Leibniz-Institut für Astrophysik Potsdam

AIP

Cosmological zoom simulations of merging galaxy clusters

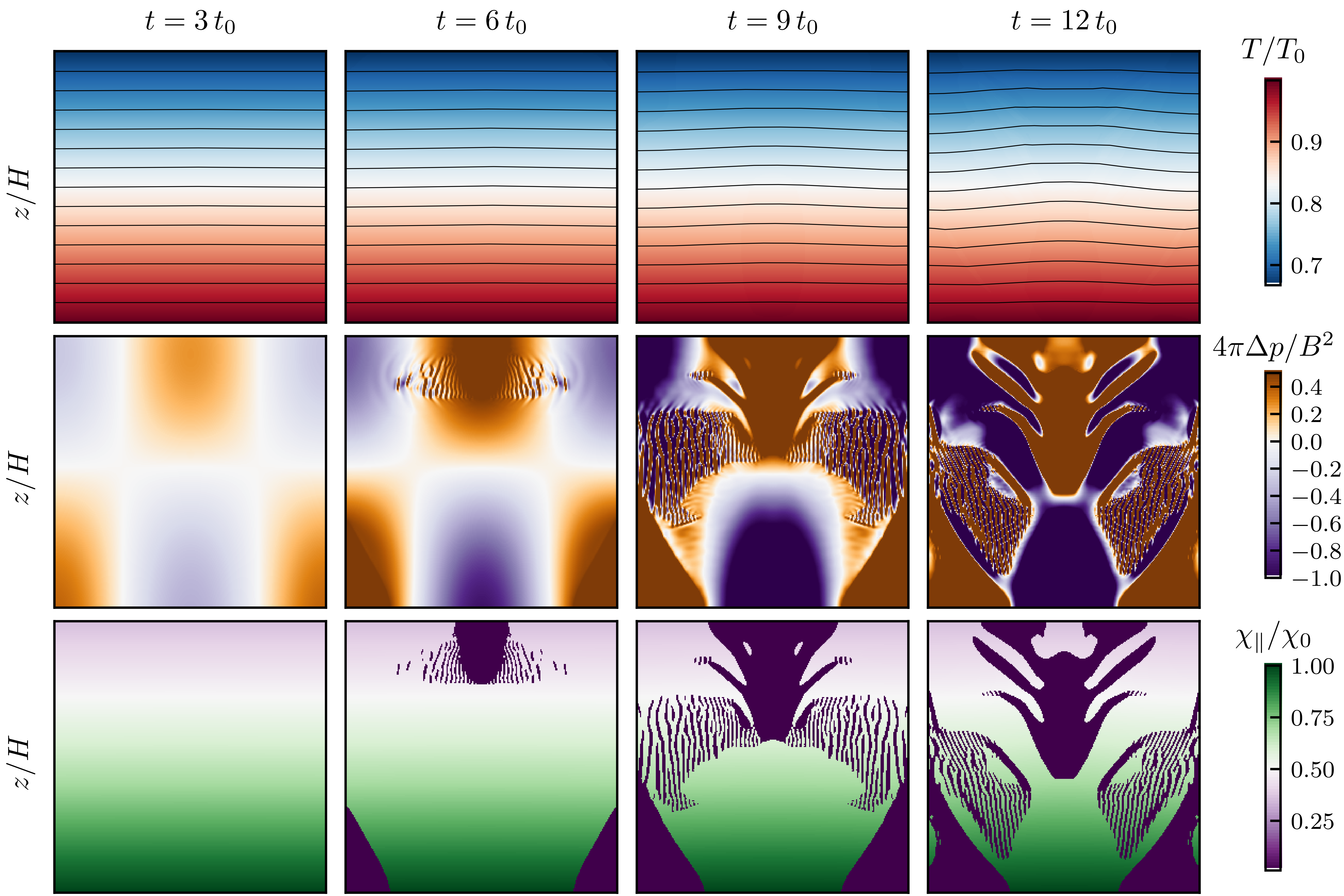
Thomas Berlok

6th ICM Theory and Computation Workshop, 19/8/22

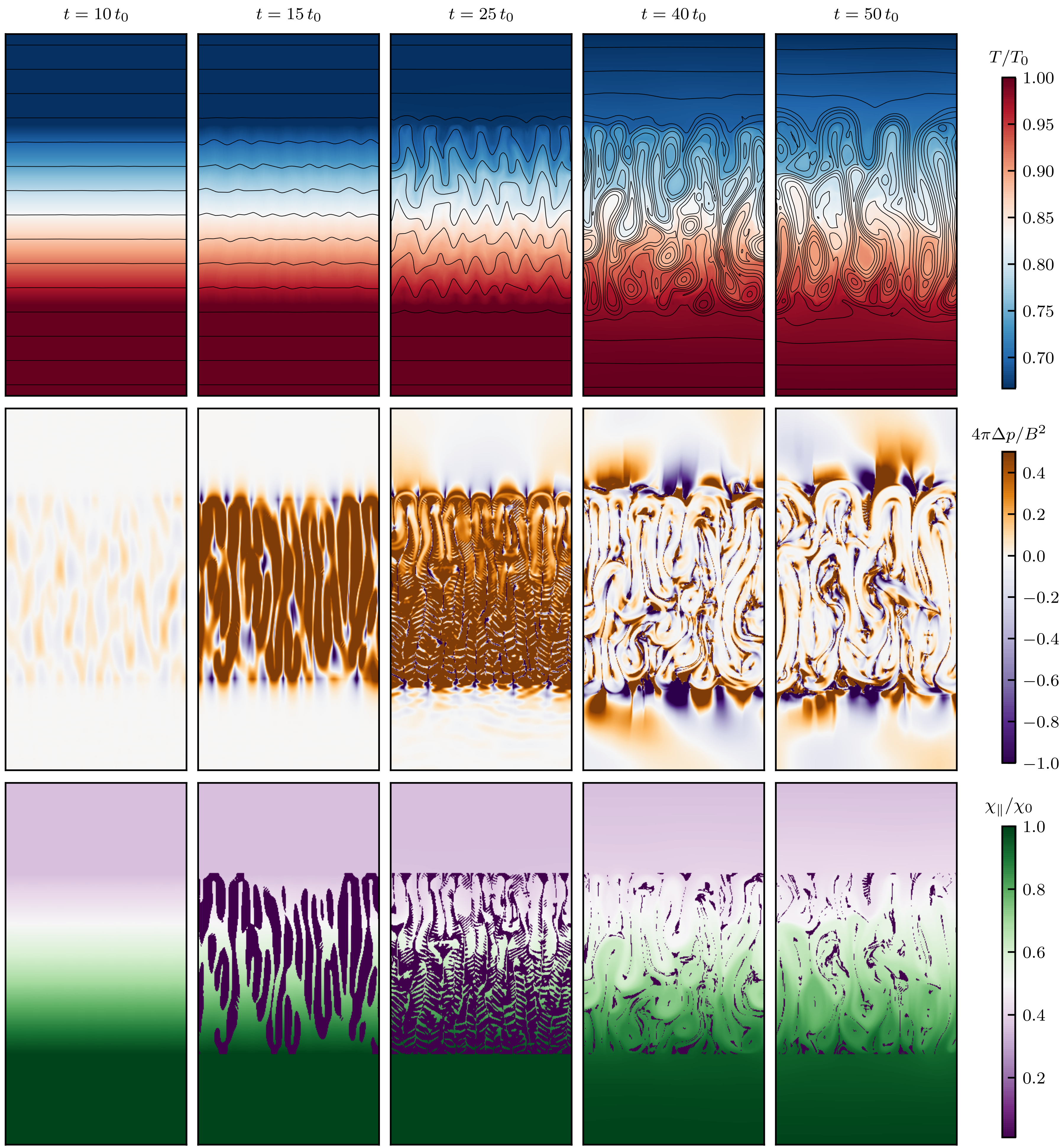
Part I

Magneto-thermal instability with suppressed heat conductivity in mirror-unstable regions

