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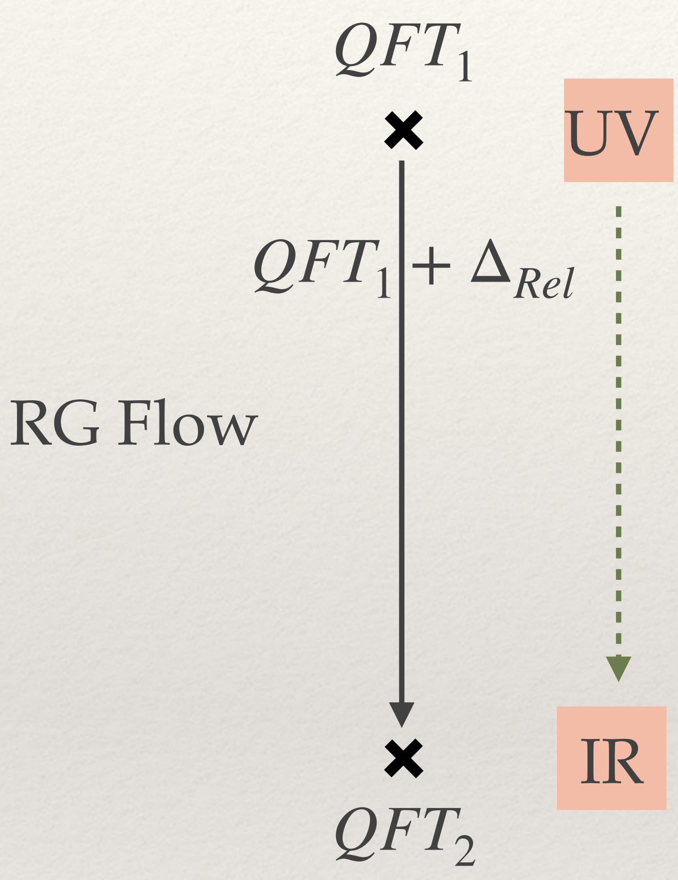
IR dualities across dimensions

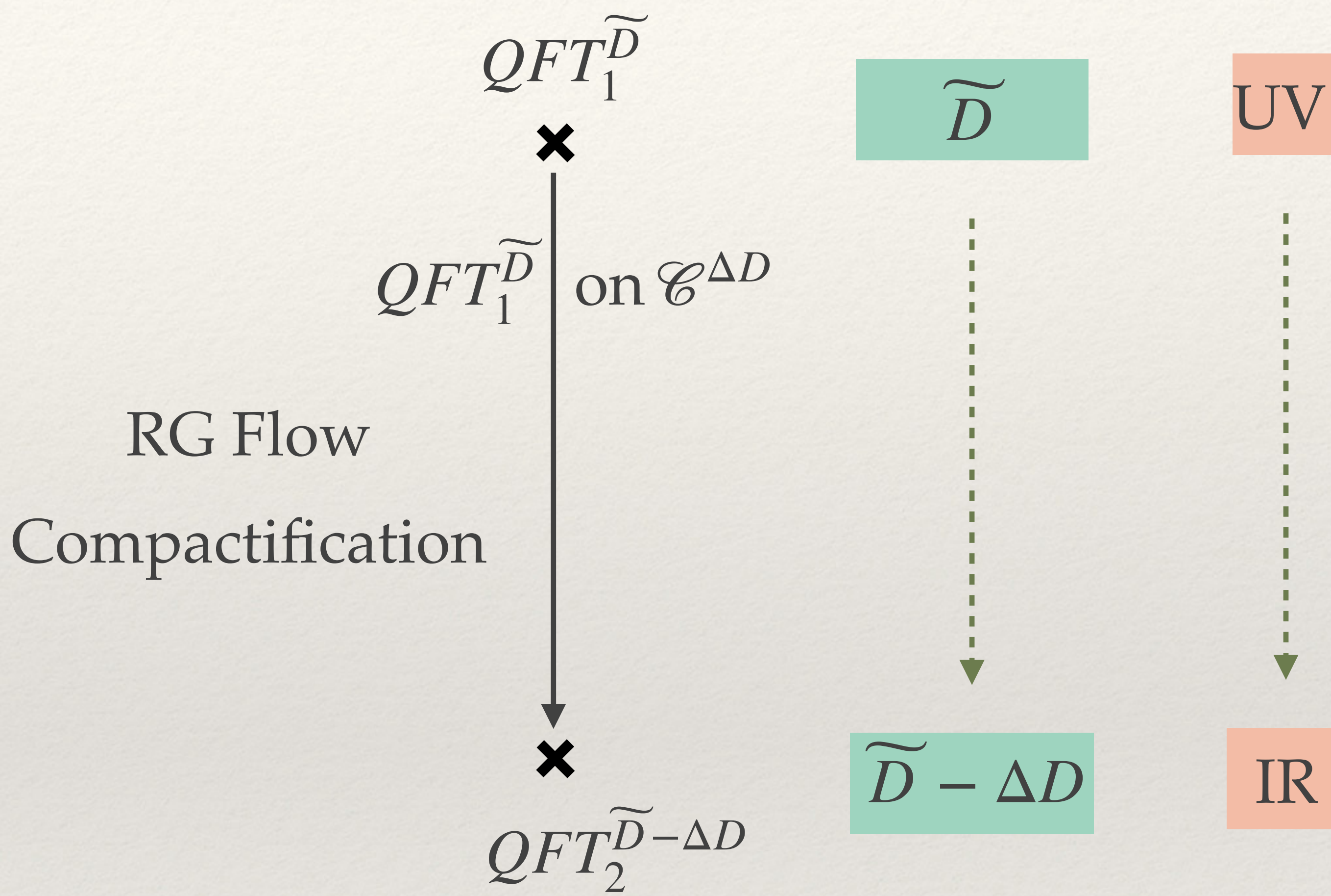
14/11/2022

31st Nordic Network Meeting on **Strings, Fields and Branes**
NBI, Copenhagen

Based roughly on 2203.06880 with E. Sabag, O. Sela, and G. Zafrir

RG Flows





- RG flows in a given dimension
- RG flows between dimensions

- *Our discussion will be supersymmetric, at least $\mathcal{N} = 1$ in 4D*

Outline:

A: Dualities

B: Dualities across dimensions

C: 4D dualities from geometry

D: Dualities from dualities

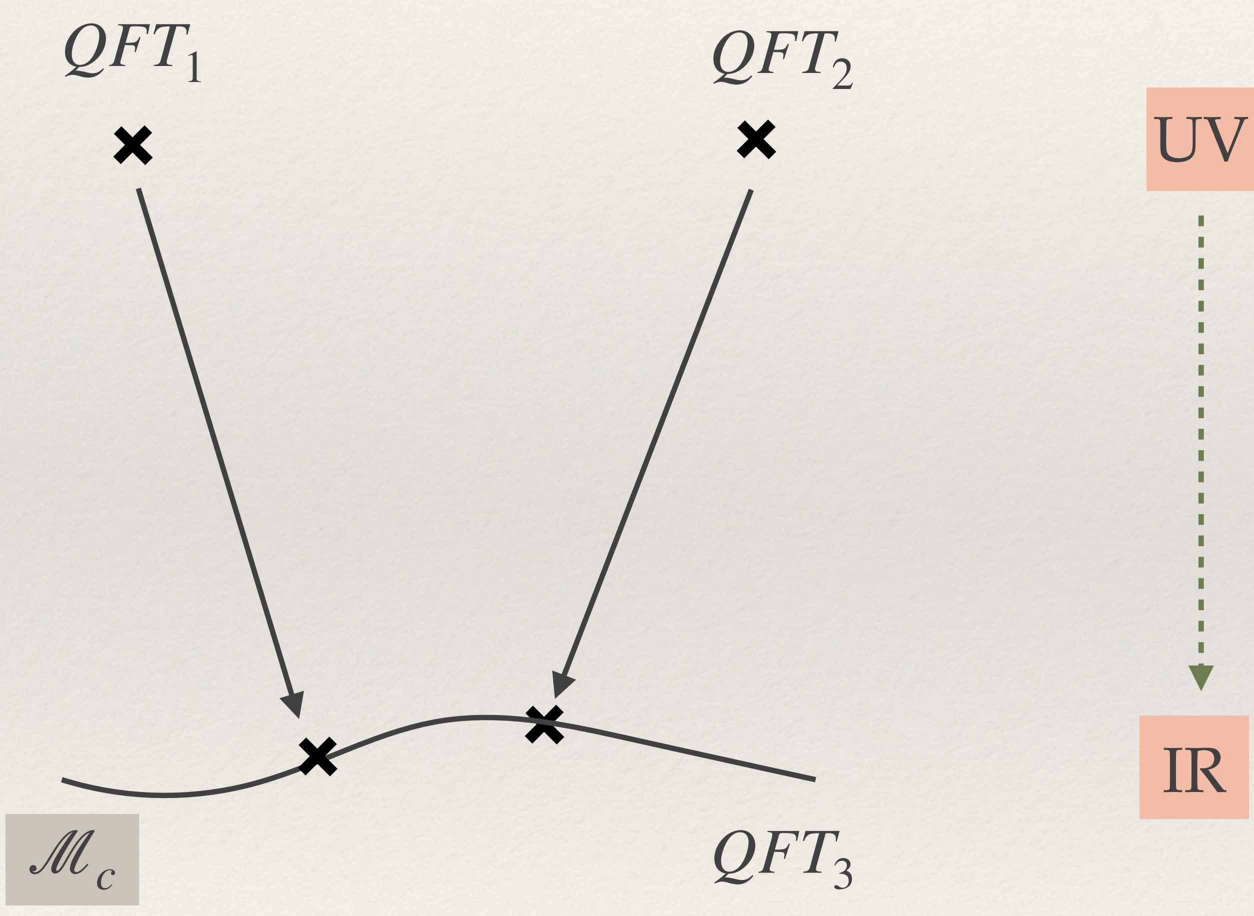
E: 4D flows from 6D flows

F: 6D Dualities

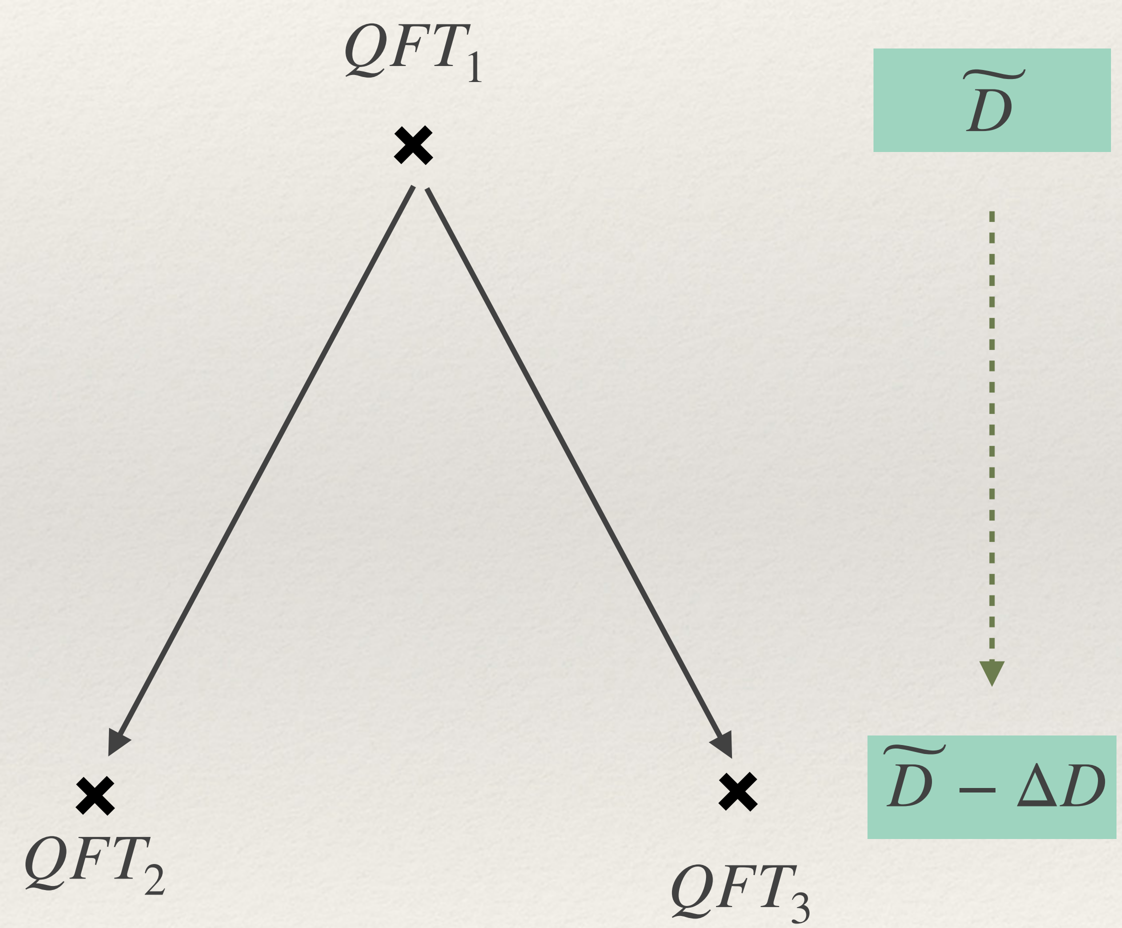
G: Deconstruction and fractons

A. Dualities

- IR Duality



- UV Duality

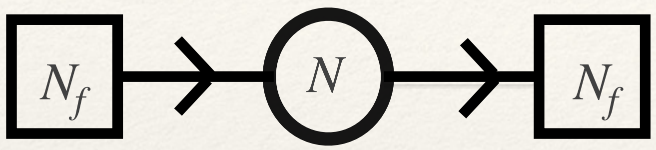


Eg: Dualities

- IR Duality

Seiberg 94

QFT_1 ×

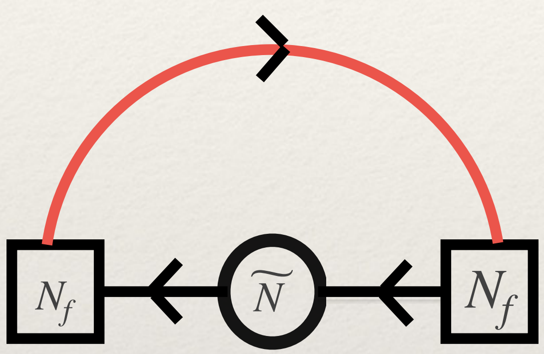


$N_f < 3N, N_f < 3\widetilde{N}$

CFT_3 ×

- Strongly coupled SCFT

QFT_2 ×



$\widetilde{N} = N_f - N$

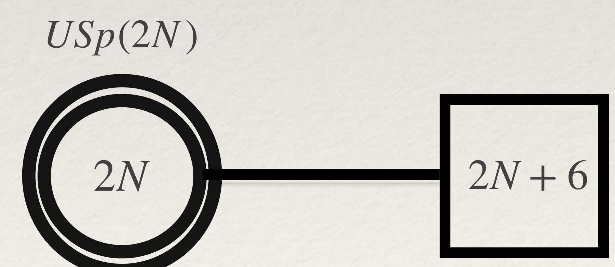
UV ⟷ IR

QFT_1 ×

- UV Duality

$\widetilde{D} = 6$

$\Delta D = 1$



QFT_2 ×

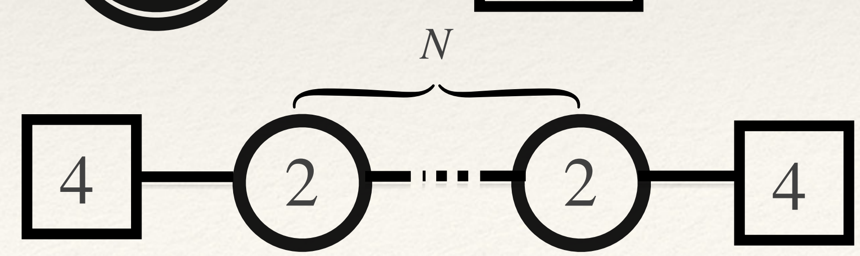
- Strongly coupled SCFT

CFT_1 ×

$\mathcal{N} = (1,0) \ \ (D_{N+3}, D_{N+3})$

Minimal Conformal Matter

QFT_3 ×



Hayashi, Kim, Lee, Taki, Yagi 15

The power of counting: no SUSY

- Anomalies and Symmetries

- Symmetries are tricky: (assume no spontaneous breaking)

  - G_{IR} can be bigger than G_{UV}: emergence of symmetry
  - G_{IR} can be smaller than G_{UV}: act not faithfully

- Anomalies are robust: `t Hooft Anomaly matching
- Eg: Weakly coupled Lagrangians count massless Weyl fermions weighed with charges/representations
- Higher form/Higher group/Categorical generalization

The power of counting: SUSY I

- Superconformal algebra

$$\{Q, \widetilde{Q}\} \sim P\,, \qquad \{Q, S\} \sim \Delta + J + R$$

- Conformal anomalies and R symmetry

$$\langle T^{\mu}{}_{\mu} \rangle \sim c\,W^2 - a\,E_4 \qquad a = \frac{9\mathrm{Tr}R^3 - 3\mathrm{Tr}R}{32}\,,\ c = \frac{9\mathrm{Tr}R^3 - 5\mathrm{Tr}R}{32}$$

- For this to be true R has to be the superconformal R-symmetry:
  a-maximization

$$a(\lambda_i) = \frac{9}{32} Tr(R + \lambda_i U(1)_i)^3 - \frac{3}{32} Tr(R + \lambda_i U(1)_i)$$

a is maximized

Intriligator, Wecht 03

The power of counting: SUSY II

- Power of holomorphy: non-renormalization theorems

- And various exact statements

- Eg marginal operators can be only either exactly marginal or marginally irrelevant

- Theories labeled by continuous parameters: conformal manifold $\mathscr{M}_c$

- Dimension of the conformal manifold also a counting problem

$$\mathscr{M}_c = \{\lambda_i\}/G_{\mathbb{C}}$$

(λ_i marginal couplings in W)

Green, Komargodski, Seiberg, Tachikawa, Wecht

The power of counting: SUSY III

Kinney, Maldacena, Minwalla, Raju 05; Romelsberger 05

- Counting local operators: Witten indices

$$\mathcal{I}(q,p,u_i) = \mathrm{Tr}_{\mathcal{H}} \left[(-1)^F q^{\frac{1}{2}R+J_1-J_2} p^{\frac{1}{2}R+J_1+J_2} \prod_{i=1}^{\mathrm{Rank}\, G_F} u_i^{Q_i} \right] e^{-\beta \{Q, Q^\dagger\}}$$

