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Scattering Kerr black holes via amplitudes

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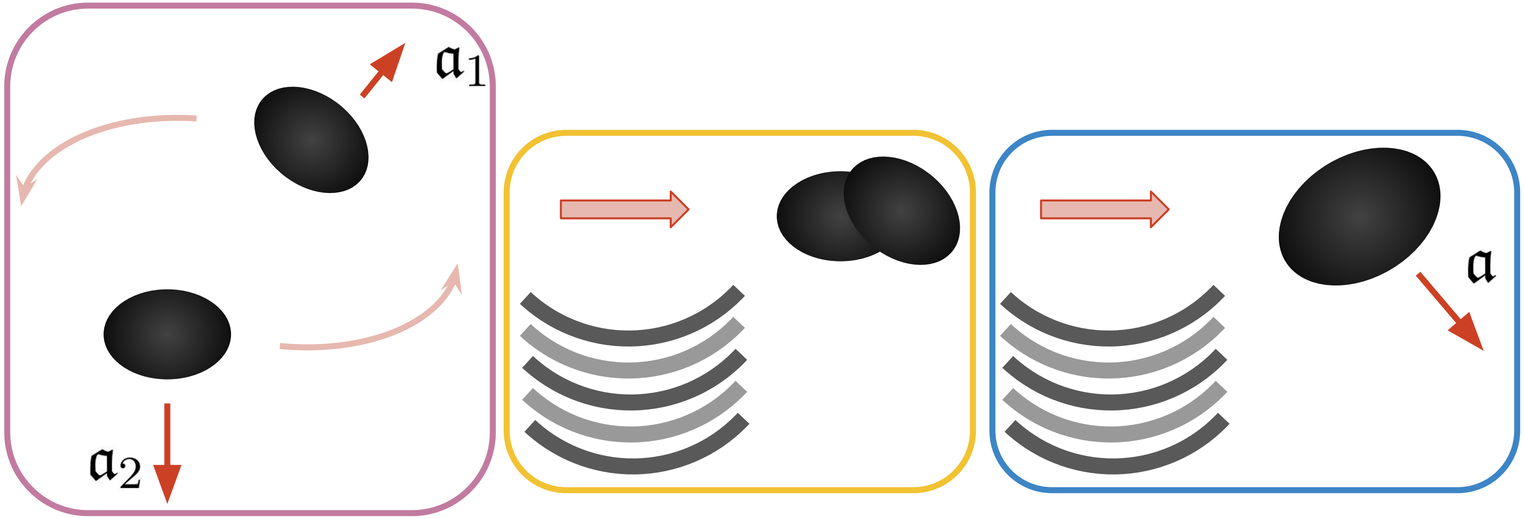
