Leading Singularities of Gravity Amplitudes

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September, 11th 2008 - NBIA, Copenhagen

P.B. and Freddy Cachazo - work-in-progress

Outline

1. Motivations
2. Leading Singularity Technique
3. $\mathcal{N} = 8$ SUGRA
4. Conclusion

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- Perturbative structures of field theories seems to be simpler that what appears from Feynman diagrams
- S-matrix analysis
  1. tree level singularities: poles only

     $$\downarrow$$

     a class of theory is *fully* determined by the 3-particle amplitude
  2. loop singularities: poles and branch cuts

     $$\downarrow$$

     at one loop, a class of theories is determined by the quadruple cuts

Attention on $\mathcal{N} = 8$ Supergravity

Investigating loop-orders

- unitarity-based method (Bern, Carrasco, Dixon, Dunbar, Johansson, Kosower, Morgan, Roiban,...)
- leading singularity:
  1. $\mathcal{N} = 4$ SYM (Cachazo & Skinner; Cachazo; Cachazo, Spradlin & Volovich; Spradlin, Volovich & Wen)
  2. $\mathcal{N} = 8$ Supergravity (Cachazo & Skinner; Arkani-Hamed, Cachazo, Kaplan)

$\mathcal{N} = 8$ Supergravity at 1-loop

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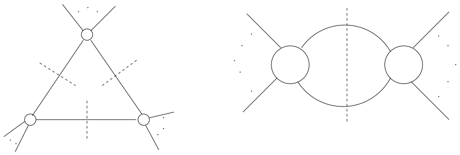
N. Arkani-Hamed, F. Cachazo, J. Kaplan



$$\Delta_3 = \sum_i \Delta_4^{(i)}$$

$$\Delta_2 = \sum_i \Delta_3^{(i)}$$

Triple and double cuts in terms of quadruple cuts

$\downarrow$

Boxes reproduce Feynman diagrams singularities

(No-triangle hypotesys proven by E. Bjerrum-Bohr & P. Vanhove)

$\mathcal{N} = 8$ Supergravity at higher loops

- at 1 loop: absence of triangles & bubbles $\equiv$ amplitude fully determined by leading singularities
- what about higher loops?

$$\downarrow$$

Conjecture (Arkani-Hamed, Cachazo, Kaplan):
The full S-matrix is determined by its leading singularities

$$\downarrow$$

Consequences, if true:

- tree level amplitudes determine the *full* S-matrix
- no UV-divergencies

Worth to explore!

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Leading Singularity Technique

Amplitude as sum of Feynman diagrams and in terms of an integral basis $\{I_i\}_{i\in\mathcal{I}}$, $I_i(k) = \int\prod_{i=1}^{L} d^d l^{(i)}\, \mathfrak{I}_i(k,l)$

$$M_n = \sum_{f\in\mathcal{F}}\int\prod_{i=1}^{L} d^d\, l^{(i)}\, F_f(k,l) = \sum_{i\in\mathcal{I}} c_i(k)\, I_i(k)$$

Singularities: poles and branch cuts.

Leading Singularities : Highest codimension singularities

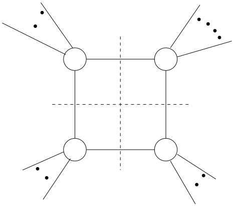
In d-dimensions the discontinuity across the leading singularity is computed by a d-dimensional cut

$$\downarrow$$

Quadruple-cut computes the leading singularity in 4-dimensions

Leading Singularity Technique

Quadruple cut



$$\frac{1}{l^2 + i\epsilon} \longrightarrow \delta^{+}(l^2)$$

4 δ-functions $\quad\longrightarrow\quad$ l is localized in $l^{(i)} \in \mathbb{C}^4$

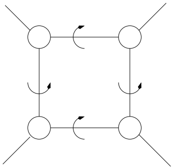
$$\downarrow$$

the integral is given by the jacobian of the change of variable from l to the argument of δ evaluated at $l^{(i)}$

Leading Singularity Technique

Contour integral point of view

$$\sum_{f\in\mathcal{F}} \int_\gamma \prod_{i=1}^{L} d^4\, l^{(i)}\, F_f(k,l) \;=\; \sum_{i\in\mathcal{I}} c_i(k) \int_\gamma \prod_{i=1}^{L} d^4 l^{(i)}\, \mathfrak{I}_i(k,l)$$



$$\gamma = T^{4L} \subset \mathbb{C}^{4L}$$

Massless case: $\forall$ loop, T^4 localizes the l-integral onto $l^{(i)} \in \mathbb{C}^4$ $(i = 1, 2)$