- Invariant of RG flow and continuous deformations

- Easy window to non-perturbative IR physics: can easily read off the spectrum of supersymmetric relevant and exactly marginal deformations, and global symmetry

$$\text{Marginals-Currents} = \#\, q\, p \in \mathcal{I}$$

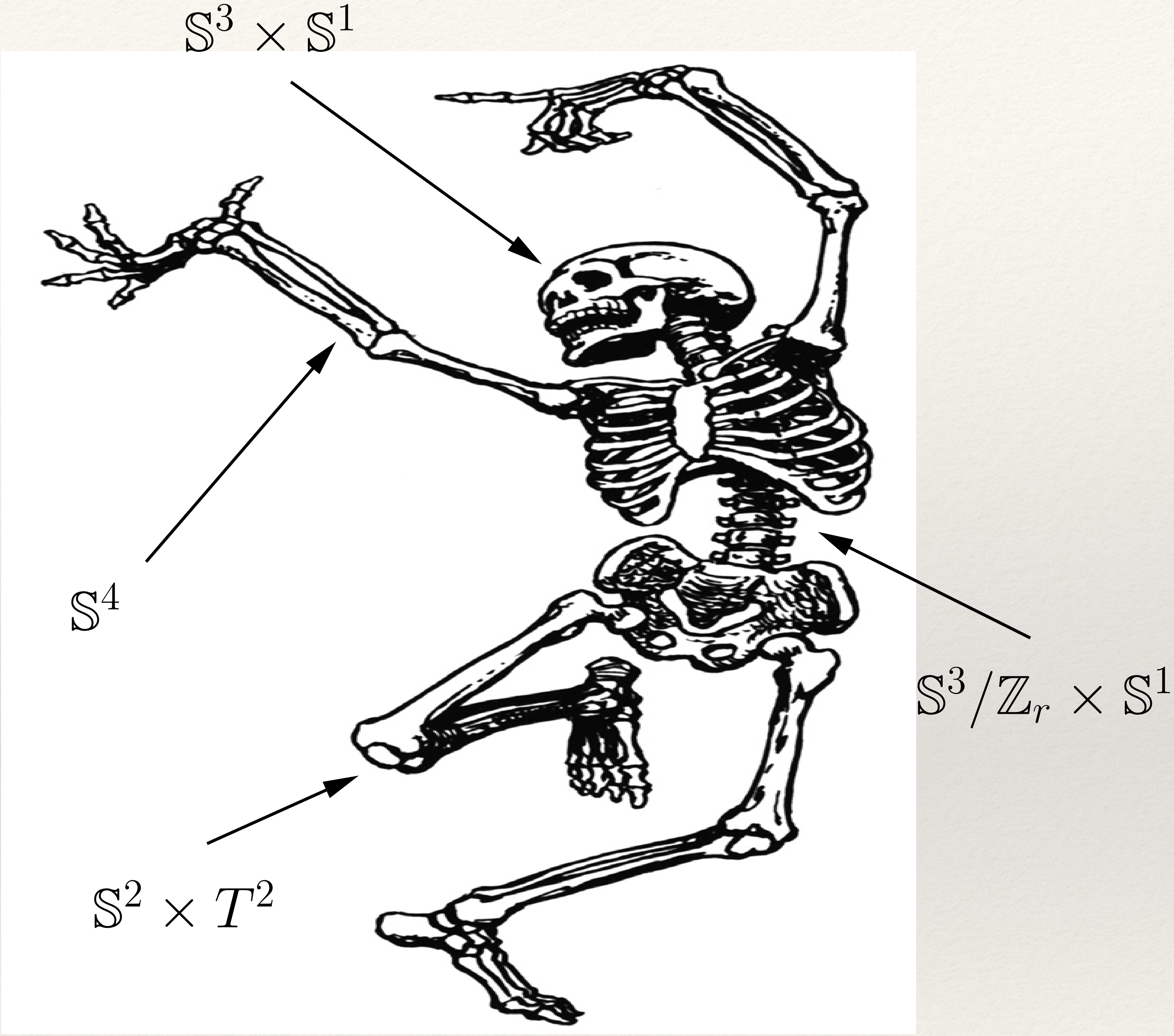
Beem, Gadde 12

- Eg often determines the IR symmetry group (*)

- Various countings give us the skeleton of the (IR) possibly strongly coupled theories

- We then are able to deduce some properties (such as symmetries)

- and Conjecture more interesting dynamics such as IR dualities



Consistency checks

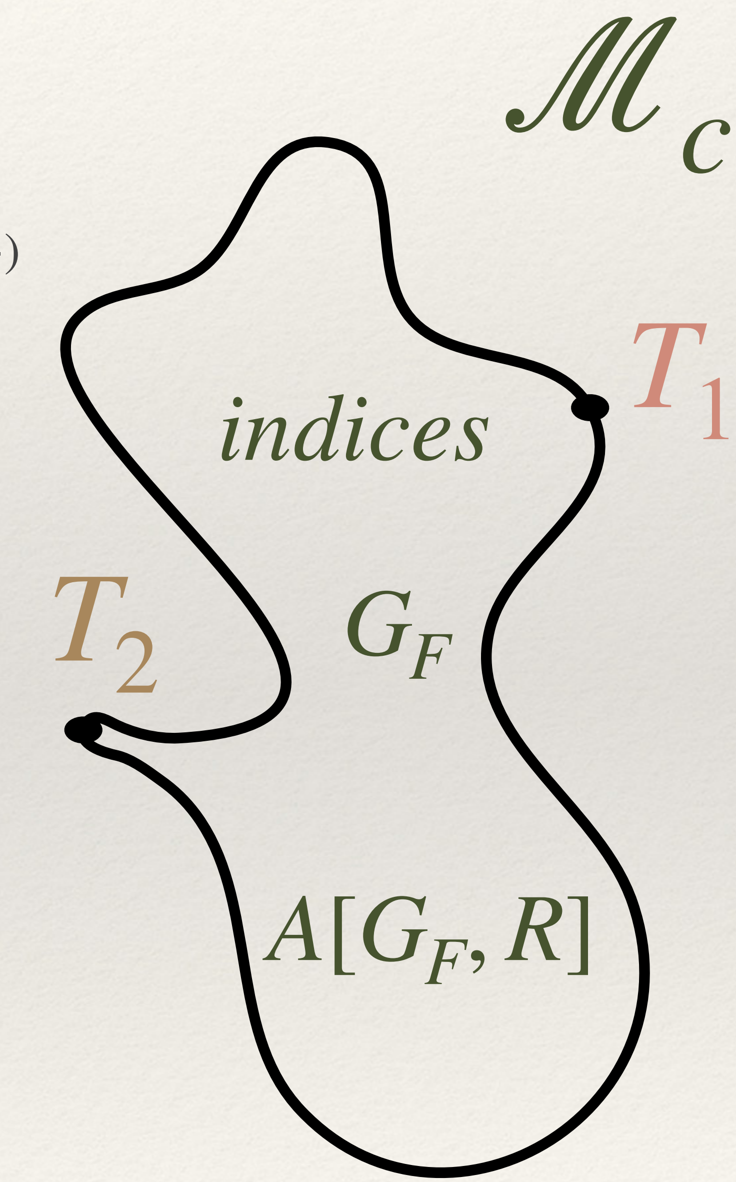
- Once a duality/emergence of symmetry/etc is conjectured
- one can perform a plethora of consistency checks
- RG flows/Moduli spaces of vacua (often included in counting)
- Obtain the same thing in two different way following the same logic

Eg: Conformal duals from counting

- Given an SCFT T_1 with a and c central charges
- $a = \frac{9\mathrm{Tr}R^3 - 3\mathrm{Tr}R}{32}, \; c = \frac{9\mathrm{Tr}R^3 - 5\mathrm{Tr}R}{32}$

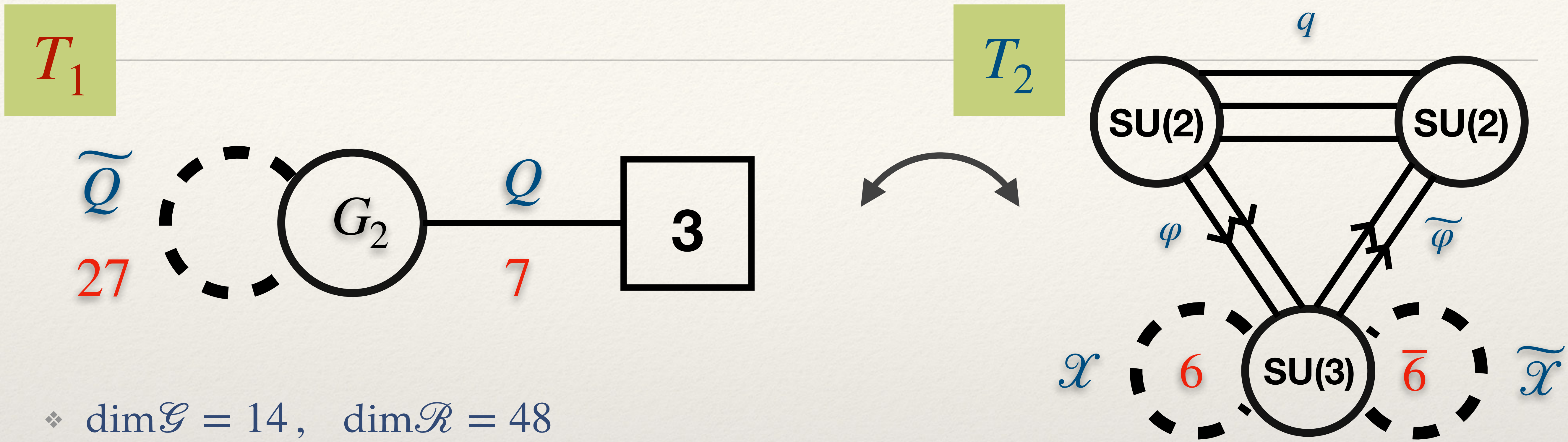
$(a_v = \frac{3}{16}, c_v = \frac{1}{8})$
$(a_\chi = \frac{1}{48}, c_\chi = \frac{1}{24})$

- Assume conformal gauge theory dual on $\mathscr{M}_c$, T_2
- $a = a_v \dim \mathscr{G} + a_\chi \dim \mathscr{R}, \;\; c = c_v \dim \mathscr{G} + c_\chi \dim \mathscr{R}$
- Seek for conformal gauge theories with free dim $\mathscr{R}$ chiral fields and dim $\mathscr{G}$ vector fields
- Out of the finite set of such theories seek for models with same $\mathscr{M}_c$ invariants: e.g. symmetry on $\mathscr{M}_c$, `t Hooft anomalies, indices etc
- If such models exist they are putative conformal duals

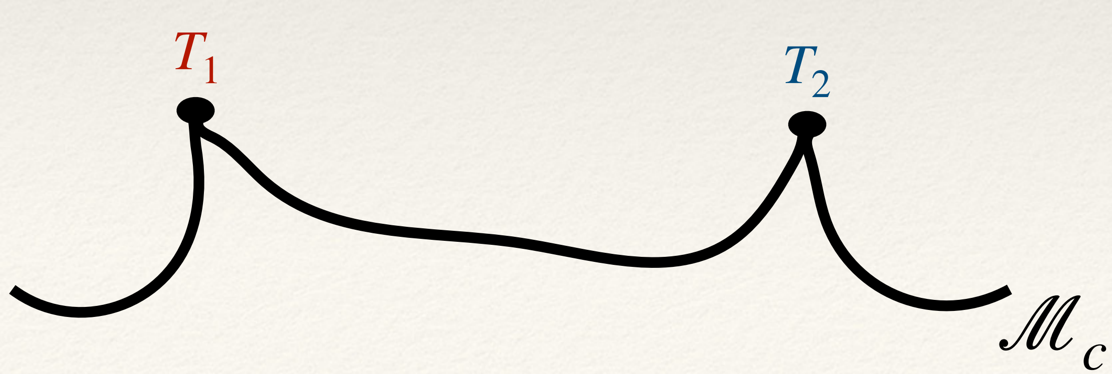


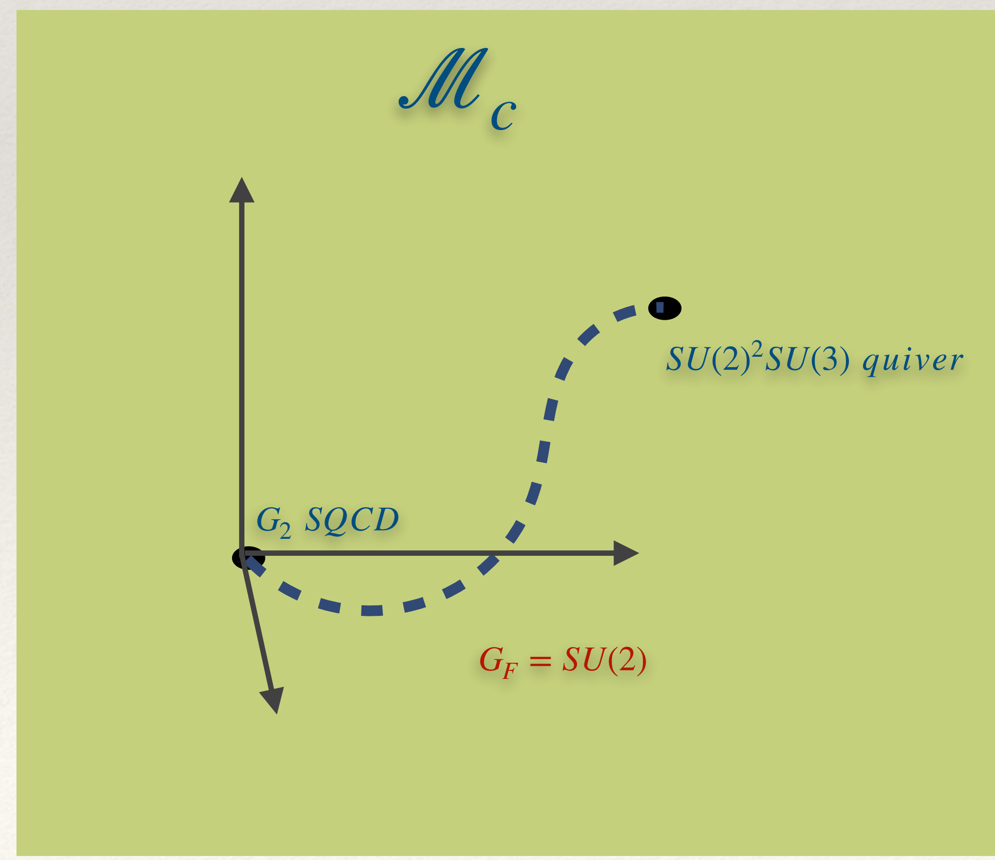
SSR, Zafrir 19

Example: Conformal dual of an exceptional SQCD



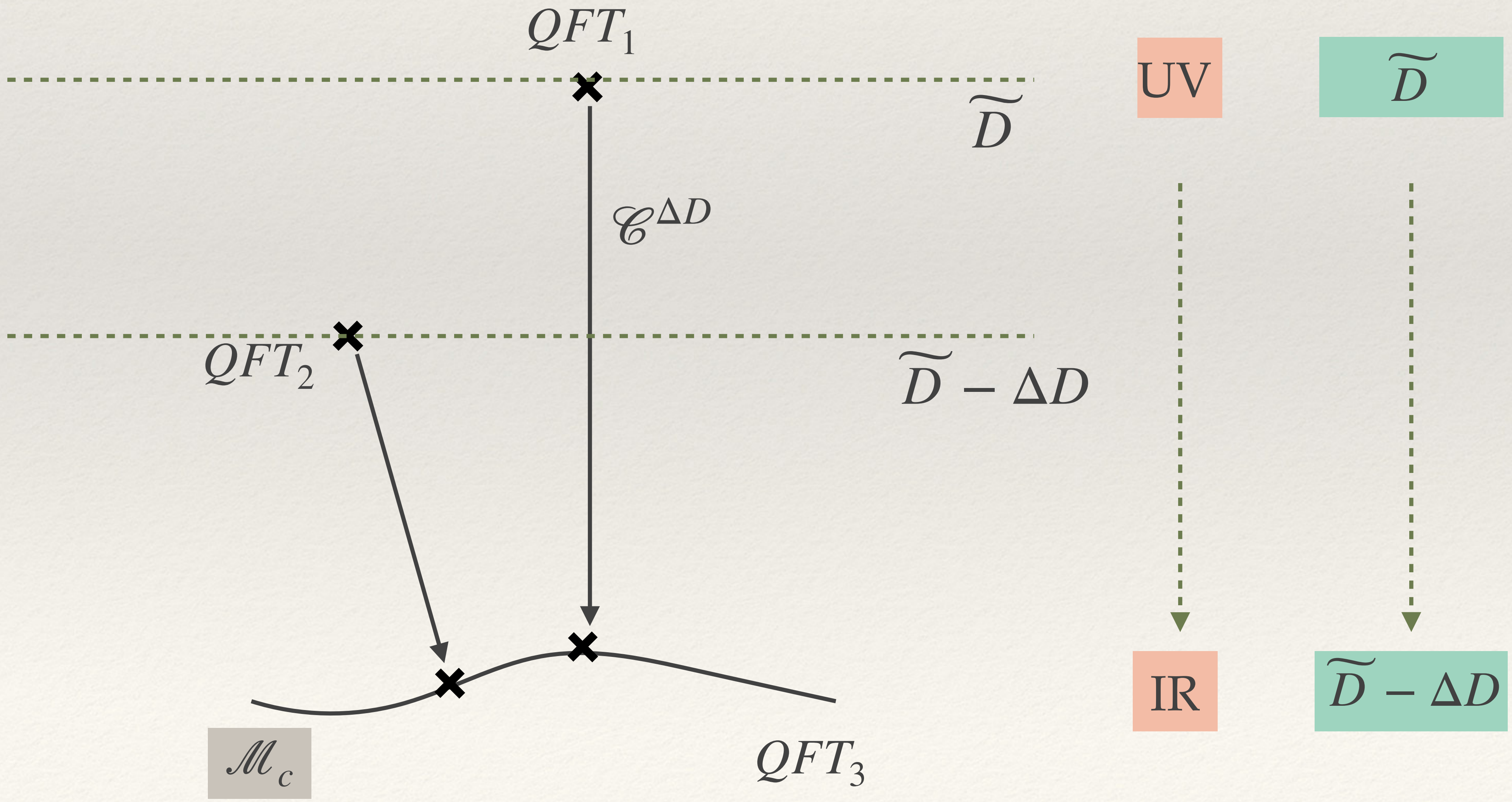
- $\dim \mathscr{G} = 14\,, \quad \dim \mathscr{R} = 48$
- $\dim \mathscr{M}_c = 3\,, \;\; G_F = SU(2)\,, \;\; \mathrm{Tr}\, R\, SU(2)^2 = -\dfrac{14}{3}$
- Relevant operators $1 + 1 + \mathbf{5}$ of $SU(2)$
- 14=3+3+8





B. Dualities across dimensions

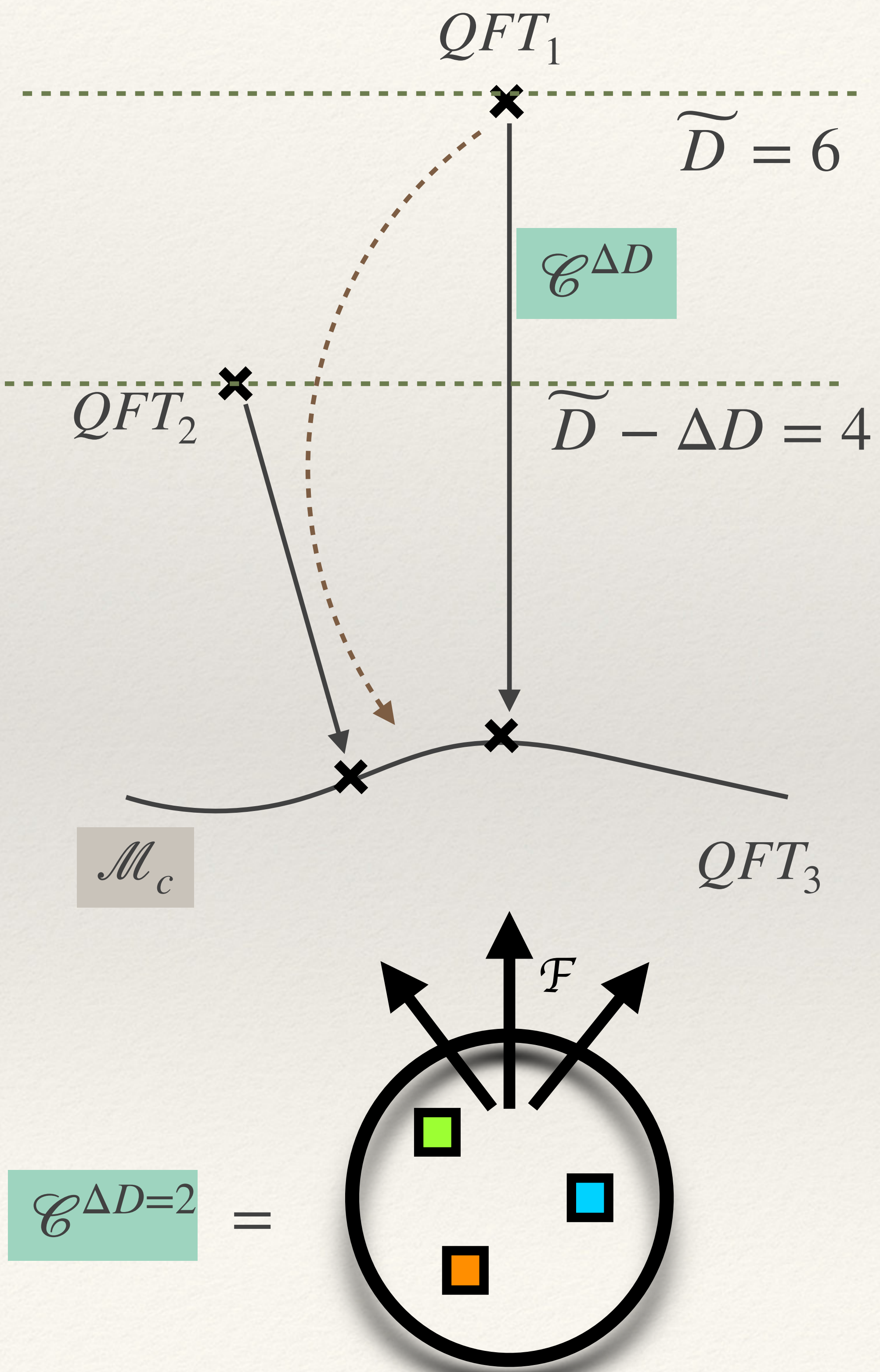
- Can consider a higher dimensional theory ``deformed'' by compact geometry
- Flowing to a lower dimensional QFT in the IR
- The IR QFT might have a different, dual, description starting from a weakly coupled QFT in a lower dimension



Independent descriptions

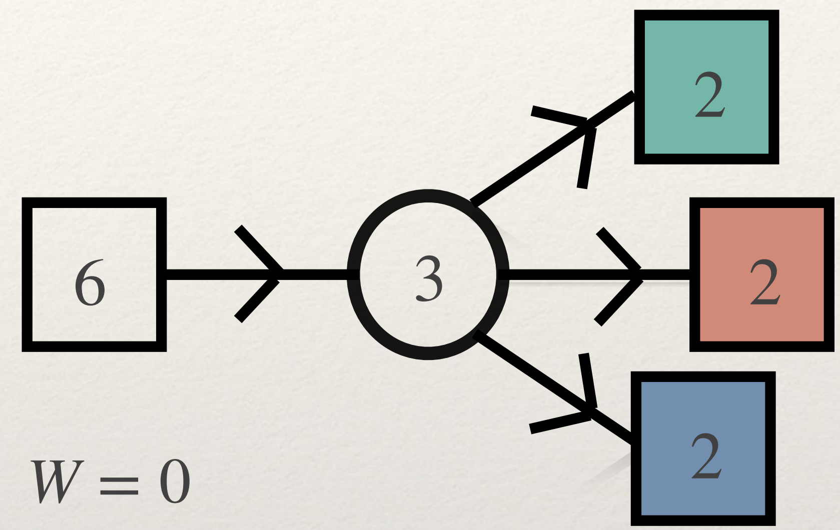
- We assume QFT_1 and QFT_2 have independent descriptions
- These can be Lagrangian or Stringy, but independent
- (Otherwise the duality has no content)
- Eg without this the lower dimensional theories are sometimes called
- Non-Lagrangian CFTs:
- only defined through compactification

Examples



- **Ex 1:** $QFT_1 = (D_4, D_4)$ min. conf. matt.

