Maximal Supersymmetry and Higher loop Amplitudes in Supergravity

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Septembre 10, 2008 @ Niels Bohr International Academy

based on

- 0802.0868, 0805.3682 and 0806.1726 with N.E.J. Bjerrum-Bohr
- hep-th/0610299, hep-th/0611273 and 0807.0389 with M.B. Green, J.G. Russo

$\mathcal{N} = 8$ supergravity

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The real implementation of these symmetry in perturbative computations is obscure. And we would like to understand better

- the role of the all symmetries of $\mathcal{N} = 8$ supergravity
    - gauge invariance
    - supersymmetry
    - non-perturbative symmetries
- the relation to string theory [Green, Ooguri, Schwarz]

Outline

1. On-shell supersymmetry in $\mathcal{N} = 8$ supergravity amplitudes

2. No triangle hypothesis

3. Conclusion & Outlook

Perturbative structure of $\mathcal{N} = 8$ sugra

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- Depending on the various implementation of on-shell supersymmetry

$$6 \leq c_L = 6 + 2\beta_L \leq 18$$

- Which gives a possible first divergence in $D = 4$

$$3 \leq L \leq 9$$

Effective action

Supergravity effective action for higher derivative terms

$$\mathcal{L} = \frac{1}{\kappa_{(4)}^2} \int d^4x \sqrt{-g} \, [\mathcal{R}_{(4)} + \sum_k f_k(\varphi_i) \, D^{2k} \mathcal{R}^4]$$

The functions $f_k(\varphi_i)$

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Local counter-term to a UV divergence read

$$\delta^{(L)}\mathcal{L} = \frac{1}{\kappa_{(4)}^4} \int d^4x \sqrt{-g}\, \kappa_{(4)}^{2(L+1)}\, [D^{2(L-3)}\, \mathcal{R}^4 + \cdots + \mathcal{R}^{L+1}]$$

Non-renormalisation theorems

The non-renormalisation theorems [Green,Russo,Vanhove; Berkovits] and the explicit expressions for the 1-,2- and 3-loop amplitudes [Bern,Dixon,Perelstein,Rozowsky; Bern, Carrasco, Dixon, Johansson, Kosower, Roiban] imply the following

- 1-loop non-renormalisation of R^4: $\beta_L \geq 2 \qquad L \geq 2$

$$\text{UV divergence: } L \geq 3 + \beta_L \quad \geq \quad 5\,\text{loops}$$

- 2-loop non-renormalisation of $D^4 R^4$: $\beta_L \geq 3 \qquad L \geq 3$

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- 3-loop non-renormalisation of $D^6 R^4$: $\beta_L \geq 4 \qquad L \geq 4$

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First possible UV divergence of $\mathcal{N} = 8$ supergravity

- The 7-loop counter-term

$$\delta^{(7)}\mathcal{L} = \int d^4x \int d^{32}\theta\, \varphi_{ijkl}^4 = \int d^4x \sqrt{g}\, D^8 R^4$$

Forbidden by the non-linear $E_{7(7)}/SU(8)$ symmetries because φ_{ijkl} is a vielbein for the coset [Howe/Lindstrom; Kallosh]

First possible UV divergence of $\mathcal{N}=8$ supergravity

- The 8-loop counter-term [Kallosh]

$$\delta^{(8)}\mathcal{L} = \int d^4x \int d^{32}\theta\, \chi^4_{ijk\alpha} = \int d^4x\, \sqrt{g}\, D^{10} R^4$$

This superfield starts with the 56 dilatini

First possible UV divergence of $\mathcal{N} = 8$ supergravity

- The 9-loop counter-term [Green et al.]

$$\delta^{(9)}\mathcal{L} \;=\; \int d^D x \int d^{32}\theta \, W_{ij}^4 = \int d^D x \sqrt{g}\, D^{12} R^4$$

is expressed in terms of the spin 1 superfield build from the field-strengths (all gauge invariant quantities)

The R^4 case

The famous saturated R^4 has a $\frac{1}{2}$-superspace expression

$$\delta\mathcal{L} = \int d^D x \int d^{16}\theta\, \varphi^4_{ijkl}$$

- This is a valid 1-loop counter-term in $D = 8$ with local (harmonic) superspace construction
  (φ_{ijkl} stands for the chiral superfield and the linear superfield of $D = 8$ harmonic superspace [Berkovits])

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  (φ_{ijkl} stands for the chiral superfield and the linear superfield of $D = 8$ harmonic superspace [Berkovits])

- But this not a valid 3-loop counter-term in $D = 4$ because it is not invariant under the $E_{7(7)}/SU(8)$ symmetries.
- In terms of $E_{7(7)}/SU(8)$ this R^4 is a *non local* quantity

$$\delta\mathcal{L} = \int d^4x \int d^{16}\theta \, (D\bar{D})^{-4} \, W^4_{ij} \sim \int d^4x \frac{1}{\Box^2} \mathcal{R}^4$$

viz. the one-loop effective action of $\mathcal{N} = 8$

Supersymmetry and what else?

