Dual superconformal symmetry from $AdS_5 \times S^5$

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arXiv:0711.0707 R.R., A.A.Tseytlin and M.Wolf

arXiv:0807.3228 N.Beisert, R.R., A.A.Tseytlin and M.Wolf

Outline

- Introduction and motivation
- Review of Green-Schwarz superstring in $AdS_5 \times S^5$
- Worldsheet duality transformations
  - i) Bosonic T-duality
  - ii) "Fermionic" T-duality
- Combination of worldsheet transformations $\longrightarrow$ dual superconformal symmetry

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Introduction

Scattering amplitudes at strong coupling have been related to a lightlike Wilson loop computation in string theory.

Alday, Maldacena

The loop lives in a "dual" AdS_5 space obtained by formal T-duality on the isometric directions of the corresponding bosonic sigma model

$$\frac{1}{Y^2}\left(\mathrm{d}X^\mu \mathrm{d}X^\mu + \mathrm{d}Y^2\right) \quad \rightarrow \quad \frac{1}{\tilde{Y}^2}\left(\mathrm{d}\tilde{X}^\mu \mathrm{d}\tilde{X}^\mu + \mathrm{d}\tilde{Y}^2\right)$$

$$\mathrm{d}X^\mu \rightarrow Y^2 * \mathrm{d}\tilde{X}^\mu, \qquad Y \rightarrow \tilde{Y}^{-1}$$

(i) The geometry is again AdS_5

(ii) An immediate consequence is that a *dual conformal* symmetry appears

An analogous symmetry appears in the computation of scattering amplitudes in planar $\mathcal{N} = 4$ SYM at weak coupling.

Drummond, Henn, Korchemsky, Sokatchev; Brandhuber, Heslop, Travaglini

It is tantalizing to conjecture that these two symmetries are actually two sides of the same coin.

- In gauge theory this symmetry has been promoted to a full dual superconformal symmetry

  Drummond, Henn, Korchemsky, Sokatchev; Paul's talk

- Can we use string theory to understand its origin?

The answer seems to be positive and the result is that the dual superconformal symmetry is related to the integrability of the superstring. To uncover the precise relation we will need to consider the full superstring theory on $\mathrm{AdS}_5 \times S^5$.

$\mathrm{AdS}_5 \times S^5$ superstring

It is based on the supercoset $\frac{G}{H} = \frac{PSU(2,2|4)}{SO(1,4)SO(5)} \supset AdS_5 \times S^5$

Metsaev, Tseytlin

$$S \sim \sqrt{\lambda} \int G_{MN} \partial X^M \partial X^N + \bar{\theta}(D + F_5)\theta \partial X + \cdots$$

- Classical bosonic string theory on $\mathrm{AdS}_5 \times S^5$ is integrable
  Lüscher, Pohlmeyer
- Classical integrability extends to the full superstring
  Bena, Polchinski, Roiban
- Infinite tower of conserved charges:
  local (Noether)+non-local charges

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More formally we can use the $\mathbb{Z}_4$-grading of the superconformal algebra $\mathfrak{g} = \mathfrak{psu}(2,2|4)$

$$\mathfrak{g} \;\cong\; \mathfrak{h} \oplus \mathfrak{g}_{(1)} \oplus \mathfrak{g}_{(2)} \oplus \mathfrak{g}_{(3)}\,, \qquad [\mathfrak{g}_{(m)}, \mathfrak{g}_{(n)}\} \;\subset\; \mathfrak{g}_{(m+n)}$$

to decompose the supercoset current as follows

$$j \;=\; g^{-1}\mathrm{d}g \;=\; A + j_{(1)} + j_{(2)} + j_{(3)}, \quad A \;\in\; \mathfrak{h}, \quad j_{(m)} \;\in\; \mathfrak{g}_{(m)}.$$

so that the GS string can be compactly written as

$$S \;=\; -\tfrac{T}{2} \int \mathrm{str}\left[j_{(2)} \wedge * j_{(2)} + j_{(1)} \wedge j_{(3)} \right]$$

