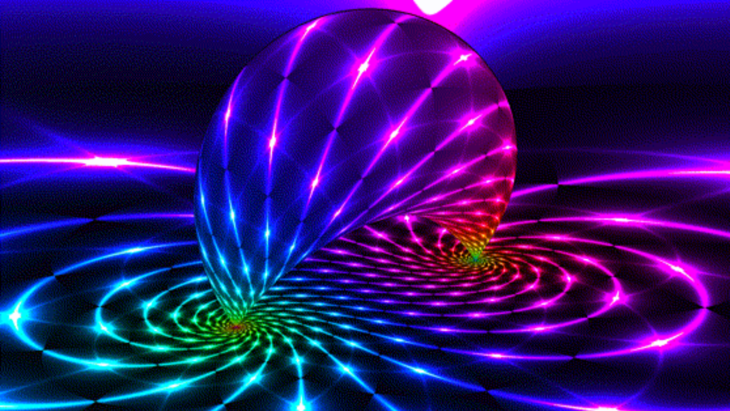

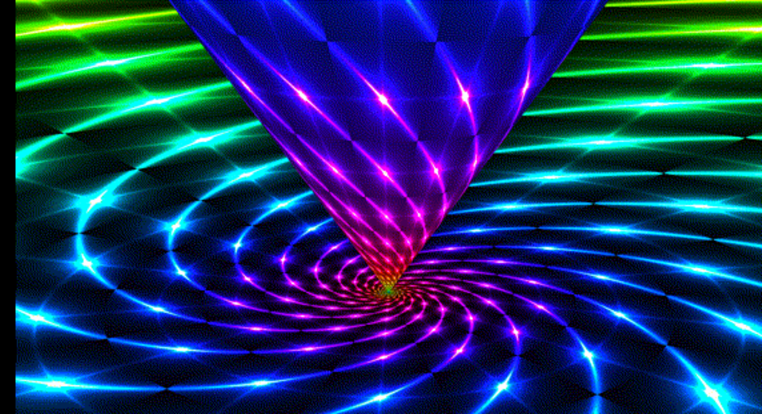

A Selberg zeta function for warped AdS3 black holes

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Based on: VLM, Poddar, Þórarinsdóttir 2210.01118

VLM, Poddar, Svesko, Þórarinsdóttir WIP

The Niels Bohr International Academy

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Selberg zeta: What and why

Let's start with the Riemann zeta function for comparison:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}}$$

Selberg zeta function: Replace prime numbers with prime geodesics.

$$Z_\Gamma(s) = \prod_p \prod_{n=0}^{\infty} \left(1 - N(p)^{-s-n}\right)$$

Selberg zeta functions are defined for hyperbolic quotient spacetimes.

$$\mathbb{H}^n/\Gamma \qquad \partial_\phi$$

Example: The BTZ black hole

Perry, Williams 2003

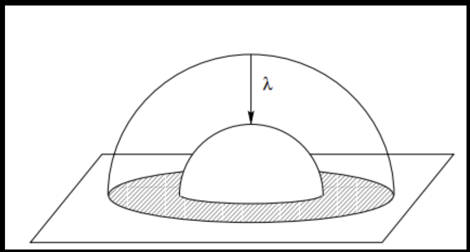
$$Z_\Gamma(s) = \prod_{k_1,k_2=0}^{\infty} \left[1 - e^{2ibk_1} e^{-2ibk_2} e^{-2a(k_1+k_2+s)}\right]$$

IMPORTANT PART: The Selberg zeta function for BTZ encodes quantum corrections and quasinormal modes.

Keeler, Martin, Svesko 2018

$$\log Z_\Gamma(\Delta) \propto \log \det \nabla^2_{reg}$$

$$s^\star \leftrightarrow \omega_{QN}$$



Krasnov 2000

Warped spacetimes: What and why

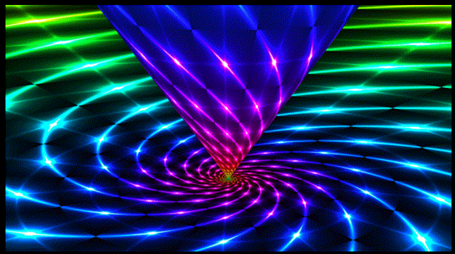
Spacelike WAdS3 is constructed by stretching or squashing the length of a fibre over AdS2. The symmetry group is SL(2,R)xU(1).

$$ds^2 = \frac{L^2}{\nu^2+3}\left(-\cosh^2\sigma d\tau^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2+3}(du + \sinh\sigma d\tau)^2\right)$$

