Bjarne 50++ Symposium

Finite–Time Thermodynamics of Endoreversible Systems

Karl Heinz Hoffmann
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Germany

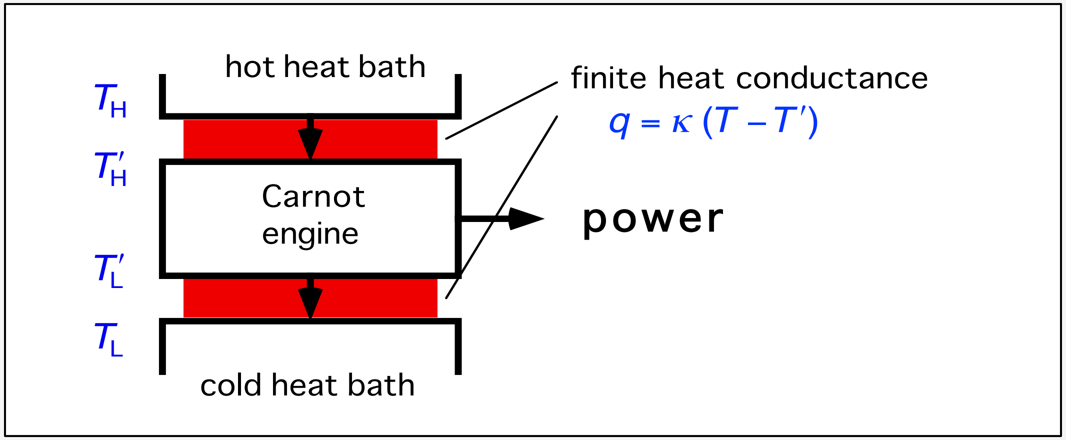
Finite Time Thermodynamics

	Efficiency	Process Type
Classical Thermodynamics	Carnot $\eta_{\text{Carnot}} = \frac{T_H - T_L}{T_H}$	reversible, quasistatic, infinitely slowly
Reality	?	Finite rates Finite times

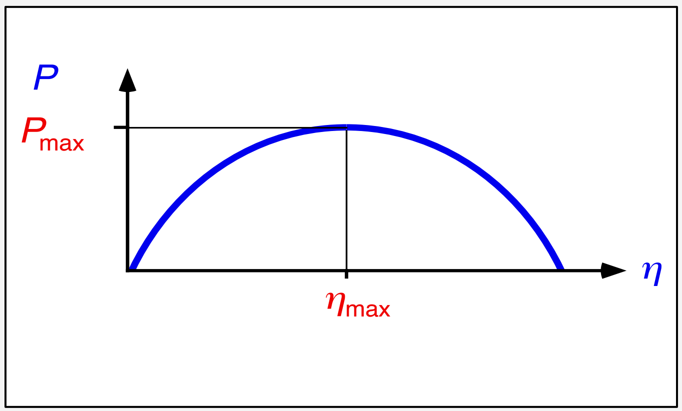
- Is there a price for finite rates ?
- How large is the price ?
- What is the process with the minimal price ?

Maximum Power Processes

- An irreversible power station model: The Curzon-Ahlborn Engine



- Maximize power !!



$$\eta_{CARNOT} = 1 - \frac{T_L}{T_H}$$

Maximum Power Processes

- Efficiency at maximum power:

$$\eta_{\mathrm{maxP}} = 1 - \sqrt{\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}}$$

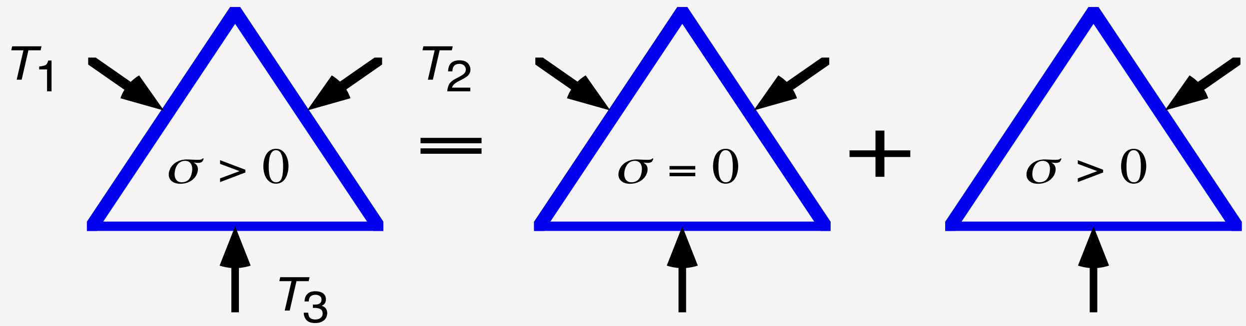
- Application to real power stations

Power station	T_{L}	T_{H}	η_{Carnot}	η_{maxP}	η_{real}
West Thurock (Coal)	25	565	0.64	0.40	0.36
CANDU (nuclear)	25	300	0.48	0.28	0.30
Laderello (geothermal)	80	250	0.32	0.17	0.16

F. L. Curzon and B. Ahlborn. Efficiency of a Carnot engine at maximum power output. Am. J. Phys., 43:22–24, 1975.

An Early Finite Time Thermodynamics Concept

- Tricycle Formalism (B. Andresen, P. Salamon, RS Berry, 1977)



- Express the dissipative flows in terms of the reversible one

→ Quantify dissipation

Endoreversible Thermodynamics

A non-equilibrium
Finite-Time
Thermodynamics theory

Symposium Finite-Time Thermodynamics
Copenhagen
May 23 – 25, 2022

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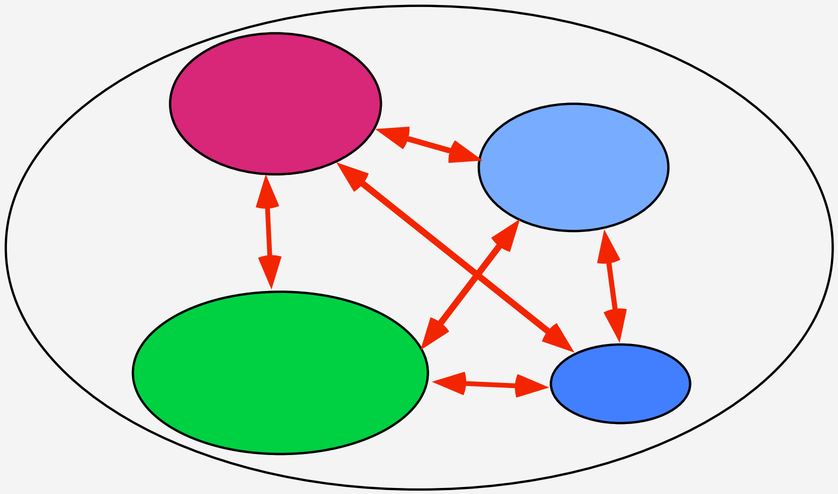
Endoreversible Thermodynamics

- Nonequilibrium → entropy production → losses



SUBSYSTEMS exchange extensities, e.g. volume, chemical compunds, charge, etc

- Subsystems **reversibel** (endoreversible!)
- Transport **irreversibel**

Quantifying entropy production!

