Quantum finite-time thermodynamics

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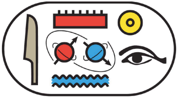


**International Symposium on Finite-Time Thermodynamics –
Past, Present, and Future**

May 24, 2022

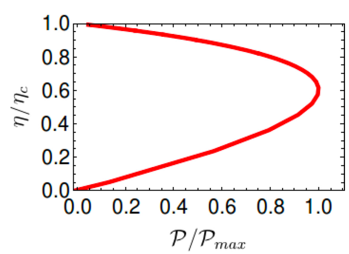
Copenhagen when ?

Finite time thermodynamics

Inserting Dynamics into Thermodynamics

Power or efficiency?

$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}}$$



$$\eta_c = 1 - \frac{T_c}{T_h}$$

Efficiency at maximum power

$\Delta S^u > 0$

Maximum efficiency

$\Delta S^u = 0$

Quantum Finite Time Thermodynamics: Motivation

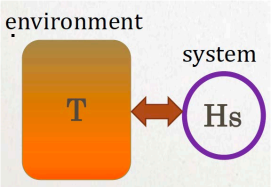
First Principle derivation of basic building blocks.

1. Heat flow.
2. Friction.
3. Heat leaks.

Methodology: Learn from example:

Study quantum heat engines and refrigerators.

The theory of open quantum systems :



Reduced description

$$\hat{H} = \hat{H}_S + \hat{H}_B + \hat{H}_{SB}$$

Separation of time scales

If the bath timescale is much faster than the system then:

$$\frac{d}{dt}\hat{\rho}_S = \mathscr{L}_S \hat{\rho}_S$$

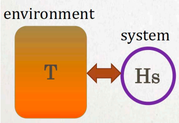
where $\mathscr{L}_S$ depends on the bath implicitly.

Dynamical equations consistent with Thermodynamics .

Isothermal Partition .

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_S + \hat{\mathbf{H}}_B + \hat{\mathbf{H}}_{SB}$$.

$$[\hat{\mathbf{H}}_{SB}, \hat{\mathbf{H}}] = 0$$



Reduced description: $\Lambda_s \hat{\rho}_S = \sum_j \hat{\mathbf{K}}_j \hat{\rho}_S \hat{\mathbf{K}}_j^{\dagger}$

.

0- law: The fixed point of the map is a Gibbs state:

$\Lambda_s \hat{\rho}_S(eq) = \hat{\rho}_S(eq) = \frac{1}{Z} e^{-\beta H_S}$ where $\beta = 1/kT_B$

.

1-law conservation of energy $dE_S = -dE_B$

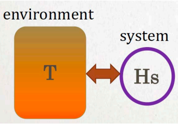
This implies: $[\Lambda, \mathscr{U}_S] = 0$ where $\mathscr{U}_S = e^{-\frac{i}{\hbar}[\hat{\mathbf{H}}_S, \bullet]t}$

.

2-law Contraction:

$$\mathscr{D}(\Lambda(\hat{\rho}_S) | \Lambda(\hat{\rho}_S(eq))) \leq \mathscr{D}(\hat{\rho}_S | \hat{\rho}_S(eq))$$

The GKLS Master Equation



.

$$\frac{d}{dt}\hat{\rho}_S(t) = -i[\hat{\mathbf{H}}_S(t), \hat{\rho}_S] + \mathscr{L}_D(\hat{\rho}_S)$$

$$\mathscr{L}_H(\hat{\rho}_S) = -i[\hat{\mathbf{H}}_S(t), \hat{\rho}_S]$$

$$\mathscr{L}_D(\hat{\rho}_S) = \sum_k \gamma_k \left(\hat{\mathbf{F}}_j \hat{\rho}_S \hat{\mathbf{F}}_j^\dagger - \frac{1}{2}\{\hat{\mathbf{F}}_j^\dagger \hat{\mathbf{F}}_j, \rho_S\} \right) \quad .$$

G. Lindblad

From the **I-law**: $[\mathscr{L}_H, \mathscr{L}_D] = 0$, this implies that $\hat{\mathbf{F}}_k$ are common eigenoperators of $\mathscr{L}_H$ and $\mathscr{L}_D$.

$$\mathscr{L}_H(\hat{\mathbf{F}}_k) = i\omega_k \hat{\mathbf{F}}_k \qquad \mathscr{L}_H(\hat{\mathbf{F}}_k) = \gamma_k \hat{\mathbf{F}}_k$$

when $\omega_k = 0$ $\hat{\mathbf{F}}_k$ is an invariant of the unitary dynamics.

.

For $\gamma_l = 0$ $\hat{\mathbf{F}}_l$ is an invariant a fixed point $\hat{\mathbf{F}}_l = \frac{1}{Z}e^{-\beta \hat{\mathbf{H}}_S}$

for $k \neq l$, $\hat{\mathbf{F}}_k$ are Lindblad jump operators.

Inserting Dynamics into Thermodynamics

Dynamical I-law of thermodynamics

The Heisenberg equations of motion:

$$\frac{d}{dt}\hat{\mathbf{X}} = \frac{i}{\hbar}[\hat{\mathbf{H}}, \hat{\mathbf{X}}] + \mathcal{L}_D(\hat{\mathbf{X}}) + \frac{\partial}{\partial t}\hat{\mathbf{X}}$$

$$\mathcal{L}_D(\hat{\mathbf{X}}) = \sum_n \hat{\mathbf{V}}_n \hat{\mathbf{X}} \hat{\mathbf{V}}_n^* - \frac{1}{2}\{\hat{\mathbf{V}}_n \hat{\mathbf{V}}_n^*, \hat{\mathbf{X}}\}$$

If we choose $\hat{\mathbf{X}} = \hat{\mathbf{H}}$ then:

$$\frac{d}{dt}\mathbf{E} = \langle \frac{\partial}{\partial t}\hat{\mathbf{H}} \rangle + \langle \mathcal{L}_D(\hat{\mathbf{H}}) \rangle$$

$$\frac{d}{dt}\mathbf{E} = \mathcal{P} + \dot{\mathbf{Q}}$$

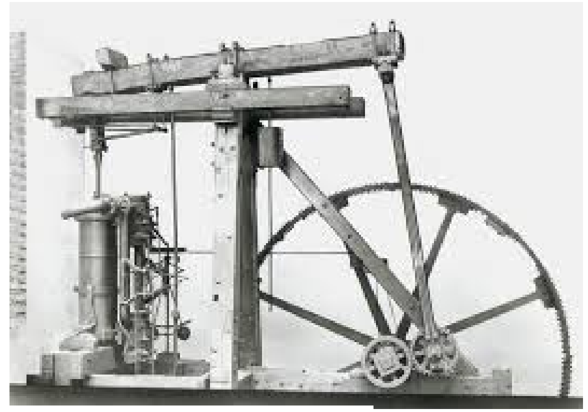
Power + Heat current

R. Alicki , J.phys. A Math. Gen. 12 L103 (1979)

Carnot cycle

1. Hot to cold adiabatic stroke Λ_{hc}
2. Cold isotherm Λ_c
3. Cold to hot adiabatic stroke Λ_{ch}
4. Hot isotherm Λ_h

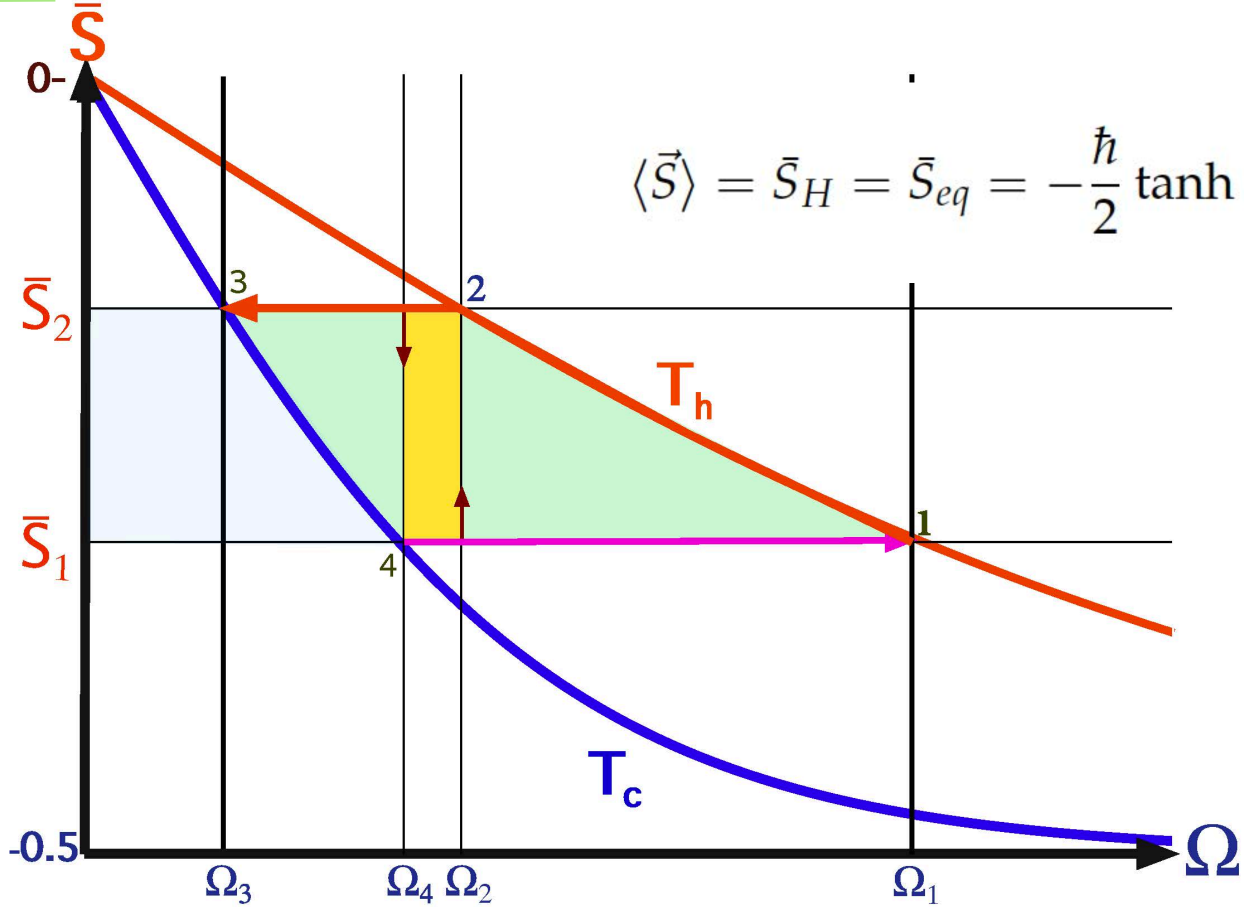
$$\eta_C = 1 - \frac{T_c}{T_h}$$



Carnot cycle:

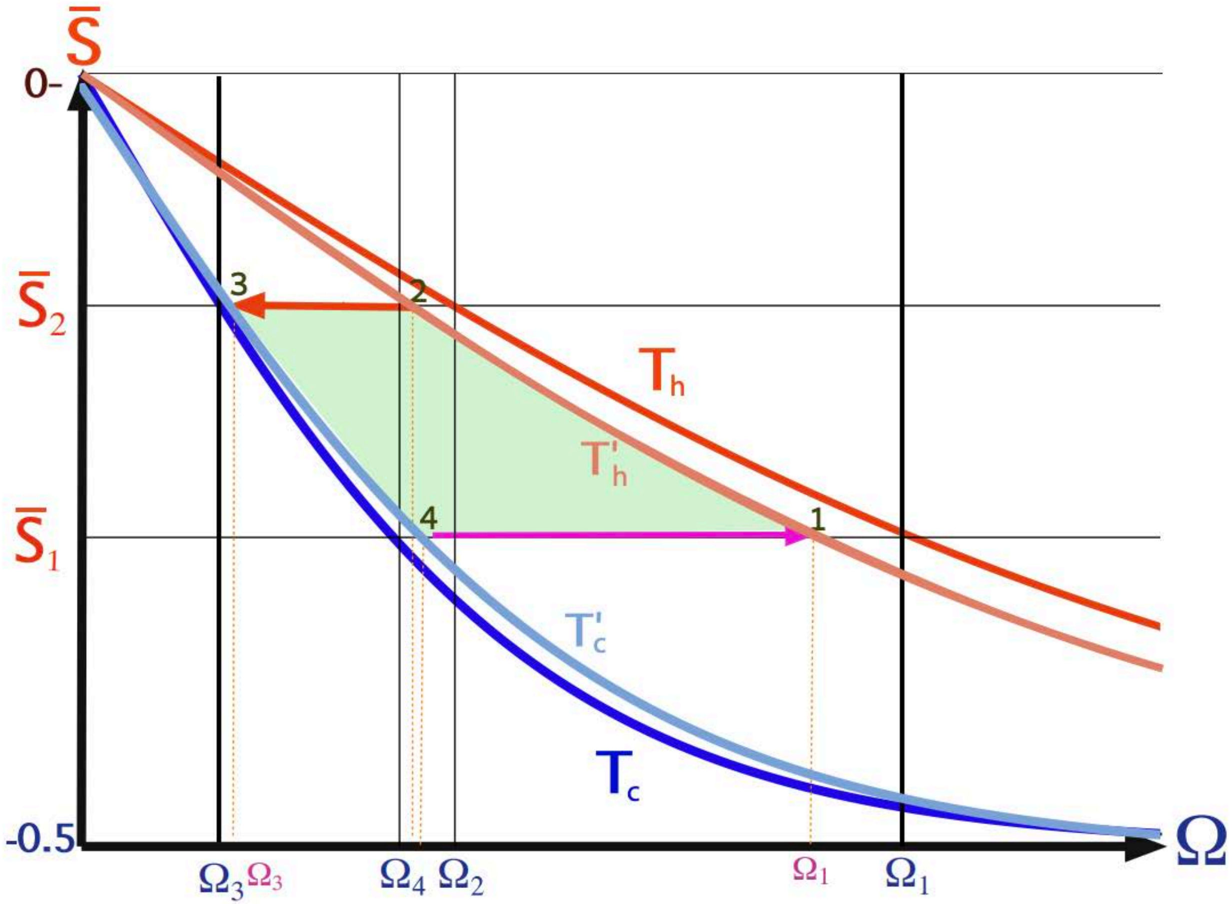
$$\Lambda_{cyc} = \Lambda_h \Lambda_{ch} \Lambda_c \Lambda_{hc}$$

$$\Lambda_{cyc} \hat{\rho}_S = 1 \hat{\rho}_S$$



Operating conditions
fixed point of CPTP map

Endoreversible carnot cycle



$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}} \ ,$$

high temperature limit

Carnot cycle

A quantum-mechanical heat engine operating in finite time. A model consisting of spin-1/2 systems as the working fluid

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(Received 28 August 1991; accepted 21 October 1991)

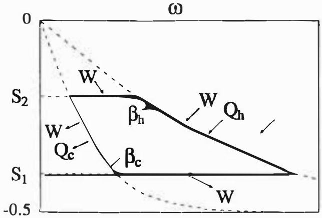


FIG. 1. The reversible Carnotcycle (solid line) in the (ω,S) plane (ω is the field and S the polarization). The cycle is composed of two reversible isotherms corresponding to the temperatures β_h and β_c $(\beta_c > \beta_h)$ and of two adiabats corresponding to the polarizations S_1 and S_2 $(S_1 < S_2;\ S_1, S_2 < 0)$. Positive net work production is obtained by going anticlockwise. The directions of work and heat flows along each branch are indicated.