Lax connection

The GS string is integrable and the associated Lax connection is

$$\mathrm{j}(z) \;=\; A + z^{-1} j_{(1)} + \tfrac{1}{2}(z^2 + z^{-2})\, j_{(2)} + z\, j_{(3)} + \tfrac{1}{2}(z^2 - z^{-2}) * j_{(2)}$$

where z is a complex spectral parameter, $\mathrm{j}(1) = g^{-1}\mathrm{d}g$.
The Lax connection is *flat*

$$\mathrm{dj}(z) + \mathrm{j}(z) \wedge \mathrm{j}(z) \;=\; 0.$$

The complete set of string equations is encoded in the flatness equation of $\mathrm{j}(z)$.

Expanding $\mathrm{j}(z)$ around $z = 1$ we can construct an infinite tower of conserved charges (Noether plus non-local).

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Now we apply the supercoset formalism to construct the explicit form of the GS action in the Poincaré parametrization:

$$\mathrm{d}s^2 \;=\; -\tfrac{1}{2}Y^2\mathrm{d}X_{\alpha\dot\beta}\mathrm{d}X^{\dot\beta\alpha} + \tfrac{1}{4Y^2}\,\mathrm{d}Y_{ij}\mathrm{d}Y^{ij}, \qquad Y^2 \;:=\; \tfrac{1}{4}Y_{ij}Y^{ij}$$

The 32 independent fermionic coordinates are $\Theta = (\theta^{i\alpha}_{\pm}, \bar\theta^{\dot\alpha}_{\pm i})$. The associated coset representative g may be chosen as

$$g(X, Y, \Theta) \;=\; B(X, Y)\,\mathrm{e}^{-F(\Theta)},$$

with

$$\begin{aligned} B(X, Y) &= \mathrm{e}^{\mathrm{i}X\,P}\; Y^{\mathrm{i}D}\,, \\ F(\Theta) &= \mathrm{i}\big[(\theta_+ Q + \theta_- S) - (\bar\theta_+\bar Q + \bar\theta_-\bar S)\big]. \end{aligned}$$

We also need to fix κ-symmetry. A convenient choice is

$$\theta_-^{i\alpha} = 0 = \bar\theta^{\dot\alpha}_{-i}, \qquad \text{“S-gauge”}$$

so that

$$F(\Theta) = \mathrm{i}\big[\theta_+ Q - \bar\theta_+ \bar Q\big]$$

does not contain terms with S-generators.

$$S = -\frac{T}{2}\int -\frac{1}{2}Y^2\Pi_{\alpha\dot\beta}\wedge *\Pi^{\dot\beta\alpha} + \frac{1}{4Y^2}\,\mathrm{d}Y_{ij}\wedge *\mathrm{d}Y^{ij}$$
$$+ \frac{1}{2}\big(\mathrm{d}Y_{ij}\wedge\theta^{i\alpha}\mathrm{d}\theta^j_\alpha - \mathrm{d}Y^{ij}\wedge\bar\theta^{\dot\alpha}_i\mathrm{d}\bar\theta_{j\dot\alpha}\big)$$

with

$$\Pi^{\dot\alpha\beta} := \mathrm{d}X^{\dot\alpha\beta} + \frac{\mathrm{i}}{2}\big(\bar\theta^{\dot\alpha}_i\mathrm{d}\theta^{i\beta} - \mathrm{d}\bar\theta^{\dot\alpha}_i\theta^{i\beta}\big).$$

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Bosonic T-duality

Now we dualize the four X isometries

$$S(\partial X, \dots) \to S(V, \dots) + \tilde{X}\, \mathrm{dV}$$

Integrate out $\tilde{X}$: $\quad \mathrm{dV} = 0 \Rightarrow \mathrm{V} = \mathrm{d}X \to$ original action