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Subsystems

- Equilibrium systems described by extensities:
  - volume
  - entropy
  - charge
  - momentum
  - angular momentum
  - particle number
  - etc

- Extensities obey balance equations

$$\frac{dX}{dt} = I_X + \Sigma_X$$

(rate of change = flux + production)

Interactions

- The Gibbs equation

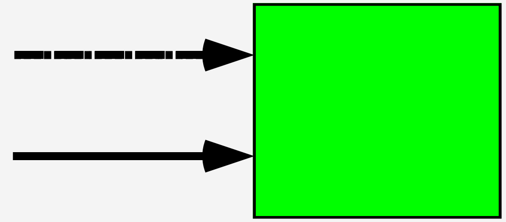
$$dE = T\,dS - p\,dV + \mu\,dn + \omega\,dL + v\,dP + \ldots$$

- Influx of carrier $\frac{dX}{dt} = J_X$ leads to influx of energy $I_X = \frac{dE}{dt}$

<u>energy comes with a carrier</u>

$$I_X = y_X J_X$$

$$J_X$$



- examples: heat comes with entropy
  work comes with volume

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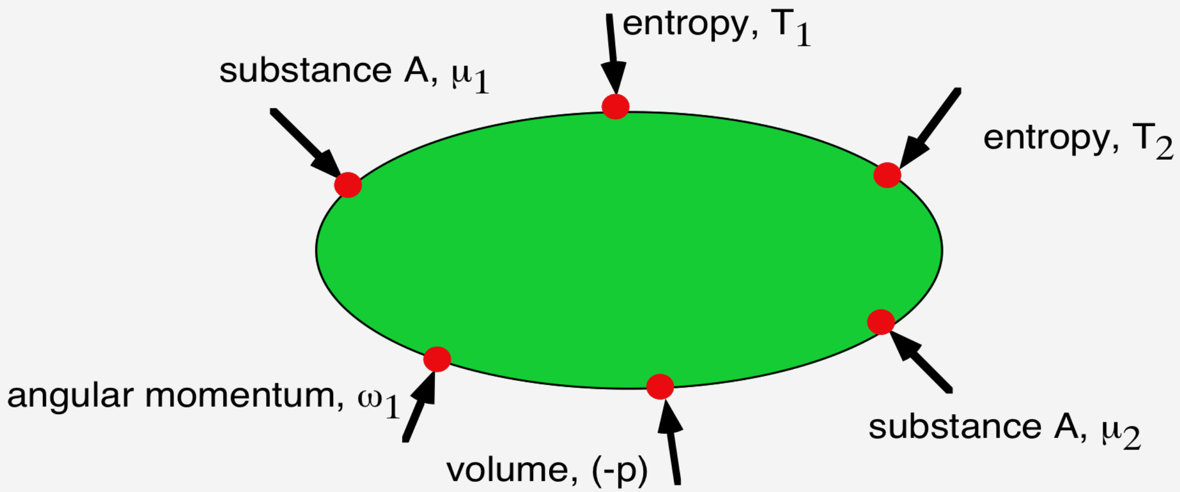
Subsystems

- Equilibrium systems described by extensities or intensities

  e.g. heat bath with temperature T

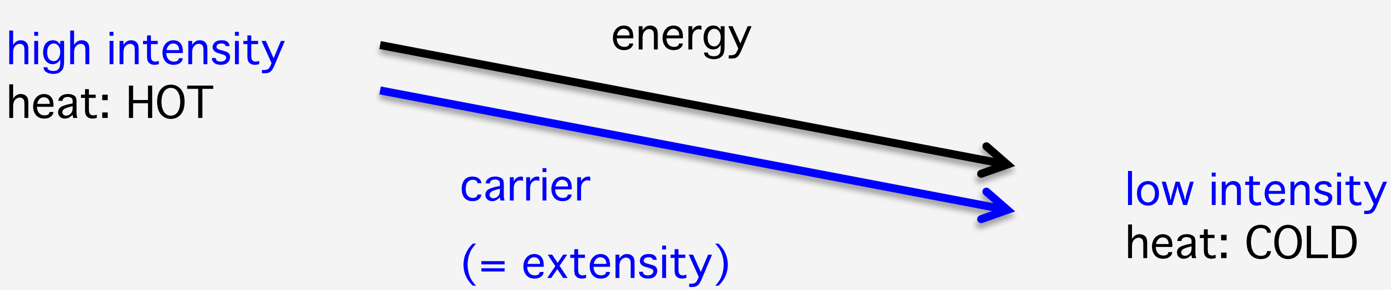
- reversible engines, e.g, Carnot engine or more general



Moving Energy

- Energy always has a carrier
  - energy — heat, electrical energy, chemical energy
  - carrier — entropy, charge, chemical compound
  - intensity — temperature, electrical potential, chemical potential

- Energy moves if there is a difference in intensity



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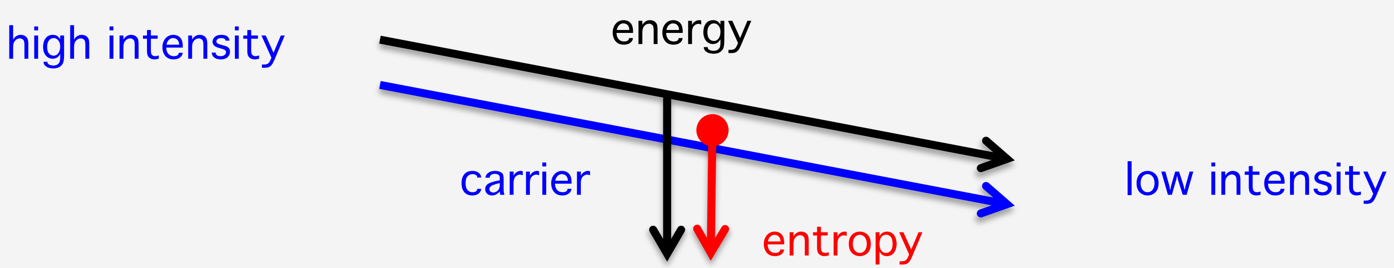
Moving Energy is Not For Free

- Dissipative interaction: entropy production = loss



- Transport laws
  - for carrier — example chemical compound — $j \propto \mu_1 - \mu_2$
  - for energy — example heat — $q \propto T_1 - T_2$

Endoreversible Thermodynamics

- System theory for nonequilibrium systems
  - subsystems
  - interactions

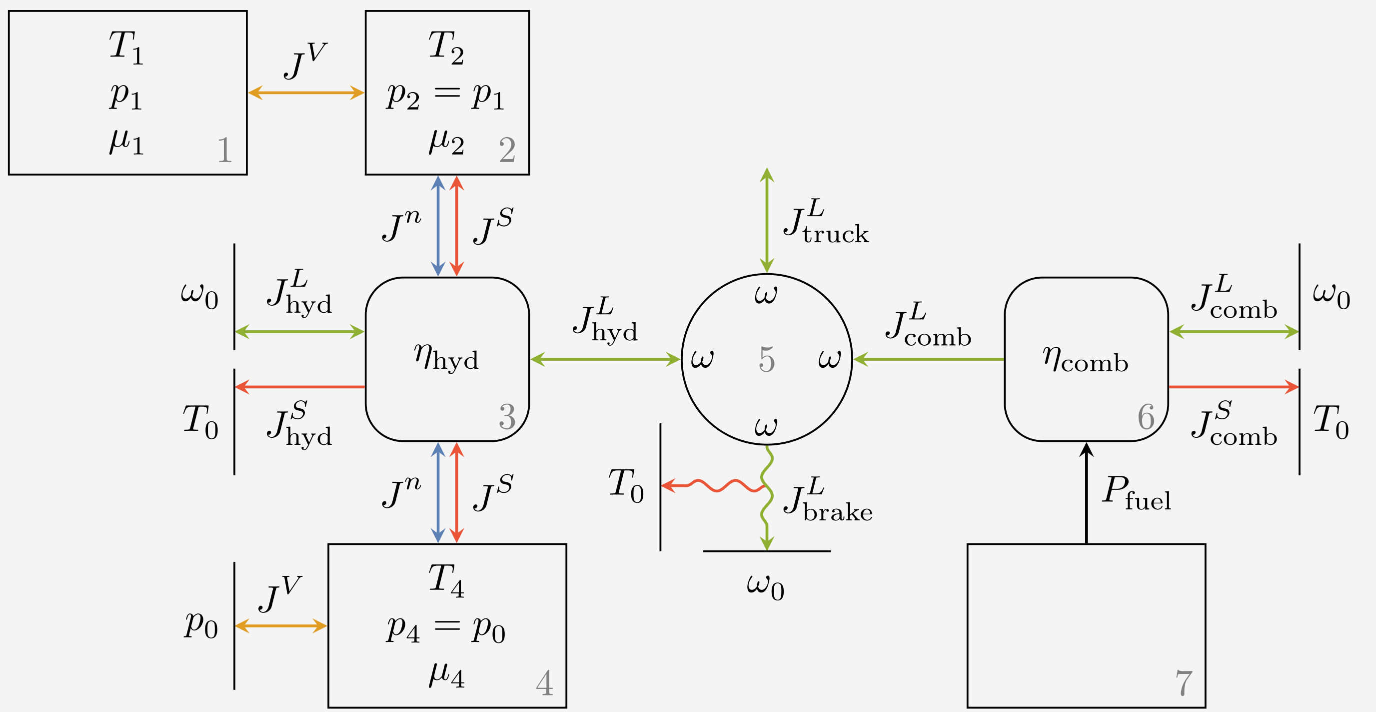
- **Subsystems**
  - equilibrium systems: Reservoirs
  - but also Reversible Engines
  - Reactors