Magneto-thermal instability

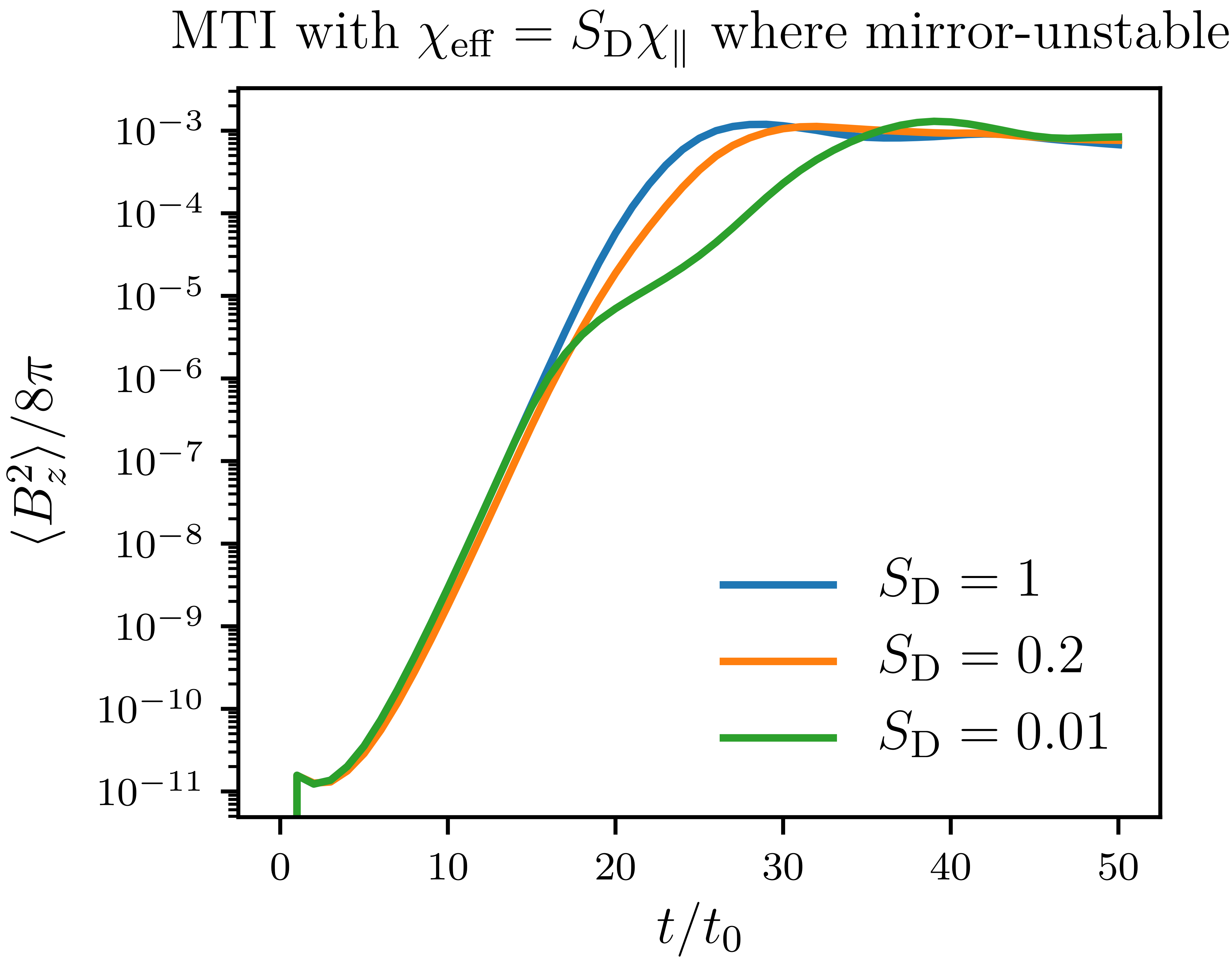


Berlok, Quataert, Pessah and Pfrommer, MNRAS, 2021



$t = 10\,t_0$
$t = 15\,t_0$
$t = 25\,t_0$
$t = 40\,t_0$
$t = 50\,t_0$
T/T_0
1.00
0.95
0.90
0.85
0.80
0.75
0.70
$4\pi\Delta p/B^2$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
$\chi_\parallel/\chi_0$
1.0
0.8
0.6
0.4
0.2

Magneto-thermal instability



Berlok, Quataert, Pessah and Pfrommer, MNRAS, 2021

Part II

Hydromagnetic waves in an expanding universe – cosmological MHD code tests using analytic solutions

New tests of comoving hydrodynamics/MHD

- Berlok 2022, MNRAS
- Python implementation of analytic solutions available: https://github.com/tberlok/comoving_mhd_waves

Ideal MHD equations

$$\frac{d \ln \rho}{dt} = -\nabla_{\boldsymbol{r}} \cdot \boldsymbol{v} \ ,$$

$$\rho \frac{d\boldsymbol{v}}{dt} = -\nabla_{\boldsymbol{r}} p - \nabla_{\boldsymbol{r}} \cdot \left(\frac{B^2}{2} \mathbb{1} - \boldsymbol{B}\boldsymbol{B} \right) - \rho \nabla_{\boldsymbol{r}} \Phi \ ,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla_{\boldsymbol{r}} \times (\boldsymbol{v} \times \boldsymbol{B}) \ ,$$

$$\frac{p}{\gamma - 1} \frac{d \ln(p \rho^{-\gamma})}{dt} = 0 \ ,$$

Comoving MHD equations

Substitutions

$$\boldsymbol{r} = a\boldsymbol{x},\ \rho_{\mathrm{c}} = \rho a^3,\ \boldsymbol{u} = a\dot{\boldsymbol{x}},$$

$$\boldsymbol{B}_{\mathrm{c}} = \boldsymbol{B}a^2,\ \varepsilon_{\mathrm{c}} = \varepsilon a^3$$

$$\varepsilon = p/(\gamma - 1)$$

$$\frac{d \ln \rho_{\mathrm{c}}}{dt} = -\frac{1}{a}\nabla_{\boldsymbol{x}}\cdot\boldsymbol{u}\ ,$$

$$a\rho_{\mathrm{c}}\frac{d\boldsymbol{u}}{dt} = -\nabla_{\boldsymbol{x}} p_{\mathrm{c}} - \frac{1}{a}\nabla_{\boldsymbol{x}}\cdot\left(\frac{B_{\mathrm{c}}^2}{2}\mathbf{1} - \boldsymbol{B}_{\mathrm{c}}\boldsymbol{B}_{\mathrm{c}}\right) - \rho_{\mathrm{c}}\nabla_{\boldsymbol{x}}\delta\Phi - \rho_{\mathrm{c}}\dot{a}\boldsymbol{u}\ ,$$

$$\frac{\partial \boldsymbol{B}_{\mathrm{c}}}{\partial t} = \frac{1}{a}\nabla_{\boldsymbol{x}} \times (\boldsymbol{u} \times \boldsymbol{B}_{\mathrm{c}})\ ,$$

$$\frac{d\varepsilon_{\mathrm{c}}}{dt} = -3\frac{\dot{a}}{a}(\gamma - 1)\varepsilon_{\mathrm{c}} - \gamma\varepsilon_{\mathrm{c}}\frac{1}{a}\nabla_{\boldsymbol{x}}\cdot\boldsymbol{u}\ ,$$

$$\nabla_{\boldsymbol{x}}^2\delta\Phi = \frac{4\pi G}{a}(\rho_{\mathrm{tot,c}} - \bar{\rho}_{\mathrm{tot,c}})$$

Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\bar{\rho}_{\mathrm{tot}}$$

$$\bar{\rho}_{\mathrm{tot}} = \left(\frac{\Omega_{\mathrm{r},0}}{a^4} + \frac{\Omega_{\mathrm{m},0}}{a^3} + \Omega_{\Lambda,0}\right)\rho_{\mathrm{crit},0}$$

$$\rho_{\mathrm{crit},0} = 3H_0^2/(8\pi G)$$

Linear theory (Berlok, MNRAS, 2022)

$$c_{\mathrm{s}} \equiv \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma p_{\mathrm{c},0}}{\rho_{\mathrm{c}}} a^{-3(\gamma-1)/2}} \ ,$$

$$v_{\mathrm{A}} \equiv \frac{B}{\sqrt{\rho}} = \frac{B_{\mathrm{c}}}{\sqrt{\rho_{\mathrm{c}}}} a^{-1/2},$$

$$v_{\mathrm{g}} \equiv \frac{\sqrt{4\pi G\rho}}{k'} = \frac{\sqrt{4\pi G\rho_{\mathrm{c}}}}{k} a^{-1/2}$$

$$k' = k/a$$

Standard MHD result

Alfvén wave

$$\omega = k' v_{\mathrm{A}}$$

Magnetosonic wave

$$\omega = k' \left(c_{\mathrm{s}}^2 + v_{\mathrm{A}}^2 - v_{\mathrm{g}}^2\right)^{1/2}$$

Useful definitions

$$\mathcal{V}_{\mathrm{s}} \equiv \sqrt{\frac{\gamma p_{\mathrm{c},0}}{\rho_{\mathrm{c}}}} \ , \quad \mathcal{V}_{\mathrm{A}} \equiv \frac{B_{\mathrm{c}}}{\sqrt{\rho_{\mathrm{c}}}} \ , \quad \mathcal{V}_{\mathrm{g}} \equiv \frac{\sqrt{4\pi G\rho_{\mathrm{c}}}}{k} \ ,$$

$$\Omega_{\mathrm{s}} \equiv \frac{k\mathcal{V}_{\mathrm{s}}}{H_0} \ , \quad \Omega_{\mathrm{A}} \equiv \frac{k\mathcal{V}_{\mathrm{A}}}{H_0} \ , \quad \Omega_{\mathrm{g}} \equiv \frac{k\mathcal{V}_{\mathrm{g}}}{H_0} \ .$$

Comoving Alfvén wave

$$\frac{\partial}{\partial t}\frac{\delta B_{\rm c}}{B_{\rm c}} = \frac{{\rm i}k}{a}\delta u$$

$$\frac{\partial(a\delta u)}{\partial t} = {\rm i}k\frac{B_{\rm c}^2}{a\rho_{\rm c}}\frac{\delta B_{\rm c}}{B_{\rm c}}$$

Einstein-de-Sitter → $\dot{a} = H_0/\sqrt{a}$

$$\frac{\partial}{\partial a}\frac{\delta B_{\rm c}}{B_{\rm c}} = \frac{{\rm i}k}{H_0 a^{1/2}}\delta u$$

$$\frac{\partial(a\delta u)}{\partial a} = \frac{{\rm i}k\mathcal{V}_{\rm A}^2}{H_0 a^{1/2}}\frac{\delta B_{\rm c}}{B_{\rm c}}$$

