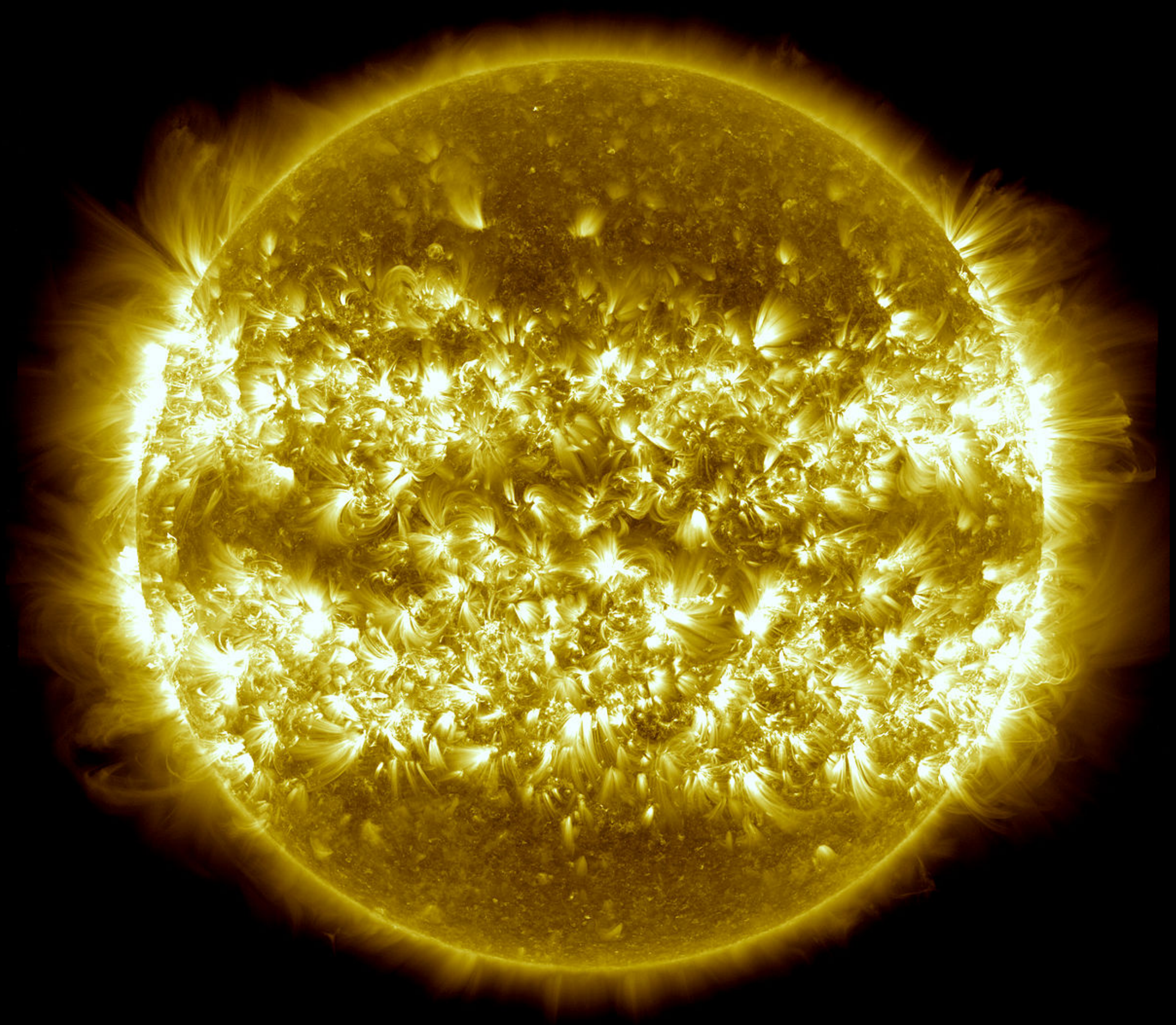
A STABLE AND CAUSAL MODEL OF MAGNETOHYDRODYNAMICS



Based on:

arXiv: 1808.01939 (PRL) & 1811.04913 (JHEP) by JA & A. Jain
arXiv: 2201.06847 by JA & F. Camilloni

