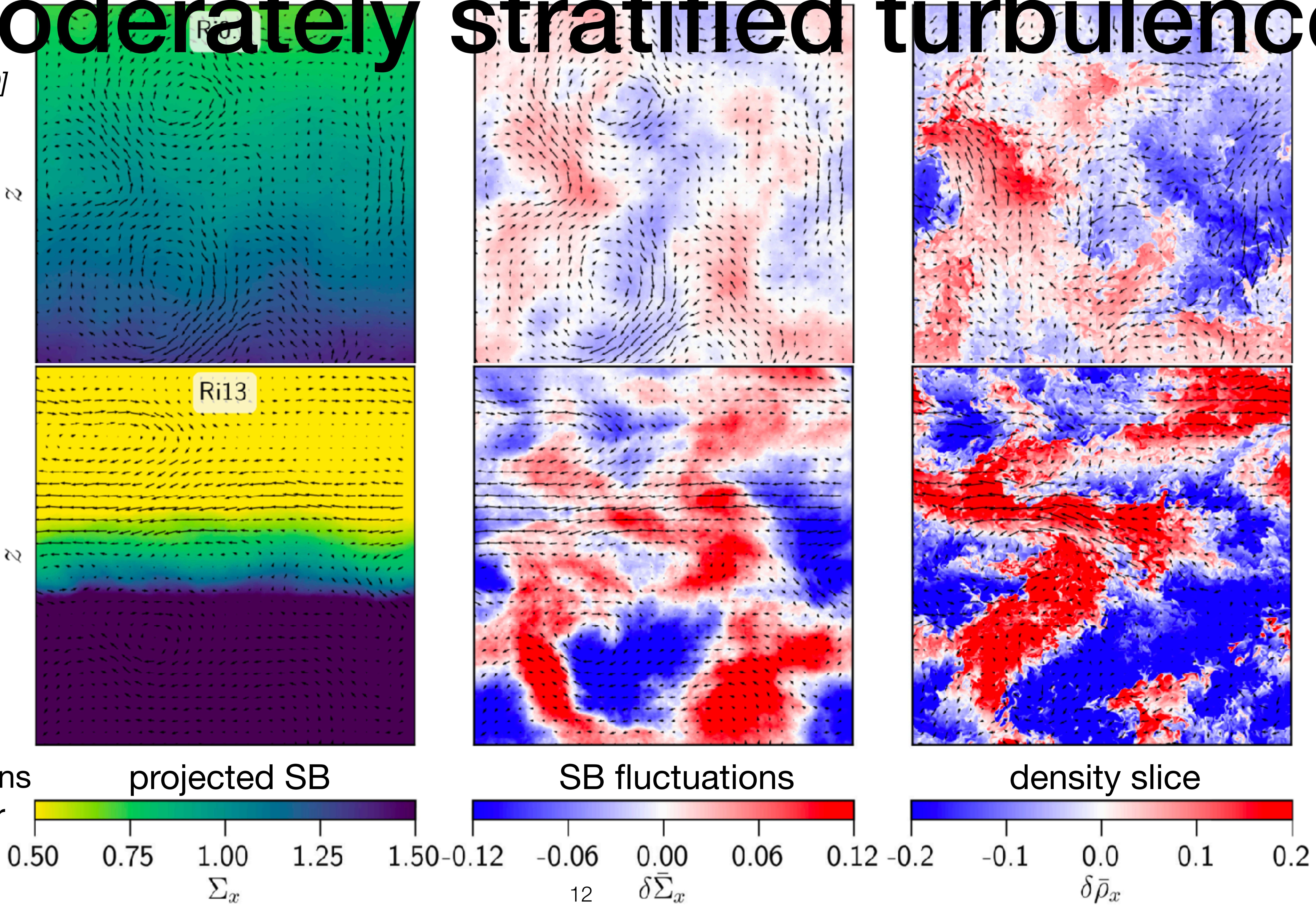
Moderately stratified turbulence

[Mohapatra + 2020]