Euler equation

$$\frac{\partial^2}{\partial a^2}\frac{\delta B_{\rm c}}{B_{\rm c}} + \frac{3}{2a}\frac{\partial}{\partial a}\frac{\delta B_{\rm c}}{B_{\rm c}} + \frac{\Omega_{\rm A}^2}{a^2}\frac{\delta B_{\rm c}}{B_{\rm c}} = 0$$

$$\frac{\delta B_{\rm c}}{B_{\rm c}} = a^{-1/4}\left(c_1{\rm e}^{{\rm i}\kappa\ln a} + c_2{\rm e}^{-{\rm i}\kappa\ln a}\right)$$

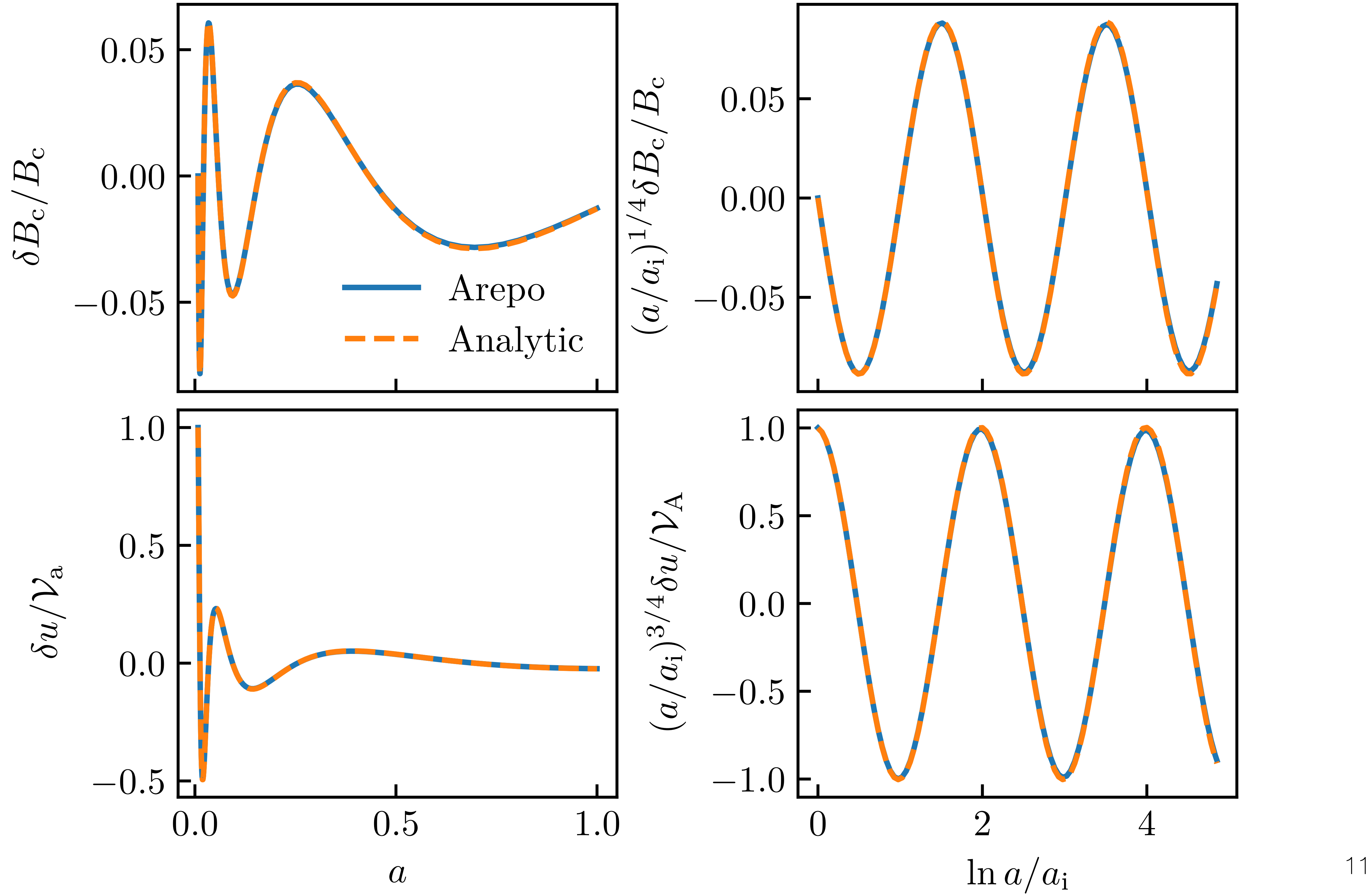
$$\kappa \equiv \sqrt{\Omega_{\rm A}^2 - \frac{1}{16}}$$

Comoving Alfvén wave

$$\psi = \kappa \ln(a/a_\mathrm{i})$$

$$\frac{\delta B_\mathrm{c}(x,a)}{B_\mathrm{c}} = -A_u \left(\frac{a}{a_\mathrm{i}}\right)^{-1/4} \frac{\sqrt{a_\mathrm{i}}\Omega_\mathrm{A}}{\kappa} \sin(\psi)\sin(kx),$$

$$\frac{\delta u(x,a)}{\mathcal{V}_\mathrm{A}} = A_u \left(\frac{a}{a_\mathrm{i}}\right)^{-3/4} \left(\cos(\psi) - \frac{\sin(\psi)}{4\kappa}\right)\cos(kx)$$



Magnetosonic wave

$$\frac{\partial}{\partial a}\frac{\delta\rho_{\mathrm{c}}}{\rho_{\mathrm{c}}} = -\frac{\mathrm{i}k}{H_0\sqrt{a}}\delta u \ ,$$

$$\frac{\partial(a\delta u)}{\partial a} = -\frac{\mathrm{i}k\sqrt{a}}{H_0}\left(\frac{\mathcal{V}_{\mathrm{s}}^2}{a^{3(\gamma-1)}} + \frac{\mathcal{V}_{\mathrm{A}}^2 - \mathcal{V}_{\mathrm{g}}^2}{a}\right)\frac{\delta\rho_{\mathrm{c}}}{\rho_{\mathrm{c}}}$$

Gravitational instability

$\gamma = 1$ Thermal pressure term does not decay

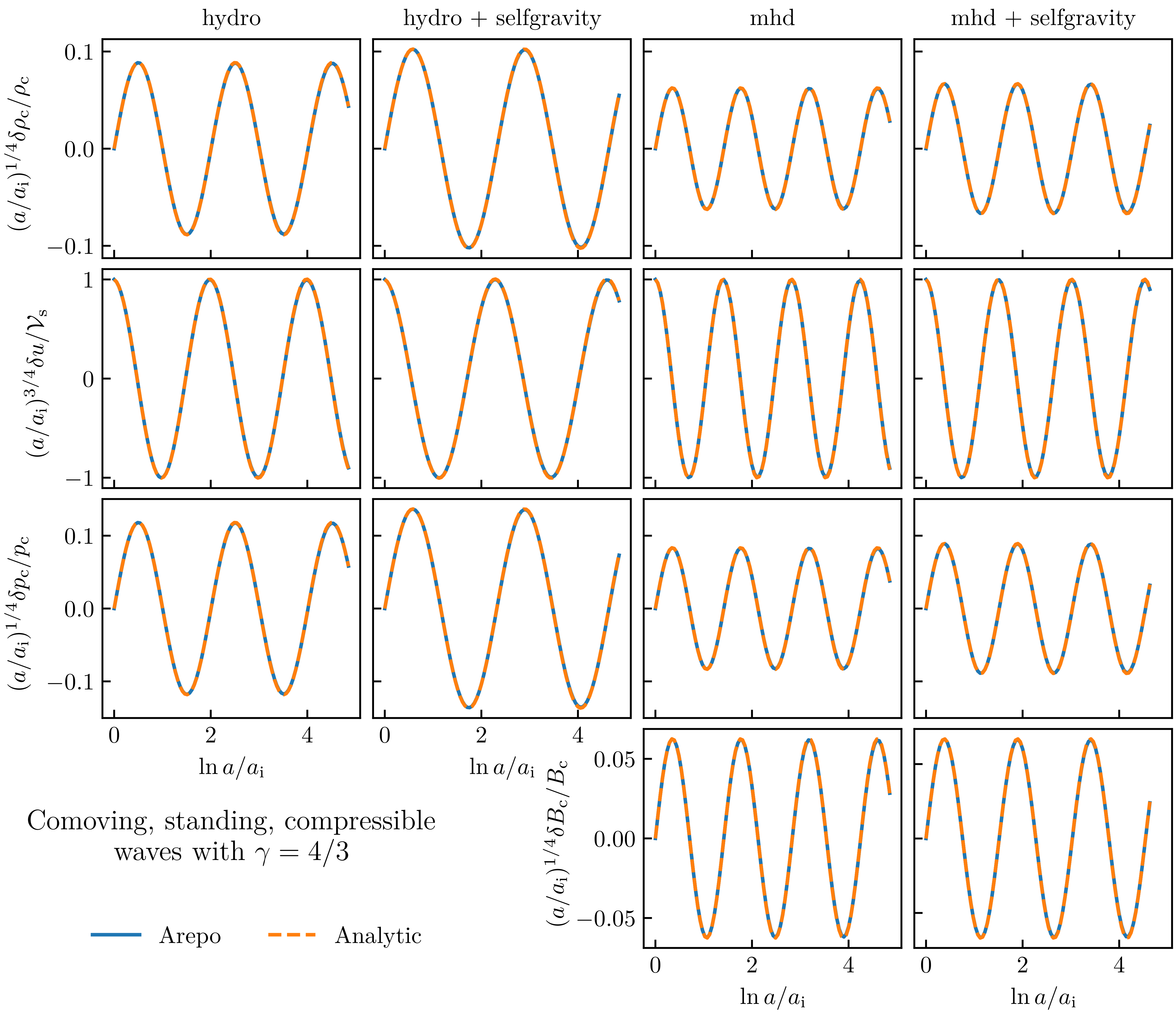
$\gamma = 4/3$ Thermal pressure term decays at same rate as other terms