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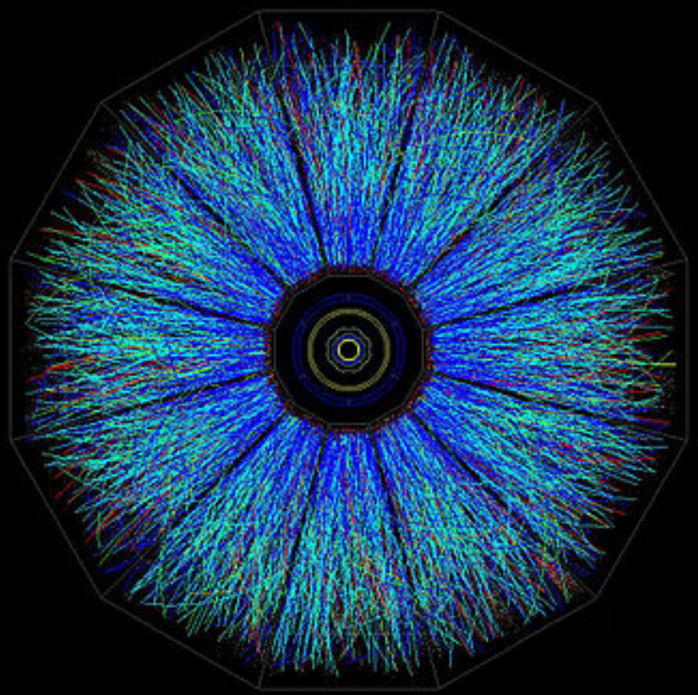
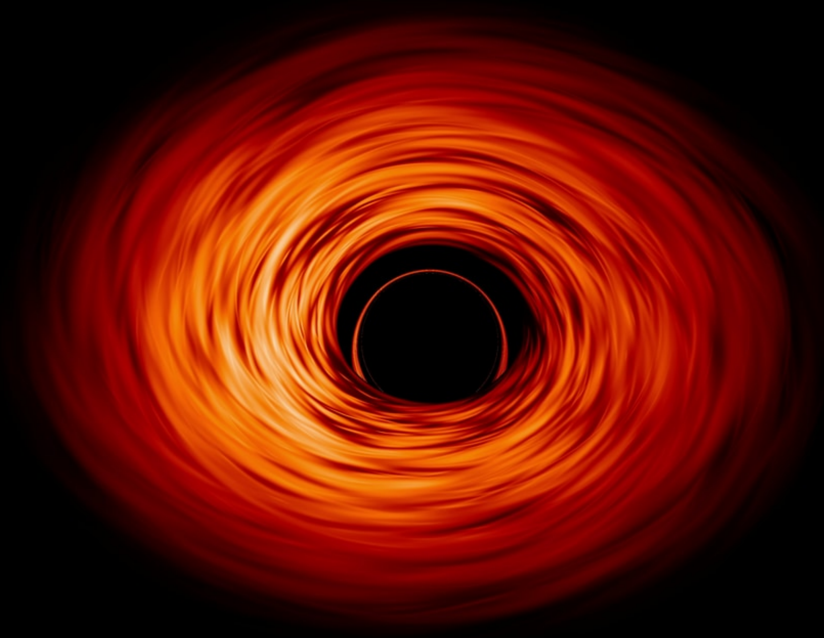
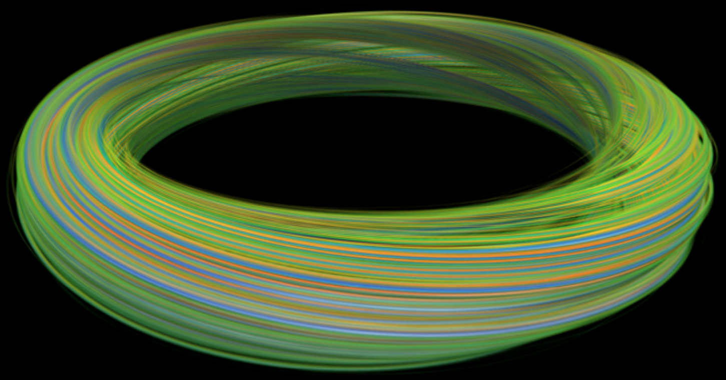
MOTIVATIONS