$\mathrm{Ri} = 0.2$

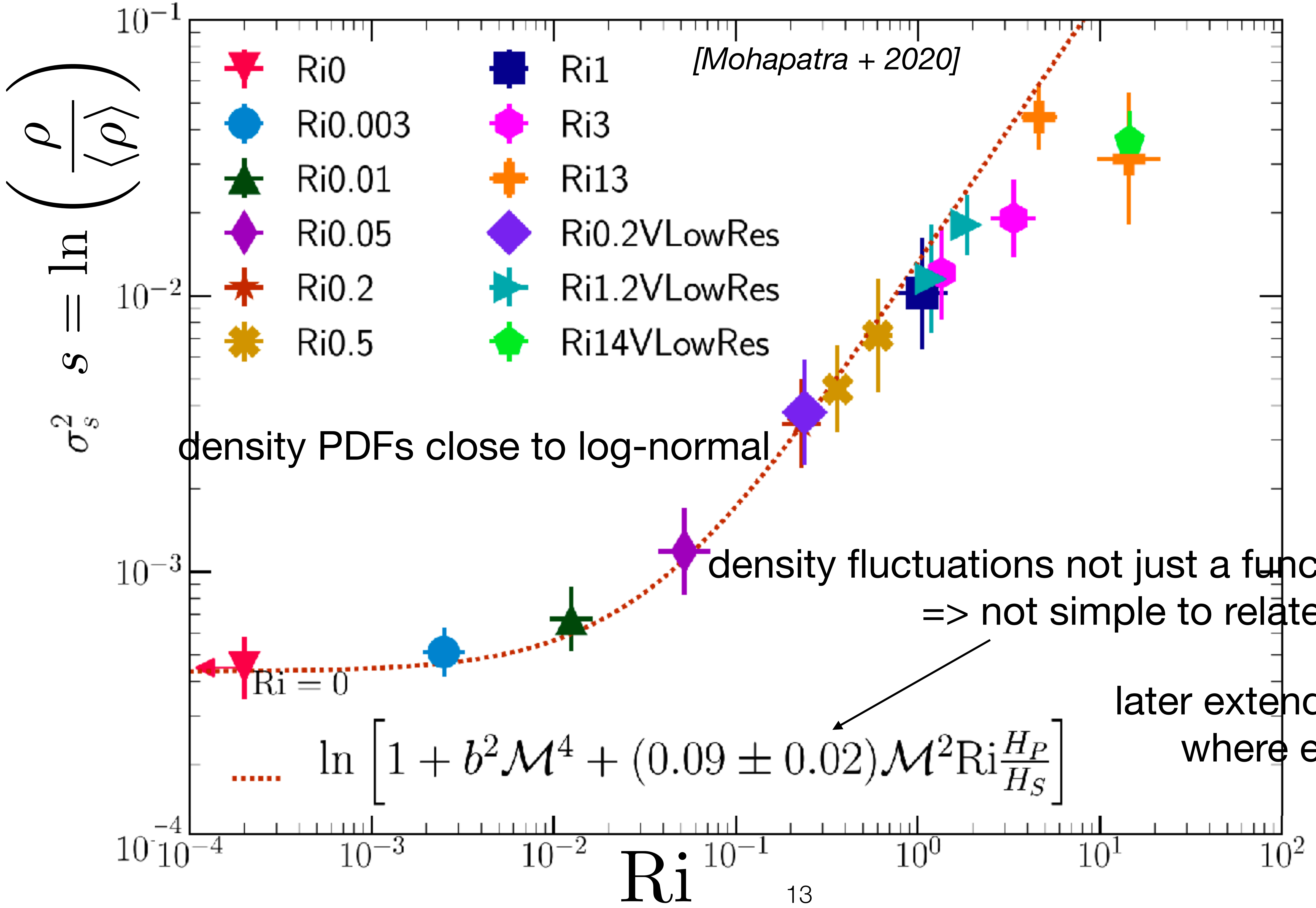
increasing stratification

$\mathrm{Ri} = 13$



density fluctuations
higher for larger
stratification

Density fluctuation scaling



variance of log-density

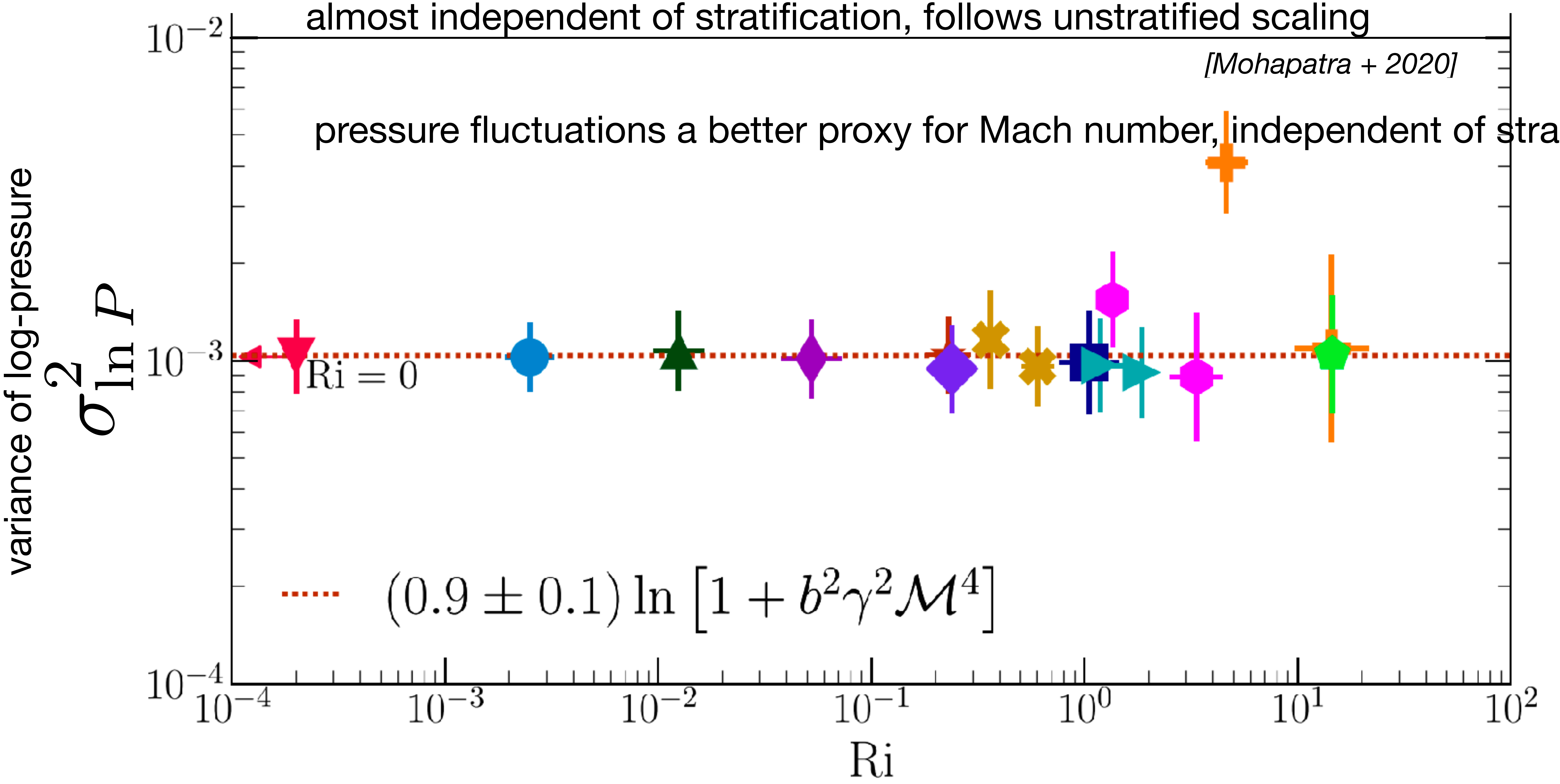
density PDFs close to log-normal

density fluctuations not just a function of Mach number
=> not simple to relate the two!

later extended to strong stratification
where eddies flatten & $v_z < v_\perp$

What about pressure fluctuations?

almost independent of stratification, follows unstratified scaling



[Mohapatra + 2020]

pressure fluctuations a better proxy for Mach number, independent of stratification

Multiphase turbulence

again using FLASH code; no explicit dissipation

hydro equations with global thermal balance:

resolutions up to ~1500^3

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v} + P^* I - \mathbf{B} \otimes \mathbf{B}) = \rho \mathbf{F},$$

$$\frac{\partial E}{\partial t} + \nabla \cdot ((E + P^*)\mathbf{v} - (\mathbf{B} \cdot \mathbf{v})\mathbf{B}) = \rho \mathbf{F} \cdot \mathbf{v} + Q - \mathcal{L},$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0,$$

$$P^* = P + \frac{\mathbf{B} \cdot \mathbf{B}}{2},$$

$$E = \frac{\rho \mathbf{v} \cdot \mathbf{v}}{2} + \frac{P}{\gamma - 1} + \frac{\mathbf{B} \cdot \mathbf{B}}{2},$$

solenoidal driving

cooling rate density $n_e n_i \Lambda(T)$

thermal heating rate density

power injection rate density by turbulent forcing

isotropic
homogeneous
low-k driving

$$\int \rho \mathbf{F} \cdot \mathbf{v} \mathrm{d}V = f_{\mathrm{turb}} \int \mathcal{L} \mathrm{d}V$$

$$Q_{\mathrm{mw}}(\mathbf{x}, t) = \rho(\mathbf{x}, t)(1 - f_{\mathrm{turb}}) \frac{\int \mathcal{L} \mathrm{d}V}{\int \rho \mathrm{d}V},$$

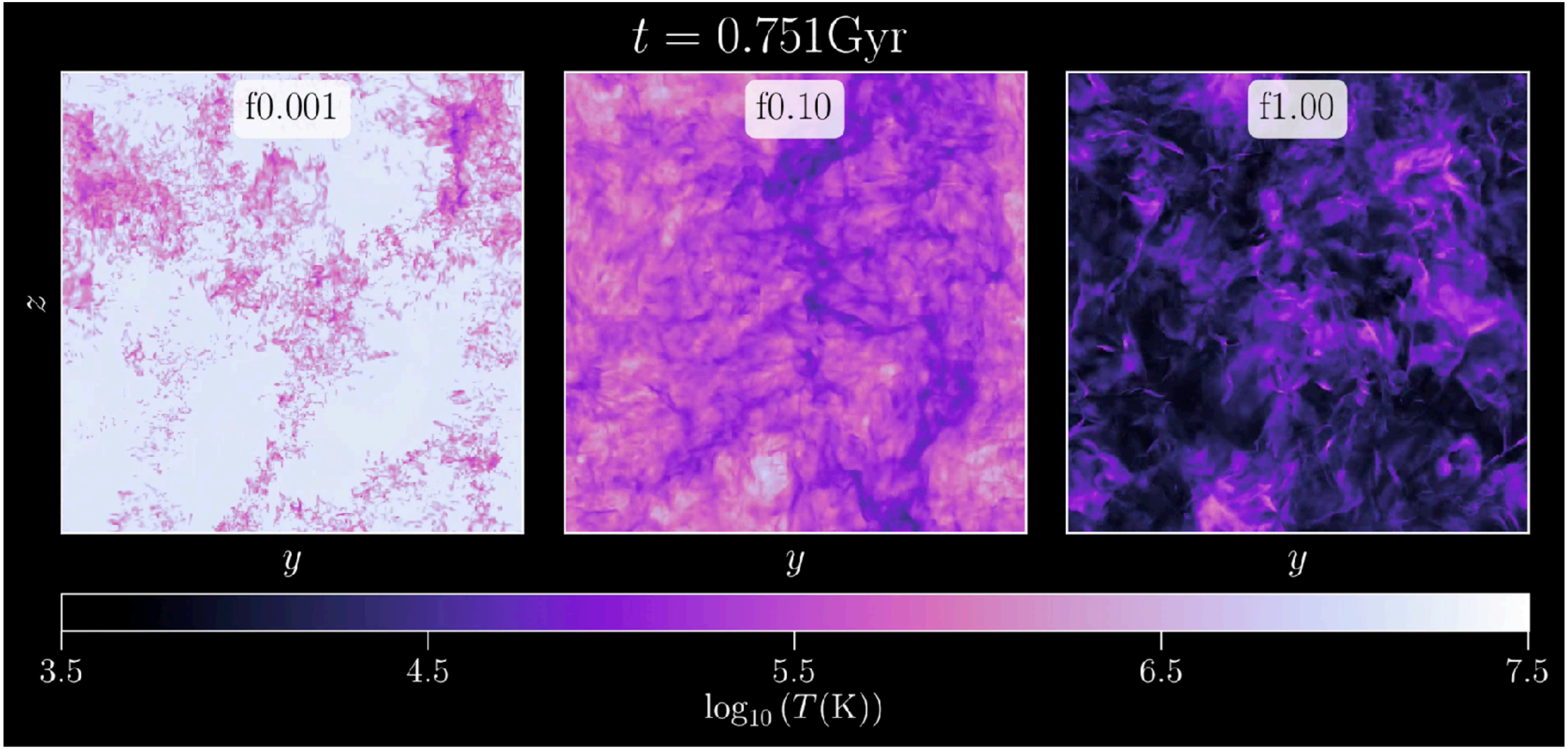
$$Q_{\mathrm{vw}}(\mathbf{x}, t) = (1 - f_{\mathrm{turb}}) \frac{\int \mathcal{L} \mathrm{d}V}{\int \mathrm{d}V}.$$