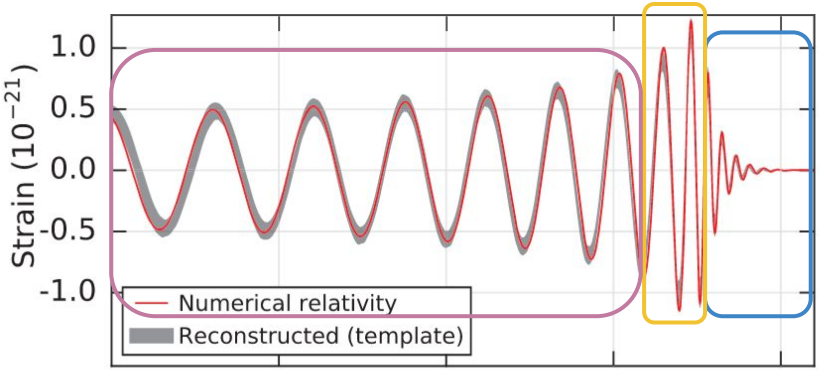
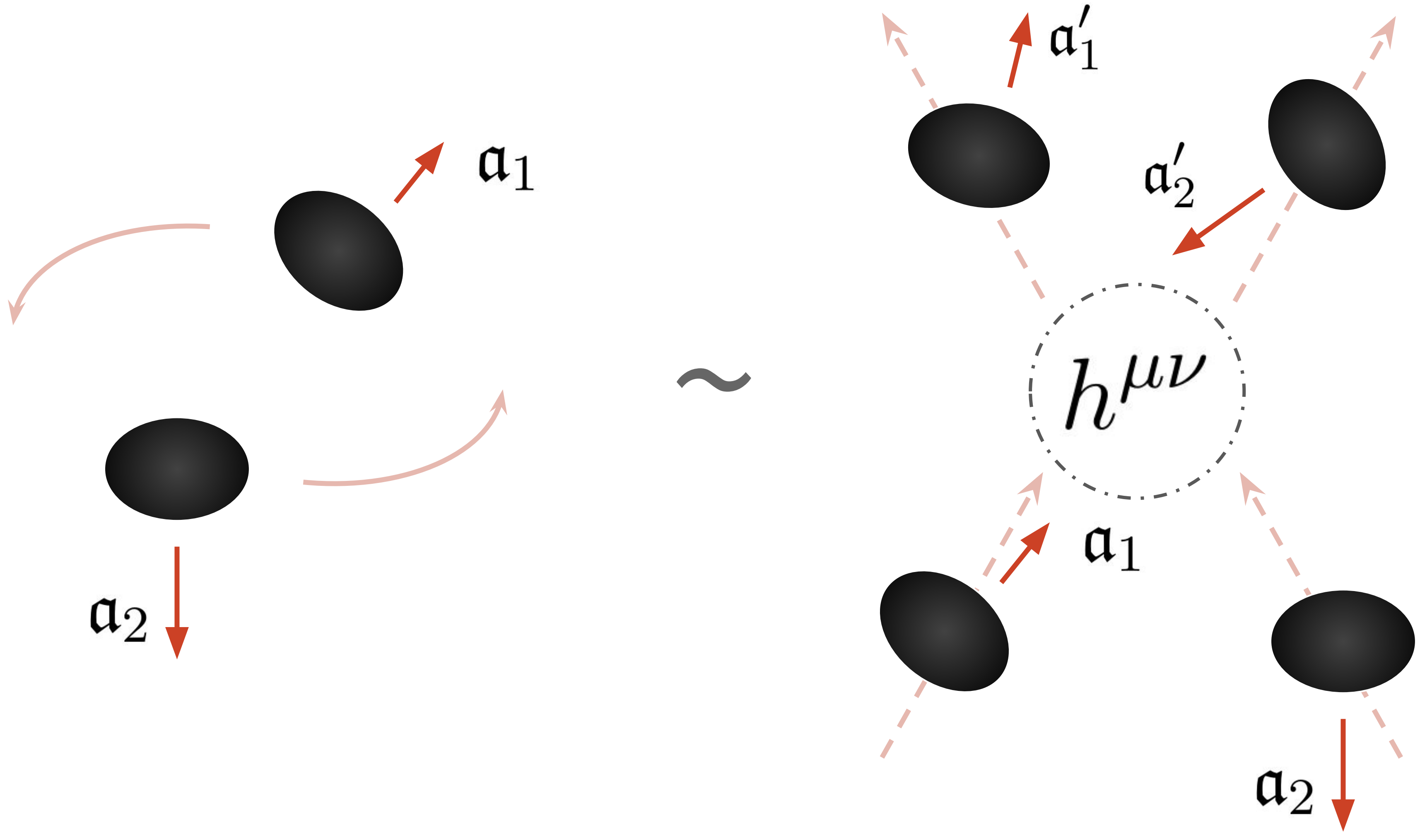
2203.06197 and **2205.02809** with Rafael Aoude & Andreas Helset



