Stony Brook University | The State University of New York

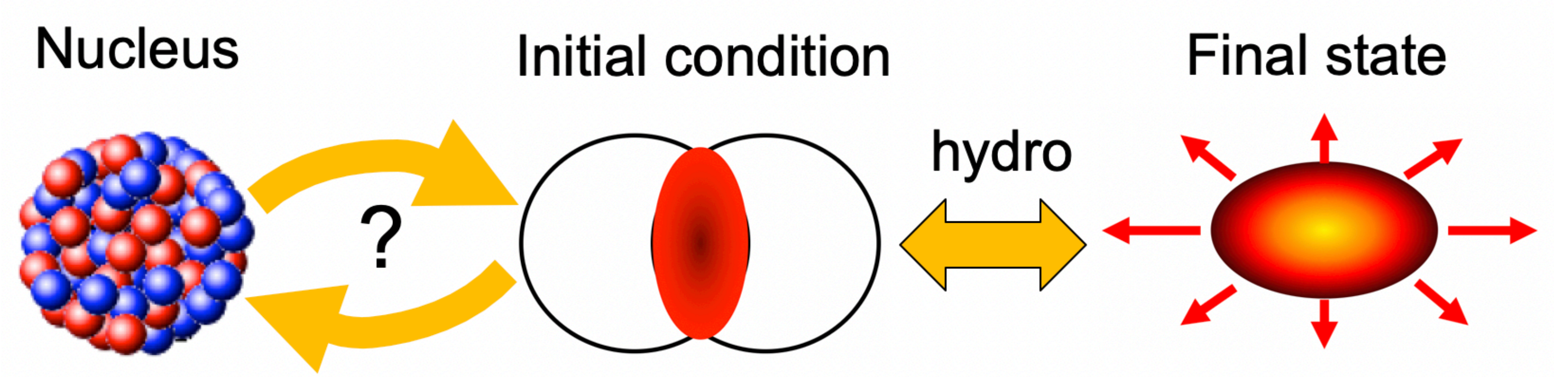
ExtreMe Matter Institute EMMI
EMMI Rapid Reaction Task Force
Nuclear Physics Confronts
Relativistic Collisions of Isobars
Open Symposium:
May 30, 2022, 1:45 p.m., Grosser Hoersaal, Heidelberg University,
Philosophenweg 12, 69120 Heidelberg/Germany

Imaging nuclear structure in high-energy heavy-ion collisions

Jiangyong Jia

Nuclear Structure

High-energy heavy-ion collisions

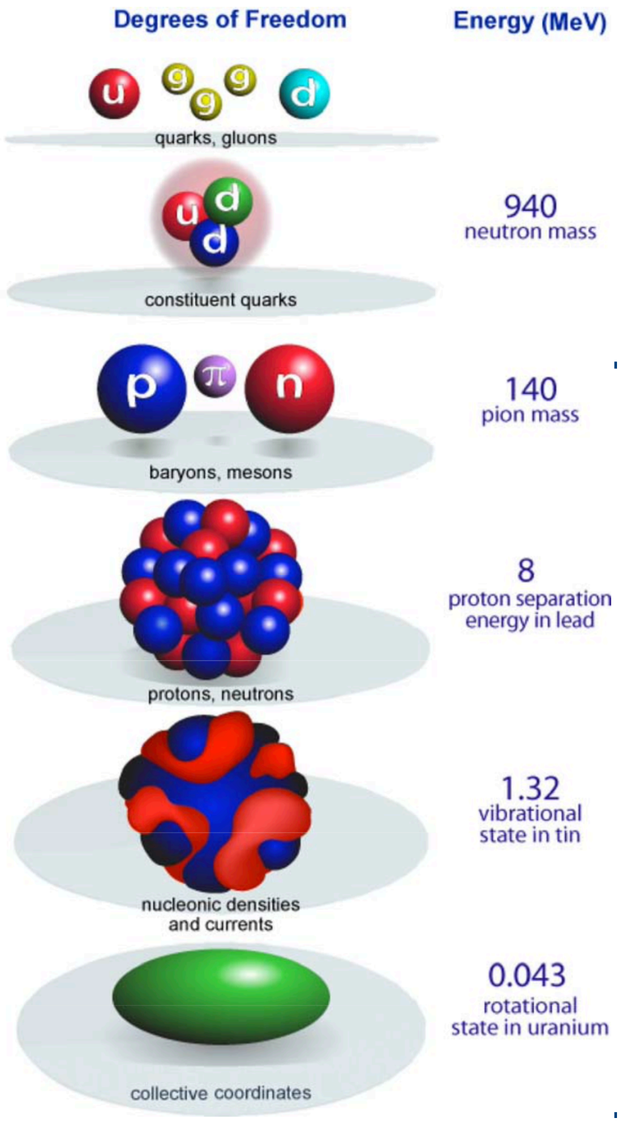


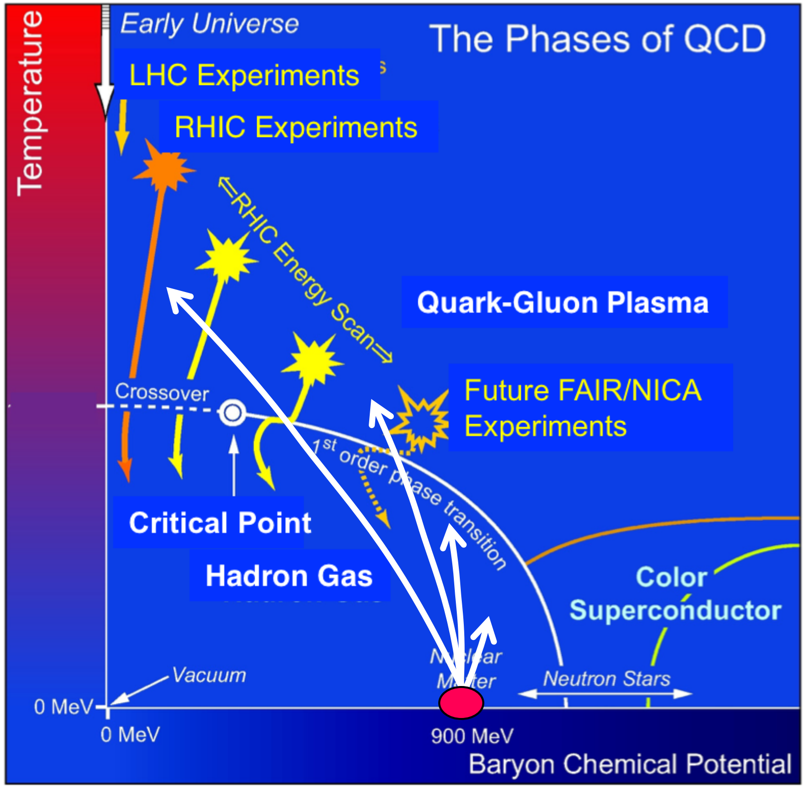
Landscape of nuclear physics

Quark-gluon plasma

hadrons

nuclei

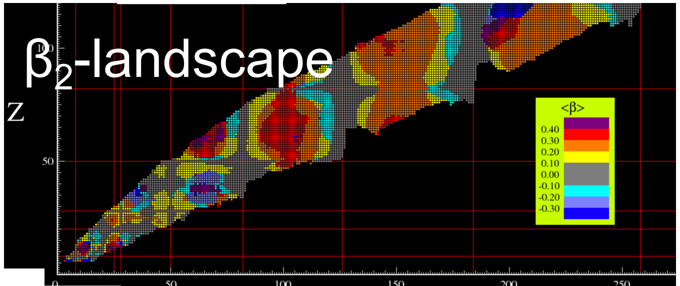


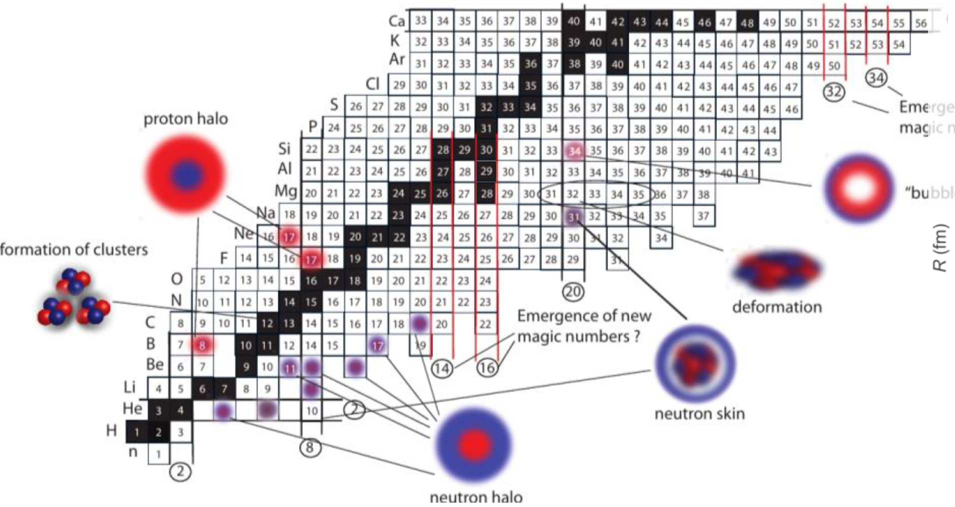


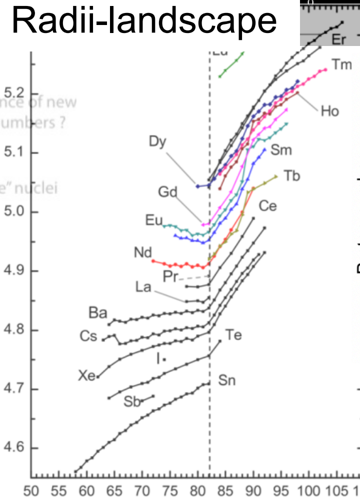
Most nuclear experiments starts with nuclei

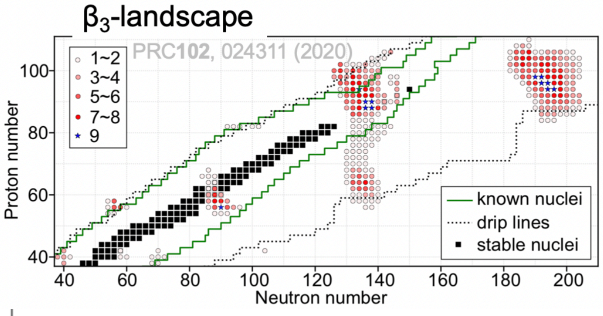
Rich structure of atomic nuclei

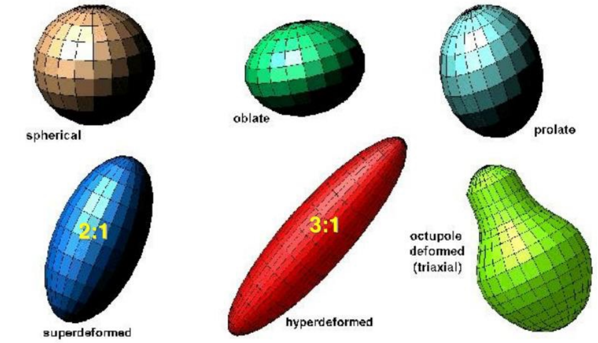
- Collective phenomena of many-body quantum system
  - clustering, halo, skin, bubble…
  - quadrupole/octupole/hexdecopole deformations
  - Nontrivial evaluation with N and Z.





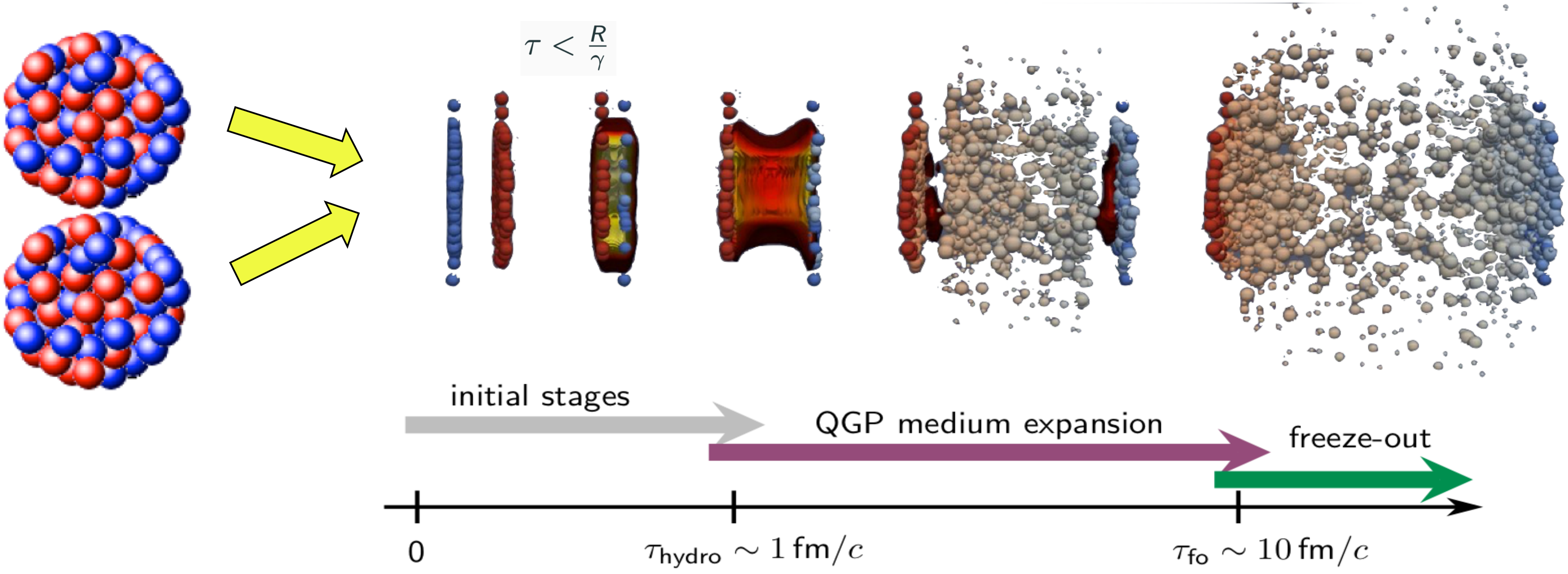






- Understanding via effective nuclear theories
  - Lattice, Ab.initio (starting from NN interaction)
  - Shell models (configuration interaction)
  - DFT models (non-relativistic and covariant)

High-energy heavy ion collision



400 nucleons → 30000 hadrons

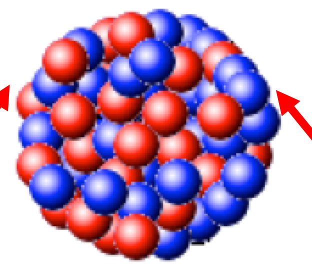
in 10^{-23} seconds

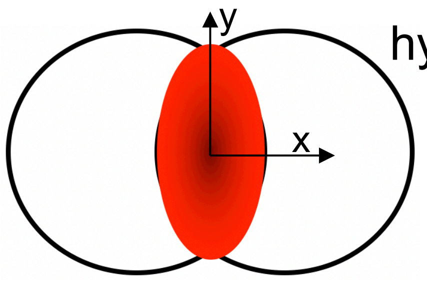
High-energy heavy ion collision

Nucleus

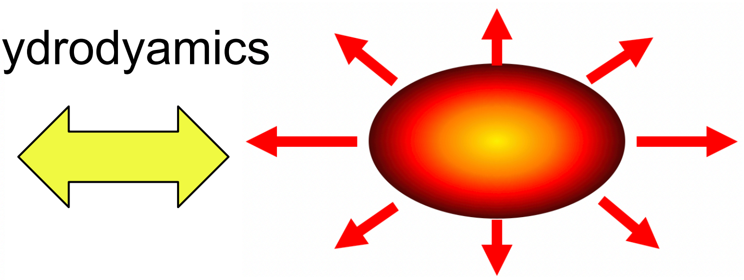
Initial condition

Final state



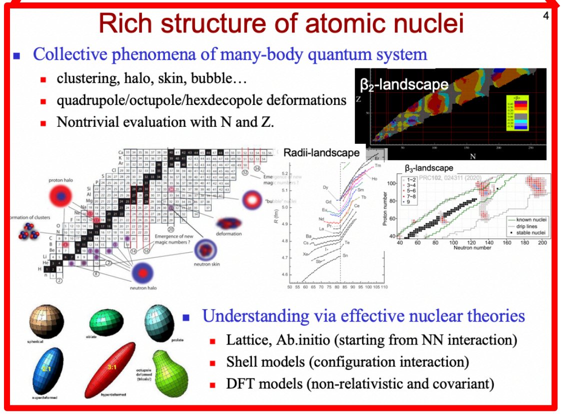


hydrodyamics



Asymmetric distribution

Anisotropic expansion



Rich structure of atomic nuclei

- Collective phenomena of many-body quantum system
  - clustering, halo, skin, bubble…
  - quadrupole/octupole/hexdecopole deformations
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- Understanding via effective nuclear theories
  - Lattice, Ab.initio (starting from NN interaction)
  - Shell models (configuration interaction)
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1) Are nuclear structures important for HI initial condition and final state evolution?

