Weyl Anomalies of 4d Conformal Boundaries and Defects

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Based on 2111.14713

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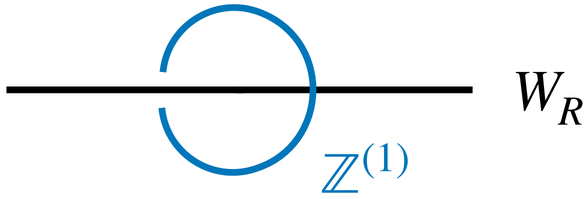
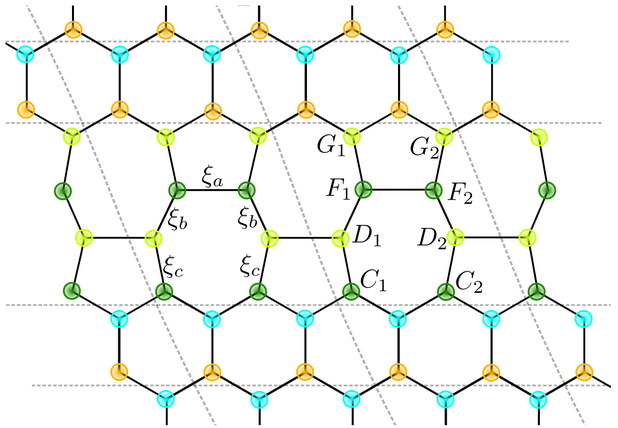
Motivation

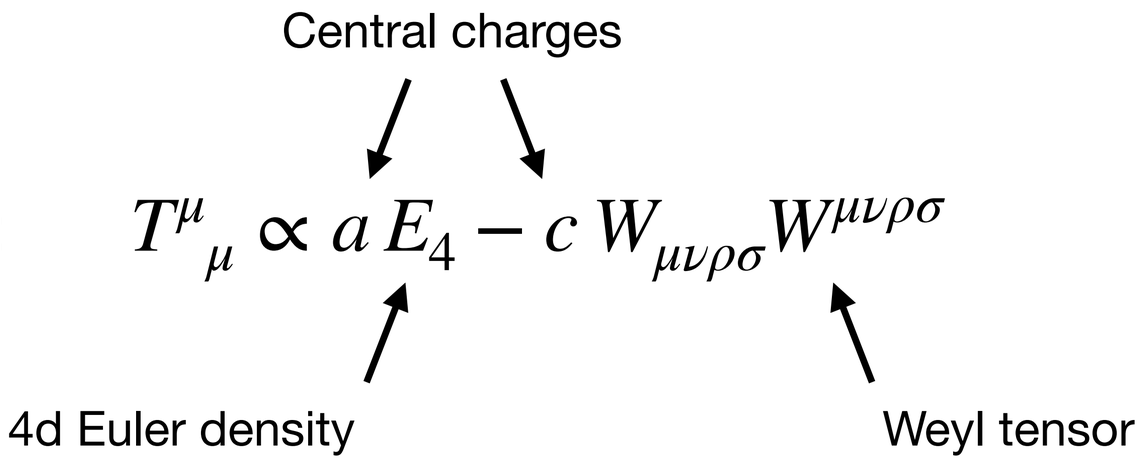
- Boundaries and defects in the lab
- Defects in QFT
- Boundaries and defects in string theory





Weyl Anomaly in Standard CFT

- On a curved background Weyl symmetry $g \to e^{2\omega} g$ is anomalous
- Anomaly is captured by $T^{\mu}{}_{\mu} \neq 0$ as a function of g
- E.g. for CFT_4 :