$QFT_2 =$

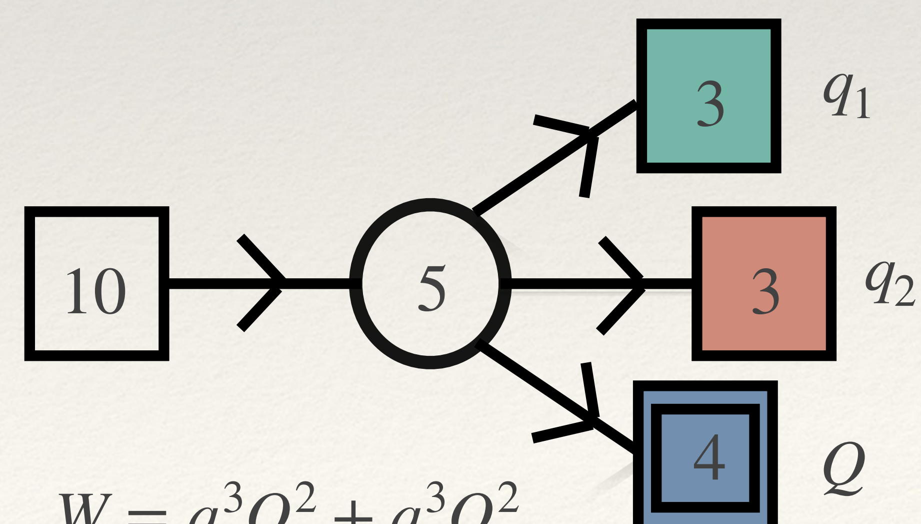


SR, Sabag 2020

$W = 0$

- **Ex 2:** $QFT_1 = (D_5, D_5)$ min. conf. matt.

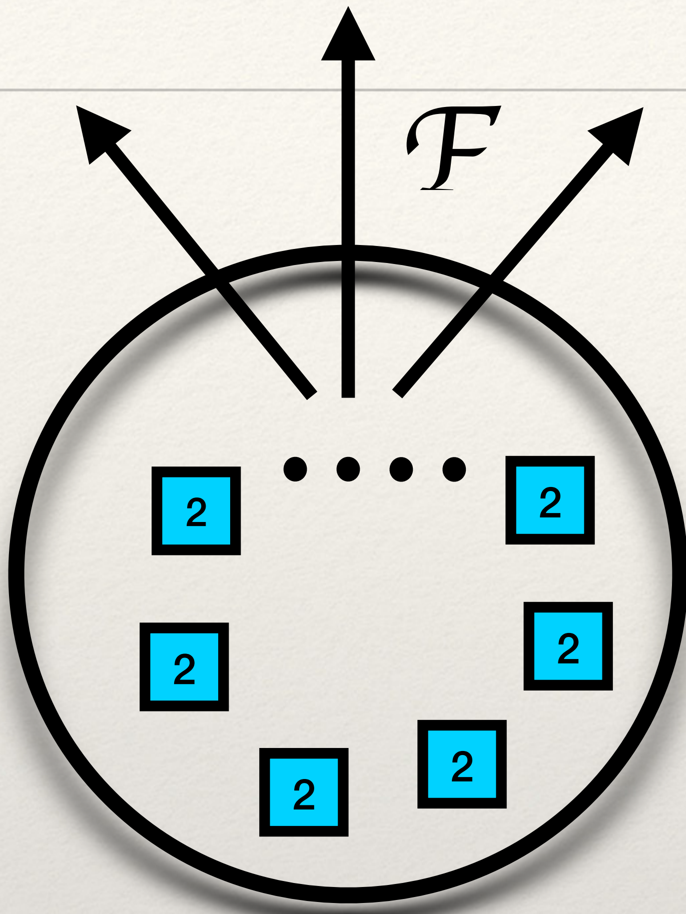
$QFT_2 =$



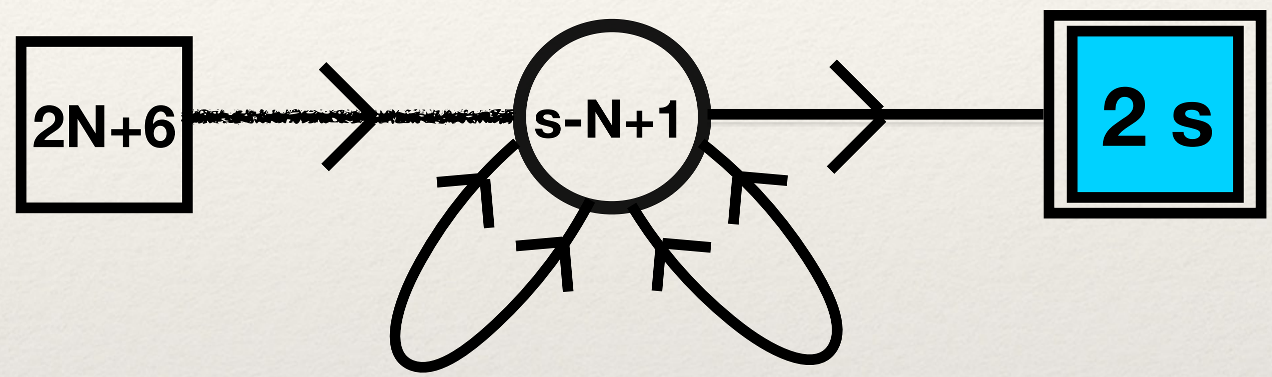
Nazzal, Nedelin, SR 2022

$W = q_1^3 Q^2 + q_2^3 Q^2$

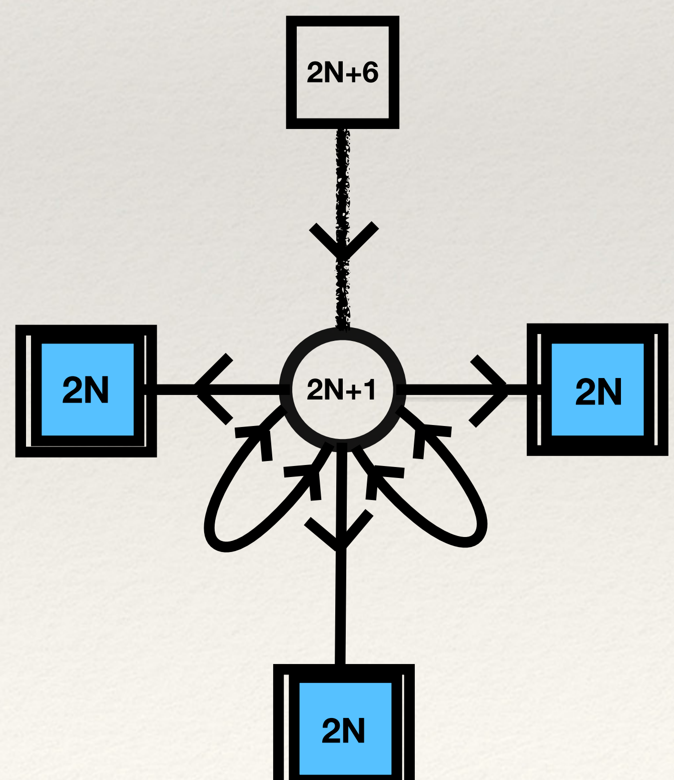
Examples

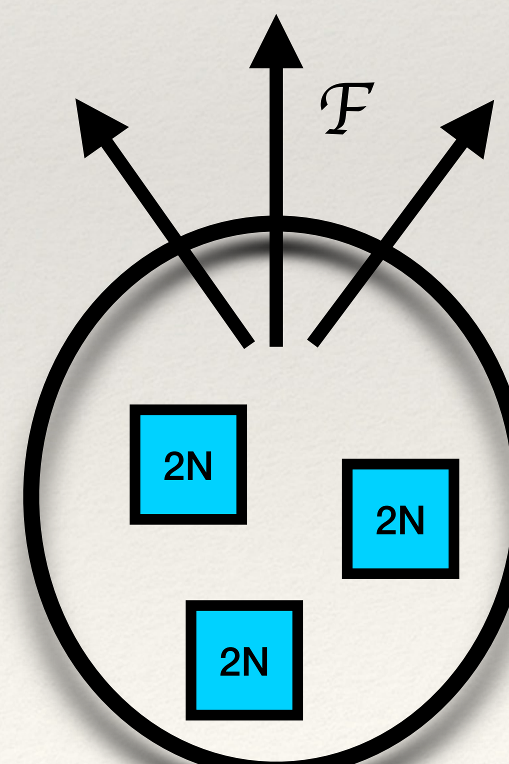


- **Ex 3:** $QFT_1 = (D_{N+3}, D_{N+3})$ min. conf. matt.



- **Ex 4:** Special case — three $USp(2N)$
  Maximal punctures



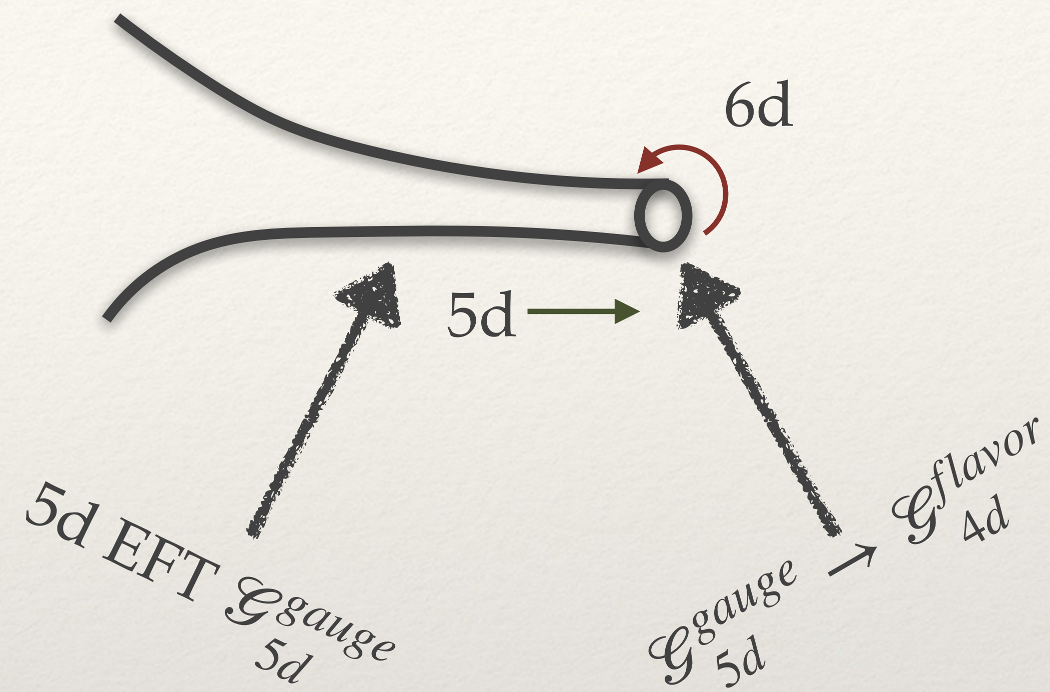


- Completely Lagrangian trinions

Kim, SR to appear

Interlude: "Caricature" of Punctures and 5d

- Compactifying on a surface with punctures we can elongate the region near the puncture into a long cylinder with a boundary
- On a cylinder, with suitable holonomies, get sometimes effective description as a 5d gauge theory
- Natural boundary conditions freezing the 5d gauge group and makes it 4d global symmetry (maximal puncture)
- The matter fields with Neumann boundary condition give a natural set of 4d operators charged under this symmetry
- Different choices of bc can lead to a variety of punctures (colors)



A_{N-1}(2,0) 5d EFT:

$$\mathcal{G}_{5d}^{gauge} = SU(N)\,,\ \oplus \text{Adj.}$$

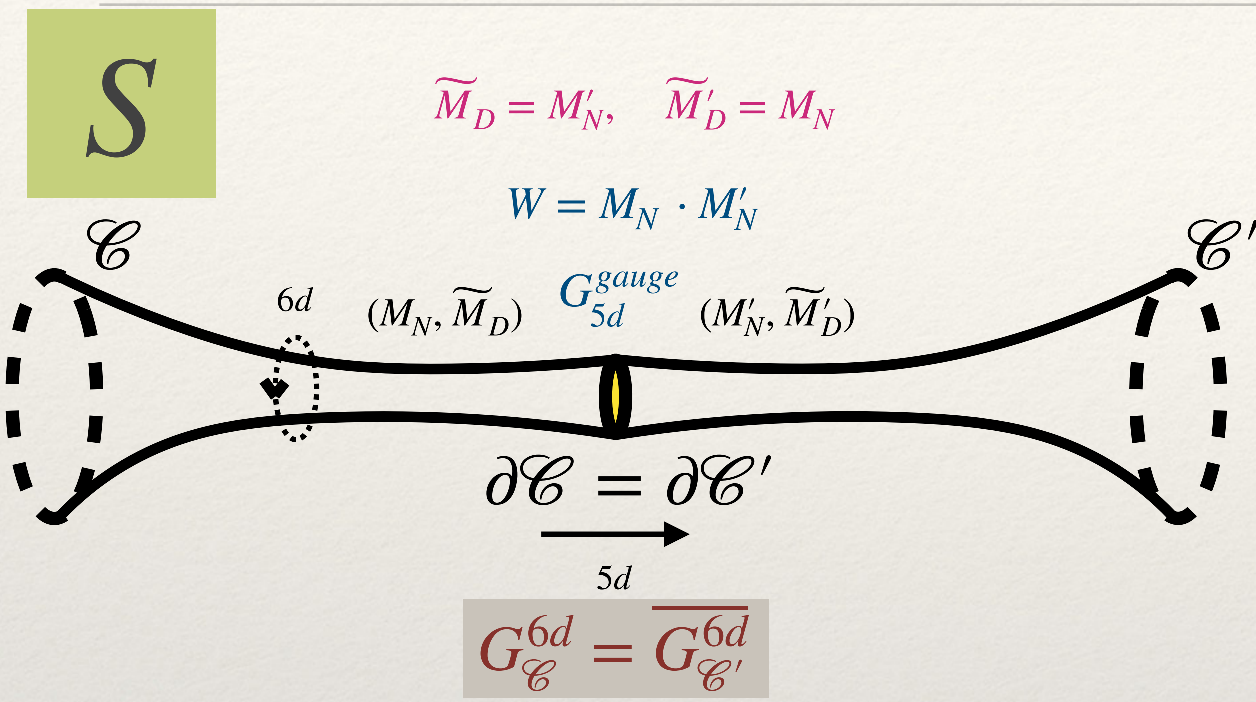
Moment maps: 1 Adj χ-op.

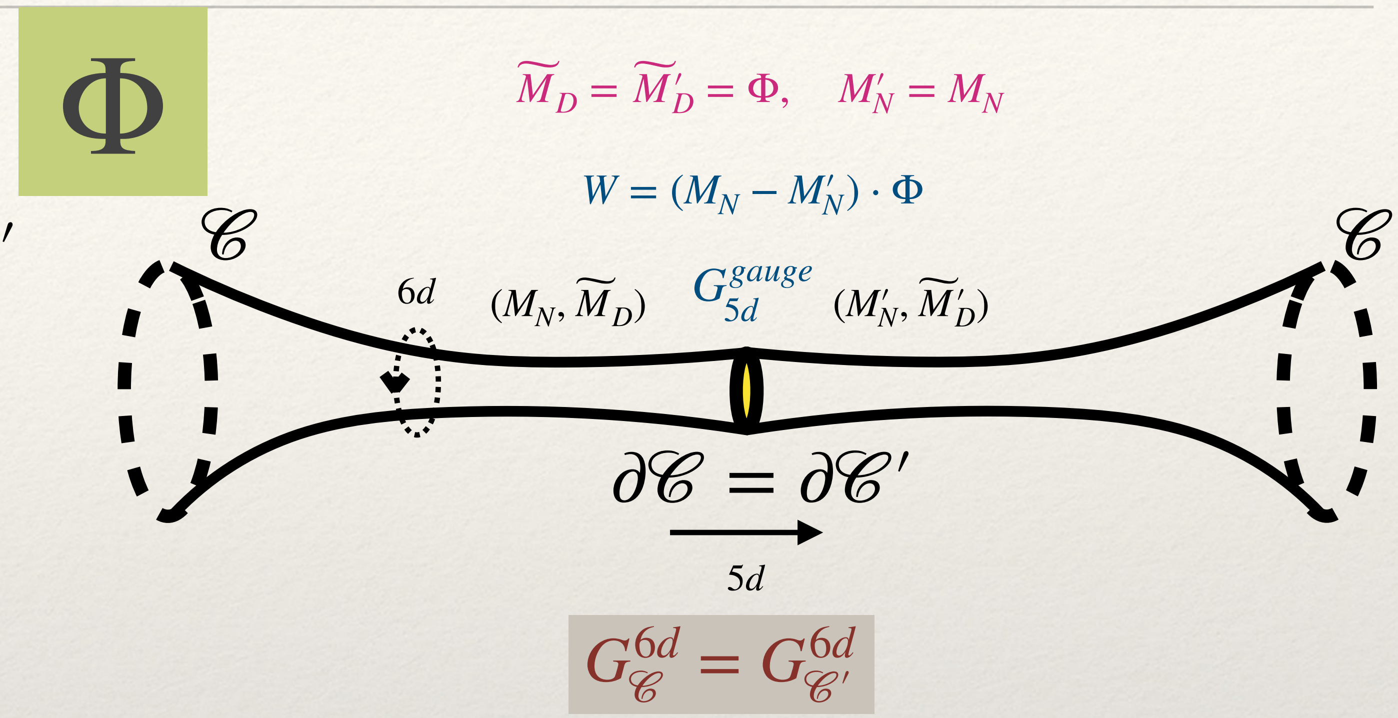
E-string 5d EFT: Ganor, Morrison, Seiberg 1996

$$\mathcal{G}_{5d}^{gauge} = SU(2)\,,\ N_f = 8$$

"Moment maps": 8 $\square$ χ-op.