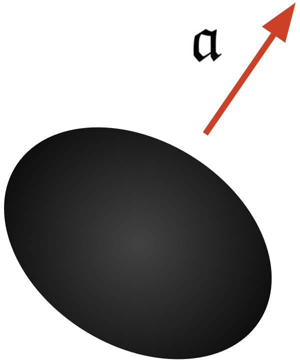


[LIGO Collaboration, ‘16]

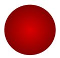


$\mathfrak{a}_1$
$\mathfrak{a}_2$
$\sim$
$\mathfrak{a}'_1$
$\mathfrak{a}'_2$
$h^{\mu\nu}$
$\mathfrak{a}_1$
$\mathfrak{a}_2$

How do we model Kerr scattering?

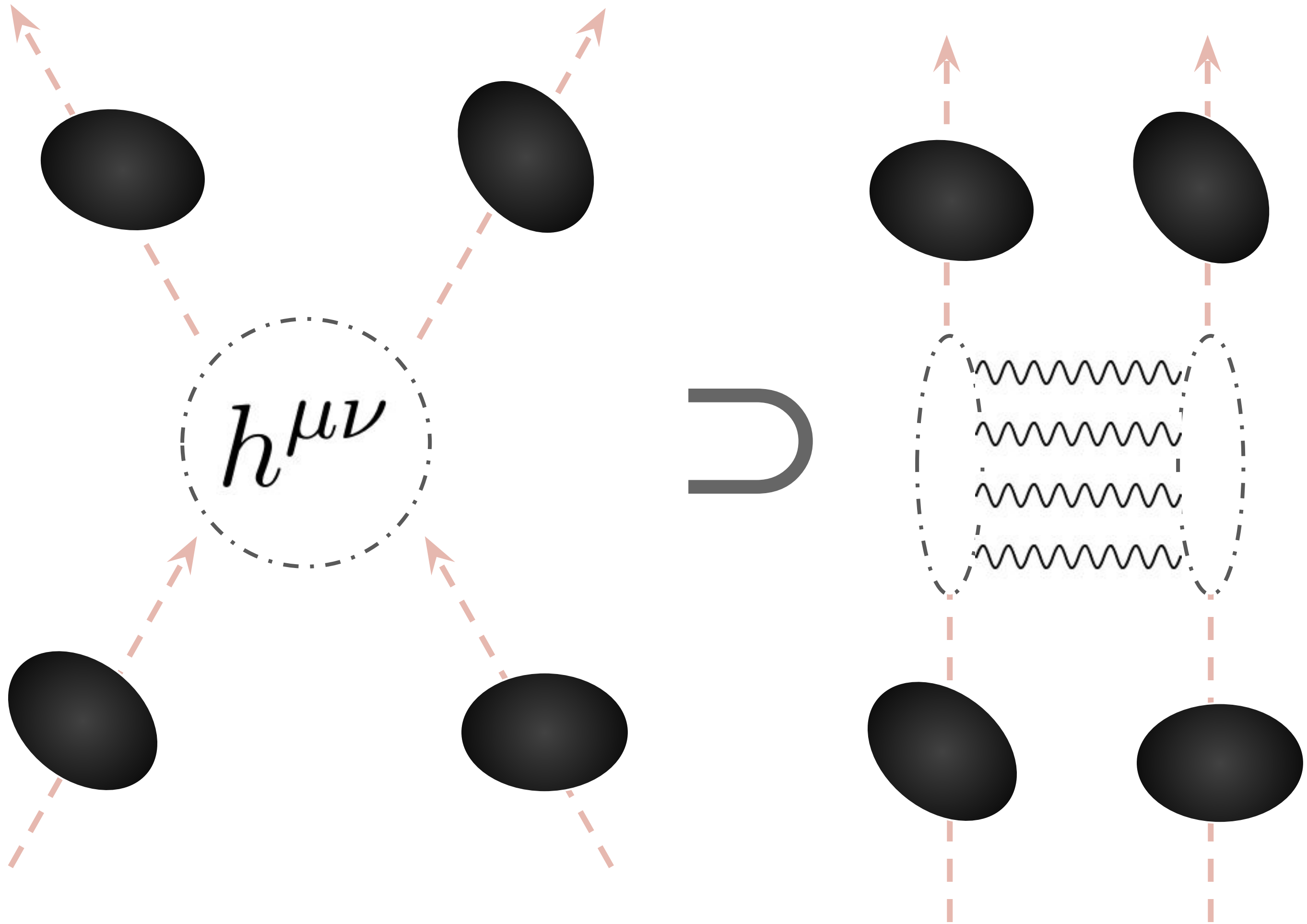


observable $\sim f_0 + f_1 \mathfrak{a} + f_2 \mathfrak{a}^2 + \ldots$



observable $\sim g_0 + g_1 \frac{\langle S \rangle}{m} + g_2 \frac{\langle S^2 \rangle}{m^2} + \cdots + g_{2s} \frac{\langle S^{2s} \rangle}{m^{2s}}$

classical limit, $\hbar \to 0,\ s \to \infty$: $\lim_{\hbar \to 0} \langle S^n \rangle = (m\mathfrak{a})^n, \quad \frac{d}{ds}\left(\lim_{\hbar \to 0} g_i\right) = 0, \quad \lim_{\hbar \to 0} g_i = f_i$