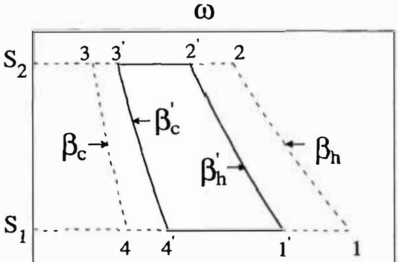


FIG. 5. The cycle $1' \to 2' \to 3' \to 4' \to 1'$ is of the Curzon–Ahlborn

Heisenberg picture, reads as follows:

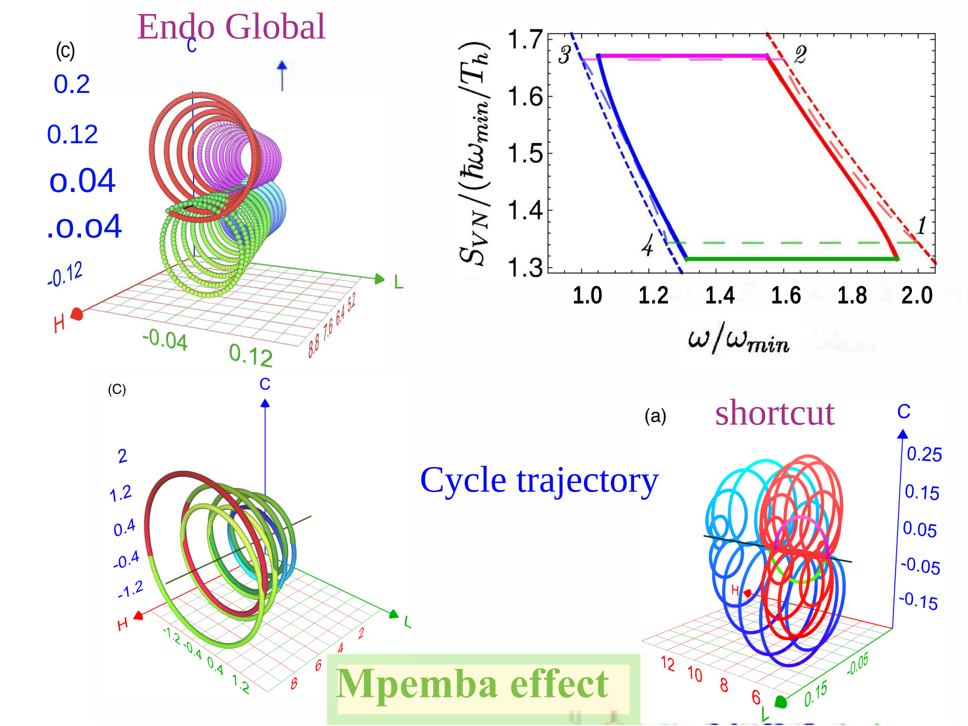
$$\dot{\mathbf{X}} = i[\mathbf{H},\mathbf{X}] + \frac{\partial \mathbf{X}}{\partial t} + \mathscr{L}_D(\mathbf{X}),$$

$$\mathscr{L}_D(\mathbf{X}) = \sum_\alpha \gamma_\alpha (\mathbf{V}_\alpha^\dagger [\mathbf{X},\mathbf{V}_\alpha] + [\mathbf{V}_\alpha^\dagger,\mathbf{X}]\mathbf{V}_\alpha).$$

Adiabatic approach

No coherence considered

J. Chem. Phys., Vol. 96, No. 4, 15 February 1992



Endo Global
(c)
C
0.2
0.12
o.04
.o.04
-0.12
L
H
-0.04
0.12
8.8 7.6 6.4 5.2
$S_{VN}/(\hbar\omega_{min}/T_h)$
1.7
1.6
1.5
1.4
1.3
1.0
1.2
1.4
1.6
1.8
2.0
ω/ω_{min}
1
2
3
4
(c)
C
2
1.2
0.4
-0.4
-1.2
H
L
-1.2 -0.4 0.4 1.2
8 6 4 2
Cycle trajectory
(a)
shortcut
C
0.25
0.15
0.05
-0.05
-0.15
H
L
12 10 8 6
0.15 -0.05
Mpemba effect

Moving to Otto

Heat engines in finite time governed by master equations

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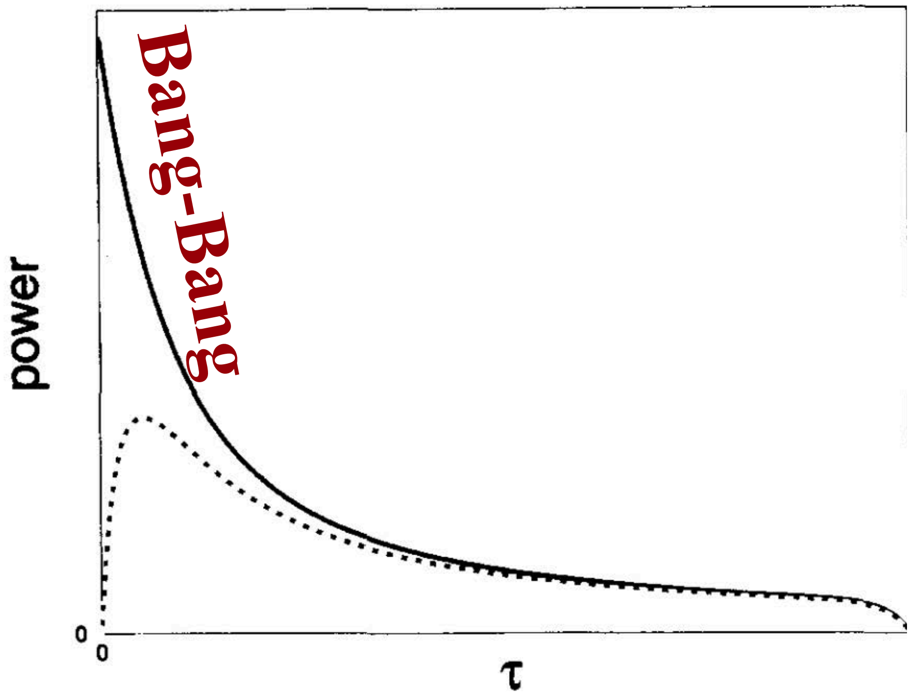
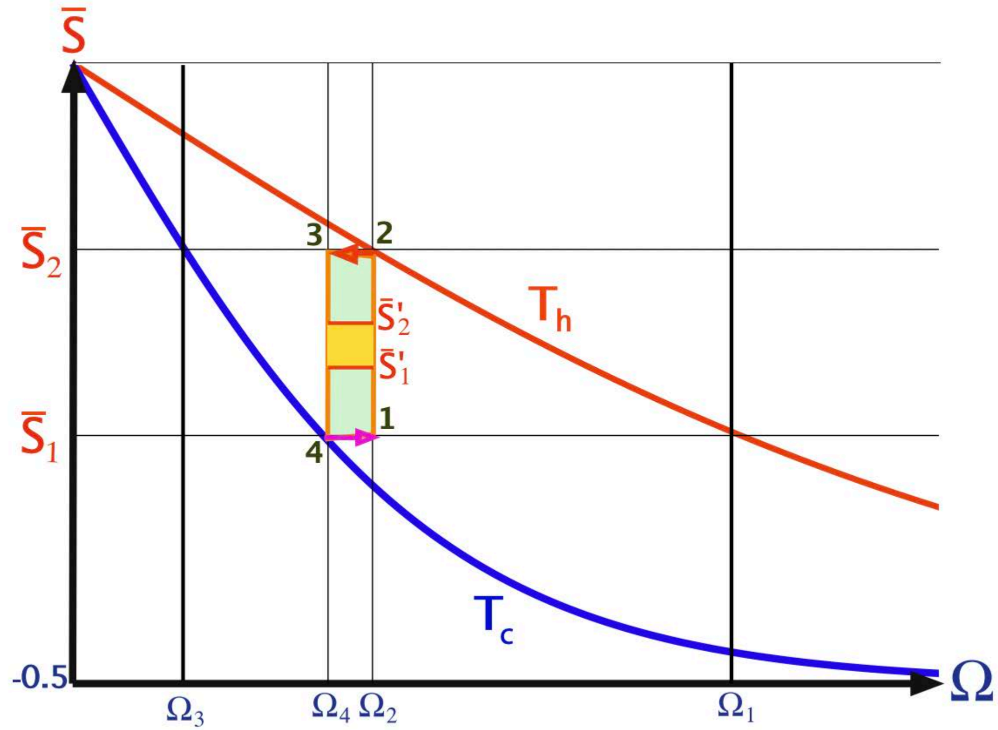
(Received 10 April 1995; accepted 29 June 1995)

$$\eta_{Otto} = 1 - \frac{\Omega_c}{\Omega_h} \ ,$$

$$\mathcal{W}_{Otto} = \Delta\Omega\Delta\bar{S} \ ,$$





RAPID COMMUNICATIONS

PHYSICAL REVIEW E, VOLUME 65, 055102(R)

Discrete four-stroke quantum heat engine exploring the origin of friction

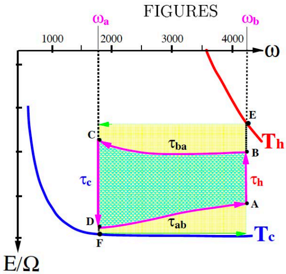
Ronnie Kosloff and Tova Feldmann
Department of Physical Chemistry, The Hebrew University, Jerusalem 91904, Israel
(Received 19 October 2001; published 16 May 2002)

NFWQGA?J PCTGCU C 46* ./4 ./ &0..1'

Os_lrsk dmsp+qrpmic fc_r cleglc8 Rfcpkmbwl_kga m`qcpt_`jcq gl _ kmbcj ugrf glrpglqga dpgargml

Rmt_ Dcjbk_ml _mb Pmllgc Imqjmdd'
Bcn_prkclr mdNfwqga_j Afckgqrpw Rfc Fc`pcu Slgtcpqgrw* Hcplq_jck 7/7.2* Gqp_cj*
yPcacgtcb // Dc`ps_pw 0..19 ns`jgqfcb 1 **July** 0..1'

Quantum origin of Friction



Control Hamiltonian

$$\hat{H}(t) = \hat{H}_{int} + \hat{H}_{cont}(t)$$

As a result:

$$[\hat{H}(t), \hat{H}(t')] \neq 0$$

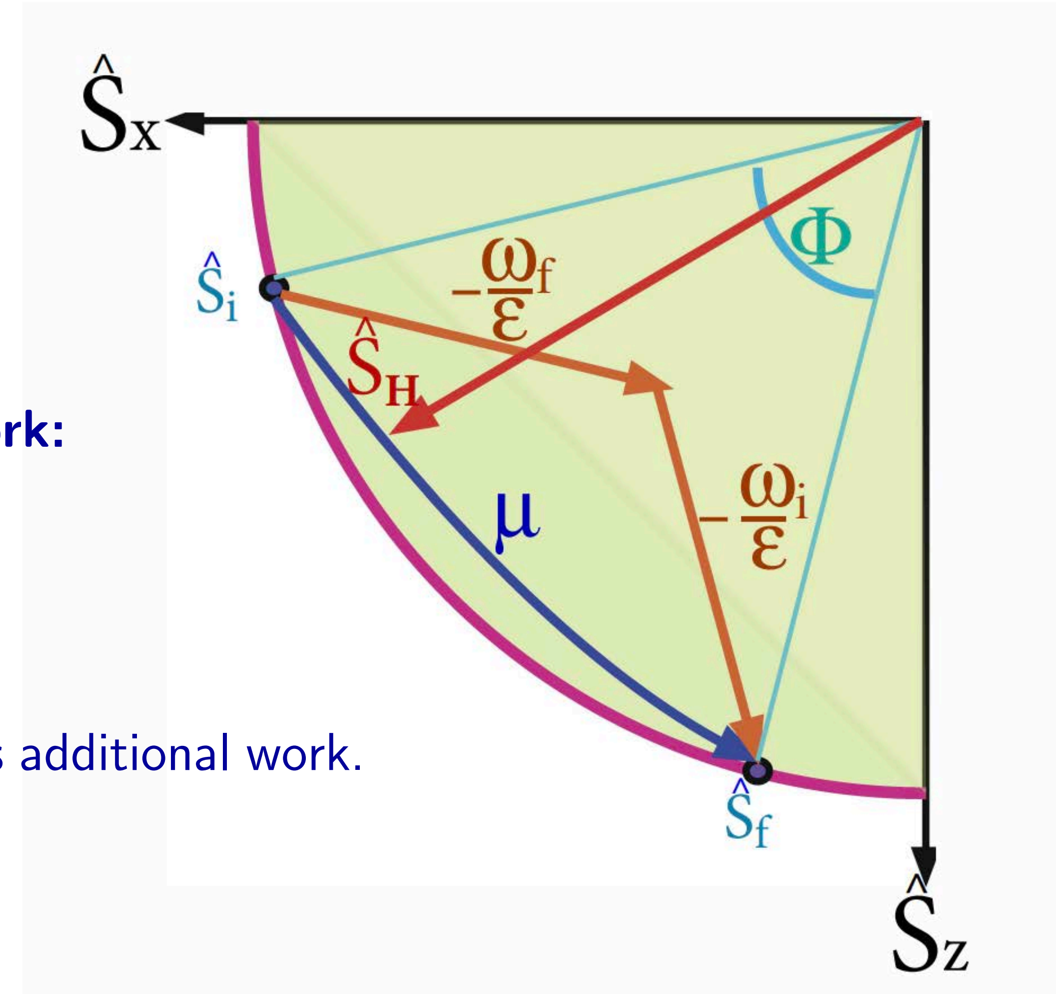
PAPER www.rsc.org/pccp | Physical Chemistry Chemical Physics

Maximum work in minimum time from a conservative quantum system

Peter Salamon,*[a] Karl Heinz Hoffmann,[b] Yair Rezek[c] and Ronnie Kosloff[c]

Received 15th September 2008, Accepted 30th October 2008
First published as an Advance Article on the web 18th December 2008
DOI: 10.1039/b816102j

Generating coherence cost work



We can reveres the process and recover the work: Unitary Shortcuts.