Interlude: Gluing punctures

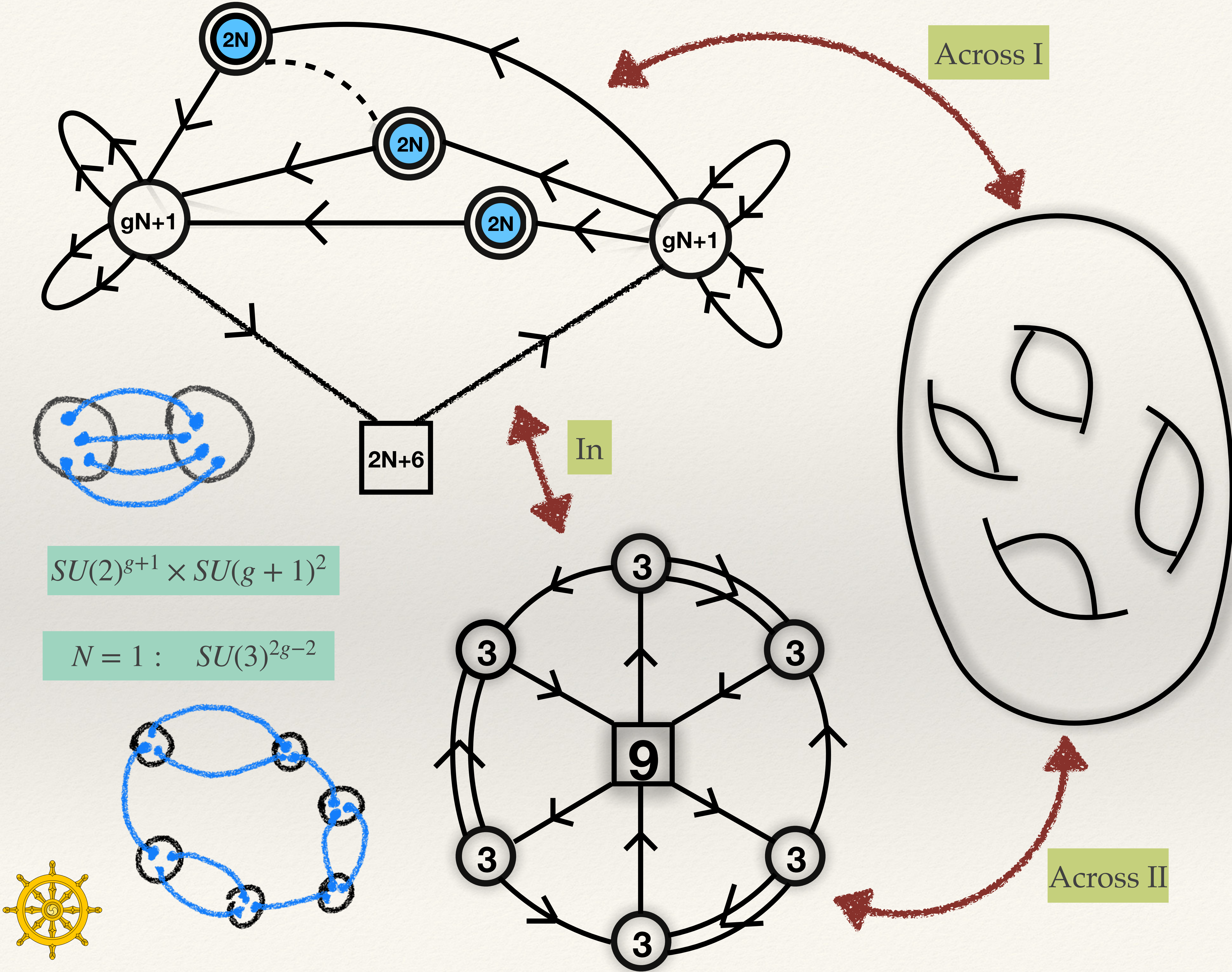




- Gluing punctures we gauge the puncture G_{5d}^{gauge} symmetry, add charged fields, and turn on a superpotential
- There are choices how to glue related to choices of identifying the symmetries of the glued theories
- More general gluings: $W = \sum_i M_i M'_i + \sum_j \Phi_j \cdot (M_j - M'_j)$

Flip fields

(Can have global obstructions: in 4d due to gauge and Witten anomalies)


2N
2N
2N
gN+1
gN+1
2N+6
Across I
In
$SU(2)^{g+1} \times SU(g+1)^2$
$N = 1: \quad SU(3)^{2g-2}$
3
3
3
9
3
3
3
Across II

Derivation from counting: E-string $((D_4, D_4))$

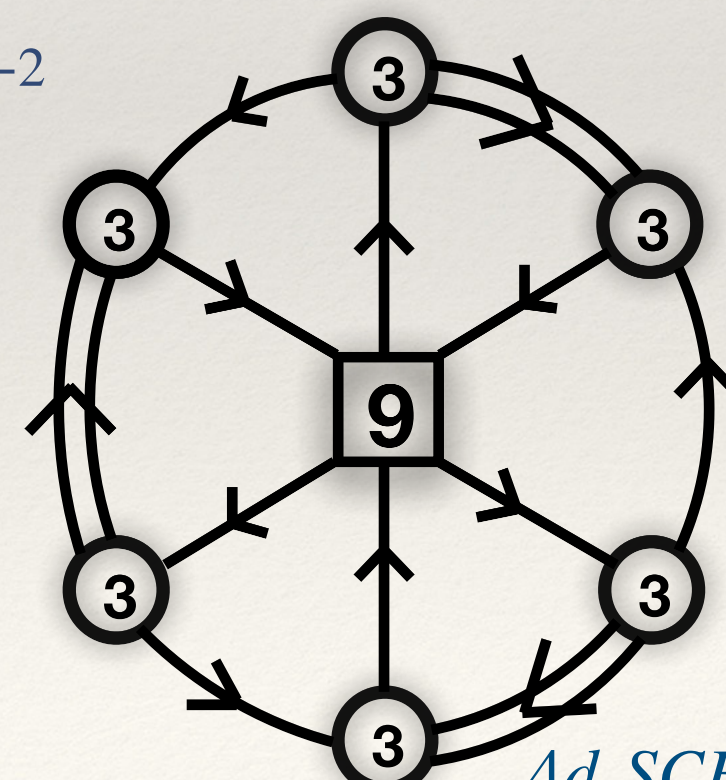
- Take $6d\ SCFT_{UV}$ to be rank 1 E-string and $\mathscr{C}_g$ genus $g > 1$ surface $(\mathscr{F} = 0)$
- Anomaly in **4d**: $$I_6 = \int_{\mathscr{C}_g} I_8 \to \quad a = \frac{75}{16}(g-1), \quad c = \frac{43}{8}(g-1)$$
- Assume in 4d described by **Conformal** Gauge Theory
- $\to \quad \dim \mathscr{G} = 16\,(g-1)\,, \quad \dim \mathscr{R} = 81\,(g-1),$ 16=8+8
- **Fits a circular** $\mathscr{N} = 1$ **quiver with** $\mathscr{G} = SU(3)^{2g-2}$

E-string on:

$\mathscr{C}_g$

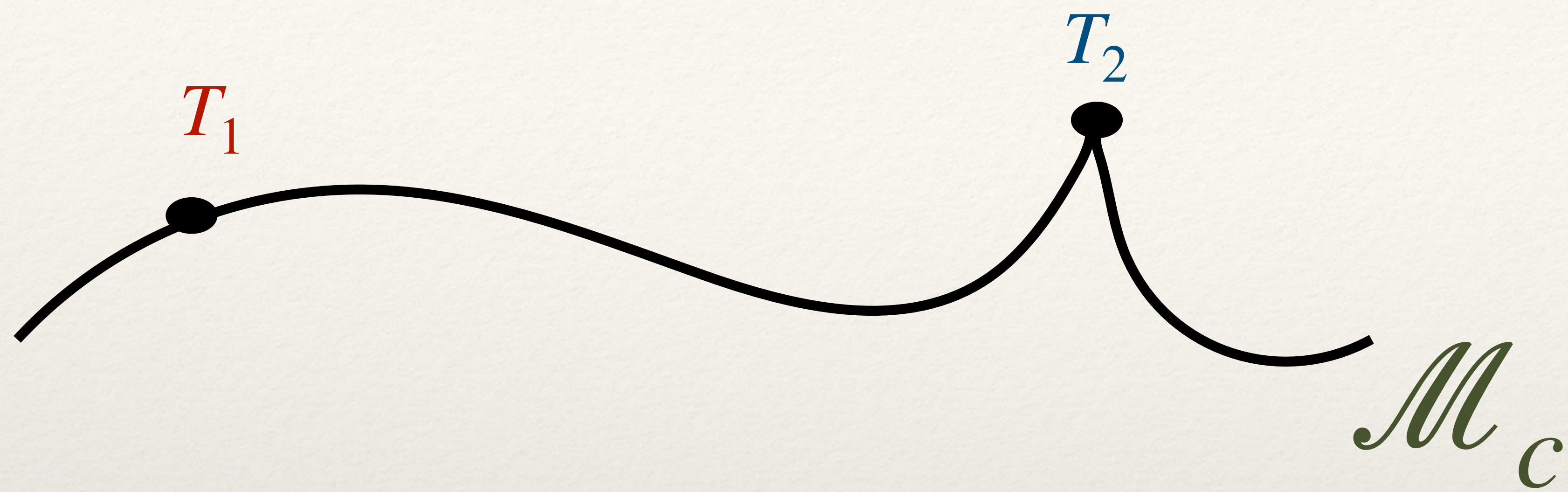
Conformal

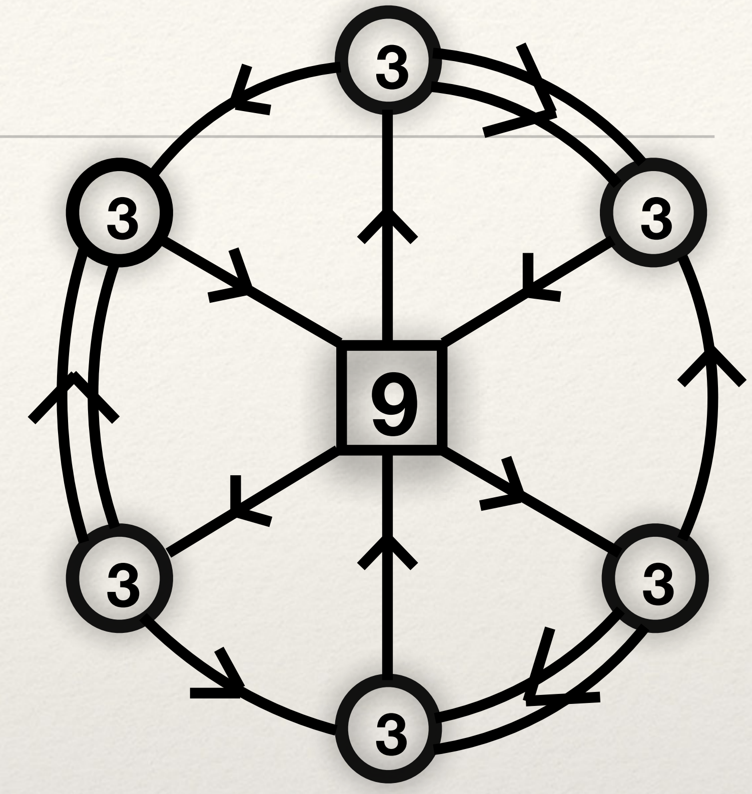
dual



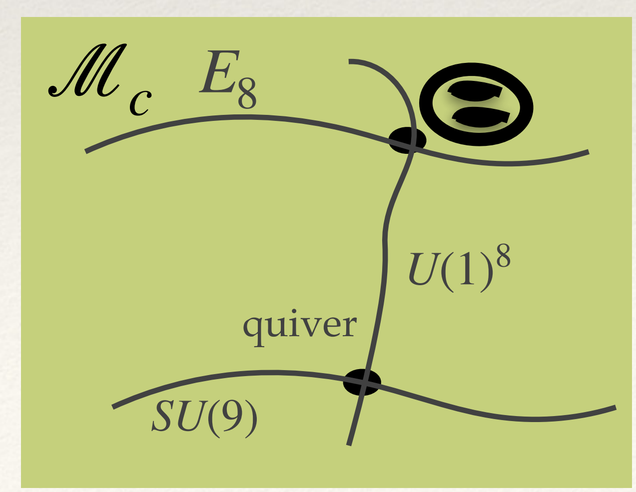
$4d\ SCFT_{UV}[\mathscr{C}_g]$

Basic evidence for the conjecture

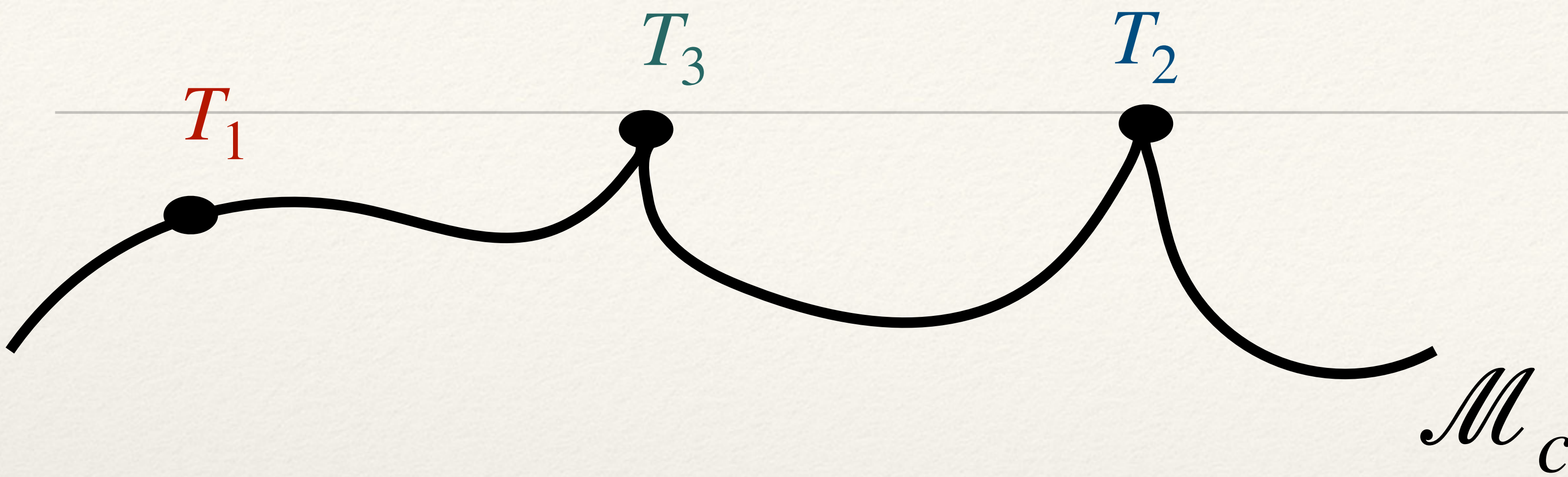




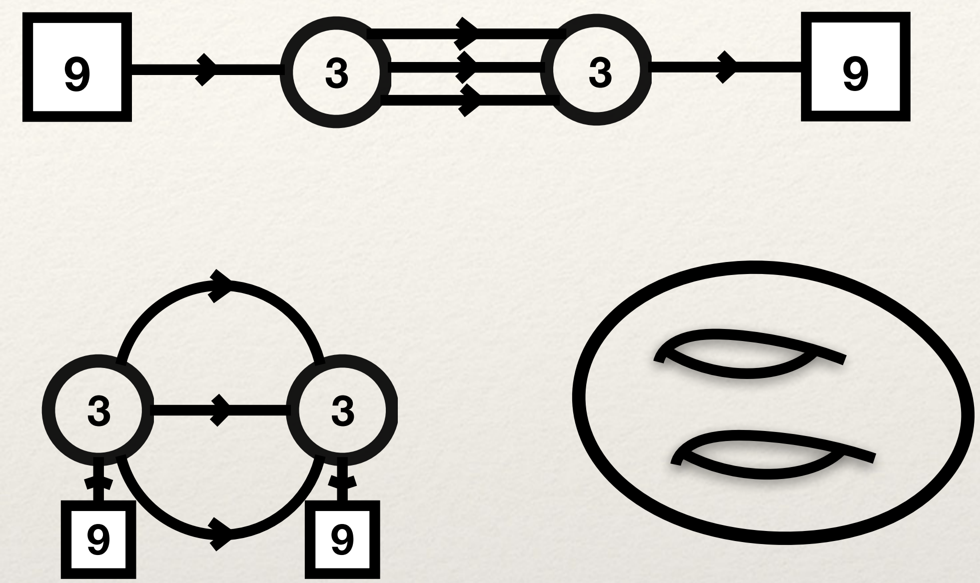
- 6d: Symmetry preserved during the flow is E_8
- $\dim \mathscr{M}_{conf.} = (3g-3) + (g-1)\,\mathbf{248},\qquad G_F = \varnothing,\qquad \mathrm{Tr}\, R\, E_8^2 = -(g-1)$
- 4d: The above is indeed the conformal manifold of the quiver theory: Superpotentials from Baryons and triangles: $\mathbf{248} \to \mathbf{80} + \overline{\mathbf{84}} + \mathbf{84}$
- Cartan of $SU(9) \quad \to \quad E_8\,,\qquad \mathrm{Tr}\, R\, SU(9)^2 = -(g-1)$
- Superconformal index matches expectations



More duals: $g = 2$

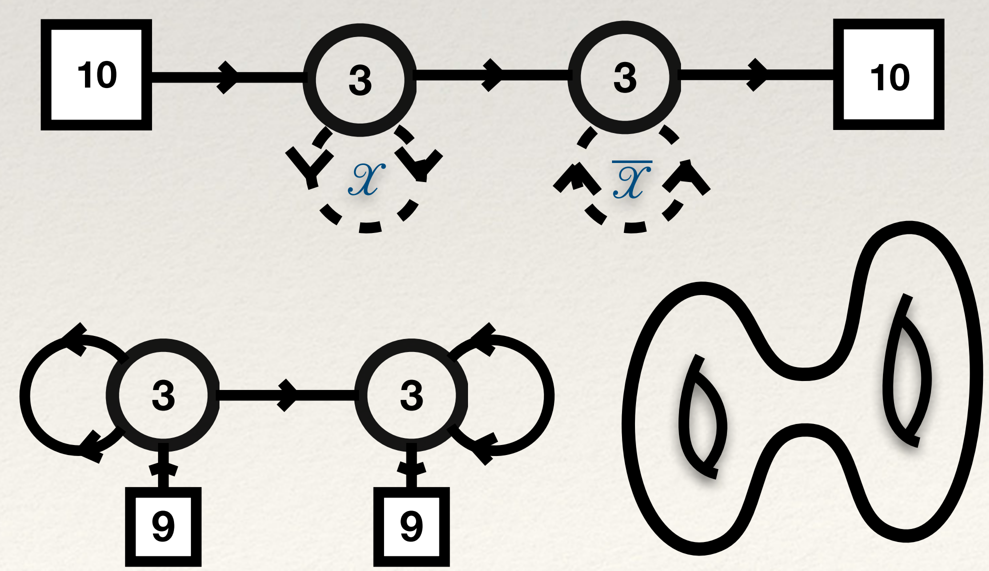


T_2

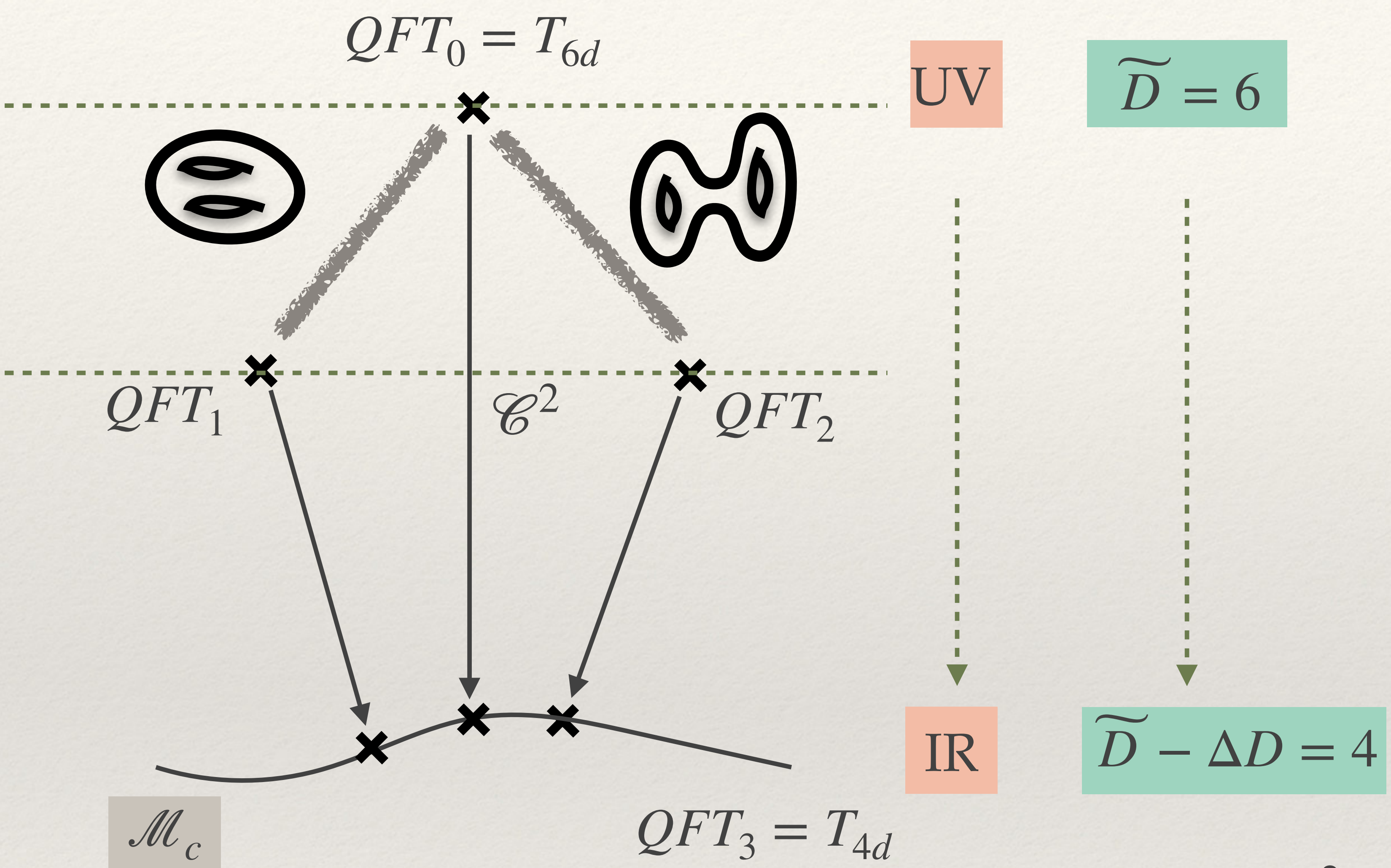


- We can find different Lagrangians fitting the bill
- These then all are conjectured to be dual to each other: novel looking conformal duality
- Looking at the duals T_2 and T_3 there is a hint of ``pairs of pants'' decomposition ($\mathbf{3} \times \mathbf{3} = \overline{\mathbf{3}} + \mathbf{6}$)
- The dual frames come from two different splittings of the surface into pairs of pants