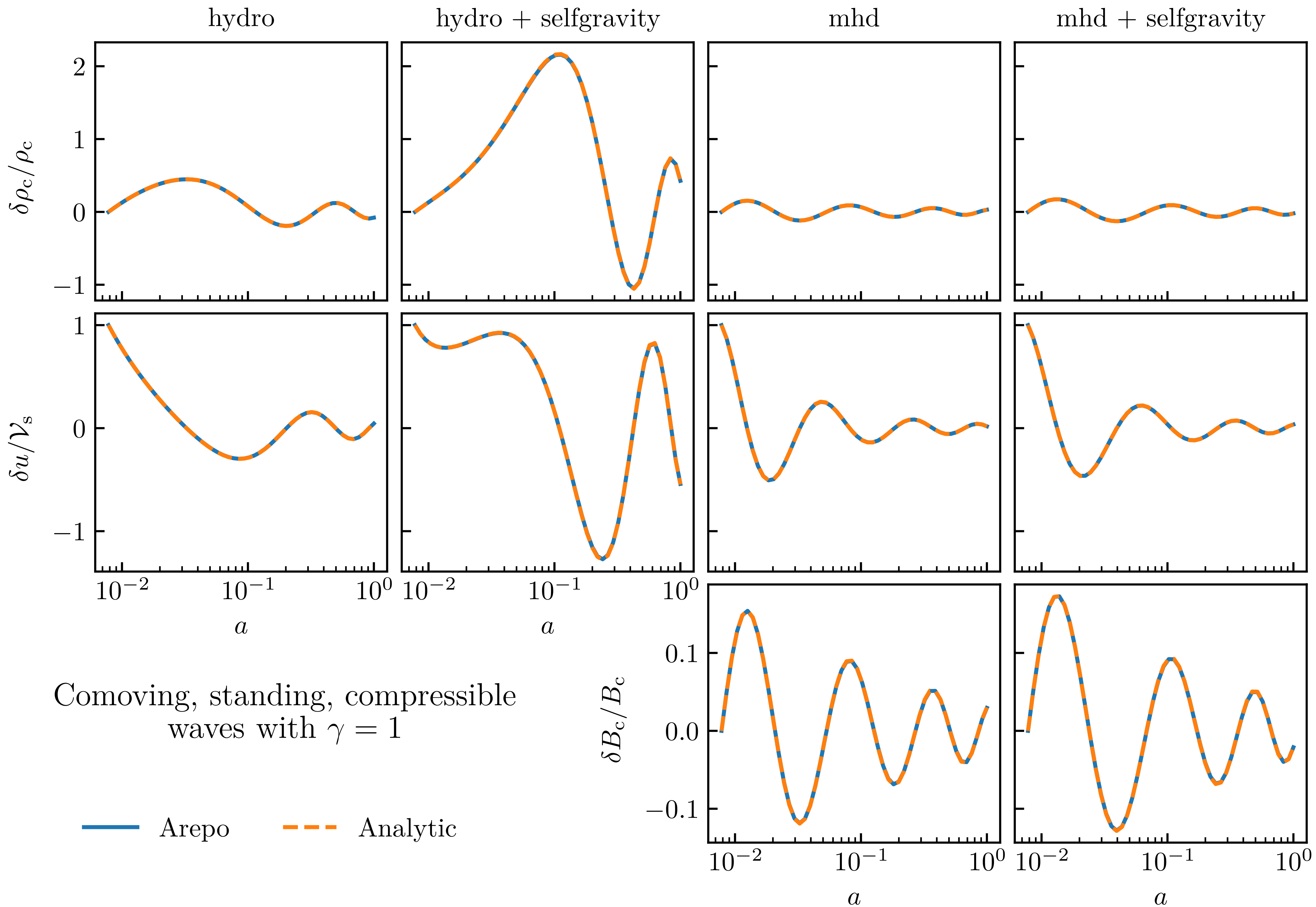
$\gamma = 5/3$ Thermal pressure term decays faster than other terms

Differential equation

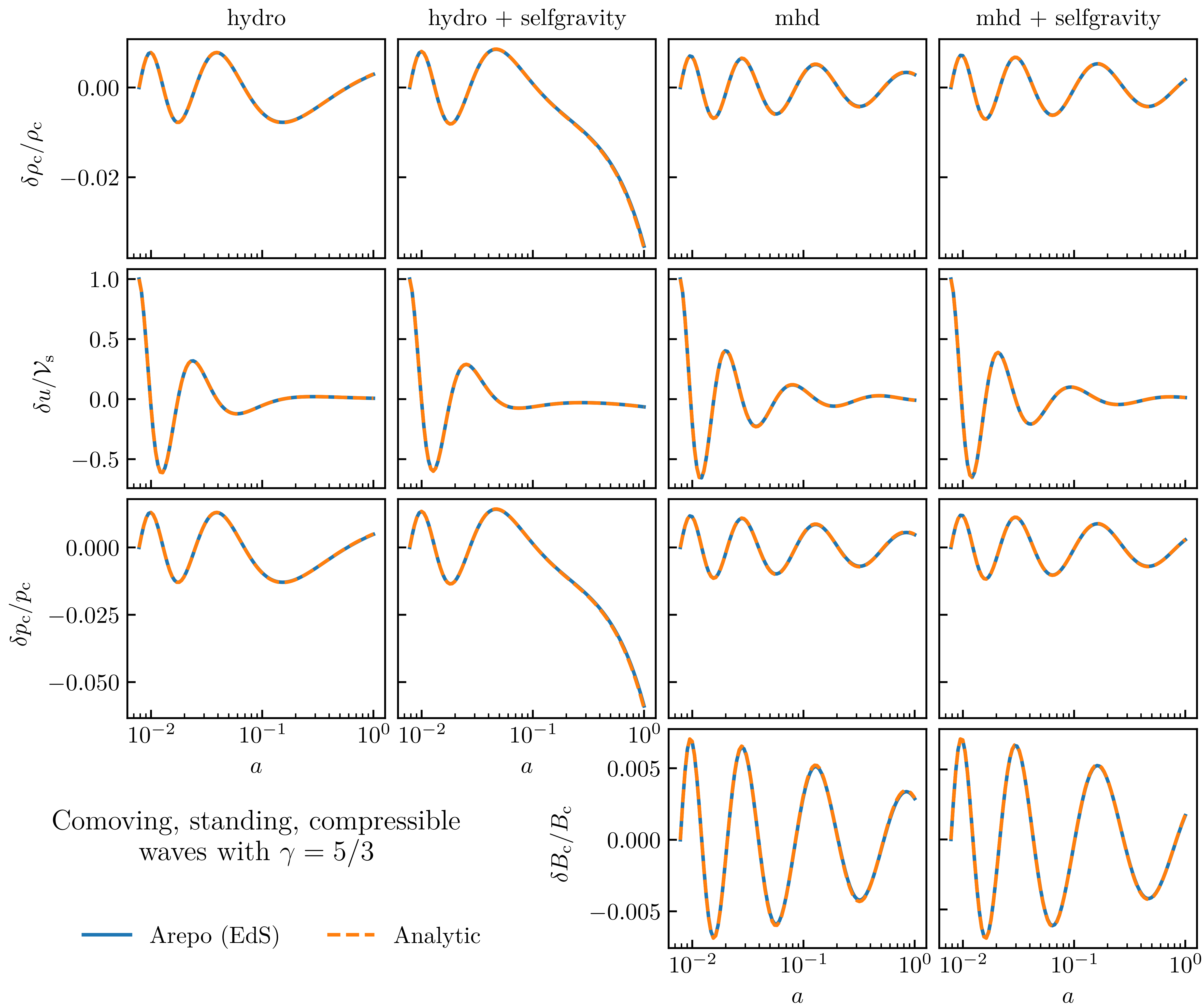
$$\frac{\partial^2}{\partial a^2}\frac{\delta\rho_{\mathrm{c}}}{\rho_{\mathrm{c}}} + \frac{3}{2a}\frac{\partial}{\partial a}\frac{\delta\rho_{\mathrm{c}}}{\rho_{\mathrm{c}}} + \left(\frac{\Omega_{\mathrm{s}}^2}{a^{3\gamma-2}} + \frac{\Omega_{\mathrm{A}}^2 - \Omega_{\mathrm{g}}^2}{a^2}\right)\frac{\delta\rho_{\mathrm{c}}}{\rho_{\mathrm{c}}} = 0$$