- **Interactions**
  - reversible
  - irreversible
  - for instance: regenerator

An Endoreversible Thermodynamics description allows to quantify the dissipation

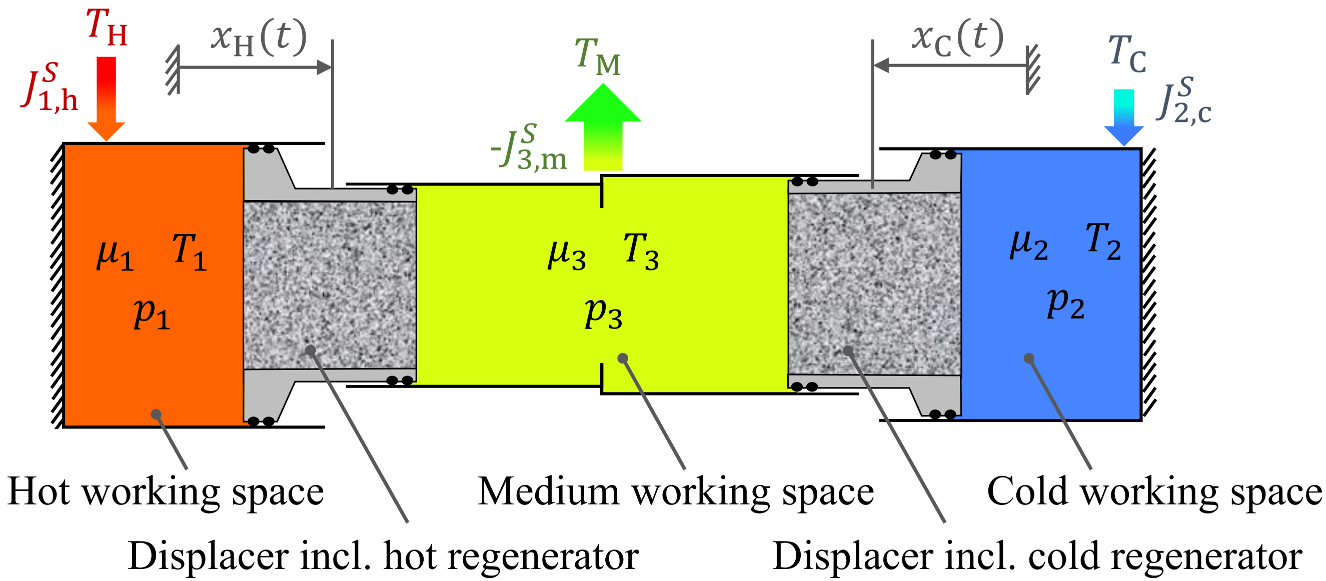
Endoreversible Thermodynamics: Applications

- Hydraulic recuperation systems for trucks (Robin Masser, KHH)



Endoreversible Thermodynamics: Applications

- Vuilleumier refrigerators (Raphael Paul, Abdellah Khodja, Andreas Fischer, KHH)



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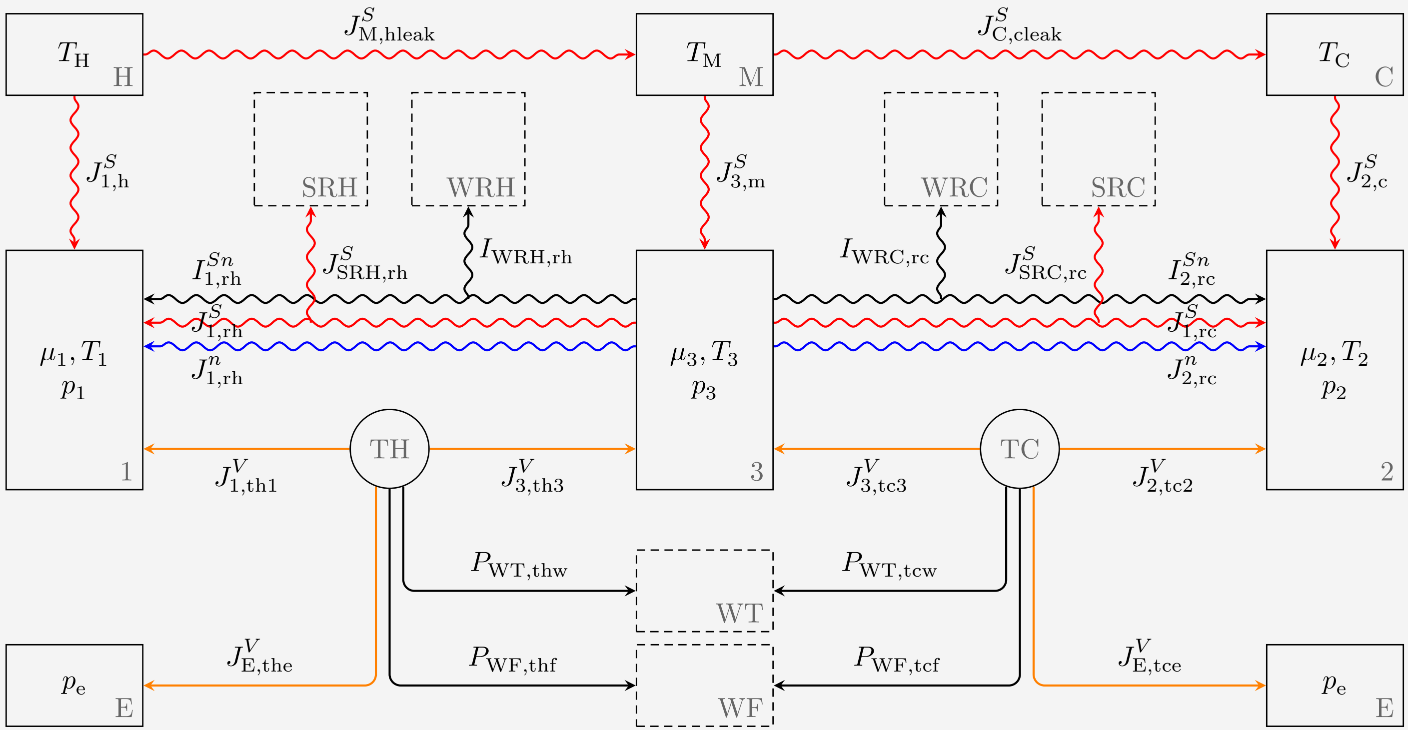

Performance Measures

- Description of systems:
  - transport equations
  - state equations
  - balance equations

known dynamics !!

