31st Nordic String Meeting

NBI Copenhagen

The Many Avatars of Carroll CFTs

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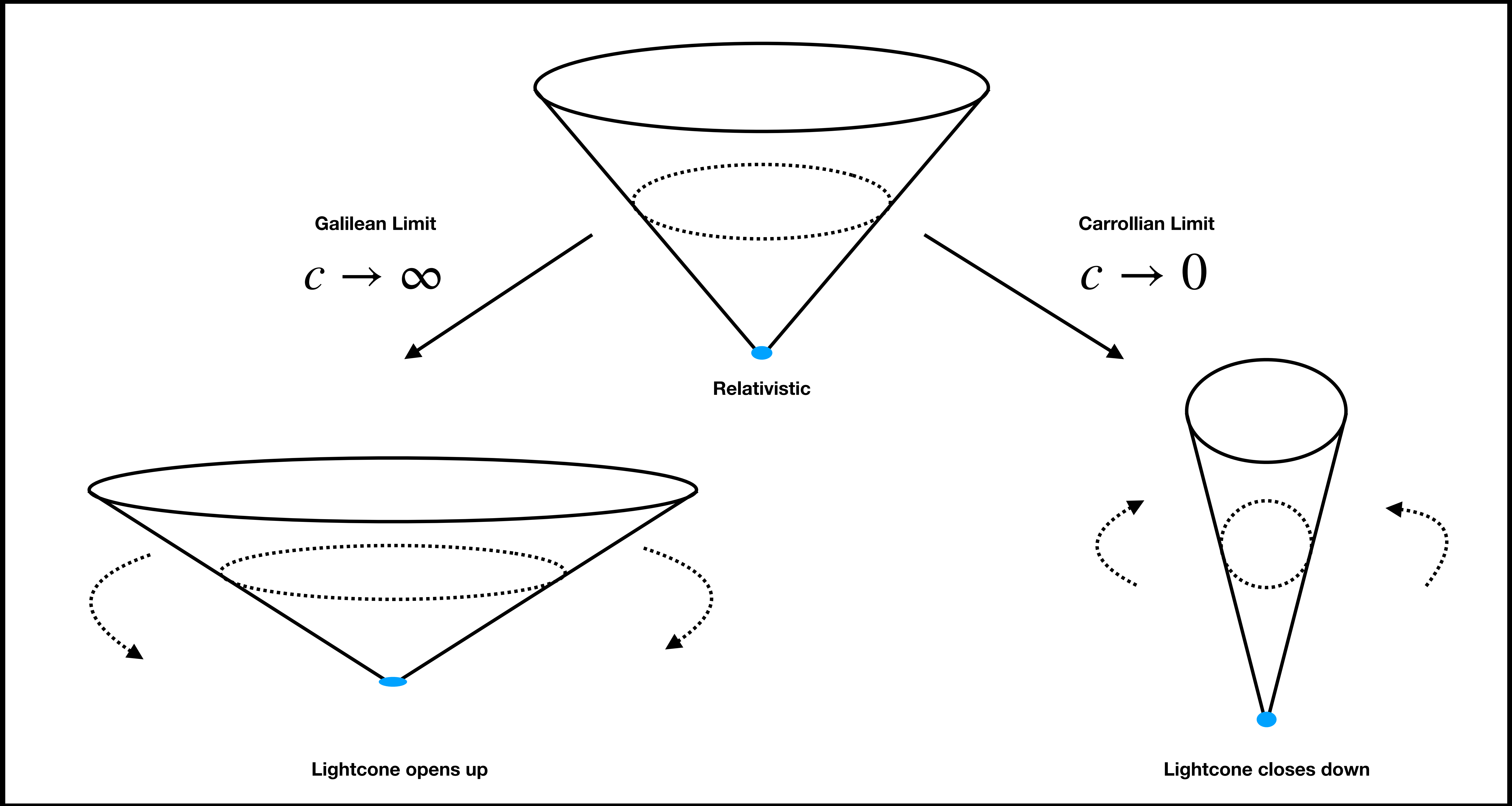
Well... not quite this!
Although it seems a sequel is coming next month!

This is more of a journey down a rabbit hole …

Non Lorentzian Limits



* We are familiar with Galilean limits.
* Here we would be interested in the diametrically opposite one, the Carroll limit.


Holography
Non-Lorentzian Holography
Flat Holography
Carroll Fluids
Cosmology
Condensed Matter
Carrollian CFTs
Black Holes
Fractons
Superconductivity
Luttinger Liquids
String Theory
Tensionless/null strings
Carroll strings

Today ...

We will give a brief overview of:

* Flat holography from a Carroll Perspective
* Tensionless or null strings

Flat Holography: A Carroll Perspective

Carroll and Conformal Carroll Symmetry: The algebraic way

* Carroll algebra: Inonu-Wigner contraction of Poincare algebra when $c \to 0$
* This can be achieved by $x^i \to x^i, \quad t \to \epsilon t, \quad \epsilon \to 0$
* Carroll generators: $H = \partial_t, \quad P_i = \partial_i, \quad C_i = x_i \partial_t, \quad J_{ij} = x_i \partial_j - x_j \partial_i.$
* The algebra: $[J_{ij}, J_{kl}] = 4\delta_{[i[k}J_{l]j]}, \ [J_{ij}, P_k] = 2\delta_{k[j}P_{i]}, \ [J_{ij}, C_k] = 2\delta_{k[j}C_{i]}, \ [C_i, P_j] = -\delta_{ij}H.$
* Crucially: $[C_i, C_j] = 0$. Reflects non-Lorentzian nature of the algebra.
* Conformal extension: $D = t\partial_t + x_i \partial_i, \quad K_0 = x_i x_i \partial_t, \quad K_i = 2x_i(t\partial_t + x_j \partial_j) - x_j x_j \partial_i.$
* Conformal Carroll algebra: $[D, P_i] = -P_i, \ [D, H] = -H \ [D, K_i] = K_i, \ [D, K_0] = K_0,$
  $[K_0, P_i] = -2C_i \ [K_i, H] = -2C_i, \ [K_i, P_j] = -2\delta_{ij}D - 2J_{ij}.$
* Can be given an infinite dimensional lift in all dimensions.

Carroll & Conformal Carroll Symmetry: The geometric way

* Start with Minkowski spacetime: $ds^2 = -c^2dt^2 + (dx^i)^2$ and send speed of light to zero.
* Metric degenerates

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & 0 \\ 0 & I_{d-1} \end{pmatrix} \to \tilde{h}_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & I_{d-1} \end{pmatrix}, \quad \eta^{\mu\nu} = \begin{pmatrix} -1/c^2 & 0 \\ 0 & I_{d-1} \end{pmatrix} \quad -c^2\eta^{\mu\nu} \to \Theta^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0_{d-1} \end{pmatrix}$$

* Also: $\Theta^{\mu\nu} = \theta^\mu\theta^\nu \qquad \tilde{h}_{\mu\nu}\theta^\nu = 0.$ Henneaux 1979
* A Carroll manifold is defined by a quadruple $(\mathcal{C}, \tilde{h}, \theta, \nabla)$ Duval, Gibbons, Horvathy 2014
  - $\mathcal{C}$ is a d dimensional manifold, on which one can choose a coordinate chart (t, x^i).
  - $\tilde{h}$ is a covariant, symmetric, positive, tensor field of rank $d-1$ and of signature $(0, \underbrace{+1, \ldots, +1}_{d-1})$.
  - θ is a non-vanishing vector field which generates the kernel of $\tilde{h}$.
  - ∇ is a symmetric affine connection that parallel transports both $\tilde{h}_{\mu\nu}$ and θ^ν.
* Carroll Lie algebra: $\mathcal{L}_\xi \tilde{h}_{\mu\nu} = 0, \quad \mathcal{L}_\xi\theta = 0.$ Conformal Carroll Lie algebra: $\mathcal{L}_\xi\tilde{h} = \lambda\tilde{h}, \quad \mathcal{L}_\xi\theta = -\frac{\lambda}{2}\theta.$

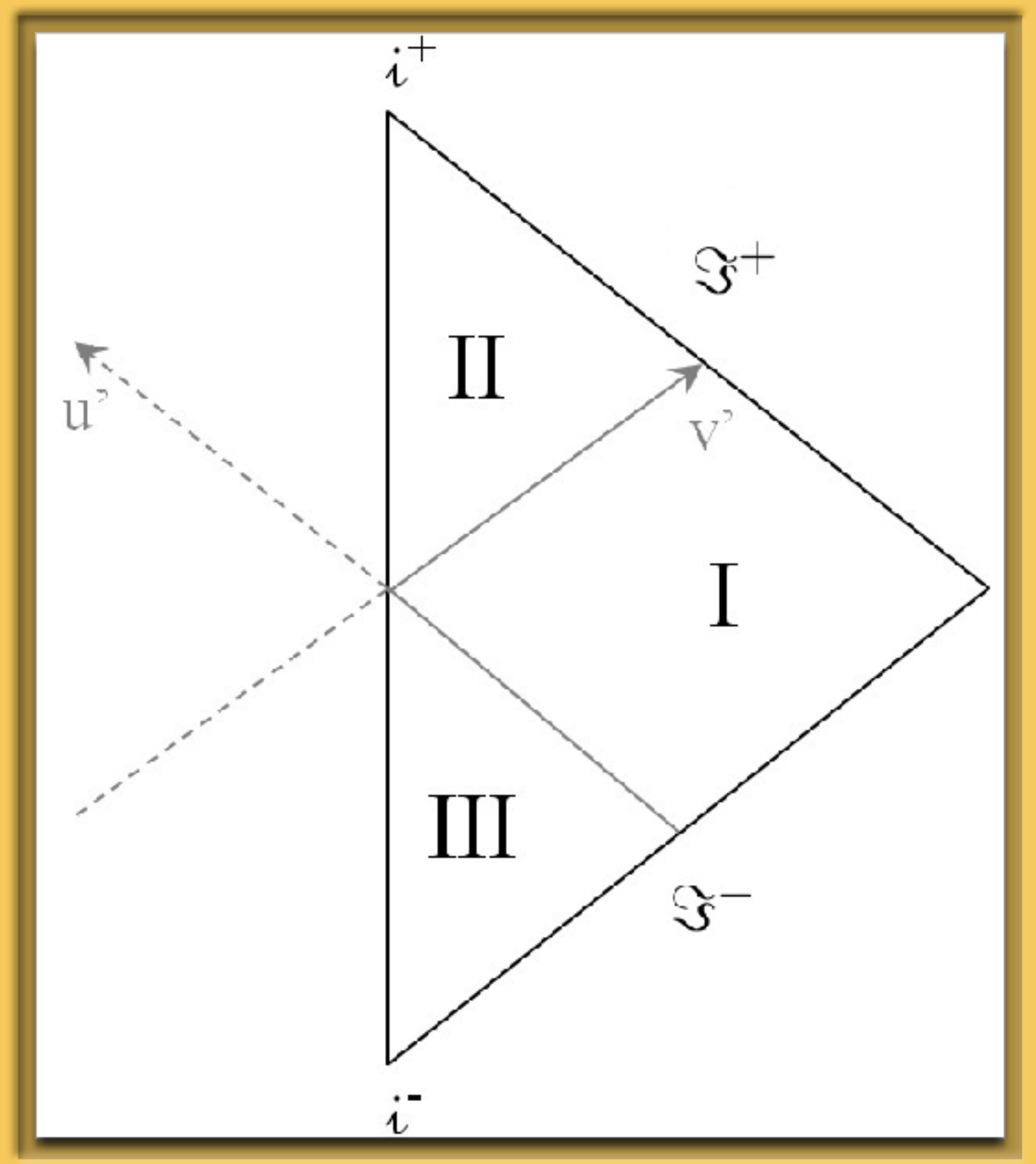
Flat space and BMS symmetries

- Asymptotic symmetries of flat space at null infinity is given by the Bondi-Metzner-Sachs (BMS) group.
- In 3 and 4 dimensions, the BMS group is infinite dimensional.
- In 3 dimensions, the BMS_3 algebra reads:

$$[L_n, L_m] = (n-m)L_{m+n} + \frac{c_L}{12}\delta_{n+m,0}(n^3-n)$$
$$[L_n, M_m] = (n-m)M_{m+n} + \frac{c_M}{12}\delta_{n+m,0}(n^3-n)$$
$$[M_n, M_m] = 0.$$

- M's: supertranslations. Angle dependent translations along the null direction.
- L's: superrotations. Diffeos of the circle at infinity.
- For Einstein gravity, $c_L = 0, \quad c_M = \frac{3}{G}$

Barnich, Compere 2006



Penrose Diagram of Minkowski spacetime

Asymptotic Symmetries of 4d Flat Spacetime

* In 4d, the BMS_4 algebra is a bit more involved.

$$[L_n, L_m] = (n-m)L_{n+m}, \quad [\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m}$$
$$[L_n, M_{r,s}] = \left(\frac{n+1}{2} - r\right) M_{n+r,s}, \quad [\bar{L}_n, M_{r,s}] = \left(\frac{n+1}{2} - s\right) M_{r,n+s}$$
$$[M_{r,s}, M_{t,u}] = 0.$$

* Two Virasoros and supertranslations with two legs.
* Complications regarding central charges, which we will studiously avoid for now.

The Connection

AB 2010;
Duval, Gibbons, Horvathy 2014.

$$\mathfrak{CCarr}_d = \mathfrak{bms}_{d+1}.$$

Conformal Carroll algebra in d-dimensions is isomorphic to the BMS algebra in (d+1) dimensions

From AdS to Flatspace

* Can obtain flat space by taking the radius of AdS to infinity.
* Start with 2 copies of Virasoro algebra that form asymptotic symmetries of AdS3.

$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m} + \frac{c}{12}\delta_{n+m,0}(n^3 - n).$$
$$[\bar{\mathcal{L}}_n, \bar{\mathcal{L}}_m] = (n-m)\bar{\mathcal{L}}_{n+m} + \frac{\bar{c}}{12}\delta_{n+m,0}(n^3 - n).$$
$$[\mathcal{L}_n, \bar{\mathcal{L}}_m] = 0$$

* The central terms of the left and right copies: $c = \bar{c} = \frac{3\ell}{2G}$
* We take the following limit: $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$
* Easy to see that this contracts 2 copies of Virasoro algebra to BMS3 algebra.
* The central terms $c_L = c - \bar{c} = 0$ and $c_M = \epsilon(c + \bar{c}) = \frac{3}{G}$

Barnich, Compere 2006

* **Flatspace limit in bulk = Carroll limit on boundary.**

AB, Fareghbal 2012

Carrollian road to Minkowskian holography

* Field theory dual to Minkowski spacetimes should inherit its asymptotic symmetries.
* For D-dim Minkowski spacetimes, the dual theory should be a (D-1)-dim field theory living on the null boundary of flatspace. It should be a **(D-1)-dimensional Carrollian CFT**.
* We would have two separate tools to study these field theories.
  * The intrinsic way: use only symmetries of BMS.
  * The limiting way: use the Carrollian limit from relativistic CFTs.
* We will be attempting to understand aspects of flatspace from a field theory on $\mathcal{I}_+$.

Carrollian Holography: some checks of proposal

* Asymptotic density of states from field theory and bulk [AB, Detournay, Fareghbal, Simon 2012; Barnich 2012; AB, Basu 2013.]