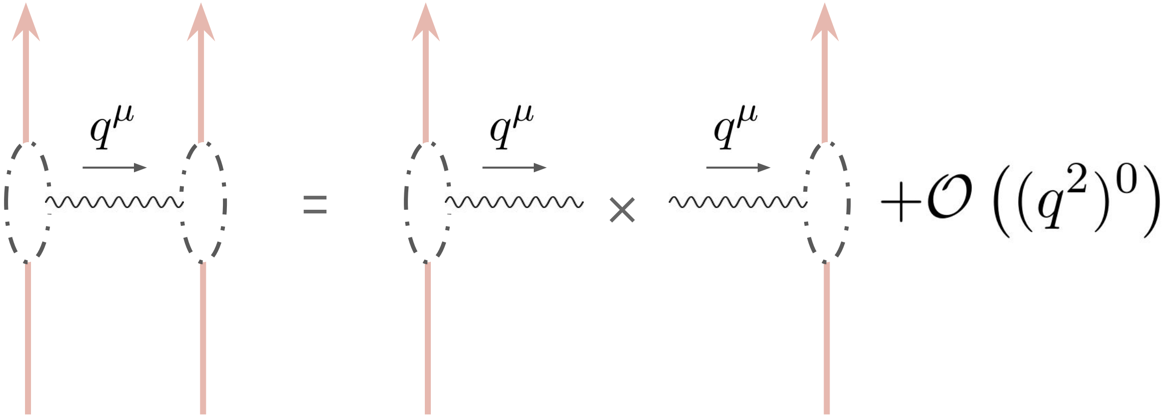


n gravitons

$n-1$ loops

$\mathcal{O}(G^n v^\infty)$

nPM order

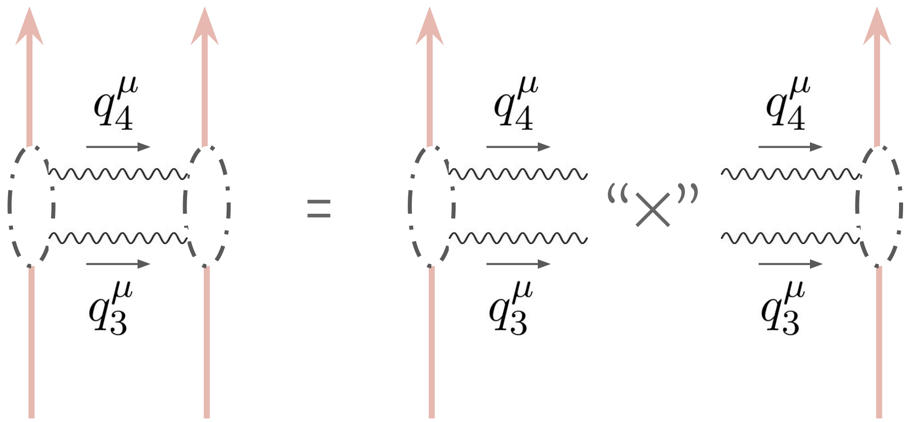


"minimal coupling" amplitude:
[Arkani-Hamed, Huang, Huang, '17]

$$\mathcal{M}_3^{(s)} = \frac{\langle \mathbf{1}\mathbf{1}' \rangle^{\odot 2s}}{m^{2s}} \mathcal{M}_3^{(0)}$$

[Guevara, Ochirov, Vines, '18] based on [Vines, '17]
[Chung, Huang, Kim, Lee, '18] based on [Levi, Steinhoff, '15]

$$\lim_{s\to\infty} \mathcal{M}_3^{(s)} = \mathcal{M}_3^{\text{Kerr}} = e^{\pm q\cdot\mathfrak{a}} \mathcal{M}_3^{\text{Schw}}$$



$$\mathcal{M}^s_{4,-+} = \frac{y^4}{s_{34} t_{13} t_{14}} \left(\frac{\langle \mathbf{31} \rangle [\mathbf{42}] - \langle \mathbf{32} \rangle [\mathbf{41}]}{y} \right)^{\odot 2s}$$

[Arkani-Hamed, Huang, Huang, '17]

$$\lim_{\substack{\hbar \to 0 \\ s \to \infty}} \mathcal{M}^s_{4,-+} = \frac{y^4}{s_{34} t_{13} t_{14}} \exp \left[\left(q_4 - q_3 + \frac{t_{14} - t_{13}}{y} w \right) \cdot \mathfrak{a} \right]$$

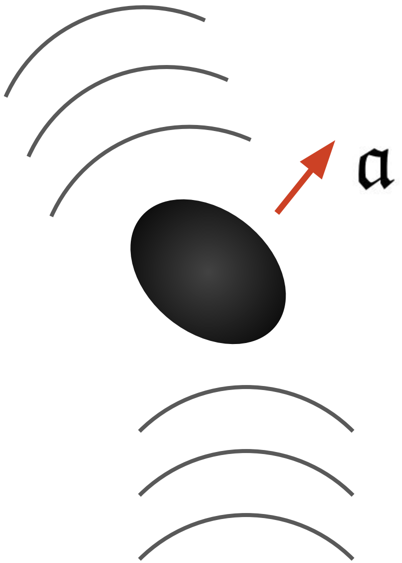
[Aoude, KH, Helset, '20, '22]

