An Effective Collision Operator for Heat-Flux-Generated Whistler Turbulence

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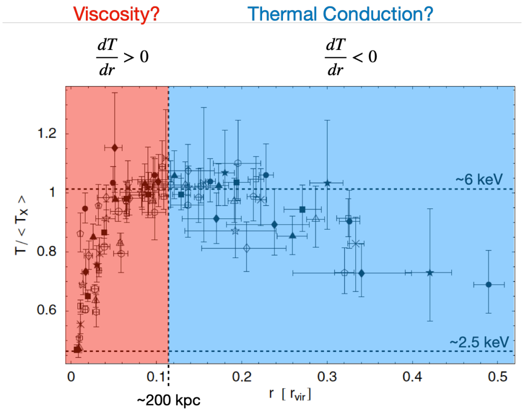
Microphysics Affects Large-Scale Structure of the ICM



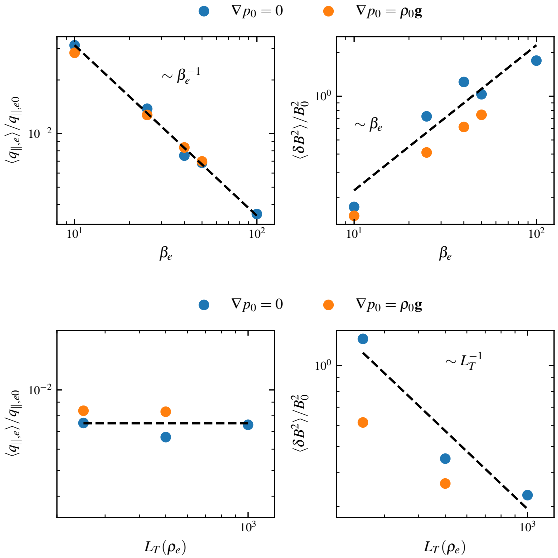
Piffaretti et al. 2005

Plasma is

- high-β
- magnetized ($\rho_i / L_T \ll 1$)
- weakly collisional

1. Pressure-Anisotropy-Driven Mirror and Fire Hose Instabilities ($k\rho_i \sim 1$)
   - Reduced viscosity (Kunz et al. 2014)
   - Collisionless sound wave propagation (Kunz et al. 2020)
2. Heat-Flux-Driven Whistler Instability ($k\rho_e \sim 1$)
   - Affects large-scale instabilities (MTI, HBI)

Simulation Results



$$\frac{q_{\parallel,e}}{q_{\parallel,e0}} \sim \beta_e^{-1} \quad \text{and} \quad \frac{\delta B^2}{B_0^2} \sim \frac{v_{\mathrm{th},e}\beta_e}{|\Omega_e| L_T}$$

$$q_{\parallel} \sim -n \frac{v_{\mathrm{th},e}^2}{\nu_{\mathrm{w}}} \nabla T$$

$$\Rightarrow \nu_{\mathrm{w}} \sim \frac{\beta_e v_{\mathrm{th},e}}{L_T} \sim \frac{\delta B^2}{B_0^2} |\Omega_e|.$$

Agrees with previous work:
Komarov et al. 2018,
Roberg-Clark et al. 2018

Effective Collision Operator

We use the Fokker-Planck method:

$$\frac{\partial f(t,\boldsymbol{x},\boldsymbol{v})}{\partial t} = -\frac{\partial}{\partial \boldsymbol{v}} \cdot \boldsymbol{A}(t,\boldsymbol{x},\boldsymbol{v}) f(t,\boldsymbol{x},\boldsymbol{v}) + \frac{\partial^2}{\partial \boldsymbol{v} \partial \boldsymbol{v}} : \boldsymbol{B}(t,\boldsymbol{x},\boldsymbol{v}) f(t,\boldsymbol{x},\boldsymbol{v})$$

$$\boldsymbol{A}(t,\boldsymbol{x},\boldsymbol{v}) \doteq \lim_{\Delta t \to \text{``0''}} \frac{\langle \Delta \boldsymbol{v}(t,\boldsymbol{x},\boldsymbol{v},\Delta t) \rangle}{\Delta t}$$

$$\boldsymbol{B}(t,\boldsymbol{x},\boldsymbol{v}) \doteq \frac{1}{2} \lim_{\Delta t \to \text{``0''}} \frac{\langle \Delta \boldsymbol{v}(t,\boldsymbol{x},\boldsymbol{v},\Delta t) \Delta \boldsymbol{v}(t,\boldsymbol{x},\boldsymbol{v},\Delta t) \rangle}{\Delta t}$$

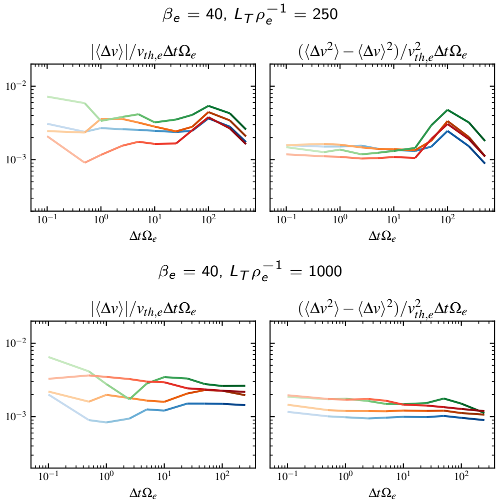
where

$$\Delta \boldsymbol{v}(t,\boldsymbol{x},\boldsymbol{v},\Delta t) = \boldsymbol{v}(t+\Delta t,\boldsymbol{x}) - \boldsymbol{v}(t,\boldsymbol{x})$$