* Multipoint correlation functions of EM tensor in boundary and bulk.
  * Novel phase transitions from zero-point functions. [AB, Detournay, Grumiller, Simon'13].
  * Matching of higher point correlations [AB, Grumiller, Merbis '15].

* Construction and matching of Entanglement Entropy [AB, Basu, Grumiller, Riegler '14; Jiang, Song, Wen '17; Hijano-Rabideau '17].

* Holographic Reconstruction of 3d flatspace [Hartong '15].

* Construction of bulk-boundary dictionary, matching of correlation functions of primary operators [Hijano-Rabideau '17; Hijano '18]

* BMS Characters & matching with 1-loop partition function [Oblak '15; Barnich, Gonzalez, Oblak, Maloney '15; AB, Saha, Zodinmawia '19]

* Asymptotic Structure constants from boundary and bulk [AB, Nandi, Saha, Zodinmawia '20]

* Generalisations
  * Flat Space Chiral Gravity: CS Gravity dual to chiral half of CFT. [AB, Detournay, Grumiller '12]
  * Higher spin theories in flat space. [Afshar, AB, Fareghbal, Grumiller, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13]

* Fluid-Gravity correspondence for flat space [Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18].

* Carrollian holography for d=4 [Donnay et al '22].

Ancient History

AB, Detournay, Fareghbal, Simon 2012.

See also Barnich 2012.

S=Area/4G for Flat Holography?

* Important early checks of AdS/CFT: CFT reproduces Black Hole entropy.
* Entropy of BTZ black holes = Entropy from Cardy formula in CFT2.
* Can we do something similar for holography in flat spacetimes?
* Yes! AB, Detournay, Fareghbal, Simon 2012. (See also Barnich 2012)
* We will quickly review this old work to remind people of one of the early successes of this programme.

BTZ Black holes and 2d CFT

* The non-extremal BTZ black hole is given by

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2\ell^2}dt^2 + \frac{r^2\ell^2}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2 + r^2\left(d\phi + \frac{r_+ r_-}{\ell r^2}dt\right)^2$$

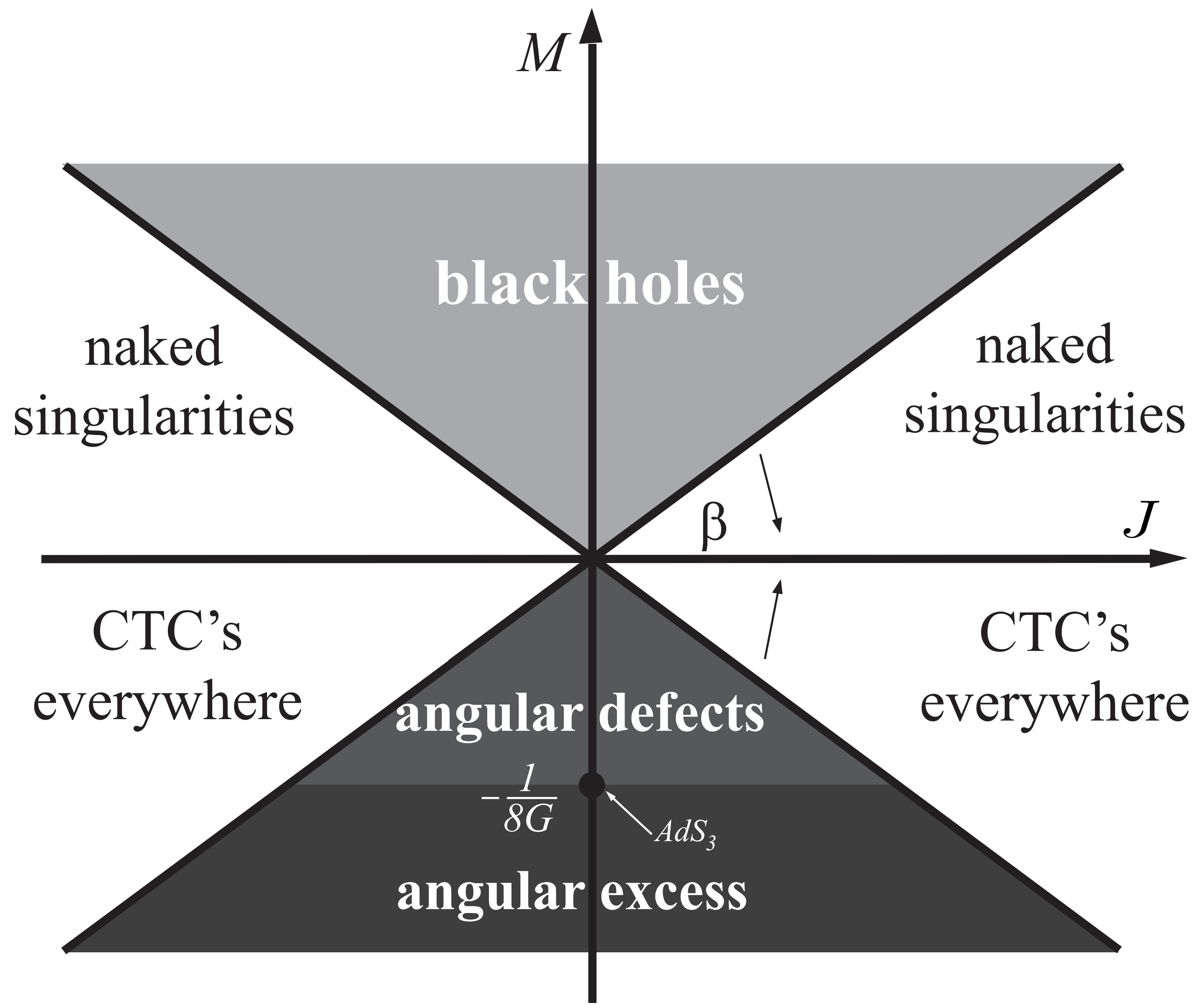
$$r_\pm = \sqrt{2G\ell(\ell M + J)} \pm \sqrt{2G\ell(\ell M - J)};$$

* Bekenstein-Hawking entropy: $S_{BH} = \frac{\text{Area of Horizon}}{4G} = \frac{\pi r_+}{2G}.$

* Cardy formula for 2d CFTs: $S_{CFT} = 2\pi\left(\sqrt{\frac{c\,h}{6}} + \sqrt{\frac{\bar{c}\,\bar{h}}{6}}\right).$

* Central terms for AdS3 and weights: $c = \bar{c} = \frac{3\ell}{2G}.\quad h = \frac{1}{2}(\ell M + J) + \frac{c}{24},\qquad \bar{h} = \frac{1}{2}(\ell M - J) + \frac{\bar{c}}{24}$

* So ultimately: $S_{BH} = S_{CFT}$



Phase space of AdS3 solutions

Flat Space Cosmologies

* Take the radius of AdS to infinity. No Black holes in 3d flat spacetimes. What is happening?

* Outer horizon goes to infinity. Left with inside of BTZ black hole.

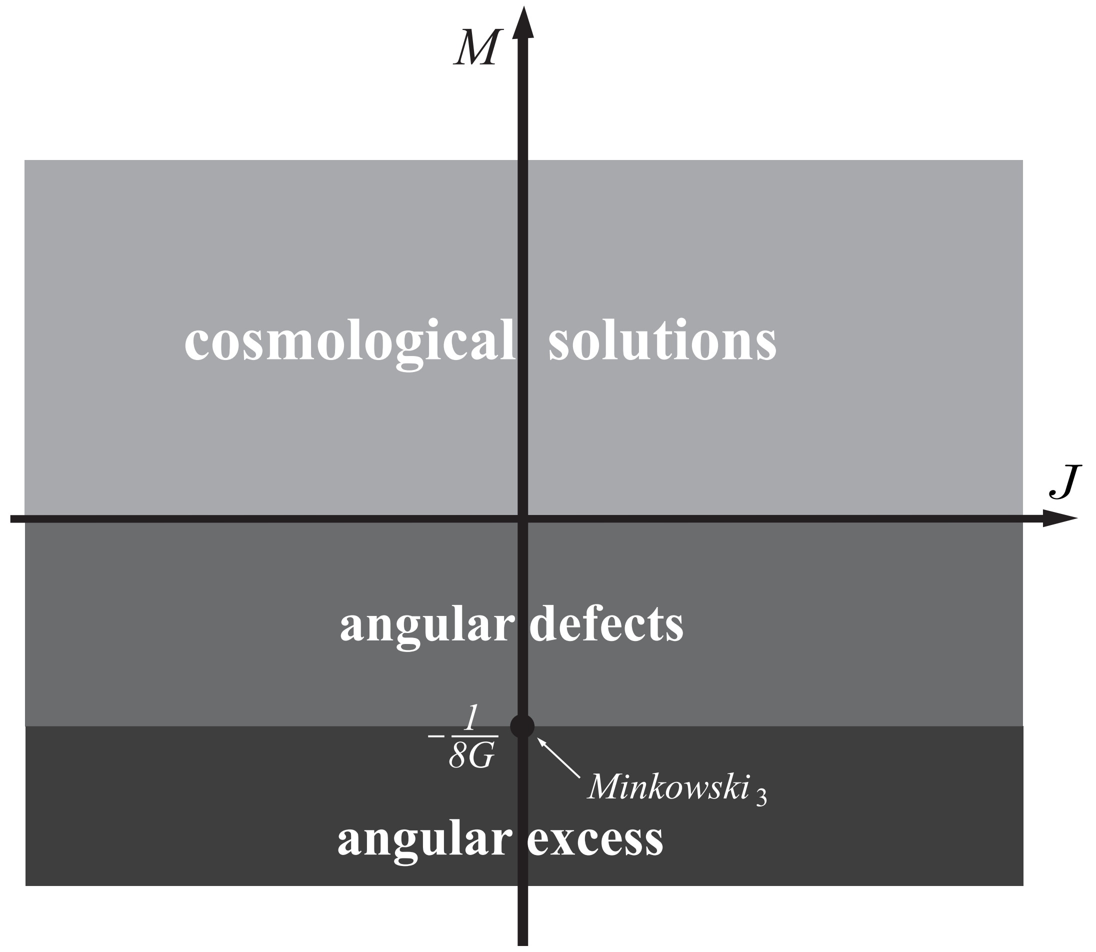
$$\ell \to \infty : r_+ \to \ell\sqrt{2GM} = \ell \hat{r}_+, \quad r_- \to r_0 = \sqrt{\frac{2G}{M}}\, J.$$

* Inner horizon survives. Cosmological solution with horizon. Flat Space Cosmology.

$$ds^2_{\rm FSC} = \hat{r}_+^2 dt^2 - \frac{r^2\, dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 d\phi^2 - 2\hat{r}_+ r_0 dt d\phi$$

* Entropy:

$$\boxed{S_{\rm FSC} = \frac{\text{Area of horizon}}{4G} = \frac{\pi r_0}{2G} = \frac{\pi J}{\sqrt{2GM}}}$$



Phase space of Min_3 solutions

BMS-Cardy formula and Entropy matching

* Label states of the 2d Carroll CFT: $L_0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle,\ \ M_0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle$

* Partition function: $Z_{\text{CarrollCFT}} = \text{Tr}\exp\{2\pi i\,(\sigma L_0 + \rho M_0)\}$

* Carroll modular transformations: $\sigma \to \frac{a\sigma + b}{c\sigma + d}, \quad \rho \to \frac{\rho}{(c\sigma + d)^2}$

* Demand invariance of Z to derive BMS-Cardy formula

$$\boxed{S^{(0)} = \ln d(\Delta,\xi) = 2\pi\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right).}$$

* Carroll Weights: $\xi = GM, \quad \Delta = J.$ Central Charges: $c_M = \frac{3}{G},\ c_L = 0.$

* Putting things together: $S_{FSC} = S_{BMS-Cardy}$

Flat Holography : Aspects of dual theory

- Symmetry of 2d Carroll CFT: $$[L_n, L_m] = (n-m)L_{m+n} + \frac{c_L}{12}\delta_{n+m,0}(n^3-n)$$ $$[L_n, M_m] = (n-m)M_{m+n} + \frac{c_M}{12}\delta_{n+m,0}(n^3-n)$$ $$[M_n, M_m] = 0.$$
- Label states of the theory with $L_0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle,\ M_0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle$
- We will build highest weight representations.
- BMS Primaries: $L_n|\Delta,\xi\rangle_p = M_n|\Delta,\xi\rangle_p = 0,\ \forall n > 0.$
- BMS modules are built out of these primary states by acting with raising operators.
- A general descent is of the form $L_{-1}^{k_1}L_{-2}^{k_2}....L_{-l}^{k_l}M_{-1}^{q_1}M_{-2}^{q_2}....M_{-r}^{q_r}|\Delta,\xi\rangle \equiv L_{\vec{k}}M_{\vec{q}}|\Delta,\xi\rangle$

Carroll CFT: Partition functions.

- Can define the theory on a cylinder. $L_n = ie^{in\phi}(\partial_\phi + in\tau\partial_\tau), \quad M_n = ie^{in\phi}\partial_\tau$
- The mapping from the plane to the cylinder: $x = e^{i\phi}, \quad t = i\tau e^{i\phi}$
- We can identify the end of the cylinder to define the theory on the torus.
- Partition function: $Z_{\text{CarrollCFT}} = \text{Tr}\exp\{2\pi i\left(\sigma L_0 + \rho M_0\right)\}$
- Look at Carroll limit of CFTs. 2d CFT partition function: $Z_{\text{CFT}} = \text{Tr}\ e^{2\pi i\zeta L_0}e^{-2\pi i\bar{\zeta}\bar{L}_0}$
- Relation between weights: $\Delta = h - \bar{h},\ \xi = \epsilon(h + \bar{h}).$
- In a convenient basis: $Z_{\text{CFT}} = \sum d_{\text{CFT}}(h,\bar{h})e^{2\pi i(\zeta h - \bar{\zeta}\bar{h})} = \sum d(\Delta,\xi)e^{2\pi i(\sigma\Delta - \frac{\rho}{\epsilon}\xi)}$
- Here $2\sigma = \zeta - \bar{\zeta}, \quad 2\rho = \zeta + \bar{\zeta}$
- We work with the assumption that $Z_{\text{CFT}} \to Z_{\text{CarrollCFT}}$ as $\epsilon \to 0$
- To keep the partition function finite, we need to scale $\rho \to \epsilon\rho$

Modular invariance in 2d Carroll CFTs

* BMS Partition function: $Z_{\text{BMS}} = \sum d(\Delta, \xi) e^{2\pi i(\sigma\Delta - \rho\xi)}$
* Any notion of BMS modular invariance? We again investigate the limit.
* Modular transformation in the original CFT: $\zeta \to \frac{a\zeta + b}{c\zeta + d}$ with $ad - bc = 1$
* In the BMS basis:

$$\sigma + \rho \to \frac{a(\sigma+\rho)+b}{c(\sigma+\rho)+d} = \frac{a\sigma+b}{c\sigma+d} + \frac{(ad-bc)\rho}{(c\sigma+d)^2} + \frac{(ad-bc)c\rho^2}{(c\sigma+d)^3} + \dots$$

* The contracted modular transformation reads: $$\boxed{\sigma \to \frac{a\sigma+b}{c\sigma+d}, \quad \rho \to \frac{\rho}{(c\sigma+d)^2}}$$
* This is what we will call the **Carroll modular transformation**.
* Intrinsic interpretation=> S-transformation: Exchange of circles on the Euclidean torus.
[ala **Detournay-Hartman-Hofmann** for warped CFT. See e.g. Song et al 2017]

Invariance of Partition function

- Demand partition function is invariant under Carroll modular transformation and find consequences.

$$Z^0_{\text{BMS}}(\sigma,\rho) = \text{Tr}\ e^{2\pi i\sigma(L_0-\frac{c_L}{2})}e^{2\pi i\rho(M_0-\frac{c_M}{2})} = e^{\pi i(\sigma c_L+\rho c_M)} Z_{\text{BMS}}(\sigma,\rho)$$

- Carroll S-transformation: $(\sigma,\rho)\to\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)$
- Invariance of the above quantity: $Z^0_{\text{BMS}}(\sigma,\rho) = Z^0_{\text{BMS}}\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)$
- This translates to: $Z_{\text{BMS}}(\sigma,\rho) = e^{2\pi i\sigma\frac{c_L}{2}}e^{2\pi i\rho\frac{c_M}{2}}e^{-2\pi i(-\frac{1}{\sigma})\frac{c_L}{2}}e^{-2\pi i(\frac{\rho}{\sigma^2})\frac{c_M}{2}} Z_{\text{BMS}}\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)$
- The density of states can be found with an inverse Laplace transformation

$$d(\Delta,\xi) = \int d\sigma d\rho\ e^{2\pi i\tilde{f}(\sigma,\rho)} Z\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right).$$

where $\tilde{f}(\sigma,\rho) = \frac{c_L\sigma}{2}+\frac{c_M\rho}{2}+\frac{c_L}{2\sigma}-\frac{c_M\rho}{2\sigma^2}-\Delta\sigma-\xi\rho.$

- In the limit of large charges, this integration can be done with a saddle point approximation.