- Performance measures
  - maximum power
  - efficiency
  - minimum entropy production
  - maximum availability / exergy
  - etc

optimize performance !!

The importance of optimization goals and interactions (transport equations)

in an Endoreversible Thermodynamics description

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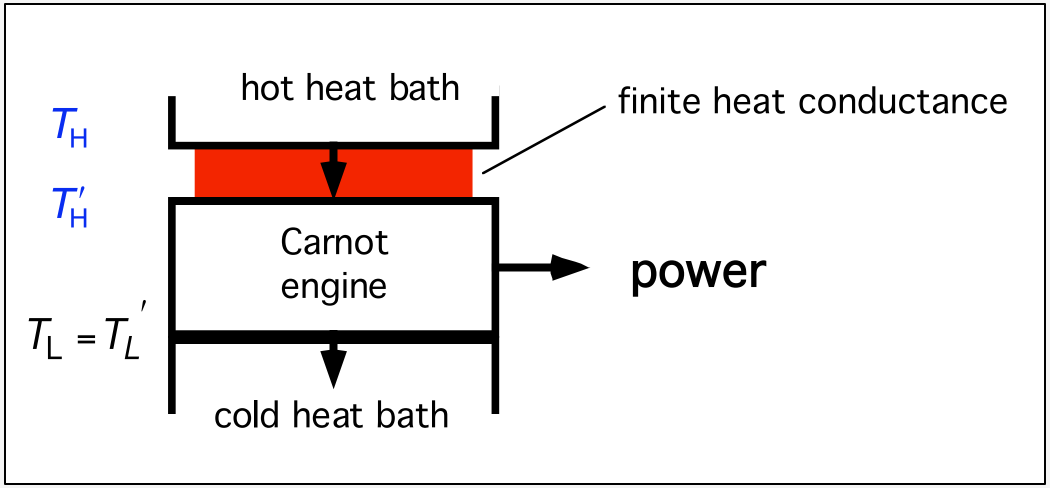
Maximum Power Processes

- An irreversible power station model: The Novikov Engine



- Newtonian heat conduction

$$\eta_N = 1 - \sqrt{\frac{T_L}{T_H}}$$

- Fourier heat conduction

$$q = \kappa\left(\frac{1}{T} - \frac{1}{T'}\right)$$

$$\eta_F = \frac{1}{2}\left(1 - \frac{T_L}{T_H}\right) = \frac{1}{2}\eta_{Carnot}$$

Maximum Power Processes

- Efficiency at maximum power

- Application to real power stations

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Maximum Power vs. Minimum Entropy Production

- For some people, these are equivalent optimization goals. Are they?

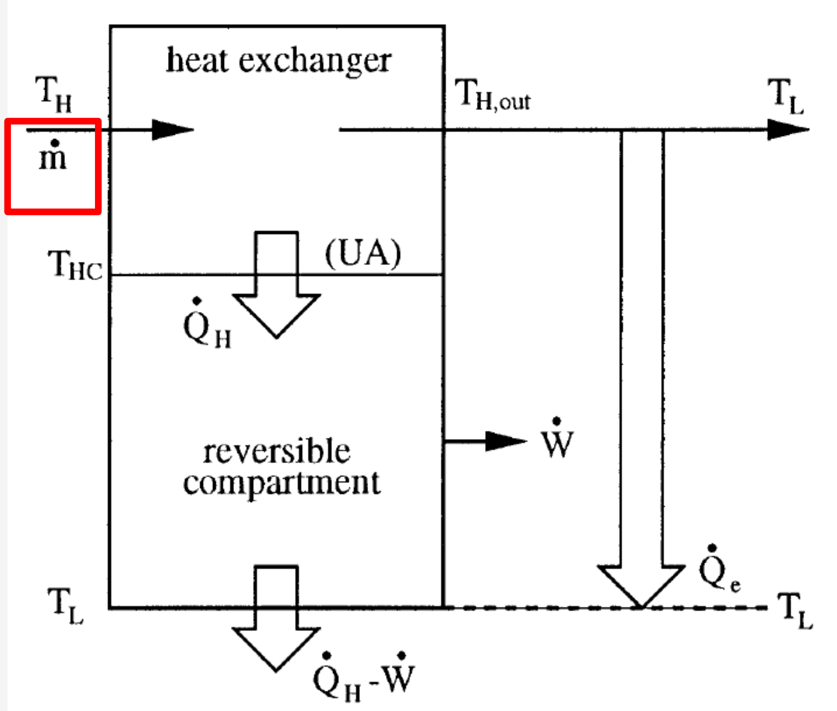
What Conditions Make Minimum Entropy Production Equivalent to Maximum Power Production?

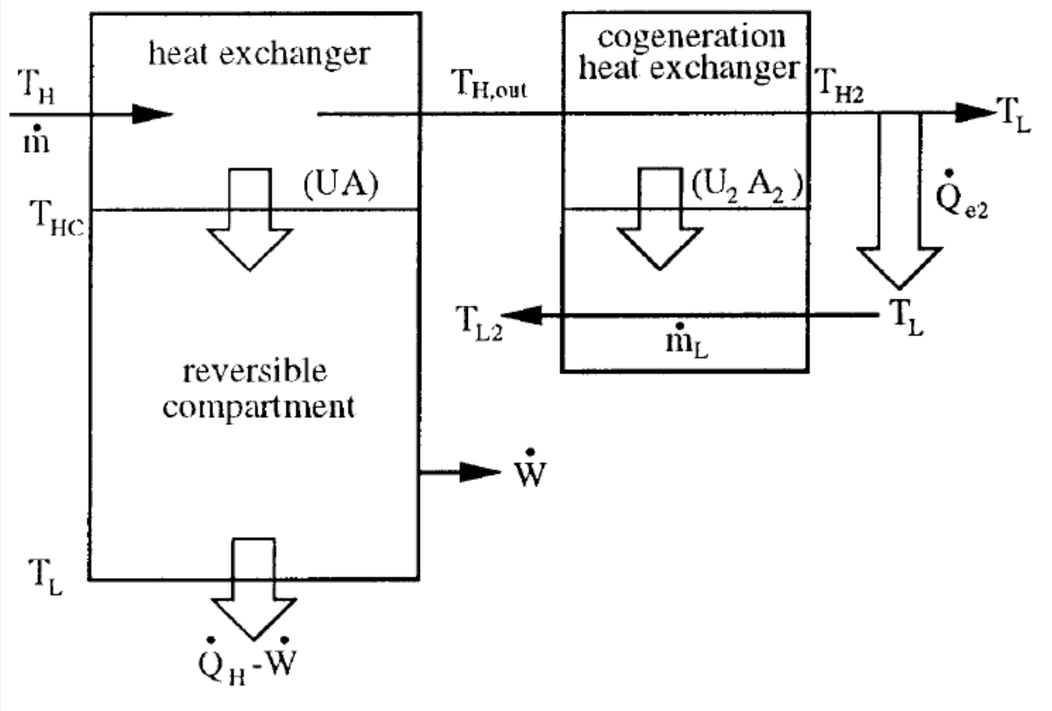
Peter Salamon[1], Karl Heinz Hoffmann[2], Sven Schubert[2], R. Stephen Berry[3], Bjarne Andresen[4]
[1]Department of Mathematical Sciences, San Diego State University, San Diego, California, USA
[2]Institut für Physik, Technische Universität Chemnitz, Chemnitz, Germany
[3]Department of Chemistry and the James Frank Institute, The University of Chicago, Chicago, Illinois, USA
[4]Ørsted Laboratory, University of Copenhagen, Copenhagen, Denmark