T_3



C. 4d dualities from geometry



Geometric operation

$$(\mathscr{C}^2 = \oplus_i \mathscr{C}_i^2)$$

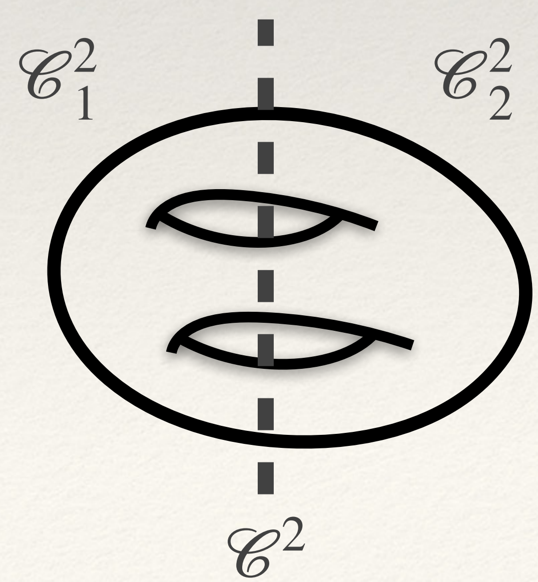
$$T_{4d}[T_{6d}; \mathscr{C}^2] =$$

$$= \otimes_i T_{4d}[T_{6d}; \mathscr{C}_i^2]$$

QFT operation

- Duality across dimensions might ``explain'' in-dimension dualities through a geometric decomposition

- Eg:



$\oplus$ =

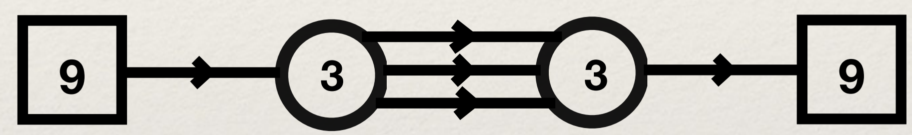
Gluing surfaces and Summing the fluxes

$\otimes$ =

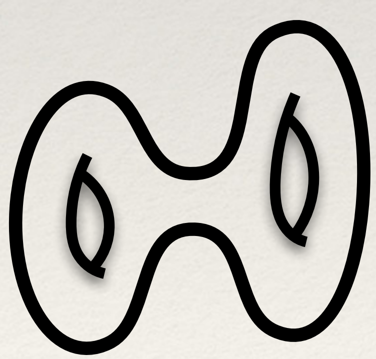
Gauging and coupling

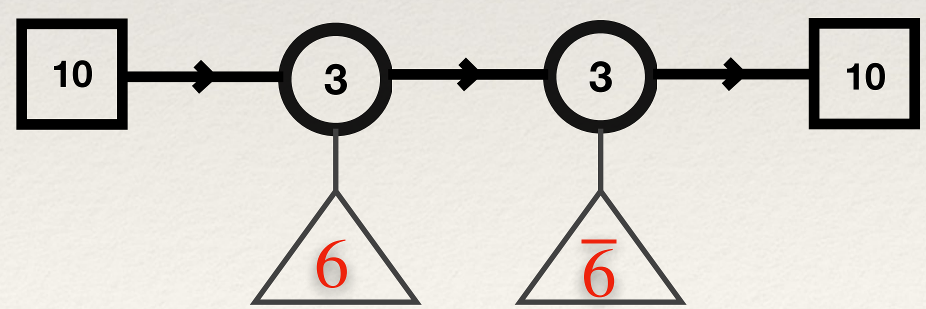
Example 1

- Ex 1: (D_4, D_4) min. conf. matt. on $\mathscr{C}^2_{g=2}$



(Conformal duality)

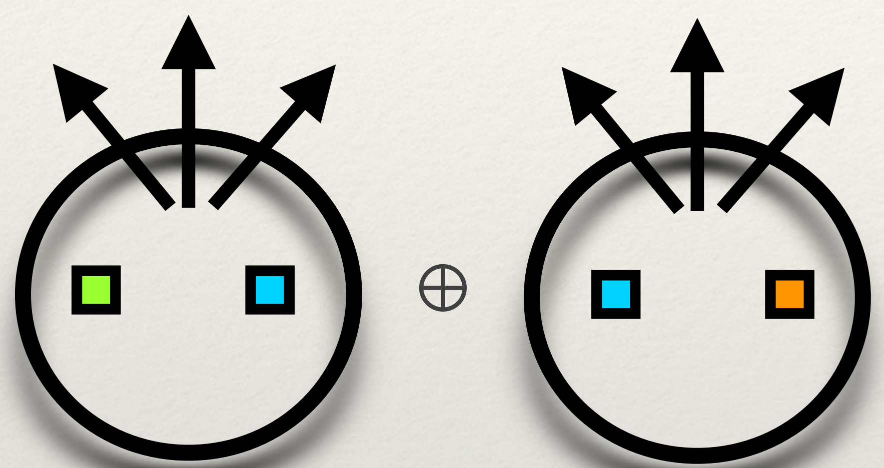




SR, Zafrir 19

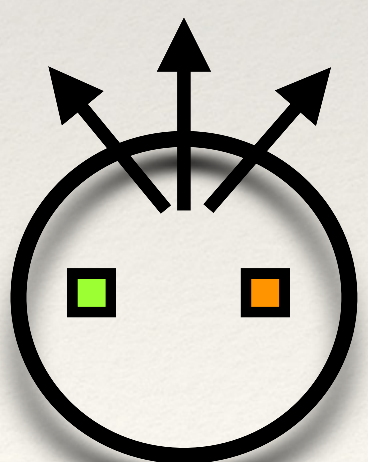
Example 2

- Ex 2: (D_4, D_4) min. conf. matt. on $\mathcal{C}^2_{g=1,s=2}$ and flux $(G_{6d} = E_8,\ \text{flux for Cartan})$

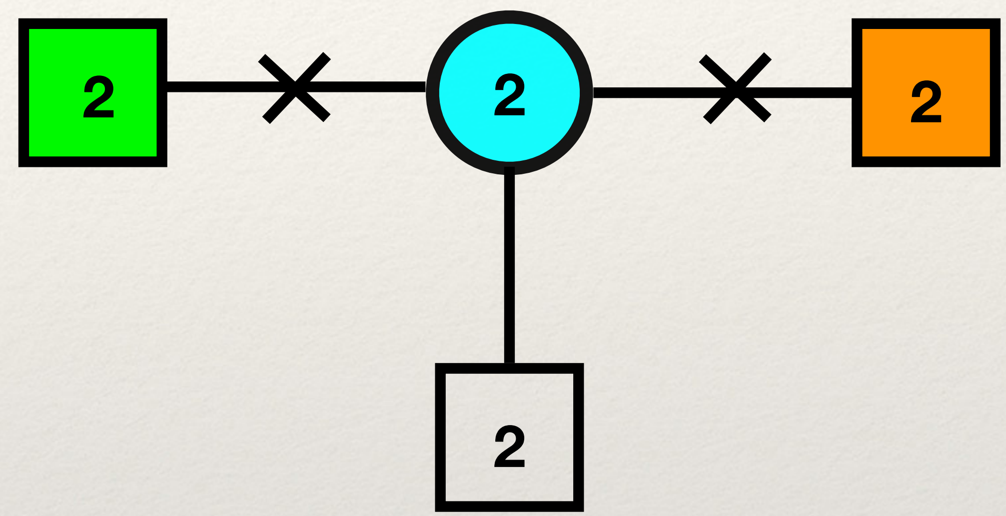


$\mathcal{F}_1 = (\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$ $\mathcal{F}_2 = (\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$

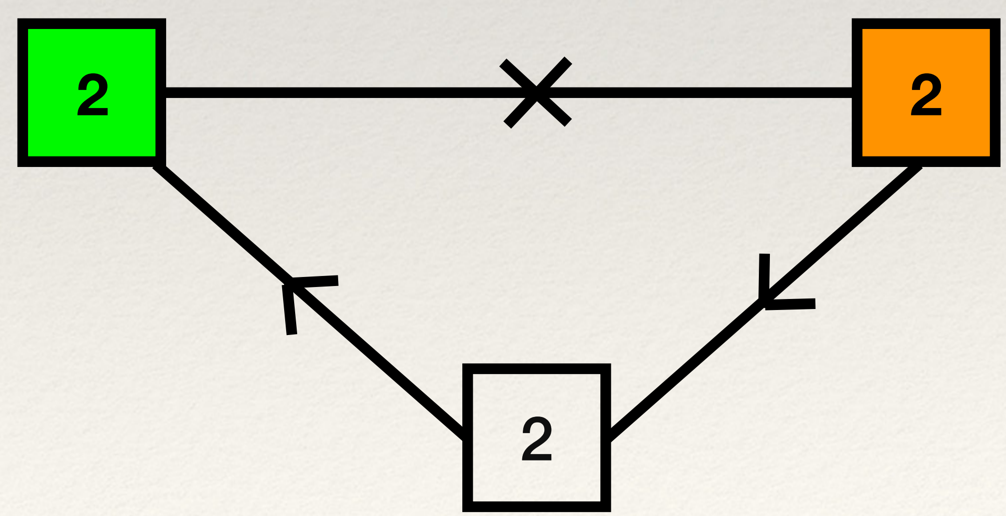
$\mathcal{F}_1 + \mathcal{F}_2 = (1,1,0,0,0,0,0,0)$



$\mathcal{F} = (\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$

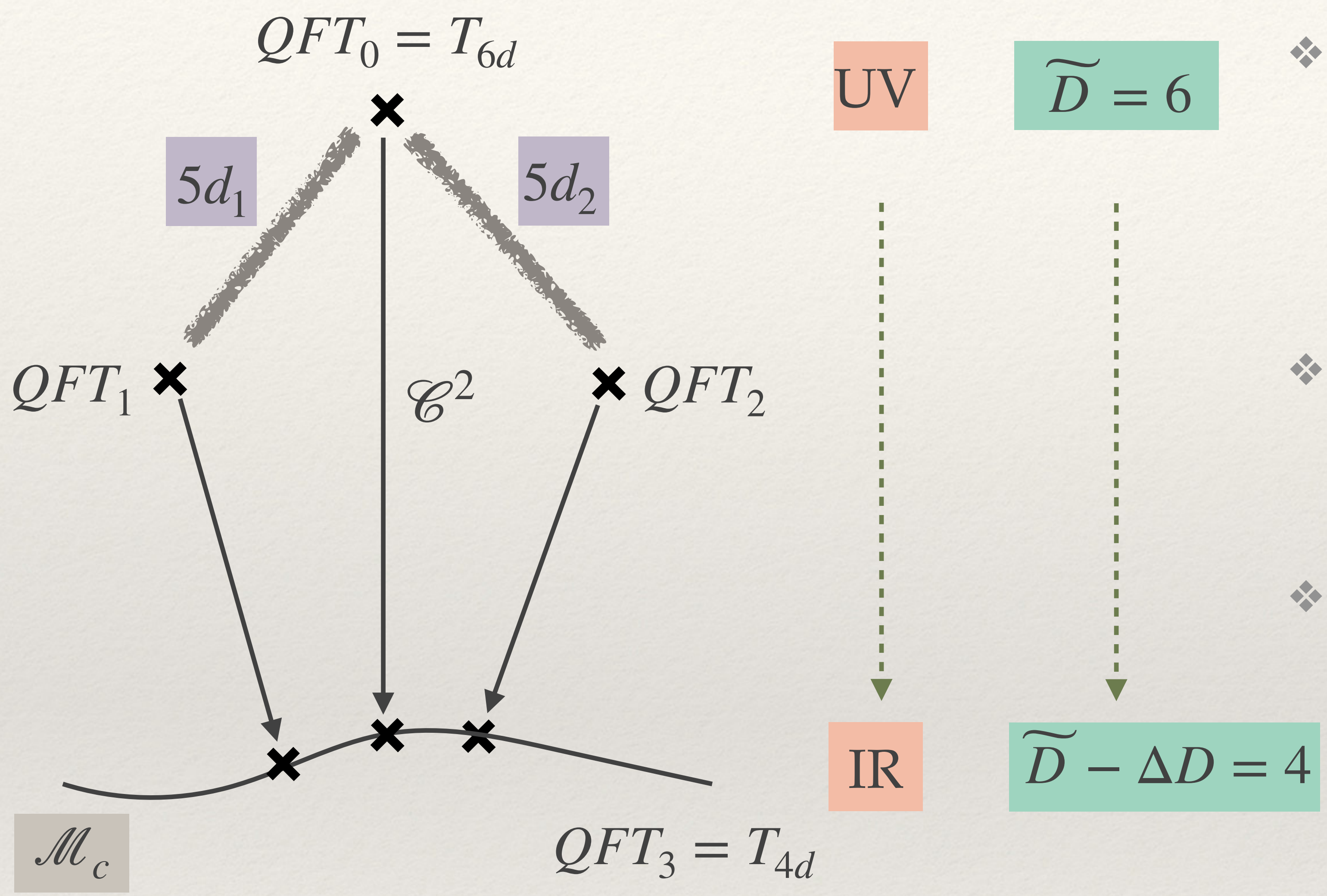


- Seiberg duality Seiberg 94



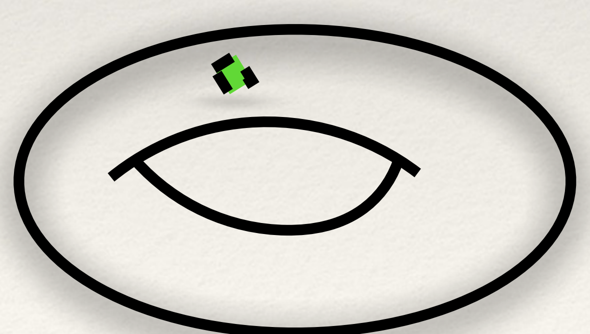
Kim, SR, Vafa, Zafrir 17

4d dualities from 5d dualities

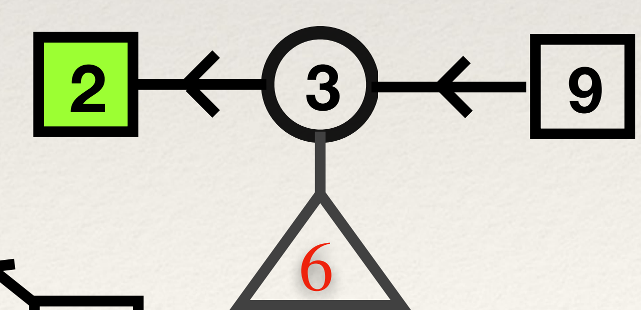


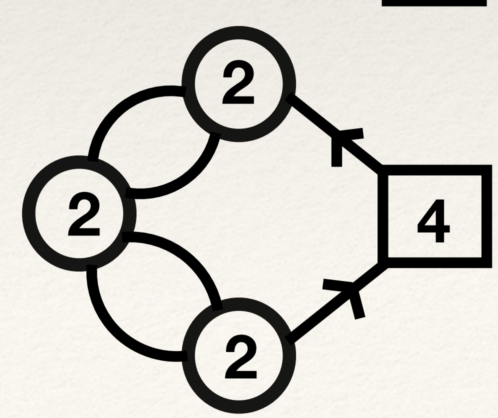
- 5d UV dualities can lead to different cross dimensional dualities
- Which symmetry gauged when $\otimes$?
- The different UV dualities can be used to compose equivalent 4d theories

- Eg:

$\mathscr{C}^2 =$ 

$T_{6d} = (D_4, D_4)$ min. conf. matt.