Integrate out $V \to$ dual action: *Kallosh and Tseytlin*

$$\begin{aligned} \tilde{S} \;&= \int -\tfrac{1}{2Y^2}\mathrm{d}\tilde{X}_{\alpha\dot{\beta}} \wedge *\mathrm{d}\tilde{X}^{\dot{\beta}\alpha} + \tfrac{1}{4Y^2}\,\mathrm{d}Y_{ij} \wedge *\mathrm{d}Y^{ij} \\ &+ \tfrac{\mathrm{i}}{2}\mathrm{d}\tilde{X}_{\beta\dot{\alpha}} \wedge (\bar{\theta}^{\dot{\alpha}}_i \mathrm{d}\theta^{i\beta} - \mathrm{d}\bar{\theta}^{\dot{\alpha}}_i \theta^{i\beta}) + \tfrac{1}{2}(\mathrm{d}Y_{ij} \wedge \theta^{i\alpha}\mathrm{d}\theta^j_\alpha - \mathrm{d}Y^{ij} \wedge \bar{\theta}^{\dot{\alpha}}_i \mathrm{d}\bar{\theta}_{j\dot{\alpha}}) \end{aligned}$$

The relation between the original and dual coordinates is

$$\mathrm{d}X^{\dot{\alpha}\beta} + \tfrac{\mathrm{i}}{2}(\bar{\theta}^{\dot{\alpha}}_i \mathrm{d}\theta^{i\beta} - \mathrm{d}\bar{\theta}^{\dot{\alpha}}_i \theta^{i\beta}) \;=\; Y^{-2} *\mathrm{d}\tilde{X}^{\dot{\alpha}\beta}.$$

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Observations

- The bosonic geometry is again $\text{AdS}_5 \times S^5$.
  This is a special property of the Poincaré parametrization of AdS_5.
- In general T-duality breaks some of the original global isometries.

Example: Bosonic sigma model on S^2

$$ds^2 = d\theta^2 + \sin(\theta)^2 d\phi^2 \quad \rightarrow \quad d\tilde{s}^2 = d\theta^2 + \sin(\theta)^{-2} d\tilde{\phi}^2$$

$$SO(3) \quad \rightarrow \quad U(1)$$

- The bosonic sigma-model on AdS_5 is mapped into itself: a dual conformal symmetry becomes manifest after T-duality.
- Can we find a larger dual symmetry if we study the full superstring on $\text{AdS}_5 \times S^5$?

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Fermionic duality transformation

Let us now perform a further $2d$-duality, this time applied to the fermionic coordinates $\theta^{i\alpha}$ (but not to their conjugates $\bar{\theta}_i^{\dot{\alpha}}$).
The duality is implemented as in the bosonic case:

$$S(\dots, \partial\theta, \dots) \to S(\dots, \mathcal{V}, \dots) + \tilde{\theta}_{i\alpha} \wedge \mathrm{d}\mathcal{V}^{i\alpha}$$

Note that in this case the relation between θ and $\tilde{\theta}$

$$\mathrm{d}\theta^{i\alpha} \;=\; -\frac{1}{Y^2} Y^{ij}(\mathrm{d}\tilde{\theta}_j^{\alpha} - \mathrm{i}\tilde{X}_{\dot{\alpha}}^{\alpha}\mathrm{d}\bar{\theta}_j^{\dot{\alpha}})$$

does not involve the Hodge $*$.

The dual fermionic action is again the GS action albeit in a *complex* κ gauge:

Roiban, Siegel

$$\theta_- \;=\; 0 \;=\; \bar{\theta}_+, \qquad \bar{\mathrm{Q}}S - \mathrm{gauge}$$

Fermionic duality was performed only on θ and not on $\bar{\theta}$.