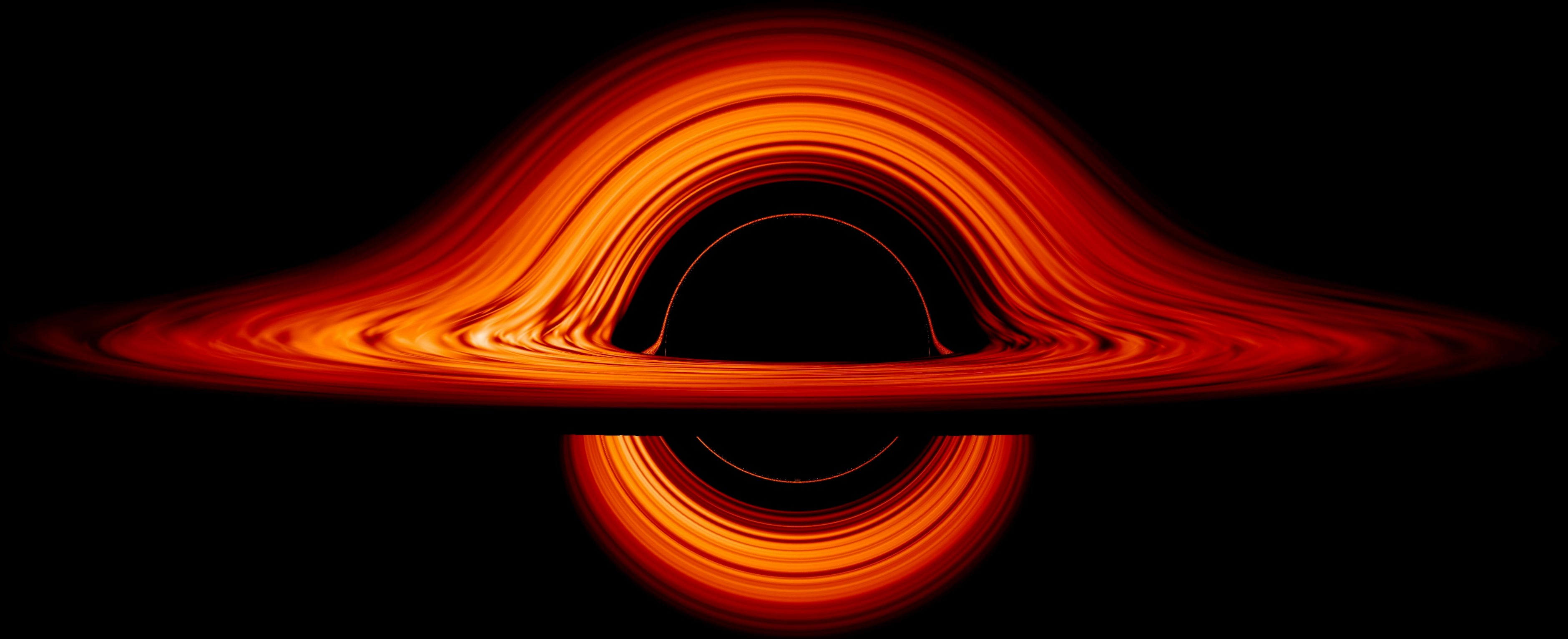






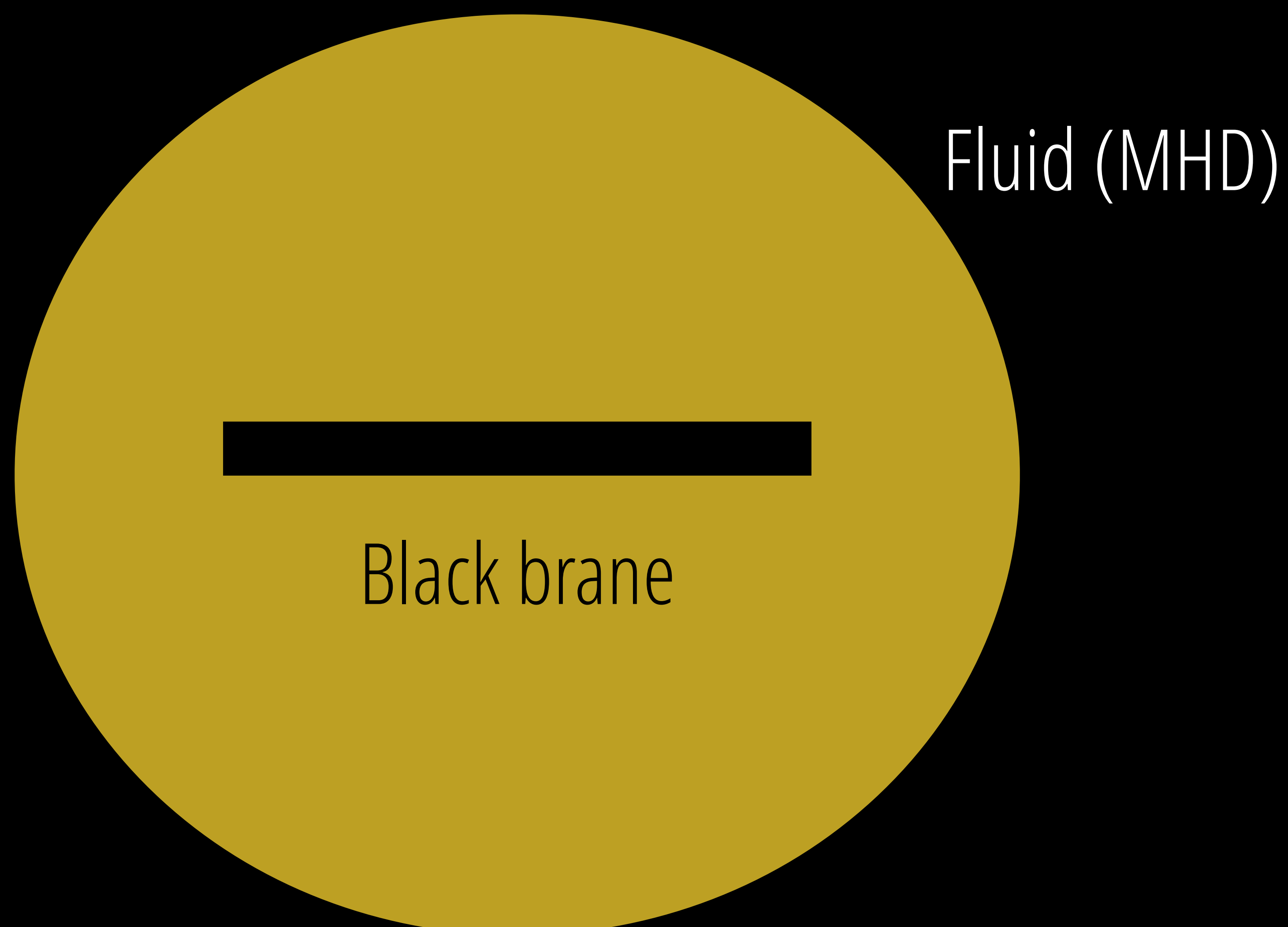
MHD works at various length scales.