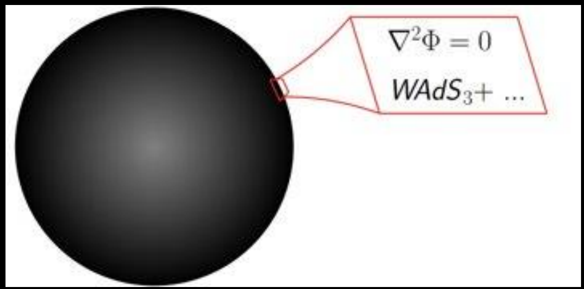
Just like the BTZ black hole is a discrete quotient of AdS3, WAdS3 black holes are discrete quotients of WAdS3.

Anninos, Li, Padi, Song, Strominger 2008

$$\partial_\phi = \frac{\nu^2+3}{8}\left[\left(r_+ - r_- - \frac{\sqrt{(\nu^2+3)r_+r_-}}{\nu}\right)J_2 - (r_+ - r_-)\,\tilde{J}_2\right]$$



Why interesting?: WAdS quotients appear in many important contexts, such Kerr/CFT.



How to make Z_Γ from quotient

Perry, Williams 2003

Example: Euclidean BTZ $$ds^2 = N(r)^2 d\tau^2 + \frac{dr^2}{N(r)^2} + r^2 \left(N^\phi(r) d\tau + d\phi\right)^2$$

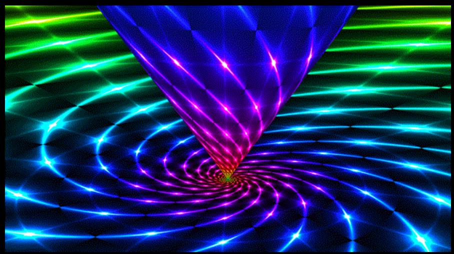
To a point $(\tau, r, \phi) \in \mathbb{R}^3$ one associates the point $(x, y, z) \in \mathbb{H}^3$ given by

$$x = A(r) \cos f(\phi, \tau) \exp(r_+ \phi - |r_-|\tau)$$
$$y = A(r) \sin f(\phi, \tau) \exp(r_+ \phi - |r_-|\tau)$$
$$z = B(r) \exp(r_+ \phi - |r_-|\tau)$$

$$ds^2 = \frac{L^2}{z^2}\left(dx^2 + dy^2 + dz^2\right)$$

When $\phi \to \phi + 2\pi n$ we have $(x, y, z) \to (x', y', z')$, where

$$x' = e^{2\pi n r_+}\left[x \cos(2\pi n |r_-|) - y \sin(2\pi n |r_-|)\right]$$
$$y' = e^{2\pi n r_+}\left[y \cos(2\pi n |r_-|) + x \sin(2\pi n |r_-|)\right]$$
$$z' = e^{2\pi n r_+} z$$



The zeta function parameters are the coefficients of the dilation and rotation.

$$a = \pi r_+$$
$$b = \pi |r_-|$$

$$Z_\Gamma(s) = \prod_{k_1, k_2 = 0}^{\infty}\left[1 - e^{2ibk_1} e^{-2ibk_2} e^{-2a(k_1 + k_2 + s)}\right]$$

Warped Poincare Patch

Build conformal coordinates $w^+ = x + iy \qquad w^- = x - iy$

$$w^+ = \sqrt{\frac{r^2 - r_+^2}{r^2 - r_-^2}} e^{\alpha\phi + \beta t} \qquad w^- = \sqrt{\frac{r^2 - r_+^2}{r^2 - r_-^2}} e^{\gamma\phi + \delta t} \qquad z = \sqrt{\frac{r_+^2 - r_-^2}{r^2 - r_-^2}} e^{1/2((\alpha+\gamma)\phi + (\beta+\delta)t)}$$

From these coordinates we can make generators of SL(2,R)xSL(2,R):

$$H_1 = i\partial_+, \qquad H_0 = i\left(w^+\partial_+ + \frac{1}{2}z\partial_z\right), \qquad H_{-1} = i((w^+)^2\partial_+ + w^+ z\partial_z - z^2\partial_-)$$

$$\bar{H}_1 = i\partial_-, \qquad \bar{H}_0 = i\left(w^-\partial_- + \frac{1}{2}z\partial_z\right), \qquad \bar{H}_{-1} = i((w^-)^2\partial_- + w^- z\partial_z - z^2\partial_+)$$