BMS Cardy formula

- In the large charge limit, $\tilde{f}(\sigma,\rho) \to f(\sigma,\rho) = \frac{c_L}{2\sigma} - \frac{c_M \rho}{2\sigma^2} - \Delta\sigma - \xi\rho.$
- Value at the extremum is $f^{max}(\sigma,\rho) = -i\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right).$
- BMS-Cardy formula is given by

$$\boxed{S^{(0)} = \ln d(\Delta,\xi) = 2\pi\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right).}$$

Bagchi, Detournay, Fareghbal, Simon 2012.

- One can calculate leading logarithmic corrections to this.

$$S = 2\pi\left(c_L\sqrt{\frac{\xi}{2c_M}} + \Delta\sqrt{\frac{c_M}{2\xi}}\right) - \frac{3}{2}\log\left(\frac{\xi}{c_M^{1/3}}\right) + \text{constant} = S^{(0)} + S^{(1)}.$$

Bagchi, Basu 2013.

FSC entropy from dual theory

- The weights for the FSC: $\xi = GM + \frac{c_M}{24} \sim GM, \quad \Delta = J$
- Putting this back into the BMS-Cardy formula, we get $S_{\rm FSC} = \frac{\pi J}{\sqrt{2GM}}$

Bagchi, Detournay, Fareghbal, Simon 2012; Barnich 2012

which is precisely what we obtained from the gravitational analysis.

- The log-correction is of the form $S^{(1)}_{\rm FSC} = -\frac{3}{2}\log(2GM)$
- Total entropy: $\boxed{S_{\rm FSC} = \frac{2\pi r_0}{4G} - \frac{3}{2}\log(\frac{2\pi r_0}{4G}) - \frac{3}{2}\log\kappa + \text{ constant}}$

Bagchi, Basu 2013.

Here $\kappa = \frac{\hat{r}^2}{r_0} = \frac{8GM}{r_0}$ is the surface gravity of FSC.

- Can also be obtained in the limit from the "inner" Cardy formula.

Riegler 2014; Fareghbal, Naseh 2014.

Bulk Scattering from Carroll CFTs

AB, Banerjee, Basu, Dutta 2022 (PRL)

What's new? Bulk Scattering from Carroll CFTs

AB, Banerjee, Basu, Dutta 2022 (PRL)

* In asymptotically flat spaces, S-matrices are the observables of interest.
* Especially true in d>=4, where one has propagating DOF.
* Can we connect Carroll CFT correlations to S-matrix? YES!
* Interesting branches of correlators. "Weird" branch gives correct answer.
* We show this for d=3 boundary theory and d=4 bulk.
* Inspired by Pasterski-Shao map for Celestial CFTs. Use modified Mellin transformations.

3d Carrollian CFTs

Algebra on $\mathscr{I}^+$: $[L_n, L_m] = (n-m)L_{n+m}, \quad [\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m}$

$$[L_n, M_{r,s}] = \left(\frac{n+1}{2} - r\right) M_{n+r,s}, \quad [\bar{L}_n, M_{r,s}] = \left(\frac{n+1}{2} - s\right) M_{r,n+s} \qquad [M_{r,s}, M_{t,u}] = 0.$$

Representation (vector fields): $L_n = -z^{n+1}\partial_z - \frac{1}{2}(n+1)z^n u\partial_u \quad \bar{L}_n = -\bar{z}^{n+1}\partial_{\bar{z}} - \frac{1}{2}(n+1)\bar{z}^n u\partial_u \quad M_{r,s} = z^r \bar{z}^s \partial_u$

Here z: stereographic coordinate on sphere, u: null direction.

Labelling of operators: $[L_0, \Phi(0)] = h\Phi(0), \quad [\bar{L}_0, \Phi(0)] = \bar{h}\Phi(0).$

Assume existence of Conformal Carroll primaries on $\mathscr{I}^+$

Highest weight representations: $[L_n, \Phi(0)] = 0, \quad [\bar{L}_n, \Phi(0)] = 0, \quad \forall n > 0, \quad [M_{r,s}, \Phi(0)] = 0, \quad \forall r, s > 0.$

Transformation rules for Carrollian primaries: $\delta_{L_n}\Phi_{h,\bar{h}}(u,z,\bar{z}) = \epsilon\left[z^{n+1}\partial_z + (n+1)z^n\left(h + \frac{1}{2}u\partial_u\right)\right]\Phi_{h,\bar{h}}(u,z,\bar{z})$

$$\delta_{M_{r,s}}\Phi_{h,\bar{h}}(u,z,\bar{z}) = \epsilon z^r \bar{z}^s \partial_u \Phi_{h,\bar{h}}(u,z,\bar{z}).$$

Scattering in 4d flatspace: Connections to 2d CFT

Consider massless particles. 4-momenta parametrised as:

$$p^\mu = \omega\left(1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z}\right), \; p^\mu p_\mu = 0$$

Mellin transformation: We also introduce a symbol ϵ which is equal to ± 1 if the particle is (outgoing) incoming.

$$\mathcal{M}\left(\{z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \prod_{i=1}^{n} \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} S\left(\{\epsilon_i \omega_i, z_i, \bar{z}_i, \sigma_i\}\right), \; \Delta \in \mathbb{C}, \;\; \sigma \in \frac{\mathbb{Z}}{2}$$

S is the S-matrix element for n massless particle scattering.

Also: $h = \dfrac{\Delta + \sigma}{2}, \; \bar{h} = \dfrac{\Delta - \sigma}{2}$

Using Lorentz transformation properties of the S-matrix, it can be shown that the LHS transforms like a correlation function of n primary operators of a 2d CFT.

[Pasterski-Shao(-Strominger), 2016]

4d Scattering: Modified Mellin Transformation

Under supertranslations: $u \to u' = u + f(z,\bar{z}), \;\; z \to z' = z, \;\; \bar{z} \to \bar{z}' = \bar{z}$

Under superrotations: $u \to u' = \left(\frac{dw}{dz}\right)^{\frac{1}{2}} \left(\frac{d\bar{w}}{d\bar{z}}\right)^{\frac{1}{2}} u, \;\; z \to z' = w(z), \;\; \bar{z} \to \bar{z}' = \bar{w}(\bar{z})$

Modified Mellin transformation:

$$\tilde{\mathcal{M}}\left(\{u_i, z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \prod_{i=1}^{n} \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} e^{-i\epsilon_i \omega_i u_i} S\left(\{\epsilon_i \omega_i, z_i, \bar{z}_i, \sigma_i\}\right), \;\; \Delta \in \mathbb{C}$$

[Banerjee 2017, Banerjee-Ghosh-Paul 2020]

Now defined in a 3d space with coordinates $(u, z, \bar{z})$. Transforms covariantly under BMS transformations

Used in Celestial holography since original Mellin transformation is not convergent due to bad UV behaviour of gravitation scattering amplitudes.

4d Scattering: Modified Mellin Transformation

Define: $\phi^{\epsilon}_{h,\bar{h}}(u, z, \bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{-i\epsilon\omega u} a(\epsilon\omega, z, \bar{z}, \sigma).$

where $a(\epsilon\omega, z, \bar{z}, \sigma)$ is the momentum space (creation) annihilation operator of a massless particle with helicity σ when $(\epsilon = -1)$ $\epsilon = 1$. In terms of these fields we can write

$$\tilde{\mathcal{M}}\left(\{u_i, z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \langle \prod_{i=1}^{n} \phi^{\epsilon_i}_{h_i,\bar{h}_i}(u_i, z_i, \bar{z}_i) \rangle.$$

The field $\phi^{\epsilon}_{h,\bar{h}}(u, z, \bar{z})$ transforms under BMS transformations as:

Supertranslation: $\phi^{\epsilon}_{h,\bar{h}}(u, z, \bar{z}) \to \phi^{\epsilon}_{h,\bar{h}}(u + f(z, \bar{z}), z, \bar{z})$

Superrotation: $\phi^{\epsilon}_{h,\bar{h}}(u, z, \bar{z}) \to \left(\frac{dw}{dz}\right)^{h} \left(\frac{d\bar{w}}{d\bar{z}}\right)^{\bar{h}} \phi^{\epsilon}_{h,\bar{h}}(u', z', \bar{z}')$

These are exactly the same as the Carrollian CFT primaries that were defined earlier.

This is a central observation of what is to follow.

Proposal: Scattering Amplitude = Carroll CFT Correlator

It is natural to identify **the time-dependent correlation functions of primary fields in a Carrollian CFT** with **the modified Mellin transformation:**

$$\tilde{\mathcal{M}}\left(\{u_i, z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \prod_i \langle \phi^{\epsilon_i}_{h_i,\bar{h}_i}(u_i, z_i, \bar{z}_i)\rangle.$$

The time-dependent correlators of a 3d Carroll CFT compute the 4d scattering amplitudes in the Mellin basis.

Carrollian CFT and Correlation functions

- We are interested in vacuum correlation of Carroll primary fields.
- As in CFTs, possible to fix 2 and 3-point fns by the "global" or Poincare sub-algebra of the BMS4.

  Poincare sub-algebra: $(\{M_{l,m}, L_n\}$ with $l, m = 0, 1$ and $n = 0, \pm 1)$
- Consider the 2-point function $G(u, z, \bar{z}, u', z', \bar{z}') = \langle 0|\Phi(u, z, \bar{z})\Phi'(u', z', \bar{z}')|0\rangle$.

  Here $\Phi(u, z, \bar{z})$ and $\Phi'(u', z', \bar{z}')$ are primaries with weight (h, h') and $(\bar{h}, \bar{h}')$ respectively.
- Invariance under Carroll time translations: $\left(\frac{\partial}{\partial u} + \frac{\partial}{\partial u'}\right) G(u, z, \bar{z}, u', z', \bar{z}') = 0$
- Under Carroll boosts $(u \to u + bz + \bar{b}\bar{z})$: [Note: 3d Carroll boosts are translations in Mink_4]

$$\left(z\frac{\partial}{\partial u} + z'\frac{\partial}{\partial u'}\right) G(u, z, \bar{z}, u', z', \bar{z}') = 0, \quad \left(\bar{z}\frac{\partial}{\partial u} + \bar{z}'\frac{\partial}{\partial u'}\right) G(u, z, \bar{z}, u', z', \bar{z}') = 0.$$

Carroll correlation functions: Two branches

- Combining previous equations we get

$$(z-z')\frac{\partial}{\partial u}G(u-u',z-z',\bar{z}-\bar{z}')=0,\ (\bar{z}-\bar{z}')\frac{\partial}{\partial u}G(u-u',z-z',\bar{z}-\bar{z}')=0.$$

- This equation has two branches.
- Branch 1: Corresponds to choice $\frac{\partial}{\partial u}G(u-u',z-z',\bar{z}-\bar{z}')=0$
- Using invariances under other global generators we get

$$G(u,z,\bar{z},u',z',\bar{z}')=\frac{\delta_{h,h'}\delta_{\bar{h},\bar{h}'}}{(z-z')^{2h}(\bar{z}-\bar{z}')^{2\bar{h}}}.$$

Bagchi, Basu, Kakkar, Mehra 2016.

- This is the 2-pt function of a usual 2d CFT. Also natural when thinking of limits from 3d CFTs.
- We will not be interested in this branch in this context.

Carroll correlations: Delta function branch

- The second class of solutions correspond to $\frac{\partial}{\partial u} G(u-u', z-z', \bar{z}-\bar{z}') \propto \delta^2\left(z-z'\right)$
- Thus: $G(u, z, \bar{z}, u', z', \bar{z}') = f(u-u')\delta^2(z-z')$.
- Demanding invariance under the subalgebra $\{L_{0,\pm1}, \bar{L}_{0,\pm1}\}$ of BMS_4, we get

$$(\Delta + \Delta' - 2) f(u-u') + (u-u')\partial_u f(u-u') = 0, \quad (\sigma+\sigma')f(u-u') = 0.$$

Here $\Delta = (h+\bar{h})$ is the scaling dimension and $\sigma = (h-\bar{h})$ is spin.

- Solving we get: $$G(u, z, \bar{z}, u', z', \bar{z}') = \frac{C\,\delta^2(z-z')}{(u-u')^{\Delta+\Delta'-2}}\delta_{\sigma+\sigma',0}.$$

The constraint equation coming from M_{11} is trivially satisfied.
(See also [de Boer, Hartong, Obers, Sybesma, Vandoren '21])

- Notice that the correlation does not require equal weights to be non-zero. Very different from a usual CFT. Not obtainable as a limit (?).