$\gamma = 4/3$ is an Euler ODE, in general a transformed Bessel equation!


hydro
hydro + selfgravity
mhd
mhd + selfgravity
$(a/a_\mathrm{i})^{1/4}\delta\rho_\mathrm{c}/\rho_\mathrm{c}$
$(a/a_\mathrm{i})^{3/4}\delta u/\mathcal{V}_\mathrm{s}$
$(a/a_\mathrm{i})^{1/4}\delta p_\mathrm{c}/p_\mathrm{c}$
$(a/a_\mathrm{i})^{1/4}\delta B_\mathrm{c}/B_\mathrm{c}$
$\ln a/a_\mathrm{i}$
Comoving, standing, compressible waves with $\gamma = 4/3$
Arepo
Analytic



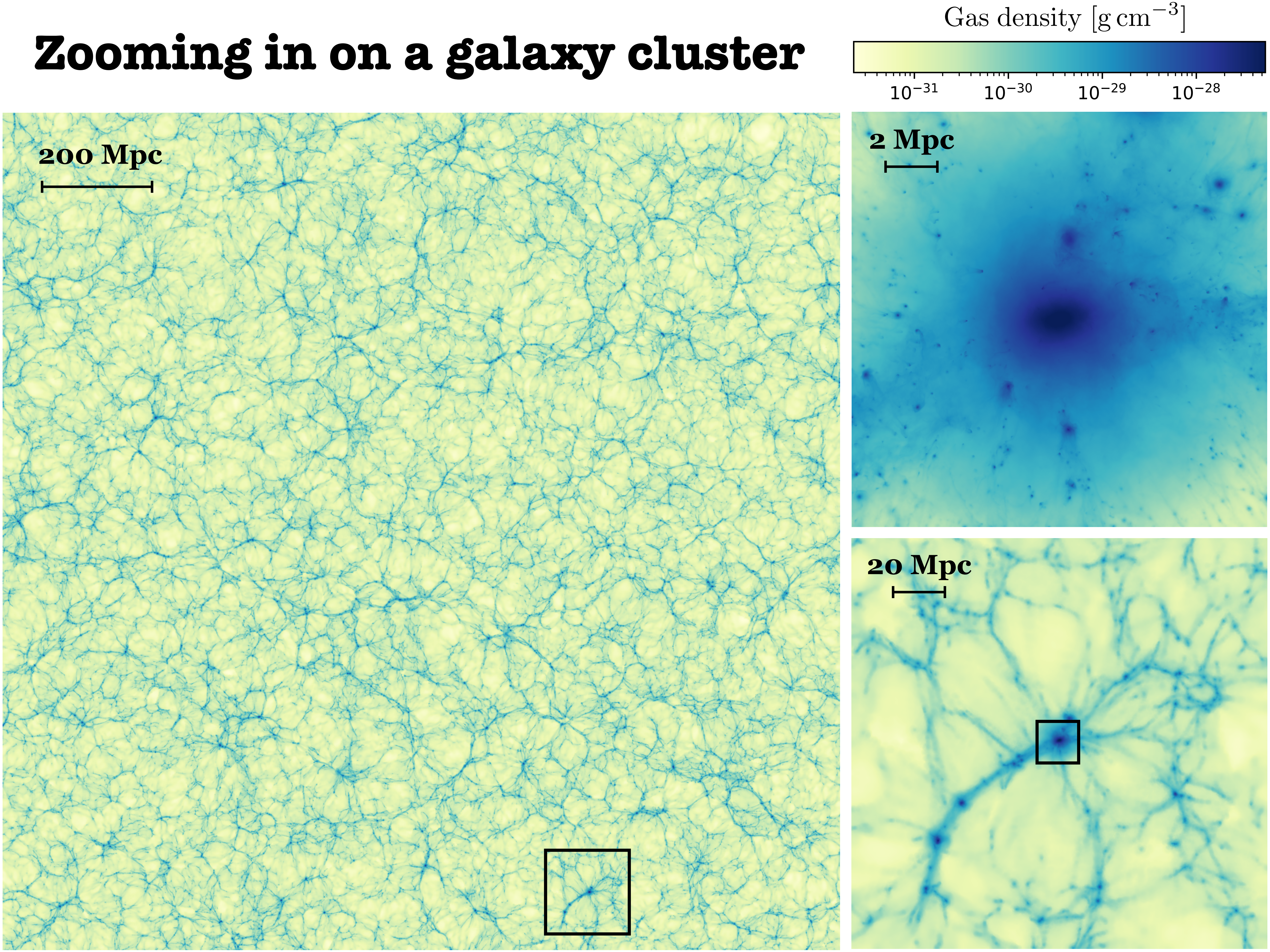
Comoving, standing, compressible waves with $\gamma = 1$


hydro
hydro + selfgravity
mhd
mhd + selfgravity
$\delta\rho_c/\rho_c$
0.00
−0.02
$\delta u/\mathcal{V}_s$
1.0
0.5
0.0
−0.5
$\delta p_c/p_c$
0.000
−0.025
−0.050
$\delta B_c/B_c$
0.005
0.000
−0.005
10^{-2}
10^{-1}
10^{0}
a
Comoving, standing, compressible waves with $\gamma = 5/3$
Arepo (EdS)
Analytic