2) What HI experimental observables can be used to infer structure information?

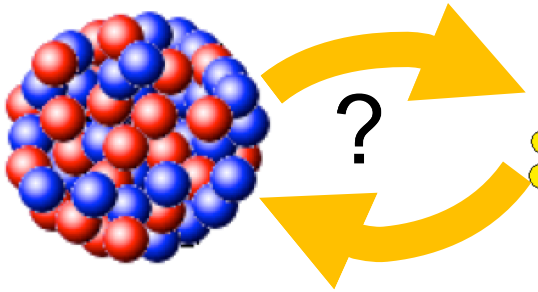
3) Can HI provides competitive constraints on nuclear shape and radial profile? can consideration of nuclear structure improves understanding of HI initial condition?

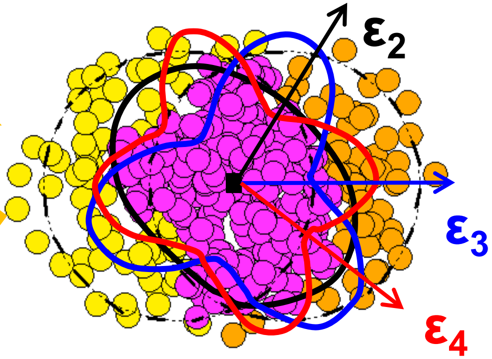
Collective flow in fluctuating events

Nucleus

Initial condition

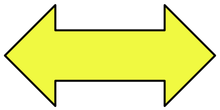
Final state

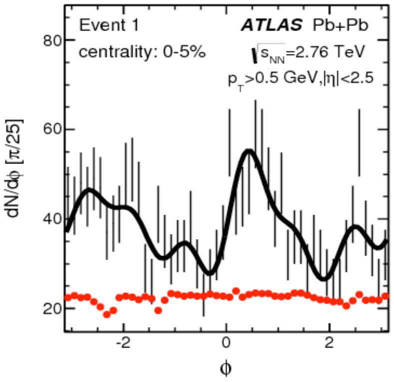




Fixed impact parameter

hydro

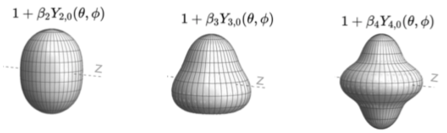




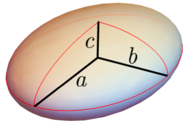
Nuclear structure

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}}$$

$$R(\theta,\phi) = R_0\left(1+\beta_2[\cos\gamma Y_{2,0}+\sin\gamma Y_{2,2}] + \beta_3\sum_{m=-3}^{3}\alpha_{3,m}Y_{3,m} + \beta_4\sum_{m=-4}^{4}\alpha_{4,m}Y_{4,m}\right)$$



Triaxial spheroid: $a \neq b \neq c$.



$$0 \leq \gamma \leq \pi/3$$

Initial volume

N_{part}

Initial Size

$$R_\perp^2 \propto \langle r_\perp^2 \rangle,$$

Initial Shape

$$\mathcal{E}_2 \propto \left\langle r_\perp^2 e^{i2\phi} \right\rangle$$

$$\mathcal{E}_3 \propto \left\langle r_\perp^3 e^{i3\phi} \right\rangle$$

$$\mathcal{E}_4 \propto \left\langle r_\perp^4 e^{i4\phi} \right\rangle$$

$$\cdots$$

Multiplicity

N_{ch}

Radial Flow

Harmonic Flow

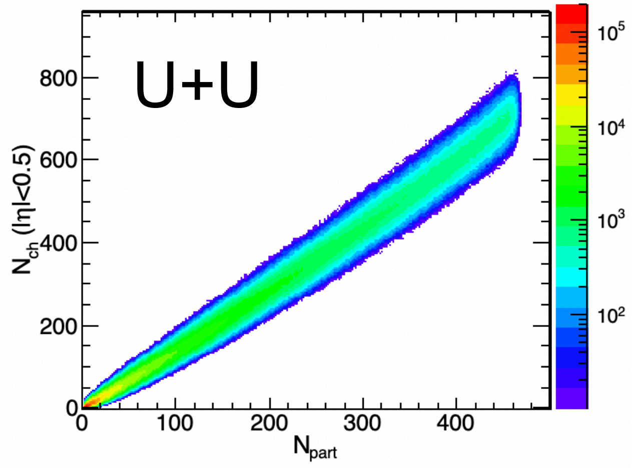
$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n\, e^{-in\phi}\right)$$

High energy: approx. linear response in each event:

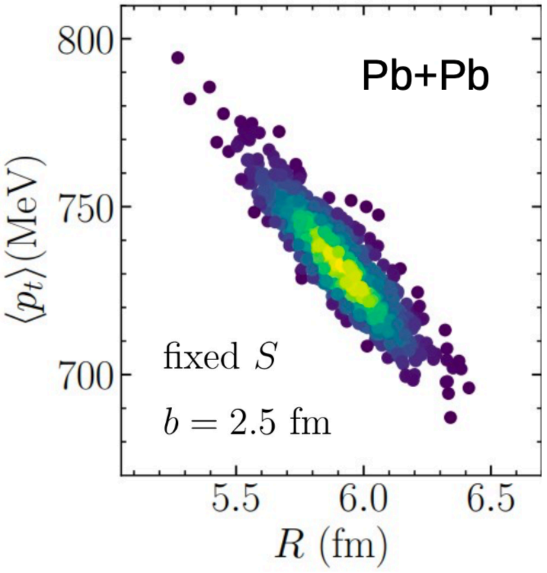
$$N_{ch} \propto N_{part} \qquad \frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \qquad V_n \propto \mathcal{E}_n$$

Linear corr. between initial & final state

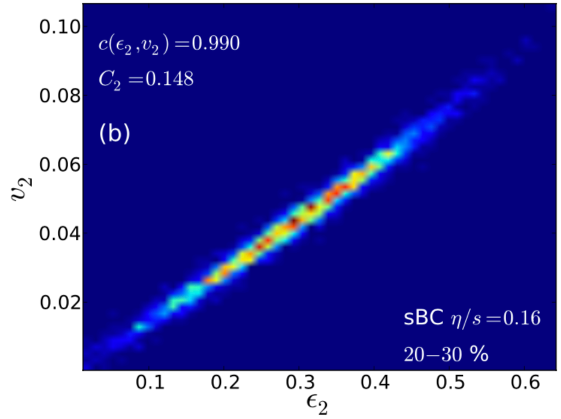
$$N_{ch} \propto N_{part}$$

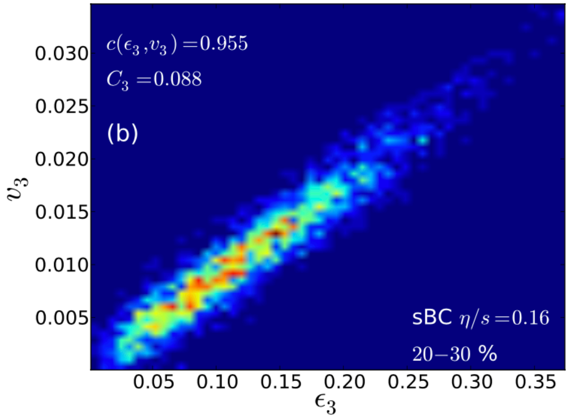


$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp}$$



$$V_n \propto \mathcal{E}_n$$





nice correlation at high energy

Zoo of Flow observables

- Single particle distribution

Flow vector: $\boldsymbol{V}_n = v_n e^{in\Psi_n}$

$$\frac{d^2N}{d\phi dp_{\mathrm{T}}} = N(p_T)\left[1 + 2\sum_n v_{\mathrm{n}}(p_T)\cos n(\phi - \Psi_n(p_T))\right]$$

$$= N(p_T)\left[\sum_{n=-\infty}^{\infty} V_{\mathrm{n}}(p_T)e^{in\phi}\right]$$

Radial flow

Anisotropic flow

- Two-particle correlation function

$$\left\langle \frac{d^2N_1}{d\phi dp_{\mathrm{T}}}\frac{d^2N_2}{d\phi dp_{\mathrm{T}}}\right\rangle \Rightarrow \langle \boldsymbol{V}_n(p_{T1})\boldsymbol{V}_n^*(p_{T2})\rangle \qquad n - n = 0$$

- Multi-particle correlation function

$$\left\langle \frac{d^2N_1}{d\phi dp_{\mathrm{T}}}\cdots\frac{d^2N_m}{d\phi dp_{\mathrm{T}}}\right\rangle \Rightarrow \left\langle V_{n_1}V_{n_2}...V_{n_m}\right\rangle \qquad n_1 + n_2 + ... + n_m = 0$$

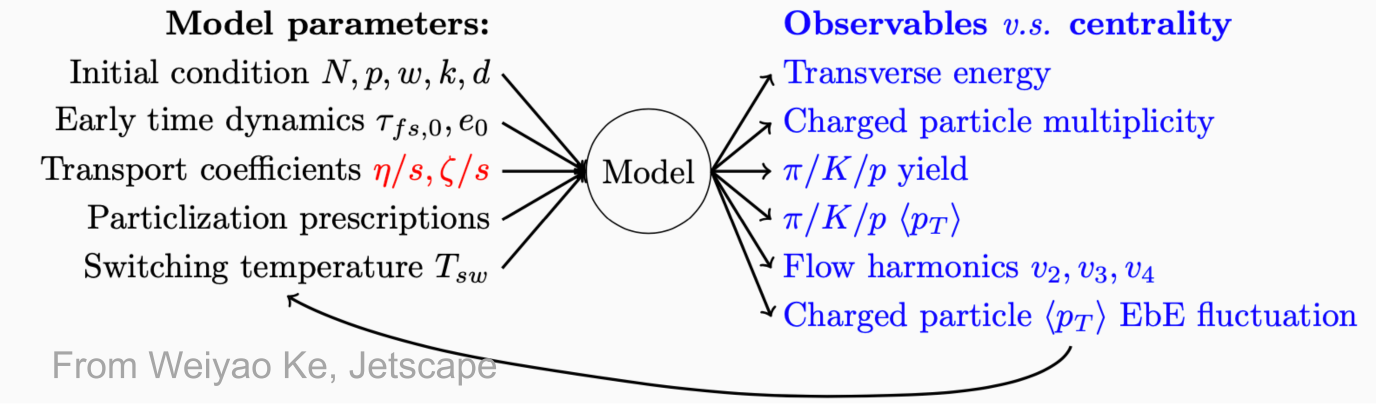
$$\downarrow$$

$$p(\delta[p_T], \boldsymbol{V}_2, \boldsymbol{V}_3\ldots) = \frac{1}{N_{\mathrm{evts}}}\frac{dN_{\mathrm{evts}}}{d\delta[p_T]d\boldsymbol{V}_2 d\boldsymbol{V}_3\ldots}$$

State-of-the-art modeling of HI collisions

- Data-model comparison via Bayesian inference to optimize constraining power.

Only a subset observables is used



Model parameters:

Initial condition N, p, w, k, d

Early time dynamics $\tau_{fs,0}, e_0$

Transport coefficients $\eta/s, \zeta/s$

Particlization prescriptions

Switching temperature T_{sw}

Model

Observables *v.s.* centrality

Transverse energy

Charged particle multiplicity

$\pi/K/p$ yield

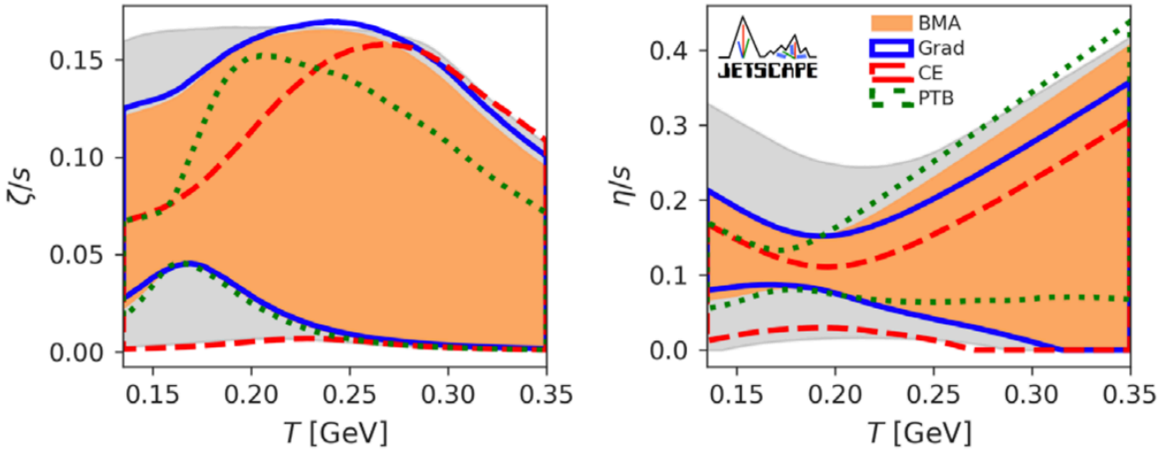
$\pi/K/p$ $\langle p_T \rangle$

Flow harmonics v_2, v_3, v_4

Charged particle $\langle p_T \rangle$ EbE fluctuation

From Weiyao Ke, Jetscape

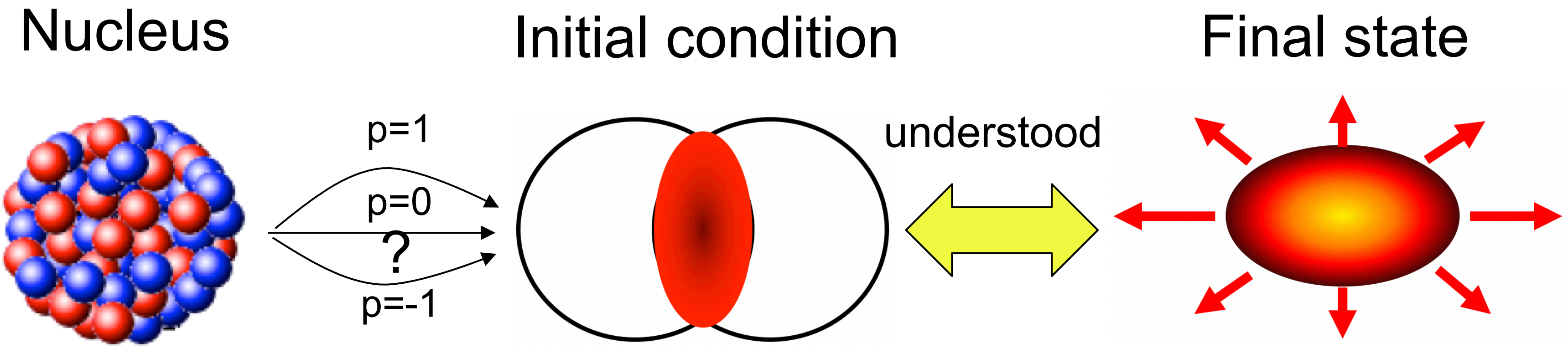
- Detailed temperature dependence of viscosity!