Connection to 4d Scattering

- Of course in case of the 2 point function, the scattering amplitude is trivial.
- Two point function is given by the inner product $\langle p_1, \sigma_1 | p_2, \sigma_2 \rangle = (2\pi)^3 2E_{p_1} \delta^3 (\vec{p}_1 - \vec{p}_2) \delta_{\sigma_1+\sigma_2,0}$

  Notation is standard except we label helicity of external particle as if it were outgoing.
- With our earlier parametrisation: $\langle p_1, \sigma_1 | p_2, \sigma_2 \rangle = 4\pi^3 \frac{\delta(\omega_1 - \omega_2) \delta^2 (z_1 - z_2)}{\omega_1} \delta_{\sigma_1+\sigma_2,0}$
- Mellin transformed 2 point function:

$$\tilde{\mathcal{M}} \left(u_1, z_1, \bar{z}_1, u_2, z_2, \bar{z}_2, h_1, \bar{h}_1, h_2, \bar{h}_2, \epsilon_1 = 1, \epsilon_2 = -1\right) = 4\pi^3 \delta_{\sigma_1+\sigma_2,0} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \omega_1^{\Delta_1 - 1} \omega_2^{\Delta_2 - 1} e^{-i\omega_1 u_1} e^{i\omega_2 u_2} \frac{\delta(\omega_1 - \omega_2) \delta^2 (z_1 - z_2)}{\omega_1}$$

$$\tilde{\mathcal{M}} = 4\pi^3 \Gamma(\Delta_1 + \Delta_2 - 2) \frac{\delta^2 (z_1 - z_2)}{(i(u_1 - u_2))^{\Delta_1 + \Delta_2 - 2}} \delta_{\sigma_1+\sigma_2,0}$$

- Spatial delta function has dual interpretation that momentum direction of a free particle.

More on Scattering and Carroll

- In the same way as above, we can compute the three-point function and show that in the time-dependent branch this is zero.
- This has the dual interpretation that in Minkowski signature the scattering amplitude of three massless particles vanishes due to momentum conservation.
- So we see that **the peculiarities of the delta-function branch of correlations of a Carroll CFT are exactly what is required to connect to scattering amplitudes in the bulk Minkowski spacetime.**

Example: Carroll Massless Scalar

- Simplest of examples to illustrate our findings: the Carroll massless scalar.

$$\mathcal{S} = \int dud^2x^i \, \tau^\mu \tau^\nu \partial_\mu \Phi \partial_\nu \Phi$$

- Flat Carroll backgrounds: $\tau^\mu = (1,0)$ and $g_{ij} = \delta_{ij}$. So, $\mathcal{S} = \int dud^2x^i \, \frac{1}{2}(\partial_u \Phi)^2$
- Green's function: $\partial_u^2 G(u-u, z^i - z'^i) = \delta^3(u-u', z^i - z'^i)$.
- Solved in the usual way by going to Fourier space: $\tilde{G}(k_u, k_i) = -\frac{1}{k_u^2}$.
- Position space: $G(u-u', z^i - z'^i) = -\int \frac{dk_u}{k_u^2 + \mu^2} e^{ik_u(u-u')} \int d^2z \; e^{ik_i(z^i - z'^i)} = \frac{i}{2}\left[\frac{1}{\mu} - (u-u')\right] \delta^{(2)}(z^i - z'^i)$.
- Regulating: $G(u-u', z^i - z'^i) = -\frac{i}{2}(u-u')\delta^2(z-z', \bar{z}-\bar{z}'))$
- Scaling dimensions: $h = \frac{1}{4}, \quad \bar{h} = \frac{1}{4}$. Answer exactly matches with previous symmetry analysis.

Example: Carroll Massless Scalar

- Can also use canonical methods. Put the Carroll scalar on a sphere times the null line.
- Action: $\mathcal{S} = \int du d^2 z \sqrt{q} \left[\frac{1}{2} (\partial_u \Phi)^2 - k^2 \Phi^2 \right]$ Here k is related to the radius of the sphere R by $k = \frac{1}{2R}$.
- EOM: $\ddot{\Phi} + k^2 \Phi^2 = 0$. Generic real solutions: $\Phi(u, z, \bar{z}) = \frac{1}{\sqrt{k}} \left(C^\dagger(z, \bar{z}) e^{iku} + C(z, \bar{z}) e^{-iku} \right)$.
- Commutation relations: $[C(z, \bar{z}), C^\dagger(z', \bar{z}')] = \frac{1}{2} \delta^2(z - z')$
- Hamiltonian: $H = k \int d^2 z \sqrt{q} \left(2C^\dagger(z, \bar{z}) C(z, \bar{z}) + \frac{1}{2} \delta^2(0) \right)$. Unphysical zero point energy. Neglect.
- Ground state: $C(z, \bar{z}) |0\rangle = 0, \ \text{for} \ (z, \bar{z}) \in \mathbb{S}^2$.
- Use usual methods to calculate correlation functions: $G(u, u', z^i, z'^i) = \langle 0 | T \Phi(u, z, \bar{z}) \Phi(u', z', \bar{z}') | 0 \rangle$.
- 2 point function: $G(u, u', z^i, z'^i) = -\frac{1}{2k} [\cos k(u - u') + i \sin k(u - u')] \delta^2(z - z', \bar{z} - \bar{z}')$
- Large radius limit: $$G(u, u', z^i, z'^i) = -\left[\frac{1}{2k} + \frac{i}{2}(u - u') \right] \delta^2(z - z', \bar{z} - \bar{z}')$$

What have we learnt so far?

* Carrollian physics emerges in the vanishing speed of light limit of Lorentzian physics.
* Carrollian CFTs are natural holographic duals of flat spacetimes as they inherit the asymptotic symmetries of the bulk theory.
* Over the years, a lot of evidence has been gathered about especially the duality between 3d flatspace and 2d Carroll CFTs.
* In particular, a BMS-Cardy formula in a 2d Carroll CFT reproduces the entropy of the cosmological horizon of Flatspace Cosmologies, providing one of the most important checks of the holographic analysis in flatspace.
* A stumbling block was the formulation of scattering in Carroll CFTs.

What have we learnt so far?

* The S-matrix is the most important observable for Quantum gravity in flatspace.

* Carroll CFT correlation functions have two branches. One of them is time-independent and gives correlations of a 2d CFT. The other one gives spatial delta functions and depends on the null time direction.

* Using modified Mellin transformations, can show this delta-function branch has the correct properties for reproducing scattering amplitudes in the bulk.

* So scattering amplitudes are connected to Carroll CFT correlations in a rather non-trivial and non-obvious way.

Open questions: Flat Holography

* Why is the "electric" leg important for scattering?
* Going beyond 2 and 3 point functions. 4 point? Can we construct an interacting theory and make the connection concrete? Input from gravity?
* Limit from AdS/CFT for flatspace scattering? Does not seem to work at first sight.
* Bootstrap for Carroll CFT for d>2. [Bootstrap for d=2 (AB, Gary, Zodinmawia 2016)]
* Connection to the picture of Donnay et al.
  Celestial Holography as a "restriction" of Carrollian Holography?
* Addressing the question of S=A/4G for d=4.
* Vacuum degeneracy and memory in Carroll CFTs.

Tensionless Strings

Null Strings?! What? Why?

* Massless point particles move on null geodesics. Worldlines are null.

* Null strings: extended analogues of massless point particles.
  Massless point particles => Tensionless strings.

* Tensionless or null strings: studied since **Schild** in 1970's.

* Tension $T = \frac{1}{2\pi\alpha'} \to 0$: point particle limit of string theory => Classical gravity.

* Tensionless regime: $T = \frac{1}{2\pi\alpha'} \to \infty$: ultra-high energy, ultra-quantum gravity!

Null strings are vital for:

A. Strings at **very high temperatures**: Hagedorn Phase.

B. Strings near **spacetime singularities:** Strings near Black holes, near the Big Bang.

C. Connections to **higher spin theory**.

Summary of Results

* 2d Conformal Carrollian (or BMS3) and its supersymmetric cousins arise on the worldsheet of the tensionless string replacing the two copies of the (super) Virasoro algebra.

* Classical tensionless strings: properties can be derived intrinsically or as a limit of usual tensile strings.

* Quantum tensionless strings: many surprising new results.

Classical Tensionless Strings

Isberg, Lindstrom, Sundborg, Theodoridis 1993

AB 2013; AB, Chakrabortty, Parekh 2015.

Going tensionless

Isberg, Lindstrom, Sundborg,Theodoridis 1993

Start with Nambu-Goto action:

$$S = -T \int d^2\xi \sqrt{-\det \gamma_{\alpha\beta}}. \qquad (1)$$

To take the tensionless limit, first switch to Hamiltonian framework.

- Generalised momenta: $P_m = T\sqrt{-\gamma}\gamma^{0\alpha}\partial_\alpha X_m$.
- Constraints: $P^2 + T^2\gamma\gamma^{00} = 0, \;\; P_m\partial_\sigma X^m = 0$.
- Hamiltonian: $\mathcal{H}_T = \mathcal{H}_\mathcal{C} + \rho^i(\text{constraints})_i = \lambda(P^2 + T^2\gamma\gamma^{00}) + \rho\, P_m\partial_\sigma X^m$.

Action after integrating out momenta:

$$S = \frac{1}{2}\int d^2\xi\, \frac{1}{2\lambda}\left[\dot{X}^2 - 2\rho\dot{X}^m\partial_\sigma X_m + \rho^2\partial_\sigma X^m\partial_\sigma X_m - 4\lambda^2T^2\gamma\gamma^{00}\right] \qquad (2)$$

Identifying

$$g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 + 4\lambda^2T^2 \end{pmatrix},$$

action takes the familiar Weyl-invariant form

$$S = -\frac{T}{2}\int d^2\xi \sqrt{-g}g^{\alpha\beta}\partial_\alpha X^m\partial_\beta X^n\eta_{mn}. \qquad (3)$$

Going Tensionless ...

Isberg, Lindstrom, Sundborg,Theodoridis 1993

- Tensionless limit can now be taken systematically.
- $T \to 0 \Rightarrow$

$$g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 \end{pmatrix}.$$

- Metric is degenerate. $\det g = 0$.
- Replace degenerate metric density $T\sqrt{-g}g^{\alpha\beta}$ by a rank-1 matrix $V^\alpha V^\beta$ where V^α is a vector density

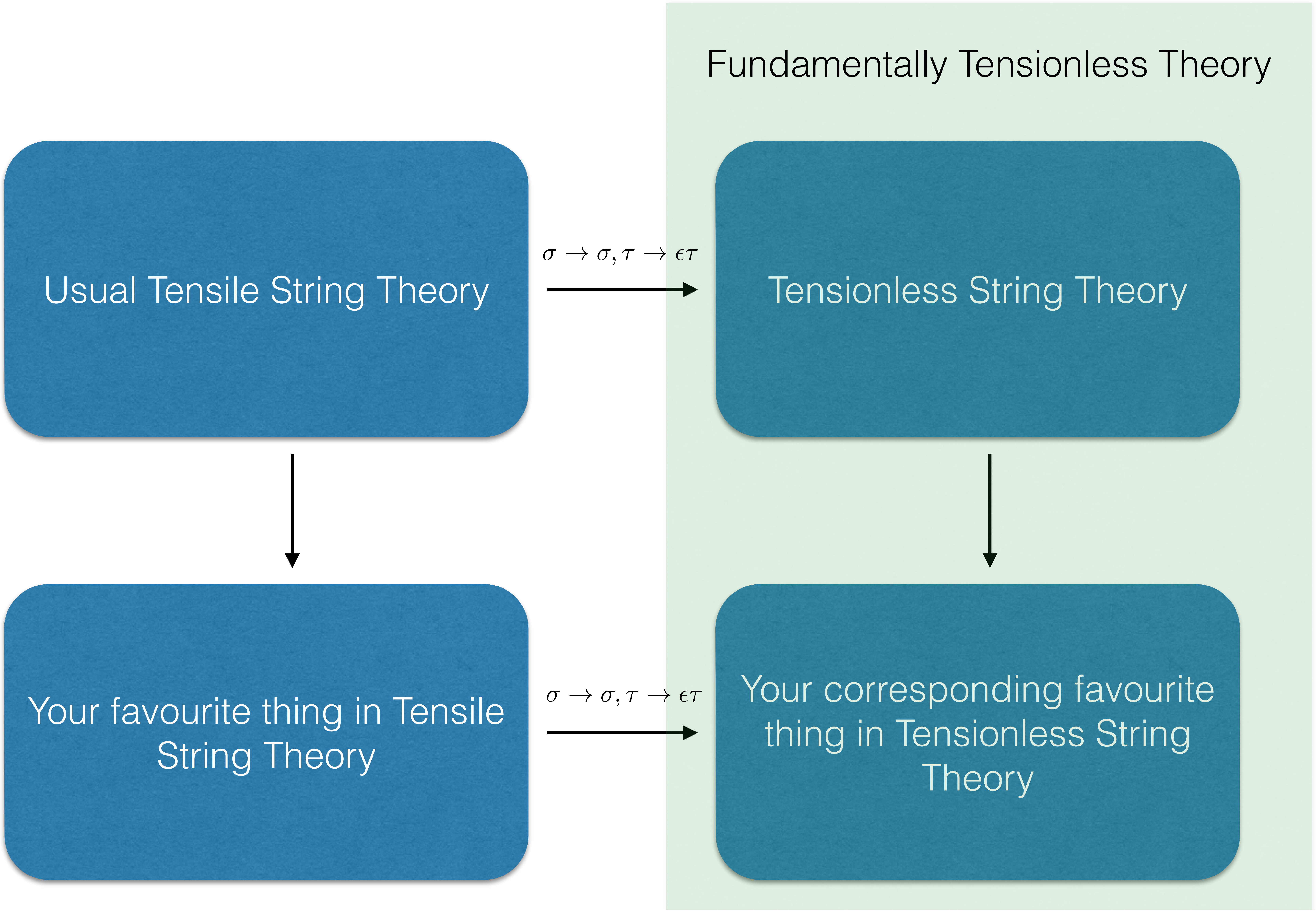
$$V^\alpha \equiv \frac{1}{\sqrt{2\lambda}}(1, \rho) \tag{4}$$

- Action in $T \to 0$ limit

$$\boxed{S = \int d^2\xi \; V^\alpha V^\beta \partial_\alpha X^m \partial_\beta X^n \eta_{mn}.} \tag{5}$$

- Starting point of tensionless strings.
- Need not refer to any parent theory. Treat this as action of fundamental objects.

Completing the square?



Gauge and Residual Gauge Symmetries

Tensionless action is invariant under world-sheet diffeomorphisms.

Fixing gauge: "Conformal" gauge: $V^\alpha = (v, 0)$ (v: constant).

Tensile: Residual symmetry after fixing conformal gauge = Vir $\otimes$ Vir. Central to understanding string theory.

Tensionless: Similar residual symmetry left over after gauge fixing.