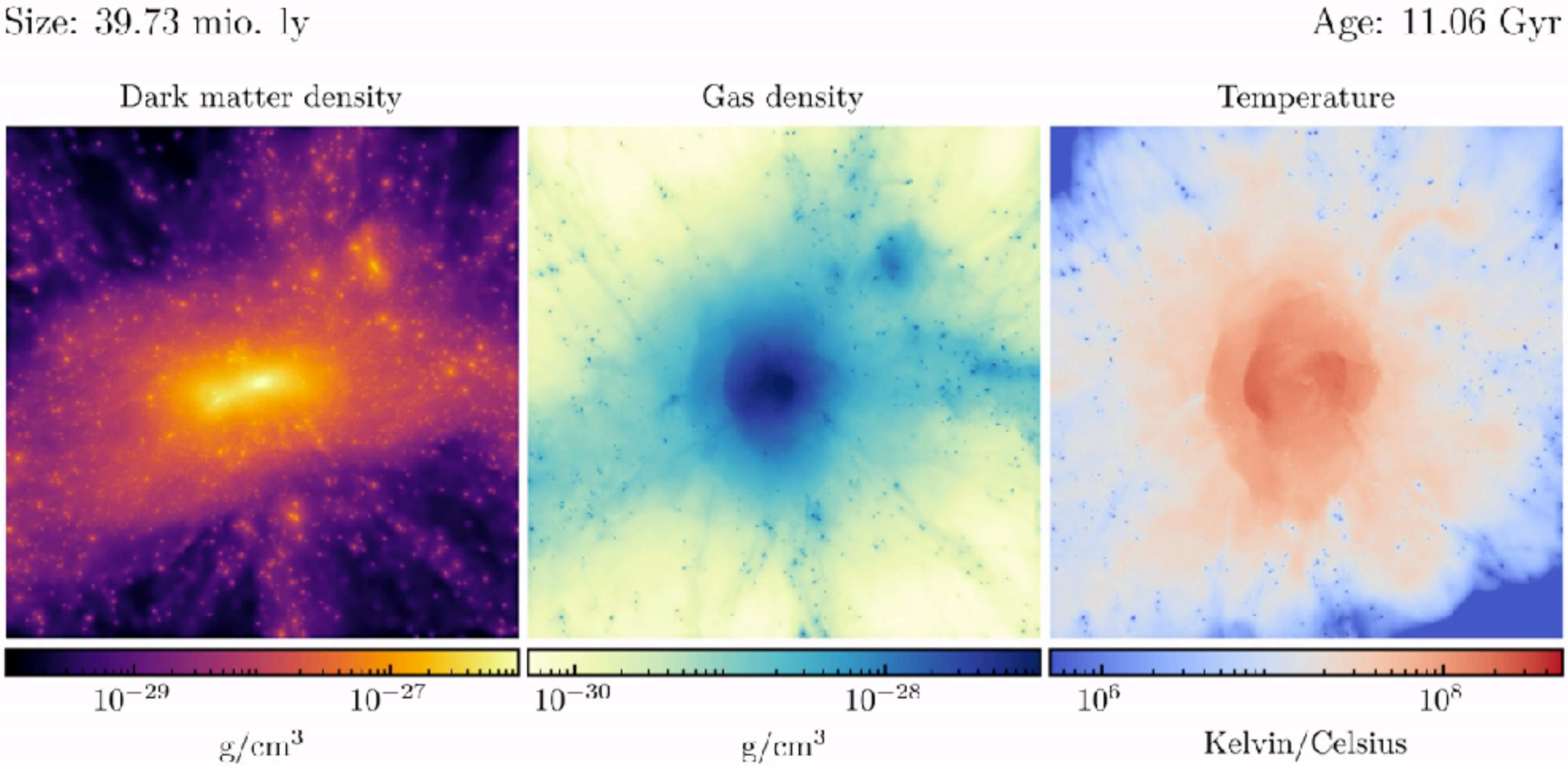
Part III

Zoom simulations of merging galaxy clusters

Zooming in on a galaxy cluster

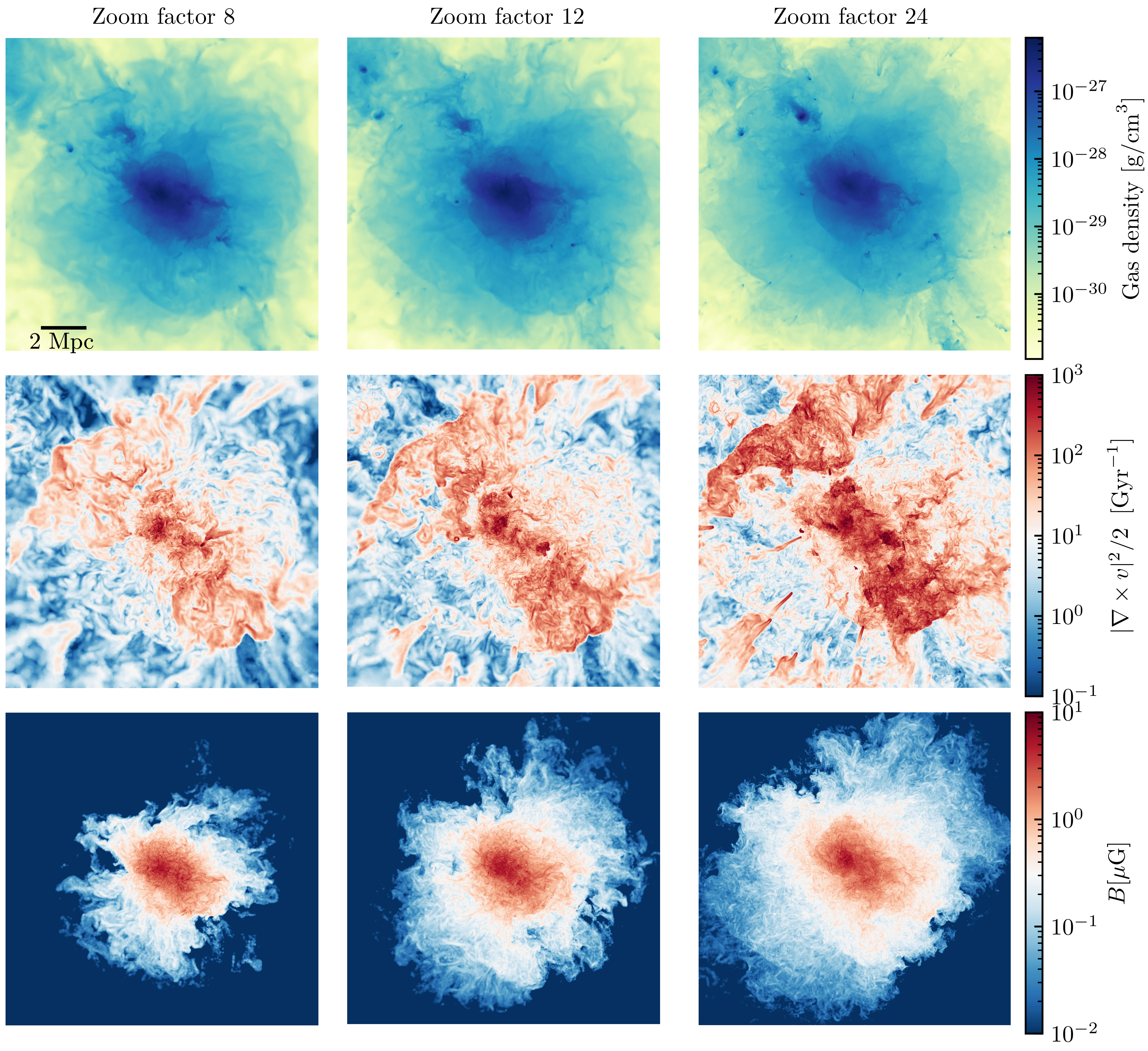


Major merger including galaxy formation



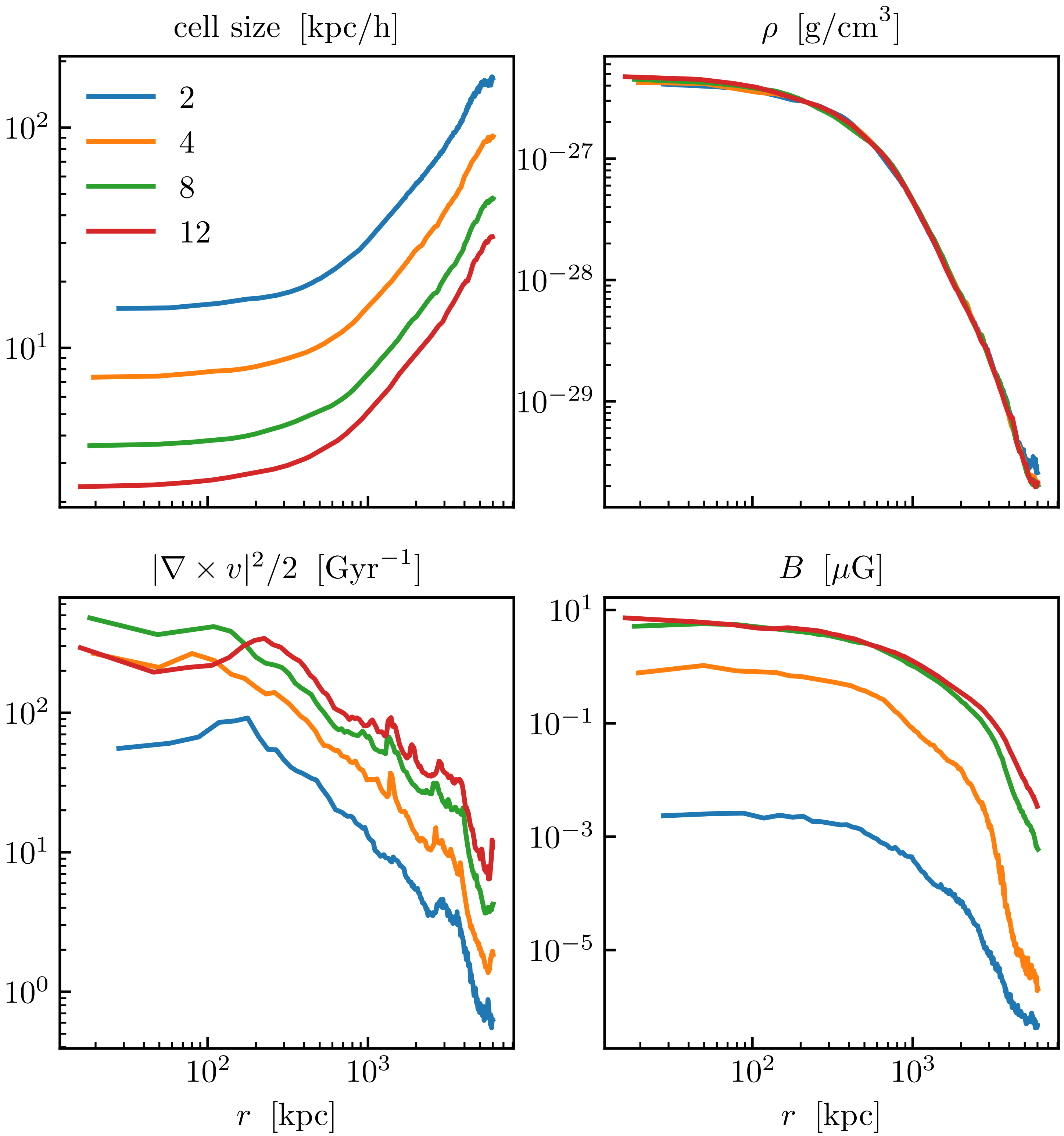
11 mio cpu-h on SuperMUC-NG in Germany (PI: T. Berlok)