$E = \rho v^2/2 + P/(\gamma - 1)$, $\gamma=5/3$

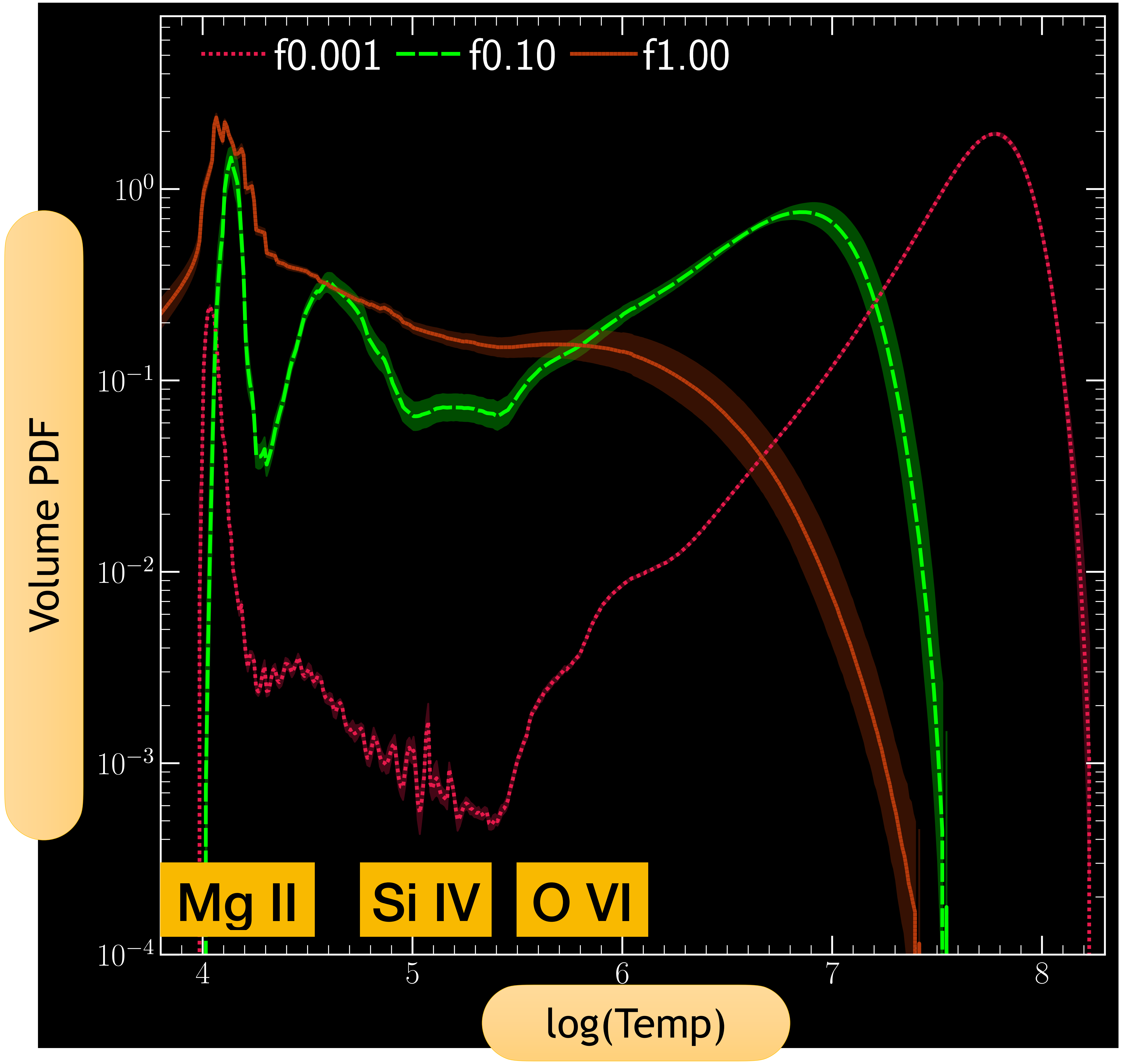
[Mohapatra+ 2022a, b, c]

Projected temperature

cold gas forms much faster with stronger turbulence, turbulence seeds density perturbations



Temperature PDFs



f- feedback energy fraction in form of turbulence

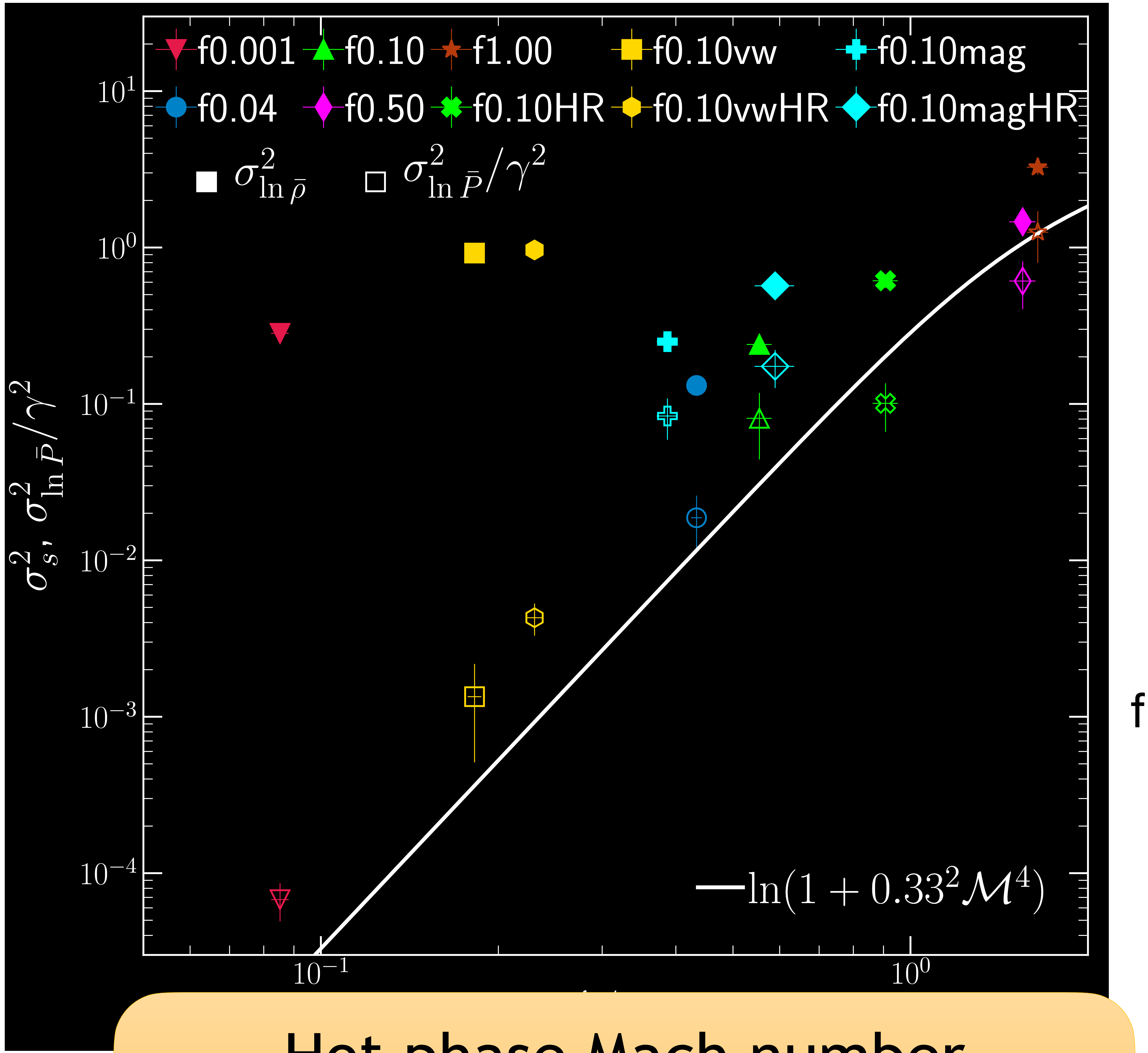
turbulence enhances intermediate-T gas by mixing hot & cold gas