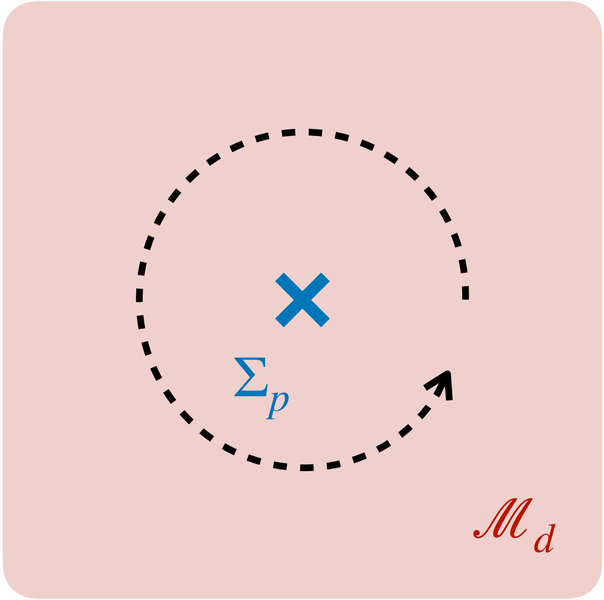


Conformal Defects

- A p-dim conformal defect in CFT$_d$ breaks conformal group

$$SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$$

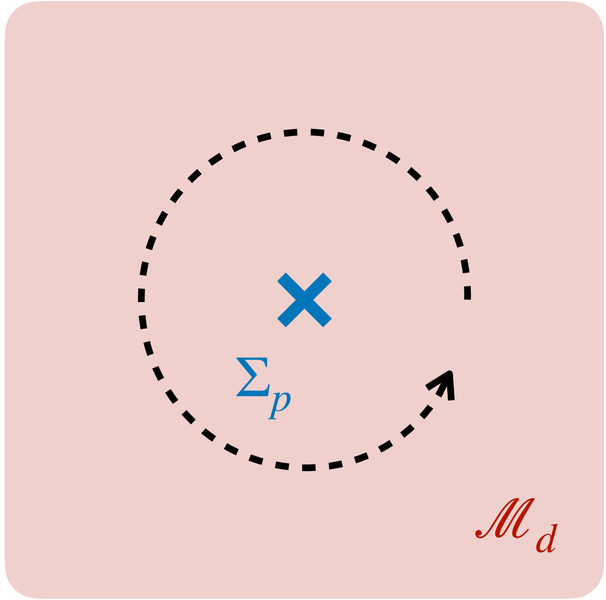
- Residual symmetry still strongly constrains correlators
- E.g. $\langle \mathcal{O}_\Delta(x_\perp, x_\parallel) \rangle = \dfrac{a_\mathcal{O}}{|x_\perp|^\Delta}$



Conformal Defects

- $\nexists$ conserved defect stress tensor $\hat{T}^{ab}_{\Sigma}$
- Full stress tensor $T^{\mu\nu} = T^{\mu\nu}\Big|_{\mathscr{M}} + T^{\mu\nu}\Big|_{\Sigma}$

  satisfies $\partial_\mu T^{\mu a} = 0$ everywhere
- Ward identity $\partial_\mu T^{\mu i} = \delta^{(d-p)}(x_\perp)\, \hat{D}^i(x_\parallel)$
- Displacement operator $\hat{D}^i$ is universal



Weyl Anomaly in DCFT

- Defect contribution is captured by $T^{\mu}{}_{\mu}\Big|_{\Sigma} \neq 0$ as a function of $g, X^{\mu}(x_{||})$

- E.g. 2d defect

$$T^{\mu}{}_{\mu}\Big|_{\Sigma_2} \propto a_{\Sigma}\,\bar{E}_2 + d_1\,\mathring{\mathrm{I\!I}}^{\mu}_{ab}\mathring{\mathrm{I\!I}}^{ab}_{\mu} + d_2\,W_{ab}{}^{ab}$$

Defect central charges

2d Euler density of the induced connection

Traceless 2nd fundamental form (aka extrinsic curvature)

Pullback of ambient Weyl tensor

Weyl Anomaly of 4d Conformal Defects

Based on 2111.14713 — Chalabi, Herzog, O'Bannon, Robinson, Sisti

- For a 4d defect ∃ 23 parity-even terms!

$$
\begin{aligned}
T^{\mu}{}_{\mu}|_{\Sigma_4} \propto & - a_\Sigma \overline{E}_4 + d_1 \mathcal{J}_1 + d_2 \mathcal{J}_2 + d_3 \overline{W}_{abcd} \overline{W}^{abcd} + d_4 (W_{ab}{}^{ab})^2 \\
& + d_5 W_{aibj} W^{aibj} + d_6 W^b{}_{iab} W_c{}^{iac} + d_7 W_{ijkl} W^{ijkl} + d_8 W_{aijk} W^{aijk} \\
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Weyl Anomaly of 4d Conformal Defects