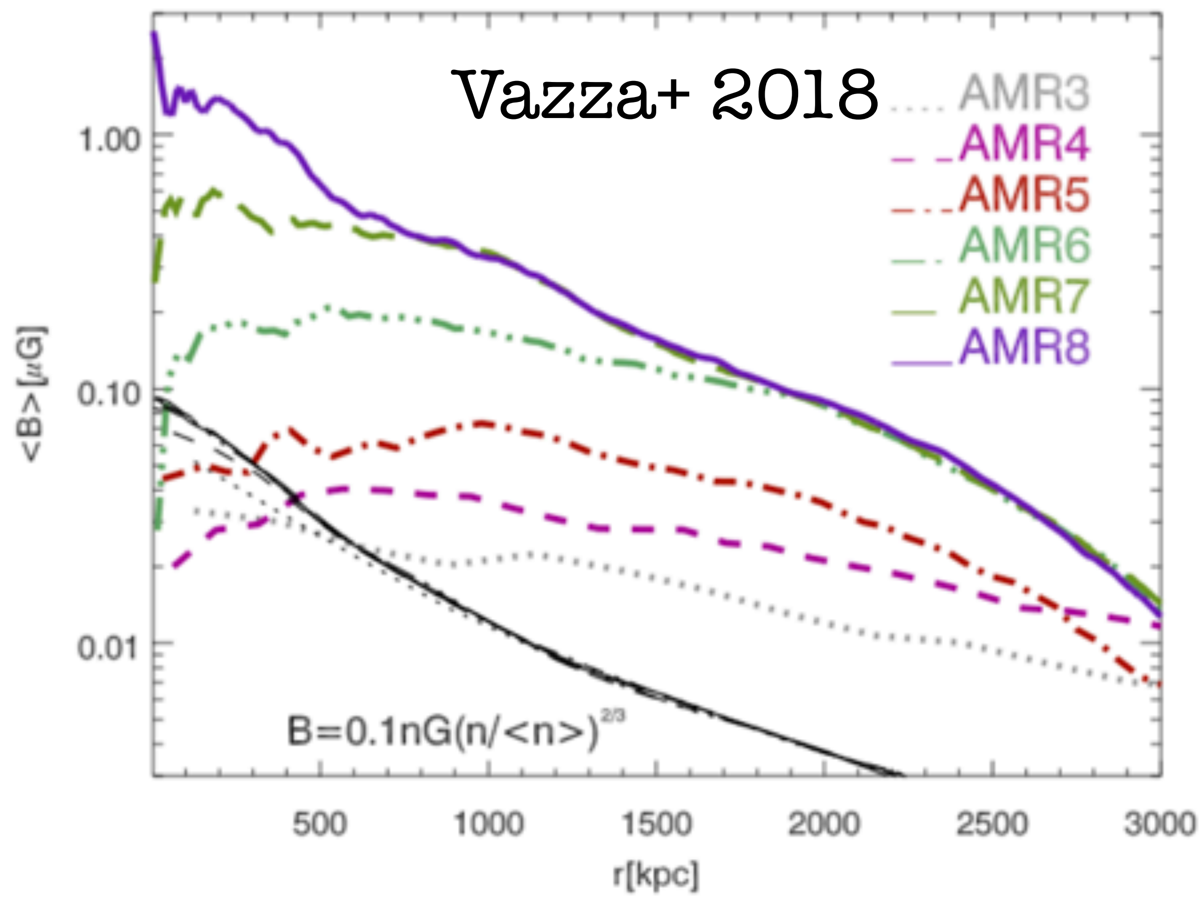
In collaboration with:
Joseph Whittingham, Léna Jlassi, Larissa Tevlin, Martin Sparre, Rainer Weinberger, Ewald Puchwein, Rüdiger Pakmor and Christoph Pfrommer



Zoom factor 8
Zoom factor 12
Zoom factor 24
2 Mpc
Gas density [g/cm^3]
10^{-27}
10^{-28}
10^{-29}
10^{-30}
$|\nabla \times v|^2/2$ [Gyr^{-1}]
10^3
10^2
10^1
10^0
10^{-1}
$B[\mu G]$
10^1
10^0
10^{-1}
10^{-2}

Resolving the magnetic dynamo is expensive





Part IV

Braginskii viscosity in Arepo

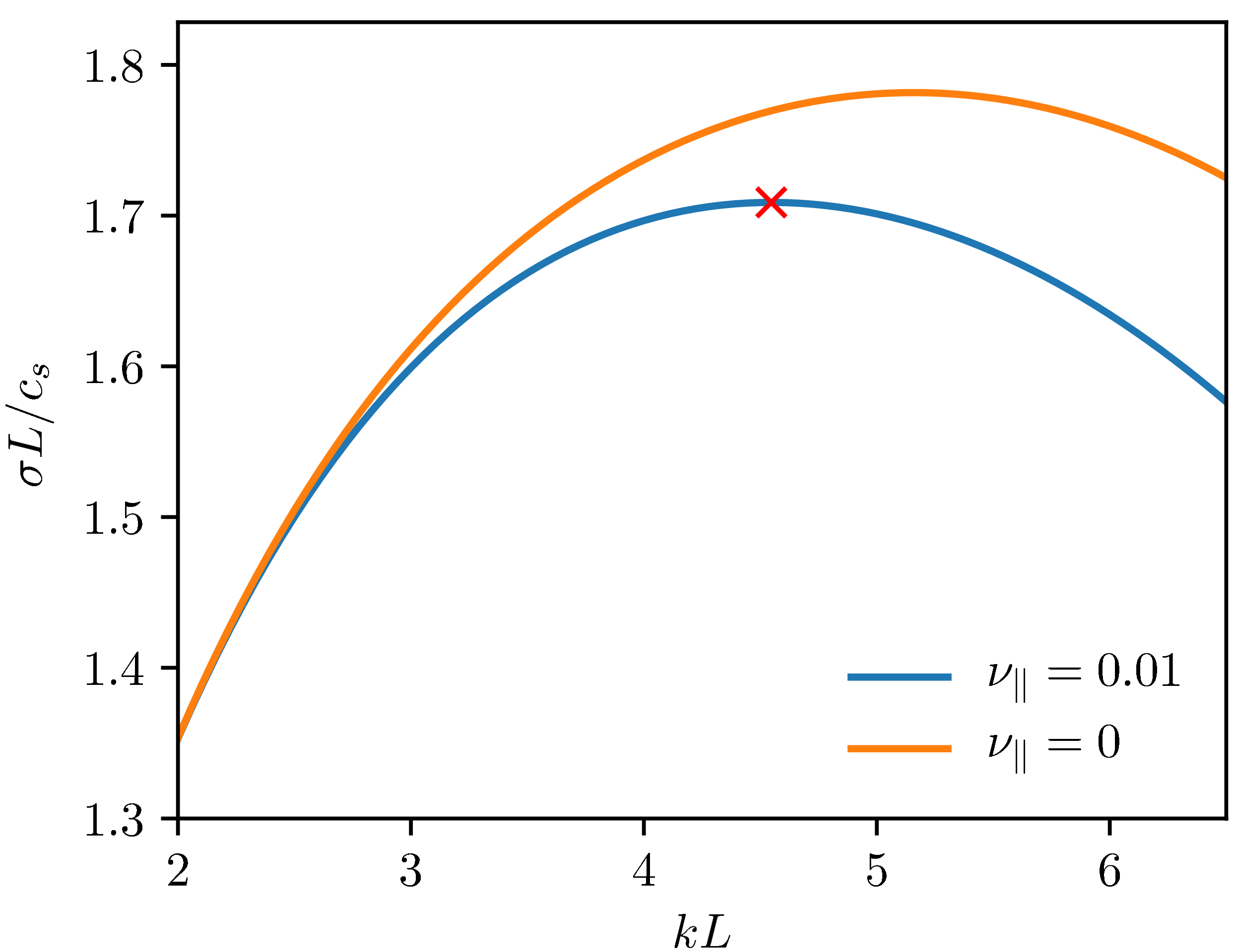
Braginskii viscosity in Arepo

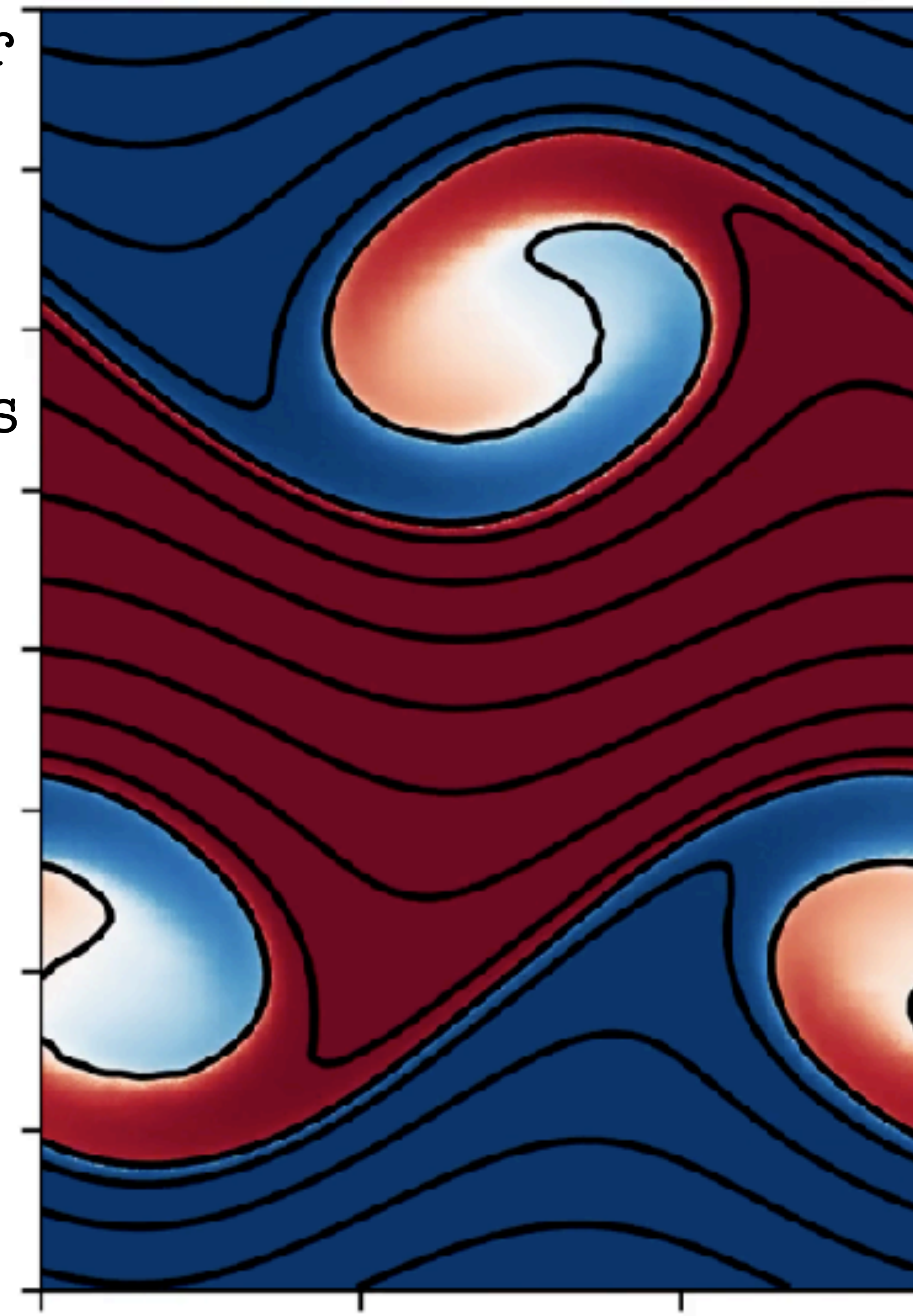
$$\mathbf{\Pi} = -\Delta p \left(\boldsymbol{bb} - \frac{\mathbf{1}}{3} \right)$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} = -\nabla \cdot \mathbf{\Pi}$$