hot peak cooler with higher turbulence

Density & pressure fluctuations vs M

Pressure/γ fluctuations squared

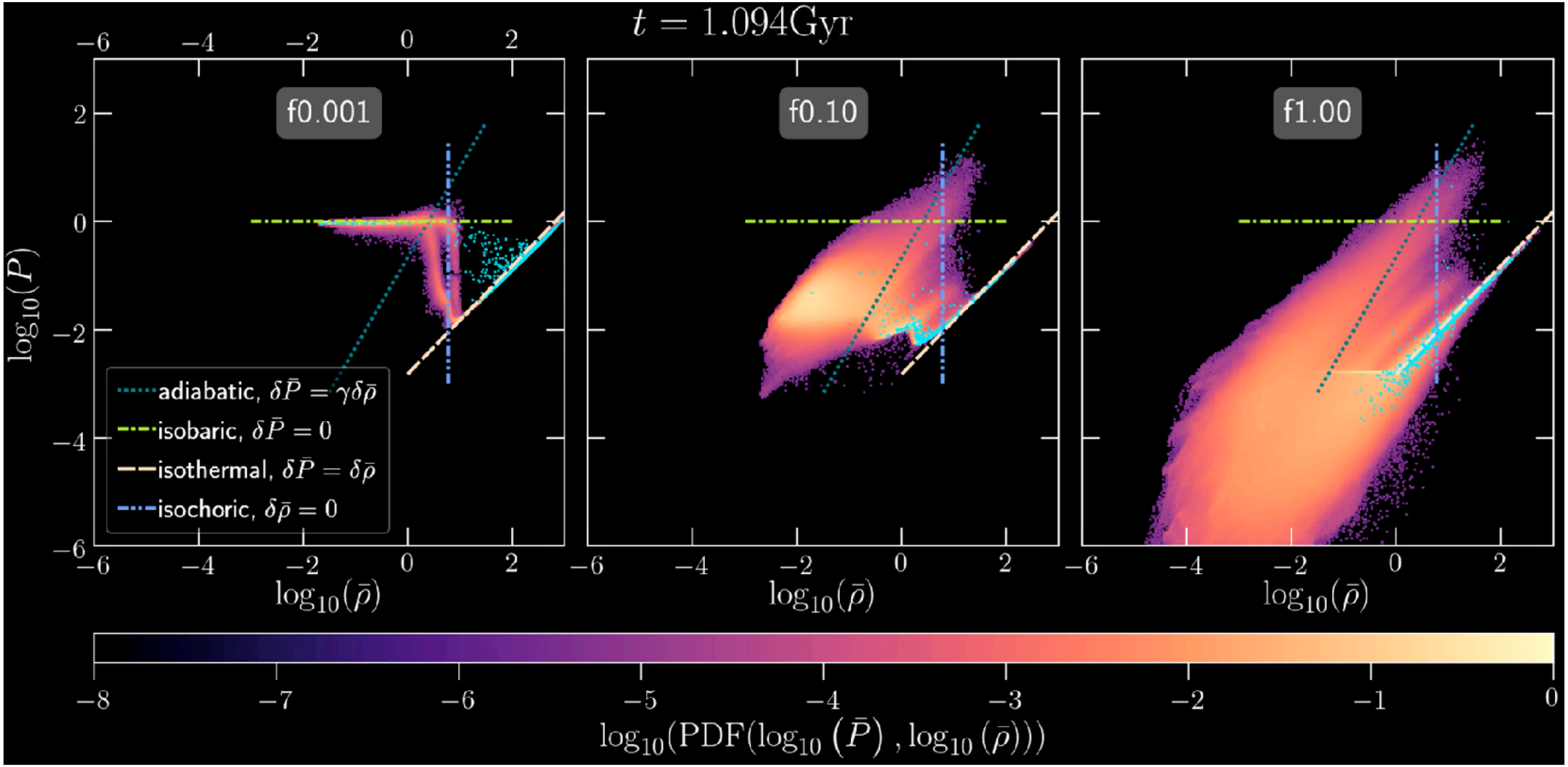
Density fluctuations squared



Hot phase Mach number

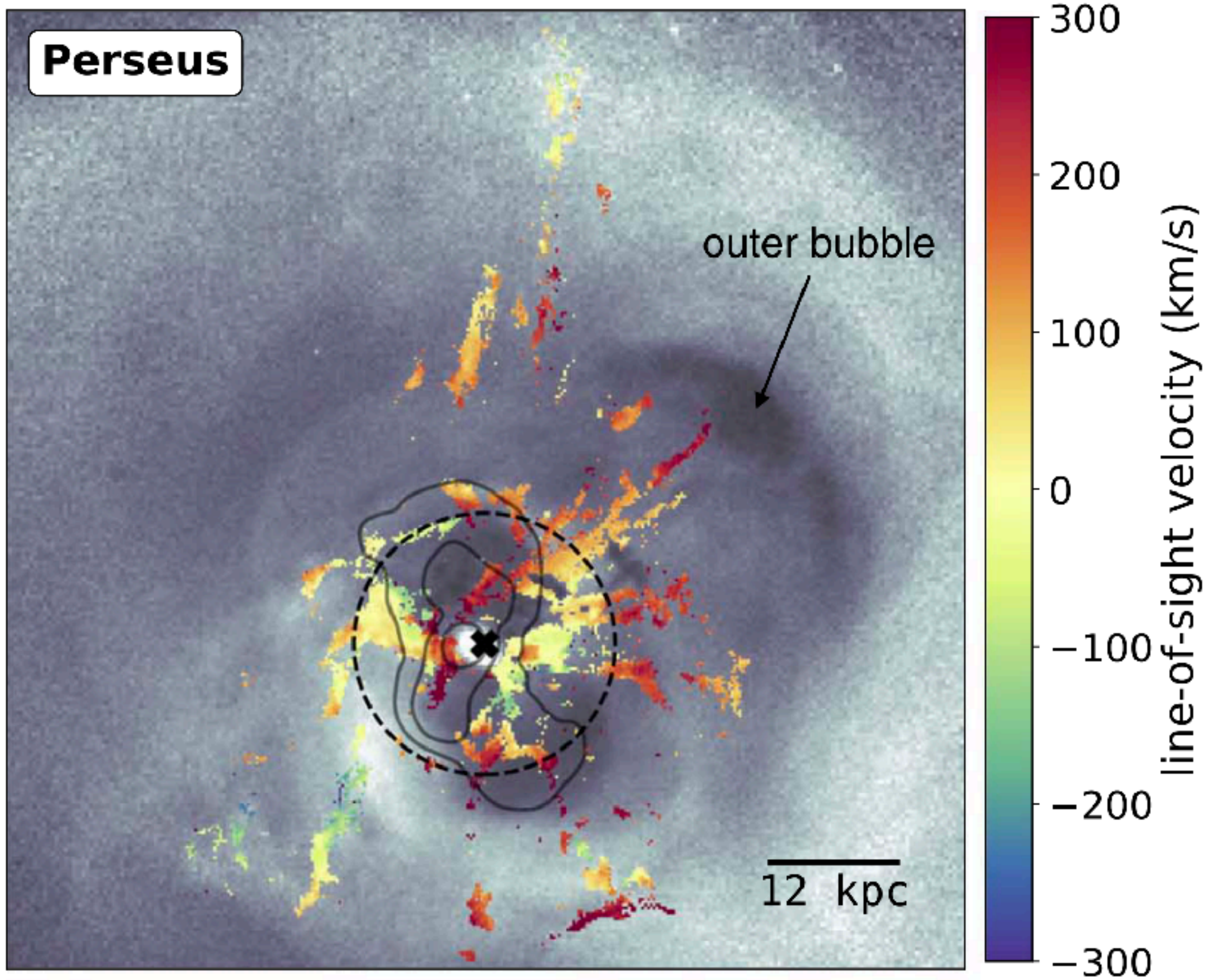
for multiphase turbulence, density fluctuations larger but pressure fluctuations similar to non-radiative runs

Modes of perturbation



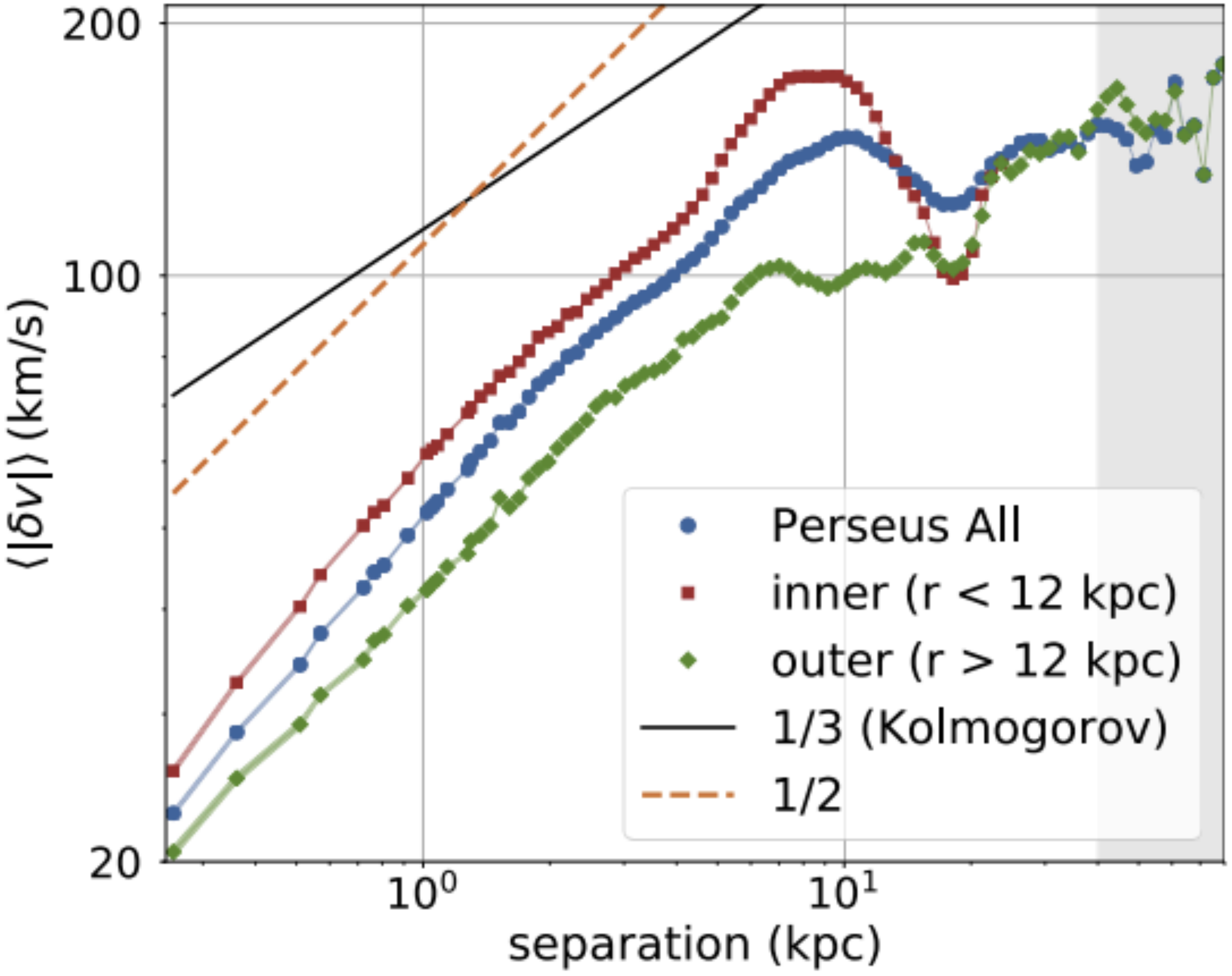
can be probed observationally, in principle