Jetscape PRL.126.242301
Trjactum PRL.126.202301

Major uncertainty: initial condition and pre-hydro phase

The role of nuclear structure



$$T_A(x,y) = \int \rho(x,y,z)dz$$

- Different ways of depositing energy $T \propto \left(\frac{T_A^p + T_B^p}{2}\right)^{q/p}$

$$e(x,y) \sim \begin{cases} T_A + T_B & N_{\rm part} - \text{scaling}, p = 1 \\ T_A T_B & N_{\rm coll} - \text{scaling}, p = 0, q = 2 \\ \sqrt{T_A T_B} & \text{Trento default}, p = 0 \\ \min\{T_A, T_B\} & \text{KLN model}, p \sim -2/3 \\ T_A + T_B + \alpha T_A T_B & \text{two-component model}, \\ & \text{similar to quark-glauber model} \end{cases}$$

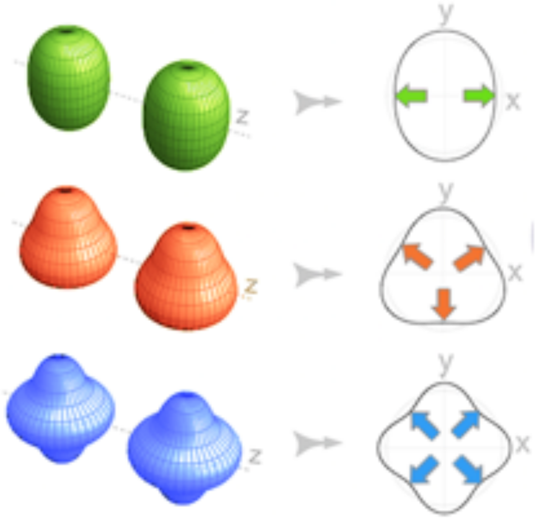
- Use nuclear structure to provide extra lever-arm for initial condition？

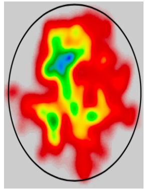
The role of nuclear structure

Initial condition

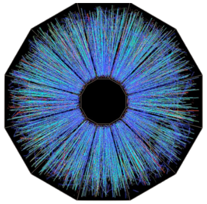
Final state

Nuclear shape:





ε_2↗



v_2↗

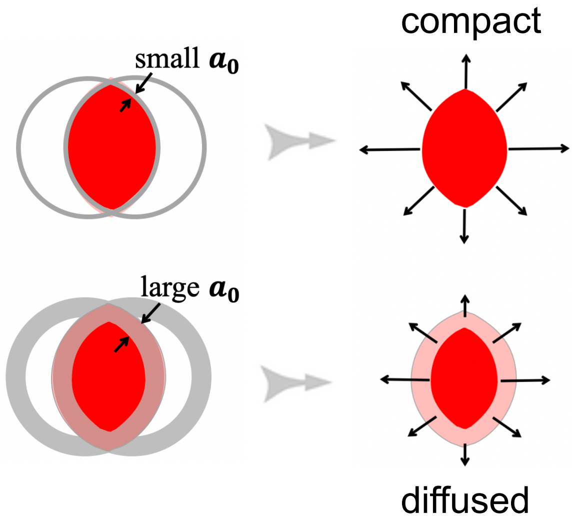
ε_3↗

v_3↗

ε_4↗

v_4↗

Radial profile:



ε_2↗ R↘

v_2↗ p_T↗

ε_2↘ R↗

v_2↘ p_T↘

Parametric form

- In principle, can measure any moments of p(1/R, ε_2, ε_3…)

Mean	$\langle d_\perp\rangle$		$\langle p_\mathrm{T}\rangle$
Variances:	$\langle\varepsilon_n^2\rangle, \left\langle(\delta d_\perp/d_\perp)^2\right\rangle$ $\quad d_\perp \equiv 1/R_\perp$	⇒	$\langle v_n^2\rangle, \left\langle(\delta p_\mathrm{T}/p_\mathrm{T})^2\right\rangle$
Skewness	$\langle\varepsilon_n^2\delta d_\perp/d_\perp\rangle, \left\langle(\delta d_\perp/d_\perp)^3\right\rangle$		$\langle v_n^2\delta p_\mathrm{T}/p_\mathrm{T}\rangle, \left\langle(\delta p_\mathrm{T}/p_\mathrm{T})^3\right\rangle$
Kurtosis	$\langle\varepsilon_n^4\rangle - 2\langle\varepsilon_n^2\rangle^2, \left\langle(\delta d_\perp/d_\perp)^4\right\rangle - 3\left\langle(\delta d_\perp/d_\perp)^2\right\rangle^2$		$\langle v_n^4\rangle - 2\langle v_n^2\rangle^2, \left\langle(\delta p_\mathrm{T}/p_\mathrm{T})^4\right\rangle - 3\left\langle(\delta p_\mathrm{T}/p_\mathrm{T})^2\right\rangle^2$
…			

- All with rather simple connection to deformation, for example:
  - Variances

$$\langle\varepsilon_n^2\rangle \approx a_n + \sum_{m,m'} b_{n;m,m'}\beta_m\beta'_m$$

$$\left\langle(\delta d_\perp/d_\perp)^2\right\rangle \approx a_0 + \sum_{m,m'} b_{0;m,m'}\beta_m\beta'_m$$

Specifically:

$$\langle\varepsilon_2^2\rangle \sim a_2 + b_2\beta_2^2 + b_{2,3}\beta_3^2$$

$$\langle\varepsilon_3^2\rangle \sim a_3 + b_3\beta_3^2 + b_{3,2}\beta_2^2 + b_{3,4}\beta_4^2$$

$$\langle\varepsilon_4^2\rangle \sim a_4 + b_4\beta_4^2 + b_{4,2}\beta_2^2$$

$$\left\langle(\delta d_\perp/d_\perp)^2\right\rangle \sim a_0 + b_0\beta_2^2 + b_{0,3}\beta_3^2$$

  - Skewness

$$\left\langle\varepsilon_2^2\delta d_\perp/d_\perp\right\rangle \sim a_1 - b_1\cos(3\gamma)\beta_2^3$$

$$\left\langle(\delta d_\perp/d_\perp)^3\right\rangle \sim a_2 + b_2\cos(3\gamma)\beta_2^3$$

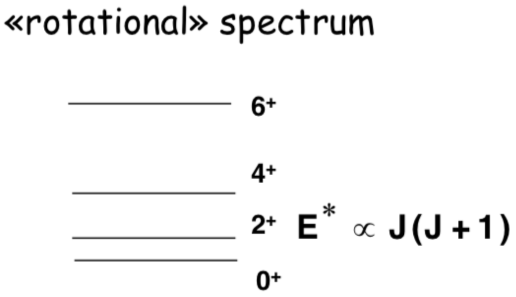
  - Kurtosis

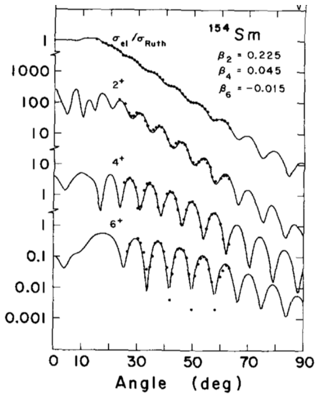
$$\left\langle\varepsilon_2^4\right\rangle - 2\left\langle\varepsilon_2^2\right\rangle^2 \sim a_3 - b_3\beta_2^4$$

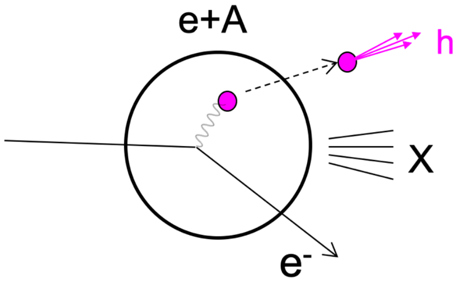
$$\left\langle(\delta d_\perp/d_\perp)^4\right\rangle - 3\left\langle(\delta d_\perp/d_\perp)^2\right\rangle^2 \sim a_4 - b_4\beta_2^4$$

Low-energy vs high-energy HI method

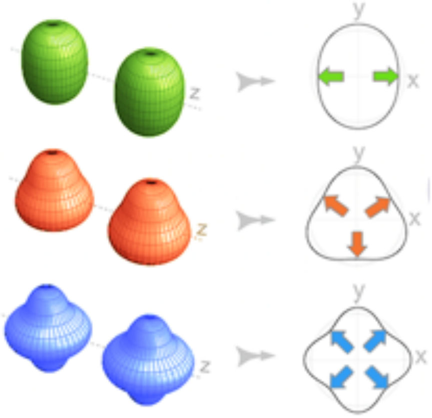
- Shape from B(En), radial profile from e+A or ion-A scattering



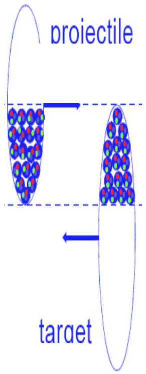


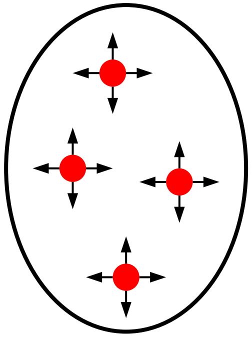


- Shape frozen in crossing time ($<10^{-24}$s), probe entire mass distribution via multi-point correlations.



Collective flow response to nuclear structure

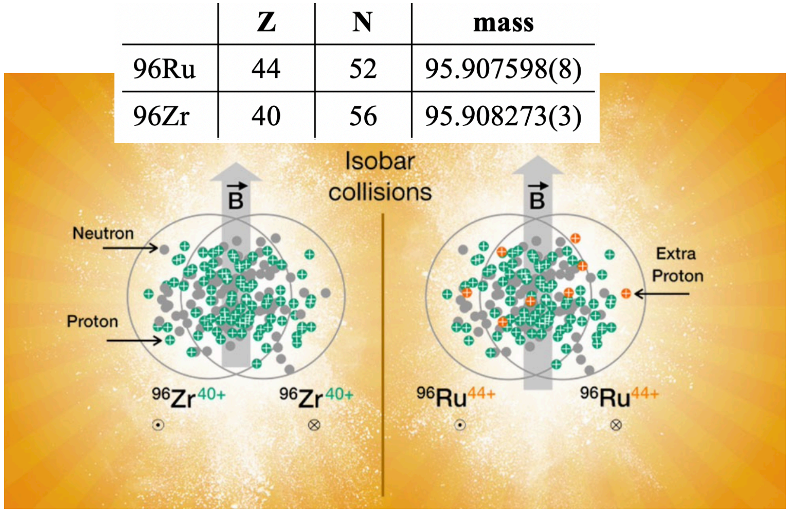




$$
\begin{aligned}
S(\mathbf{s}_1,\mathbf{s}_2) &\equiv \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2)\rangle \\
&= \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2)\rangle - \langle \rho(\mathbf{s}_1)\rangle\langle \rho(\mathbf{s}_2)\rangle .
\end{aligned}
$$

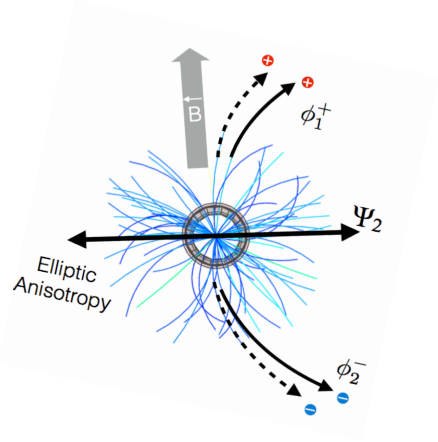
But how to achieve precision?

Isobar collisions at RHIC: context



	Z	N	mass
96Ru	44	52	95.907598(8)
96Zr	40	56	95.908273(3)

arXiv:2109.00131



Voloshin, hep-ph/0406311

- Designed to search for the chiral magnetic effect: strong P & CP violation in the presence of EM field. Experimental signature is a spontaneous separation of + and - hadrons along EM direction, vertical to $\mathcal{E}_2$

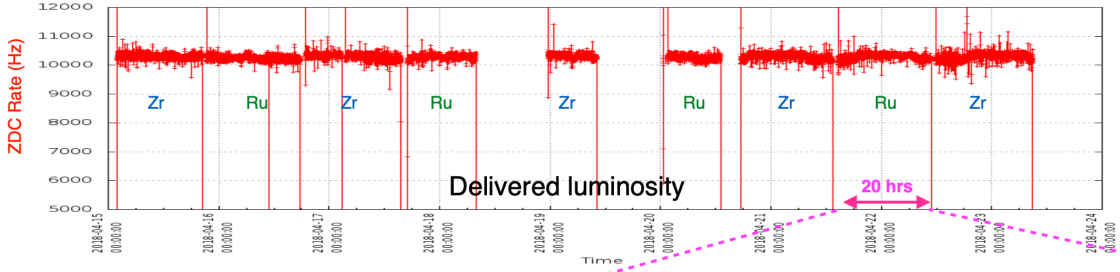
- Turns out the CME signal is small, and isobar-differences are dominated by the nuclear structure differences.