MOTIVATIONS



MOTIVATIONS

Relevant for holography, string theory, Kachru-Pearson-Verlinde constructions, etc



MOTIVATIONS

>> How do we consistently formulate MHD?

[dissipative effects, gradient expansion, no weird assumptions such as infinite conductivity, strength of magnetic fields, etc]

>> How do we characterise plasmas?

[symmetries, how many transport coefficients, responses coefficients, etc]

>> How do we describe all equilibrium states in MHD?

[effective action, partition functions, etc]

>> How do we deal with near-equilibrium dynamics?

[causality and stability issues, MIS or BDNK completions, etc]

OTHER MOTIVATIONS

>> How do we deal with far-from equilibrium dynamics?

[MIS or BDNK completions, initial conditions, nonlinear evolution, global hyperbolicity, breakdown of expansion, turbulence, etc]

>> What phases of hot electromagnetism can we describe?

[non-conducting plasmas, additional currents, anomalies, etc]

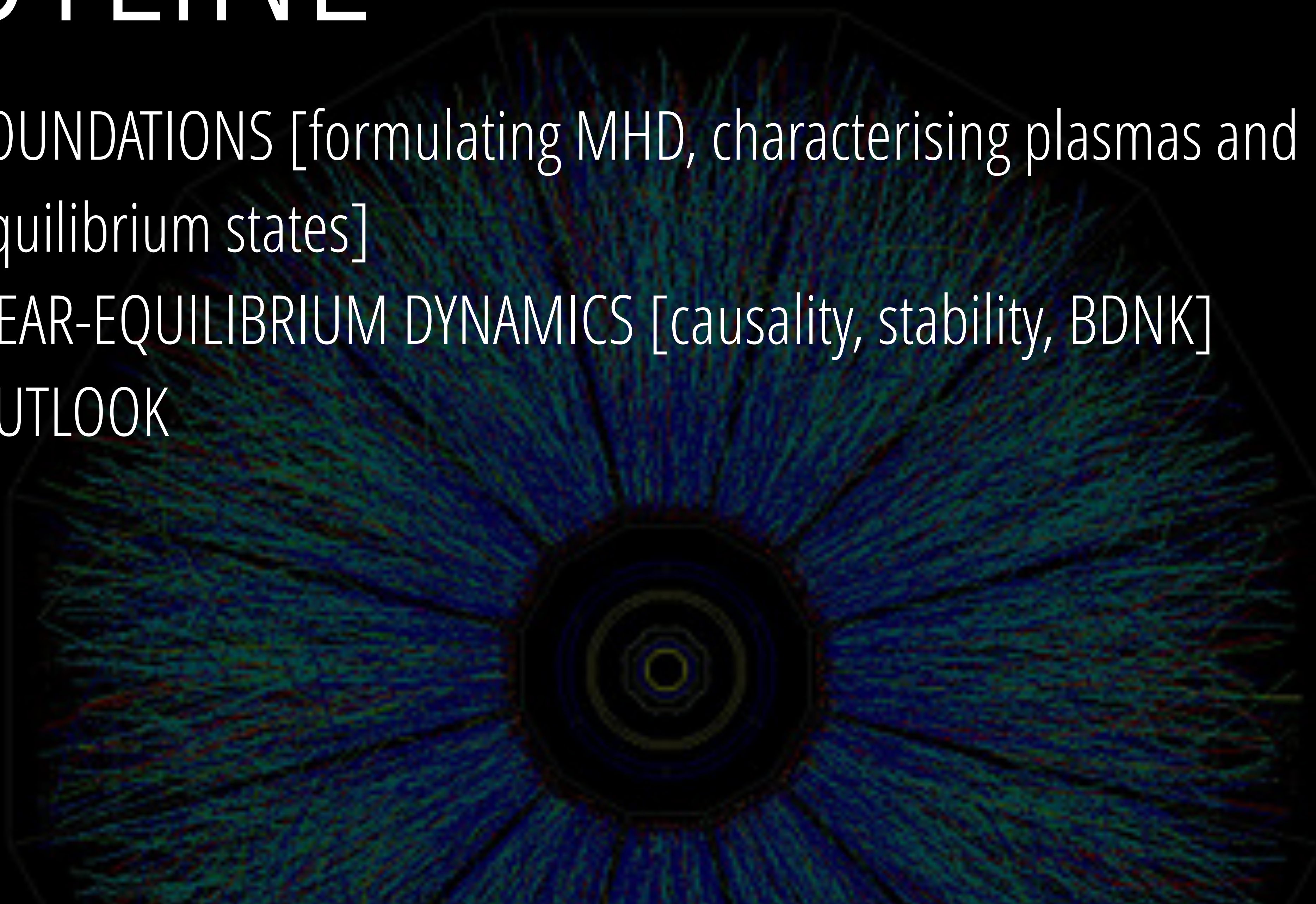
>> What is the relation between MHD and FFE*?