(underlying formalism: on-shell heavy spinors)

see also [Guevara, Ochirov, Vines, '18]

$$w^\mu \equiv \frac{1}{2} [4 | \bar{\sigma}^\mu | 3 \rangle, \quad y \equiv 2 p_1 \cdot w$$

$$\mathcal{M}^{\infty}_{4,-+} \xrightarrow{\mathcal{O}(\mathfrak{a}^{n\leq 4})}$$



[Saketh, Vines, ‘22]
[Fabian Bautista’s thesis, ‘22]

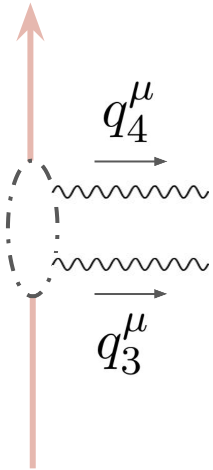
$$\mathcal{M}^{\infty}_{4,-+} \supset \frac{(t_{14}-t_{13})^n}{s_{34}t_{13}t_{14}}\frac{(w\cdot\mathfrak{a})^n}{y^{n-4}}, \qquad \mathcal{O}(\mathfrak{a}^{n>4})$$

A viable Compton amplitude

1. no spurious poles
2. factorizes onto three-point Kerr

see also [Chung, Huang, Kim, Lee, ‘18; Falkowski, Machado, ‘20; Chiodaroli, Johansson, Pichini, ‘21]

Gravity



$$\mathcal{M}^{\infty}_{4,-+} = e^{(q_4 - q_3)\cdot \mathfrak{a}} \sum_{n=0}^{\infty} \frac{1}{n!} K_n$$

$$K_n = \frac{y^4}{s_{34} t_{13} t_{14}} \left(\frac{t_{14} - t_{13}}{y} w \cdot \mathfrak{a} \right)^n$$

$$K_{n>4} = -\frac{4}{s_{34}} \frac{(t_{14} - t_{13})^{n-2}}{y^{n-4}} (w \cdot \mathfrak{a})^n$$

Gravity

vanishing Gram determinant:

$$K_{n\geq 5} = 16m^2 \frac{(t_{14}-t_{13})^{n-4}}{y^{n-4}}(w\cdot\mathfrak{a})^n + 2\mathfrak{s}_1 K_{n-1} - \mathfrak{s}_2 K_{n-2}$$

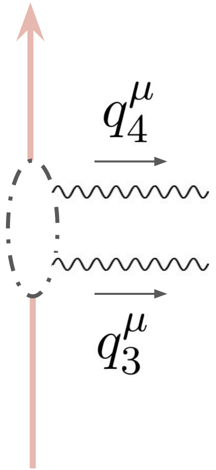
$$\mathfrak{s}_1 \equiv (q_3 - q_4)\cdot\mathfrak{a},$$
$$\mathfrak{s}_2 \equiv -4(q_3\cdot\mathfrak{a})(q_4\cdot\mathfrak{a}) + s_{34}\mathfrak{a}^2$$

solution:

$$\bar{K}_n \equiv \begin{cases} K_n, & n \leq 4, \\ K_4 L_{n-4} - K_3 \mathfrak{s}_2 L_{n-5}, & n > 4. \end{cases}$$

$$L_m \equiv \sum_{j=0}^{\lfloor m/2 \rfloor} \binom{m+1}{2j+1} \mathfrak{s}_1^{m-2j} (\mathfrak{s}_1^2 - \mathfrak{s}_2)^j$$

Gravity



$$\mathcal{M}^{\infty}_{4,-+} = e^{(q_4 - q_3)\cdot \mathfrak{a}} \sum_{n=0}^{\infty} \frac{1}{n!} \bar{K}_n$$

$$\mathcal{M}^{\text{Kerr}}_{4,-+} = \mathcal{M}^{\infty}_{4,-+} + m^2 \mathcal{C} \text{ where } \text{Res}_{t_{13}=0}\mathcal{C} = \text{Res}_{t_{14}=0}\mathcal{C} = \text{Res}_{s_{34}=0}\mathcal{C} = 0$$