Lubrication:

Also possible when subject to dissipation but requires additional work.

PHYSICAL REVIEW E **97**, 062153 (2018)

Quantum heat engines: Limit cycles and exceptional points

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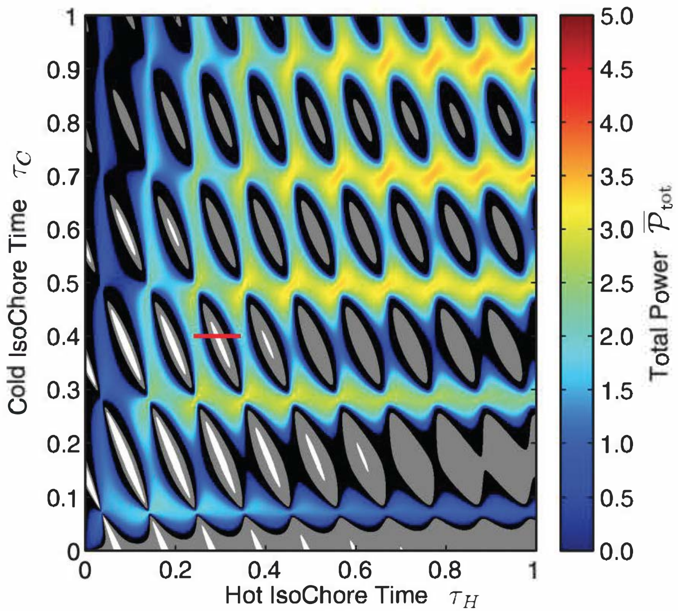
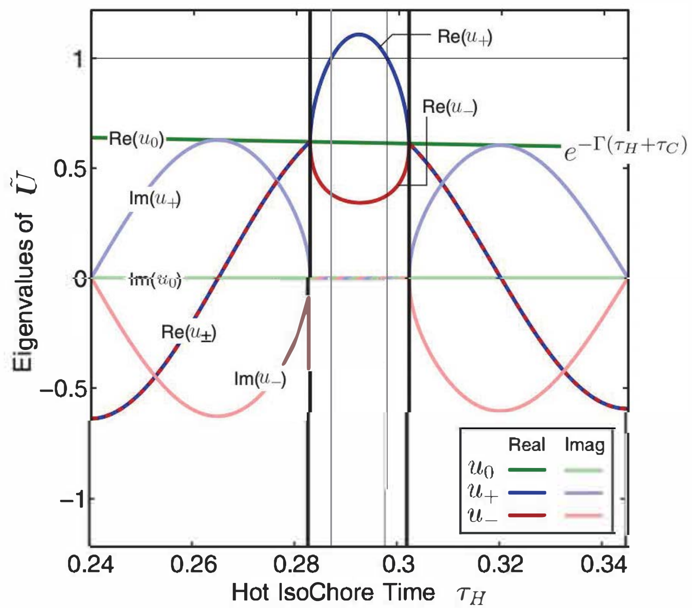
Institute of Chemistry, The Hebrew University, Jerusalem 91904, Israel

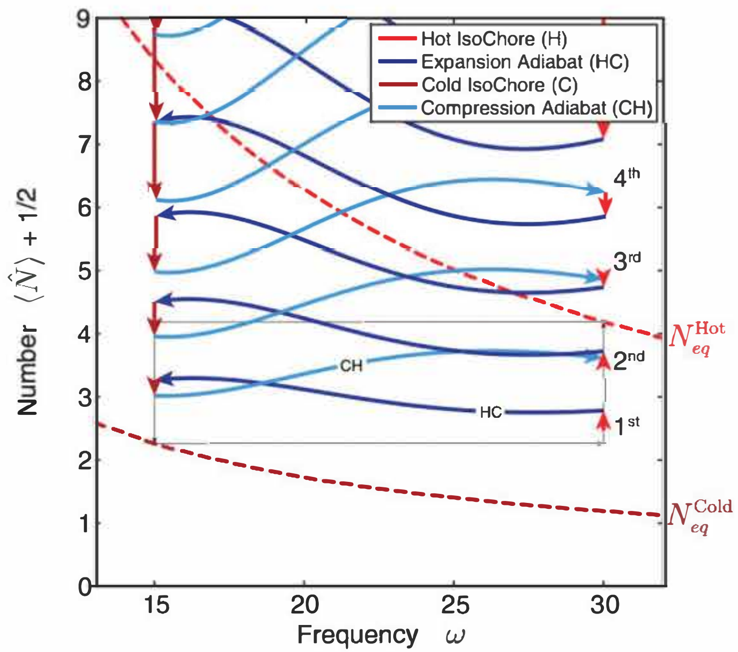
The spectrum of the propagator

$$\Lambda_{cyc} = \mathcal{U}_{hc} \cdot \Lambda_c \cdot \mathcal{U}_{ch} \cdot \Lambda_h$$

$$\Lambda_{cyc} \hat{\mathbf{X}} = \lambda \hat{\mathbf{X}}$$





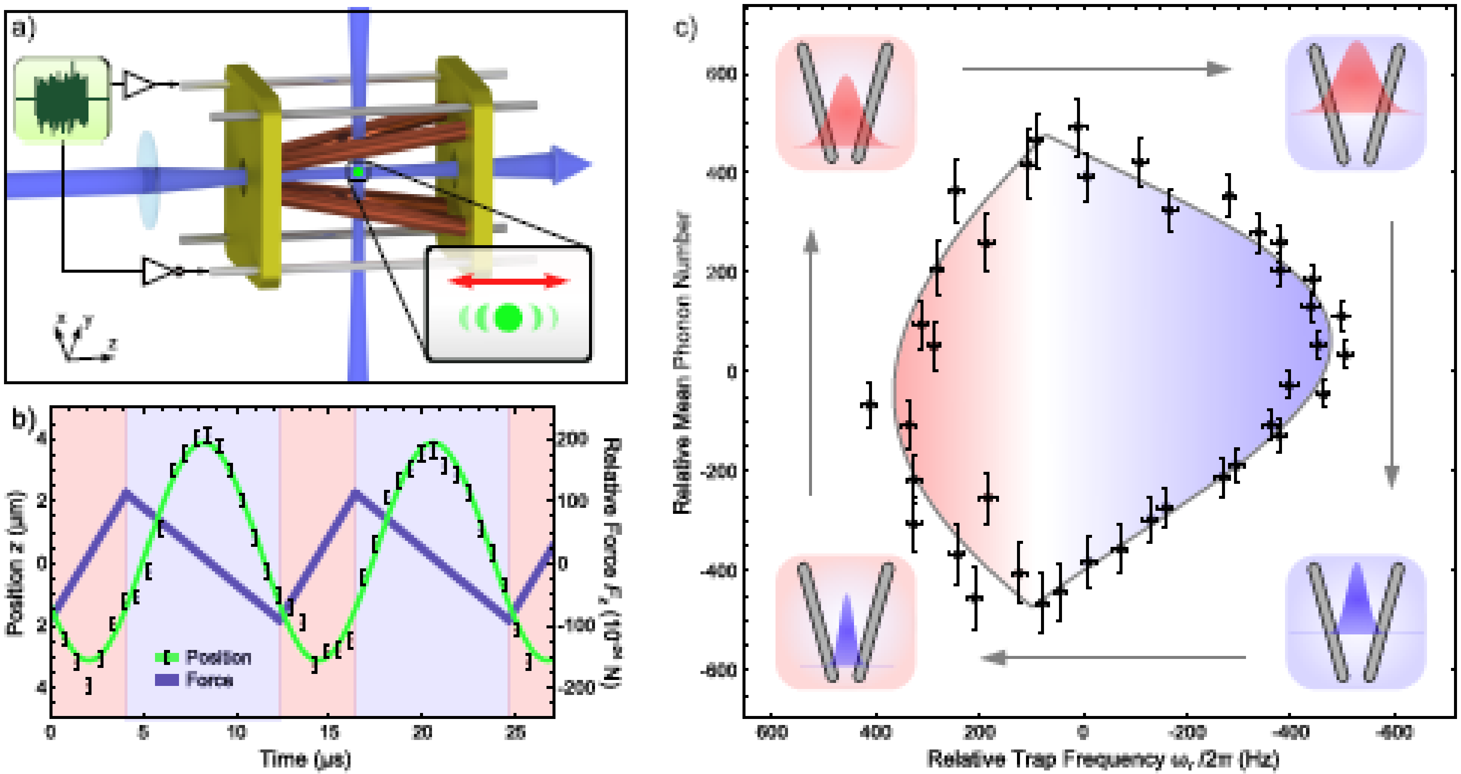


Realizations 2016

Science 352 , 325 (2016)

A single-atom heat engine

Johannes Roßnagel,[1,*] Samuel Thomas Dawkins,[1] Karl Nicolas Tolazzi,[1]
Obinna Abah,[2] Eric Lutz,[2] Ferdinand Schmidt-Kaler,[1] and Kilian Singer[1,3]



PHYSICAL REVIEW E, VOLUME 64, 056130

Heat leak

Quantum thermodynamic cooling cycle

José P. Palao* and Ronnie Kosloff
Department of Physical Chemistry and the Fritz Haber Research Center for Molecular Dynamics, Hebrew University, Jerusalem 91904, Israel

Jeffrey M. Gordon
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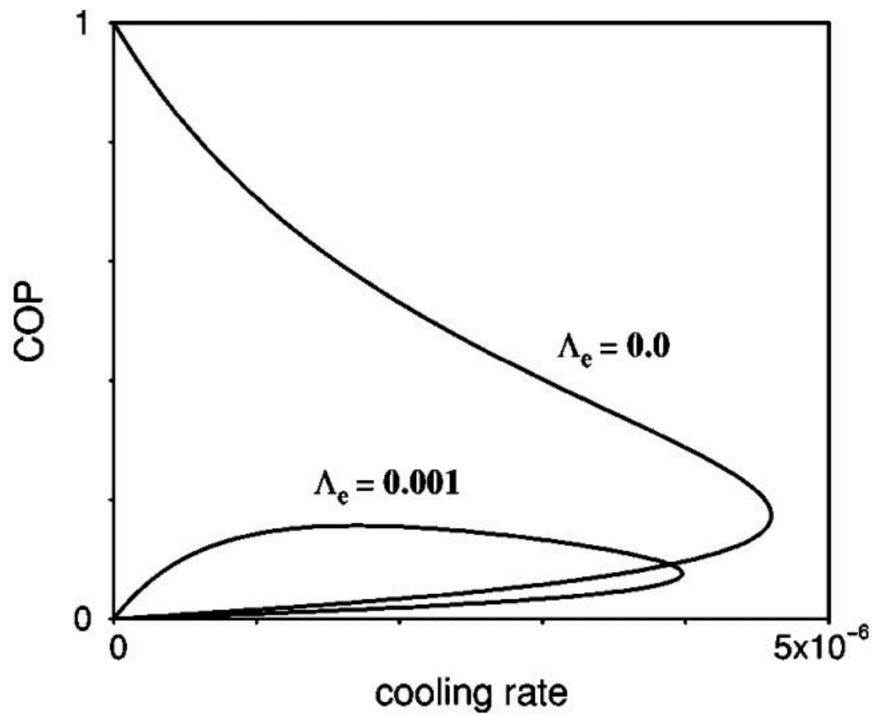
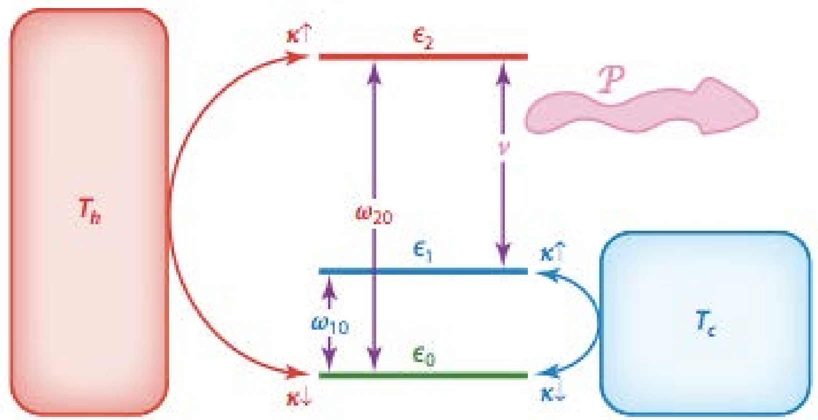




Quantum absorption refrigerator

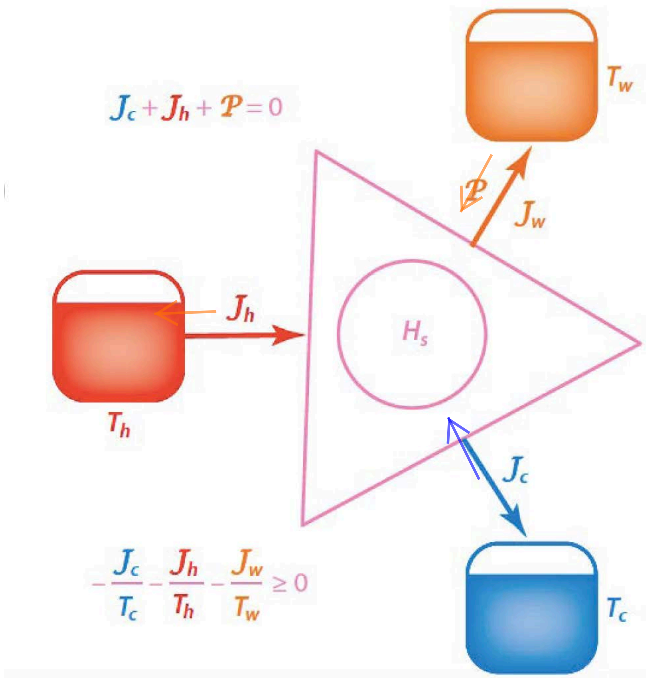
Absorption refrigerator Using heat to cool!