[symmetry enhancement, limits, etc]

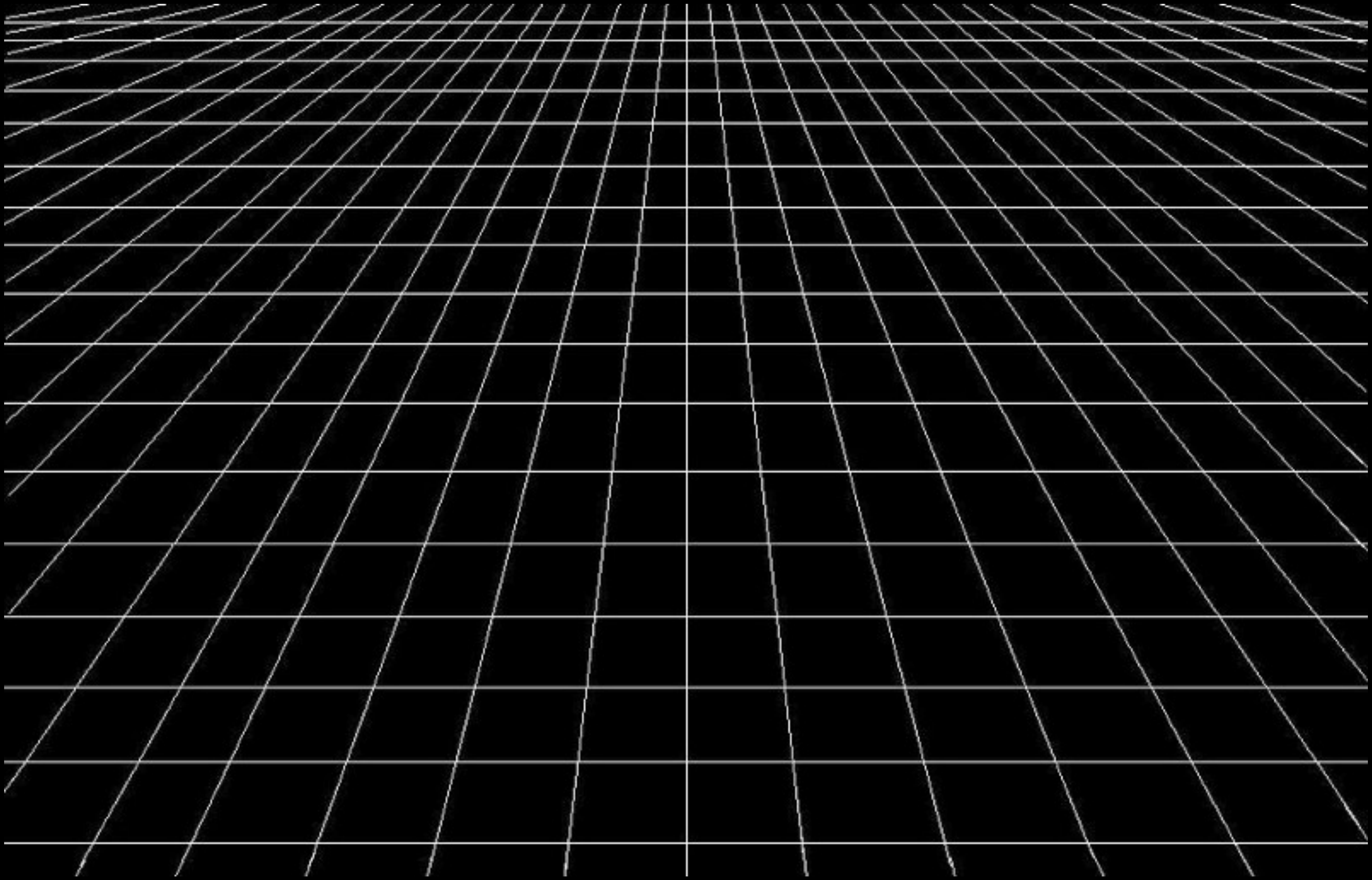
*Force Free Electrodynamics

OUTLINE

(1) FOUNDATIONS [formulating MHD, characterising plasmas and equilibrium states]
(2) NEAR-EQUILIBRIUM DYNAMICS [causality, stability, BDNK]
(3) OUTLOOK



I- FOUNDATIONS



$g_{\mu\nu}$ In many cases $g_{\mu\nu} \sim \eta_{\mu\nu}$

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

Ideal stress tensor:

$$T^{\mu\nu} = \epsilon(T) u^\mu u^\nu + P(T)(g^{\mu\nu} + u^\mu u^\nu)$$

$$\nabla_\mu T^{\mu\nu} = 0$$
$$\nabla_\mu S^\mu \geq 0$$

Ideal stress tensor:

$$T^{\mu\nu} = \epsilon(T) u^\mu u^\nu + P(T)(g^{\mu\nu} + u^\mu u^\nu)$$

Entropy current:

$$S^\mu = s u^\mu \quad , \quad \epsilon + P = Ts$$

What are the equilibrium solutions?

$$\mathcal{L}_K g_{\mu\nu} = 0$$

The fluid fields are determined:

$$u^\mu = \frac{K^\mu}{|K|} \qquad T = \frac{T_0}{|K|}$$

Free energy functional (purely geometric)

$$F[g_{\mu\nu}] = -\int d^4x\sqrt{-g}P(T)$$

$$T = \frac{T_0}{|K|}$$

Variation gives

$$\delta F[g_{\mu\nu}] = \frac{1}{2}\int d^4x\sqrt{-g}T^{\mu\nu}\delta g_{\mu\nu}$$