Kerr Compton amplitude for $\mathcal{O}(\mathfrak{a}^{n\leq 4})$ invariant under shift (see also [Bern, Kosmopoulos, Luna, Roiban, Teng, '22])

$$\mathfrak{a}^\mu \to \mathfrak{a}^\mu + \xi(q_3^\mu + q_4^\mu)$$

require same symmetry of contact terms:*

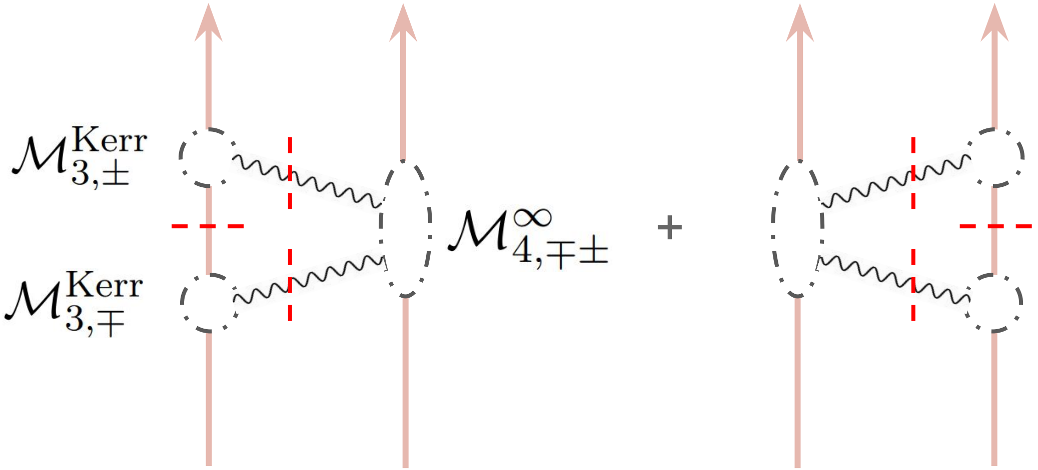
$$\mathcal{C} \equiv (w\cdot\mathfrak{a})^4 \sum_{n=0}^{\infty}\sum_{j=0}^{\infty} d_{n,j}\mathfrak{s}_1^n(\mathfrak{s}_1^2-\mathfrak{s}_2)^j$$

only $\lfloor k/2 \rfloor - 1$ terms at $\mathcal{O}(\mathfrak{a}^{k\geq 4})$.

***does not necessarily describe Kerr above fourth spin order**

High-spin scattering at 2PM

spinning object × Schwarzschild



no $\mathcal{M}_{4,\pm\pm}^{\infty}$ if shift symmetry extends to 2PM

$$\mathcal{M}_{\rm 2PM} = \frac{2G^2\pi^2 m_1^2 m_2^2}{\sqrt{-q^2}}\left(\mathcal{M}_{\rm 2PM}^{\rm even} + i\omega\mathcal{E}_1\mathcal{M}_{\rm 2PM}^{\rm odd}\right)$$

$$\omega \equiv v_1\cdot v_2$$
$$\mathcal{E}_1 \equiv \epsilon^{\mu\nu\alpha\beta}v_{1\mu}v_{2\nu}q_\alpha\mathfrak{a}_{1\beta}$$
$$Q \equiv (q\cdot\mathfrak{a}_1)^2 - q^2\mathfrak{a}_1^2$$
$$V \equiv q^2(v_2\cdot\mathfrak{a}_1)^2$$

$$\begin{aligned}\mathcal{M}_{\rm 2PM}^{\rm even} = m_1\Bigg[&3(5\omega^2-1)\mathcal{F}_0 + \frac{1}{4}(\omega^2-1)\mathcal{F}_2 Q + \frac{8\omega^4-8\omega^2+1}{\omega^2-1}\mathcal{F}_1 Q\\ &-\frac{1}{2}\mathcal{F}_2 V + \sum_{k=1}^{\infty}\frac{(8\omega^4-8\omega^2+1)}{(\omega^2-1)^{k+1}}\frac{(-1)^k 2\Gamma[k]}{\Gamma[2k]}\mathcal{F}_{k-1}V^k\Bigg]\\ -m_2\Bigg[&-3(5\omega^2-1)\sqrt{\pi}\mathcal{F}_{-1/2} - \frac{3\sqrt{\pi}}{4}\mathcal{F}_{1/2}Q - \frac{1}{\omega^2-1}\mathcal{F}_1 Q + \frac{15\sqrt{\pi}}{4}\mathcal{F}_{1/2}V\\ &+6\sqrt{\pi}\sum_{k=1}^{\infty}\frac{\omega^{2k}}{(\omega^2-1)^{k+1}}\frac{(-1)^k\mathcal{F}_{k-1}V^k}{\Gamma[2k+1]\Gamma[5/2-k]}\\ &\times\left[{}_2F_1\left(\frac{1}{2}-k,-k;\frac{5}{2}-k;\frac{1}{\omega^2}\right) - \left(k+\frac{3}{2}\right){}_2F_1\left(\frac{3}{2}-k,-k;\frac{5}{2}-k;\frac{1}{\omega^2}\right)\right]\\ &-\frac{1}{64}(3Q^2+30QV+35V^2)\sum_{k=0}^{\infty}c_k^{(0)} + \frac{1}{16}(Q+7V)(Q+V)\sum_{k=1}^{\infty}c_k^{(1)} - \frac{1}{64}(Q+V)^2\sum_{k=2}^{\infty}c_k^{(2)}\Bigg]\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{\rm 2PM}^{\rm odd} = -m_1&\left[4\mathcal{F}_1 + \sum_{k=0}^{\infty}\frac{(2\omega^2-1)}{(\omega^2-1)^{k+1}}\frac{(-1)^k 8\Gamma[k+1]}{\Gamma[2k+1]}\mathcal{F}_k V^k\right]\\ -m_2&\Bigg[\frac{15\sqrt{\pi}}{2}\mathcal{F}_{1/2} + \sum_{k=0}^{\infty}\frac{\omega^{2k}}{(\omega^2-1)^{k+1}}\frac{4^{1-k}\mathcal{F}_k V^k}{(1)_k(2k-1)}\\ &\times\left[{}_2F_1\left(-\frac{1}{2}-k,-k;\frac{3}{2}-k;\frac{1}{\omega^2}\right) - \left(k+\frac{5}{2}\right){}_2F_1\left(\frac{1}{2}-k,-k;\frac{3}{2}-k;\frac{1}{\omega^2}\right)\right]\Bigg]\end{aligned}$$