Susy alone cannot be responsible for the finiteness of $\mathcal{N} = 8$ supergravity.

The other symmetries of the theory will play an important role

- gauge and crossing symmetries?
- Duality symmetries (local $SU(8)_R$ and/or global $E_{7(7)}$)?

In the following we will focus on the gauge and crossing symmetries which are typical of gravity theories.

The no triangle property in $\mathcal{N} = 8$

- Gravity is cross symmetric (colorless theory)
  - sum over all the permutations at one-loop
  - sum over all the planar and nonplanar diagrams at higher loop order

- Gauge invariance $\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + \partial_\mu v_\nu + \partial_\nu v_\mu$

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=> Simplification of tree-level amplitudes [Bern,Carrasco,Johansson]

$$\mathfrak{M}_n = \sum_r \frac{n_r^L n_r^R}{\prod_j (p_j)_r^2}$$

=> Cancellations in loop amplitudes [Bjerrum-Bohr, Vanhove]

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Unordered amplitudes are more than just the sum over all ordering of color ordered amplitudes.
All the various ordering have the same tensorial structure

$$\mathfrak{M} = \sum_r t_r \int_0^\infty \frac{dT}{T} \int_0^1 \prod_{i=1}^{n-1} d\nu_i \, T^{-D/2+n} \exp(-T \sum_{r,s} (k_r \cdot k_s) \, G_{r,s})$$

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Loop momentum is a total derivative $k_i \cdot \ell \sim \partial_{\nu_i} Q_n$ which can be freely integrated

$$\int_0^\infty \frac{dT}{T} \int_0^1 d^{n-1}\nu \, \partial_{\nu_i} Q_n(\cdots) = - \int_0^\infty \frac{dT}{T} \int_0^1 d^{n-1}\nu \, Q_n \partial_{\nu_i}(\cdots)$$

Since $\partial_{\nu_i}\partial_{\nu_j} Q_n \sim (k_i \cdot k_j)[\delta(\nu_i - \nu_j) - 1]$ does not contain any loop momenta

two powers of loop momentum are cancelled at each steps

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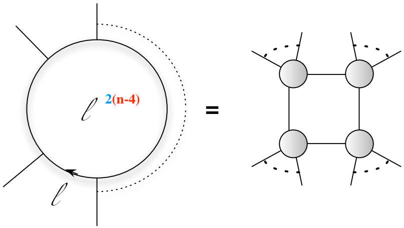
Unordered amplitudes are more than just the sum over all ordering of color ordered amplitudes.

Improved reductions formulas with two powers of loop momentum are cancelled at each steps => $\mathcal{N} = 8$ sugra no triangle property

[Bjerrum-Bohr, Vanhove; Emil's talk]



The absence of rational term

All the rational terms are push into *UV and IR finite* dimension shifted integrals

$$I_n^{[4+2m]} = \int \frac{d^4\ell d^{2m}\ell_\perp}{\prod_{r=1}^{4+m}(\ell - q_r)^2 + \ell_\perp^2}$$

- These contributions decouple by gauge invariance [Bjerrum-Bohr, Vanhove]

$$\prod_{1\leq i\neq j\leq n} (\varepsilon_i \cdot \varepsilon_j)\, I_n^{[4+2n]}$$

In $\mathcal{N} = 8$ amplitude all the coefficient of these contributions have extra powers of momentum needed for gauge invariance (connection $\Gamma_{(i)} \sim k_\mu \varepsilon_{(i)\nu}$) and the dimension shifted integral cannot arise by dimension analysis (but they arise for sugra amplitudes with low or no supersymmetries)

Summary & Outlook

The present situation regarding the role of supersymmetry in perturbative $\mathcal{N} = 8$ in $D = 4$ is still not understood

- Important to include the role of the extended lorentz symmetries and use the higher dimensional origin of the theory
(The expressions for the loop amplitudes obtained by [Bern et al.] are valid in all dimensions)
- Finiteness of $\mathcal{N} = 8$ in $D = 4$ requieres extra symmetries beyond supersymmetry. Role of no triangle property at higher loop order ?

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