QFT_1 : 

QFT_2 : 

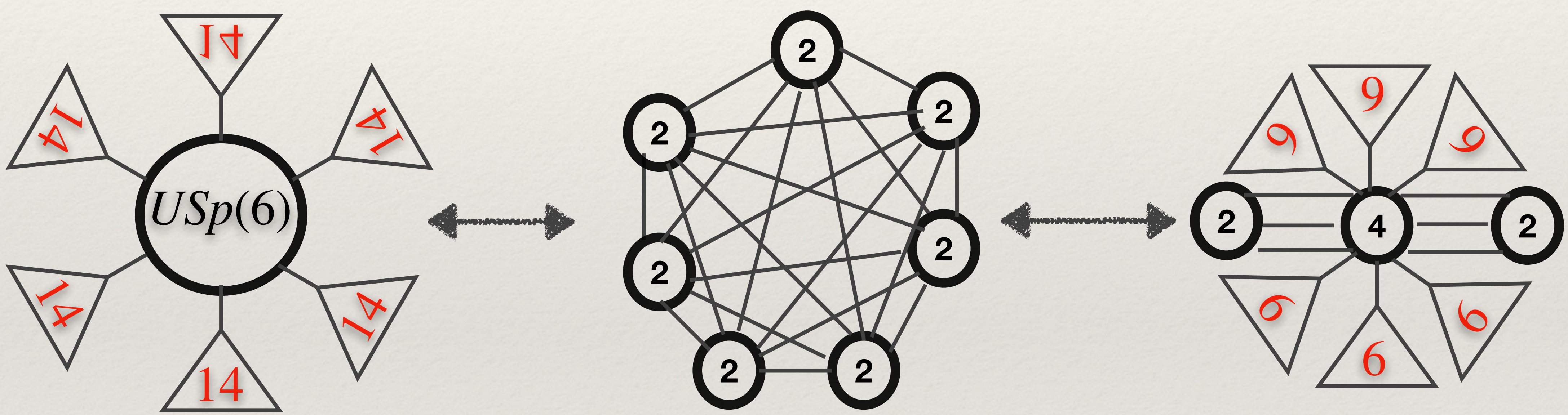
SR, Sabag 20

Examples of geometrically mysterious 4d dualities

❖ Ex1: Kutasov-Schwimmer / Brodie / Kutasov-Lin dualities (ADE classification)

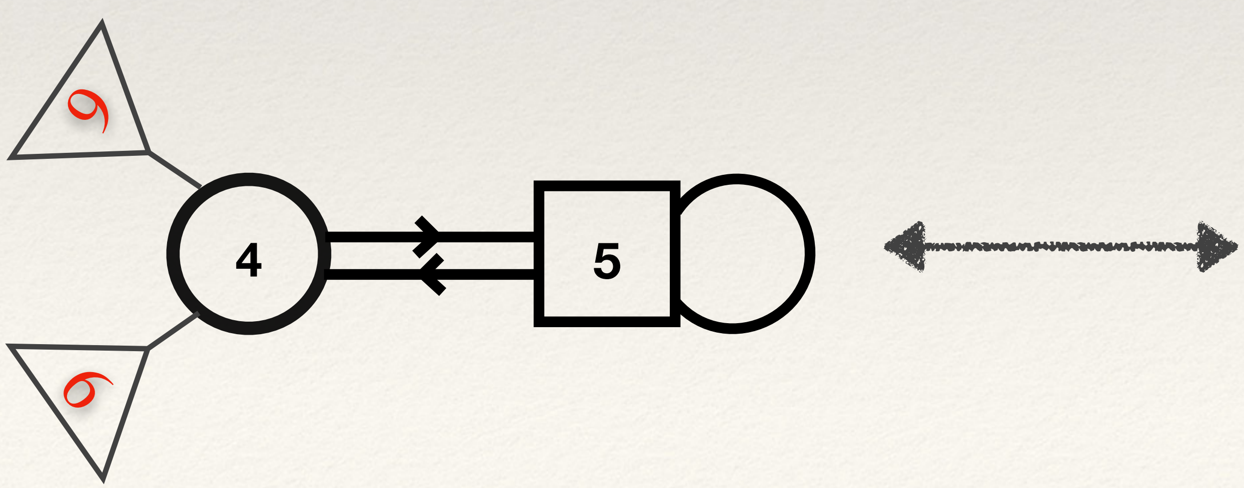
❖ Ex2:

SR, Sabag, Zafrir 20



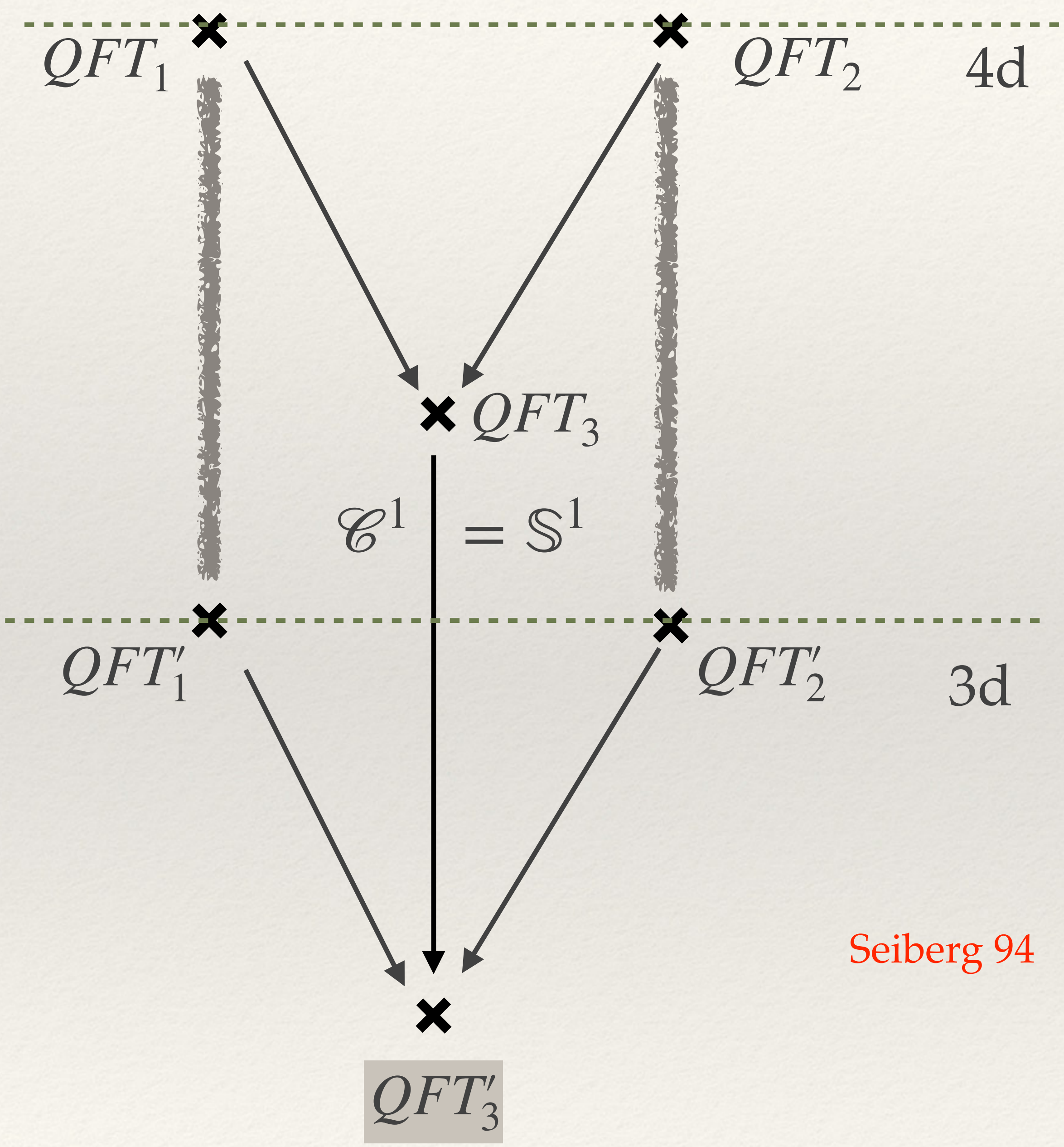
❖ Ex3:

Zafrir 19



E_6 Minahan-Nemeschansky
$\mathcal{N} = 2$ SCFT

D. Dualities from dualities



- Duality in higher dimensions might reduce to dualities in lower dimensions
- When reducing dualities proper care needs to be taken
- Eg: 4d to 3d generally leads to monopole superpotentials in 3d

  Aharony, SR, Seiberg, Willett 13
  Niarchos 12; Gadde, Yan 11; Spiridonov, Vartanov 11

- Ex:

Seiberg 94 $4d:$ $USp(2N)$ with $2N_f \leftrightarrow USp(2N_f - 2N - 4)$ with $2N_f + W$

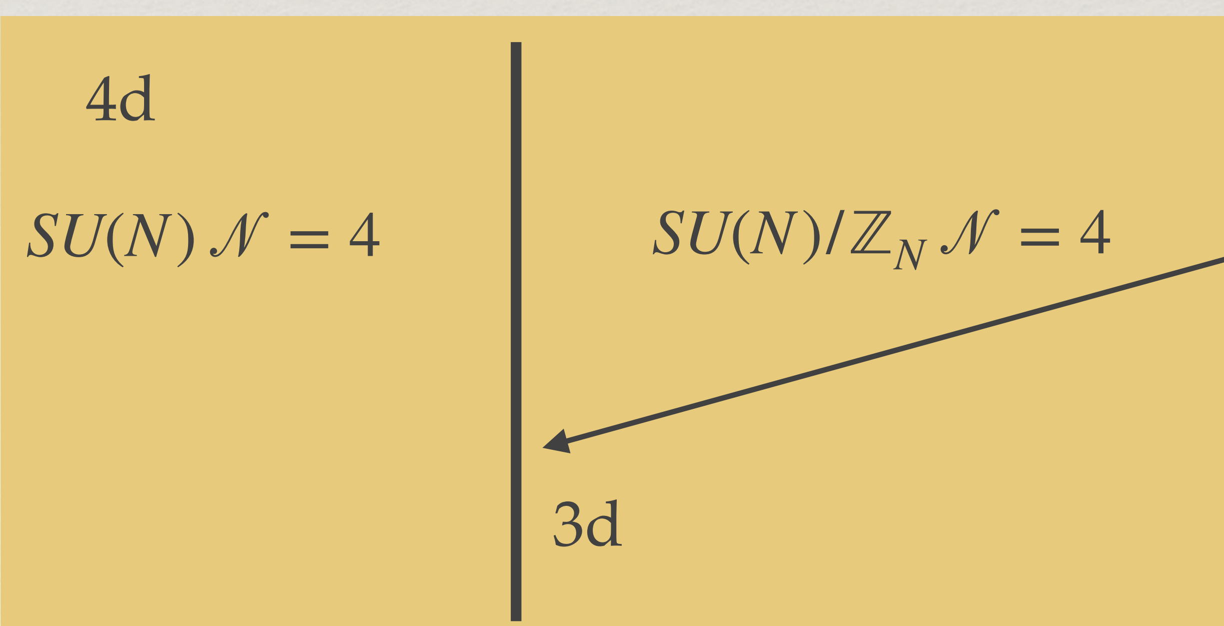
Aharony 97 $3d:$ $USp(2N)$ with $2N_f \leftrightarrow USp(2N_f - 2N - 2)$ with $2N_f + \widetilde{W}$

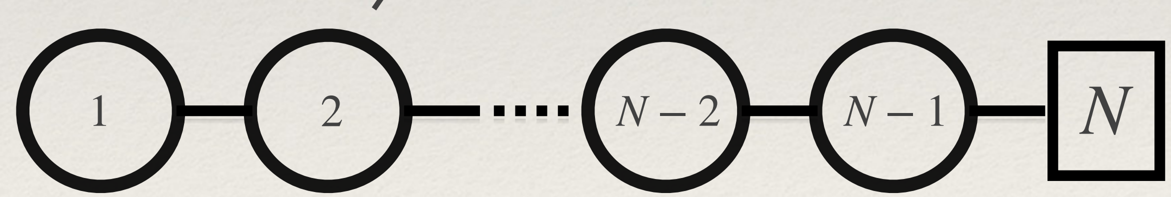
Do all 3d dualities have 4d uplift?

- Eg: 3d $\mathcal{N}=4$ Mirror symmetry
- R-symmetry in 3d $SU(2)_H \times SU(2)_C$ exchanged under mirror duality
- $\mathcal{N}=2$ R-symmetry in 4d only $SU(2)\times U(1)$
- Seems problematic to uplift insisting on supersymmetry

- **Ex:** $\mathcal{N}=4$ SYM S-duality walls

Unitary groups





$T[SU(N)]$

Gaiotto, Witten 08

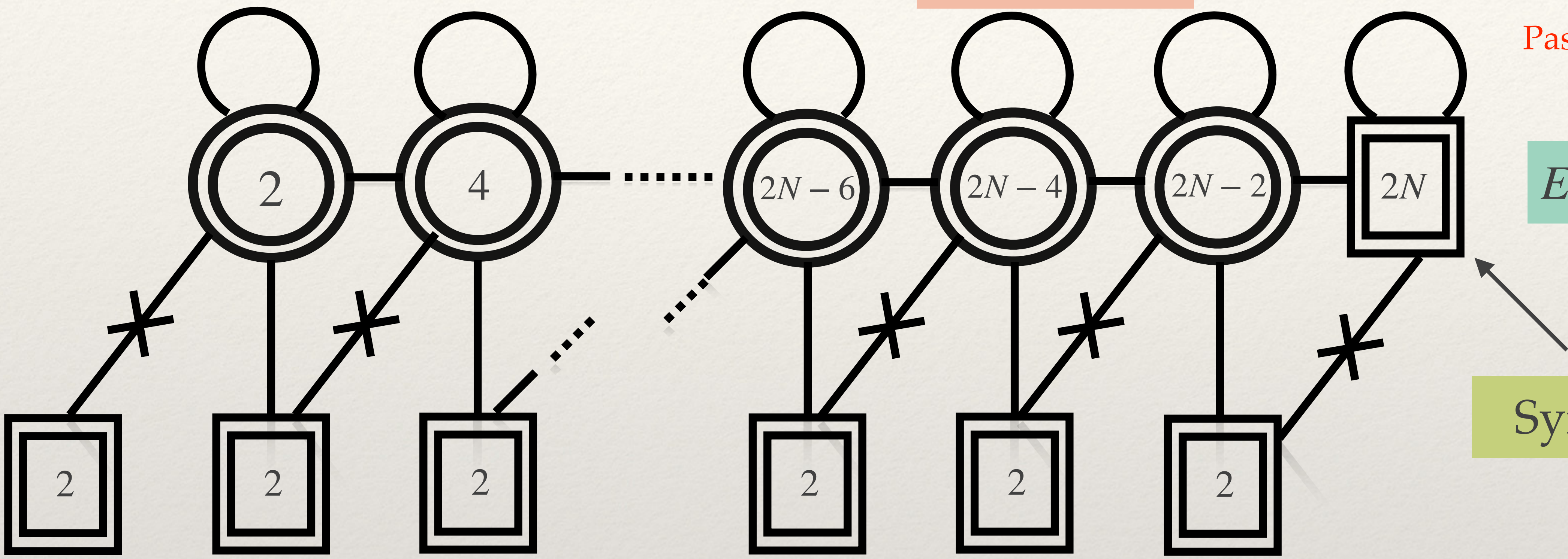
The global symmetry is:
$SU(N)_H \times SU(N)_C$

The mirror symmetry exchanges the Higgs and the Coulomb branches

The 4d uplift of $T[SU(N)]$

- One can uplift this model to 4d giving up supersymmetry to $\mathcal{N} = 1$

Pasquetti, SR, Sacchi, Zafrir 19



$E[USp(2N)]$ model

Symplectic groups

- The global symmetry is $USp(2N) \times USp(2N)$
- One copy of $USp(2N)$ emerges in the IR
- The theory is self-dual exchanging the two symmetry factors (Rains 14)
- Upon reduction (and deformation) to 3d one gets $T[SU(N)]$
- (* This model appears in compactifications of rank N E-string)
- (** Can be thought of as a domain wall theory in 5d)

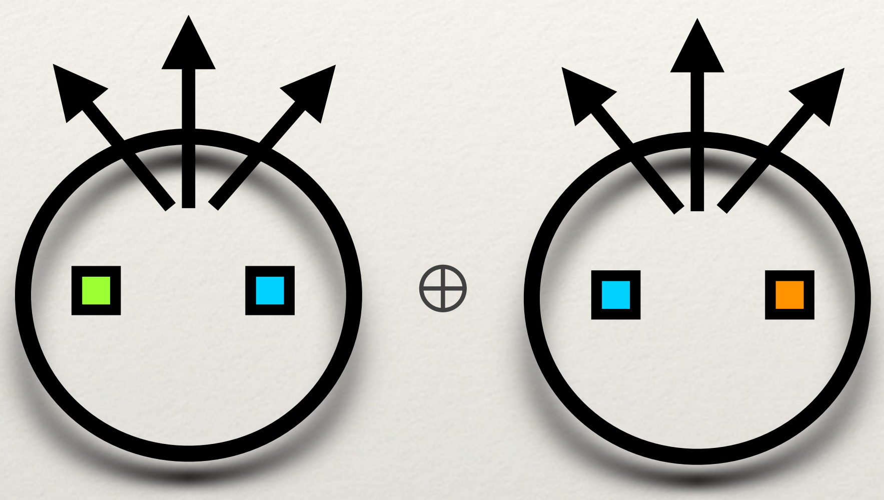
Hwang, Pasquetti, Sacchi 20

Hwang, SR, Sabag, Sacchi 21

* Generalization of Seiberg duality

Rank N E-string on $\mathscr{C}^2_{g=1,s=2}$ and flux

$(G_{6d} = E_8 \times SU(2),$ flux for Cartan)

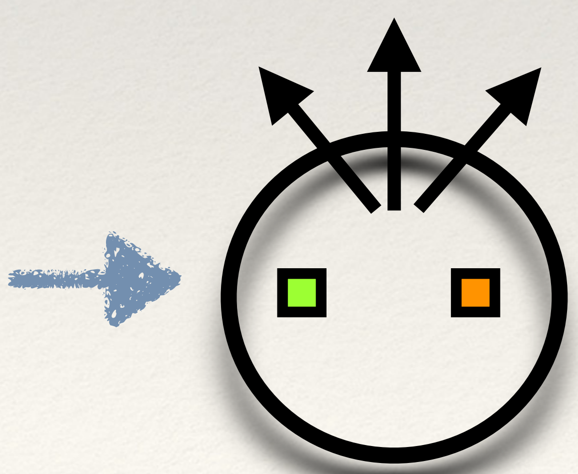


$\mathscr{F}_1 = (0; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ $\mathscr{F}_2 = (0; \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$

$\mathscr{F}_1 + \mathscr{F}_2 = (0; 1,1,0,0,0,0,0,0)$

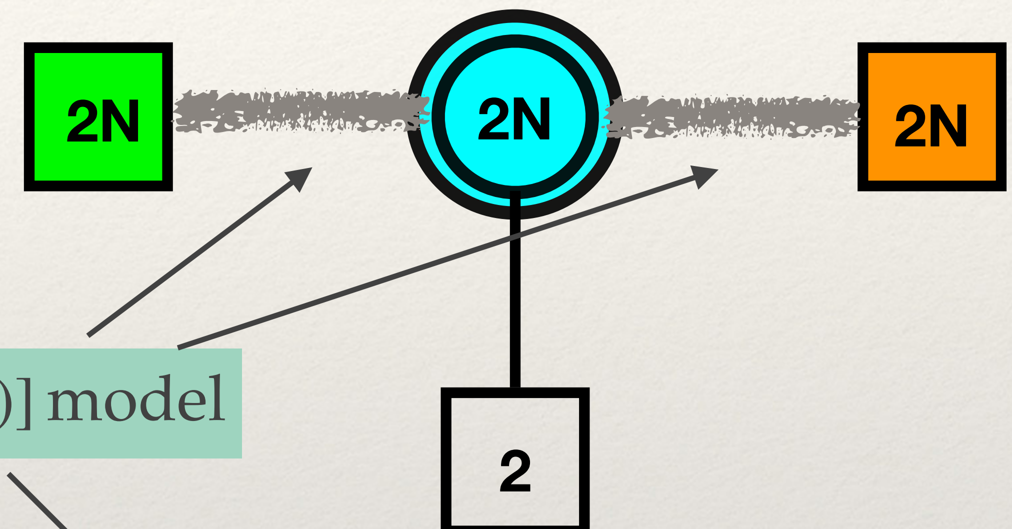
Domain wall in 5d

Kim, SR, Vafa, Zafrir 17



Equivalent in E_8

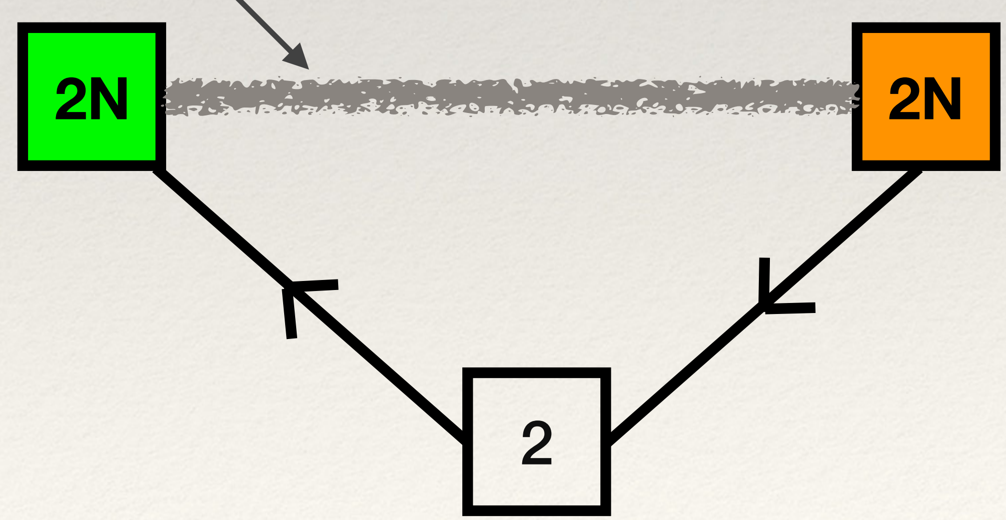
$\mathscr{F} = (0; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$