Journal of Non-Equilibrium Thermodynamics 26, 73 (2001)

Maximum Power vs. Minimum Entropy Production

- It depends !! For instance on
  - whether the heat input is variable or
  - whether unused heat is degraded to the environment

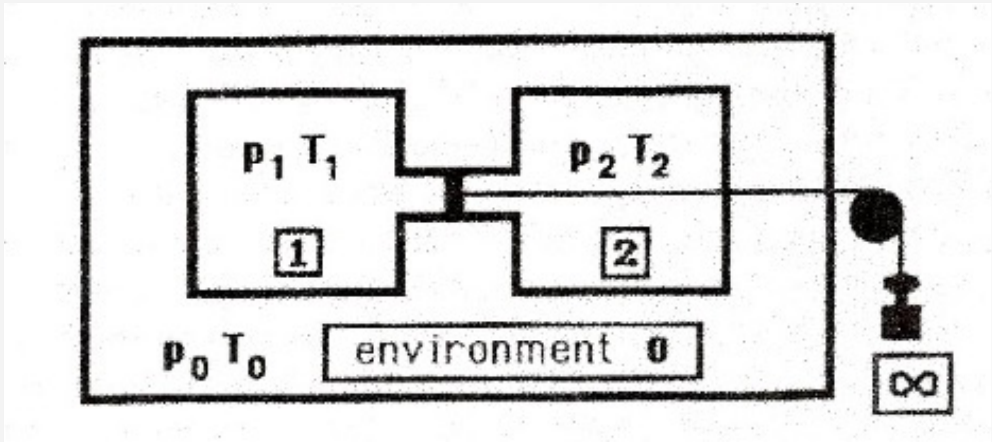




Work Deficiency

- Measures of Dissipation (KHH, Bjarne Andresen, Peter Salamon, 1989)
- Availability: always with respect to the environment
- Entropy production: only determined once the heat is deposited
- Work Deficiency:
  - Work disspated to heat
  - But where does the heat go to?

$$\mathrm{d}W^{\mathrm{d}} = \frac{1}{2}\sum_{i,j}(\mathbf{y_i} - \mathbf{y_j})\mathrm{d}\mathbf{X}_{ij}$$



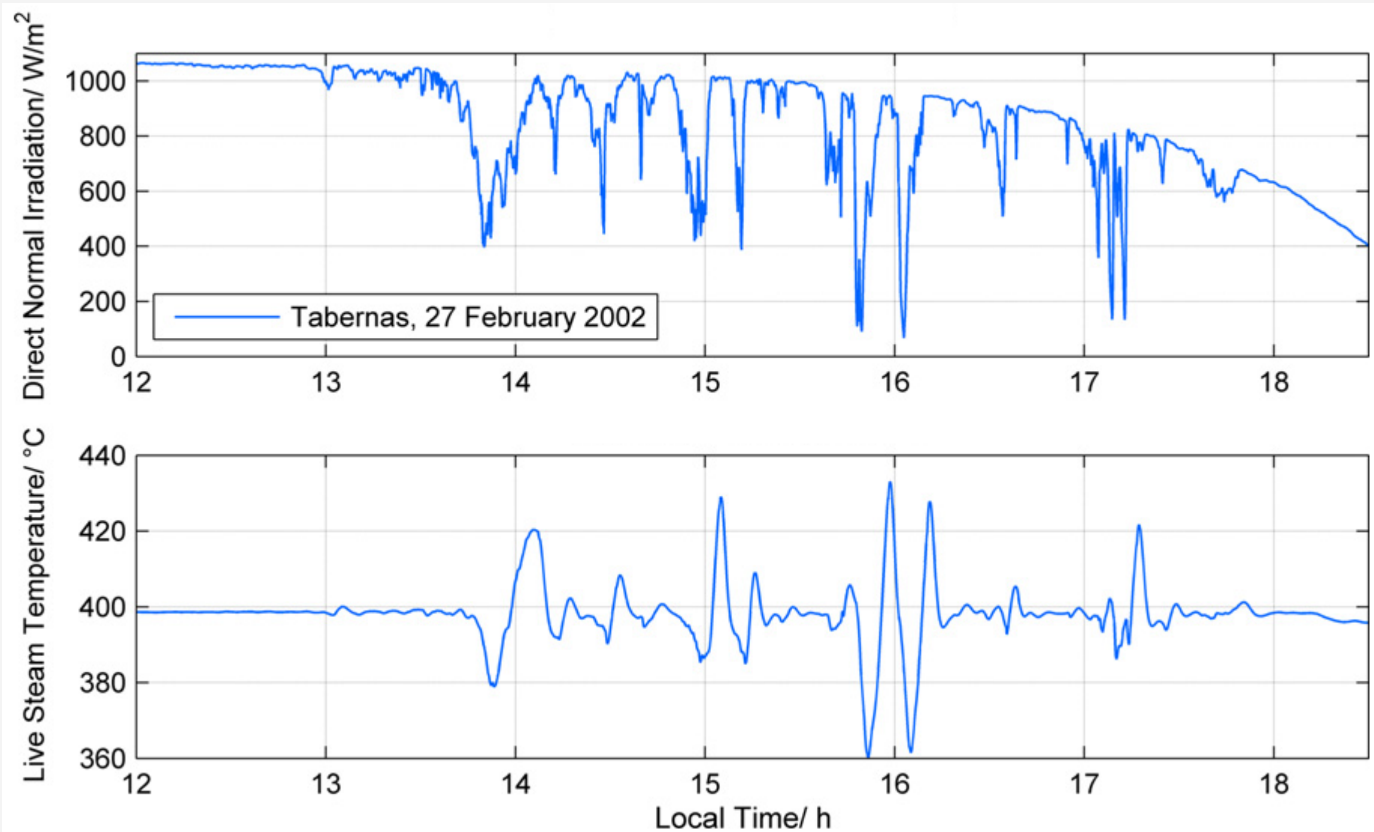
Maximum Power Processes: Solar Power Stations

Solar power station in Tabernas (Spain) (Karsten Schwalbe, KHH, 2018)

www.dlr.de/sf/en/Portaldata/73/Resources/images/psa/DLR2007-516_PSA_Panrorama_1.78.jpg [Jan 6, 2017]