Out of equilibrium:

$$T^{\mu\nu} = \epsilon(T)u^\mu u^\nu + P(T)(g^{\mu\nu} + u^\mu u^\nu) + T^{\mu\nu}_{(1)} + \mathcal{O}\left(\partial^2\right)$$

Temperature and velocity can be redefined:

$$T \to T + \delta T$$

$$u^\mu \to u^\mu + \delta u^\mu$$

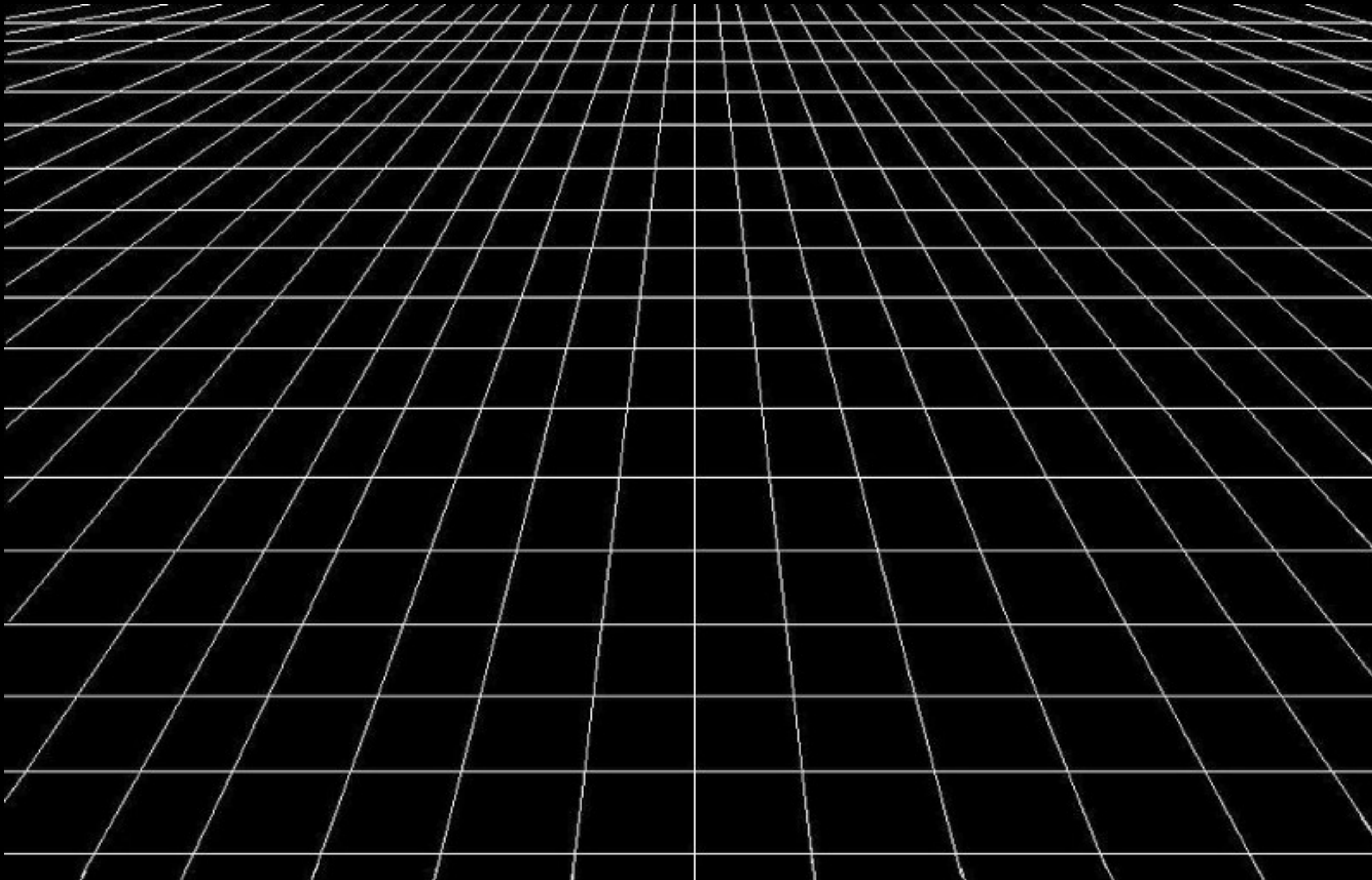
Landau frame:

$$T^{\mu\nu}_{(1)} u_\nu = 0$$

+ second law fixes the form to be:

$$\boxed{T^{\mu\nu}_{(1)} = -\zeta \nabla_\lambda u^\lambda P^{\mu\nu} - \eta \sigma^{\mu\nu}}$$

$P^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}$



$g_{\mu\nu}$ A_μ

Charged fluids:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu$$

$$\nabla_\mu J^\mu = 0$$

$$\nabla_\mu S^\mu \geq 0$$

Global U(1) symmetry

Ideal stress tensor:

$$T^{\mu\nu} = \epsilon(T,\mu)u^\mu u^\nu + P(T,\mu)(g^{\mu\nu} + u^\mu u^\nu)$$

Ideal current:

$$J^\mu = q(T,\mu)u^\mu$$

Stress tensor correction

$$T^{\mu\nu}_{(1)} = -\zeta \nabla_\lambda u^\lambda P^{\mu\nu} - \eta\sigma^{\mu\nu}$$