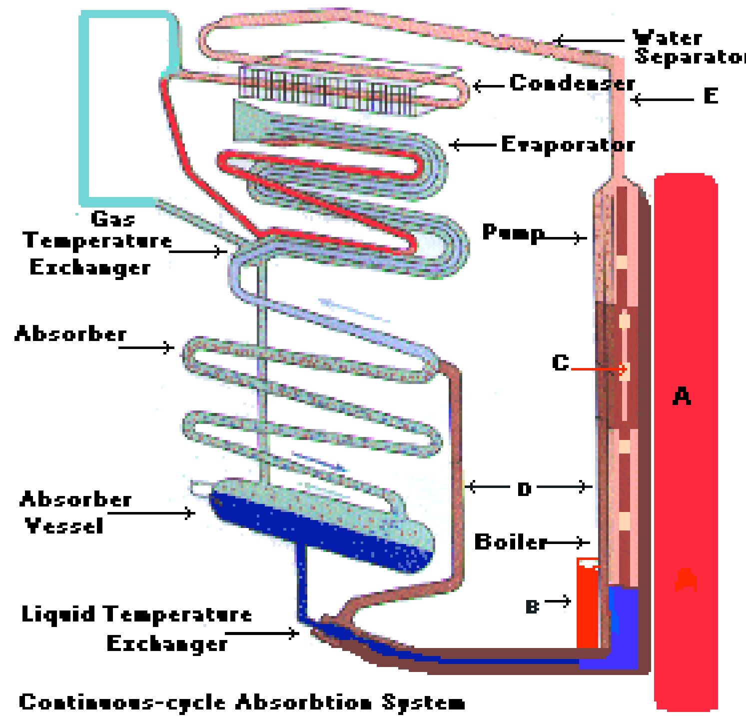
Leo Szillard

Autonomous heat device

José P. Palao, Ronnie Kosloff, and Jeffrey M. Gordon,
Quantum thermodynamic cooling cycle .
Phys. Rev. E 64, 056130 (2001)

Coupling a flow from a hot bath to a cooler intermediate one to a flow from the cold bath to the intermediate one, heat is pumped from the cold bath.



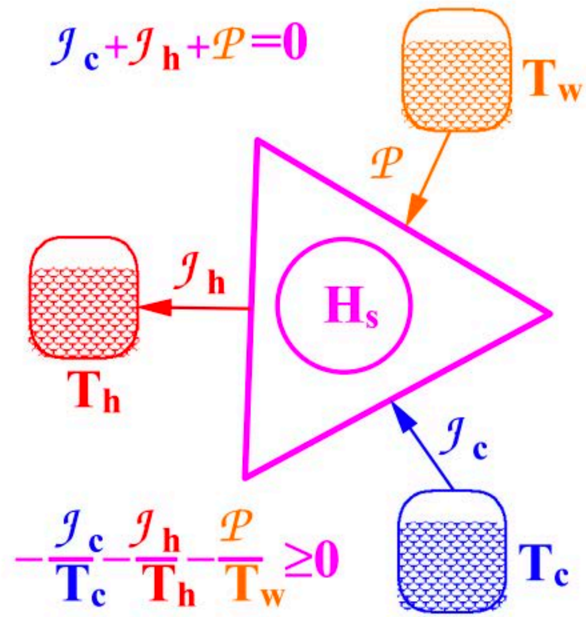


No moving parts

?How small can a refrigerator be?

e

Autonomous Refrigerator



.Palao, José P., Ronnie Kosloff, and Jeffrey M. Gordon. Quantum thermodynamic cooling cycle
Physical Review E 64.5 (2001): 056130

Linden, Noah, Sandu Popescu, and Paul Skrzypczyk. How small can thermal machines be? The smallest
. possible refrigerator
. Physical review letters 105.13 (2010): 130401

. Levy, Amikam, and Ronnie Kosloff. Quantum absorption refrigerator
.Physical review letters 108.7 (2012): 07060

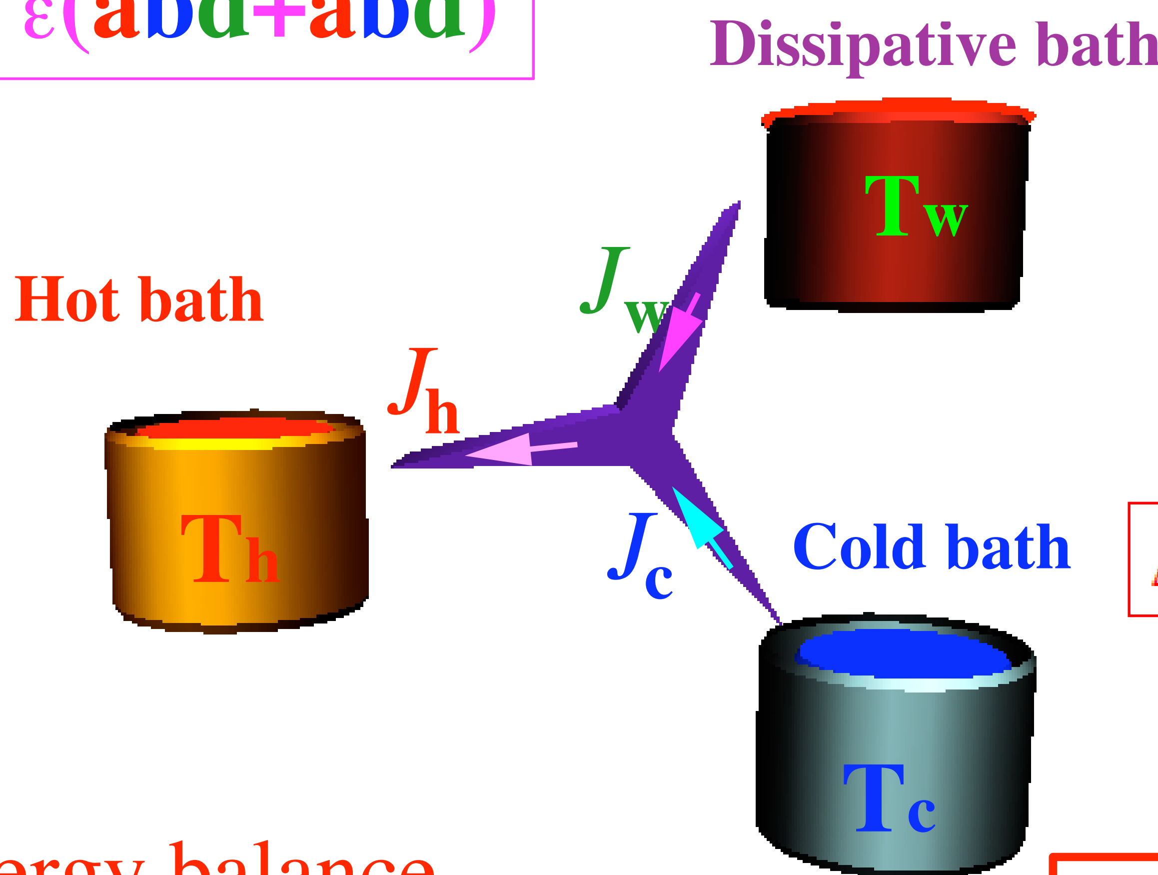
The quantum trickle

$$H = H_0 + H_{int}$$

$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b + \omega_d d^\dagger d$$

$$H_{int} = \varepsilon(a^\dagger b d + a b^\dagger d^\dagger)$$

Levi & Kosloff, PRL 108, 070604 (2012)



Entropy production

$$\Delta S_h + \Delta S_c + \Delta S_d > 0$$

$$\frac{J_h}{T_h} + \frac{J_c}{T_c} + \frac{J_d}{T_w} \geq 0$$

Energy balance

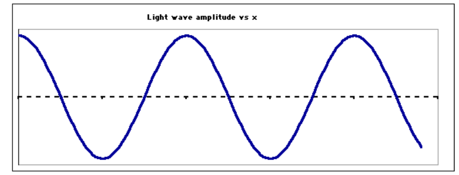
$$\dot{J}_h + \dot{J}_c + \dot{J}_d = 0$$

The quantum trickle semiclassical limit

$H = H_0 + H_{int}$

$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b$

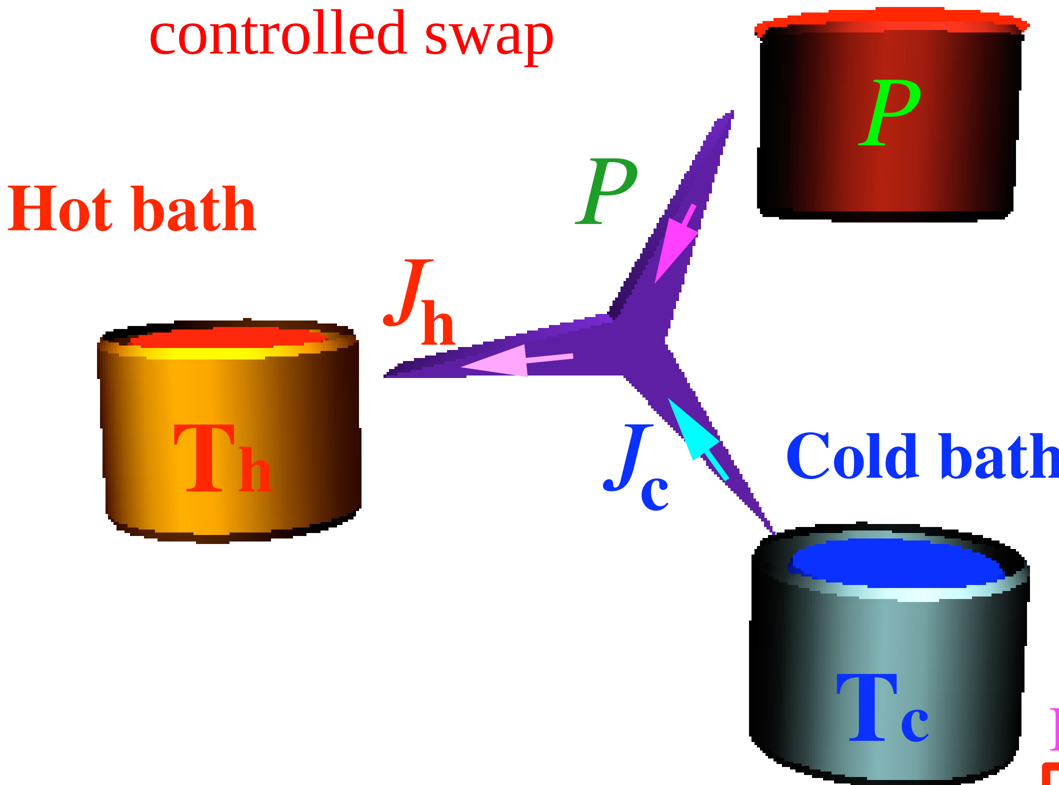
$d \Rightarrow q e^{-i\nu t}$

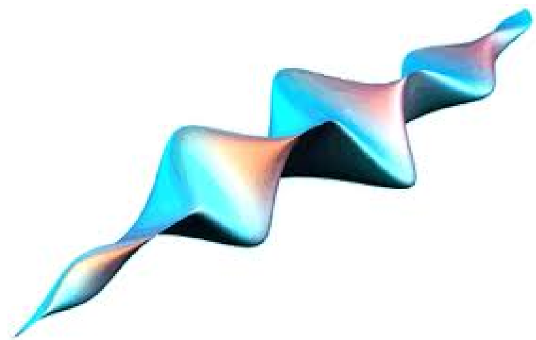


$H_{int} = \varepsilon(a^\dagger b e^{-i\nu t} + a b^\dagger e^{+i\nu t})$

controlled swap

Power source





$\eta = 1 - \frac{\omega_c}{\omega_h}$

Entropy production

$\frac{J_h}{T_h} + \frac{J_c}{T_c} \geq 0$

Energy balance

$J_h + J_c + P = 0$

The quantum trickle semiclassical limit

$$H = H_0 + H_{int}$$

As an Engine

$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b$$

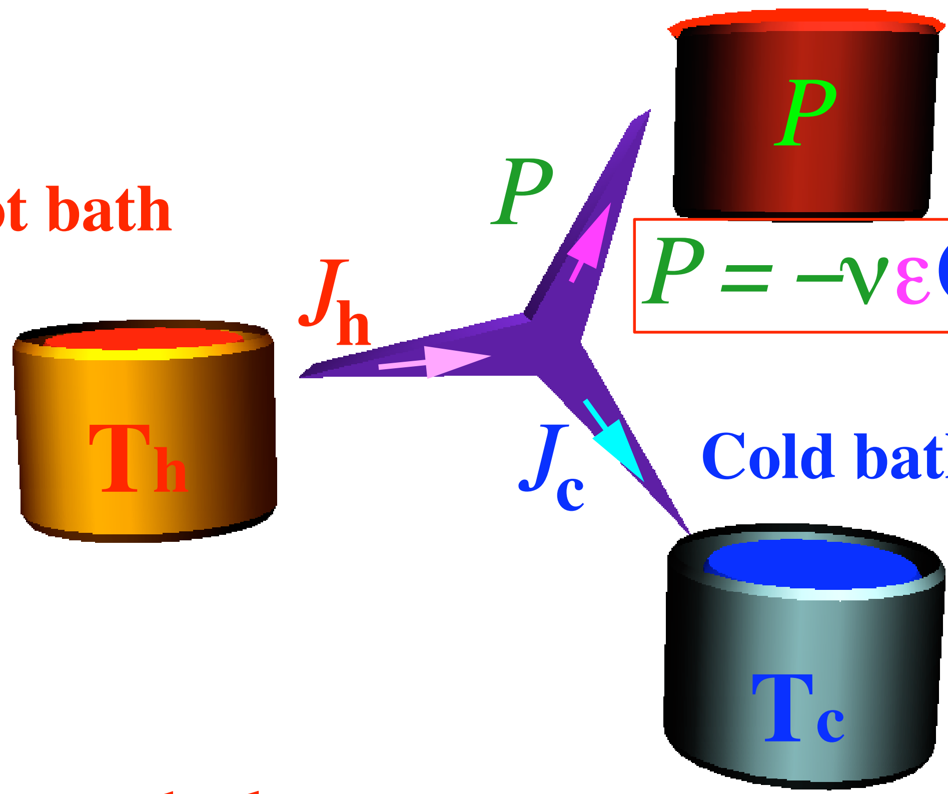
$$H_{int} = \varepsilon\left(a^\dagger b e^{-i\nu t} + a b^\dagger e^{+i\nu t}\right)$$