consistent with [Chen, Chung, Huang, Kim, ‘21] up to fourth order in spin.

agrees with [Bern, Kosmopoulos, Luna, Roiban, Teng, ‘22] at fifth order in spin

Summary

Compton amplitude needed for 2PM Kerr $\times$ Kerr scattering amplitude

- known for Kerr up to fourth order in spin; unknown for higher spin, amplitude has spurious poles

derived viable classical amplitude to all spin orders; differs from Kerr only by contact terms

- fixed contact terms assuming Kerr-scattering shift symmetry at low spins applies to all spins

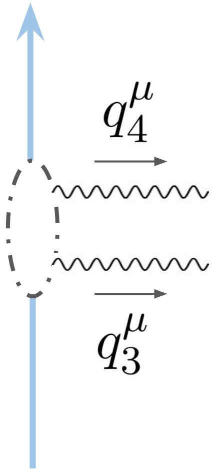
spinning $\times$ Schwarzschild amplitude derived to all spins

- ultrarelativistic limit selects Kerr at fourth spin order; can use limit to fix remaining coefficients for even-in-spin contact terms (not shown)

Kerr is not determined by this analysis... what is Kerr???

Backup

QED



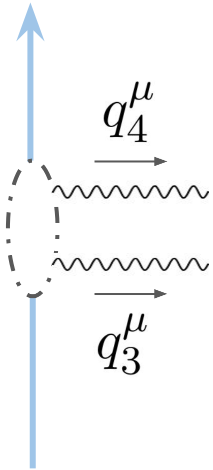
$$\mathcal{A}^{\infty}_{4,-+} = \frac{y^2}{t_{13}t_{14}} \exp\left[\left(q_4 - q_3 + \frac{t_{14}-t_{13}}{y} w\right)\cdot \mathfrak{a}\right]$$

$$= e^{(q_4-q_3)\cdot\mathfrak{a}} \sum_{n=0}^{\infty} \frac{1}{n!} I_n$$

$$I_n \equiv \frac{y^2}{t_{13}t_{14}} \left(\frac{t_{14}-t_{13}}{y} w \cdot \mathfrak{a}\right)^n$$

$$I_{n\geq 2} = -4 \left(\frac{t_{14}-t_{13}}{y}\right)^{n-2} (w\cdot\mathfrak{a})^n$$

QED



$$\mathcal{A}^{\infty}_{4,-+} = e^{(q_4 - q_3)\cdot\mathfrak{a}} \sum_{n=0}^{2} \frac{1}{n!} I_n$$

$$\mathcal{M}_{4,-+}^{\infty} + m^2 \mathcal{C}$$

decompose contact term into Wilson coefficients and functions of momenta and spin vector:

$$\mathcal{C} = \sum_{j,k} c_{j,k} \mathcal{C}_j(\mathfrak{a}^k)$$

three relevant scales: $c_{j,k} \sim G^{n_1} m^{n_2} \hbar^{n_3}$

$$\left.\begin{aligned} \mathcal{M}_{4,-+}^{\infty} &\sim \mathcal{O}(\hbar^0) \\ [\mathcal{M}_{4,-+}^{\infty}] &= [m]^2 \end{aligned}\right\} \Longrightarrow c_{j,k} \sim \left(\frac{Gm}{\hbar}\right)^n$$