Based on 2111.14713

- Two non-trivial Weyl invariants

$$\mathcal{J}_1 = \frac{1}{d-1} R \mathring{\mathrm{II}}^i_{ab} \mathring{\mathrm{II}}^{ab}_i - \frac{1}{d-2} N^{\mu\nu} R_{\mu\nu} \mathring{\mathrm{II}}^i_{ab} \mathring{\mathrm{II}}^{ab}_i - \frac{2}{d-2} R^a{}_b \mathring{\mathrm{II}}^i_{ac} \mathring{\mathrm{II}}^{bc}_i - \frac{1}{2} W^c{}_{acb} \mathrm{II}_i \mathring{\mathrm{II}}^{iab}$$
$$+ \frac{4}{9} W^c{}_{ica} \overline{D}^b \mathring{\mathrm{II}}^i_{ab} + \mathring{\mathrm{II}}^{iab} D_i W^c{}_{acb} - \frac{1}{2} \mathrm{II}^i \mathrm{Tr}\ \mathring{\mathrm{II}}_i \mathring{\mathrm{II}}^j \mathring{\mathrm{II}}_j + \frac{1}{16} \mathrm{II}^i \mathrm{II}_i \mathrm{Tr}\ \mathring{\mathrm{II}}^j \mathring{\mathrm{II}}_j + \frac{2}{9} \overline{D}^b \mathring{\mathrm{II}}^i_{ab} \overline{D}^c \mathring{\mathrm{II}}_{ic}{}^a$$

- d_1 determined by $\langle \hat{D} \hat{D} \rangle$
- Unitarity $\implies$ $d_1 \leq 0$

Weyl Anomaly of 4d Conformal Defects

Based on 2111.14713

- Two non-trivial Weyl invariants

$$\begin{aligned}\mathcal{J}_2 = &\frac{d-4}{d-2} W_{ab}{}^{ab} N^{\mu\nu} R_{\mu\nu} - \frac{d-4}{d-1} R W_{ab}{}^{ab} + \frac{4(d-5)}{3(d-2)} R_{ab} W_c{}^{acb} \\ &- \frac{5(d-4)}{48} W_{ab}{}^{ab} \Pi^i \Pi_i + \frac{2(d-5)}{3} W^c{}_{ica} \overline{D}^b \mathring{\Pi}^i_{ab} + \frac{4(d+1)}{9} \mathring{\Pi}^{iab} D_i W^c{}_{acb} \\ &- \frac{1}{3} W^{ac}_{ic} \overline{D}_a \Pi^i - \frac{2(d-5)}{3} \Pi^i \mathrm{Tr}\, \mathring{\Pi}_i \mathring{\Pi}^j \mathring{\Pi}_j + \frac{(d-10)}{12} \Pi^i D_i W_{ab}{}^{ab} + \frac{1}{3} D^i D_i W_{ab}{}^{ab}\end{aligned}$$

- d_2 determined by $\langle T \rangle$
- ANEC $\int_\gamma du \, \langle T_{uu} \rangle \geq 0 \implies d_2 \leq 0$

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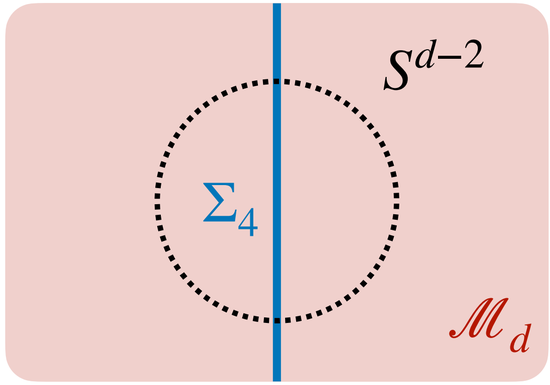
- Entanglement entropy $S_{EE}^{def}\Big|_{log} \propto a_\Sigma + \dfrac{(d-5)(d-4)}{4(d-1)} d_2$



Take-Home Messages

- Conformal defects contribute to the Weyl anomaly
- 23 parity even defect central charges for 4d conformal defects
- Defect central charges encode various physical quantities

  E.g. $\langle \hat{D}\hat{D} \rangle$, $\langle T \rangle$, S_{EE}^{def}, …
- Bounds on defect central charges

Outlook

- Other physical quantities controlled by 4d defect central charges?
- Constraints on central charges, e.g. conformal collider or bootstrap?
- Parity-odd central charges? Non-vanishing examples?
- Does SUSY impose relations between 4d defect central charges?
- Examples of 4d defects in interacting CFTs, e.g. M5 brane intersection?