$$d \Rightarrow q e^{-i\nu t}$$

$$\eta = 1 - \sqrt{\frac{T_c}{T_h}}$$

Efficiency at Maximum power

R.K. JCP 80 1625 (1984)



$$P = -\nu\varepsilon G$$

Entropy production

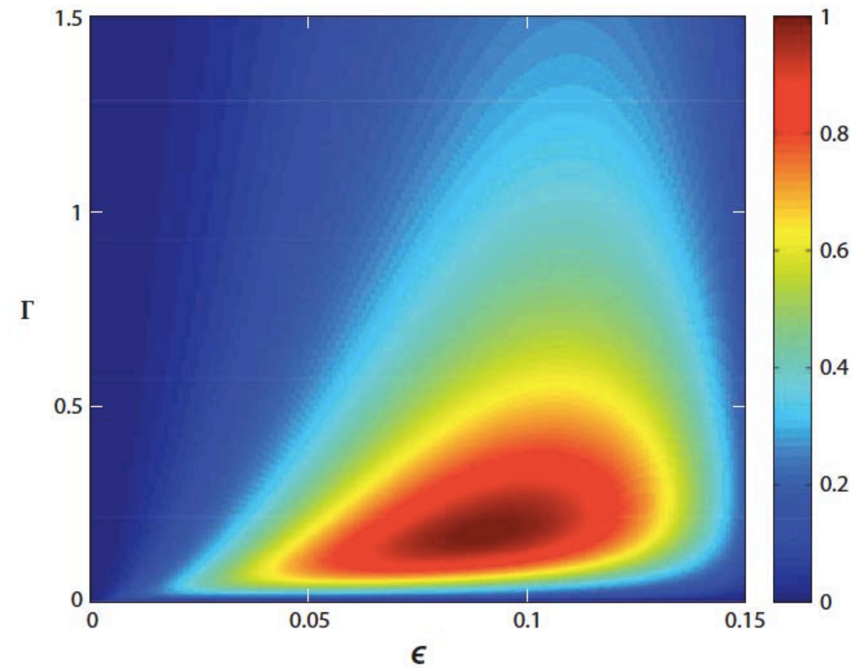
$$\frac{J_h}{T_h} + \frac{J_c}{T_c} \geq 0$$

Energy balance

$$J_h + J_c + P = 0$$

Engine at maximum power

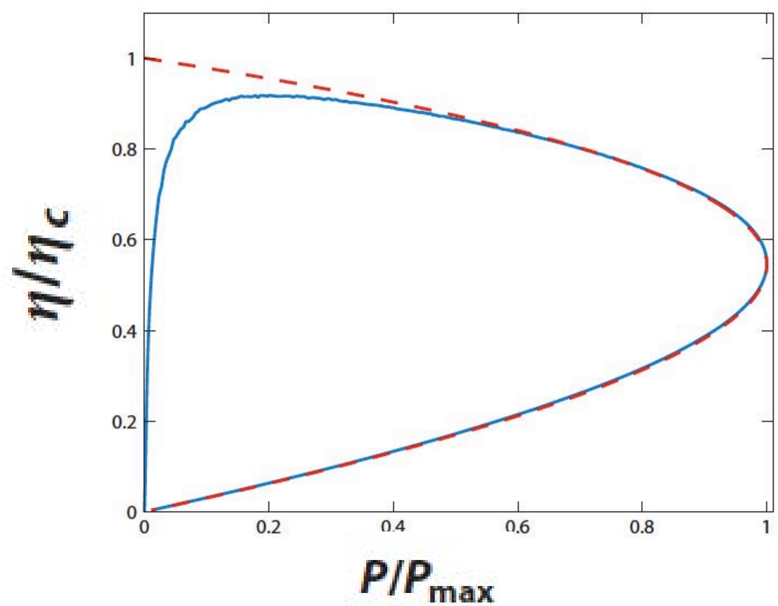
$$\mathcal{P} = -\frac{\hbar\omega\epsilon^2\Gamma G}{4\epsilon^2+\Gamma^2}$$



Further optimization

$$\mathcal{P} = -\frac{1}{2}\hbar\omega G$$

$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}}$$



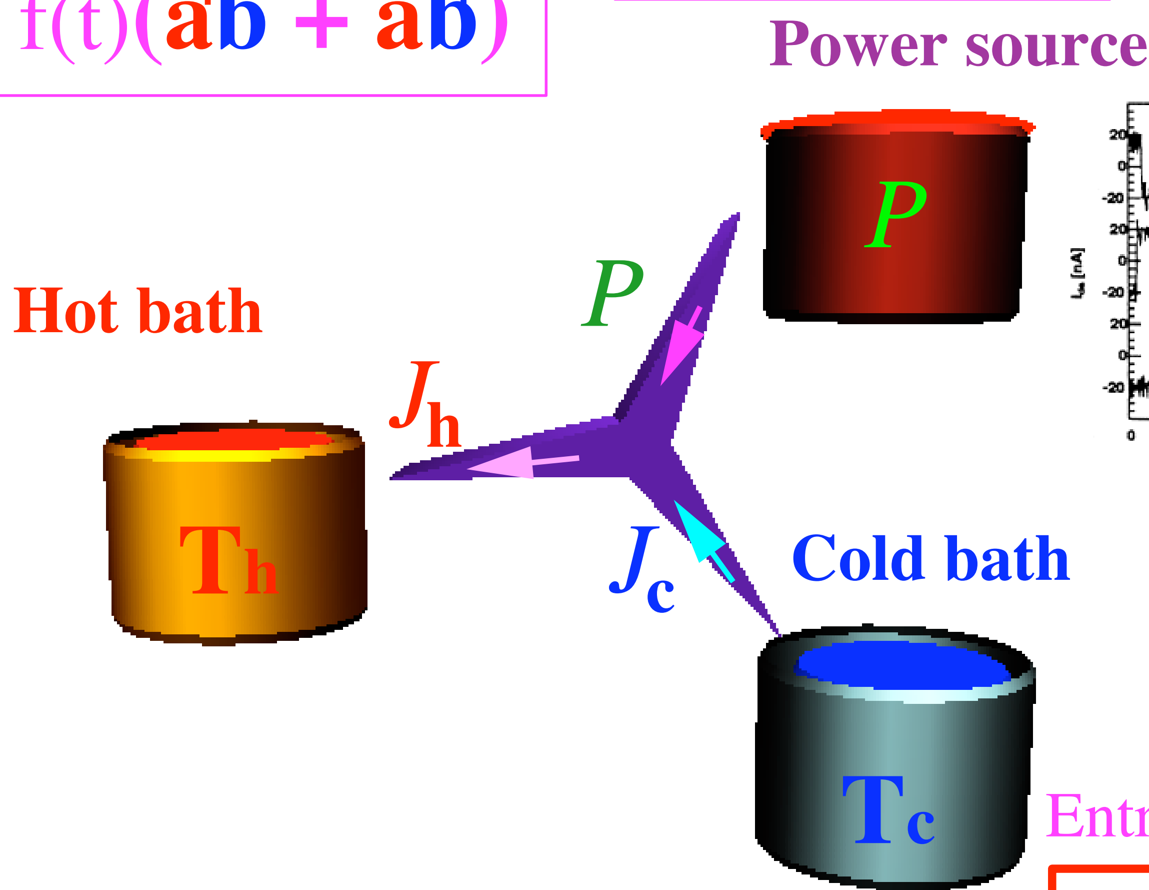
The quantum trickle *absorption refrigerator*

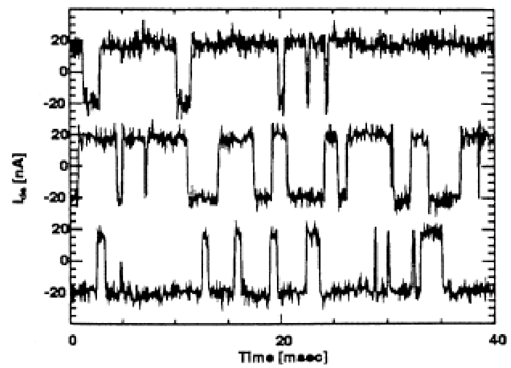
$$H = H_0 + H_{int}$$

$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b$$

$$H_{int} = f(t)(a^\dagger b + a b^\dagger)$$

$f(t)$ noise field





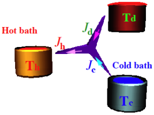
Energy balance

$$J_h + J_c + P = 0$$

Entropy production

$$\frac{J_h}{T_h} + \frac{J_c}{T_c} \geq 0$$

All types of refrigerators have universal properties as $T_c \to 0$.



In the power driven refrigerators the cold current becomes:

$$\mathcal{J}_c \approx \hbar\omega_c^- \frac{2\varepsilon^2\bar{\Gamma}}{4\varepsilon^2 + \Gamma_c\Gamma_h} \cdot G \;, \quad \textit{where the gain } G = N_c^- - N_h^-$$

and $\bar{\Gamma} = \frac{\Gamma_c\Gamma_h}{\Gamma_c+\Gamma_h}$.

In the 3-level absorption refrigerator:

$$\mathcal{J}_c \;=\; \hbar\omega_c \frac{\Gamma_c\Gamma_h\Gamma_w}{\Delta} \cdot G \quad \textit{where } G = e^{-\frac{\hbar\omega_w}{k_B T_w}} e^{-\frac{\hbar\omega_c}{k_B T_c}} - e^{-\frac{\hbar\omega_h}{k_B T_h}}$$

.

In the Guassian noise driven refrigerator:

$$\mathcal{J}_c \;=\; \hbar\omega_c \frac{2\eta\bar{\Gamma}}{2\eta+\bar{\Gamma}} \cdot G \quad \textit{where} \quad G = N_c - N_h$$

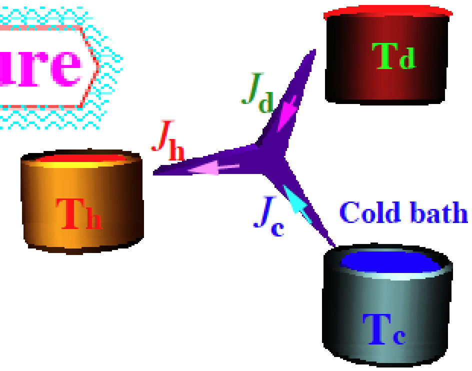
In the Poisson driven refrigerator:

$$\mathcal{J}_c \approx \hbar\Omega_- \frac{2\eta\bar{\Gamma}}{2\eta + \bar{\Gamma}} \cdot G \;, \quad \textit{where } G = (N_c^- - N_h^+) \qquad (3)$$

and $\Omega_- \approx \omega_c - \frac{\varepsilon^2}{\omega_h - \omega_c}$.

The quest to cool to the absolute zero temperature

Amikam Levy, Robert Alicki, Ronnie Kosloff



Universal optimization

$$\bar{\mathbf{J}}_{\mathbf{c}} \propto \hbar\omega_c \cdot \Gamma_c(\omega_c, T_c) \cdot (\mathbf{N_c} - \mathbf{N_h})$$

quant *coupling* *gain*

$$\omega_c \propto T_c \quad \Gamma_c \propto T_c{}^{\kappa+d-1} \quad \textit{constant}$$

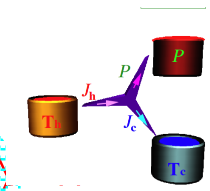
$$\frac{dT_c(t)}{dt} = -c\, T_c^{\zeta}, \quad T_c \to 0 \;\; .\zeta > 1$$

$$\Delta \dot{S}_c \sim -T_c^{\alpha} \quad , \quad \alpha > 0 \;\; .$$

$\zeta = \frac{3}{2}$ for cold Bose/Fermi gas. $\zeta = 1$ harmonic bath.

$\alpha = 2$ for Bose gas $\alpha = \frac{5}{2}$ for Fermi gas $\alpha = 3$ for harmonic bath.