For world-sheet diffeomorphism: $\xi^\alpha \to \xi^\alpha + \varepsilon^\alpha$, change in vector density: $\delta_\varepsilon V^\alpha = -V \cdot \partial \varepsilon^\alpha + \varepsilon \cdot \partial V^\alpha + \frac{1}{2}(\partial \cdot \varepsilon) V^\alpha$

Tensionless residual symmetries: for $V^\alpha = (v, 0)$, $\varepsilon^\alpha = \{f'(\sigma)\tau + g(\sigma), f(\sigma)\}$

Define: $L(f) = f'(\sigma)\tau\partial_\tau + f(\sigma)\partial_\sigma$, $M(g) = g(\sigma)\partial_\tau$. Expand: $f = \sum a_n e^{in\sigma}$, $g = \sum b_n e^{in\sigma}$

$$L(f) = \sum_n a_n e^{in\sigma}(\partial_\sigma + in\tau\partial_\tau) = \sum_n a_n L_n, \quad M(g) = \sum_n b_n e^{in\sigma}\partial_\tau = \sum_n b_n M_n.$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c_L}{12}(m^3 - m)\delta_{m+n,0}, \quad [M_m, M_n] = 0.$$

$$[L_m, M_n] = (m-n)M_{m+n} + \frac{c_M}{12}(m^3 - m)\delta_{m+n,0}.$$

Tensionless Limit from the Worldsheet

A Bagchi 2013

- Tensile string: Residual symmetry in conformal gauge $g_{\alpha\beta} = e^{\phi}\eta_{\alpha\beta}$:

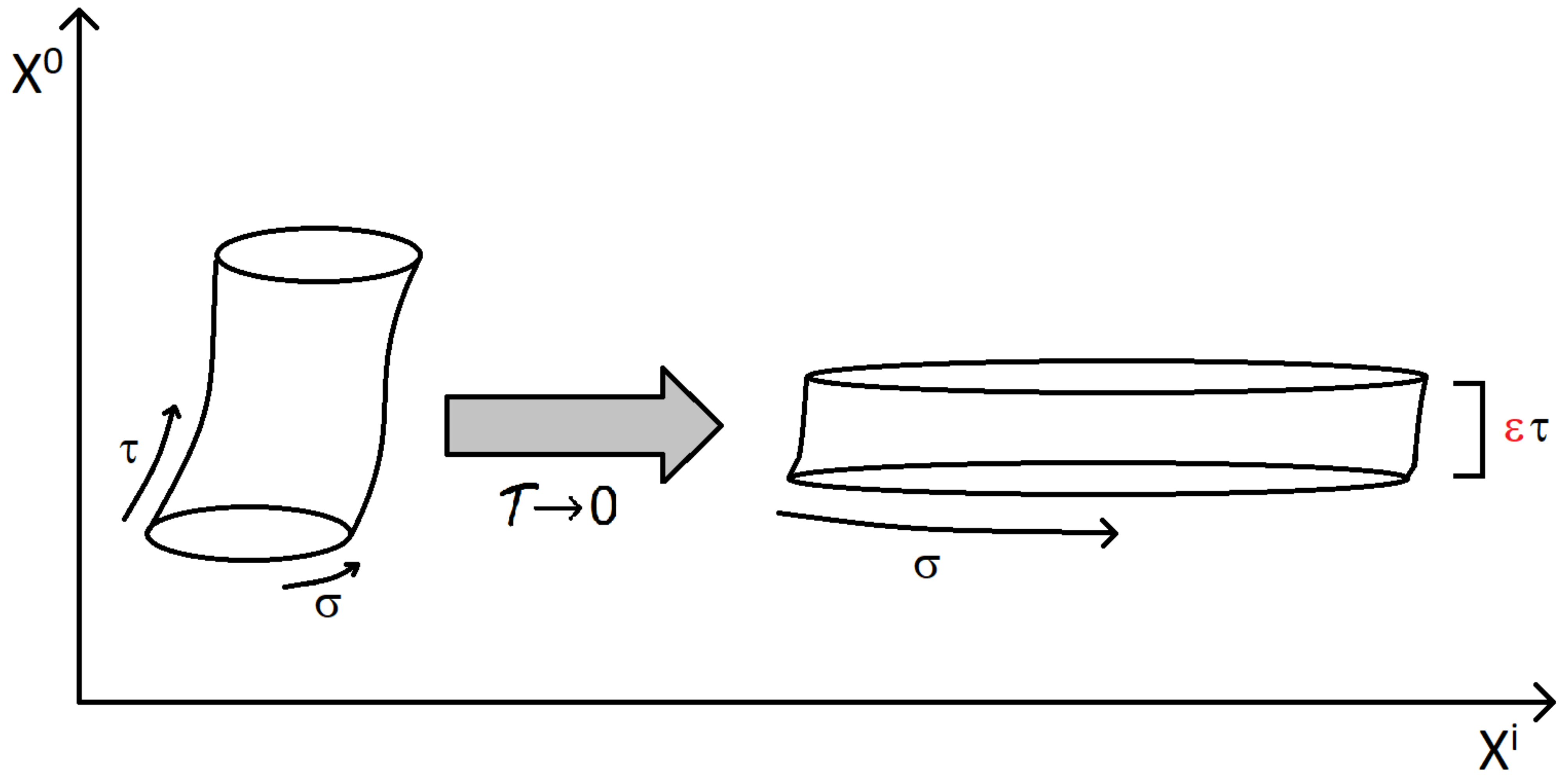
$$
\begin{aligned}
[\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0} \\
[\mathcal{L}_m, \bar{\mathcal{L}}_n] &= 0, \quad [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2-1)\delta_{m+n,0}
\end{aligned}
$$

- World-sheet is a cylinder. Symmetry best expressed as 2d conformal generators on the cylinder.

$$\mathcal{L}_n = ie^{in\omega}\partial_\omega, \quad \bar{\mathcal{L}}_n = ie^{in\bar{\omega}}\partial_{\bar{\omega}}$$

  where $\omega, \bar{\omega} = \tau \pm \sigma$. Vector fields generate centre-less Virasoros.

- Tensionless limit $\Rightarrow$ length of string becomes infinite ($\sigma \to \infty$).
- Ends of closed string identified $\Rightarrow$ limit best viewed as ($\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0$).



Tensionless Limit from the Worldsheet

A Bagchi 2013

- Define

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}).$$

- New vector fields (L_n, M_n) well-defined in limit and given by:

$$L_n = ie^{in\sigma}(\partial_\sigma + in\tau\partial_\tau), \quad M_n = ie^{in\sigma}\partial_\tau .$$

- These are *exactly the generators defined previously* . Close to form BMS_3.

$$[L_m, L_n] = (m-n)L_{m+n} \quad [L_m, M_n] = (m-n)M_{m+n} \quad [M_m, M_n] = 0.$$

- Tensionless limit on the worldsheet: $\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0$
- Worldsheet velocities $v = \frac{\sigma}{\tau} \to \infty$. Effectively, $\frac{v}{c} \to \infty$
- Hence worldsheet speed of light $\to 0$. Carrollian limit.

- Degenerate worldsheet metric.
- Riemannian tensile worldsheet $\to$ Carrollian tensionless worldsheet.

Tensionless EM Tensor and constraints

A Bagchi 2013

Spectrum of tensile string theory (in conformal gauge in flat space)

- Quantise worldsheet theory as a theory free scalar fields.
- Constraint: vanishing of EOM of metric (which is fixed to be flat).
- Op form: Physical states vanish under action of modes of E-M tensor.

EM tensor for 2d CFT on cylinder: $T_{cyl} = z^2 T_{plane} - \frac{c}{24} = \sum_n \mathcal{L}_n e^{in\omega} - \frac{c}{24}; \quad \bar{T}_{cyl} = \sum_n \bar{\mathcal{L}}_n e^{in\bar{\omega}} - \frac{\bar{c}}{24}$

Ultra-relativistic EM tensor

$$T_{(1)} = \lim_{\epsilon \to 0} \left(T_{cyl} - \bar{T}_{cyl} \right) = \sum_n (L_n - in\tau M_n) e^{in\sigma} - \frac{c_L}{24}$$

$$T_{(2)} = \lim_{\epsilon \to 0} \epsilon \left(T_{cyl} + \bar{T}_{cyl} \right) = \sum_n M_n e^{in\sigma} - \frac{c_M}{24}$$

- Classical constraint on the tensionless string: $T_{(1)} = 0, \quad T_{(2)} = 0.$

- Quantum version: physical spectrum of tensionless strings restricted by

$$\langle \text{phys} | T_{(1)} | \text{phys}' \rangle = 0, \quad \langle \text{phys} | T_{(2)} | \text{phys}' \rangle = 0.$$

Intrinsic Analysis: EOM and Mode Expansions

AB, Chakrabortty, Parekh 2015

- Equation of motion in $V^a = (v, 0)$ gauge: $\ddot{X}^\mu = 0$.
- Solution: $X^\mu(\sigma,\tau) = x^\mu + \sqrt{2c'}A_0^\mu \sigma + \sqrt{2c'}B_0^\mu \tau + i\sqrt{2c'}\sum_{n\neq 0}\frac{1}{n}\left(A_n^\mu - in\tau B_n^\mu\right)e^{in\sigma}$
- Closed string b.c.: $X^\mu(\sigma,\tau) = X^\mu(\sigma+2\pi,\tau) \Rightarrow A_0^\mu = 0$.
- Constraints: $\dot{X}^2 = 2c'\sum_{m,n} B_{-m}\cdot B_{m+n}\, e^{in\sigma} = 0, \quad \dot{X}\cdot X' = 2c'\sum_{m,n}\left(A_{-m} - in\tau B_{-m}\right)\cdot B_{m+n}\, e^{in\sigma} = 0$
- Define: $L_n = \sum_m A_{-m}\cdot B_{m+n}, \quad M_n = \sum_m B_{-m}\cdot B_{m+n}$
- Classical constraints in terms of modes: $\sum_n \left(L_n - in\tau M_n\right) e^{in\sigma} = 0 = T_{(1)}, \quad \sum_n M_n\, e^{in\sigma} = 0 = T_{(2)}$.

Familiar form obtained earlier from purely algebraic considerations.

- The algebra of the modes

$$\{A_m^\mu, A_n^\nu\} = 0, \quad \{B_m^\mu, B_n^\nu\} = 0, \quad \{A_m^\mu, B_n^\nu\} = -im\delta_{m+n,0}\,\eta^{\mu\nu}.$$

- The worldsheet symmetry algebra of tensionless strings, now constructed from the quadratics of the modes:

$$\{L_m, L_n\} = -i(m-n)L_{m+n}, \ \{L_m, M_n\} = -i(m-n)M_{m+n}, \ \{M_m, M_n\} = 0.$$

Quantization: $\{,\}_{PB} \to -\frac{i}{\hbar}[,]$ leads to the BMS_3 Algebra.

Limiting Analysis: EOM and Mode Expansions

AB, Chakrabortty, Parekh 2015

- Tensile string mode expansion: $X^\mu(\sigma,\tau) = x^\mu + 2\sqrt{2\alpha'}\alpha_0^\mu\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}[\tilde{\alpha}_n^\mu e^{-in(\tau+\sigma)} + \alpha_n^\mu e^{-in(\tau-\sigma)}].$
- The limiting procedure: $\tau \to \epsilon\tau,\ \sigma \to \sigma,\ \alpha' = c'/\epsilon$ with $\epsilon \to 0$

$$X^\mu(\sigma,\tau) = x^\mu + 2\sqrt{\frac{2c'}{\epsilon}}\alpha_0^\mu\epsilon\tau + i\sqrt{\frac{2c'}{\epsilon}}\sum_{n\neq 0}\frac{1}{n}[\tilde{\alpha}_n^\mu e^{-in\sigma}(1-in\epsilon\tau) + \alpha_n^\mu e^{in\sigma}(1-in\epsilon\tau)],$$

$$= x^\mu + 2\sqrt{2c'}(\sqrt{\epsilon})\alpha_0^\mu\tau + i\sqrt{2c'}\sum_{n\neq 0}\frac{1}{n}\left[\frac{\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu}{\sqrt{\epsilon}} - in\tau\sqrt{\epsilon}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu)\right]e^{in\sigma}.$$

- Thus we get a relation between the tensionless and tensile modes:

$$A_n^\mu = \frac{1}{\sqrt{\epsilon}}(\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu), \quad B_n^\mu = \sqrt{\epsilon}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu).$$

- The equivalent of the Virasoro contraints

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon\left[\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right]$$

Quantum Tensionless Strings

A summary of quantum results

- Novel closed to open string transition as the tension goes to zero.
  [AB, Banerjee, Parekh (PRL) 2019]

- Careful canonical quantisation leads to not one, but <u>**three different vacua**</u> which give rise to different quantum mechanical theories arising out of the same classical theory.
  [AB, Banerjee, Chakrabortty, Dutta, Parekh 2020]

- Lightcone analysis: spacetime Lorentz algebra closes for two theories for D=26. No restriction on the other theory. All acceptable limits of quantum tensile strings.
  [AB, Mandlik, Sharma 2021]

- Interpretation in terms of Rindler physics on the worldsheet.
  [AB, Banerjee, Chakrabortty (PRL) 2021]

- Carroll limit on spacetime induces tensionless limit on worldsheet. Strings become tensionless near blackhole event horizons. [AB, Banerjee, Chakrabortty, Chatterjee 2021]

Tensionless Path From Closed to Open Strings

AB, Banerjee, Parekh, Physical Review Letters 123 (2019) 111601.

BMS Induced Representations

- An important class of BMS representations: Massive modules.
- The Hilbert space of these modules contains a wavefunction $|M, s\rangle$ satisfying:

$$\boxed{M_0|M, s\rangle = M|M, s\rangle\,, \quad L_0|M, s\rangle = s|M, s\rangle\,, \quad M_n|M, s\rangle = 0,\ \forall n \neq 0.} \tag{33}$$

- This defines a 1-d rep spanned by $\{L_0, M_n, c_L, c_M\}$. Can be used to define an *induced BMS module* with basis vectors

$$|\Psi\rangle = L_{n_1} L_{n_2} \ldots L_{n_k} |M, s\rangle\,.$$

- Limit from Virasoro $\times$ Virasoro to BMS$_3$: $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n},\ M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$.
- Virasoro primary conditions:

$$\mathcal{L}_n|h, \bar{h}\rangle = 0 = \bar{\mathcal{L}}_n|h, \bar{h}\rangle\ (n > 0);\ \mathcal{L}_0|h, \bar{h}\rangle = h|h, \bar{h}\rangle,\ \bar{\mathcal{L}}_n|h, \bar{h}\rangle = \bar{h}|h, \bar{h}\rangle\,.$$

- This translates to

$$\left(L_n + \frac{1}{\epsilon}M_n\right)|h, \bar{h}\rangle = 0, \quad \left(-L_{-n} + \frac{1}{\epsilon}M_{-n}\right)|h, \bar{h}\rangle = 0,\ n > 0.$$

- In the limit, this gives (33), along with the identification: $M = \epsilon(h + \bar{h}),\ s = h - \bar{h}$.