Isobar collisions at RHIC/STAR

RHIC Running

From J Drachenberg

- Switch isobar species each time beam is inserted into RHIC
- Stable luminosity (matched between species) with long (~20 hour) beam circulation time
- Adjust and level luminosity to optimize data collection rate while minimizing backgrounds and systematics
- Restrict species-related information to those necessary for successful data-taking
- Calibration experts (recused from CME analyses) evaluate data quality "in real time"



STAR : predefined blind analysis

Step-0
Mock Data challenge
Test data structure (Au+Au data)

→

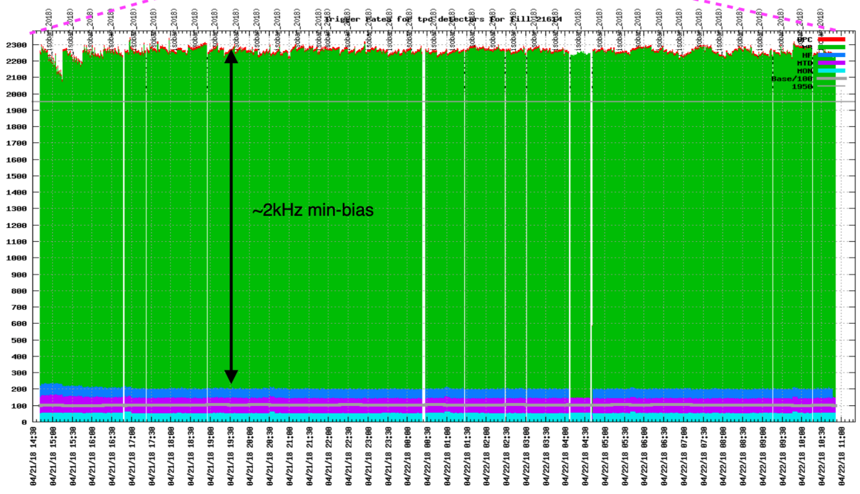
Step-1
Isobar Mixed analysis
Code freezing (Each run is Ru+Ru & Zr+Zr)

→

Step-2
Isobar Blind analysis
QA with ~ 1% data (Each run is Ru+Ru or Zr+Zr)

→

Step-3
Isobar Unblind analysis
Final analysis (Ru+Ru & Zr+Zr separated)



STAR Data Acquisition Rates

<0.4% precision is achieved in ratio of many observables between ^{96}Ru+ ^{96}Ru and ^{96}Zr+^{96}Zr systems→ precision imaging tool

Isobar collisions as precision tool

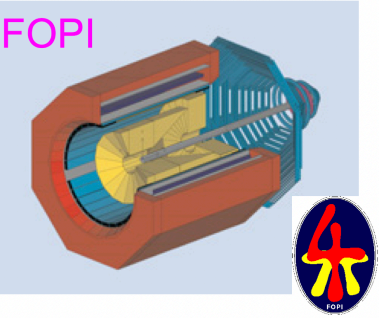
- A key question for any HI observable O:

$$\frac{O_{^{96}\mathrm{Ru}+^{96}\mathrm{Ru}}}{O_{^{96}\mathrm{Zr}+^{96}\mathrm{Zr}}} \stackrel{?}{=} 1$$

Deviation from 1 must has origin in the nuclear structure, which impacts the initial state and then survives to the final state.

- Many such pairs of isobars in the nuclear chart.
  - Small system isobar such as 36Ar and 36S.
  - Large system isobar such as 204Hg and 204Pb

- Isobar also done at low energy, e.g FOPI, with different physics focus
  - Baryon stopping and isospin sensitive observables.
  - No clear separation between initial and final state
  - Smaller hadron multiplicity limits the precision



Isobar collisions as precision tool

- A key question for any HI observable O:

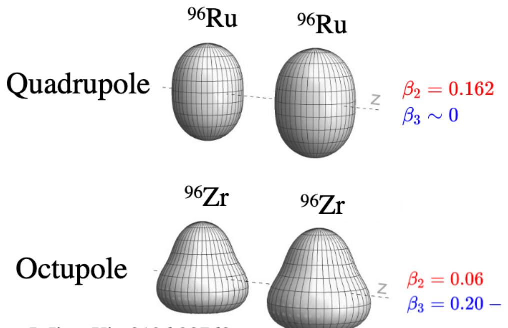
$$\boxed{\frac{O_{^{96}\mathrm{Ru}+^{96}\mathrm{Ru}}}{O_{^{96}\mathrm{Zr}+^{96}\mathrm{Zr}}} \overset{?}{=} 1}$$

Deviation from 1 must has origin in the nuclear structure, which impacts the initial state and then survives to the final state.

- Expectation

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}}$$

$$R(\theta,\phi) = R_0\left(1+\beta_2[\cos\gamma Y_{2,0}+\sin\gamma Y_{2,2}]+\beta_3\sum_{m=-3}^{3}\alpha_{3,m}Y_{3,m}+\beta_4\sum_{m=-4}^{4}\alpha_{4,m}Y_{4,m}\right)$$



J. Jia, aXiv:2106.08768

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

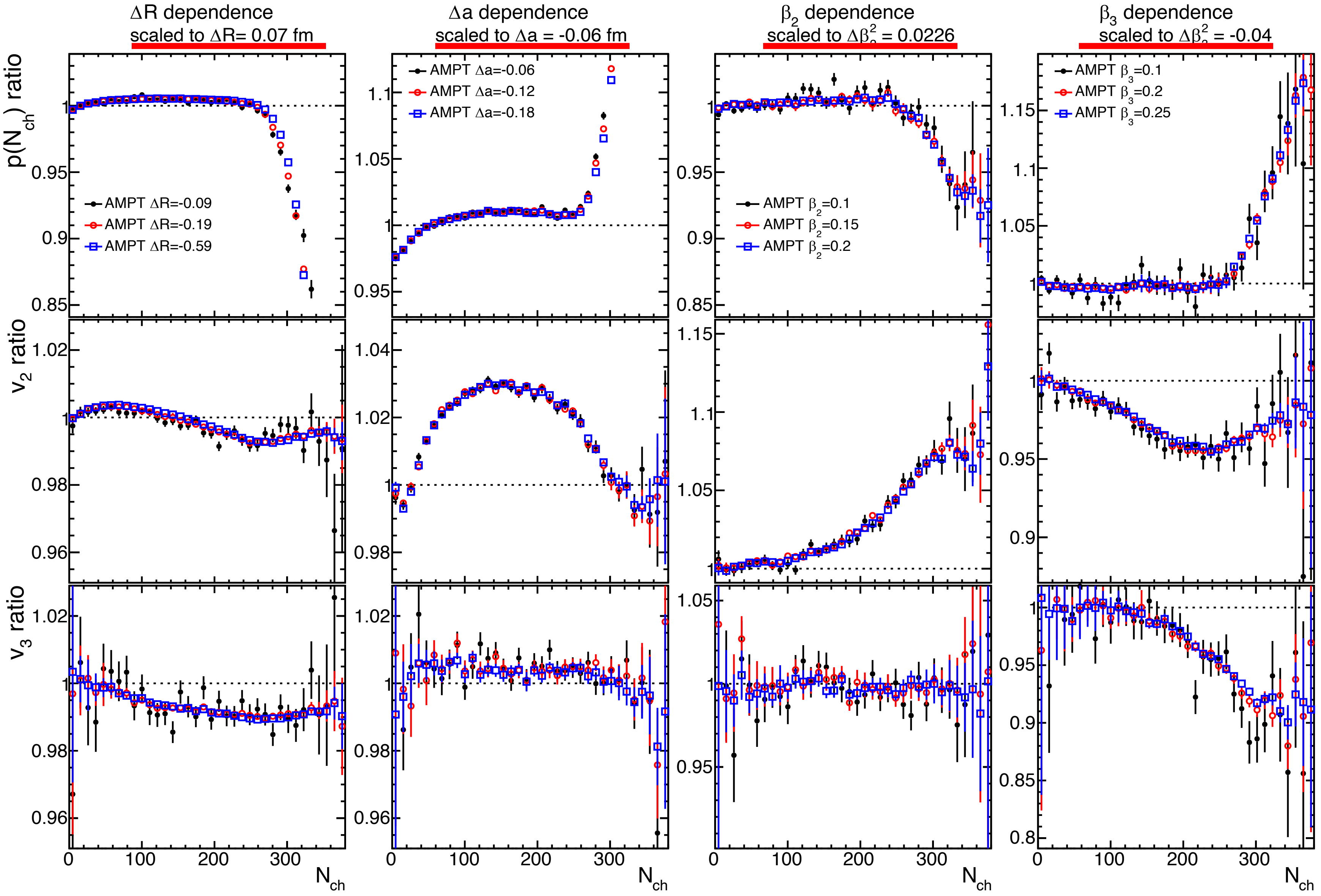
$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

Valid for most single- and two-particle observable: v_2,v_3, p(N_{ch}),<p_T>,<δp_T^2>..

Only probes isobar differences

AMPT results: scaled



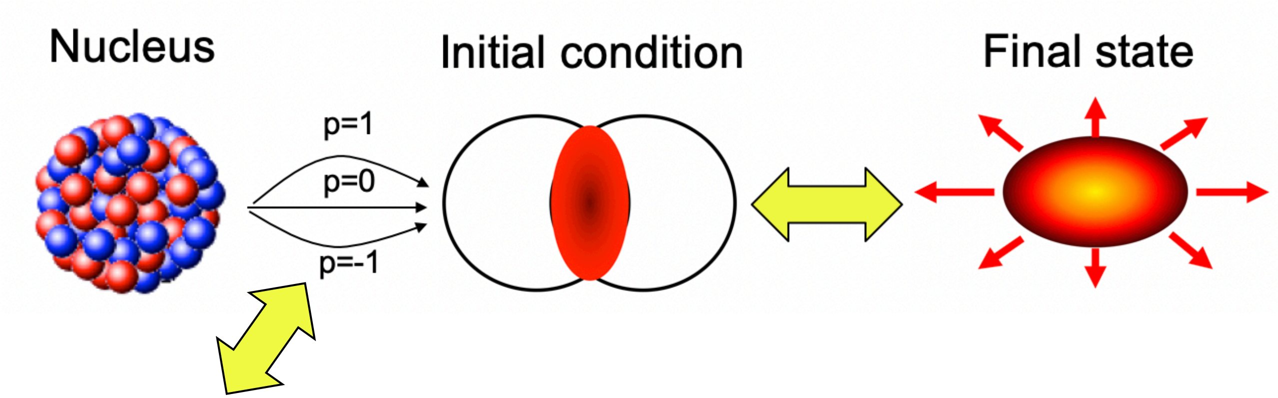
Verifies the relation: $1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta a + c_4\Delta R$

Scaling approach to nuclear structure

arXiv:2111.15559

Valid for most single- and two-particle observable: v_2,v_3, $p(N_{ch})$,$\langle p_T \rangle$,$\langle \delta p_T^2 \rangle$..

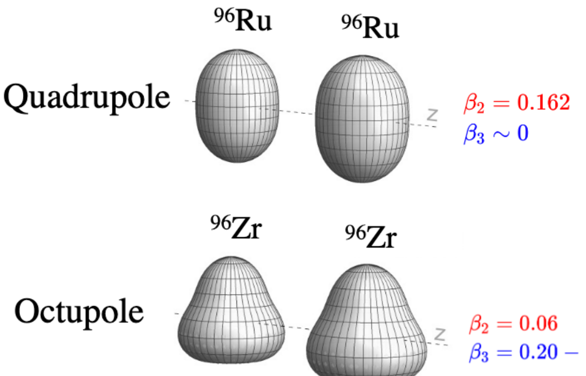
$$\frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$



c_n relates nuclear structure and initial condition

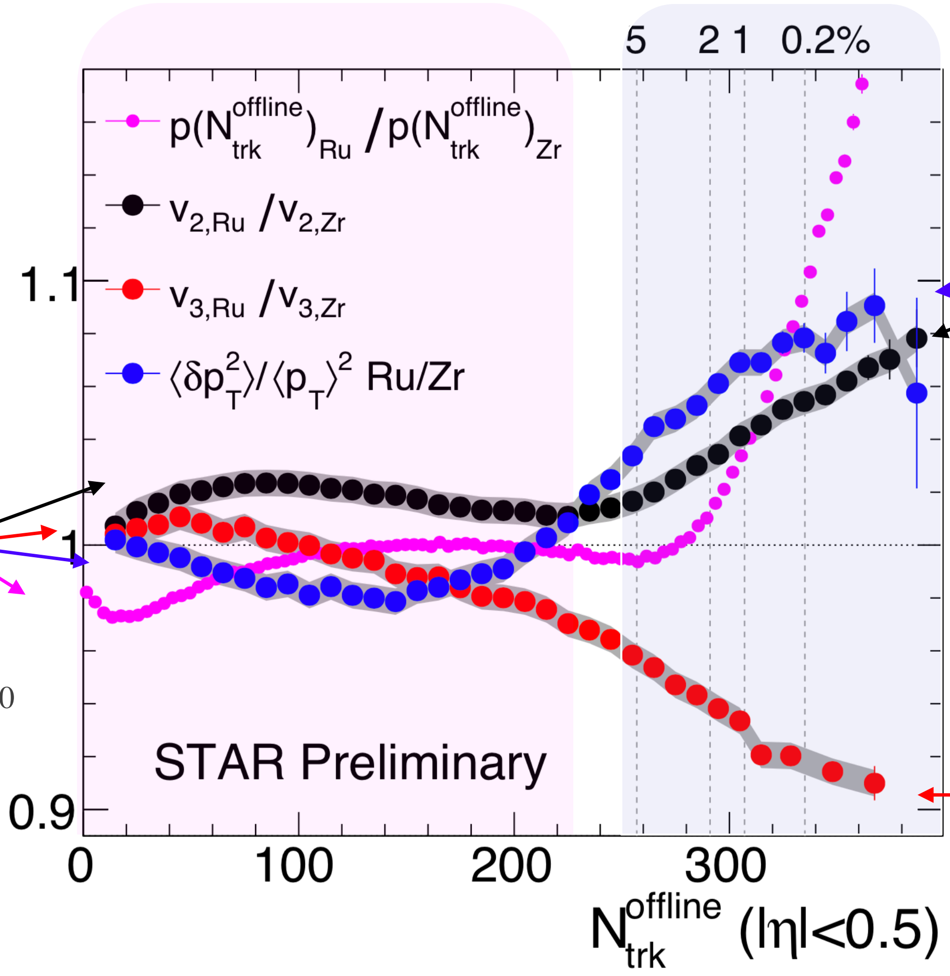
- Determine c_n once, and predict ratios for other NS parameter values.
- Constrain parameters via χ^2 analysis or Bayesian inference.

Compare with isobar data



J. Jia, aXiv:2106.08768

From chunjian DNP STAR talk



Large $\beta_{2,Ru}$

Large $\beta_{3,Zr}$

Nuclear deformation
+ Radial structures

STAR, arXiv:2109.00131

H. Xu et al., arXiv:1808.06711,1910.06170
2103.05595, 2111.14812

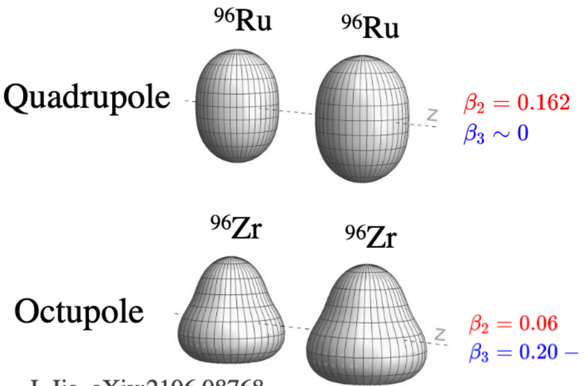
Q. Shou et al., arXiv:1409.8375

G. Giacalone et al., 2105.01638

C. Zhang et al., arXiv:2109.01631

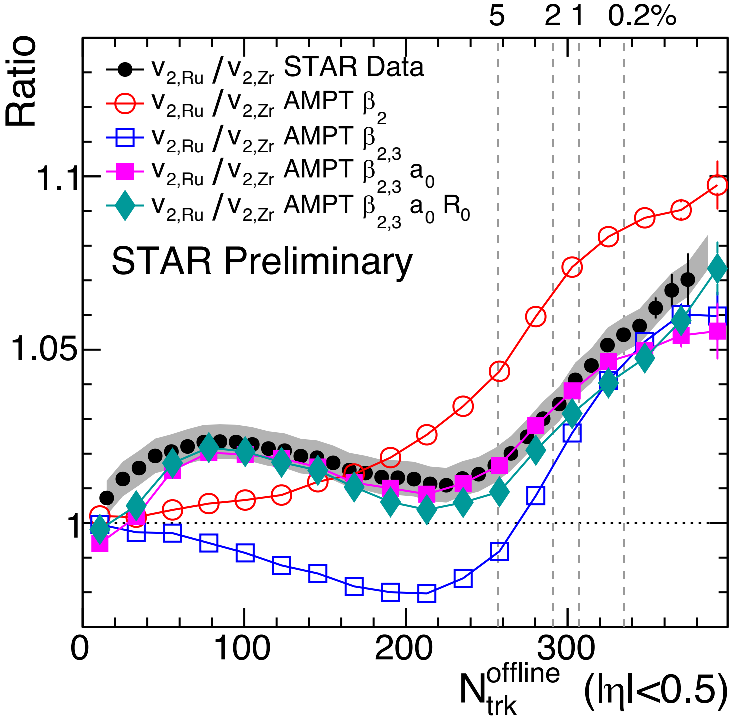
Use these ratios to probe shape and radial structure of nuclei.