The quantum trickle ***The III–law of Thermodynamics***



The quest to cool to the absolute zero temperature

3D phonon *vs.* *Bos Gas* *heat bath*

3D–Phonon

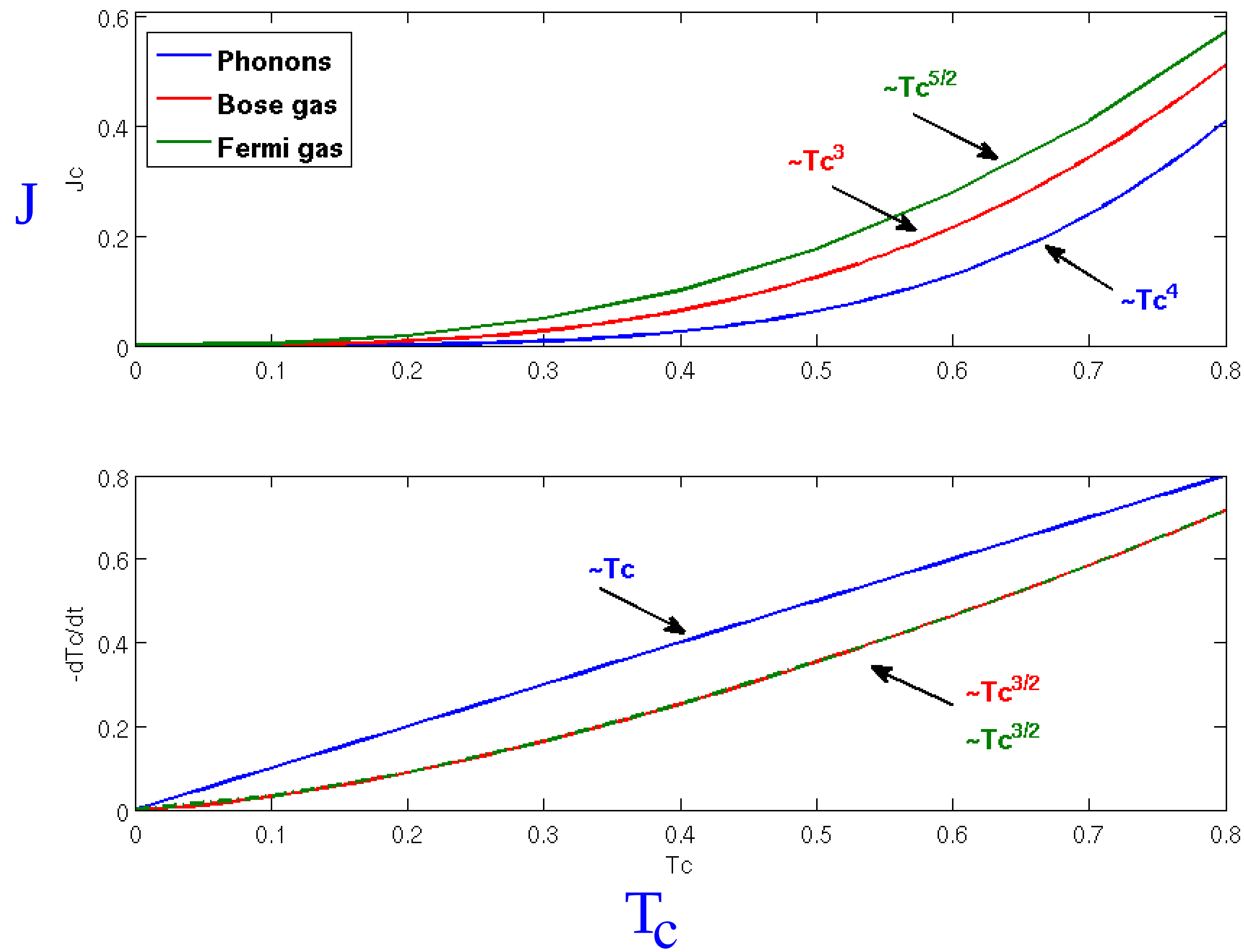
$$J_c \sim -T_c^4$$

$$\frac{dT_c}{dt} \sim -T_c^1$$

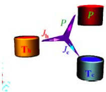
Bose gas

$$J_c \sim -T_c^3$$

$$\frac{dT_c}{dt} \sim -T_c^{3/2}$$



Realizations 2017



Quantum absorption refrigerator with trapped ions

Gleb Maslennikov,[1,*] Shiqian Ding†,[1,*] Roland Hablützel,[1] Jaren Gan,[1] Alexandre Roulet,[1] Stefan Nimmrichter,[1] Jibo Dai,[1] Valerio Scarani,[1,2] and Dzmitry Matsukevich[1,2]

[1] *Centre for Quantum Technologies, National University of Singapore, 3 Science Dr 2, 117543, Singapore*
[2] *Department of Physics, National University of Singapore, 2 Science Dr 3, 117551, Singapore*

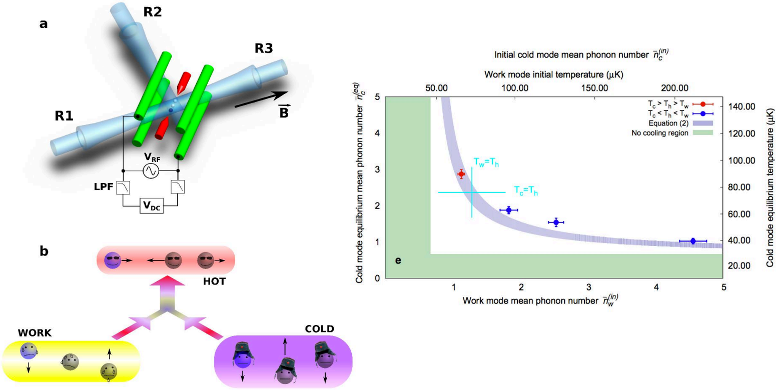


FIG. 1. **Experimental setup.** **a.** Schematic of the linear

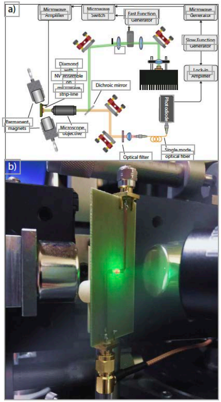
Quantum equivalence

Uzdin, Raam, Amikam Levy, and Ronnie Kosloff. "Equivalence of Quantum Heat Machines, and Quantum-Thermodynamic Signatures." Physical Review X 5, no. 3 (2015): 031044.

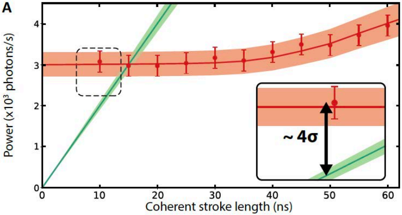
Realization 2017

Experimental demonstration of quantum effects in the operation of microscopic heat engines

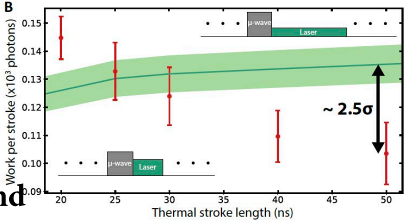
J. Klatzow,[1] C. Weinzetl,[1] P. M. Ledingham,[1] J. N. Becker,[1] D. J. Saunders,[1] J. Nunn,[1] I. A. Walmsley,[1] R. Uzdin,[2] and E. Poem[1,3,*]



NV center in Diamond



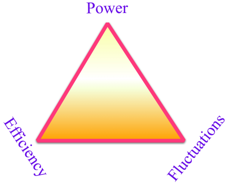
quantum signature



dephasing

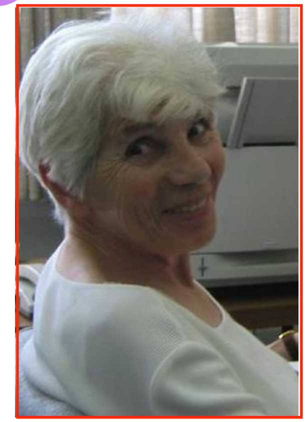
Quantum Thermodynamics: The Future

Miniaturization:



1. Tradeoff: Efficiency, Power, Fluctuations.
2. Quantum refrigerators: Laser Cooling
3. Quantum information processing.
4. Quantum enhancement: coherence, charging. :

Quantum Thermodynamics



Peter Salamon

Yair Rezek

Gil Katz

Robert Alicki

Tova Feldmann

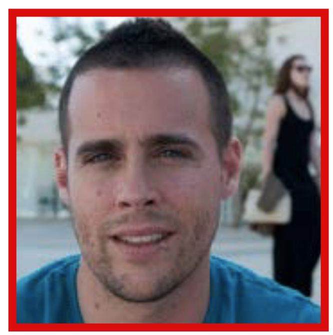
Morag Am Shalem

Jeff Gordon

Amikam Levy

Jose Palao

Raam Uzdin



Eitan Geva

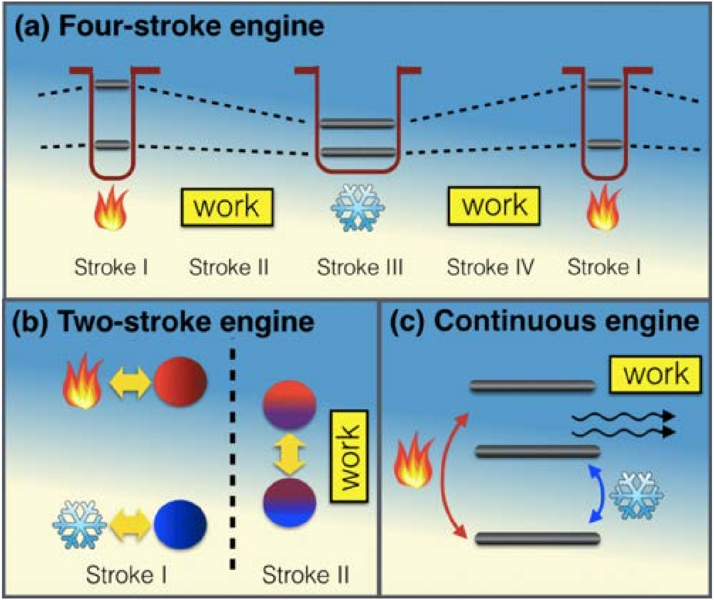
Erik Torrontegui

Roie Dann

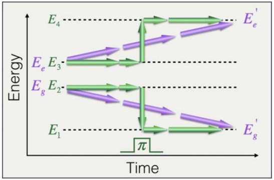
The end

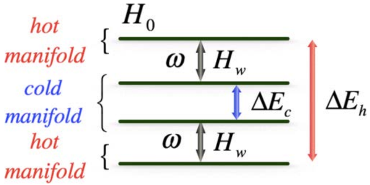
Thank you

Three types of engines

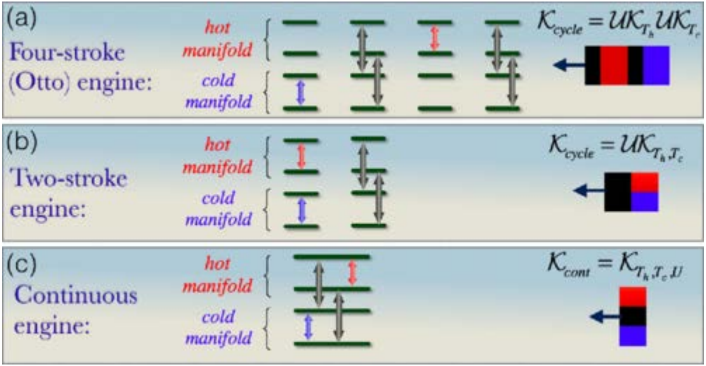


Multilevel embedding





Quantum equivalence



Quantum eqivalence

The propagator: $\mathscr{U} = e^{\mathscr{L} t}$

Four stroke cycle propagator:

$$\mathscr{U}_{cyc} = \mathscr{U}_c \mathscr{U}_{hc} \mathscr{U}_h \mathscr{U}_{ch} = e^{\mathscr{L}_c t} e^{\mathscr{L}_{hc} t} e^{\mathscr{L}_h t} e^{\mathscr{L}_{ch} t}$$

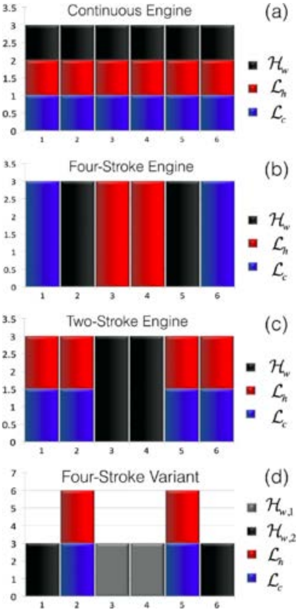
In the limit of small action: $s = ||\mathscr{L} t|| \ll \hbar$

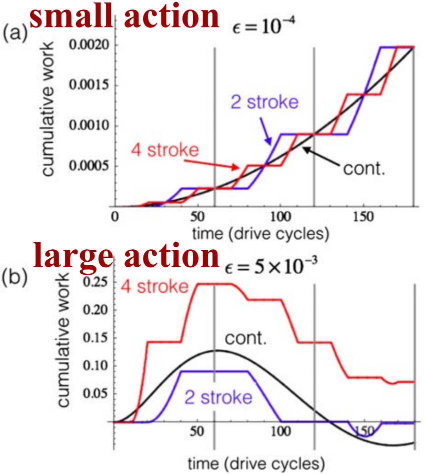
$$\mathscr{U}_{cyc} = e^{\mathscr{L}_c t/2} e^{\mathscr{L}_{hc} t} e^{\mathscr{L}_h t} e^{\mathscr{L}_{ch} t} e^{\mathscr{L}_c t/2}$$

$$\mathscr{U}_{cyc} \approx e^{(\mathscr{L}_c + \mathscr{L}_{hc} + \mathscr{L}_h + \mathscr{L}_{ch}) t} + O(s^3)$$

Raam Uzdin, Amikam Levy, and Ronnie Kosloff
Equivalence of Quantum Heat Machines, and Quantum-Thermodynamic
.(Phys. Rev. X 5, 031044 2015

Quantum equivalence



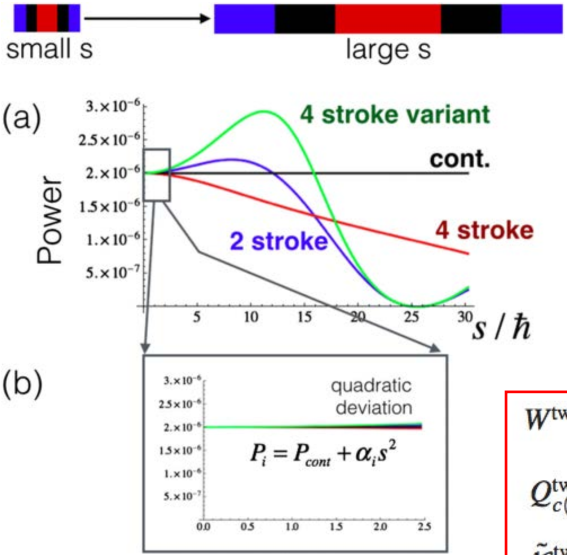


$$W^{\text{two stroke}} \cong W^{\text{four stroke}} \cong W^{\text{cont}},$$

$$Q_{c(h)}^{\text{two stroke}} \cong Q_{c(h)}^{\text{four stroke}} \cong Q_{c(h)}^{\text{cont}},$$

$$\tilde{\mathcal{K}}^{\text{two stroke}} \cong \tilde{\mathcal{K}}^{\text{four stroke}} \cong \tilde{\mathcal{K}}^{\text{cont}}.$$

Quantum equivalence



At large action:
Work extracted from population differences.