- The combination of bosonic and fermionic dualities maps the $\mathrm{AdS}_5 \times S^5$ superstring action into itself

  see also Berkovits, Maldacena

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- T-duality has been applied to the 2d fields (X, θ) dual to the $\mathcal{N} = 4$ maximal Abelian subalgebra

$$[P, P] = 0\,, \qquad \{Q, Q\} = 0$$

- The need for complexification seems natural also at weak coupling, where the dual $PSU(2, 2|4)$ symmetry becomes manifest after using a *twistor-inspired* formulation of gauge theory

$$\mathcal{A} = G^{+} + \eta^{A}\psi_{A} + \eta^{A}\eta^{B}\Phi_{AB} + \eta^{A}\eta^{B}\eta^{C}\psi_{ABCD} + \eta^{A}\eta^{B}\eta^{C}\eta^{D}G^{-}_{ABCD}$$

Drummond, Henn, Korchemsky, Sokatchev

- The dual superstring is also integrable.

Question : how are the integrable structures before and after T-duality related?

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Let us go back to the bosonic sigma model on AdS_5 for which

$$j = j_P + j_D, \qquad j_P = \mathrm{i} Y \mathrm{d}X\, P, \qquad j_D = \frac{\mathrm{i}}{Y} \mathrm{d}Y\, D.$$

By using the duality transformation

$$\mathrm{d}\tilde{X} = Y^2 {*}\mathrm{d}X \qquad \text{and} \qquad \tilde{Y} = Y^{-1}$$

we can relate the currents before and after duality

$$j_P = \mathrm{i} Y \mathrm{d}X\, P = \mathrm{i}\tilde{Y} {*}\mathrm{d}\tilde{X}\, P = {*}\tilde{j}_P,$$
$$j_D = \frac{\mathrm{i}}{Y} \mathrm{d}Y\, D = -\frac{\mathrm{i}}{\tilde{Y}} \mathrm{d}\tilde{Y}\, D = -\tilde{j}_D.$$

Applying this transformation to the bosonic Lax connection $\mathrm{j}(z)$ we can construct the dual connection $\tilde{\mathrm{j}}(z)$.

The $\mathfrak{so}(2,4)$ algebra has a natural $\mathbb{Z}_2$-automorphism (i.e. $\Omega^2 = 1$)

$$\Omega(P) = -K,\ \Omega(K) = -P,\ \Omega(L) = L,\ \Omega(D) = -D.$$

Using Ω we can construct another $\mathbb{Z}_2$-automorphism of the conformal algebra as follows

$$T \mapsto \mathcal{U}_z(T) := U_z\Omega(T)U_z^{-1}, \qquad U_z := \left(\frac{z - z^{-1}}{z + z^{-1}}\right)^{iD}.$$

This automorphism maps the Lax connections before and after duality into each other:

$$\mathcal{U}_z(\mathrm{j}(z)) = \tilde{\mathrm{j}}(z).$$

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Therefore bosonic T-duality can be abstractly understood as a symmetry of the Lax connection induced by an automorphism of the conformal algebra.

Furthermore the dual conformal charges arise from some of the non-local charges of the original model.

Ricci, Wolf, Tseytlin

Example:

$$\tilde{K} = \int d\sigma \int d\sigma' j_\tau^P(\sigma') j_\tau^D(\sigma) + \int d\sigma j_\sigma^P(\sigma) =: P_2$$

Berkovits, Maldacena

The dual conformal symmetry is intimately related to the integrability of the original string sigma model on AdS_5.

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The dual conformal symmetry is intimately related to the integrability of the original string sigma model on AdS_5.

- Does a similar interpretation apply to the superstring on $AdS_5 \times S^5$?