Current correction

$$J^\mu_{(1)} = -\sigma P^\mu{}_\nu \left(\partial^\nu \frac{\mu}{T} - E^\nu\right)$$

$$\boxed{E^\nu = F^{\mu\nu}u_\mu}$$

Superfluids:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu$$

$$\nabla_\mu J^\mu = 0$$

$$\nabla_\mu S^\mu \geq 0$$

Spontaneously broken U(1) symmetry

There is a new field:

$$\boxed{\xi_\mu = \partial_\mu \phi + A_\mu}$$

$$u^\mu \xi_\mu = \mu$$

Stress tensor:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} + f \xi^\mu \xi^\nu$$

Current:

$$J^\mu = q u^\mu - f \xi^\mu$$

Thermodynamic quantities are now functions of

$$\boxed{P(T,\mu,\chi)}$$

With $\chi = -\xi^\mu \xi_\mu$

The free energy functional is:

$$F[g_{\mu\nu}, A_\mu; \phi] = -\int d^4x\sqrt{-g}P(T, \mu, \chi)$$

$$T = \frac{T_0}{|K|} \quad \mu = u^\mu A_\mu$$

Equilibrium states are degenerate

$$\delta_\phi F \;\Rightarrow\; \nabla_\mu \left(f\xi^\mu\right) = 0$$

WHAT IS MHD?

THE GAUGE FIELD IS NO LONGER A BACKGROUND FIELD BUT A DYNAMICAL FIELD

MHD EQUATIONS:

$$\nabla_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu$$

$$J^\mu + J^\mu_{\text{ext}} = 0 \qquad \nabla_{[\mu} F_{\nu\rho]} = 0$$

$$J^\nu = \nabla_\mu F^{\mu\nu} + J^\nu_{\text{matter}}$$

Decompose the field strength as:

$$F_{\mu\nu} = 2u_{[\mu} E_{\nu]} - \epsilon_{\mu\nu\lambda\rho} u^\lambda B^\rho$$

$$E^\mu = F^{\mu\nu} u_\nu \qquad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}$$

- Minimal coupling between fluid and fields

$$T^{\mu\nu} = F^{\mu}{}_{\rho}F^{\nu\rho} - \frac{1}{4}F_{\rho\sigma}F^{\rho\sigma}g^{\mu\nu} + \epsilon(T,\mu)u^{\mu}u^{\nu} + p(T,\mu)\left(g^{\mu\nu} + u^{\mu}u^{\nu}\right)$$

$$J^{\mu} = \nabla_{\nu}F^{\nu\mu} + q(T,\mu)\,u^{\mu} - \sigma(T,\mu)P^{\mu\nu}\left(T\partial_{\nu}\frac{\mu}{T} - E_{\nu}\right) \quad ,$$

Decomposing Maxwell's equations

$q(T,\mu) = u_{\mu}$ [illegible] $F^{\nu\mu} + u_{\mu}J^{\mu}_{\text{ext}}$

$$J^{\mu} + J^{\mu}_{\text{ext}} = 0$$

$\sigma(T,\mu)E^{\mu} = -P$ [illegible] $F^{\nu\lambda} - P^{\mu}{}_{\lambda}J^{\lambda}_{\text{ext}} + T\sigma($ [illegible] $P^{\mu\nu}\partial_{\nu}\frac{\mu}{T}$

- Electric fields are Debye screened and the plasma is electrically neutral

$$q(T,\mu) = E^\mu = \mathcal{O}(\partial) \quad \text{instead of} \quad \sigma(T,\mu) \to \infty$$

Stress tensor becomes:

$$T^{\mu\nu} = (\epsilon(T,\mu) + p(T,\mu))\, u^\mu u^\nu + \left(p(T,\mu) - \frac{1}{2}B^2\right) g^{\mu\nu} + B^2 \mathbb{B}^{\mu\nu} + \mathcal{O}(\partial)$$

$$\mathbb{B}^{\mu\nu} = P^{\mu\nu} - \hat{B}^\mu \hat{B}^\nu,\ P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$\hat{B}^\mu = B^\mu / |B|$$

HISTORICAL RECORD

>> Dissipative string fluids (2014)

D. Schubring

>> Generalised global symmetries and dissipative MHD (2016)

S. Grozdanov, D. M. Hofman & N. Iqbal

>> Relativistic magnetohydrodynamics (2017)

J. Hernandez & P. Kovtun

>> Dissipative hydrodynamics with higher-form symmetry (2018)

JA, J. Gath, A. V. Pedersen & A. Jain

>> Magnetohydrodynamics as superfluidity (2018)

JA & A. Jain

>> One-form superfluids and magnetohydrodynamics (2018)

JA & A. Jain

>> EFT of MHD from generalised global symmetries (2018)