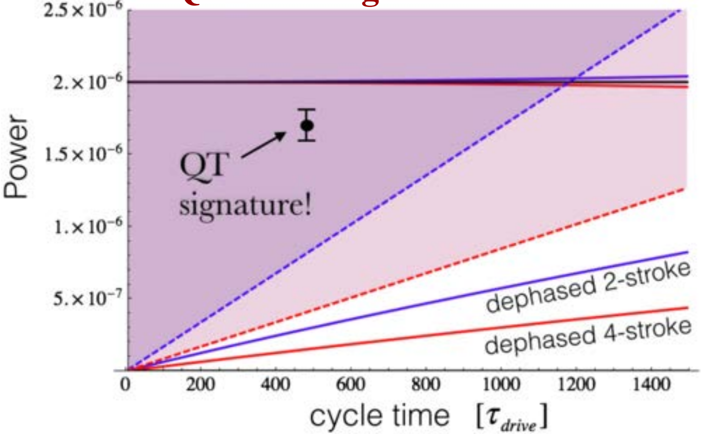
At small action:
Work can only be extracted from coherence

$$W^{\text{two stroke}} \cong W^{\text{four stroke}} \cong W^{\text{cont}},$$

$$Q_{c(h)}^{\text{two stroke}} \cong Q_{c(h)}^{\text{four stroke}} \cong Q_{c(h)}^{\text{cont}},$$

$$\tilde{\mathcal{K}}^{\text{two stroke}} \cong \tilde{\mathcal{K}}^{\text{four stroke}} \cong \tilde{\mathcal{K}}^{\text{cont}}.$$

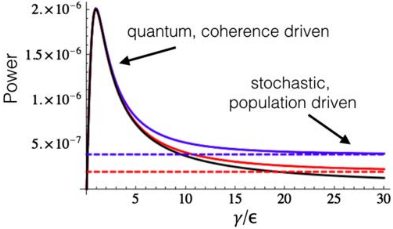
Quantum signature



$$P_{\text{stoch}} \leq \frac{z}{8\hbar^2}\sqrt{tr(H_0^2) - tr(H_0)^2}\,\Delta_w^2 d^2 \tau_{\text{cyc}},$$

$z = 1$ two-stroke,

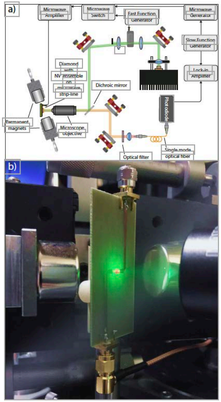
$z = 1/2$ four-stroke,

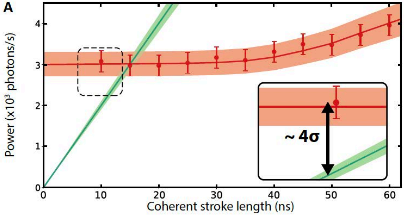


Realization 2017

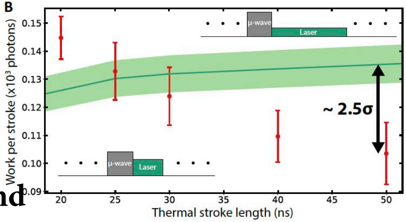
Experimental demonstration of quantum effects in the operation of microscopic heat engines

J. Klatzow,[1] C. Weinzetl,[1] P. M. Ledingham,[1] J. N. Becker,[1] D. J. Saunders,[1] J. Nunn,[1] I. A. Walmsley,[1] R. Uzdin,[2] and E. Poem[1,3,*]





quantum signature



dephasing

NV center in Diamond

Friction coherence and shortcuts

Casimir Companion invariant of unitary dynamics.

$$\bar{X} = \frac{1}{(\hbar\Omega)^2}\left(\langle\hat{H}\rangle^2 + \langle\hat{L}\rangle^2 + \langle\hat{C}\rangle^2\right) \quad ,$$

Extra work required to generate coherence.

$$\langle\hat{H}\rangle_f = \sqrt{\left(\frac{\Omega_f}{\Omega_i}\right)^2 \langle\hat{H}_i\rangle^2 - \left(\hbar\Omega_f \mathscr{C}_{\{}\right)^2} \approx \frac{\Omega_f}{\Omega_i}\langle\hat{H}_i\rangle - \frac{\hbar^2\Omega_i\Omega_f}{2\langle\hat{H}_i\rangle}\mathscr{C}_f{}^2 \quad ,$$

$$\mathscr{W}_{fric} \equiv |\mathscr{W} - \mathscr{W}_{ideal}| \approx \frac{\hbar^2\Omega_i\Omega_f}{2\langle\hat{H}_i\rangle}\mathscr{C}_f^2$$

Non-Adiabtic driving generates coherence:

$$[\hat{H}(t), \hat{H}(t')] \neq 0$$

$$\frac{1}{\Omega}\frac{d}{dt}\begin{pmatrix} \hat{H}(t) \\ \hat{L}(t) \\ \hat{C}(t) \end{pmatrix} = \left(\begin{pmatrix} 0 & \mu & 0 \\ -\mu & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} + \frac{\dot{\Omega}}{\Omega^2}\hat{I} \right) \begin{pmatrix} \hat{H}(t) \\ \hat{L}(t) \\ \hat{C}(t) \end{pmatrix} \quad ,$$

Adiabatic parameter $\mu = \frac{\dot{\omega}\varepsilon - \omega\dot{\varepsilon}}{\Omega^3}$

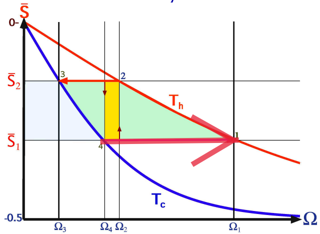
$$\mu = \frac{K}{\tau_{adi}}$$

For constant μ protocol

where $K = \left(\frac{d\omega}{ds}\varepsilon - \omega\frac{d\varepsilon}{ds}\right)/\Omega^3$, with $s = t/\tau_{adi}$.

$$\frac{\mathscr{W}_{fric}}{\mathscr{W}} \approx \mu^2 \quad .$$

Unitary stroke

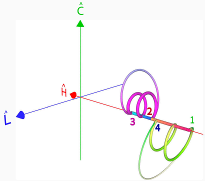


Shortcuts to Adiabticity for Unitary strokes

$$\Lambda_{adi}(t) = \mathscr{U}_1(t)\mathscr{U}_2(t),$$

where $\mathscr{U}_1(t) = \frac{\Omega(t)}{\Omega(0)}\hat{I}$ and

$\mathscr{U}_2(t)$ is the dynamical map of the polarization.

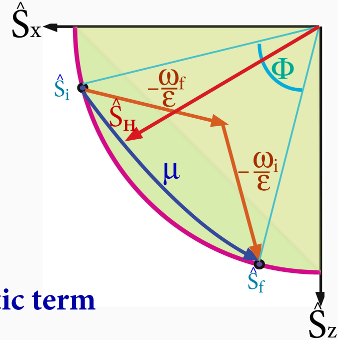


$$\mathscr{U}_2(t) = \frac{1}{\kappa^2}\begin{pmatrix} 1+\mu^2 c & \kappa\mu s & \mu(1-c) \\ -\kappa\mu s & \kappa^2 c & \kappa s \\ \mu(1-c) & -\kappa s & \mu^2 + c \end{pmatrix},$$

where $\kappa = \sqrt{1+\mu^2}$ and $s = \sin(\kappa\theta)$, $c = \cos(\kappa\theta)$ and $\theta(t) = \int_0^t \Omega(t')dt'$.

shortcut time

$$\tau_{adi}(I=1) = K\sqrt{\left(\frac{2\pi}{\Phi}\right)^2 - 1}\ .$$



Catalysis: $\hat{H}_{CA} = \upsilon(t)\hat{S}_y$. **counter adiabatic term**

Qubit basics

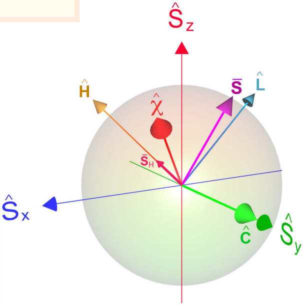
Hamiltonian $$\hat{H}_S(t) = \omega(t)\hat{S}_z + \varepsilon(t)\hat{S}_x \quad ,$$

$$\hbar\Omega(t) = \hbar\sqrt{\omega^2 + \varepsilon^2} \quad ,$$

state $$\hat{\rho} = \frac{1}{2}\hat{I} + \frac{2}{\hbar^2}\left(\langle\hat{S}_x\rangle\hat{S}_x + \langle\hat{S}_y\rangle\hat{S}_y + \langle\hat{S}_z\rangle\hat{S}_z\right)$$

time dependent set

$$\hat{H} = \omega(t)\hat{S}_z + \varepsilon(t)\hat{S}_x$$
$$\hat{L} = \varepsilon(t)\hat{S}_z - \omega(t)\hat{S}_x$$
$$\hat{C} = \Omega(t)\hat{S}_y \quad .$$

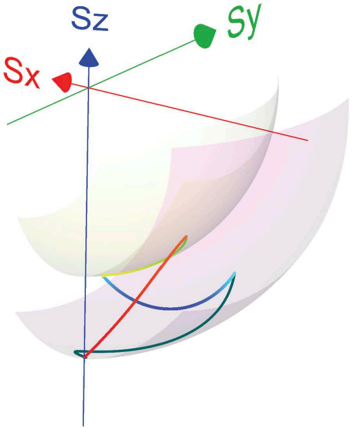


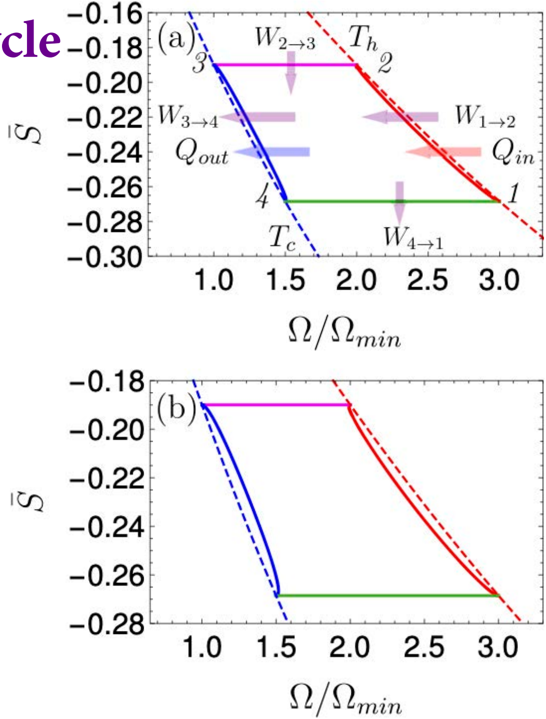
state $$\hat{\rho} = \frac{1}{2}\hat{I} + \frac{2}{(\hbar\Omega)^2}\left(\langle\hat{H}\rangle\hat{H} + \langle\hat{L}\rangle\hat{L} + \langle\hat{C}\rangle\hat{C}\right)$$

Coherence $$\mathscr{C} = \frac{1}{\hbar\Omega}\sqrt{\langle\hat{L}\rangle^2 + \langle\hat{C}\rangle^2} \quad ,$$

Isotherms and entropy generation

Shortcut Carno Cycle





Carnot cycle: The isotherms

The Problem:

Derive a dynamical description for a driven system coupled to a bath beyond the adiabatic limit:

$$\hat{H} = \hat{H}_S(t) + \hat{H}_B + \hat{H}_{SB}$$

Peter Salamon

Carnot cycle: The isotherms

$$\left[\hat{H}_S(t), \hat{H}_S(t')\right] \neq 0$$

The task: Isothermal Dynamics

Starting from a thermal initial state $\hat{\rho}_i = e^{-\beta \hat{H}_i}$

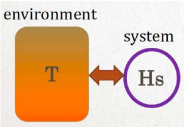
Transform as fast and accurate to the state: $\hat{\rho}_f = e^{-\beta \hat{H}_f}$

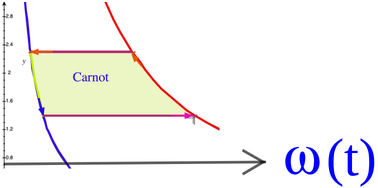
while the system is in contact with a bath of temperature $T = 1/k\beta$

The protocol: $\hat{H}_S(t)$ with $\hat{H}_S(0) = \hat{H}_i$ and $\hat{H}_S(t_f) = \hat{H}_f$

The Problem

We can control directly $\hat{H}_S(t)$ but only indirectly the relaxation rate. We need the dissipative equation of motion with a time dependent $\hat{H}_S(t)$ with a time dependent protocol.

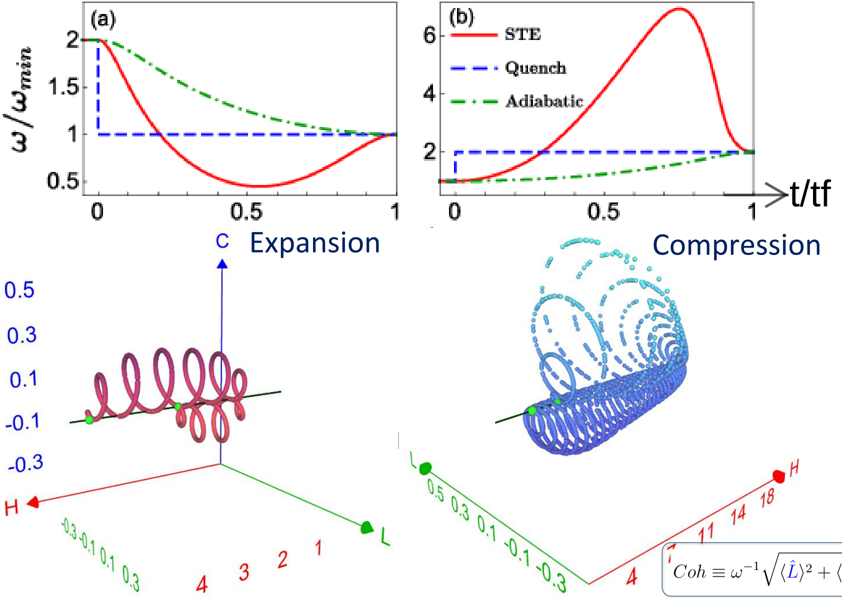




Shortcuts to Equilibrium (STE)

The shortcut protocol $\hat{H}_S(t) \rightarrow \omega(t)$:

Overshoot



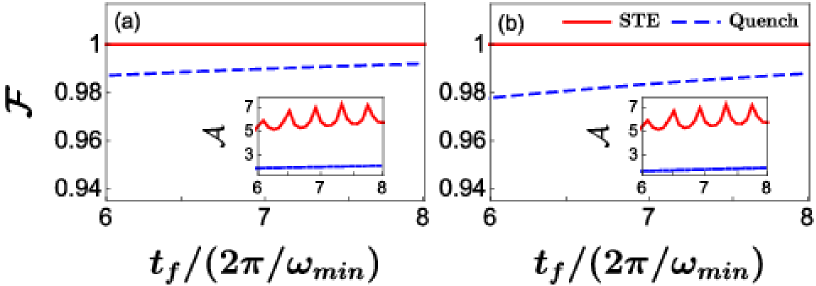
$$Coh \equiv \omega^{-1}\sqrt{\langle\hat{L}\rangle^2 + \langle\hat{C}\rangle^2}$$