$$\mathcal{C} = \sum_{j,k} c_{j,k} \mathcal{C}_j(\mathfrak{a}^k) \qquad c_{j,k} \sim \left(\frac{Gm}{\hbar}\right)^n$$

relevant at $\mathcal{O}(G^2) \Rightarrow n = 0$

$$\Rightarrow \begin{cases} c_{j,k}\mathcal{C}_j(\mathfrak{a}^k)\Big|_{k\leq 3} = \mathcal{O}(\hbar) \\ 1 \leq j \leq \frac{1}{24}(2k^3 + 9k^2 - 74k) + \begin{cases} 4, & k \text{ even} \geq 4, \\ \frac{87}{24}, & k \text{ odd} \geq 5. \end{cases} \end{cases}$$

$$\mathcal{M}_{2\mathrm{PM}} = G^2 m_1^2 m_2^2 \frac{\pi^2}{\sqrt{-q^2}} \sum_{n=0}^{\infty} \sum_{k=0}^{n} \left(M_k^{(2n)} + i\omega \mathcal{E}_1 M_k^{(2n+1)} \right) Q^{n-k} V^k$$

$$\omega \equiv v_1 \cdot v_2$$
$$\mathcal{E}_1 \equiv \epsilon^{\mu\nu\alpha\beta} v_{1\mu} v_{2\nu} q_\alpha \mathfrak{a}_{1\beta}$$
$$Q \equiv (q \cdot \mathfrak{a}_1)^2 - q^2 \mathfrak{a}_1^2$$
$$V \equiv q^2 (v_2 \cdot \mathfrak{a}_1)^2$$

fifth order in spin:

$$M_0^{(5)} = \frac{3m_2(4\omega^2 - 1) + 2m_1(13\omega^2 - 7)}{48(\omega^2 - 1)},$$
$$M_1^{(5)} = -\frac{m_2(5\omega^2 + 1) + 8m_1(2\omega^2 - 1)}{8(\omega^2 - 1)^2},$$
$$M_2^{(5)} = \frac{m_2(-7\omega^4 + 34\omega^2 - 3) + 32m_1(2\omega^2 - 1)}{48(\omega^2 - 1)^3}.$$

agrees with [Bern, Kosmopoulos, Luna, Roiban, Teng, '22]

seventh order in spin:

$$M_0^{(7)} = \frac{6m_2(8\omega^2 - 1) + 7m_1(17\omega^2 - 9)}{8064(\omega^2 - 1)},$$
$$M_1^{(7)} = -\frac{m_2(5\omega^2 + 1) + 8m_1(2\omega^2 - 1)}{192(\omega^2 - 1)^2},$$
$$M_2^{(7)} = \frac{m_2(-7\omega^4 + 34\omega^2 - 3) + 32m_1(2\omega^2 - 1)}{576(\omega^2 - 1)^3},$$
$$M_3^{(7)} = -\frac{m_2(9\omega^6 - 41\omega^4 + 95\omega^2 - 15) + 64m_1(2\omega^2 - 1)}{2880(\omega^2 - 1)^4}.$$

fourth order in spin:

$$M_0^{(4)} = \frac{m_1(239\omega^4 - 250\omega^2 + 35) + 3m_2\left[8\omega^2(5\omega^2 - 4) + 3d_{0,0}(\omega^2 - 1)\right]}{96(\omega^2 - 1)},$$

$$M_1^{(4)} = -\frac{2m_1(193\omega^4 - 194\omega^2 + 25) - 3m_2\left[8(-5\omega^4 + \omega^2 + 2) + 15d_{0,0}(\omega^2 - 1)^2\right]}{48(\omega^2 - 1)^2},$$

$$M_2^{(4)} = \frac{64m_1(8\omega^4 - 8\omega^2 + 1) + m_2\left[8(15\omega^4 + 6\omega^2 - 13) + 105d_{0,0}(\omega^2 - 1)^3\right]}{96(\omega^2 - 1)^3}.$$

Kerr has $d_{0,0} = 0$ which improves $\lim_{\omega\to\infty} M_2^{(4)}$; agrees with [Chen, Chung, Huang, Kim, '21].

requiring best $\lim_{\omega\to\infty} M_i^{(2n)}$ sets $d_{2k,j} = -\frac{16(k-j)(2k+1)}{(2j+2k+4)!}$.