Induced Reps and Tensionless String

- In term of oscillator modes, the induced modules: $B_n|M, s\rangle = 0, \ \forall n \neq 0$.
- We are interested in the vacuum module. Hence we have $B_n|I\rangle = 0$ where $|I\rangle$ is the induced vacuum.
- Wish to return to harmonic oscillator basis for the tensionless string. Define:

$$C^\mu_n = \frac{1}{2}(A^\mu_n + B^\mu_n), \quad \tilde{C}^\mu_n = \frac{1}{2}(-A^\mu_{-n} + B^\mu_{-n})$$

- The algebra: $[C^\mu_m, C^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}, \ [\tilde{C}^\mu_m, \tilde{C}^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}$.
- The tensile and tensionless raising and lowering operators are related by

$$C^\mu_n(\epsilon) = \beta_+\alpha^\mu_n + \beta_-\tilde{\alpha}^\mu_{-n}, \text{ where: } \beta_\pm = \frac{1}{2}\left(\sqrt{\epsilon} \pm \frac{1}{\sqrt{\epsilon}}\right)$$

$$\tilde{C}^\mu_n(\epsilon) = \beta_-\alpha^\mu_{-n} + \beta_+\tilde{\alpha}^\mu_n.$$

- $|0\rangle_c$: $C^\mu_n|0\rangle_c = 0 = \tilde{C}^\mu_n|0\rangle_c \quad \forall n > 0$. Different from tensile vacuum: mixing of tensile raising & lowering op in $C, \tilde{C}$.
- In the C basis, the induced vacuum is given by $(C^\mu_n + \tilde{C}^\mu_{-n})|I\rangle = 0, \quad \forall n$.
- This is precisely the condition of a Neumann boundary state $|I\rangle = \mathcal{N} \exp\left(-\sum_n \frac{1}{n} C_{-n}\tilde{C}_{-n}\right)|0\rangle_c$

Worldsheet Bogoliubov Transformations

- The relation between operators is a Bogoliubov transformation

$$\alpha_n^\mu = e^{iG} C_n e^{-iG} = \cosh\theta\, C_n^\mu - \sinh\theta\, \tilde{C}_{-n}^\mu, \quad G = i\sum_{n=1}^{\infty}\theta\left[C_{-n}.\tilde{C}_{-n} - C_n.\tilde{C}_n\right]$$

$$\tilde{\alpha}_n^\mu = e^{iG}\tilde{C}_n e^{-iG} = -\sinh\theta\, C_{-n}^\mu + \cosh\theta\, \tilde{C}_n^\mu, \quad \tanh\theta = \frac{\epsilon - 1}{\epsilon + 1}$$

- Relation between the two vacua:

$$|0\rangle_\alpha = \exp[iG]|0\rangle_c = \left(\frac{1}{\cosh\theta}\right)^{1+1+\ldots}\prod_{n=1}^{\infty}\exp[\tanh\theta C_{-n}\tilde{C}_{-n}]|0\rangle_c$$

- Using the regularisation: $1 + 1 + 1 + \ldots\infty = \zeta(0) = -\frac{1}{2}$

$$|0\rangle_\alpha = \sqrt{\cosh\theta}\prod_{n=1}^{\infty}\exp[\tanh\theta\, C_{-n}\tilde{C}_{-n}]|0\rangle_c$$

- From the point of view of $|0\rangle_c$, $|0\rangle_\alpha$ is a squeezed state.

From Closed to Open Strings

- When $\epsilon = 1$, $\tanh\theta = 0$, and we have $|0\rangle_\alpha = |0\rangle_c$. This is the closed string vacuum.
- As ϵ changes from 1, from the point of view of the C observer, the vacuum evolves. It becomes a squeezed state as shown before.

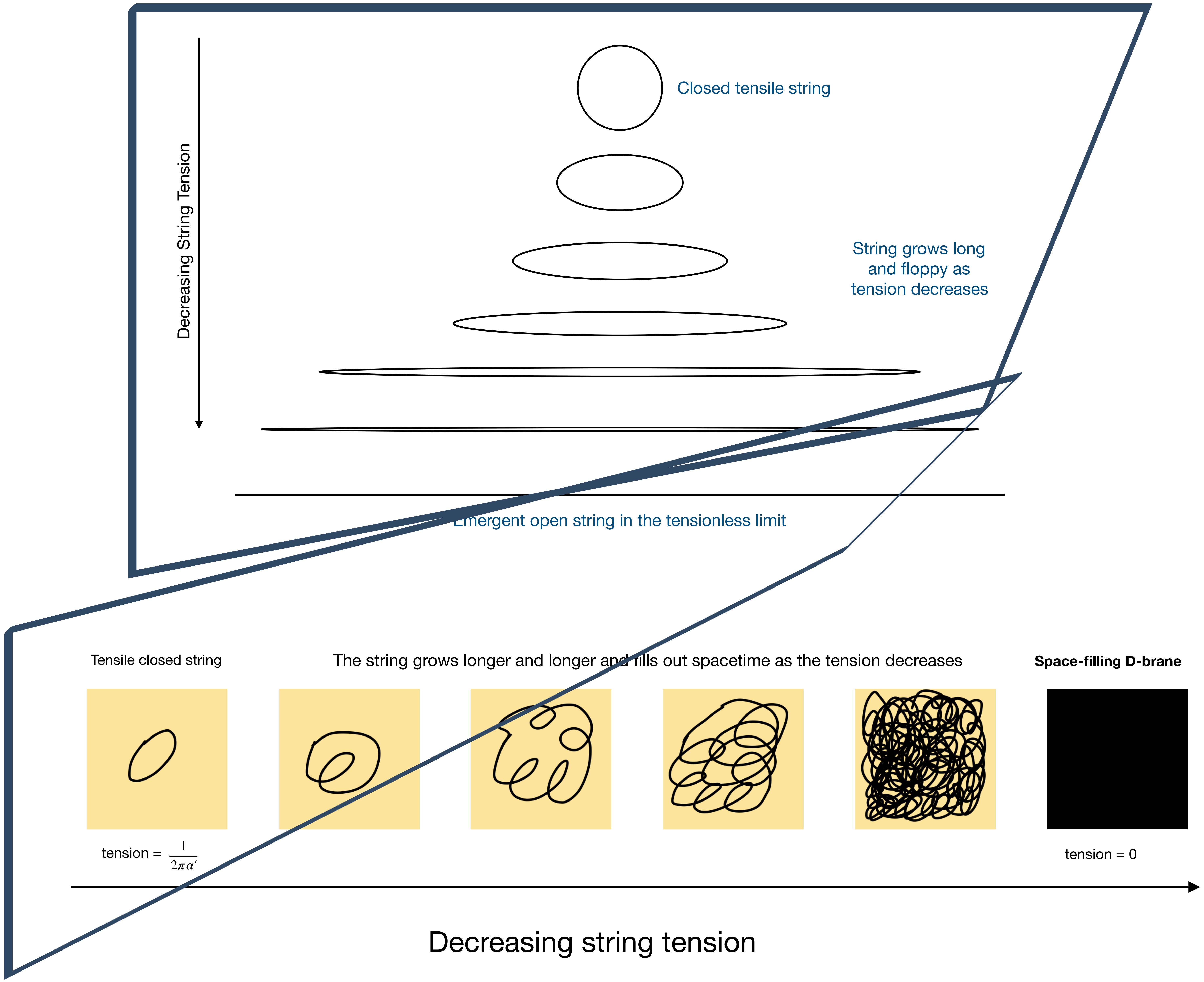
- In the limit where $\epsilon \to 0$, we have $\tanh\theta = -1$. The relation is thus:

$$|0\rangle_\alpha = \mathcal{N} \prod_{n=1}^{\infty} \exp[- C_{-n}\tilde{C}_{-n}]|0\rangle_c$$

  This is precisely the Induced vacuum $|I\rangle$ that we introduced before.
- As we said, this is a Neumann boundary state.
- This is thus an open string free to move in all dimensions (or a spacefilling D-brane).

We have thus obtained an open string by taking a tensionless limit on a closed string theory.

From Closed to Open Strings and D-branes



Bose-Einstein like Condensation on Worldsheet

- Consider any perturbative state in the original tensile theory $|\Psi\rangle = \xi_{\mu\nu}\alpha^{\mu}_{-n}\tilde{\alpha}^{\nu}_{-n}|0\rangle_\alpha$ where $\xi_{\mu\nu}$ is a polarisation tensor. Let us attempt to understand the evolution of the state as $\epsilon \to 0$.
- Close to $\epsilon = 0$, the alpha vacuum can be approximated as follows: $|0\rangle_\alpha = |I\rangle + \epsilon|I_1\rangle + \epsilon^2|I_2\rangle + \dots$
- In this limit, the conditions on the alpha vacuum translate to:

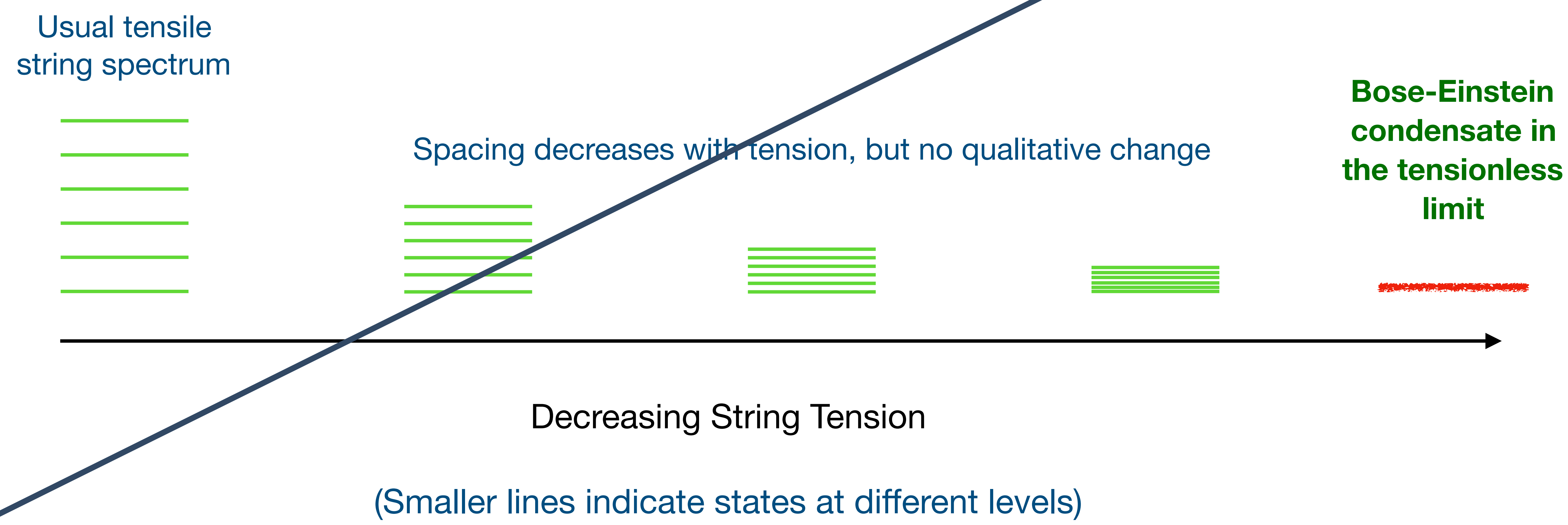
$$\alpha_n|0\rangle_\alpha = \tilde{\alpha}_n|0\rangle_\alpha = 0, \; n > 0$$

$$\Rightarrow \quad B_n|I\rangle = 0, \forall n; \quad A_n|I\rangle + B_n|I_1\rangle = 0, \; A_{-n}|I\rangle - B_{-n}|I_1\rangle = 0, \; n > 0.$$

- One can now take this limit on the state:

$$\alpha_{-n}\tilde{\alpha}_{-n}|0\rangle_\alpha = \left(\frac{1}{\sqrt{\epsilon}}B_{-n} + \sqrt{\epsilon}A_{-n}\right)\left(\frac{1}{\sqrt{\epsilon}}B_n - \sqrt{\epsilon}A_n\right)(|I\rangle + \epsilon|I_1\rangle + \dots). \to K|I\rangle$$

All perturbative closed string states condense on the open string induced vacuum.



Quantum Tensionless Strings II

Based on:

AB, Banerjee, Chakrabortty, PRL 2021.
AB, Banerjee, Chakrabortty, Dutta, Parekh, JHEP 2020.
AB, Mandlik, Sharma, JHEP 2021.
AB, Banerjee, Chakrabortty, Chatterjee, JHEP 2022.

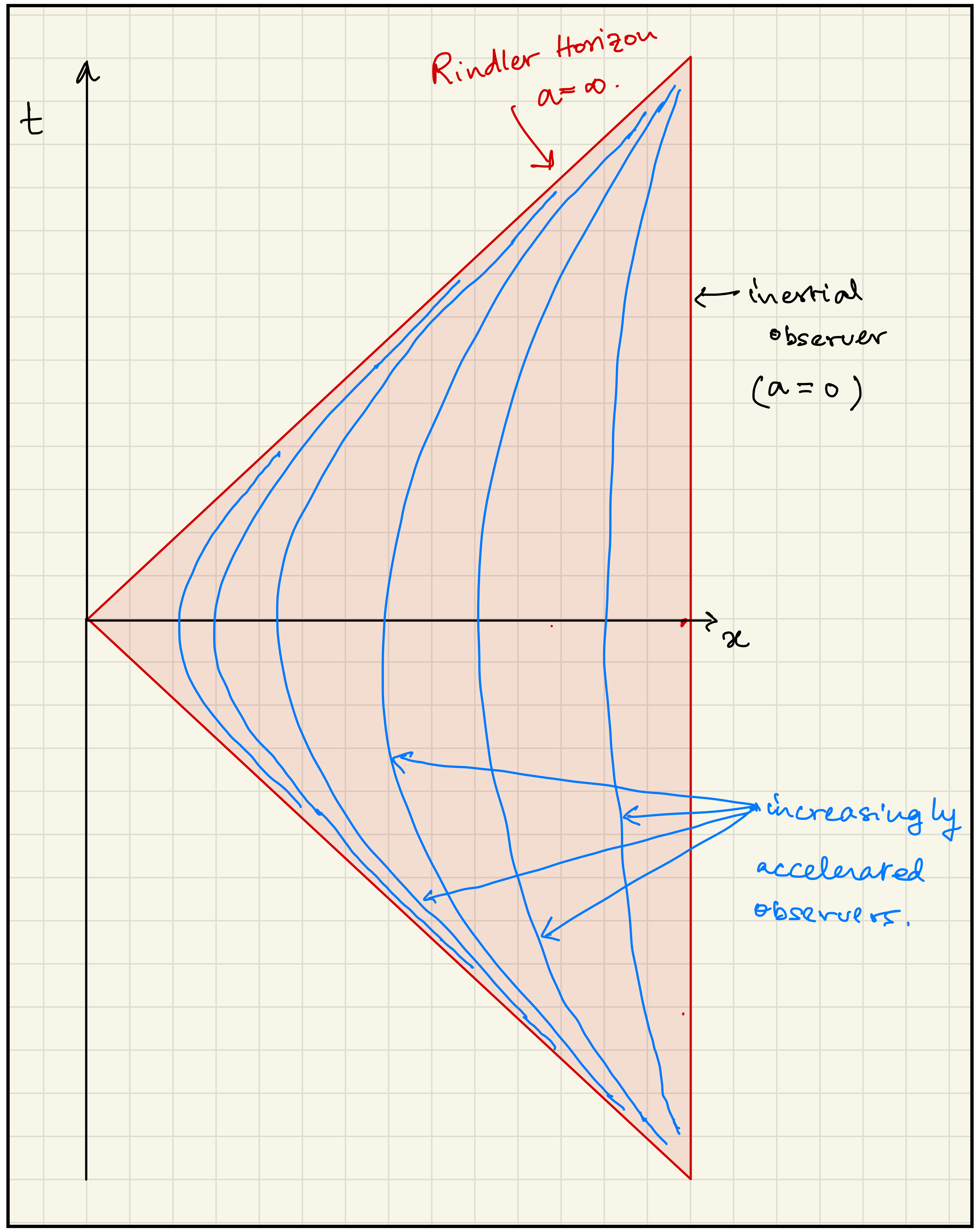
Tension and Acceleration

AB, Banerjee, Chakrabortty, Physical Review Letters 126 (2021) 3, 031601.