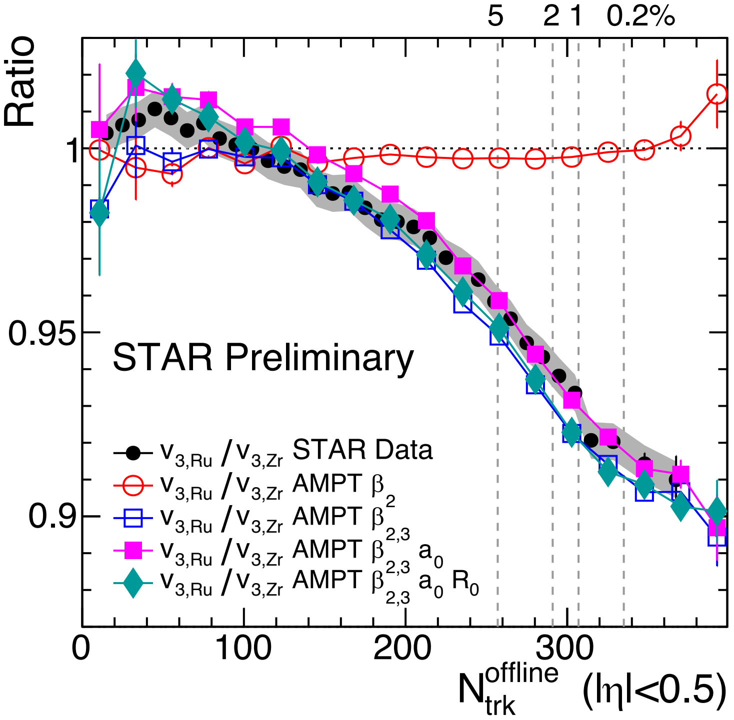
Nuclear structure via v_n-ratio



J. Jia, aXiv:2106.08768

- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio
- β_{3Zr}~0.2 decrease v_2 in mid-central, decrease v_3 ratio
- Δa_0= -0.06fm increase v_2 mid-central, small influ. on v_3.
- Radius ΔR_0=0.07fm only slightly affects v_2 and v_3 ratio.





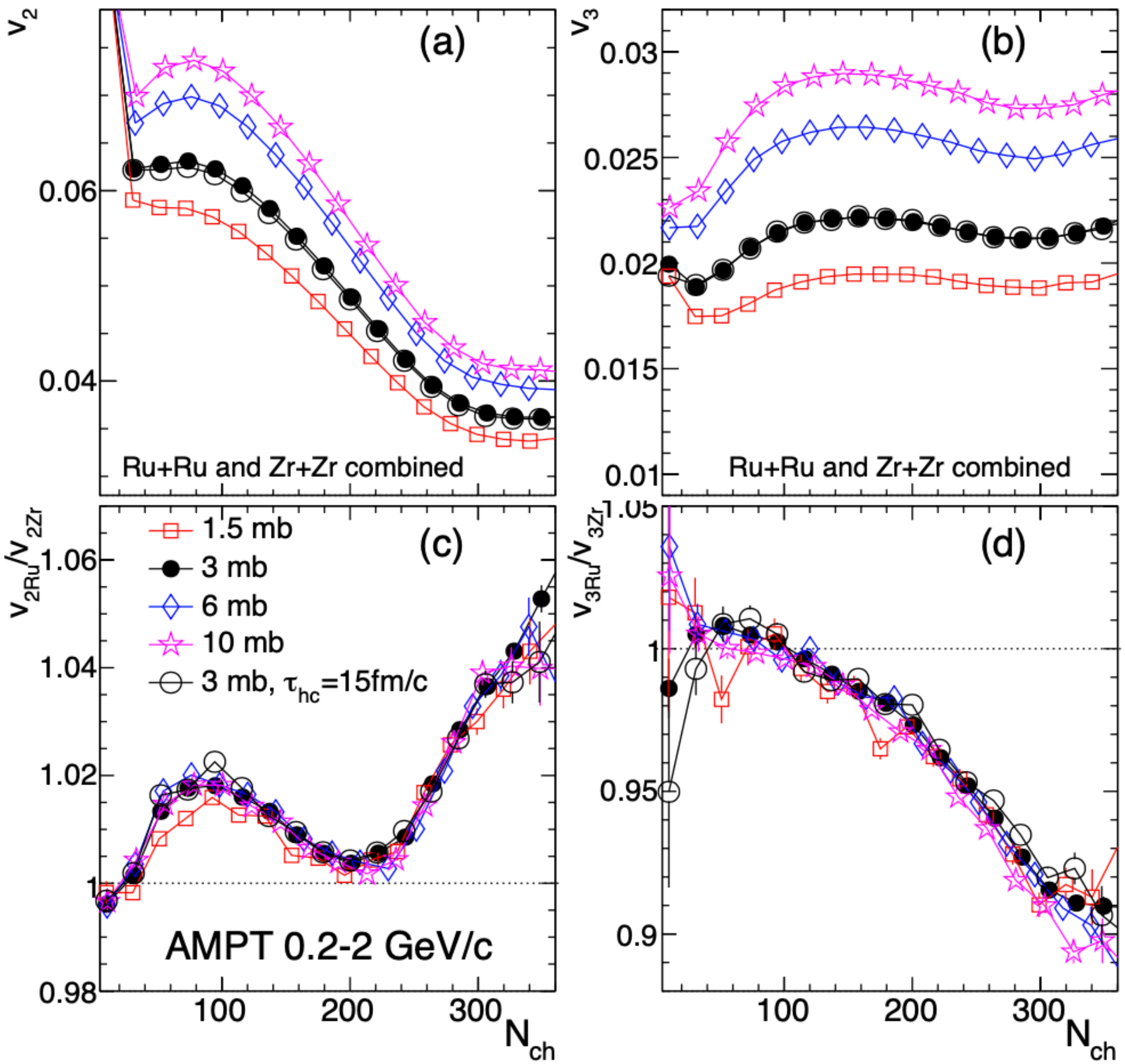
Simultaneously constrain these parameters using different N_{ch} regions

Not affected by final state

- Vary the shear viscosity via partonic cross-section
  - Flow signal change by 30-50%, the v_n ratio unchanged.

$$v_n = k_n \varepsilon_n$$

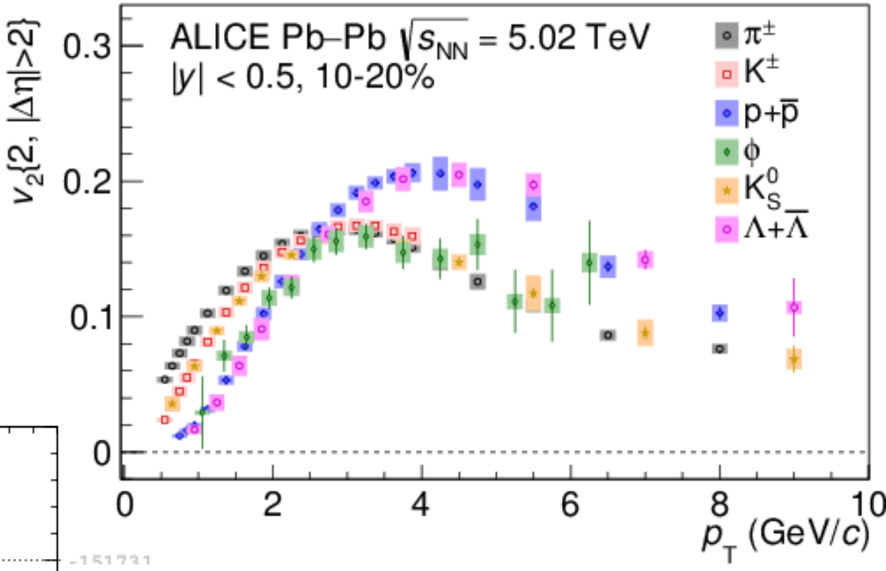
$$\frac{v_{n,\mathrm{Ru}}}{v_{n,\mathrm{Zr}}} \approx \frac{\varepsilon_{n,\mathrm{Ru}}}{\varepsilon_{n,\mathrm{Zr}}}$$

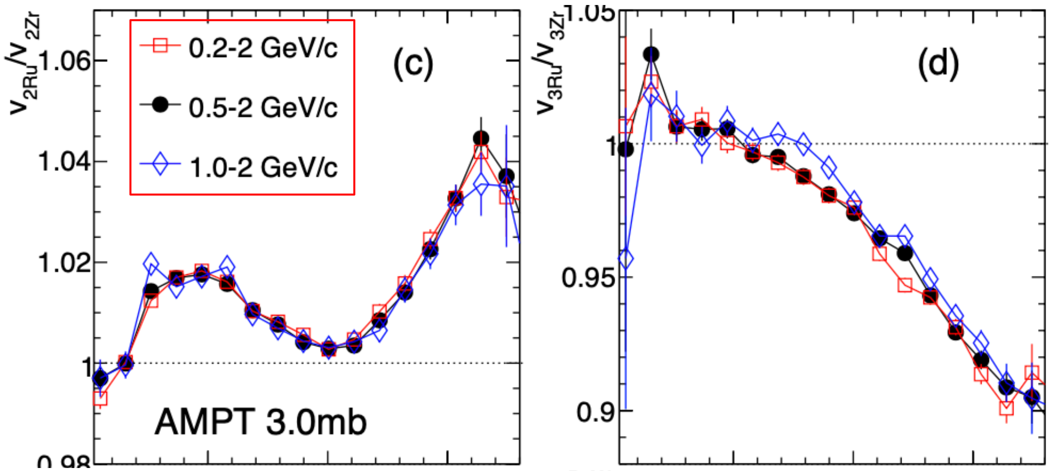


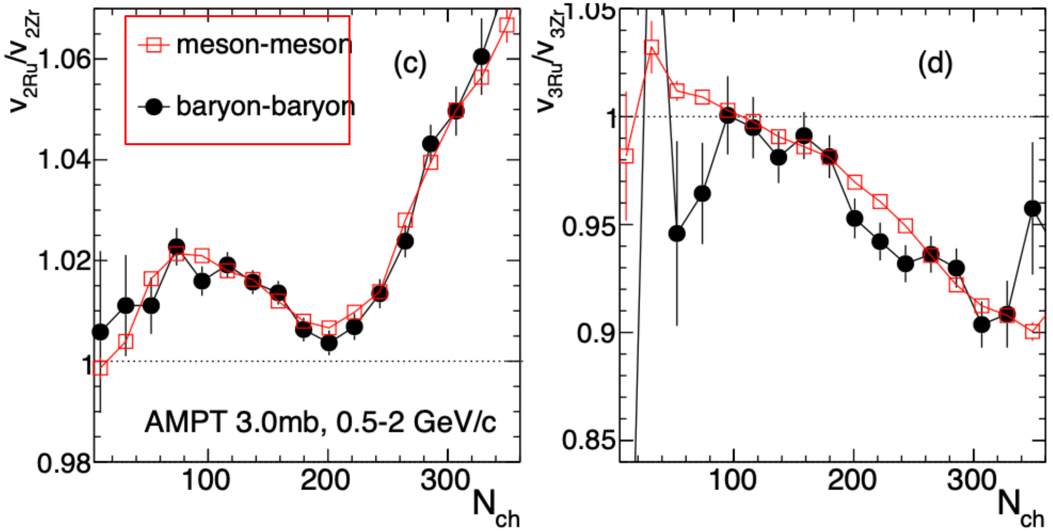
Not affected by final state

$$v_n = k_n \varepsilon_n$$

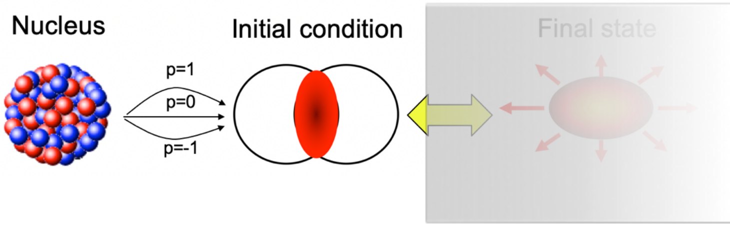
k_n is a function of p_T and PID,
But fully cancels in the ratio



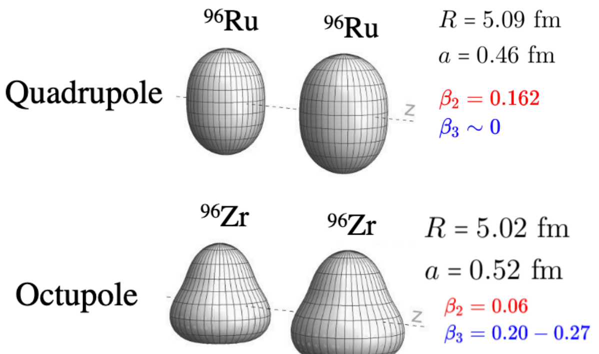




Robust probe of
initial state!

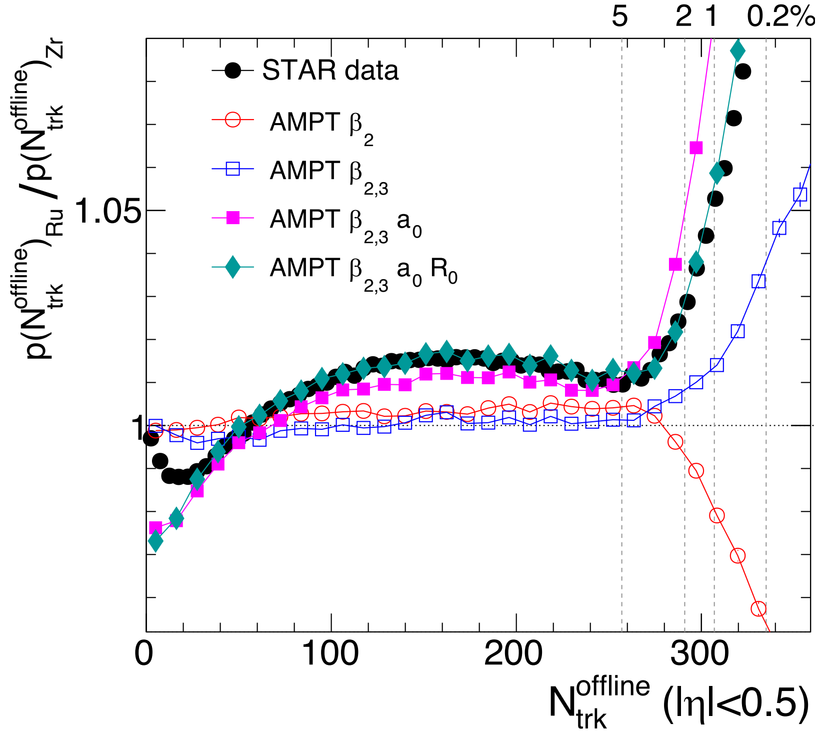


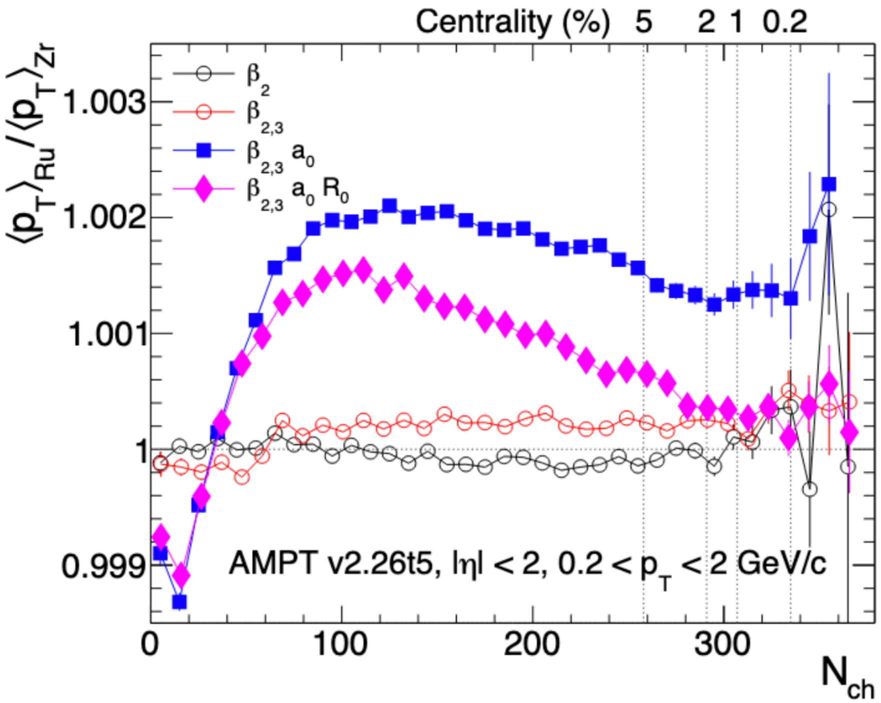
Nuclear structure via $p(N_{ch})$, <pT>-ratio