$E[USp(2N)]$ model

- Rains duality

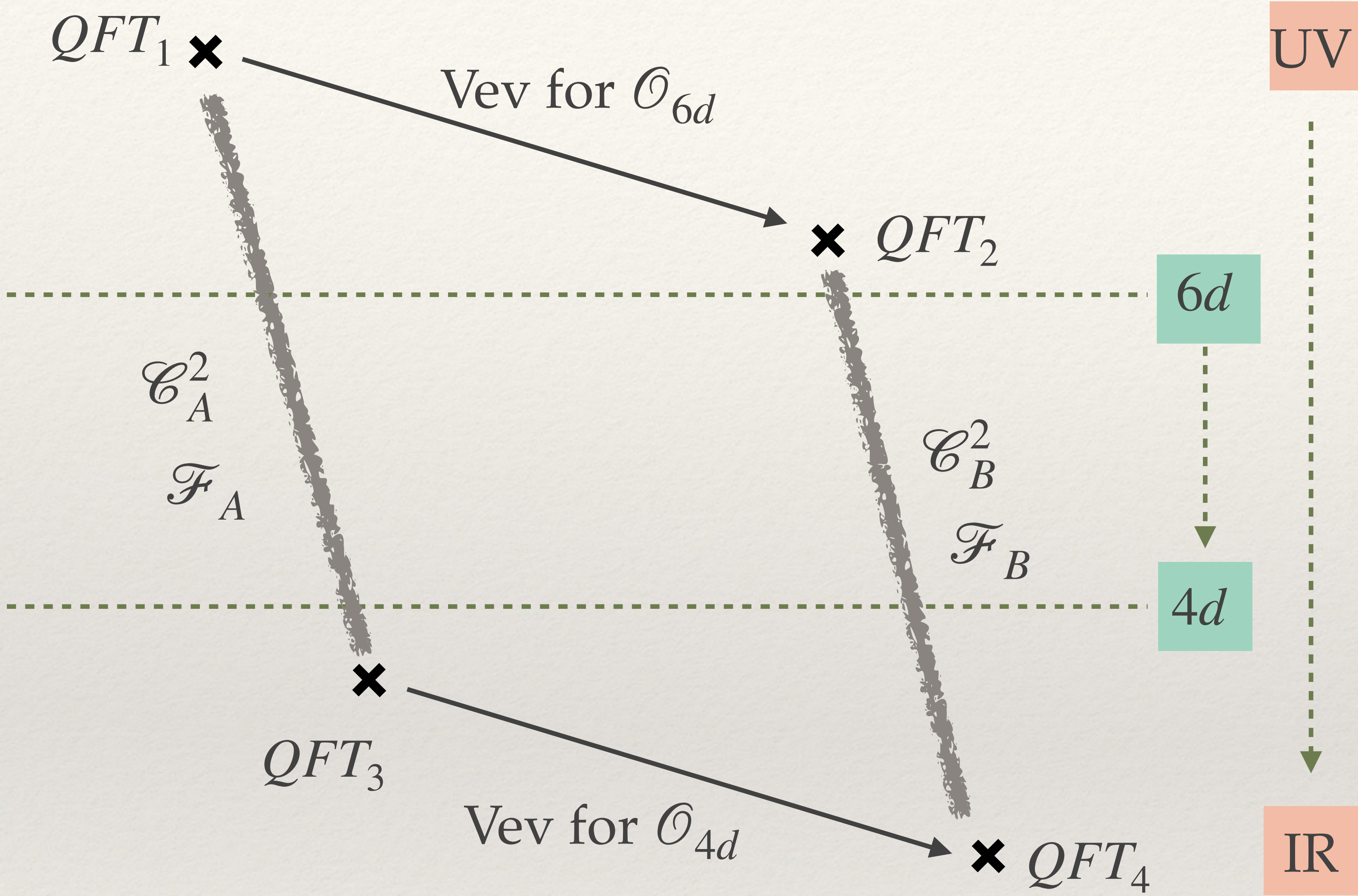
Rains 14



Pasquetti, SR, Sacchi, Zafrir 19

Bottini, Hwang, Pasquetti, Sacchi 21-22

E. 6d flows to 4d flows



- $\mathcal{C}_A^2 \neq \mathcal{C}_B^2$ in general
- $\mathcal{C}_B^2$ can have more punctures than $\mathcal{C}_A^2$
- The difference depends on the flux $\mathcal{F}_A$

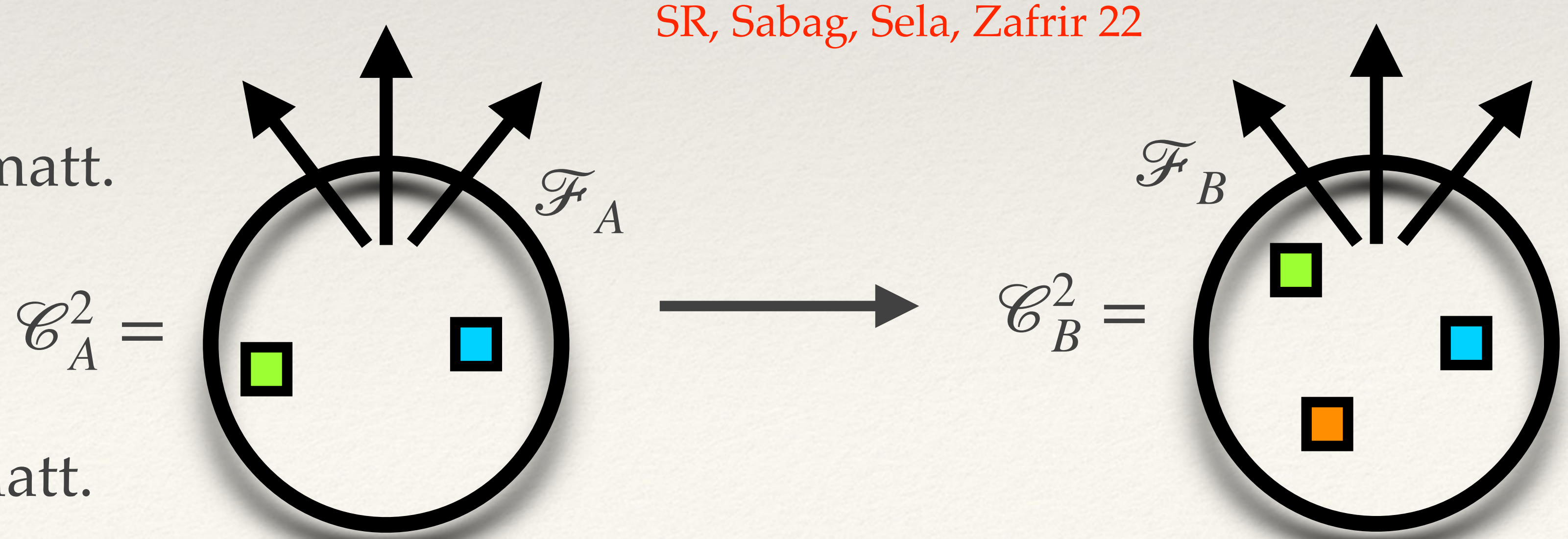
SR, Sabag, Zafrir 19

SR, Sabag 20 Kim, SR … to appear

SR, Sabag, Sela, Zafrir 22

$QFT_1 = (D_5, D_5)$ min. conf. matt.

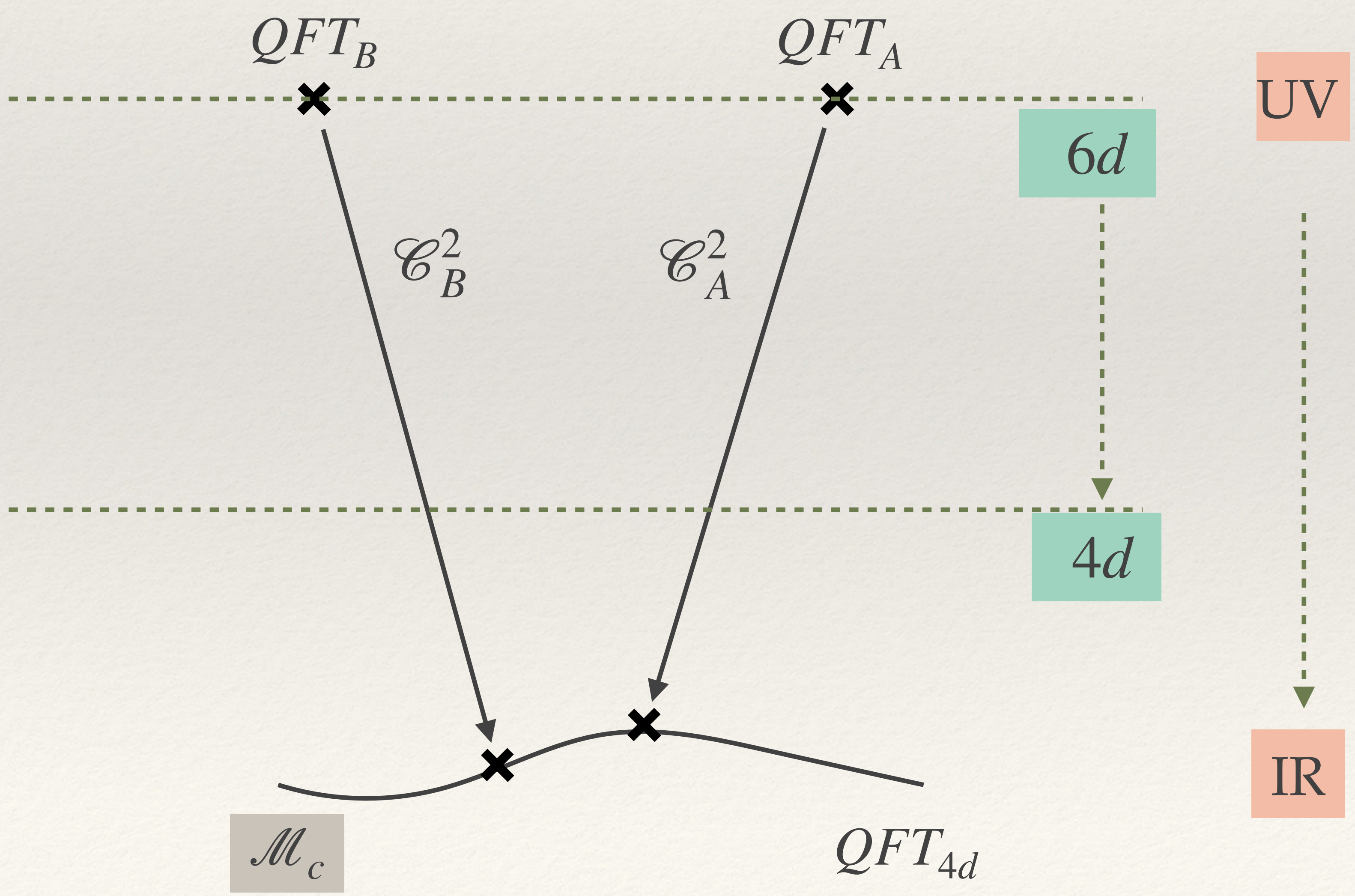
- Ex:



$QFT_2 = (D_4, D_4)$ min. conf. matt.

F. 6d dualities

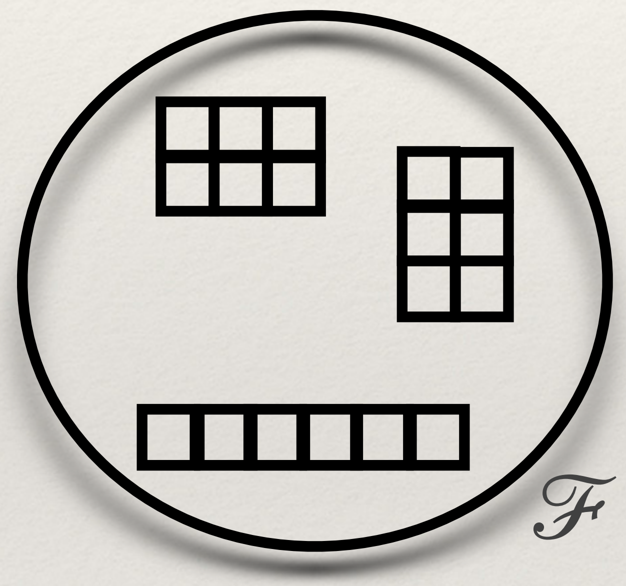
- Can start from different 6d SCFTs, QFT_A and QFT_B
- deform the two theories by different geometries, $\mathscr{C}^2_A$ and $\mathscr{C}^2_B$
- and flow to the same SCFT in 4d



An example

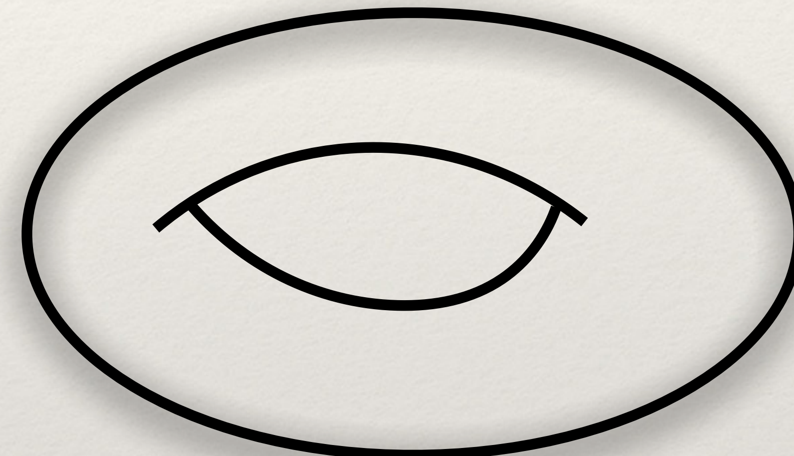
- QFT_A : 6 M5 branes
- QFT_B : (D_4, D_4) min. conf. matt. (Aka rank one E-string)

$\mathscr{C}^2_A$:



$\mathscr{F} = \frac{1}{2}$

$\mathscr{C}^2_B$:



$\mathscr{F} = 0$

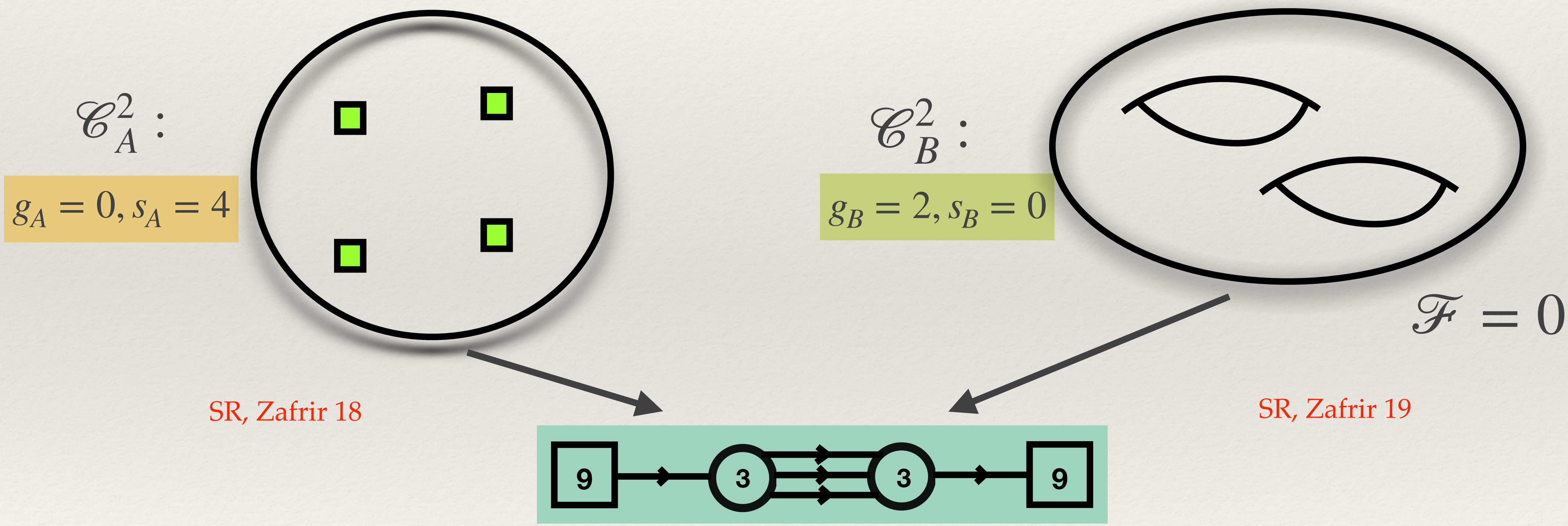
Gaiotto; Tachikawa circa 2009

Ganor, Morrison, Seiberg 96

The E_8 Minahan-Nemeschansky $\mathscr{N} = 2$ SCFT in 4d

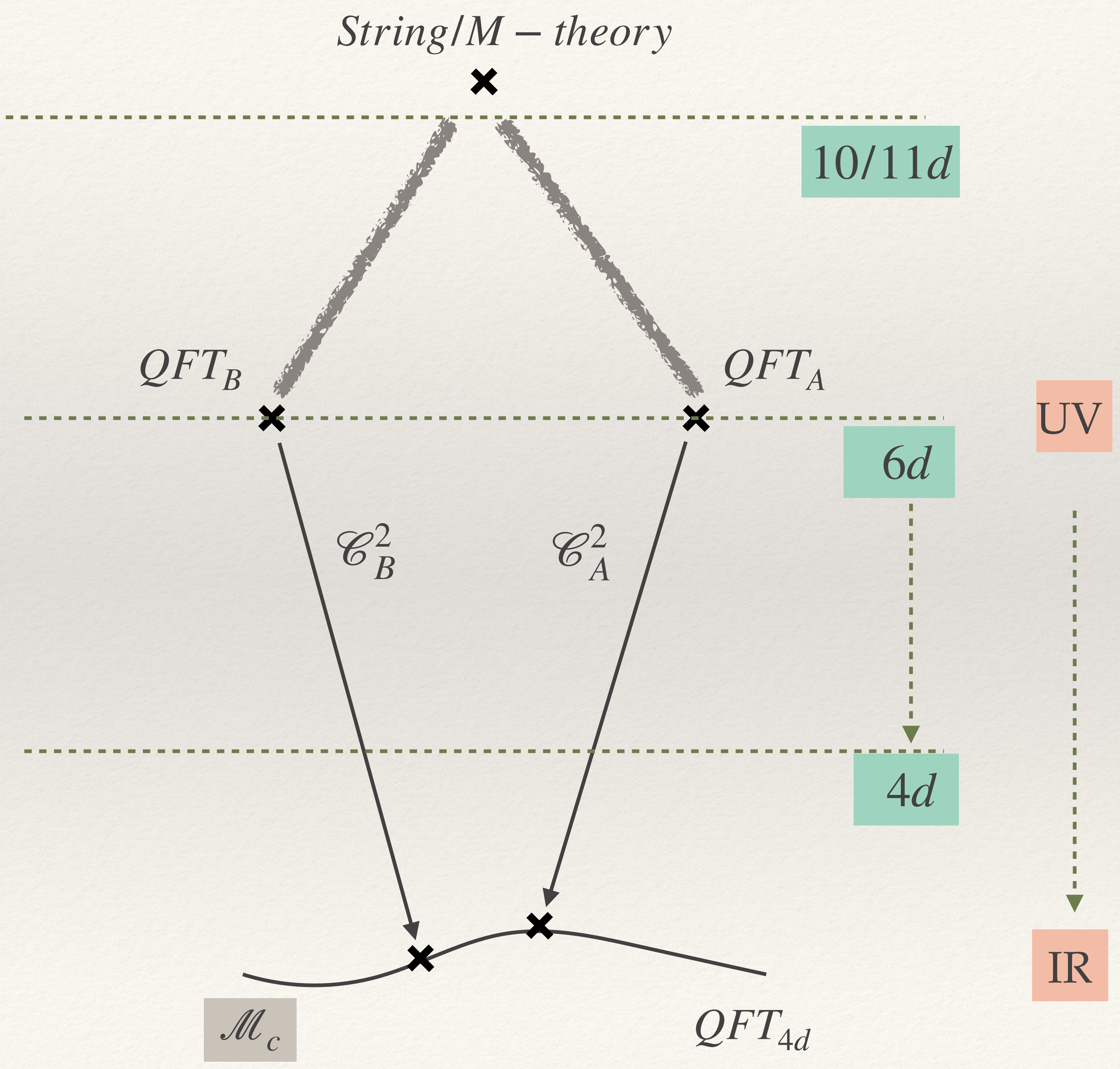
Additional Example

- QFT_A : minimal $SU(3)$ SCFT in 6d (pure $SU(3)$+tensor)
- QFT_B : (D_4, D_4) min. conf. matt. (Aka rank one E-string)



$$\dim \mathscr{M}_c = (3g_B - 3) + 248(g_B - 1) + (3g_A - 3 + s_A)$$

Explanation of 6d dualities ?