Projected vs 3D structure functions



$$\mathrm{VSF}_p(r) = \langle \delta v_r^p \rangle$$

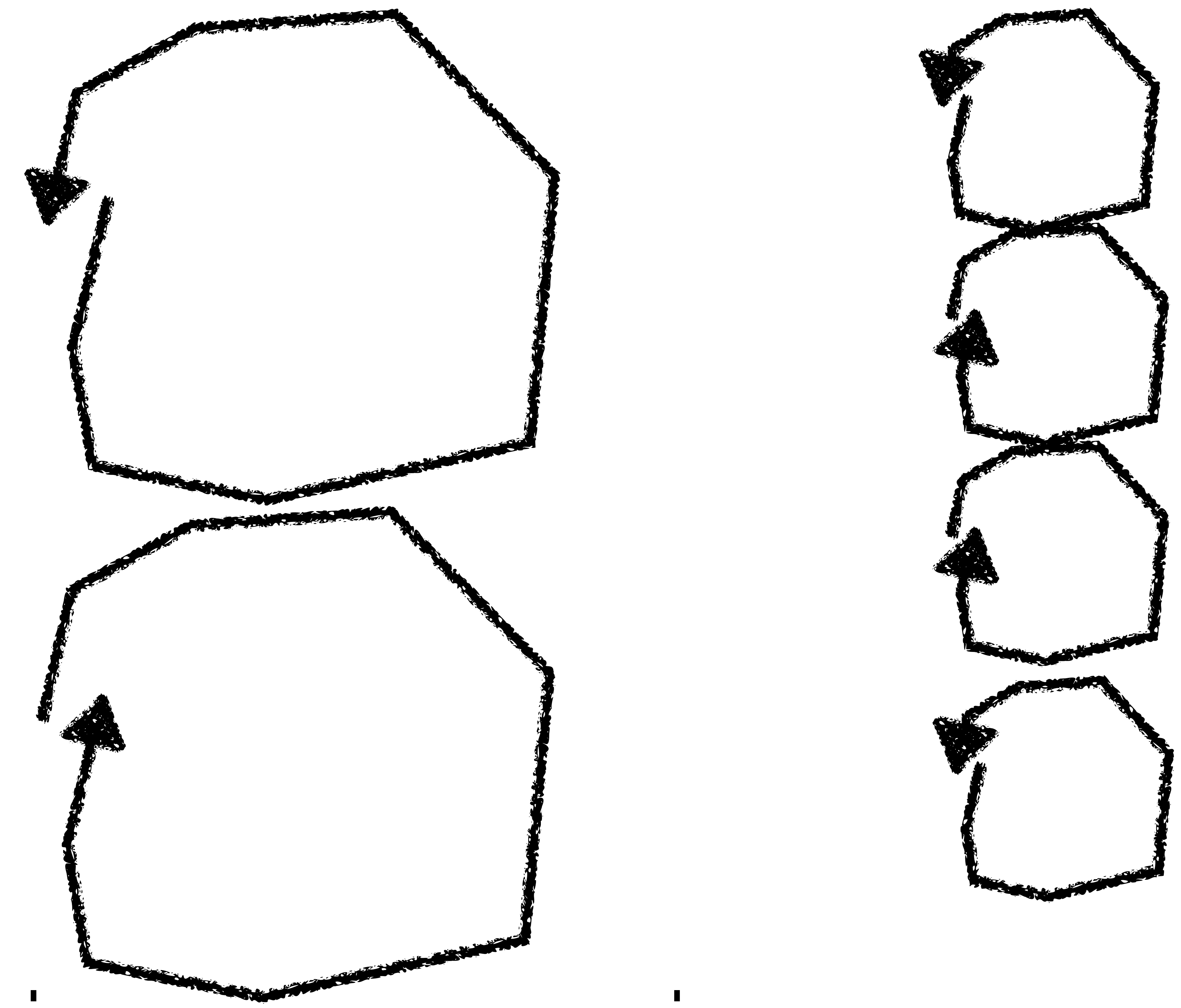
$$\delta v_r = |\mathbf{v}(\mathbf{x} + \mathbf{e}_1 r, t) - \mathbf{v}(\mathbf{x}, t)|$$



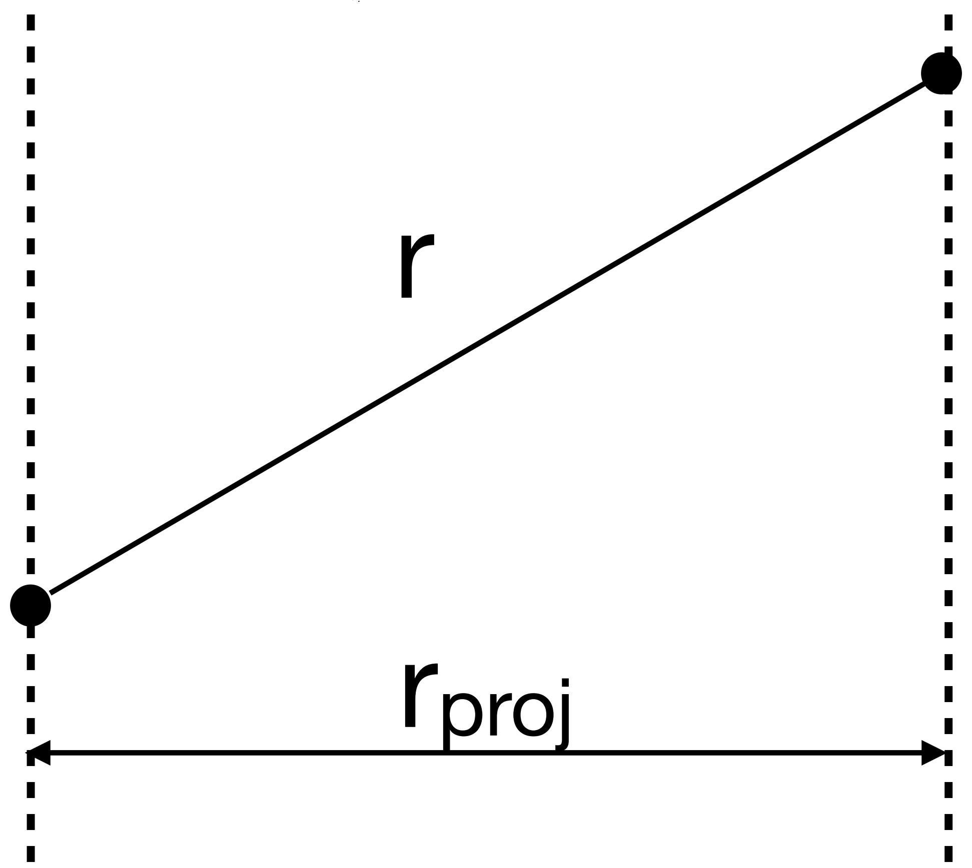
these are **projected** structure functions

Projected vs 3D structure functions

i)



ii)