Tension as Acceleration

AB, Banerjee, Chakrabortty [PRL 2021]

- One of the most common occurrences of Bogoliubov transformations is in the physics of accelerated observers vis-a-vis inertial observers.
- Minkowski spacetime <-> Rindler spacetime.
- By identifying our Bogoliubov transformations to Rindler Bogoliubov transformations, we can recast the decrease of tension to the increase of acceleration.
- So, tensionless limit of string theory can be modelled as a series of worldsheet observers with increasing acceleration.
- The tensionless or null string emerges where the accelerated observer hits the Rindler horizon. This is where the acceleration goes to infinity.



A quick Rindler tour

- 2d Rindler metric: $ds_R^2 = e^{2a\xi}(-d\eta^2 + d\xi^2)$.
- From Minkowski to Rindler $t = \frac{1}{a} e^{a\xi} \sinh a\eta, \; x = \frac{1}{a} e^{a\xi} \cosh a\eta$
- EOM: $\Box_{t,x}\phi = 0 = \Box_{\eta,\xi}\phi$.
- Minkowski mode expansion

$$\phi(\sigma,\tau) = \phi_0 + \sqrt{2\alpha'}\alpha_0\tau + \sqrt{2\pi\alpha'}\sum_{n>0}[\alpha_n u_n + \alpha_{-n}u_n^* + \tilde{\alpha}_n\tilde{u}_n + \tilde{\alpha}_{-n}\tilde{u}_n^*]$$

$$u_n = [ie^{-in(\tau+\sigma)}]/\sqrt{4\pi n}, \quad \tilde{u}_n = [ie^{-in(\tau-\sigma)}]/\sqrt{4\pi n}.$$

- Rindler mode expansion

$$\phi(\xi,\eta) = \phi_0 + \sqrt{2\alpha'}\beta_0\xi + \sqrt{2\pi\alpha'}\sum_{n>0}[\beta_n U_n + \beta_{-n}U_n^* + \tilde{\beta}_n\tilde{U}_n + \tilde{\beta}_{-n}\tilde{U}_n^*]$$

$$U_n = \frac{ie^{-in(\xi+\eta)}}{\sqrt{4\pi n}}, \quad \tilde{U}_n = \frac{ie^{-in(\xi-\eta)}}{\sqrt{4\pi n}}.$$

- The oscillators $\{\beta, \tilde{\beta}\}$ act on a new vacuum $|0\rangle_R$.
- U's act only in one wedge. To continue between them one defines smearing functions. Combinations for both wedges: $U_n^{(R)} - e^{-(\pi n/a)}U_{-n}^{(L)*}, \qquad U_{-n}^{(R)*} - e^{(\pi n/a)}U_n^{(L)}$.
- Relation between oscillators:

$$\beta_n = \frac{e^{\pi n/2a}}{\sqrt{2\sinh\frac{\pi n}{a}}}\alpha_n - \frac{e^{-\pi n/2a}}{\sqrt{2\sinh\frac{\pi n}{a}}}\tilde{\alpha}_{-n}, \quad \tilde{\beta}_n = -\frac{e^{-\pi n/2a}}{\sqrt{2\sinh\frac{\pi n}{a}}}\alpha_{-n} + \frac{e^{\pi n/2a}}{\sqrt{2\sinh\frac{\pi n}{a}}}\tilde{\alpha}_n.$$

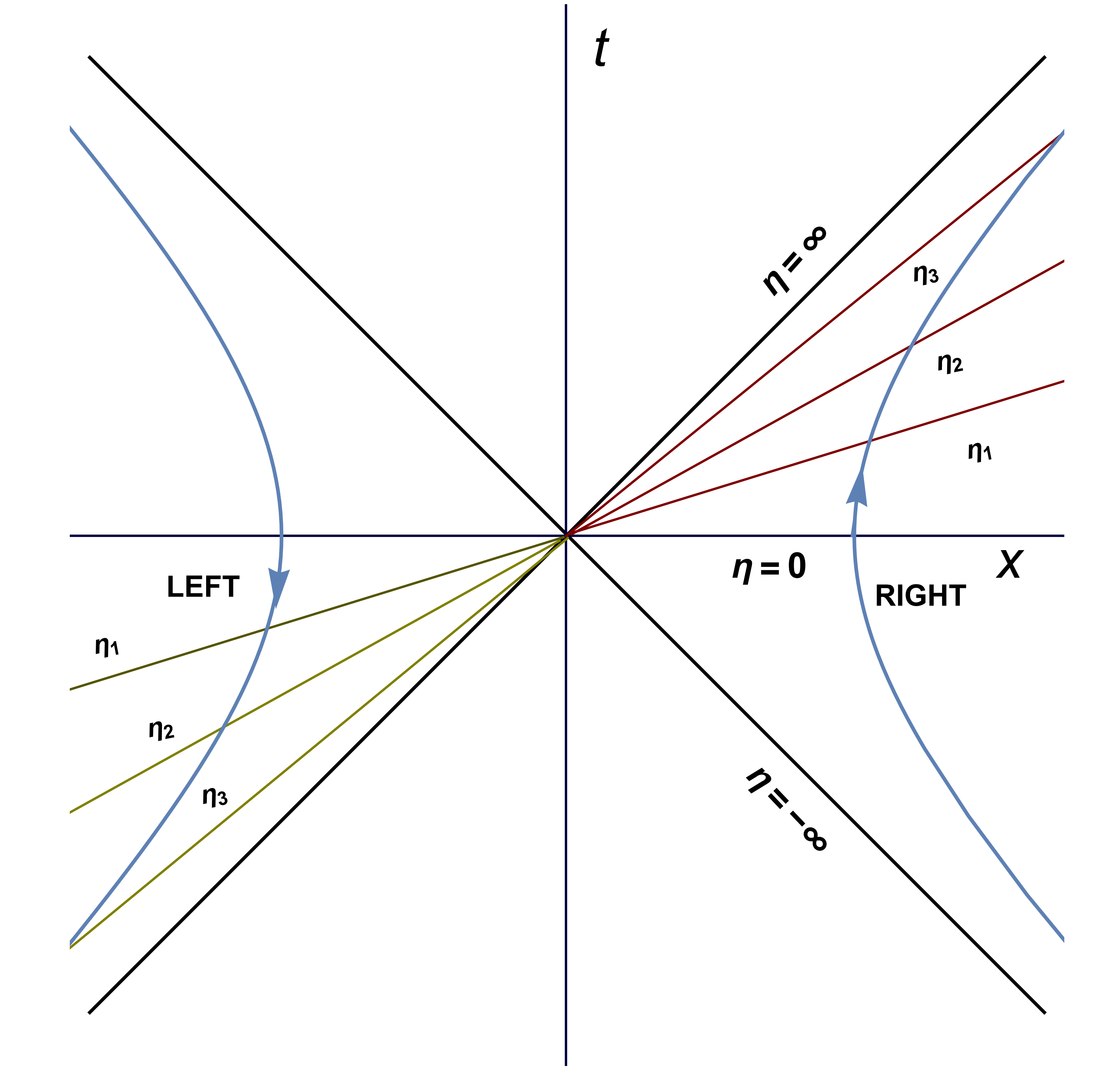
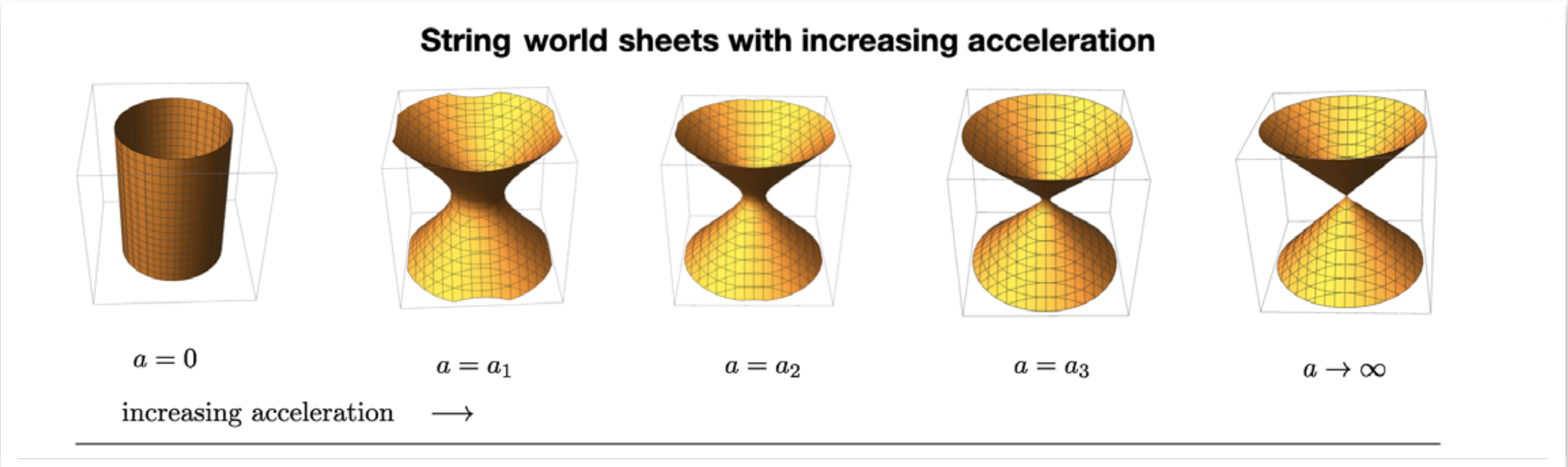


FIG. 1. Equal time slices in Rindler spacetimes.

Evolution in Acceleration

- String equivalent of Rindler observer hitting the horizon = increasingly accelerated world sheets.



- Rindler Bogoliubov transformation at large accelerations:

$$\beta_n^\infty = \frac{1}{2}\left(\sqrt{\frac{\pi n}{2a}} + \sqrt{\frac{2a}{\pi n}}\right)\alpha_n + \frac{1}{2}\left(\sqrt{\frac{\pi n}{2a}} - \sqrt{\frac{2a}{\pi n}}\right)\tilde{\alpha}_{-n}, \quad \tilde{\beta}_n^\infty = \frac{1}{2}\left(\sqrt{\frac{\pi n}{2a}} - \sqrt{\frac{2a}{\pi n}}\right)\alpha_{-n} + \frac{1}{2}\left(\sqrt{\frac{2a}{\pi n}} + \sqrt{\frac{\pi n}{2a}}\right)\tilde{\alpha}_n.$$

- Identification: $C_n = \beta_n^\infty, \qquad \tilde{C}_n = \tilde{\beta}_n^\infty, \qquad \epsilon = \frac{\pi n}{2a}.$
- The limit of zero tension is thus the limit of infinite acceleration: $\epsilon \to 0 \Rightarrow a \to \infty.$
- **Evolution:** $a = 0\colon \{\beta_n, \tilde{\beta}_n\} \to \{\alpha_n, \tilde{\alpha}_n\},\ 0 < a < \infty\colon \quad \{\beta_n(a), \tilde{\beta}_n(a)\}, \quad a \to \infty\colon \quad \{\beta_n, \tilde{\beta}_n\} \to \{C_n, \tilde{C}_n\}.$ Complete interpolating solution.

Hitting the Horizon: Evolution in Rindler Time

- We explored hitting the Rindler horizon by evolving in acceleration.
- The horizon can also be hit by evolving in Rindler time at constant acceleration.
- So the infinite time limit on the Rindler worldsheet would also generate the null string.

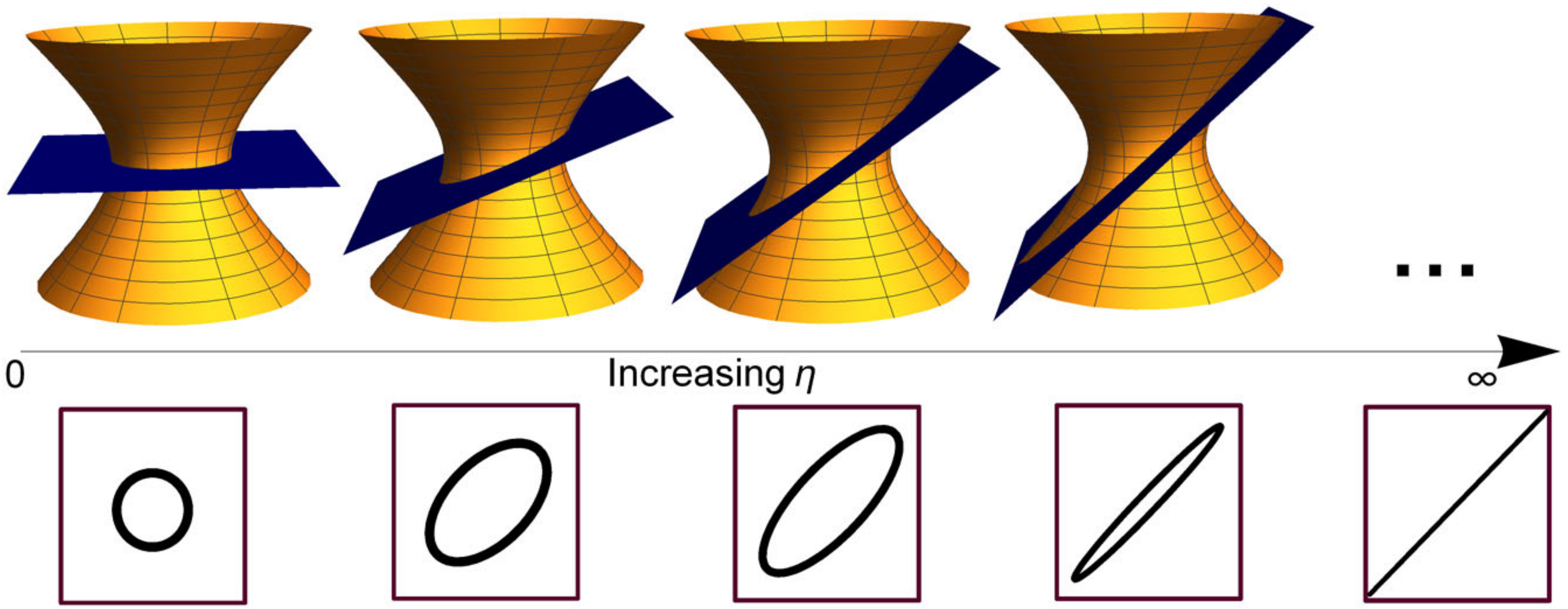


FIG. 3. Equal time slices of a Rindler worldsheet.

Hitting the Horizon: Evolution in Rindler Time

- Mathematically, this is the limit $\eta \to \infty$. Or equivalently,

$$\eta \to \eta, \qquad \xi \to \epsilon\xi, \qquad \epsilon \to 0.$$

- Conformal generators in Rindler: $\mathcal{L}_n, \bar{\mathcal{L}}_n = \pm \frac{i^n}{2} e^{n(\xi-\eta)}(\partial_\eta \mp \partial_\xi).$
- In the limit we get:

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} = i^n e^{-n\eta}(\partial_\eta - n\xi\partial_\xi),$$

$$M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}) = -i^n e^{-n\eta}\partial_\xi.$$

- These close to form the BMS algebra as expected and the null string emerges.

A Tale of Three

AB, Banerjee, Chakrabortty, Dutta, Parekh, JHEP 04 (2020) 061

A Tale of Three

AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354

- From a single classical theory, several inequivalent quantum theories may emerge. This happens when we consider canonical quantisation of tensionless string theories.