Algebraic linear equations for the coefficients $c_i(k)$

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$\mathcal{N} = 8$ Sugra: 2-loop 5-particle MHV Amplitude

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Two sectors: $\begin{cases} \text{planar} \\ \text{non-planar} \end{cases}$

Two loops $\rightarrow$ integration contour $\gamma = T^8$

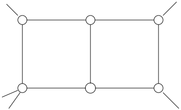
Three classes of contours T^8:

1. on diagrams with 8 internal propagators
2. on diagrams with 7 internal propagators
3. on diagrams with 6 internal propagators

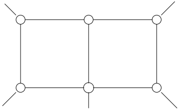
$\mathcal{N} = 8$ Sugra: Planar Sector

Three topologies

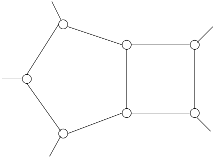
- topology 1



- topology 2

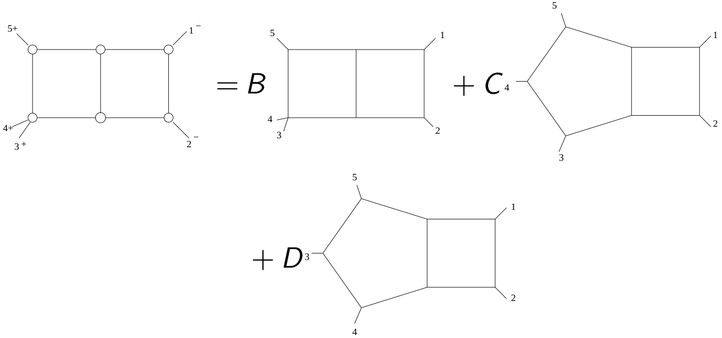


- topology 3



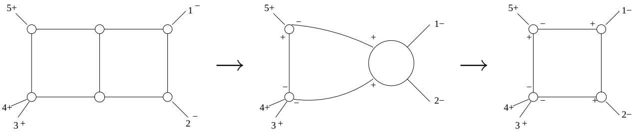
$\mathcal{N} = 8$ Sugra: Planar Sector - Topology 1

Expansion on integrals which share the same singularities



$$T^8 = \Big\{(p,q) \in \left(\mathbb{C}^4 \times \mathbb{C}^4\right) \;\Big|\; |p^2| = \varepsilon, |(p+k_1)^2| = \varepsilon, |(p+k_1+q)^2| = \varepsilon, |(p-k_2)^2| = \varepsilon,$$
$$|q^2| = \varepsilon, |(q-k_5)^2| = \varepsilon, |(q+k_1)^2| = \varepsilon, |(q+k_1+k_2)^2| = \varepsilon\Big\},$$

$\mathcal{N}=8$ Sugra: Planar Sector - Topology 1



$$\sum_{\text{multiplet}} \prod_{s=1}^{4} M_3^{\text{tree}\,(s)}\Big|_{p^{(i)}} = stu\, M_4^{\text{tree}}, \qquad I = \left(\prod_{s=1}^{4} \oint_{z_i=0} \frac{dz_i}{z_i}\right) |J|^{-1}$$

Two equations

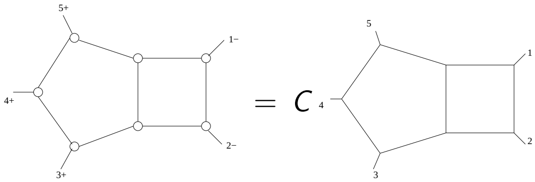
$$B - \frac{s_{25}}{\mathfrak{h}(2,5,4,3)} C - \frac{s_{25}}{\mathfrak{h}(2,5,3,4)} D = 0$$

$$B - \frac{s_{25}}{\mathfrak{h}(2,3,4,5)} C - \frac{s_{25}}{\mathfrak{h}(2,4,3,5)} D = -s_{12}^3 s_{15} \frac{\langle 1,2\rangle^8}{N(5)} \mathfrak{f}(1,3,4,2,5)$$

2 equations in 3 unknowns: not enough!