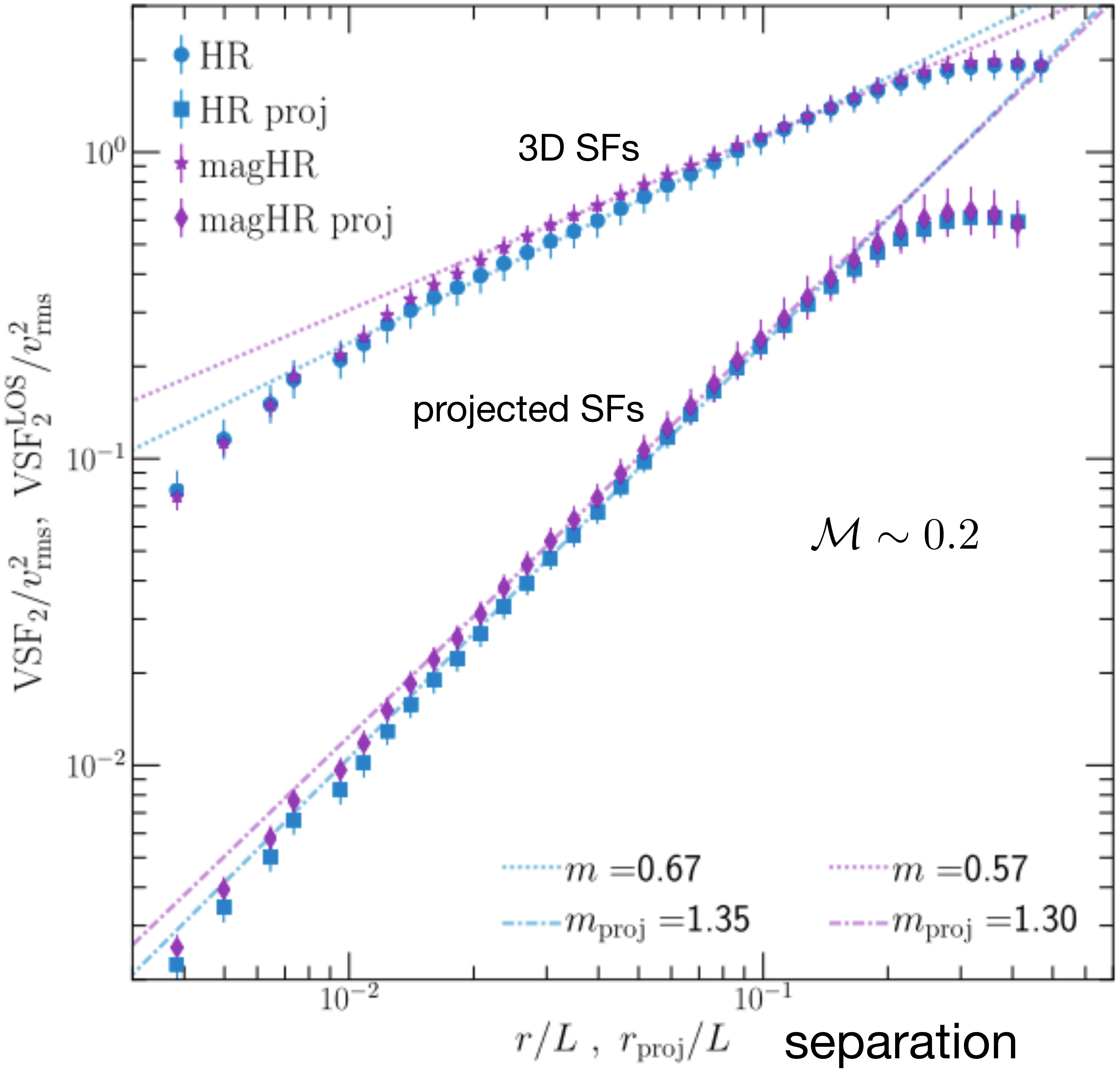
projected SFs different from 3D SFs because:

i) cancellation of velocities of many small eddies along LOS suppresses power at small scales

ii) projected distance smaller than 3D distance, hence power at small scales increases

i) steepens the VSF & ii) makes it shallower
thus projection can cause both steepening & flattening

Projection effects in K41 turbulence

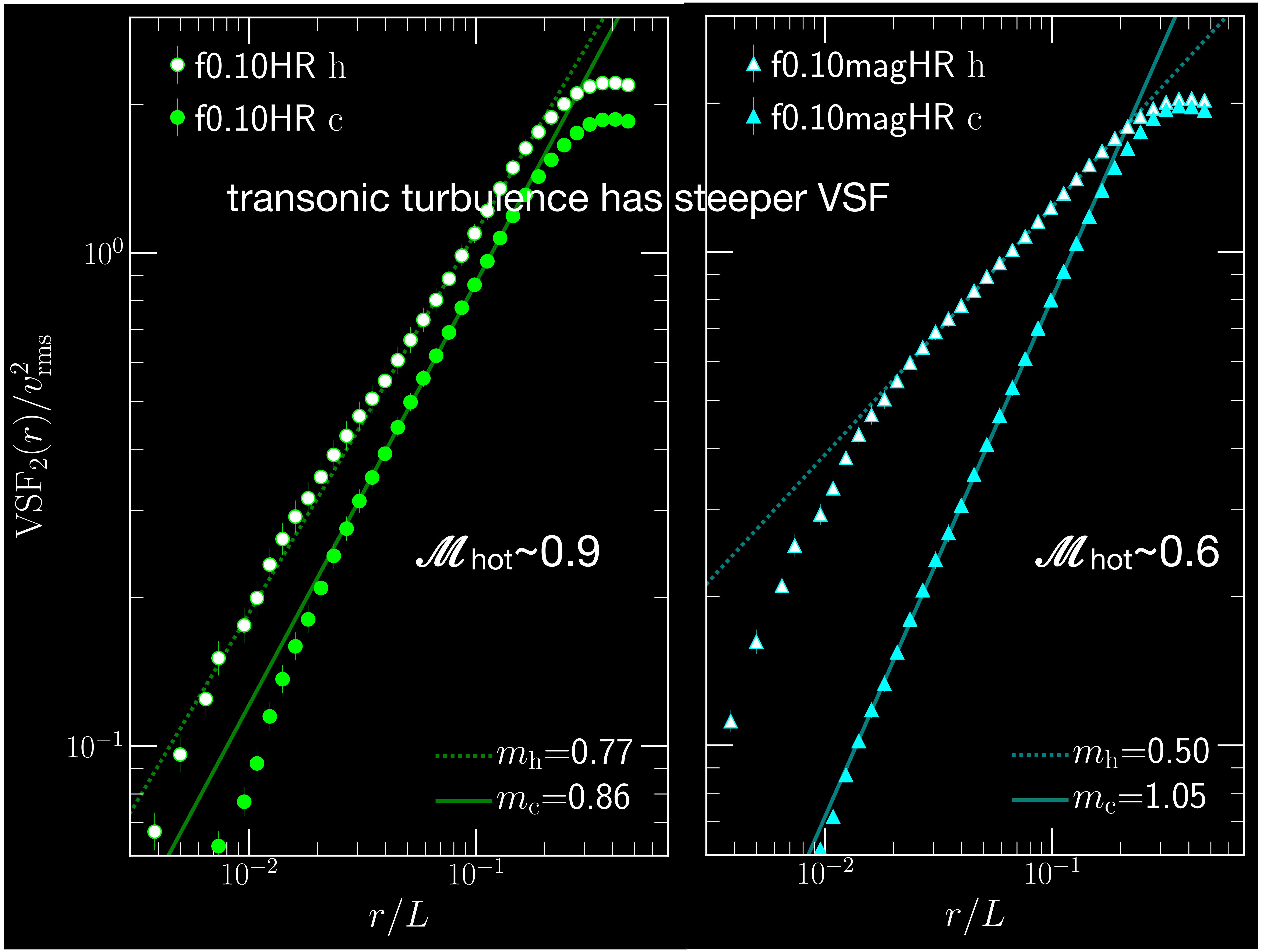


separation

projected SFs are steeper because of greater cancellation of velocities of smaller eddies along LOS (effect i) dominates)

what happens in multiphase turbulence?