$$\langle \ldots \rangle \doteq \int \mathrm{d}\boldsymbol{v} \left(\ \ldots \ \right) f(t,\boldsymbol{x},\boldsymbol{v}).$$

$$\boxed{\tau_{\mathrm{ac}} \ll \Delta t \ll \nu^{-1}}$$

Effective Collision Operator



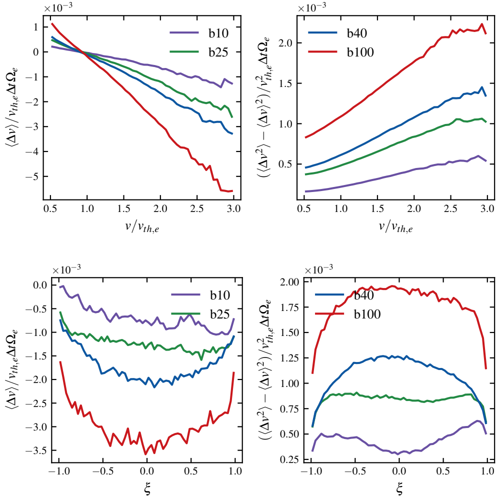
Take $\Delta t \Omega_e = 10$:

1. Parameter ordering is good:

   $$\tau_{ac}\Omega_e \sim 1 \ll \Delta t \Omega_e \sim 10 \ll \nu^{-1}\Omega_e \sim 10^3$$

2. $\langle \Delta v^{(1,2)} \rangle$ scales as expected in Δt

Effective Collision Operator



$$A(v,\xi) = -\nu_v(\xi, \beta_e, L_T)\left(v - v_{\mathrm{th},e0}\right)$$

$$B(v,\xi) = \frac{D_v(\xi, \beta_e, L_T)}{2} v$$

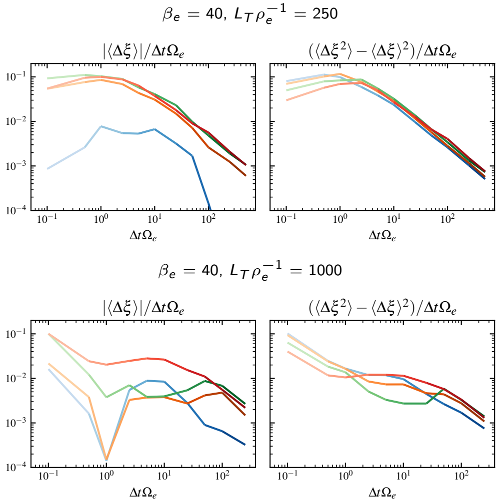
$$\nu_v(\xi, \beta_e, L_T) = \nu_{v,0}\beta_e^{.63} f_{\nu_v}(\xi)$$

$$D_v(\xi, \beta_e, L_T) = \sigma_{v,0}^2 \beta_e^{.52} f_{D_v}(\xi)$$

$f_{\nu_v}(\xi)$ and $f_{\sigma_v}(\xi)$ are nontrivial functions of $\delta B / B_0$

Does not explain $\nu_w \sim \beta_e v_{\mathrm{th},e} / L_T$

Effective Collision Operator



Pitch-angle scattering dominates velocity collision rate

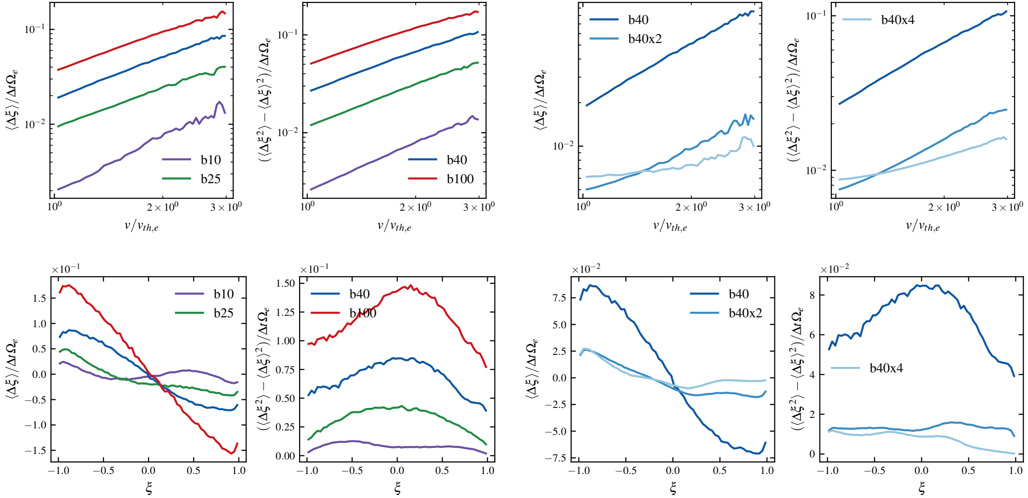
1. Parameter ordering not so good:

$$\tau_{ac}\Omega_e \sim 1 \leq \nu^{-1}\Omega_e \sim 1 - 50$$

2. $\langle \Delta\xi^{(1,2)} \rangle$ *does not* scales as expected in Δt for $\nu \Delta t \ll 1$

Effective Collision Operator

If we naïvely consider this a Fokker-Planck operator:



Conclusions

1. We expanded on and confirmed the scaling of previous numerical studies
2. Fokker-Planck method is successful in Δv, but not $\Delta\xi$
   - We lack sufficient scale separation ($\tau_{\mathrm{ac}} \sim \nu^{-1}$)
   - There is some non-diffusive physics we don't understand
3. Be careful grabbing a "collision frequency" from PIC simulations
4. Shout out to twiddle math

Future Work:

- There is an ion heat flux version of the whistler instability
- We are currently investigating this using Pegasus++ hybrid-kinetic code
- Scale separation should be much higher