$\mathcal{N} = 8$ Sugra: Planar Sector - Topology 3

Expansions on integrals which share the same singularities



$$T^8 = \left\{(p,q) \in \left(\mathbb{C}^4 \times \mathbb{C}^4\right) \Big| |p^2| = \varepsilon, |(p+k_1)^2| = \varepsilon, |(p+k_1+q)^2| = \varepsilon, |(p-k_2)^2| = \varepsilon, |q^2| = \varepsilon, |(q-k_5)^2| = \varepsilon, |(q-k_5-k_4)^2| = \varepsilon, |(q+k_1+k_2)^2| = \varepsilon\right\},$$

Two solutions: the lhs vanishes on one solution and is zero on the other one.

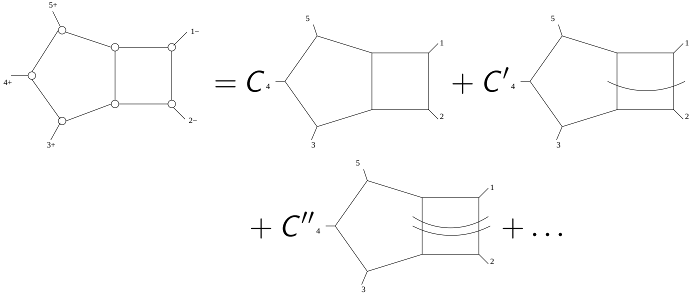
Contradiction?

$\mathcal{N} = 8$ Sugra: Planar Sector - Topology 3

NO!: The integral expansion is simply not complete!

$\downarrow$

Introduce integrals with share the same singularity BUT do not contribute to other topologies



$\mathcal{N} = 8$ Sugra: Planar Sector - Topology 3

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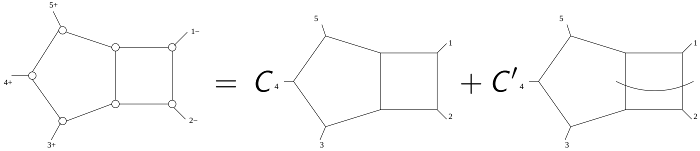
We need minimal set which makes the system consistent



Analogous expansion related to D needed $(3 \leftrightarrow 4)$

$$\downarrow$$

Eqs from top 1 + Eqs from top 3 = 6 eqs in 5 unknowns

most likely it does not have solution

$\mathcal{N} = 8$ Sugra: Planar Sector - Topology 1+3

Surprisingly, it has a unique solution!

$$B = -s_{12}^2 s_{15} s_{25} \left(s_{15} + s_{25}\right) \frac{\langle 1,2\rangle^8}{N(5)} \frac{\mathfrak{h}(1,3,4,2)\mathfrak{h}(1,4,3,2)}{\mathfrak{h}(1,2,3,5) - \mathfrak{h}(1,5,3,2)}$$

$$C = s_{12}^2 s_{23} s_{34} s_{45} s_{51} \frac{\langle 1,2\rangle^8}{N(5)} \frac{\mathfrak{h}(1,3,4,2)\mathfrak{h}(1,4,5,2)}{\mathfrak{h}(1,2,3,5) - \mathfrak{h}(1,5,3,2)}$$

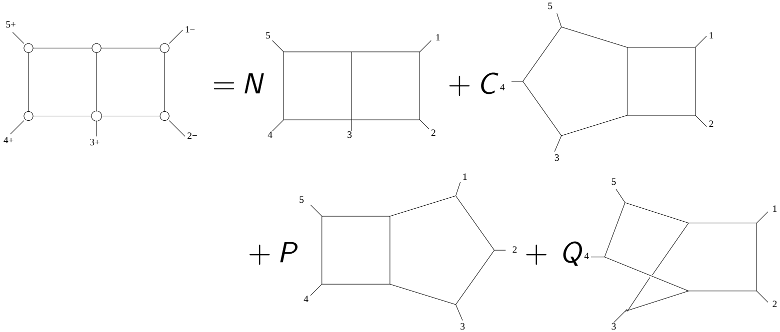
$$C' = s_{12} s_{34} s_{45} \frac{\langle 1,2\rangle^8}{N(5)} \frac{\mathfrak{h}(1,3,4,2)\mathfrak{h}(1,4,5,2)\mathfrak{h}(1,5,3,2)}{\mathfrak{h}(1,2,3,5) - \mathfrak{h}(1,5,3,2)}$$

$$D = -s_{12}^2 s_{24} s_{43} s_{35} s_{51} \frac{\langle 1,2\rangle^8}{N(5)} \frac{\mathfrak{h}(1,4,3,2)\mathfrak{h}(1,3,5,2)}{\mathfrak{h}(1,2,4,5) - \mathfrak{h}(1,5,4,2)}$$

$$D' = -s_{12} s_{34} s_{35} \frac{\langle 1,2\rangle^8}{N(5)} \frac{\mathfrak{h}(1,4,3,2)\mathfrak{h}(1,3,5,2)\mathfrak{h}(1,5,4,2)}{\mathfrak{h}(1,2,4,5) - \mathfrak{h}(1,5,4,2)}.$$

$\mathcal{N} = 8$ Sugra: Planar Sector - Topology 2

Expansion on integrals which share the same singularities



Topology 2 has a non-planarity feature built-in!