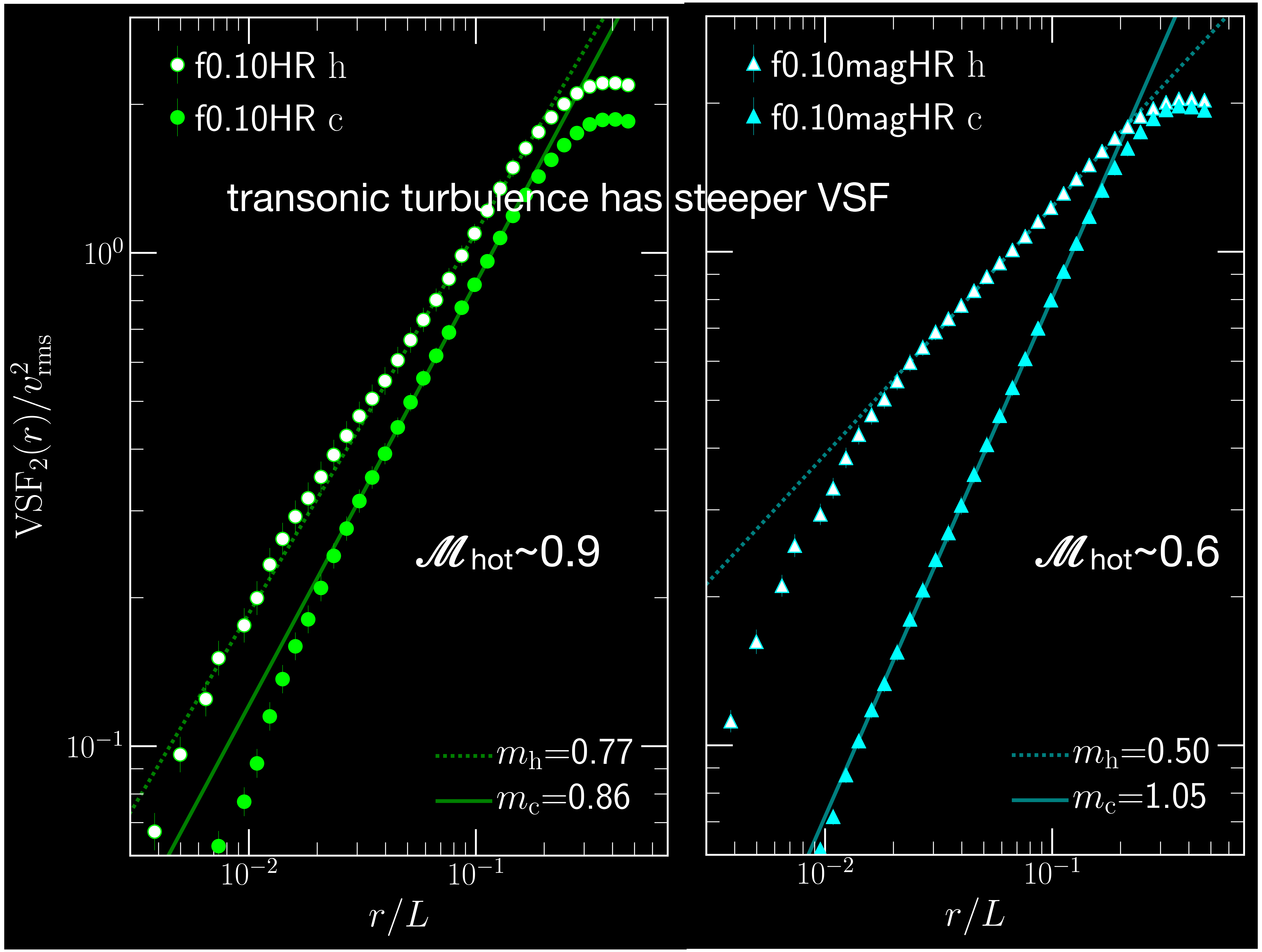
3D VSFs: hydro vs MHD



transonic turbulence has steeper VSF

Magnetic field - cold phase δv^2 much steeper than hot, since cold gas is magnetically dominated

3D VSFs: hydro vs MHD

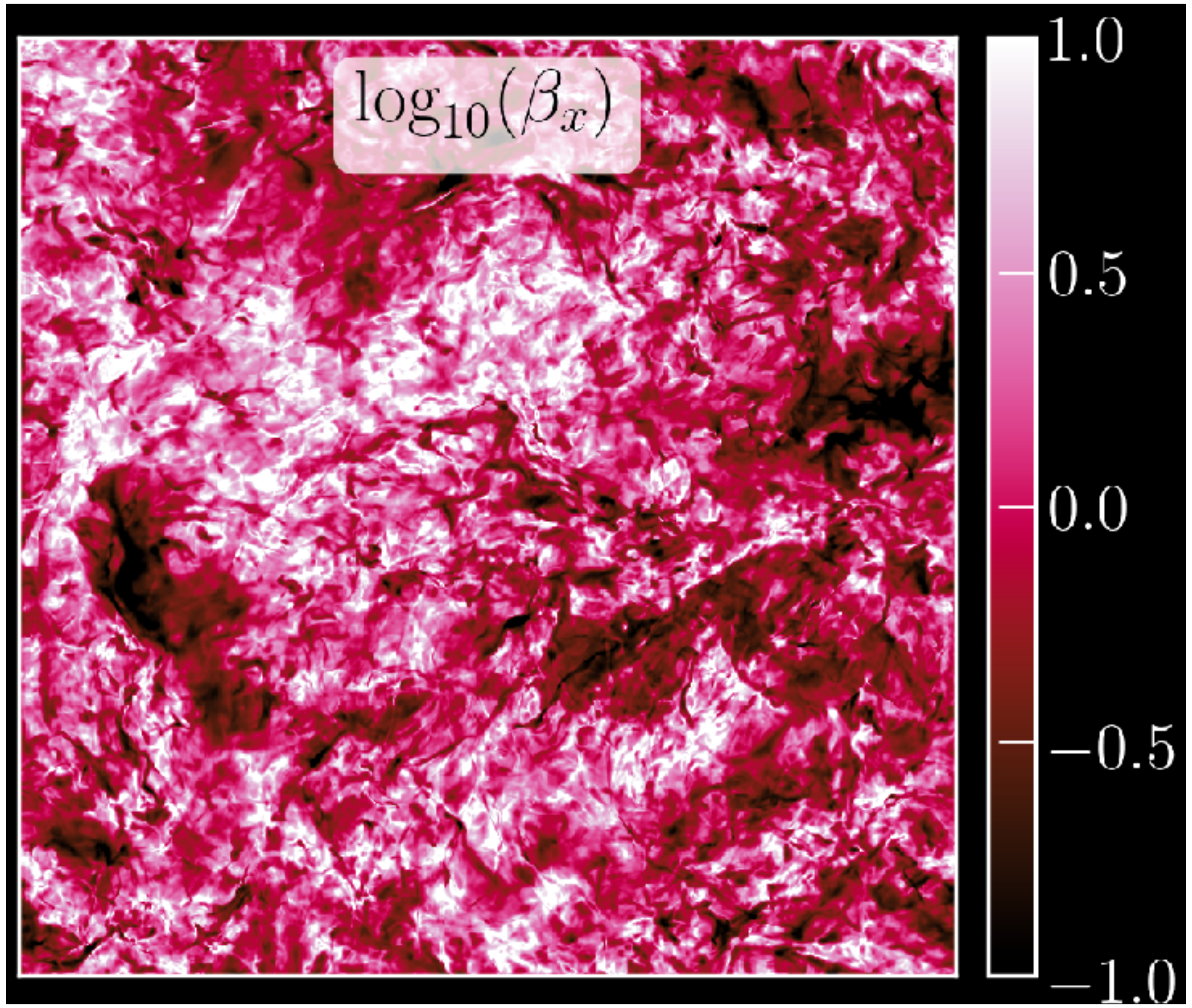


transonic turbulence has steeper VSF

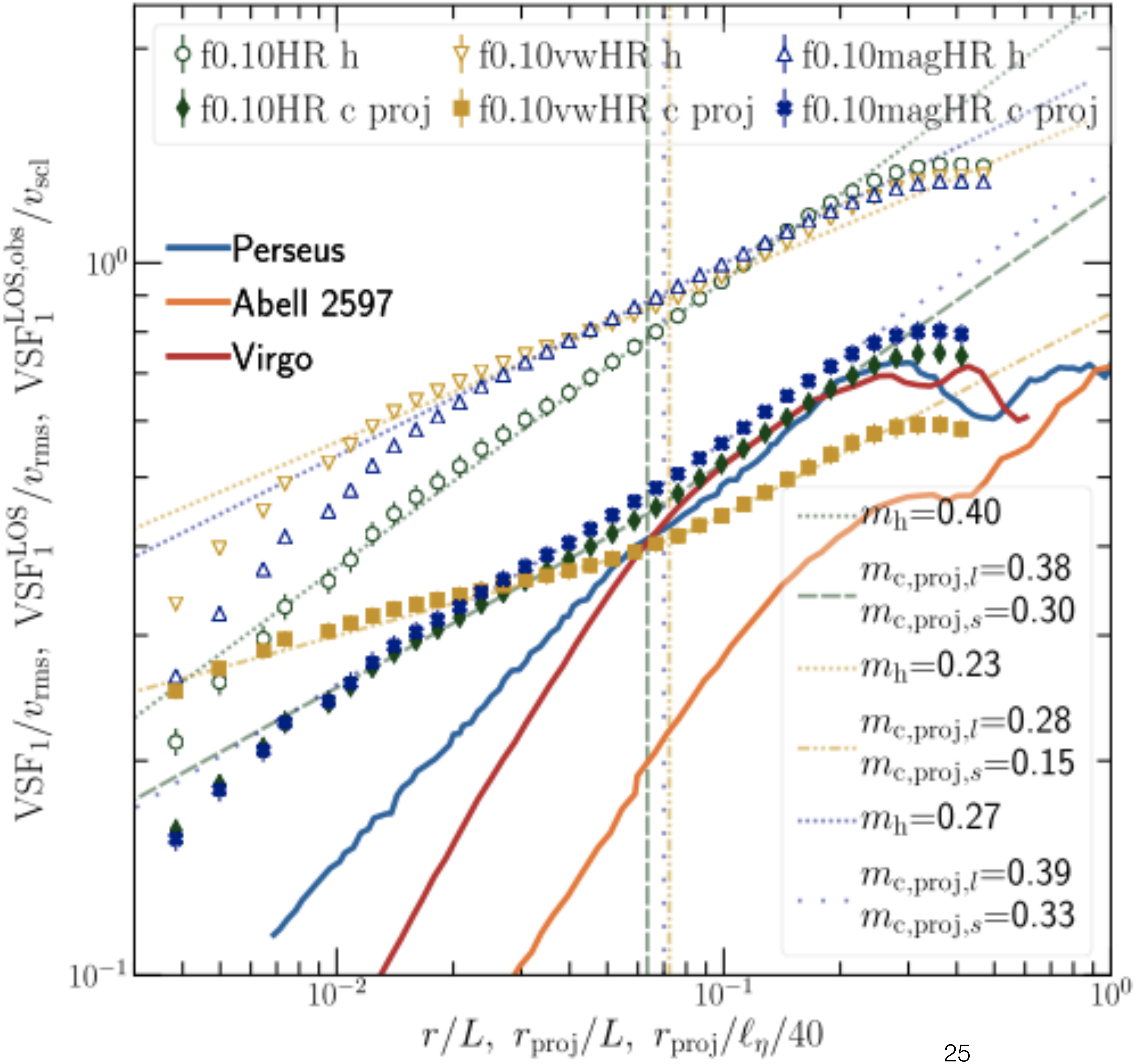
$\mathscr{M}_{\text{hot}}$~0.9

$\mathscr{M}_{\text{hot}}$~0.6

Magnetic field - cold phase δv^2 much steeper than hot, since cold gas is magnetically dominated