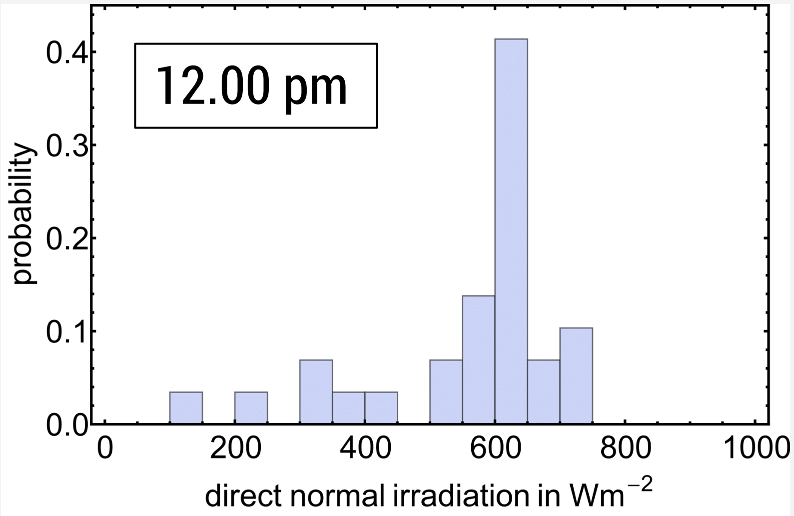
Maximum Power Processes

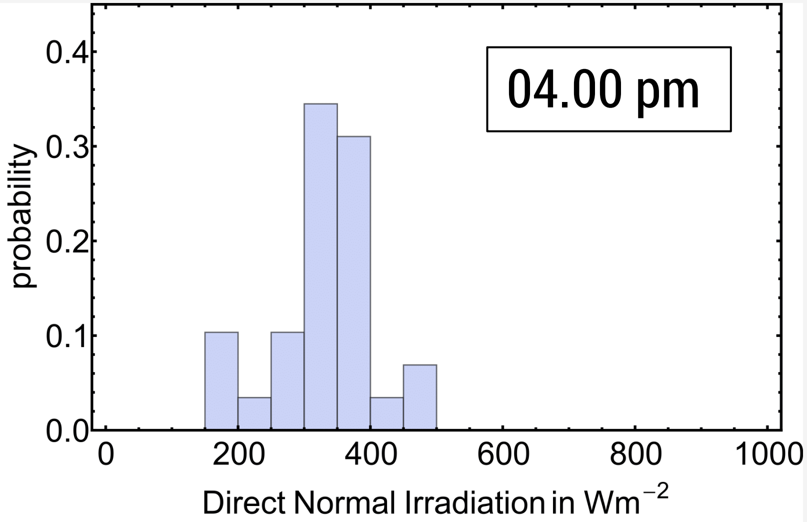
Relation between irradiation and steam temperature



www.dlr.de/sf/en/Portaldata/73/Resources/images/psa/DLR2007-516_PSA_Panrorama_1.78.jpg [Jan 6, 2017]

Time-dependent Distributions of Irradiation





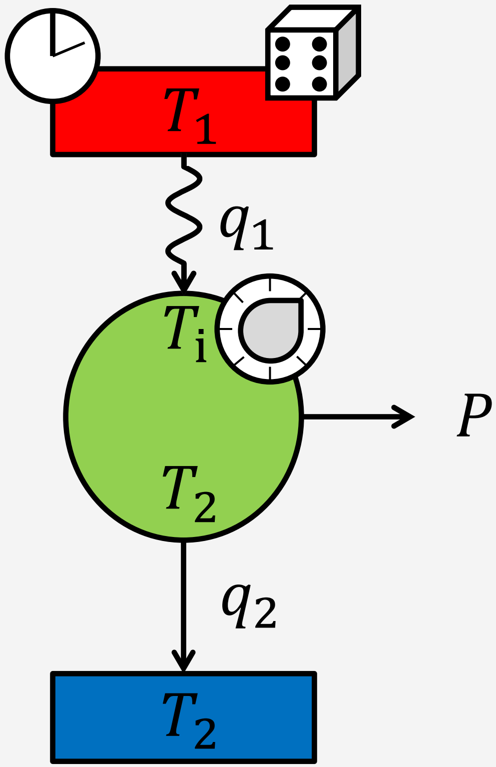
Data for Tabernas in February 2004, taken from http://www.soda-pro.com/web-services/radiation/helioclim-3-for-free [Jan 6, 2017]

Stochastic fluctuations:
- Time dependent distributions
- Relevant for power stations

→ ?

Performance measures:
- Work output W
- Efficiency η

Fluctuating Novikov Engine



Assumption: $\rho(T_1, t)$ given

Performance measures:

$$W[T_i(T_1,t)] = \tau\langle\bar{P}\rangle \to \max$$

$$Q_1[T_i(T_1,t)] = \tau\langle\bar{q}_1\rangle$$

$$\eta = \frac{W}{Q_1}$$

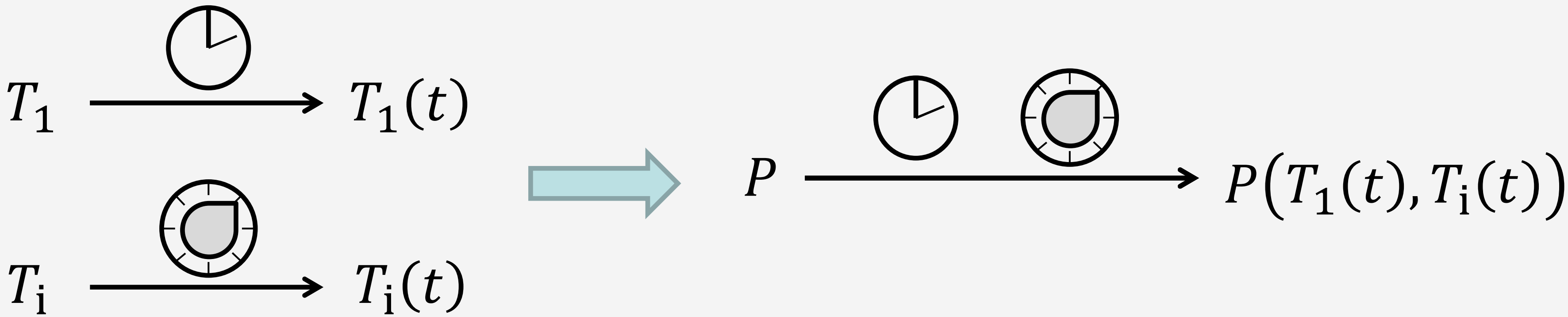
Notation:

$$\langle g\rangle = \int_0^\infty g(T_1)\rho(T_1,t)\,\mathrm{d}T_1$$

$$\bar{g} = \frac{1}{\tau}\int_0^\tau g\,(t)\mathrm{d}t$$

Control Theory

Solar tower example:



General:

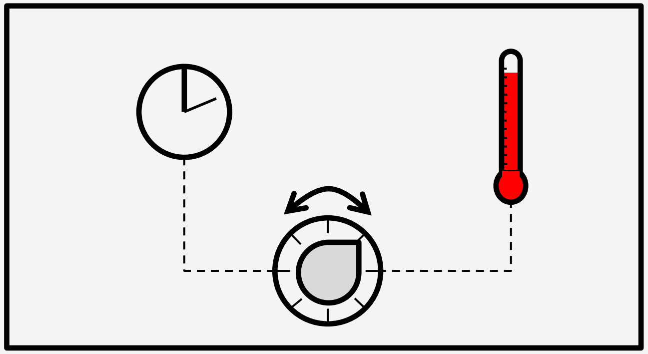
$T_1(t) \longrightarrow$ state $x(t)$

$P\big(T_1(t), T_\mathrm{i}(t)\big) \longrightarrow$ path benefits $\xi(u, x, t)$

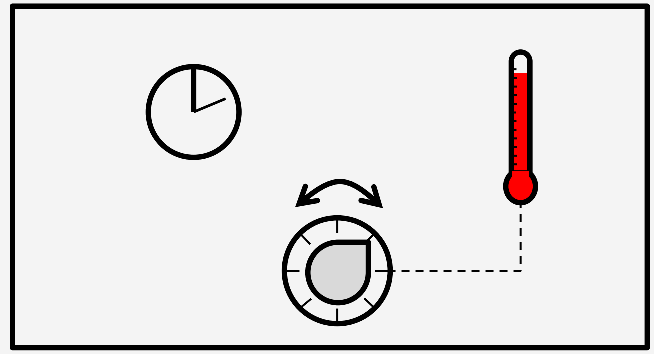
$T_\mathrm{i}(t) \longrightarrow$ control $u(t)$