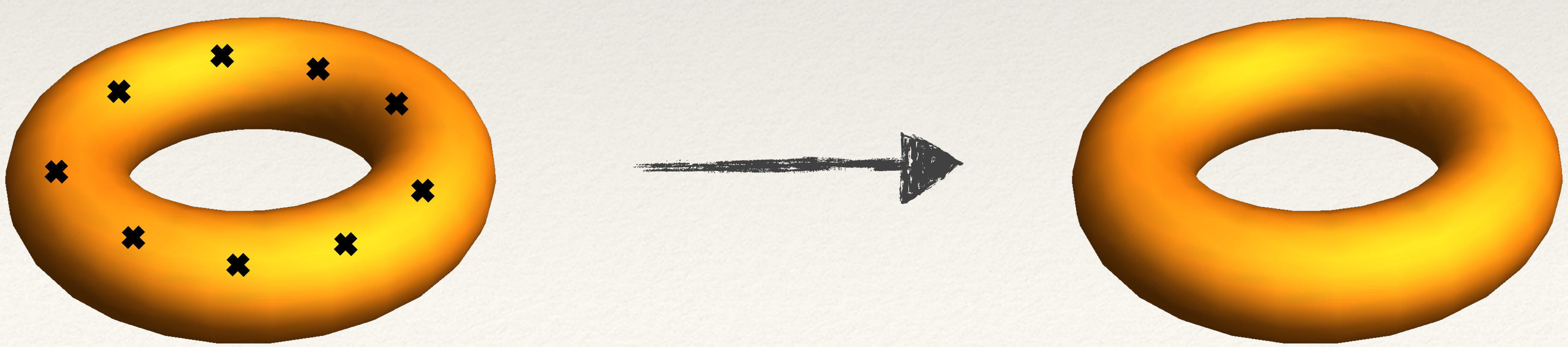


G: $\mathscr{D} = 6$ theories from $\mathscr{D} < 6$

- A_{N-1} (2,0) $\mathscr{D} = 6$ SCFT compactified on a torus with k minimal punctures is across dimensions dual to a circular quiver

Gaiotto 09

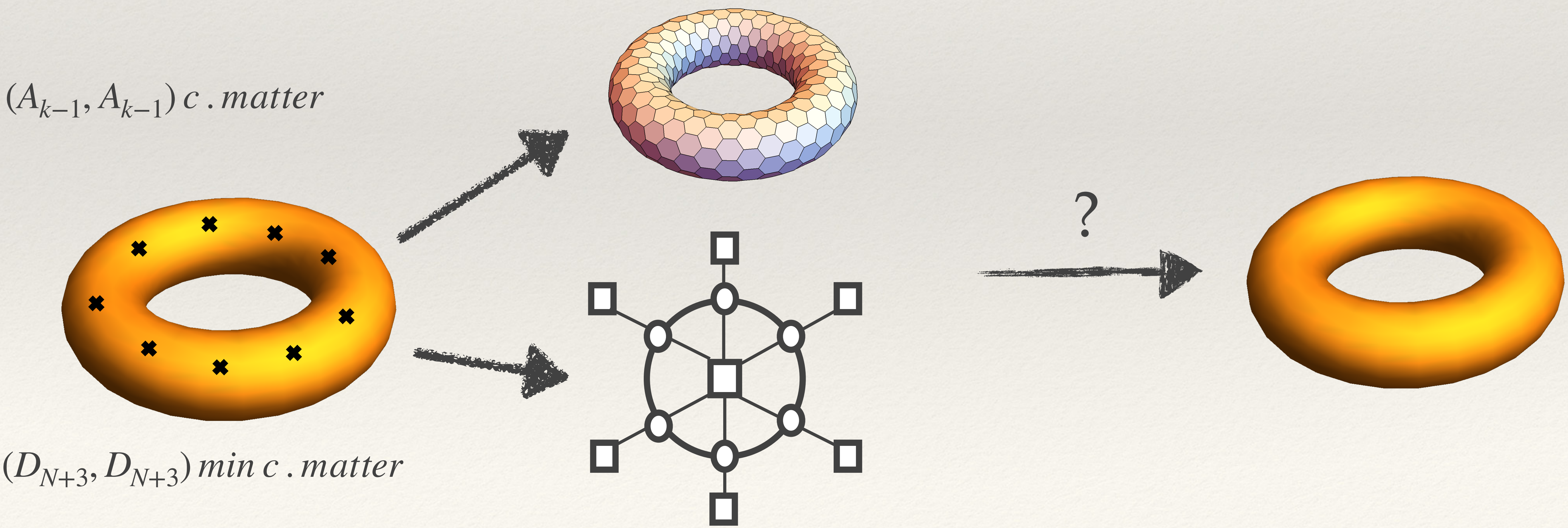
- *Conjecture (deconstruction):* Take a double scaling limit if large number of punctures and close them. Closing punctures is obtained by giving VEVs to certain operators. One then obtains the full $\mathscr{D} = 6$ SCFT on a finite size torus.



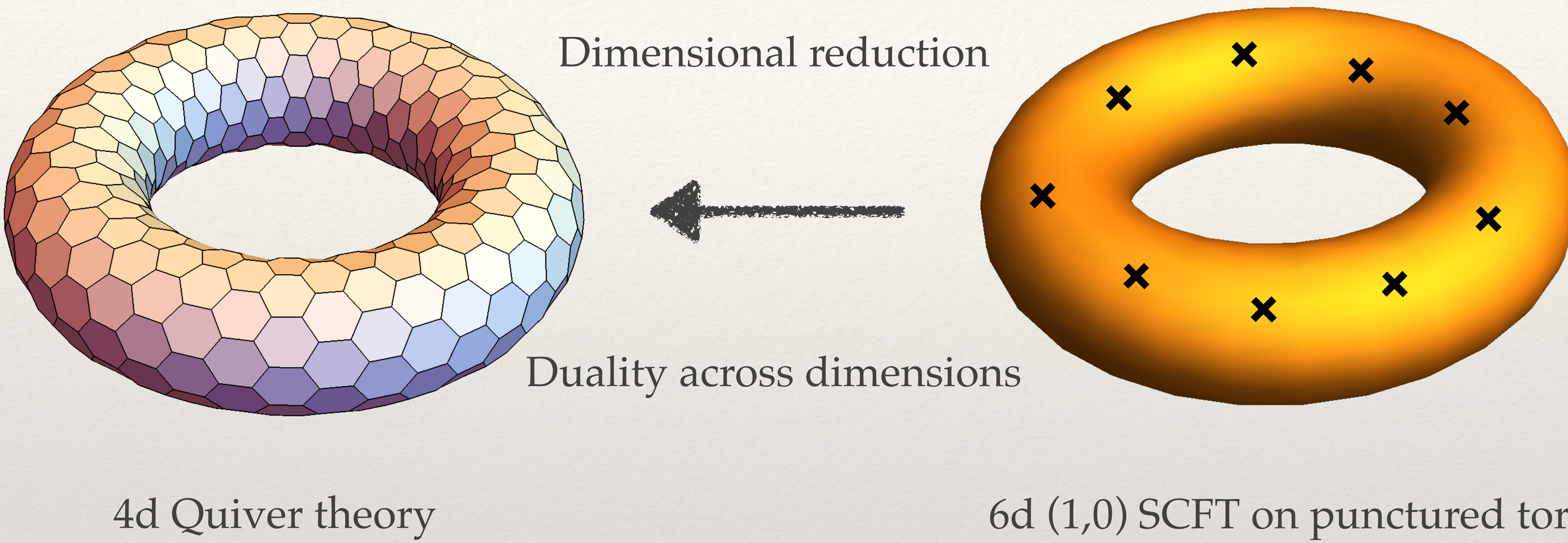
Arkani-Hamed, Cohen, Kaplan, Karch, Motl 03

More $\mathscr{D} = 6$ theories from $\mathscr{D} < 6$

- Consider an (1,0) $\mathscr{D} = 6$ SCFT compactified on a torus with k ``minimal'' punctures and find its across dimensions dual
- Take a double scaling limit of large number of punctures and close them. Does one then obtains the full $\mathscr{D} = 6$ SCFT on a finite size torus?

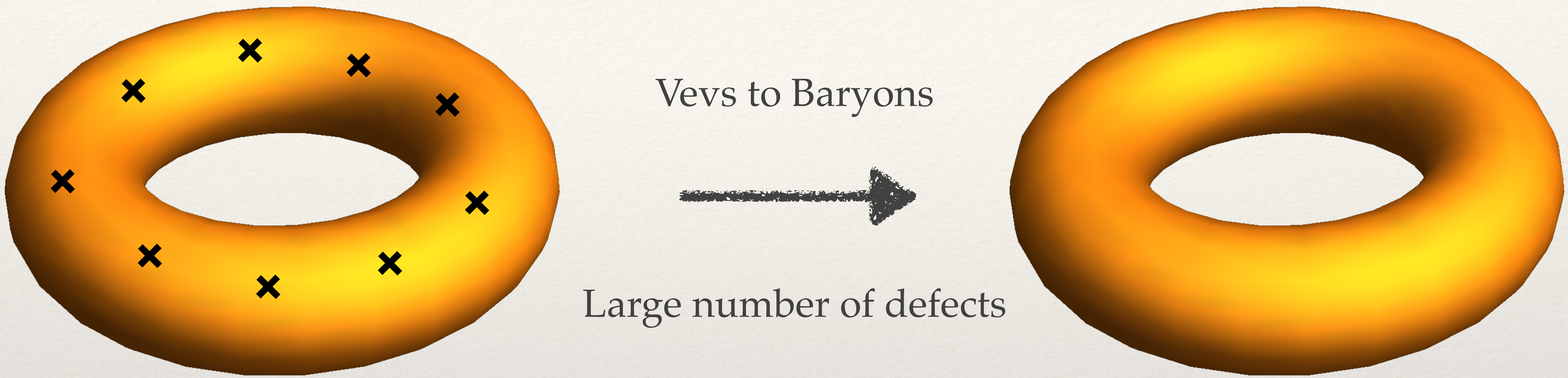


Modern view of the Quivers



- One can engineer the 4d Quivers by taking a 6d SCFT in presence of defects localised on a torus (and extended in 4d). The quivers are an IR limit of this configuration.
- The different couplings correspond eg to positions of the defects (see the duality statements before). This is very analogous to the lattice models being effective descriptions of QED with ``defects''.

Modern view on Deconstruction



6d (1,0) SCFT on punctured torus in IR

6d (1,0) on a finite torus

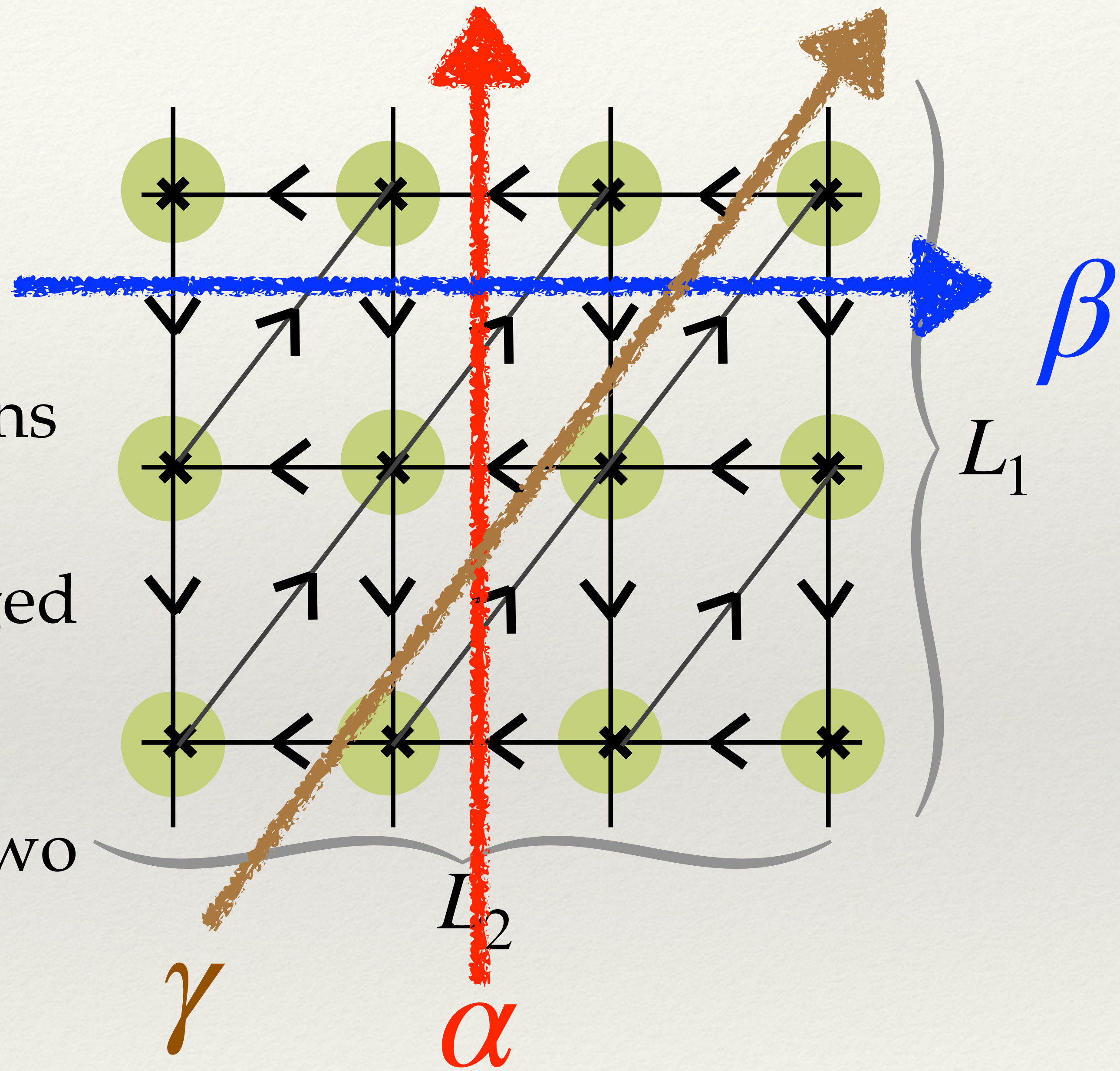
- The Higgs branch deconstruction corresponds to double scaling limit removing the punctures while taking the number of punctures to ∞
- This is again very analogous to the continuum limit of cond-mat lattices.

Sub-system symmetries

- Global symmetry:

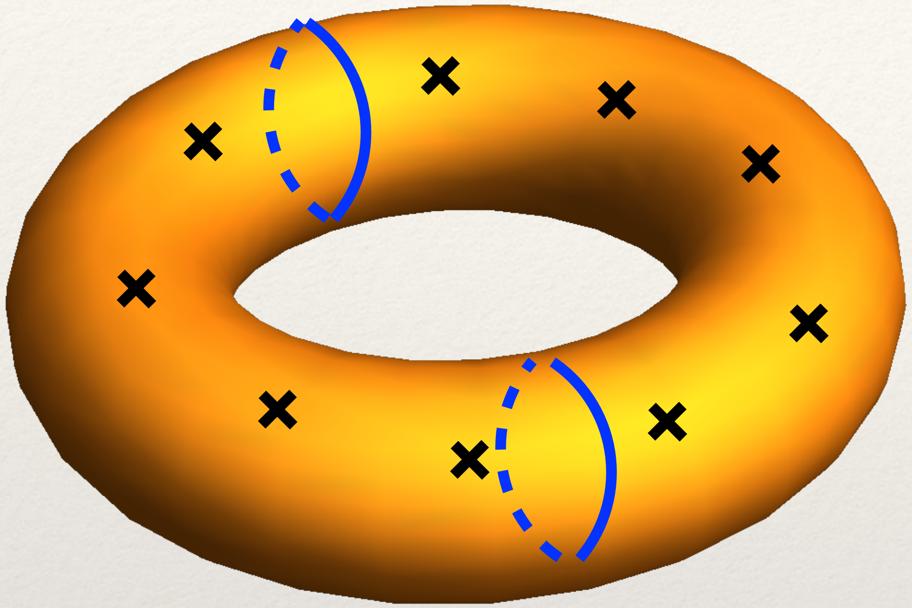
$$G = \frac{U(1)_\alpha^{L_2} \times U(1)_\beta^{L_1} \times U(1)^{GCD(L_1,L_2)}}{U(1)}$$

- This is fixed by anomaly considerations
- Fields at different sites are charged under different symmetries
- The global symmetry (except for two U(1)s) is a subsystem symmetry



$$[\mathscr{A}] = (0,1,1)\,,\ [\mathscr{B}] = (1,-1,0)\,,\ [\mathscr{C}] = (-1,0,-1)$$

Geometric view of the Coulomb branch: Fractons



Vevs to winding operators

→ ?

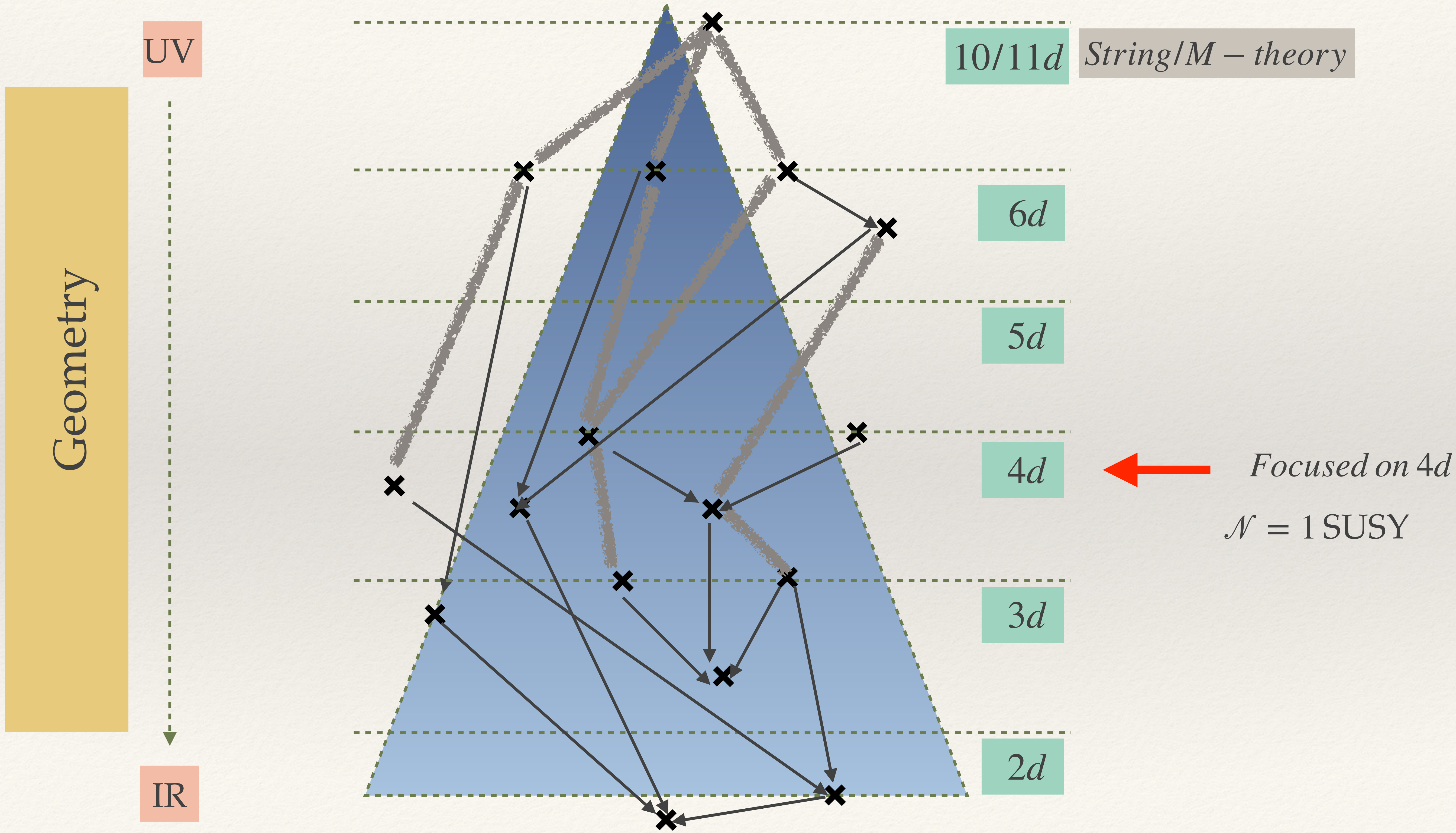
Large number of defects

6d (1,0) SCFT on punctured torus in IR

- Is there a nice geometric interpretation of this procedure
- We need to retain all the symmetries associated to the punctures
- Such setups are analogous to fractons: sub-system symmetry

SR 2021; Franco, Rodriguez-Gomez 2022

Intricate web of relations between dimensions



Outlook

- Do all 6d flows to lower d have lower d duals ?
- Gauging emergent symmetries
- Do all dualities have geometric origin in d<6 ?
- Do all dualities have string/M-theory origin?
- What is the structure of the space of flows?
- Are all SCFTs in d<5 Lagrangian?
- Do all SCFTs in d>4 have a useful field theoretic description (maybe using a lattice)

Thank You!!