Projected VSFs for cold gas & observations



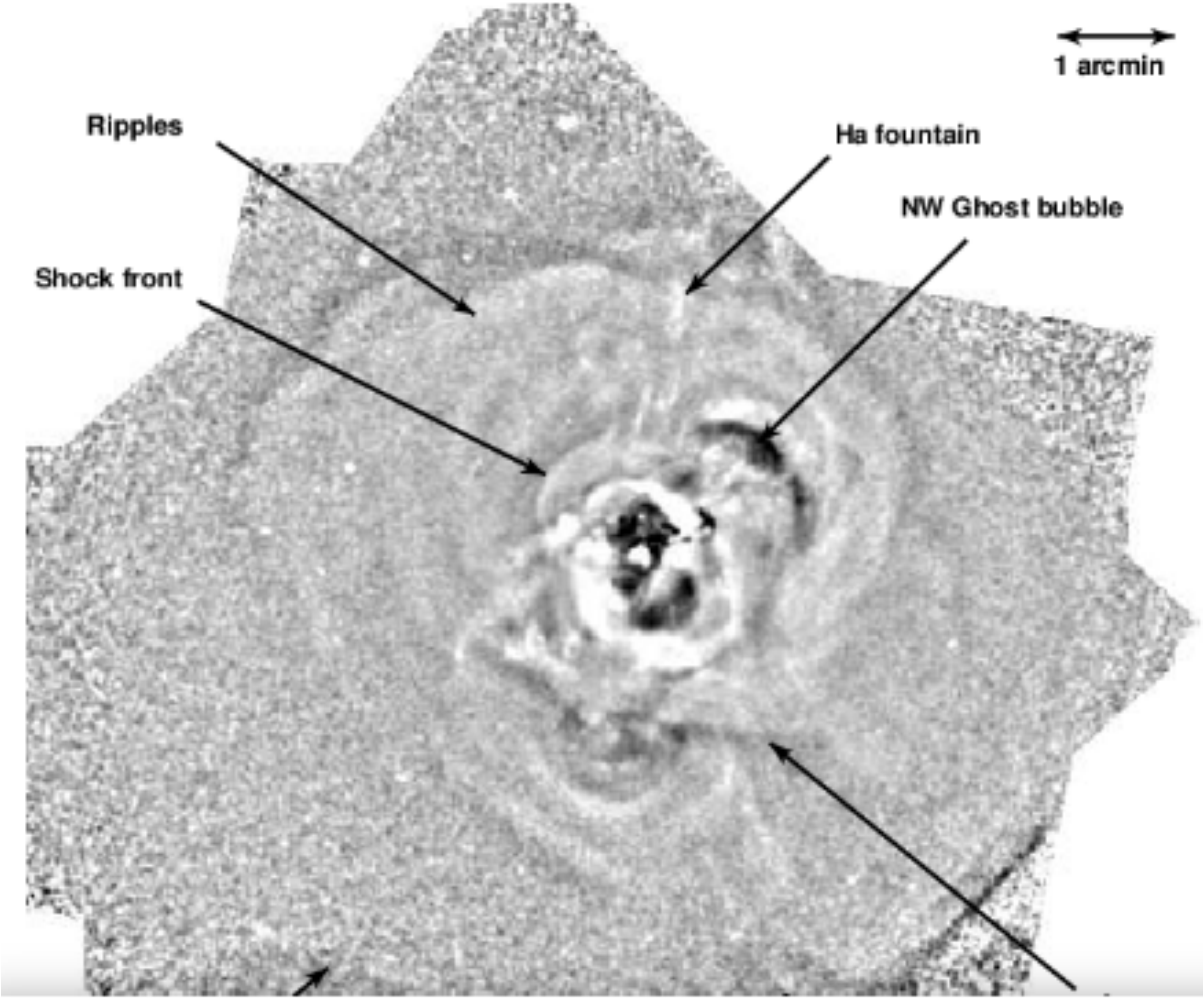
cold gas projected VSF shallow (esp. at small scales) because not many cold clouds along LOS no cancellation along LOS

projection takes power from large to small scales

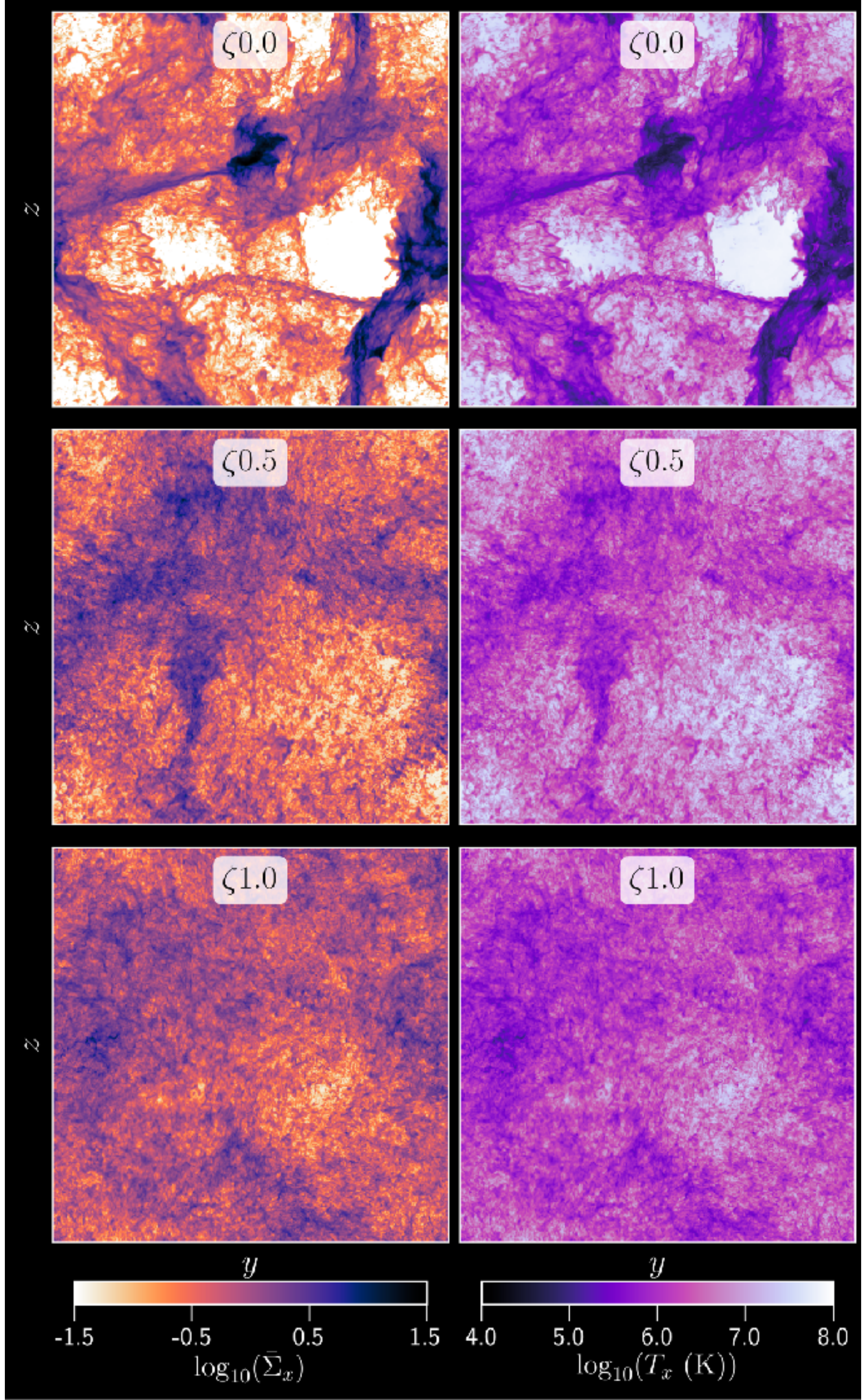
observed cold gas VSF steeper at small scales; can it be due to insufficient resolution or plasma viscosity?

ICM can have compressive driving

shocks and cavities/bubbles driven by jets



Effects of driving



Compressive
$\nabla \times \mathbf{a} = 0$

Natural
(Comp+Sol)/2

Solenoidal
$\nabla \cdot \mathbf{a} = 0$

Density

Temperature

with compressive driving, cold phase filamentary &
larger spread in density/temperature

Conclusions

- ICM turbulence is **more rich** than K41's isotropic, homogeneous, single phase turbulence
- must be borne in mind to interpret multiwavelength observations
- density fluctuations (and hence surface brightness) depend not only on Mach number but also on **stratification, cooling**
- projected VSF can be shallower or steeper than 3D VSFs; careful in interpretation
- compressive driving creates denser filaments; TI+turbulence with gravity (stay tuned!)

Thank You!