$W \longrightarrow$ benefit function J

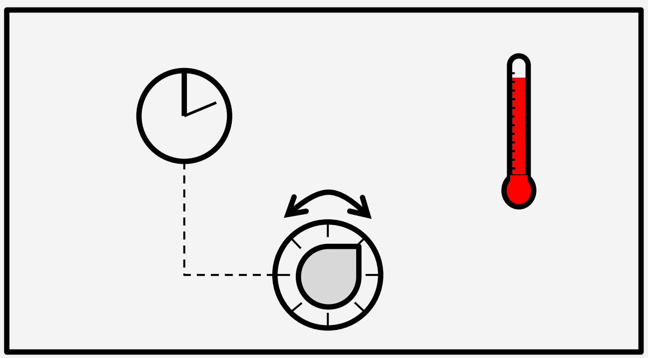
Control Types



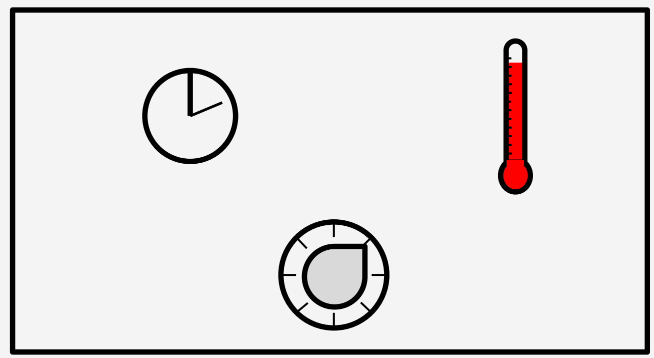
Feedback control $u(x,t)$



State dep. control $u(x)$



Time dep. control $u(t)$



Single control u^0

Fluctuating Novikov Engine

	Single control	Time dep. control	Feedback control
$\hat{T}_\mathrm{i}$	$\hat{T}_\mathrm{i}^0 = \sqrt{T_2 \overline{\langle T_1 \rangle}}$	$\hat{T}_\mathrm{i}\ (t) = \sqrt{T_2 \langle T_1 \rangle (t)}$	$\hat{T}_\mathrm{i}\ (t, T_1) = \sqrt{T_2 T_1}$
$\frac{\widehat{W}}{\kappa\tau}$	$\overline{\langle T_1 \rangle} - 2\sqrt{T_2}\sqrt{\overline{\langle T_1 \rangle}} + T_2$ $\le$	$\overline{\langle T_1 \rangle} - 2\sqrt{T_2}\,\overline{\sqrt{\langle T_1 \rangle}} + T_2$ $\le$	$\overline{\langle T_1 \rangle} - 2\sqrt{T_2}\,\overline{\langle\sqrt{T_1}\rangle} + T_2$
$\hat{\eta}$	$1 - \sqrt{\frac{T_2}{\overline{\langle T_1 \rangle}}}$	$1 - \frac{\sqrt{T_2}\,\overline{\sqrt{\langle T_1 \rangle}} - T_2}{\overline{\langle T_1 \rangle} - \sqrt{T_2}\,\overline{\sqrt{\langle T_1 \rangle}}}$	$1 - \frac{\sqrt{T_2}\,\overline{\langle\sqrt{T_1}\rangle} - T_2}{\overline{\langle T_1 \rangle} - \sqrt{T_2}\,\overline{\langle\sqrt{T_1}\rangle}}$

Fluctuating Novikov Engine

Taylor expansion of $\hat{\eta}$ for $\sigma \ll \langle T_1 \rangle$

For three investigated symmetric distributions:

$$\hat{\eta} = 1 - \sqrt{\frac{T_2}{\langle T_1 \rangle}} + \frac{1}{8}\sqrt{\frac{T_2}{\langle T_1 \rangle}} \frac{\left(1 + \sqrt{\frac{T_2}{\langle T_1 \rangle}}\right)}{\left(1 - \sqrt{\frac{T_2}{\langle T_1 \rangle}}\right)} \left(\frac{\sigma}{\langle T_1 \rangle}\right)^2 + O\left(\left(\frac{\sigma}{\langle T_1 \rangle}\right)^4\right)$$