J. Jia, aXiv:2106.08768

- For Nch ratio:
  - β_{2Ru}~0.16 decrease ratio, increase after considering β_{3Zr}~0.2
  - The bump structure in non-central region from Δa_0 and ΔR_0
- For <pT> ratio:
  - Strong influence from Δa_0 and ΔR_0





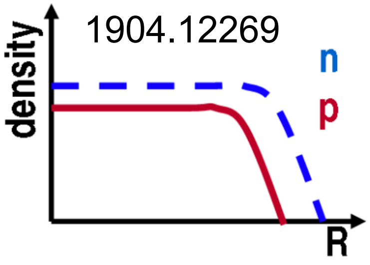
Earlier studies on this from H.Li, H.J Xu, PRL125, 222301 (2020) arXiv:2111.14812

Relating to the Neutron Skin

$$\rho = \frac{\rho_0}{1 + e^{-(r-R_0)/a}}$$

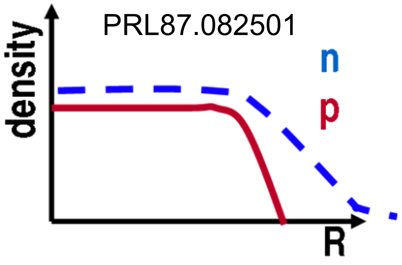
Neutron skin:

$$\Delta r_{np} = \left\langle r_n^2 \right\rangle^{1/2} - \left\langle r_p^2 \right\rangle^{1/2}$$



1904.12269

skin-type: larger R_0



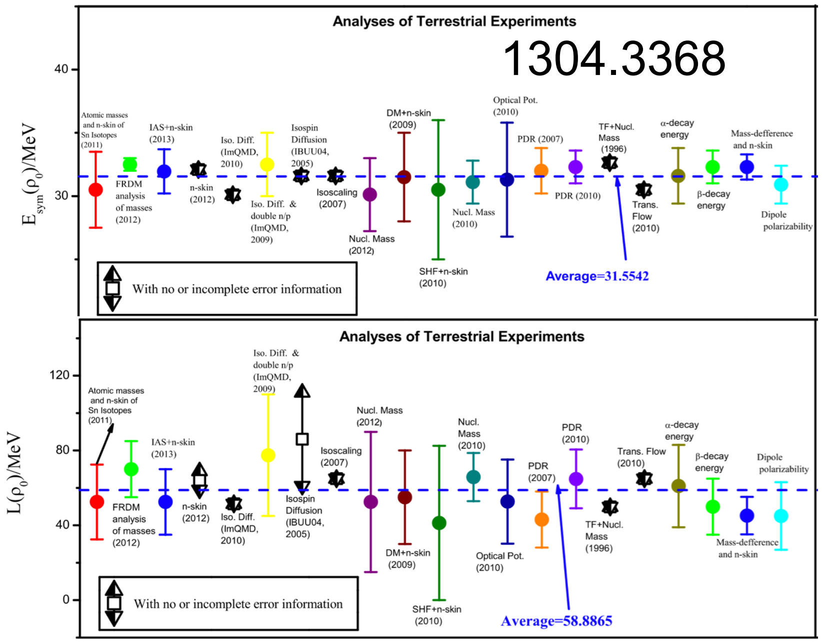
PRL87.082501

halo-type: larger a_0

Related to the EOS of symmetry energy, in particular the slope parameter “**L**”

$$E_{\rm sym}(\rho) \approx J + Lx + \frac{1}{2}K_{\rm sym}x^2 \quad x = (\rho - \rho_{\rm sat})/3\rho_{\rm sat}$$

Many constraints from structure and low-energy heavy-ion experiments



1304.3368

Relating to neutron skin: $\Delta r_{\mathrm{np}} = \langle r_n \rangle^{1/2} - \langle r_p \rangle^{1/2}$

arXiv:2111.15559

Neutron skin Δ_{np} can be expressed by R_0 and a_0 for nucleons and protons:

$$\Delta r_{np} \approx \frac{\langle r^2 \rangle - \langle r_p^2 \rangle}{\sqrt{\langle r^2 \rangle}(\delta + 1)} \qquad \delta = (N - Z)/A$$

For Woods-Saxon: $\langle r^2 \rangle \approx \left(\frac{3}{5} R_0^2 + \frac{7}{5} \pi^2 a^2 \right)$ $\langle r_p^2 \rangle \approx \left(\frac{3}{5} R_{0,p}^2 + \frac{7}{5} \pi^2 a_p^2 \right)$

Isobar collision measure "difference of neutron skin" from ΔR_0 Δa for nucleons, and known ΔR_0 Δa for protons:

$$\boxed{\Delta(\Delta r_{np}) = \Delta r_{np,1} - \Delta r_{np,2} \approx \frac{\Delta Y - \frac{7\pi^2}{3} \frac{\bar{a}^2}{\bar{R}_0^2} \left(\frac{\Delta Y}{2} + \bar{Y} \left(\frac{\Delta a}{\bar{a}} - \frac{\Delta R_0}{\bar{R}_0} \right) \right)}{\sqrt{15} \bar{R}_0 \left(1 + \bar{\delta} \right)}}$$

$$\Delta x = x_1 - x_2$$
$$\bar{x} = (x_1 + x_2)/2$$

$$Y \equiv 3(R_0^2 - R_{0,p}^2) + 7\pi^2 (a^2 - a_p^2)$$

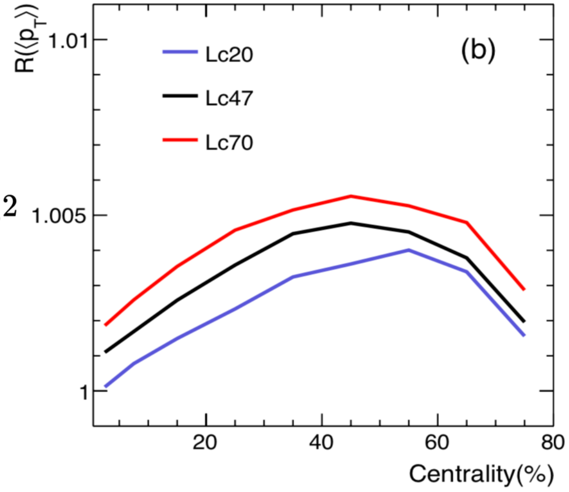
Hydro-response to Neutron skin

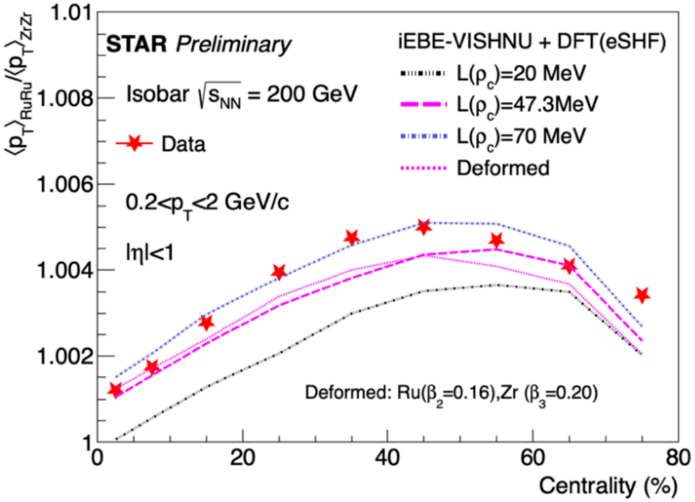
Sensitive to L parameter via hydro response: $\langle p_\perp \rangle \sim d_\perp \equiv \sqrt{N_{\text{part}} / S_\perp}$

H. Xu et.al.2111.14812

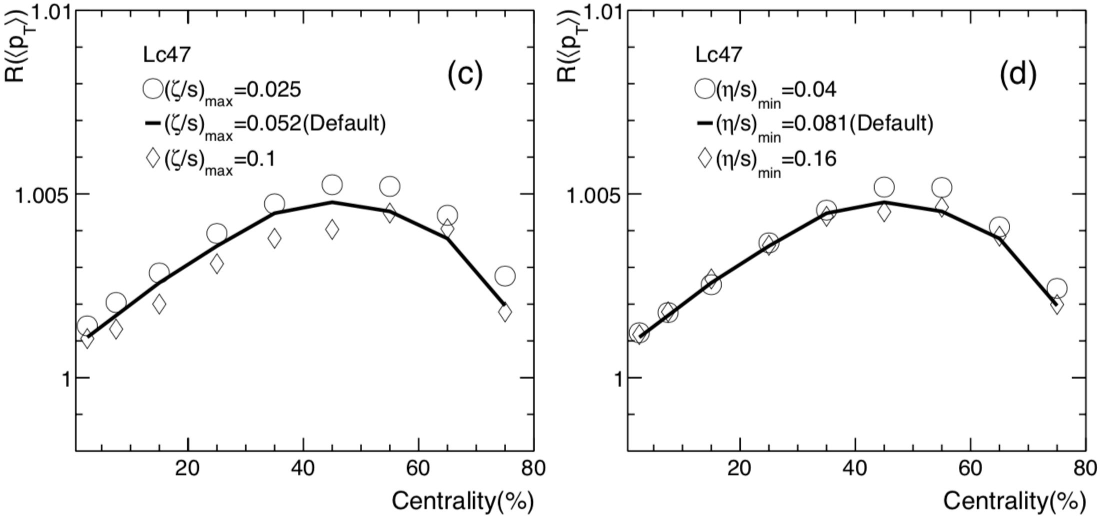
$$E_{\text{sym}}(\rho) \approx J + Lx + \frac{1}{2} K_{\text{sym}} x^2$$

$$x = (\rho - \rho_{\text{sat}})/3\rho_{\text{sat}}$$





Small sensitivity to final state effects, mostly a probe of the initial state:



H. Xu, QM2022
Extracted value from 0-5%

$L(\rho_c) = 56.8 \pm 0.4 \pm 10.4$ MeV

Need to quantify the model uncertainties

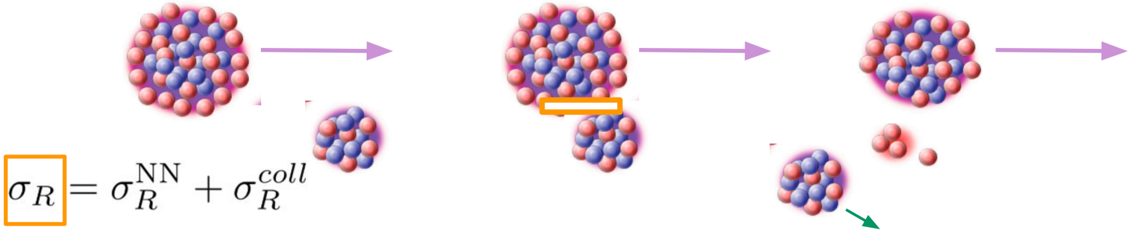
Directly peeling off the skin matter

- Similar to low energy fragmentation reaction

Andrea Jedele

TECHNISCHE
UNIVERSITÄT
DARMSTADT

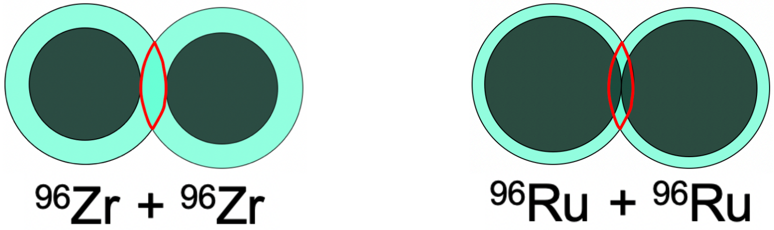
NuSym2021



$$\boxed{\sigma_R} = \sigma_R^{\mathrm{NN}} + \sigma_R^{coll}$$

- Enhanced skin contribution in peripheral collisions reduces net charge in mid-rapidity

H. Xu et.al. 2105.04052

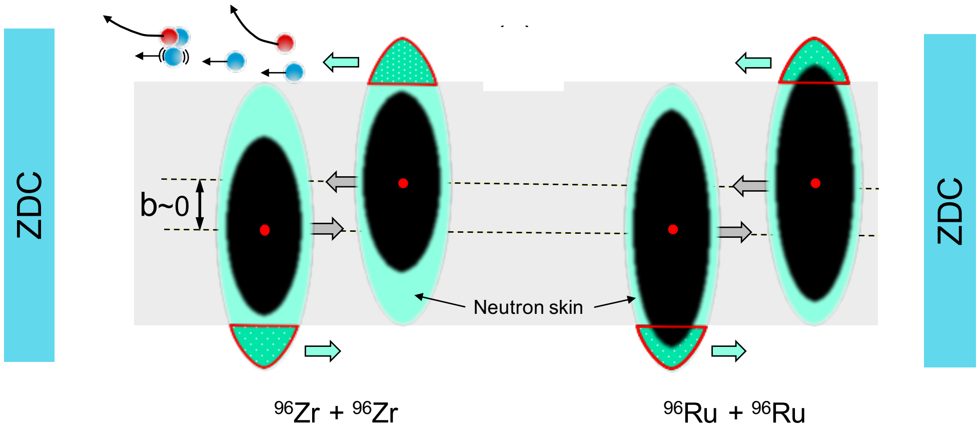


- Spectator neutrons in ultra-central isobar collisions is enhanced by neutron skin