- As we saw earlier Classical constraint on the tensionless string: $T_{(1)} = 0, \quad T_{(2)} = 0.$

  Quantum version: physical spectrum of tensionless strings restricted by $\langle \text{phys}|T_{(1)}|\text{phys}'\rangle = 0, \quad \langle \text{phys}|T_{(2)}|\text{phys}'\rangle = 0.$

- This amounts to $\boxed{\langle phys|L_n|phys'\rangle = 0, \quad \langle phys|M_n|phys'\rangle = 0.}$

- For each type of oscillator F obeying $\langle phys|F_n|phys'\rangle = 0$, there can be three types of solutions.

$$
\begin{aligned}
&1. \quad F_n|phys\rangle = 0 \quad (n > 0),\\
&2. \quad F_n|phys\rangle = 0 \quad (n \neq 0),\\
&3. \quad F_n|phys\rangle \neq 0, \;\; but \; \langle phys'|F_n|phys\rangle = 0.
\end{aligned}
$$

A Tale of Three

AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354

- Here $F_n = (L_n, M_n)$. Hence seemingly nine conditions:

$$L_m|phys\rangle = 0, \ (m>0), \begin{Bmatrix} M_n|phys\rangle = 0, \ (n>0) \\ M_n|phys\rangle = 0, \ (n \neq 0) \\ M_n|phys\rangle \neq 0, \ (\forall \ n) \end{Bmatrix}; L_m|phys\rangle = 0, \ (m \neq 0), \begin{Bmatrix} M_n|phys\rangle = 0, \ (n>0) \\ M_n|phys\rangle = 0, \ (n \neq 0) \\ M_n|phys\rangle \neq 0, \ (\forall \ n) \end{Bmatrix}; L_m|phys\rangle \neq 0, \ (\forall \ m), \begin{Bmatrix} M_n|phys\rangle = 0, \ (n>0) \\ M_n|phys\rangle = 0, \ (n \neq 0) \\ M_n|phys\rangle \neq 0, \ (\forall \ n) \end{Bmatrix}$$

- But the underlying BMS algebra also has to be satisfied. It turns out that only three of the nine choices lead to consistent solutions.

- These are three inequivalent vacua, leading to three inequivalent quantum theories.

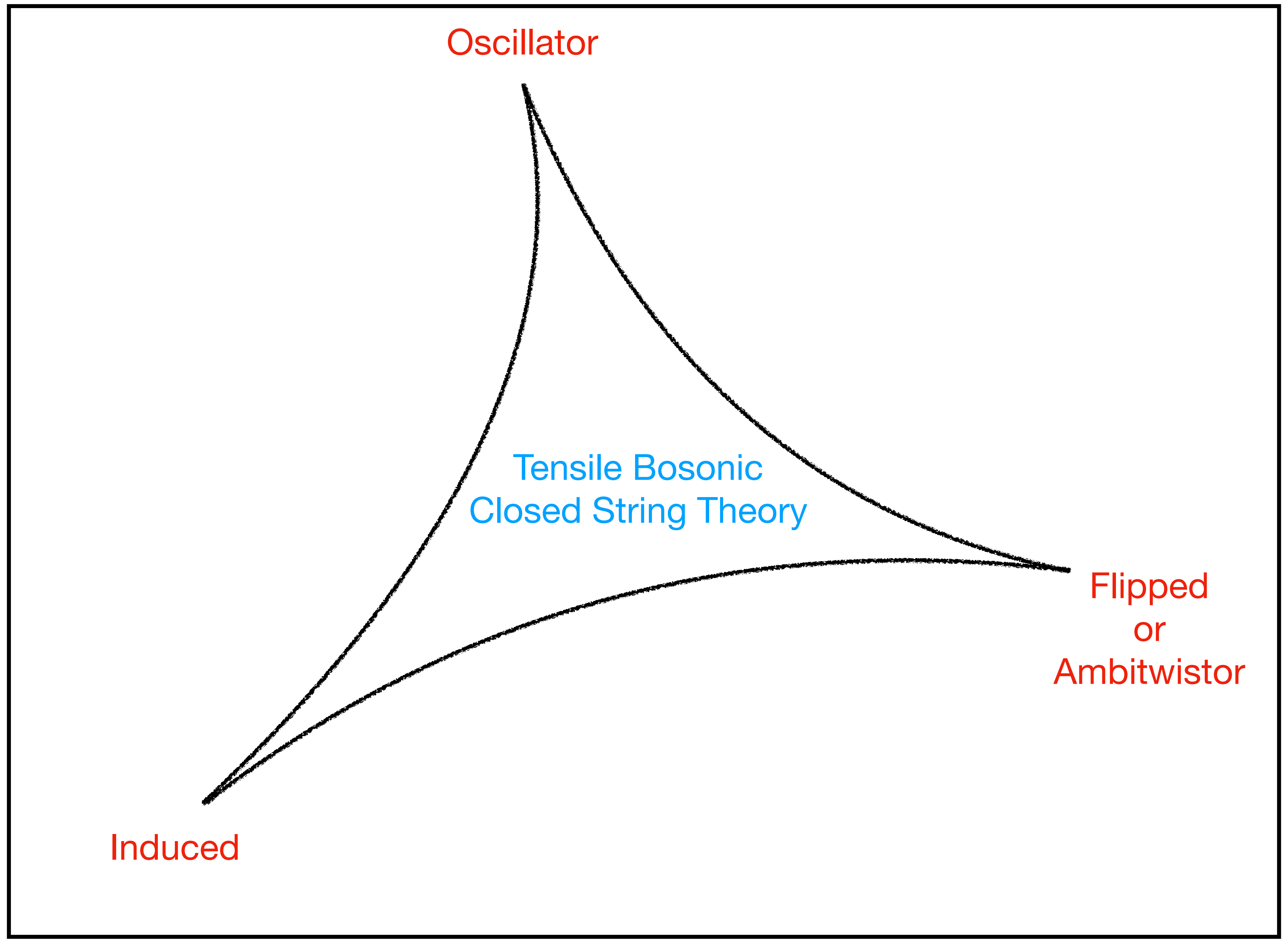
    - **Induced vacuum:** Theory obtained from the limit of usual tensile strings.

    - **Flipped vacuum:** Leads to ambitwistor strings. (See e.g. **Casali, Tourkine, (Herfray) 2016-17**)

    - **Oscillator vacuum:** Interesting new vacuum. Contains hints of huge underlying gauge symmetry.

Critical Dimensions

AB, Mandlik, Sharma. 2105.09682



Tensionless corners of Quantum Tensile String Theory

A summary of quantum results

* Novel closed to open string transition as the tension goes to zero.
  [AB, Banerjee, Parekh (PRL) 2019]

* Careful canonical quantisation leads to not one, but **<u>three different vacua</u>** which give rise to different quantum mechanical theories arising out of the same classical theory.
  [AB, Banerjee, Chakrabortty, Dutta, Parekh 2020]

* Lightcone analysis: spacetime Lorentz algebra closes for two theories for D=26. No restriction on the other theory. All acceptable limits of quantum tensile strings.
  [AB, Mandlik, Sharma 2021]

* Interpretation in terms of Rindler physics on the worldsheet.
  [AB, Banerjee, Chakrabortty (PRL) 2021]

* Carroll limit on spacetime induces tensionless limit on worldsheet. Strings become tensionless near blackhole event horizons. [AB, Banerjee, Chakrabortty, Chatterjee 2021]

Other results

* Tensionless superstrings: Two varieties depending on the underlying Superconformal Carrollian algebra.

* Homogeneous Tensionless Superstrings: Fermions scale in same way. Previous construction: Lindstrom, Sundborg, Theodoridis 1991. Limiting point of view: AB, Chakrabortty, Parekh 2016.

* Inhomogeneous Tensionless Superstrings: Fermions scale differently. New tensionless string! AB, Banerjee, Chakrabortty, Parekh 2017-18.

* Possible counting of BTZ microstates with winding null strings on the horizon. AB, Grumiller, Sheikh-Jabbari (in progress)

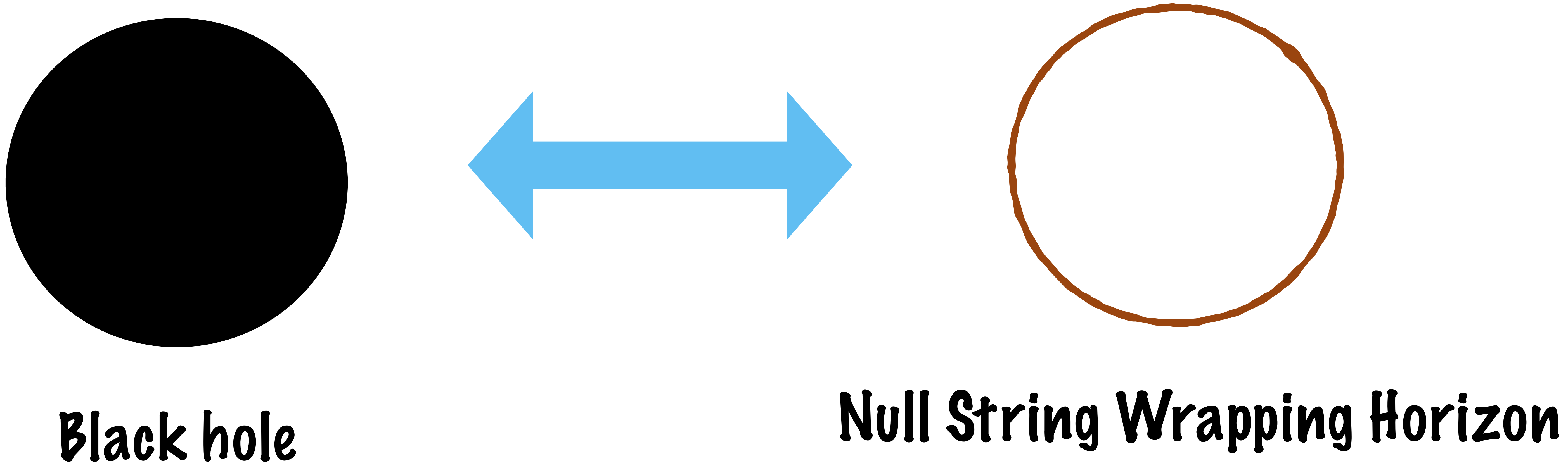
Open questions: Tensionless Strings

* Analogous calculation of beta-function=0. Consistent backgrounds?
* Linking up to Gross-Mende high energy string scattering from worldsheet symmetries.
* Attacking the Hagedorn transition from the Carroll perspective. Emergent degrees of freedom?
* Strings near black holes, strings falling into black holes?
* Extend "Tale of Three" to superstrings. Different superstring theories?
* Intricate web of tensionless superstring dualities?

Black hole Microstates from Null Strings

AB, Grumiller, Sheikh-Jabbari 2210.10794

Black holes from Null Strings?



* Event horizon of black holes are null surfaces.
* In d=3, consider BTZ black holes. Event horizon is a null circle.
* **Proposal:** A null string wrapping the event horizon contains in its spectrum the micro states of a BTZ black hole.
* We can reproduce **the Bekenstein-Hawking entropy** as well as its logarithmic corrections!
* Possible generalisations to higher dimensions.

Horizon Strings

* Proposal motivated by symmetries. Symmetries of event horizon same as symmetries of the null string worldsheet.
* Dynamic horizon on which d.o.f. live is then equivalent to a null string.
* Quantize the null string in **Oscillator Vacuum**. Use Lightcone gauge for convenience.
* Black hole states: a band of states with sufficiently high level.
* Mass is proportional to the radius of the horizon. Motivated by Near Horizon first law. [Donnay et al 2015, Afshar et al 2016].
* Complicated combinatorics leads to entropy and amazing the correct logarithmic corrections.
* Can be thought of as a precise formulation of the membrane paradigm.
* Generalization to d=4 with null membranes in progress and showing interesting signs.

Concluding remarks


Non-Lorentzian Holography
Holography
Flat Holography
Carroll Fluids
Cosmology
Condensed Matter
Carrollian CFTs
Black Holes
Fractons
Superconductivity
Tensionless/null strings
Luttinger Liquids
String Theory
Carroll strings

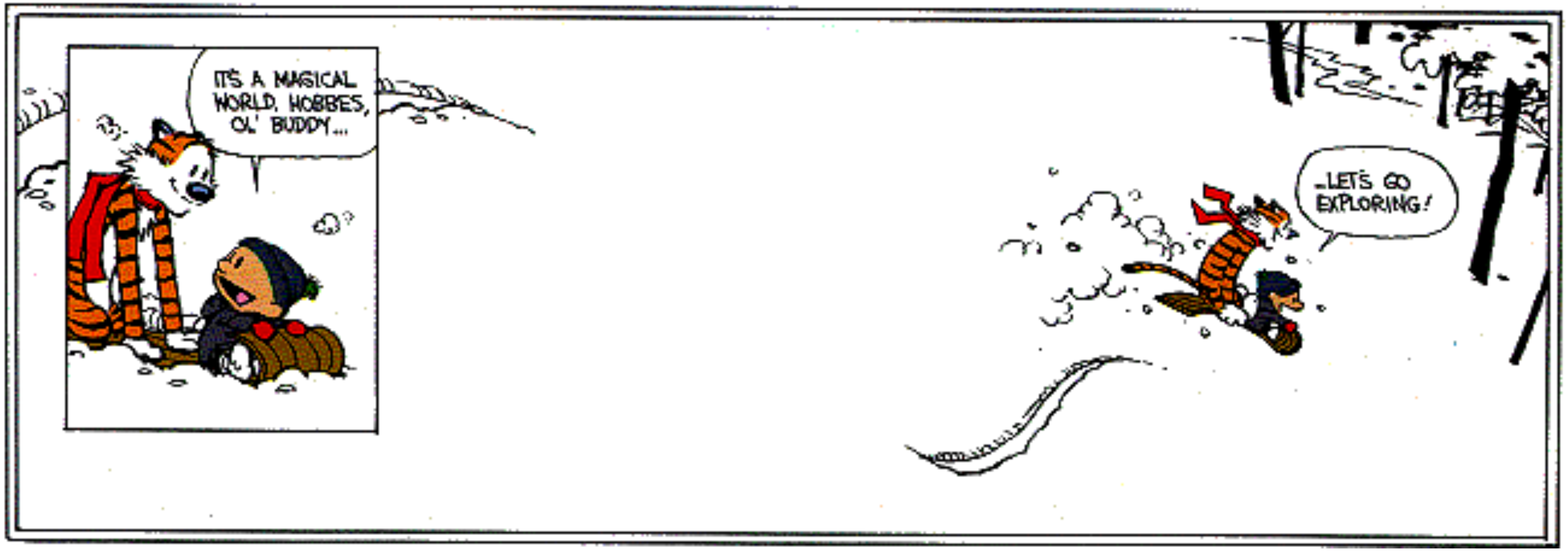
A journey that has just begun

* We have just begun to scratch the surface of what seems to be an amazingly rich subject.

* New physics, new mathematics. New ways at looking at old problems.

* Things that were previously discarded as "singular" make sense if we use correct structures and follow singular limits carefully.

* Only spoke of two applications. Many other things are afoot!



Thank you!