→ Extension of the Curzon-Ahlborn efficiency

Several Heat Baths connected to One Engine

one heat engine

operating at MAX power

connected to

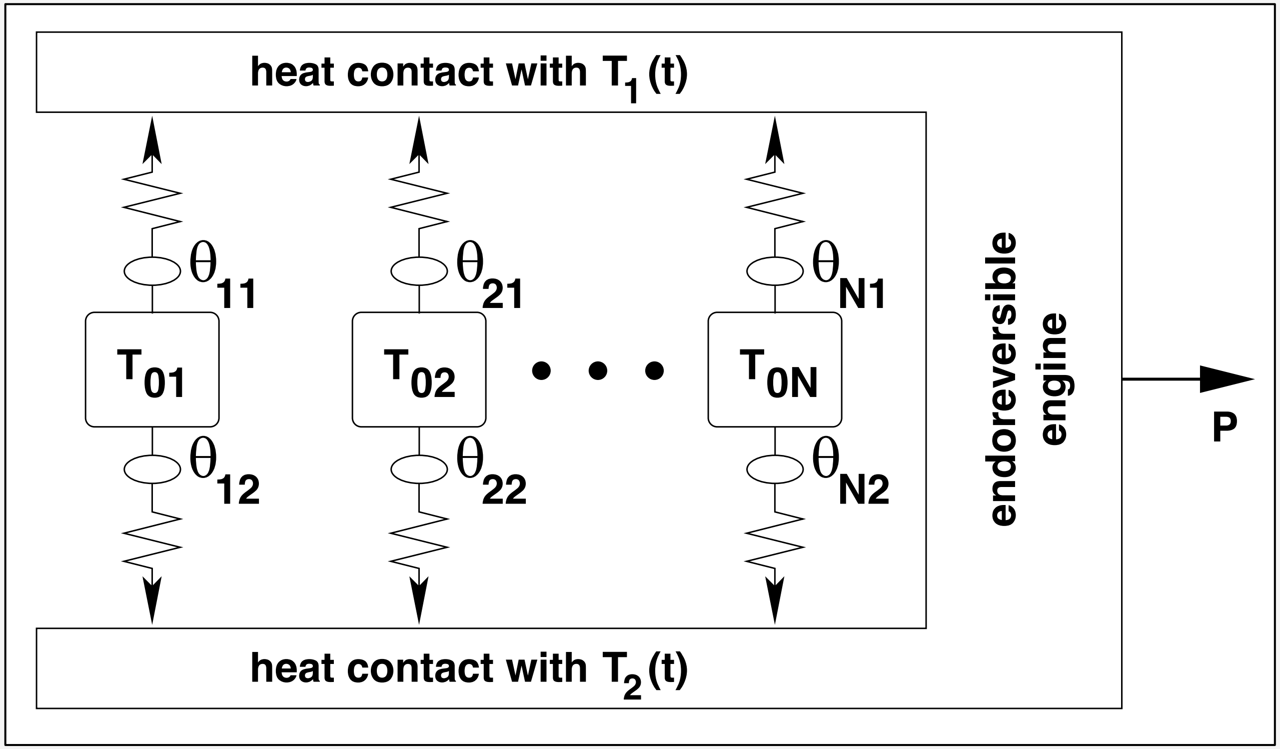
several heat baths

which

can be switched to

the hot or the cold

engine contact

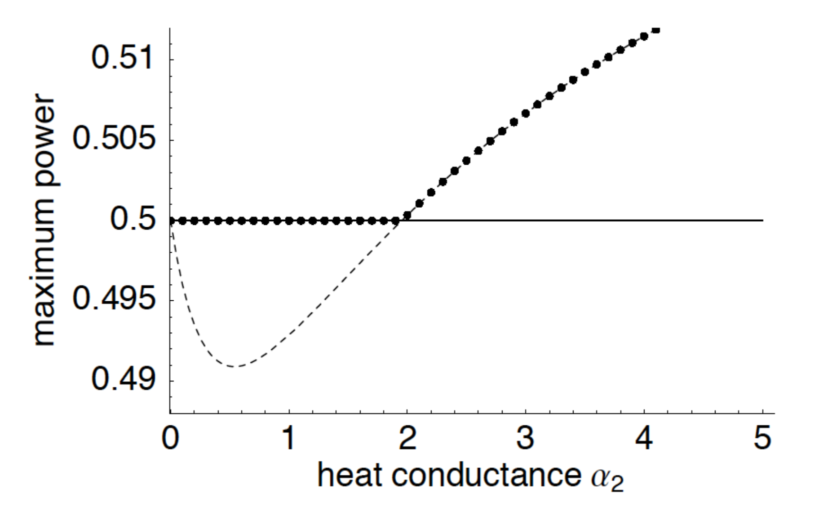


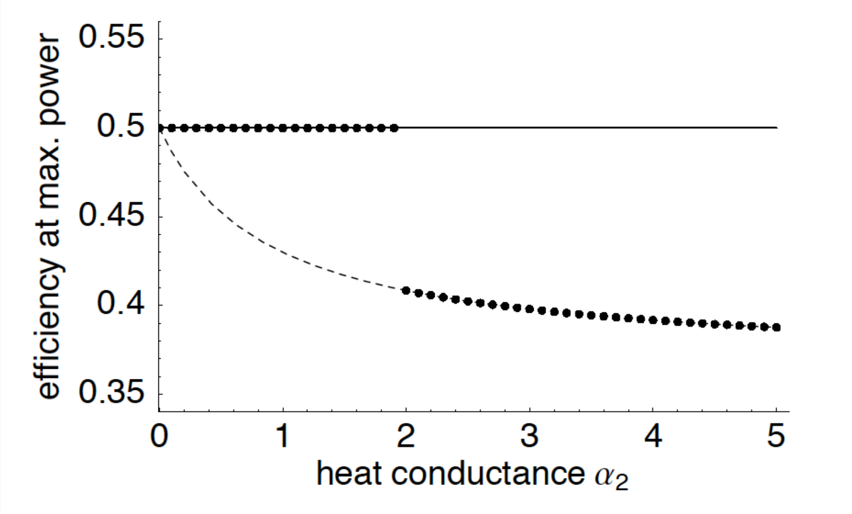
S. Amelkin, B. Andresen, J. Burzler, KHH, A. Tsirlin (2004/2005)

Several Heat Baths connected to One Engine

Example: three heat baths $T_1 = 1$ $T_2 = 1.6$ $T_3 = 4$

Newtonian heat conduction: $q_i = \alpha_i \, (T_{0i} - T)$





New phenomenon: UNUSED reservoirs

Several Heat Baths connected to One Engine

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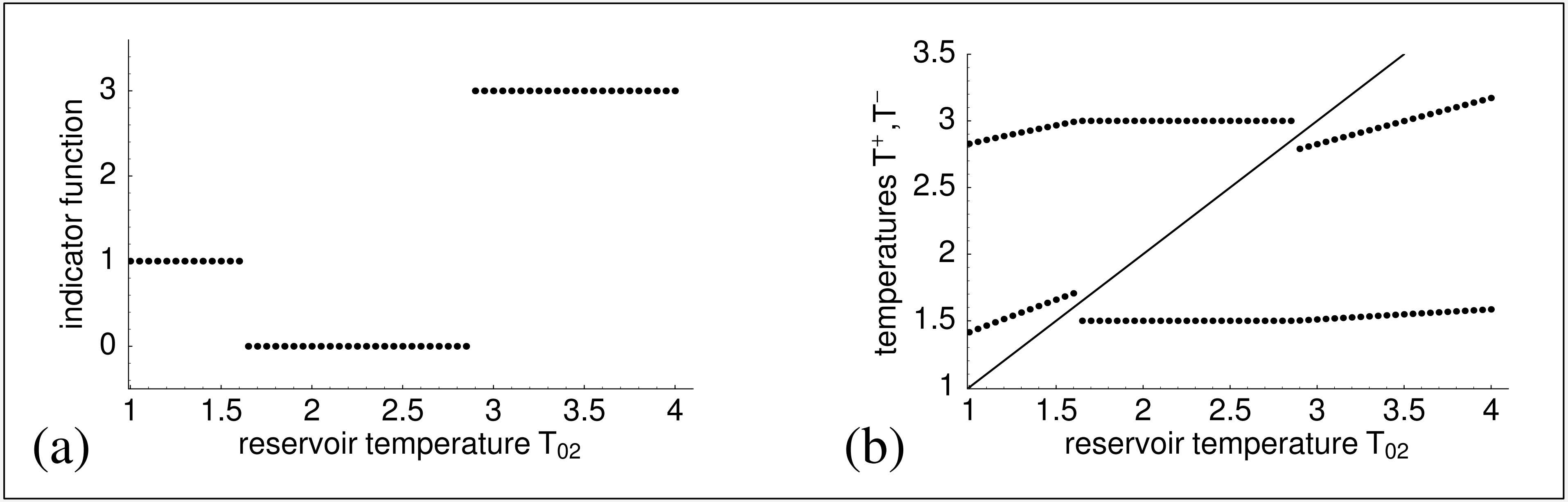

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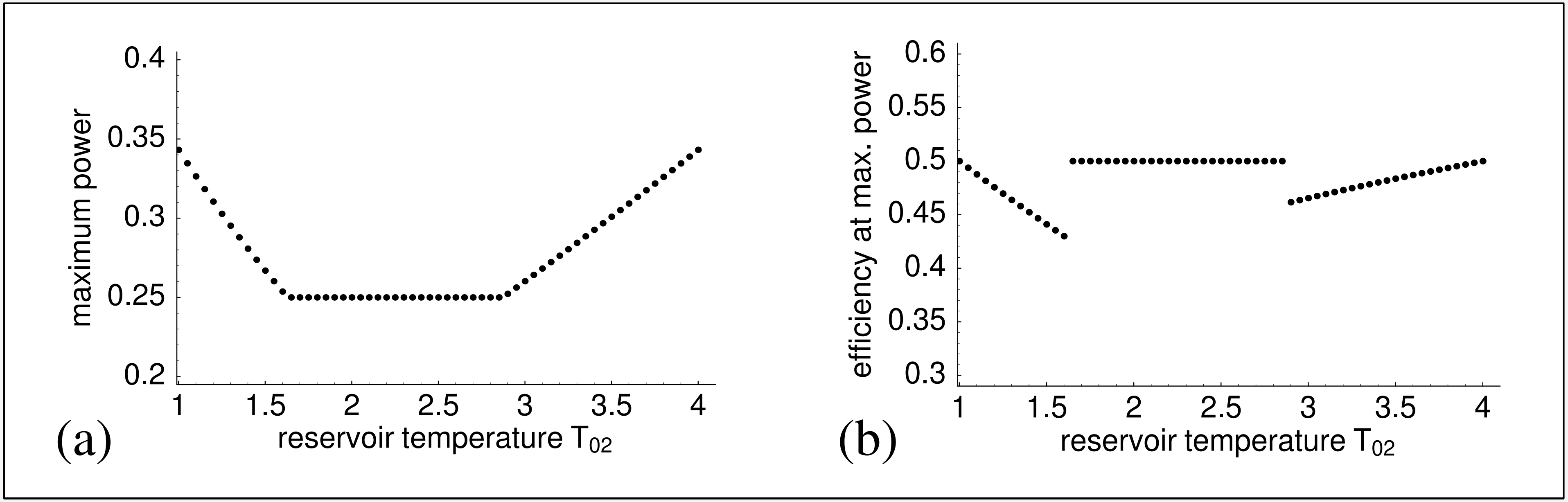

New phenomenon: UNUSED reservoirs

Summary

- Quantifying the entropy production for finite-time processes
- Endoreversible Thermodynamics
  - Reversible subsystems
  - Irreversible interactions
  - A hierarchy of models
- Measures of Dissipation
- Application: Stochastic Novikov Engine
- Application: Unused Reservoirs

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