We need to use the $\mathbb{Z}_4$-automorphism ($\Omega^4 = 1$) of the $\mathfrak{psu}(2,2|4)$ algebra:

$$\begin{aligned}
\Omega(P) &= -K,\ \Omega(K) = -P,\ \Omega(D) = -D,\\
\Omega(L) &= L,\ \Omega(R_a) = -R_a,\ \Omega(R_s) = R_s,\\
\Omega(Q) &= S,\ \Omega(\bar{Q}) = \bar{S},\\
\Omega(S) &= Q,\ \Omega(\bar{S}) = \bar{Q}.
\end{aligned}$$

- Does a similar interpretation apply to the superstring on $AdS_5 \times S^5$?

We need to use the $\mathbb{Z}_4$-automorphism ($\Omega^4 = 1$) of the $\mathfrak{psu}(2,2|4)$ algebra:

$$
\begin{aligned}
\Omega(P) &= -K, \; \Omega(K) = -P, \; \Omega(D) = -D, \\
\Omega(L) &= L, \; \Omega(R_a) = -R_a, \; \Omega(R_s) = R_s, \\
\Omega(Q) &= S, \; \Omega(\bar{Q}) = \bar{S}, \\
\Omega(S) &= Q, \; \Omega(\bar{S}) = \bar{Q}.
\end{aligned}
$$

We can now build a spectral parameter dependent automorphism of the superconformal algebra as

$$T \mapsto \mathcal{U}_z(T) := U_z \, \Omega(T) \, U_z^{-1} \, , \qquad U_z := \mathrm{e}^{-\pi \mathrm{B}} \left(\frac{z - z^{-1}}{z + z^{-1}} \right)^{\mathrm{i}(\mathrm{B}+D)}$$

where B generates a $U(1)$-automorphism.

The Lax connections before and after T-duality are related by this $\mathfrak{psu}(2,2|4)$ automorphism:

$$\tilde{\mathrm{j}}(z) = \mathcal{U}_z(\mathrm{j}(z)).$$

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If we define

$$f(z) \;:=\; \frac{z - z^{-1}}{z + z^{-1}},$$

the action of the automorphism on the generators is

$$\begin{aligned}
\mathcal{U}_z(P) &= f\,\Omega(P), & \mathcal{U}_z(K) &= f^{-1}\,\Omega(K),\\
\mathcal{U}_z(D) &= \Omega(D), & \mathcal{U}_z(R) &= \Omega(R),\\
\mathcal{U}_z(L) &= \Omega(L), & \mathcal{U}_z(L) &= \Omega(L),\\
\mathcal{U}_z(Q) &= f\,\Omega(Q), & \mathcal{U}_z(S) &= f^{-1}\,\Omega(S),\\
\mathcal{U}_z(\bar{Q}) &= -\mathrm{i}\Omega(\bar{Q}), & \mathcal{U}_z(\bar{S}) &= \mathrm{i}\Omega(\bar{S}).
\end{aligned}$$

P	$\longrightarrow$	trivial
L	$\longrightarrow$	L
K	$\longrightarrow$	non-local
D	$\longrightarrow$	D
R	$\longrightarrow$	R
Q	$\longrightarrow$	trivial
$\bar{Q}$	$\longrightarrow$	$\bar{S}$
S	$\longrightarrow$	non-local
$\bar{S}$	$\longrightarrow$	$\bar{Q}$

Observation

We have seen that the P and Q charges become trivial after T-duality. We can also understand this behavior by observing that P and Q have a trivial action on the dual coordinates $\tilde{X}$ and $\tilde{\theta}$:

$$\mathrm{d}X \sim *\mathrm{d}\tilde{X}\,, \qquad \mathrm{d}\theta \sim \mathrm{d}\tilde{\theta}\,.$$

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Conclusions

- Dual superconformal symmetry arises naturally in string theory as a consequence of a duality transformation:

  bosonic+fermionic dualities $\longrightarrow$ dual superconformal symmetry

- The Noether charges of the T-dual model have their origin in the hidden charges of the original model and vice versa.

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How to make the connection with gauge theory more precise?

- Understand the precise relation between the superstring symmetries in the bulk and the symmetries of the super-amplitude in the boundary gauge theory.
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