Shortcuts to Equilibrium (STE)

The fidelity $\mathscr{F}$ and $\mathscr{A} = -\log_{10}(1 - \mathscr{F})$:

Expansion

Compression



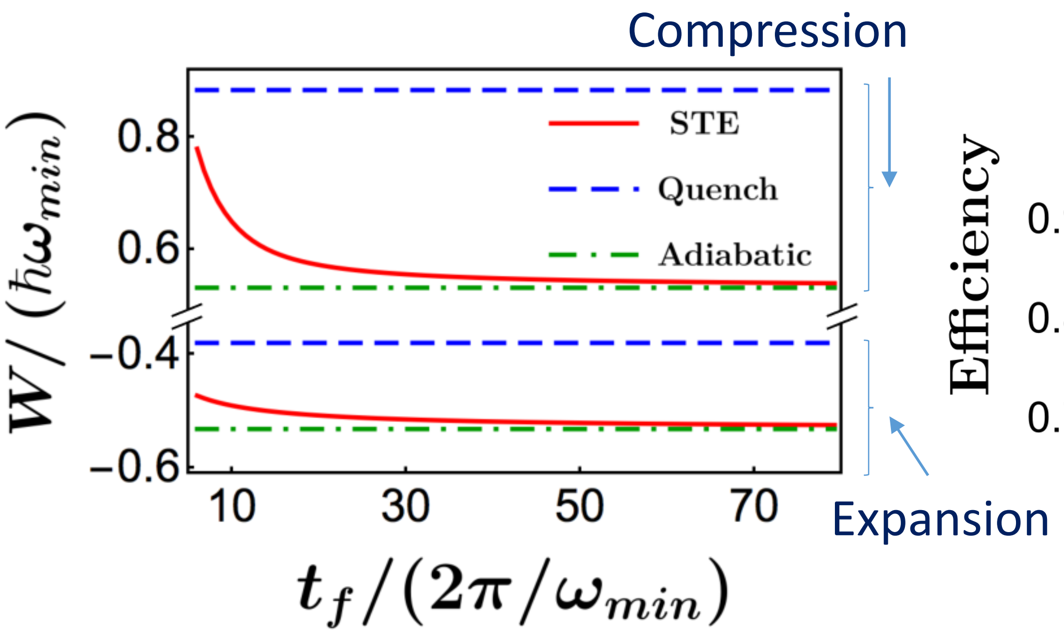
3 fold improvement in time

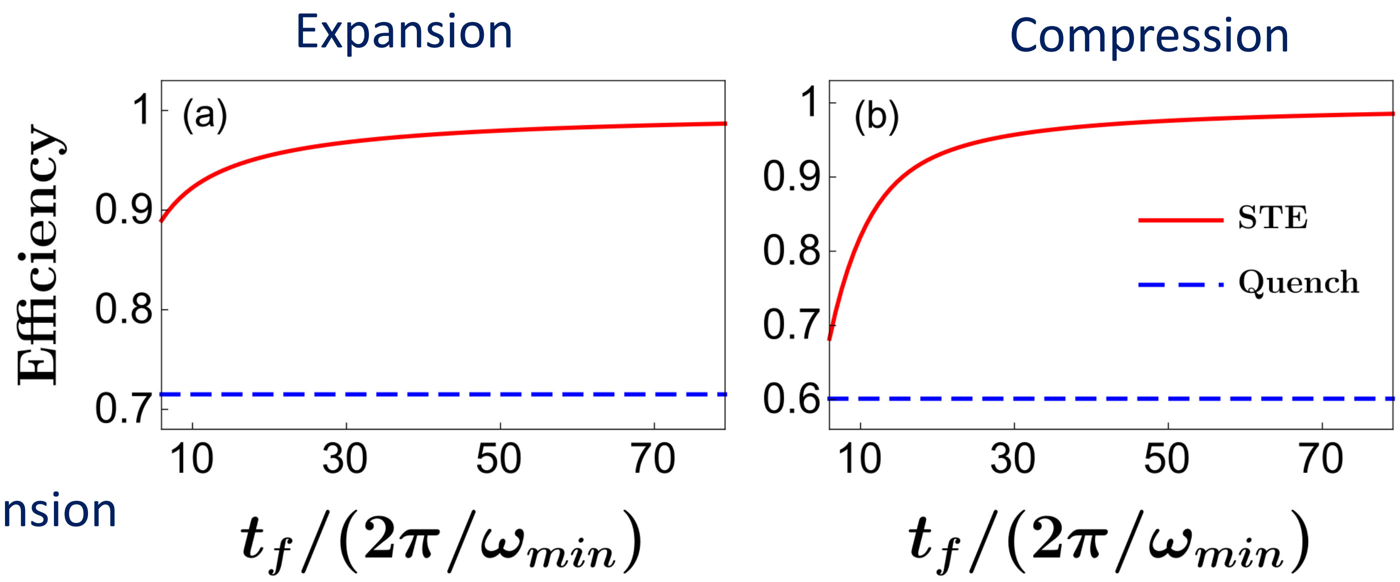
R. Dann, A. Tobalina, and R. Kosloff, *PRL* **122**, 250402 (2019)

STE- How much does it cost?

STE Quench Adiabatic

Rapid driving costs!

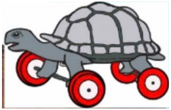




$$W = \int_0^t dt' \, \mathrm{tr}\left(\frac{\partial \hat{H}(t')}{\partial t'} \hat{\rho}_S(t')\right)$$

Efficiency: $\dfrac{W}{W_{ideal}}$

The cost of shortcuts W , S_u



Adiabatic shortcuts

Starting on the energy shell:

$$\langle \hat{\mathbf{H}} \rangle \neq 0 \, , \langle \hat{\mathbf{L}} \rangle = 0 \, , \langle \hat{\mathbf{C}} \rangle = 0$$

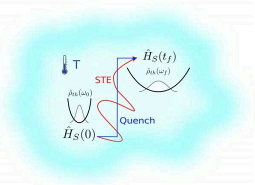
Nonadiabtic dynamics generates coherence and requires extra work. Shortcuts to Adiabaticity **STA** retrieve this work **cashing on the coherence.**

The shortcut duration s nversely related to the stored energy. The system entropy remains constant $\Delta S_{sys} = 0$.

Irreversible cost only the controller $\Delta S_U \geq 0$.

The process can be classified as catalysis.

The cost of shortcuts W , S_u



Shortcuts to Equilibrium (STE)

Starting on the energy shell:

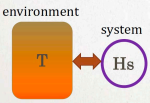
$$\langle \hat{\mathbf{H}} \rangle \neq 0 \, , \langle \hat{\mathbf{L}} \rangle = 0 \, , \langle \hat{\mathbf{C}} \rangle = 0$$

Nonadiabtic dynamics generates coherence and requires extra work.
The coherence is dissipated generating **quantum friction**
The system entropy changes $\Delta S_{sys} \neq 0$.
Irreversibility is inherent $\Delta S_U > 0$.

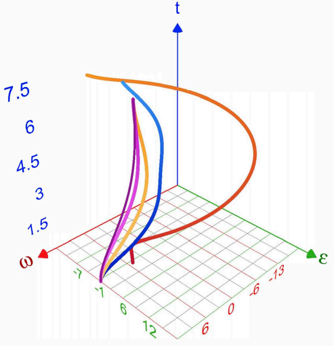
The speedup cost work and entropy production.

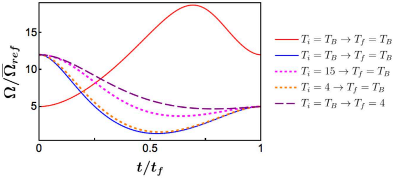
Control protocol

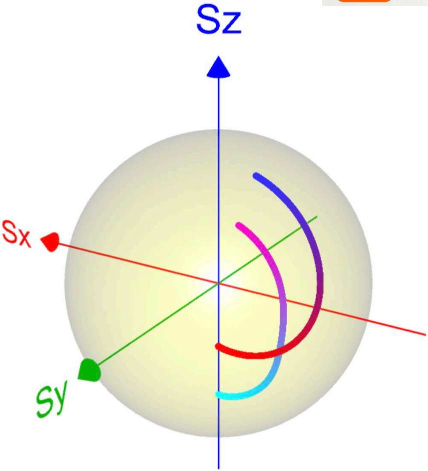
Purity



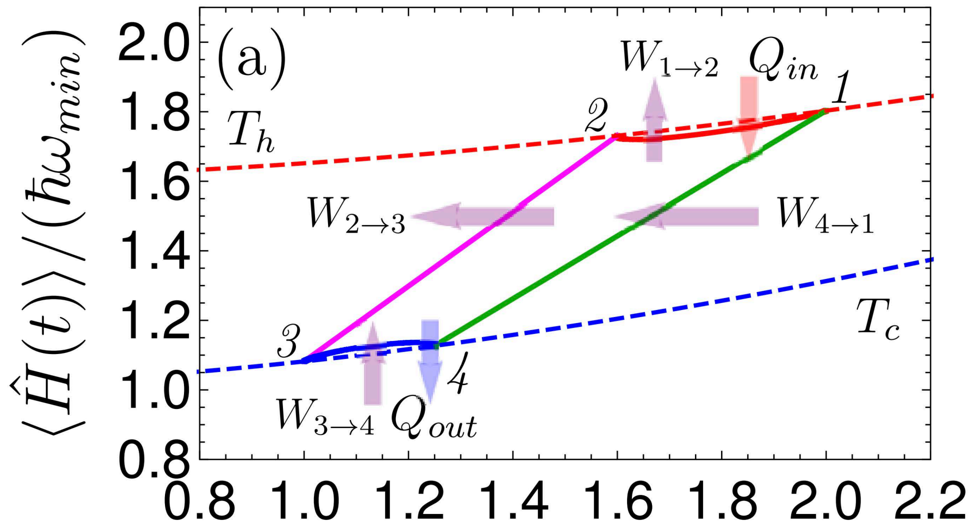
$$\hat{H}_S(t) = \omega(t)\hat{S}_z + \varepsilon(t)\hat{S}_x \quad ,$$







At last: Shortcut to four stroke Carnot cycle



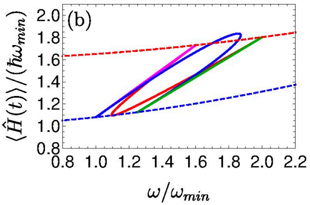
Carnot cycle:

$$\Lambda_{cyc} = \Lambda_h \Lambda_{ch} \Lambda_c \Lambda_{hc}$$

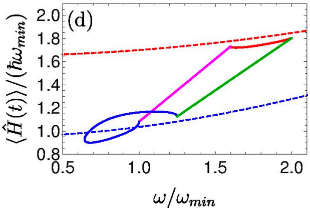
QuantumSignaturesintheQuantumCarnotCycle 20)

Performance of Shortcut to Carnot

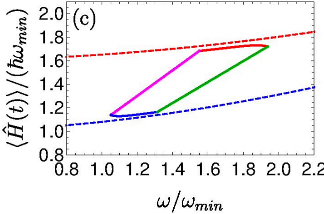
Shortcut fast

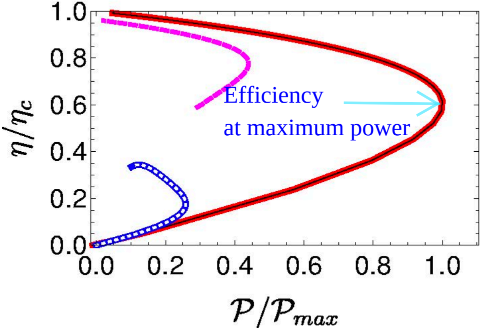


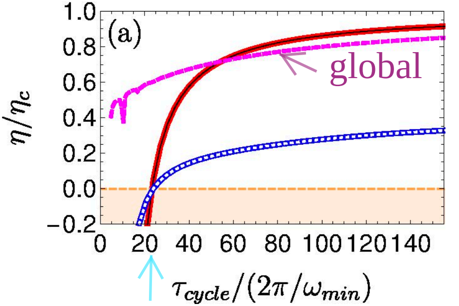
Shortcut Endo



Endo slow global



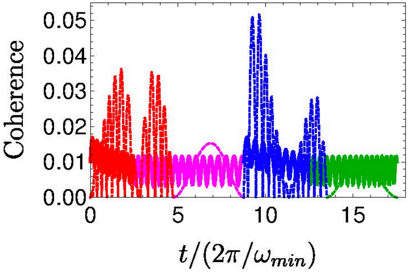




minimum cycle time

Quantum eqivalence

The propagator: $\mathscr{U} = e^{\mathscr{L} t}$



Four stroke cycle propagator:

$$\mathscr{U}_{cyc} = \mathscr{U}_c \mathscr{U}_{hc} \mathscr{U}_h \mathscr{U}_{ch} = e^{\mathscr{L}_c t} e^{\mathscr{L}_{hc} t} e^{\mathscr{L}_h t} e^{\mathscr{L}_{ch} t}$$

In the limit of small action: $s = ||\mathscr{L} t|| \ll \hbar$

$$\mathscr{U}_{cyc} = e^{\mathscr{L}_c t/2} e^{\mathscr{L}_{hc} t} e^{\mathscr{L}_h t} e^{\mathscr{L}_{ch} t} e^{\mathscr{L}_c t/2}$$

$$\mathscr{U}_{cyc} \approx e^{(\mathscr{L}_c + \mathscr{L}_{hc} + \mathscr{L}_h + \mathscr{L}_{ch}) t} + O(s^3)$$

RaamUzdin,AmikamLevy,andRonnieKoslo.
EquivalenceofQuantumHeatMachines,andQuantum-Thermodynamic .
(Phys.Rev.X5,0310442015

The Voyage:

Seeking for quantum open system description of the Carnot cycle

- Non Adiabatic Master Equation **NAME**.
- The **inertial theorem**.
- Shortcuts to non unitary maps with **entropy change.** Finite time quantum **Carnot cycle**.

Quantum signature!