To find $(\alpha, \beta, \gamma, \delta)$, we require that the SL(2,R)xU(1) Casimir is proportional to the scalar Laplacian:

$$(\mathcal{H}^2 + \lambda H_0^2)\Phi \propto \nabla^2\Phi$$

$$\mathcal{H}^2 = -\bar{H}_0^2 + \frac{1}{2}(\bar{H}_1\bar{H}_{-1} + \bar{H}_{-1}\bar{H}_1)$$

Result 1: $$ds^2 = \frac{4}{(3+\nu)^2 z^2}\left((3+\nu^2)dw_+dw_- + 4\nu^2 dz^2 + \frac{3(\nu^2-1)w_+}{z^2}(dw_-^2 + 2zdw_-dz)\right)$$

Selberg zeta function for WAdS3

The rotation and dilation generators are:

$$H_0 - \bar{H}_0 = i(w_+\partial_+ - w_-\partial_-)$$
$$H_0 + \bar{H}_0 = i(w_+\partial_+ + w_-\partial_- + z\partial_z)$$

The quotient is generated by the group element:

$$e^{-2\pi i(\gamma H_0 + \alpha \bar{H}_0)} = e^{2\pi \partial_\phi}$$

In the dilation/rotation basis:

$$e^{-2\pi i\left(\frac{\gamma+\alpha}{2}(H_0+\bar{H}_0)+\frac{\gamma-\alpha}{2}(H_0-\bar{H}_0)\right)}$$

Result 2: $$Z_\Gamma(s) = \prod_{k_1,k_2=0}^{\infty}\left[1 - e^{2ibk_1}e^{-2ibk_2}e^{-2a(k_1+k_2+s)}\right]$$

$$2a = \gamma + \alpha$$
$$2b = \gamma - \alpha$$

Result 3: We establish a map between Selberg zeros and QNMs.

$$s^* \leftrightarrow \Delta \qquad \omega_{QN} \leftrightarrow \omega_n$$

Keeler, Martin, Svesko 2018

Result 4: We calculate the thermal frequencies for WAdS3.

Castro, Keeler, Szepietowski 2017

Heat Kernel for (Euclidean) WAdS3

We would like to see whether a similar relationship holds in the warped case.

$$\log Z_\Gamma(\Delta) \propto \log \det \nabla^2_{reg}$$

To accomplish this, we set about computing the heat kernel for WAdS3.

$$\nabla^2 \psi_n = \lambda_n \psi_n \qquad (\partial_t + \nabla_x^2) K(t; x, y) = 0 \qquad K(0; x, y) = \delta(x, y)$$

$$S^{(1)} = -\frac{1}{2} \log\, \det \nabla^2 = -\frac{1}{2} \sum_n \log \lambda_n = \frac{1}{2} \int_{0^+}^{\infty} \frac{dt}{t} \int d^3x \sqrt{g} K(t; x, x)$$

For warped black holes, we employ the method of images.

Giombi, Maloney, Yin 2008

$$K^{\mathbb{H}^3/\Gamma}(t; x, y) = \sum_{\gamma \in \Gamma} K^{\mathbb{H}^3}(t; x, \gamma y)$$

We can use our warped upper-half space metric to do this calculation:

$$ds^2 = \frac{4}{(3+\nu)^2 z^2} \left((3+\nu^2) dw_+ dw_- + 4\nu^2 dz^2 + \frac{3(\nu^2-1) w_+}{z^2} (dw_-^2 + 2z dw_- dz) \right)$$

Future directions

Quotient spacetimes are everywhere! Lots of future directions.

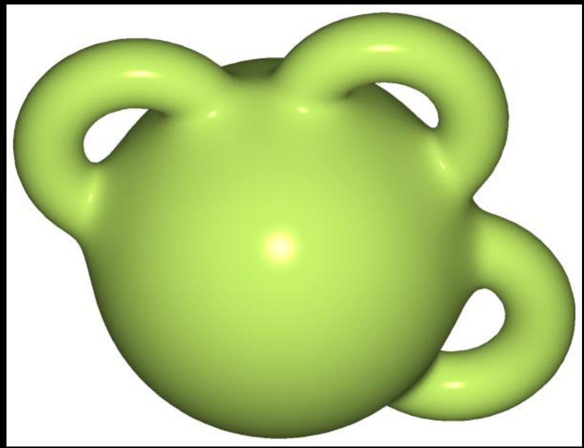
Quantum corrections in the context of Kerr/CFT (extremal and non-extremal versions).

Study quantum corrections of more generic quotients:

1. Wormholes

2. Warped de Sitter black holes

3. Holographic entanglement



Selberg trace formula as a connection between two differerent methods of calculating functional determinants: The heat kernel method and the quasinormal mode method.

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