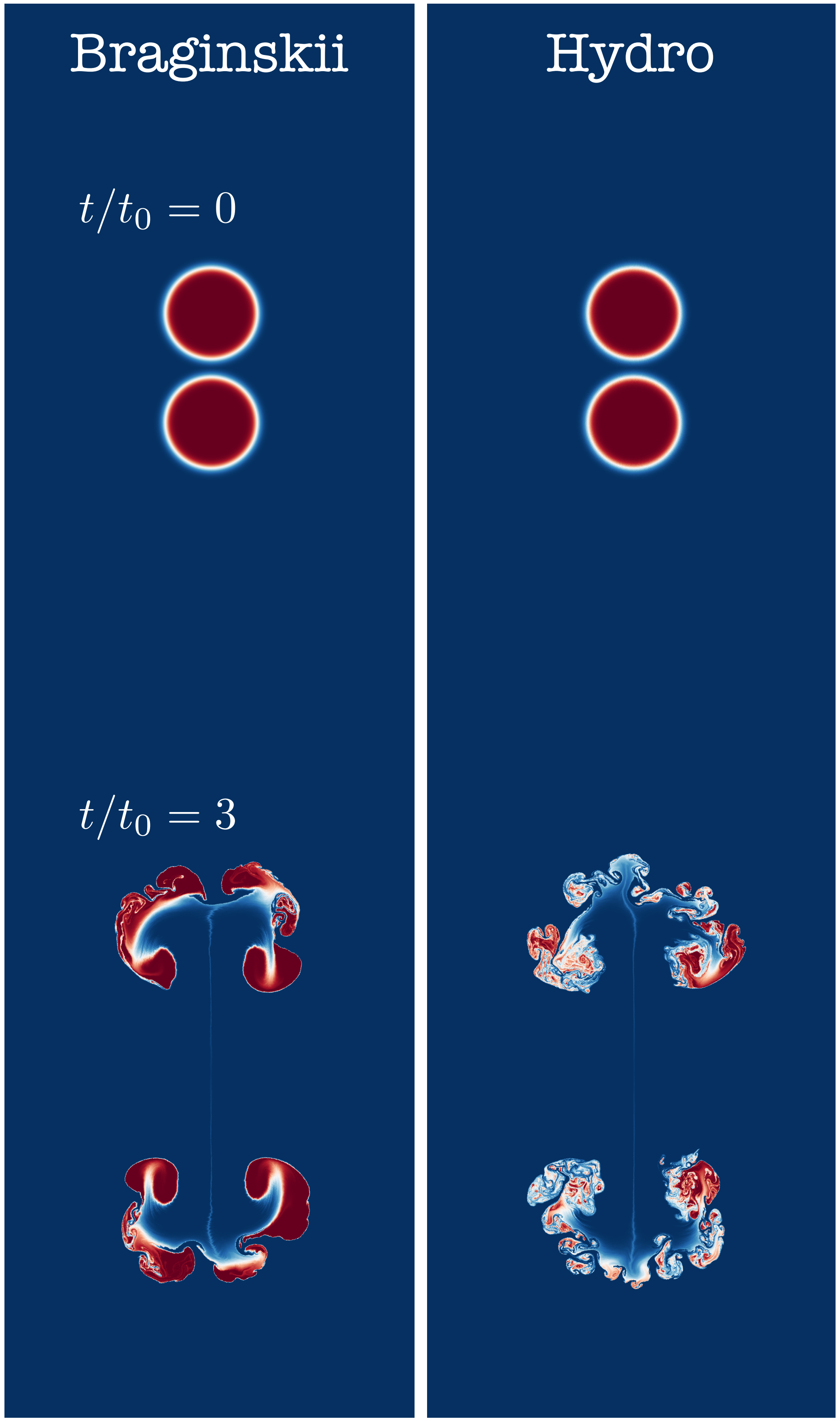
$$\frac{\partial E}{\partial t} = -\nabla \cdot (\mathbf{\Pi} \cdot \boldsymbol{v})$$

- Extensive test suite (detailed in Berlok, Pakmor & Pfrommer 2020)
- Second order accurate Super timestepping (RKL2)
- Recently extended for cosmological applications



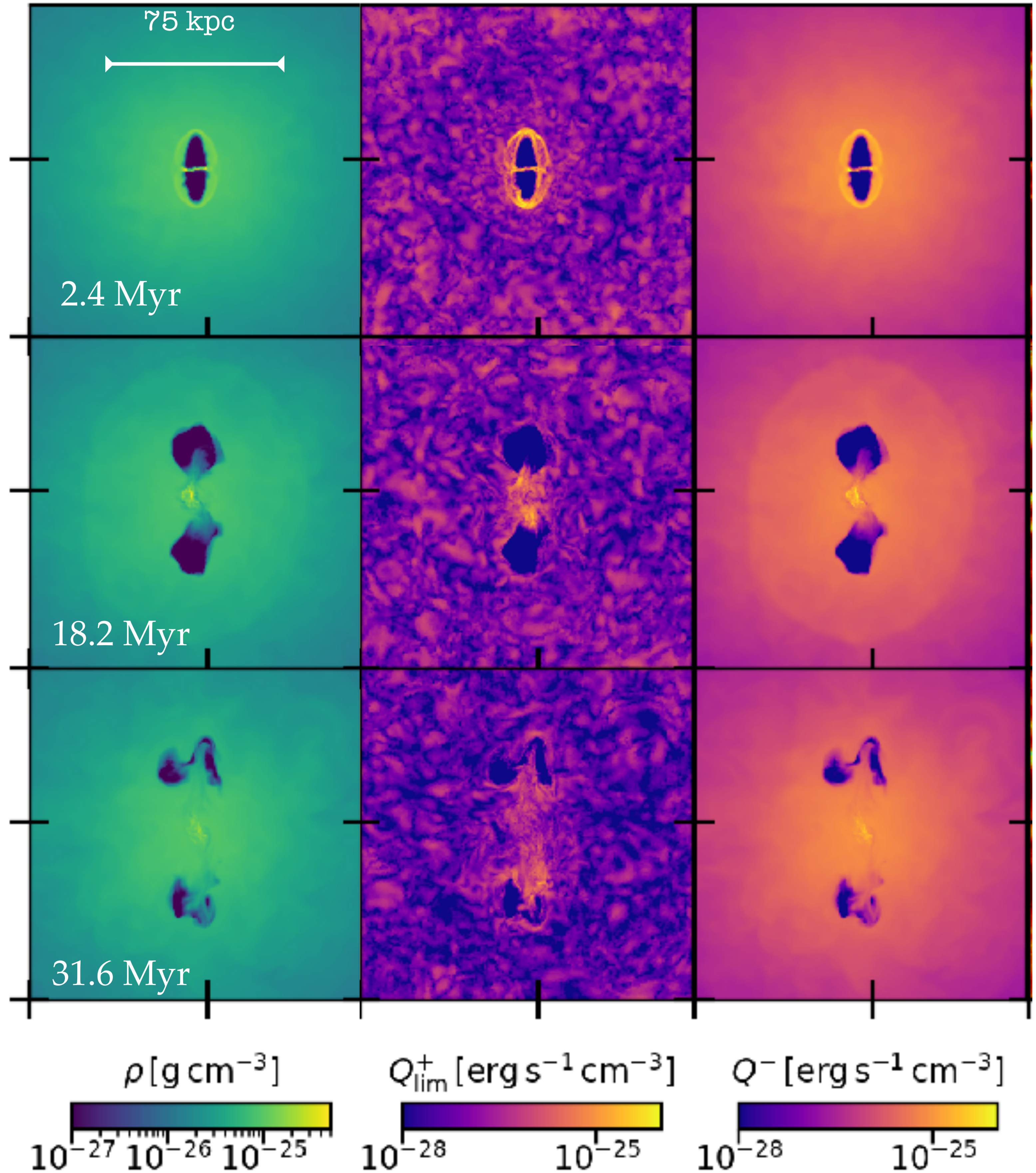


Idealized 2D bubbles



Berlok+ (unpublished)

AGN jet with Braginskii viscosity



Berlok+, in prep.

Analysis to be done!