$\mathcal{N} = 8$ Sugra: Planar Sector - Topology 2

2 equations in 2 unknowns! (C and P already determined!)

$$N = s_{12}s_{23}s_{34}s_{45}s_{51}\frac{\langle 1,2\rangle^8}{N(5)}\mathfrak{h}(1,4,5,2)\frac{s_{14}\mathfrak{h}(1,3,4,2) - s_{25}\mathfrak{h}(2,4,5,3)}{\mathfrak{f}(3,1,4,2,5) - \mathfrak{f}(3,2,5,1,4)}$$

$$Q = s_{12}s_{23}s_{34}s_{45}s_{51}s_{13}s_{35}\frac{\langle 1,2\rangle^8}{N(5)}\mathfrak{h}(1,4,5,2)\frac{\mathfrak{h}(1,3,4,2) + \mathfrak{h}(2,4,5,3)}{\mathfrak{f}(3,2,5,1,4) - \mathfrak{f}(3,1,4,2,5)}$$

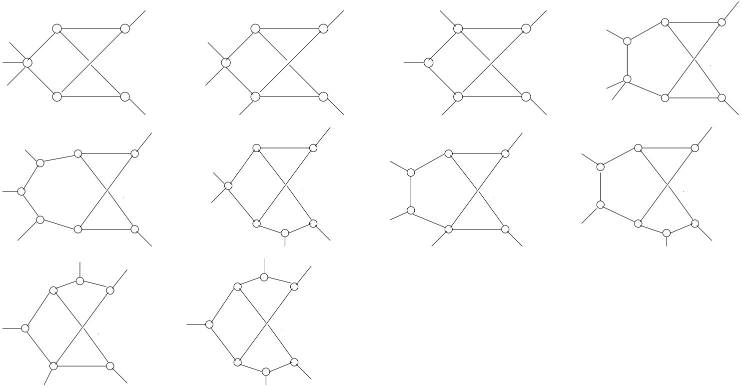
Appearence of the non-planar integral

$$\downarrow$$

the two sectors cannot be disentangled

$\mathcal{N} = 8$ Sugra: Non-Planar Sector

Higher number of topologies



$\mathcal{N} = 8$ Sugra: Non-Planar Sector

First three topologies: 6-internal propagators

Question: how can we integrate on a T^8?

Answer: integrating out 1 loop variable $\longrightarrow$ two extra propagators from the jacobian.

However

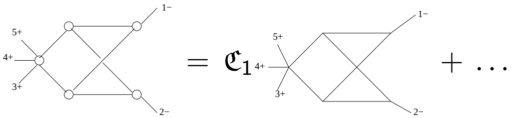
the T^8 does not have solution: a simultaneous factorization in these two channels cannot occur

$\downarrow$

These three topologies are "not relevant" (?)

$\mathcal{N} = 8$ Sugra: Non-Planar Sector

Example: first non-planar topology



The amplitude cannot show a simultaneous factorization in the channels $(q+k_1)^2$ and $(q+k_2)^2$

$$\downarrow$$

$$\mathfrak{C}_1 = 0$$

The coefficients of the integral expansion are determined by the study of the topogies from the fourth on

How to check that the final answer is correct?

Three limits to check:

- collinear limit
- multi-particle limit
- soft limit

Problem with the soft-limit: explicit expression not known for all the integrals $\longrightarrow$ IR analysis hard...

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Conclusion

- Perturbative structure of field theories is more intriguing than what appears from Feynman diagrams
- Study of poles at tree level: recursion relations
- Branch-cuts at loop level: complicated structure

$$\downarrow$$

special singularities: leading singularities

$$\uparrow$$

computation of residues!

- $\mathcal{N} = 8$ supegravity at 1-loop as well
- Other loops? 2-loop 5-particle amplitude as a first check
- Coeffs of the integral expansion never computed before
- More insights towards a proof of the leading singularity conjecture?