N.Kozyrev, I. Pshenichnov 2204.07189

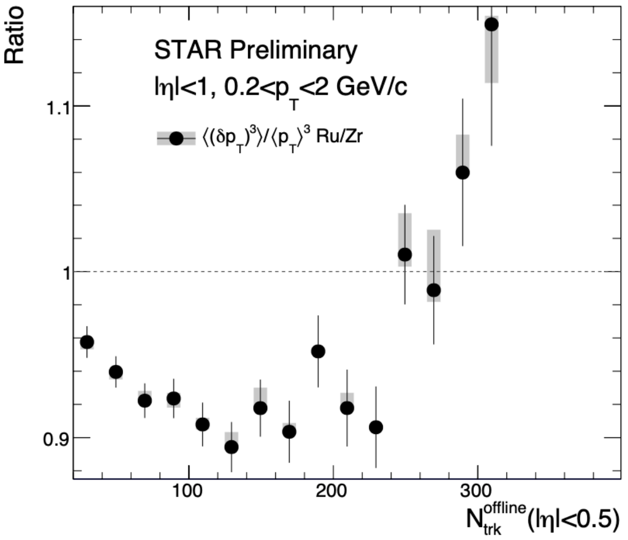
L. Liu, J. Xu et.al 2203.09924

Complete separation between participant and spectator matter

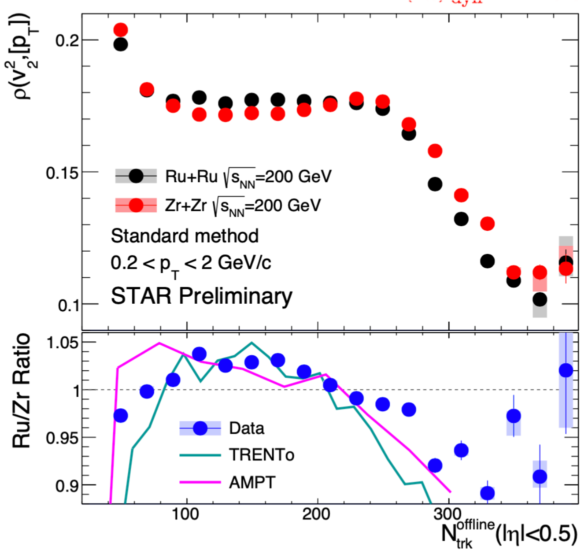


Three-particle observables

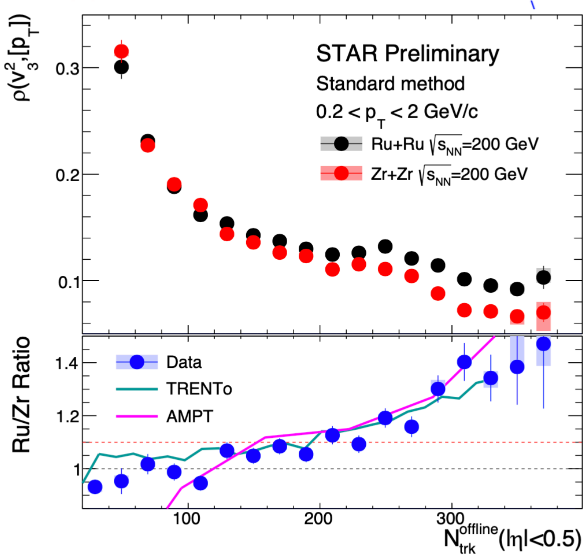
$\langle (\delta p_T)^3 \rangle$



$\langle v_2^2 \delta p_T \rangle$



$\langle v_3^2 \delta p_T \rangle$



- V$_4$ driven by $\boldsymbol{\varepsilon}_4$ plus non-linear modes

$$V_4 = V_{4L} + \chi_4 \left(V_2\right)^2$$

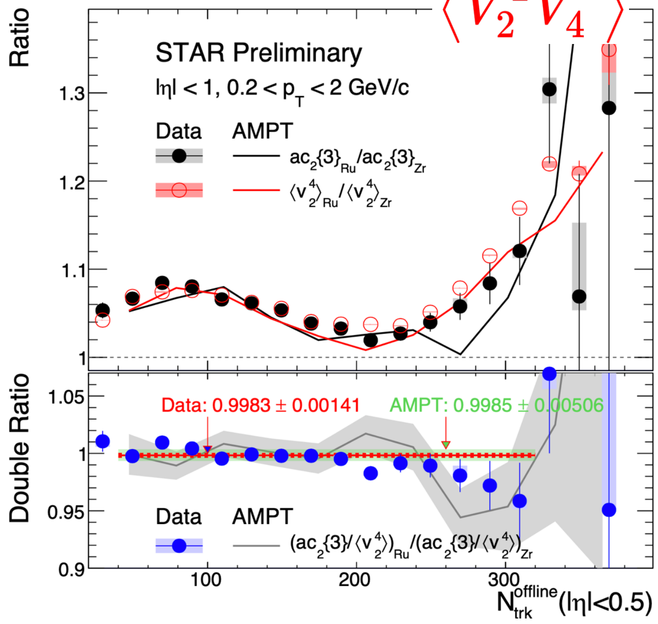
$$\Downarrow$$

$$\langle V_2^2 V_4^* \rangle = \chi_4 \langle v_2^4 \rangle$$

$$\Downarrow$$

$$\frac{\langle V_2^2 V_4^* \rangle_{\mathrm{Ru}}}{\langle V_2^2 V_4^* \rangle_{\mathrm{Zr}}} = \frac{\chi_{4\mathrm{Ru}}}{\chi_{4\mathrm{Zr}}} \frac{\langle v_2^4 \rangle_{\mathrm{Ru}}}{\langle v_2^4 \rangle_{\mathrm{Zr}}}$$

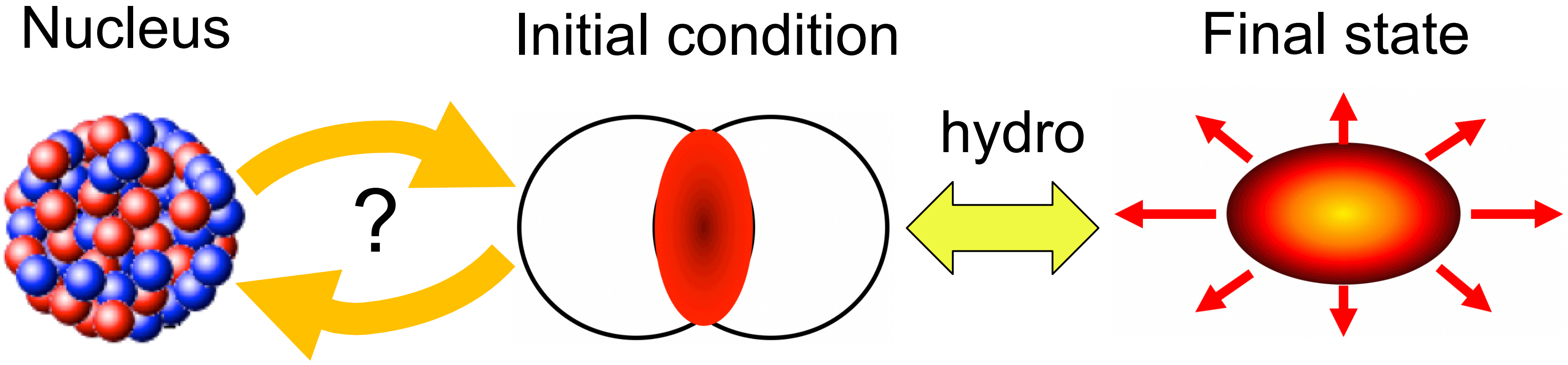
$\langle V_2^2 V_4^* \rangle$



All indicate NS effects. Much more expected from higher-order correlators

See Giuliano's talk

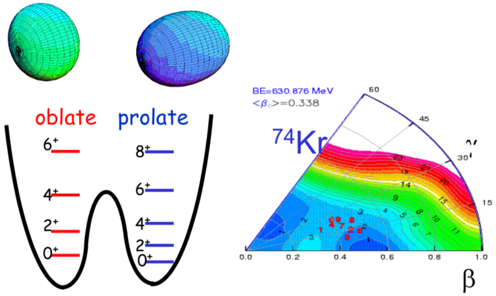
What are the next steps?



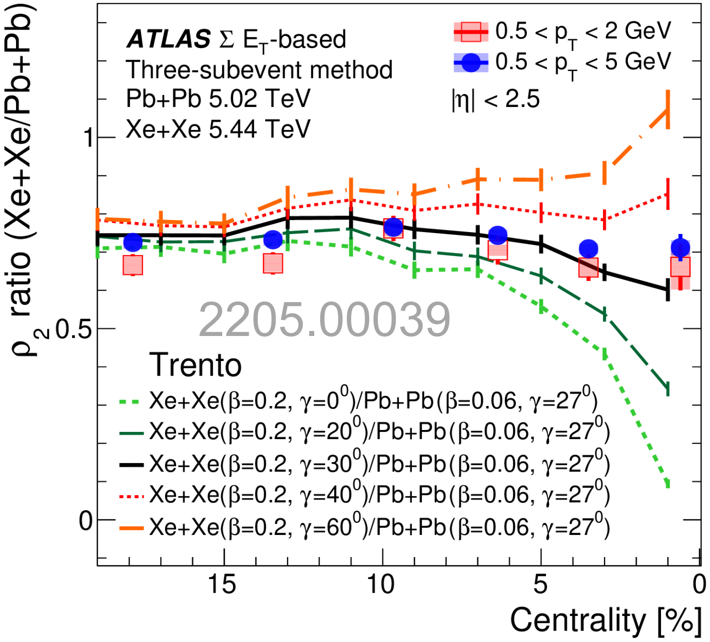
Direction I: exploration

- Further explore connections between NS and HI, more observables
    - More case studies with Ru/Zr, Au/U, Pb/Xe. cancel most final state effect
    - Shape fluctuations and shape coexistence

Shape coexistence

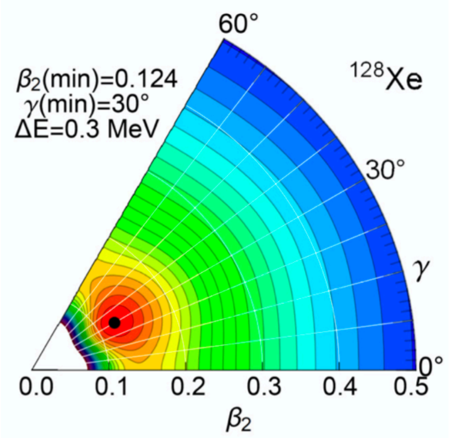


$$R = \frac{\left\langle v_2^2 \delta[p_T] \right\rangle_{^{129}Xe}}{\left\langle v_2^2 \delta[p_T] \right\rangle_{^{208}Pb}}$$



Bally et.al 2108.09578

Large shape fluct., esp along γ



PhysRevC.101.064318

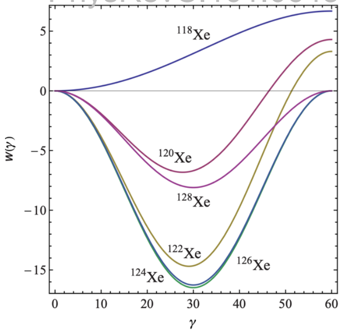
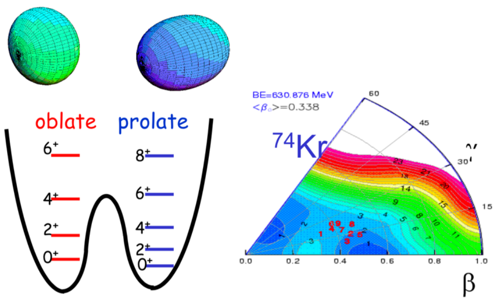


FIG. 3. The evolution of the γ potential defined by Eq. (7) with itted parameters c_1 and c_2, along the isotopic chain $^{118-128}$Xe.

Direction I: exploration

- Further explore connections between NS and HI, more observables
  - More case studies with Ru/Zr, Au/U, Pb/Xe. cancel most final state effect
  - Shape fluctuations and shape coexistence

Shape coexistence



quadrupole operator $\hat{Q}$

$$\begin{cases} \langle \beta^2 \rangle = \dfrac{16\pi^2}{9A^2R_0^4}\left\langle \hat{Q}^2 \right\rangle \\ \sigma^2(\langle \beta^2 \rangle)/\langle \beta^2 \rangle = \sigma^2\left(\langle \hat{Q}^2 \rangle\right)/\langle \hat{Q}^2 \rangle \end{cases}$$

2-p correlation

4-p correlation

1906.07542

$$\begin{cases} \langle \cos 3\gamma \rangle = -\sqrt{\dfrac{7}{2}}\dfrac{\left\langle \hat{Q}^3 \right\rangle}{\left\langle \hat{Q}^2 \right\rangle^{3/2}} \\ \dfrac{\sigma^2(\cos 3\gamma)}{(\cos 3\gamma)^2} = \dfrac{\sigma^2\langle \hat{Q}^3 \rangle}{\langle \hat{Q}^3 \rangle^2} + \dfrac{9}{4}\dfrac{\sigma^2\langle \hat{Q}^2 \rangle}{\langle \hat{Q}^2 \rangle^2} - 3\dfrac{\langle \hat{Q}^5 \rangle - \langle \hat{Q}^3 \rangle \langle \hat{Q}^2 \rangle}{\langle \hat{Q}^3 \rangle \langle \hat{Q}^2 \rangle} \end{cases}$$

3-p correlation

6-p correlation

Heavy ion observables:

$$\frac{\left\langle \varepsilon_2^2 \right\rangle}{\frac{3}{4\pi}\beta_2^2} \quad \Longleftrightarrow \quad \frac{\left\langle \varepsilon_2^4 \right\rangle - 2\left\langle \varepsilon_2^2 \right\rangle^2}{-\frac{9}{56\pi^2}\beta_2^4}$$

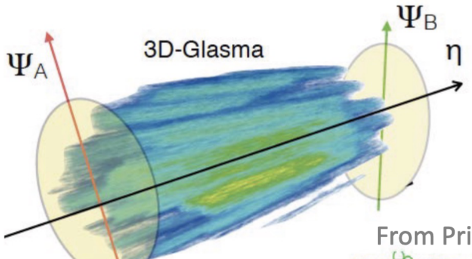
$$\frac{\left\langle \varepsilon_2^2(\delta d_\perp/d_\perp) \right\rangle}{-\frac{3\sqrt{5}}{112\pi^{3/2}}\cos(3\gamma)\beta_2^3} \quad \Longleftrightarrow \quad \frac{\left(\left\langle \varepsilon_2^6 \right\rangle - 9\left\langle \varepsilon_2^4 \right\rangle\left\langle \varepsilon_2^2 \right\rangle + 12\left\langle \varepsilon_2^2 \right\rangle^3\right)/4}{\frac{27(373-25\cos(6\gamma))}{32\times 8008\pi^3}\beta_2^6}$$

Direction I: exploration

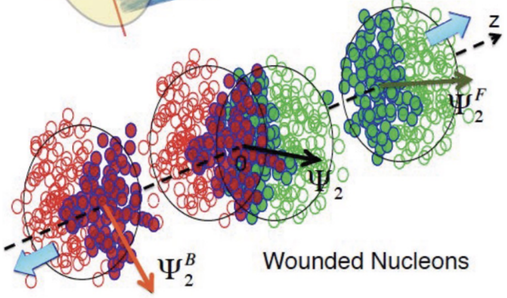
- Further explore connections between NS and HI, more observables
  - More case studies with Ru/Zr, Au/U, Pb/Xe. cancel most final state effect
  - Shape fluctuations and shape coexistence

  - Energy dependence of the nuclear structure RHIC vs LHC
    - Will gluon saturation modifies the impact of nuclear structure to initial state?

  - Longitudinal dependence?

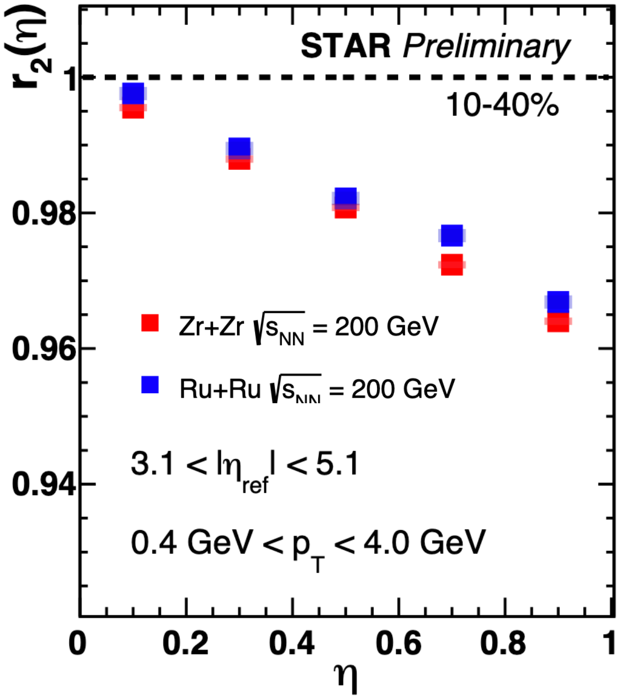


From Prithwish Tribedy



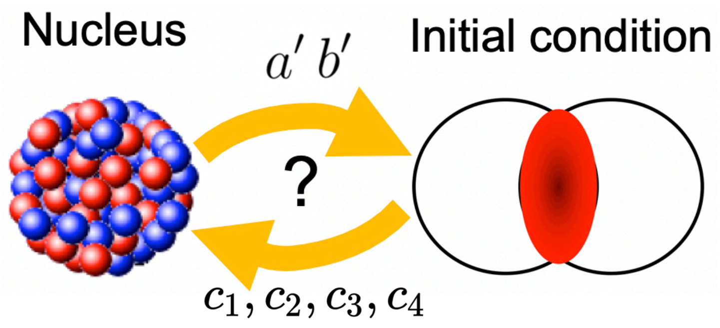
From Wei Li

STAR, QM2022



Direction II: calibration

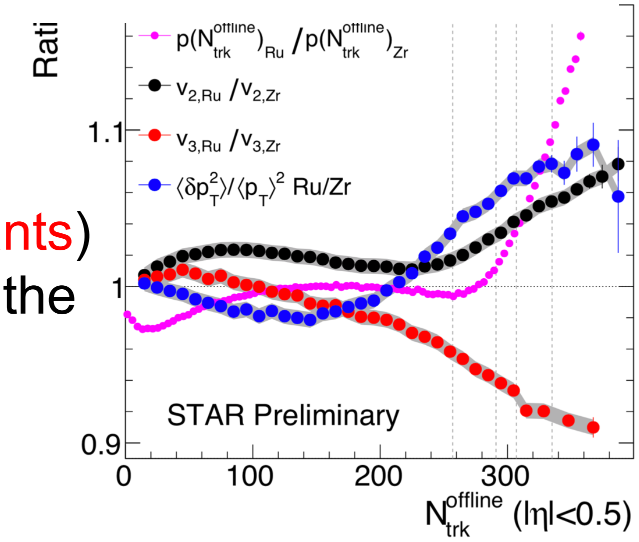
- Calibrating the response coefficients using species with well known properties such as Pb and U.
  - This is also important to understand the HI initial condition, e.g. testing different energy deposition mechanisms.
  - What kinds of ultimate precision in HI can be achieved? Require systematic theoretical efforts from both communities.



e.g. $\left\langle \epsilon_2^2 \right\rangle = a' + b'\beta_2^2$

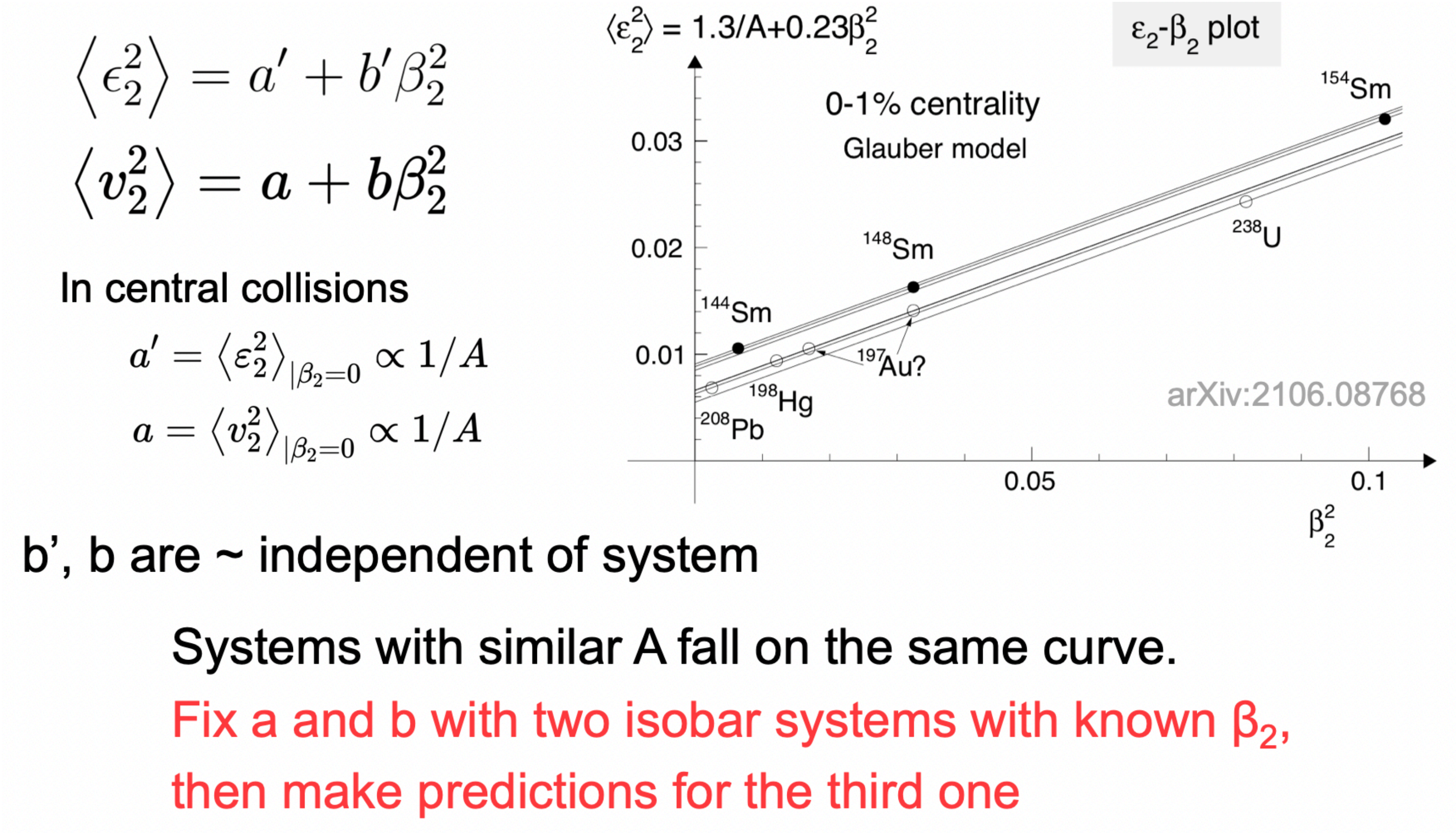
$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

Many HI observables (N_{ch},$<p_T>$, $v_{2,3,4}$, higher cumulants) + their full centrality dependence can over-constrain the collective structure parameters



Direct III : heavy system (isobar) scan

- Make predictions for “heavy species of interest”
  - Precision with isobar species: evolution of shape/skin.
  - Odd mass vs even mass given same information?



$$\left\langle \epsilon_2^2 \right\rangle = a' + b' \beta_2^2$$

$$\left\langle v_2^2 \right\rangle = a + b\beta_2^2$$

In central collisions

$$a' = \left\langle \varepsilon_2^2 \right\rangle_{|\beta_2=0} \propto 1/A$$

$$a = \left\langle v_2^2 \right\rangle_{|\beta_2=0} \propto 1/A$$

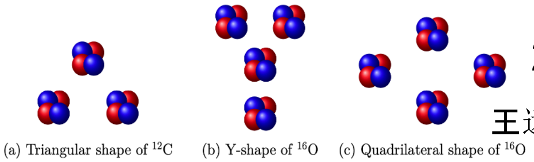
b’, b are ~ independent of system

Systems with similar A fall on the same curve.

Fix a and b with two isobar systems with known β_2, then make predictions for the third one

Direction IV: light system (isobar) scan

- Alpha clustering, halo etc from structure side
- Sub-nucleonic fluctuations will be important
- New handle on origin of collectivity, role of HI early time dynamics.

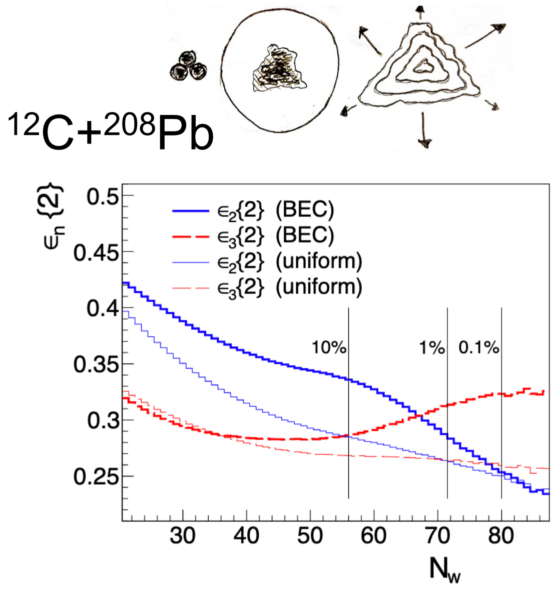


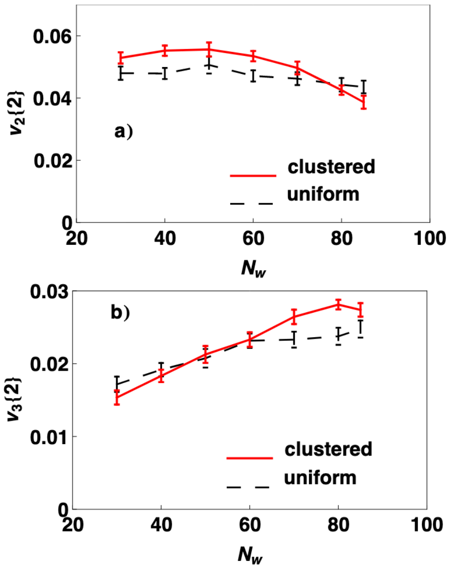
2112.08617
王远哲，张松，马余刚

Isobar (or close to) is probably the best way to achieve precision

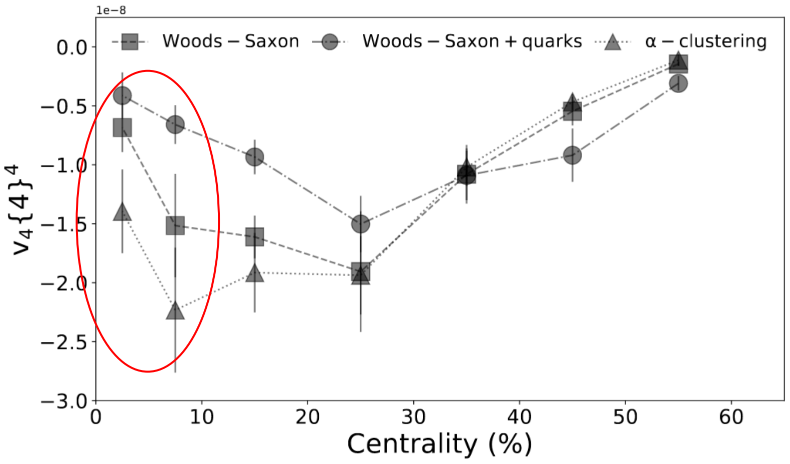
P. Bozek, W. Broniowski,E. Arriola 1312.0289, 1410.7434, 1411.5807

Analyzing ^{12}C structure via collisions with a “disk” of Pb





Flow fluctuations in ^{16}O+^{16}O,
C. Plumberg, Jaki, Lee, et.al 2103.03345



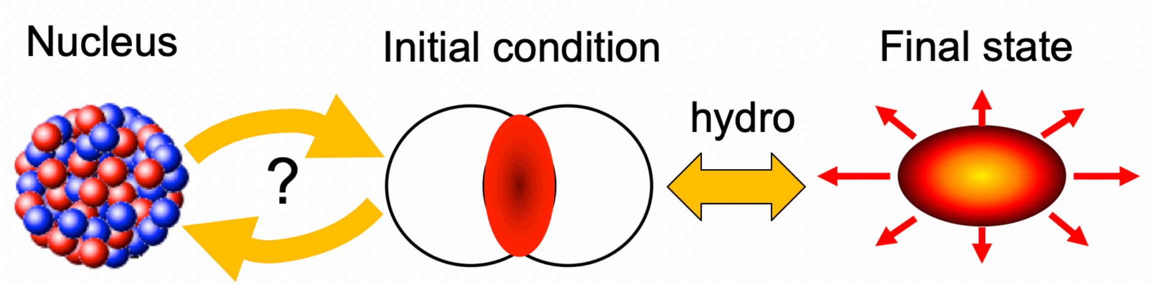
Isobar+Pb collisions?

Very subtle effects: Need light isobars to verify the sensitivity e.g. 40Ca+40Ca vs 40Ar+40Ar.

Summarizing questions

- How the nuclear shape and radial profile extracted from HI collisions relate to properties measured in nuclear structure experiments?
- How the uncertainties in nuclear structure impact the initial state of HI collisions and extraction of QGP transport properties?
- What are the most interesting stable isobar species to collide?

arXiv:2102.08158



A	isobars	A	isobars	A	isobars
36	Ar, S	106	Pd, Cd	148	Nd, Sm
40	Ca, Ar	108	Pd, Cd	150	Nd, Sm
46	Ca, Ti	110	Pd, Cd	152	Sm, Gd
48	Ca, Ti	112	Cd, Sn	154	Sm, Gd
50	Ti, V, Cr	113	Cd, In	156	Gd, Dy
54	Cr, Fe	114	Cd, Sn	158	Gd, Dy
64	Ni, Zn	115	In, Sn	160	Gd, Dy
70	Zn, Ge	116	Cd, Sn	162	Dy, Er
74	Ge, Se	120	Sn, Te	164	Dy, Er
76	Ge, Se	122	Sn, Te	168	Er, Yb
78	Se, Kr	123	Sb, Te	170	Er, Yb
80	Se, Kr	124	Sn, Te, Xe	174	Yb, Hf
84	Kr, Sr, Mo	126	Te, Xe	176	Yb, Lu, Hf
86	Kr, Sr	128	Te, Xe	180	Hf, W
87	Rb, Sr	130	Te, Xe, Ba	184	W, Os
92	Zr, Nb, Mo	132	Xe, Ba	186	W, Os
94	Zr, Mo	134	Xe, Ba	187	Re, Os
96	Zr, Mo, Ru	136	Xe, Ba, Ce	190	Os, Pt
98	Mo, Ru	138	Ba, La, Ce	192	Os, Pt
100	Mo, Ru	142	Ce, Nd	198	Pt, Hg
102	Ru, Pd	144	Nd, Sm	204	Hg, Pb
104	Ru, Pd	146	Nd, Sm		

ExtreMe Matter Institute EMMI

EMMI Rapid Reaction Task Force

Nuclear Physics Confronts Relativistic Collisions of Isobars

Open Symposium:
May 30, 2022, 1:45 p.m., Grosser Hoersaal, Heidelberg University,
Philosophenweg 12, 69120 Heidelberg/Germany

https://indico.gsi.de/event/14430

Part II in October this yar