P. Glorioso & D.T. Son

ELECTROMAGNETISM

- Maxwell's equations

$$\nabla_\mu F^{\mu\nu} = 0 \qquad \nabla_{[\mu} F_{\nu\rho]} = 0$$

$$d \star F = 0 \qquad dF = 0$$

"Dual Maxwell equations"

$$\nabla_{[\mu} J_{\nu\rho]} = 0 \qquad \nabla_\mu J^{\mu\nu} = 0$$

Hence a 2-form dipole magnetic charge:

$$Q = \int_{M_2} \star J$$

Global U(1) 1-form magnetic symmetry

MHD DUAL

- MHD Equations of motion

$$\nabla_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu$$

$$J^\mu + J^\mu_{\text{ext}} = 0 \qquad \nabla_{[\mu} F_{\nu\rho]} = 0$$

Given that Maxwell equations can be solved one can formulate the theory simply as:

$$\nabla_\mu T^{\mu\nu} = \frac{1}{2} H^{\nu\lambda\rho} J_{\lambda\rho} \qquad \nabla_\mu J^{\mu\nu} = 0$$

$$\boxed{\nabla_\mu J^\mu_{ext} = 0} \longrightarrow J^\mu_{\text{ext}} = \frac{1}{6}\epsilon^{\mu\nu\lambda\rho} H_{\nu\lambda\rho}$$

$$J^{\mu\nu} = \star F^{\mu\nu}$$

$$H_{\nu\rho\sigma} = 3\partial_{[\nu} b_{\rho\sigma]}$$

- Breaking the one-form symmetry

1-form Goldstone

$$\delta_X \varphi_\mu = \pounds_X \varphi_\mu - \Lambda^X_\mu$$

2-form superfluid velocity

$$\xi_{\mu\nu} = 2\partial_{[\mu}\varphi_{\nu]} + b_{\mu\nu}$$

1-form Josephson condition

$$\mu^\varphi_\mu = u^\nu \xi_{\nu\mu}$$

0-form Goldstone

$$\delta\phi = \mathcal{L}_X \phi - \Lambda^X$$

1-form superfluid velocity

$$\xi_\mu = \partial_\mu \phi + A_\mu$$

0-form Josephson condition

$$\mu = u^\mu \xi_\mu$$

$$\xi_{\mu\nu} = 2u_{[\mu}\zeta_{\nu]} - \epsilon_{\mu\nu\rho\sigma}u^{\rho}\bar{\zeta}^{\sigma}$$

$$\zeta_{\mu} = \xi_{\mu\nu}u^{\nu} \quad , \quad \bar{\zeta}^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_{\nu}\xi_{\rho\sigma}$$

Now one can define: $P \equiv P(T, \zeta^2, \zeta.\bar{\zeta}, \bar{\zeta}^2)$

(removal of $\bar{\zeta}^{\mu}$)

$$P \equiv P(T, \zeta^2)$$

MHD DUAL

- Can define partition function:

$$F[g_{\mu\nu}, b_{\mu\nu}; \varphi_\mu] = -\int d^4x\sqrt{-g}P(T,\zeta^2)$$

which gives the stress tensor and current:

$$T^{\mu\nu} = (\epsilon + p)\, u^\mu u^\nu + p\, g^{\mu\nu} - \varpi\rho\, h^\mu h^\nu + \mathcal{O}(\partial)$$
$$J^{\mu\nu} = 2\rho\, u^{[\mu} h^{\nu]} + \mathcal{O}(\partial) \ \ ,$$

and thermodynamic relations upon identification:

$$\zeta_\mu = -\varpi h_\mu \ \ , \ \ q = \frac{\rho}{\varpi}$$

$$\mathrm{d}p = s\,\mathrm{d}T + \rho\,\mathrm{d}\varpi \ \ , \ \ \epsilon + p = s\,T + \rho\,\varpi$$

MHD DUAL

- Variation with respect to

$$\frac{\delta S}{\delta \varphi} \Rightarrow \nabla_\mu (T \rho h^\mu) = 0$$

- At ideal order the identification is:

$$\boxed{B^\mu = \rho(T, \varpi) h^\mu + \mathcal{O}(\partial)}$$

- Higher-order partition function

$$\mathcal{N} = p - \frac{\alpha}{6}\epsilon^{\mu\nu\rho\sigma}u_\mu H_{\nu\rho\sigma} - \beta\epsilon^{\mu\nu\rho\sigma}u_\mu h_\nu \partial_\rho u_\sigma$$

$$- \tilde{\beta}_1 h^\mu \partial_\mu T - \tilde{\beta}_2 h^\mu \partial_\mu \frac{\varpi}{T} - \tilde{\beta}_3 \epsilon^{\mu\nu\rho\sigma} u_\mu h_\nu \partial_\rho h_\sigma$$

- Non-hydrostatic corrrections

$$T^{\mu\nu}_{\text{nhs}} = \delta\epsilon\ u^\mu u^\nu + \delta f\,\Delta^{\mu\nu} + \delta\tau\ h^\mu h^\nu + 2\ell^{(\mu}h^{\nu)} + 2k^{(\mu}u^{\nu)} + t^{\mu\nu}\ ,$$

$$J^{\mu\nu}_{\text{nhs}} = 2\delta\rho\ u^{[\mu}h^{\nu]} + 2m^{[\mu}h^{\nu]} + 2n^{[\mu}u^{\nu]} + \delta s\,\epsilon^{\mu\nu}\ ,$$

- Total transport coefficients:
  - >> 5 hydrostatic coefficients
  - >> 19 non-hydrostatic coefficients

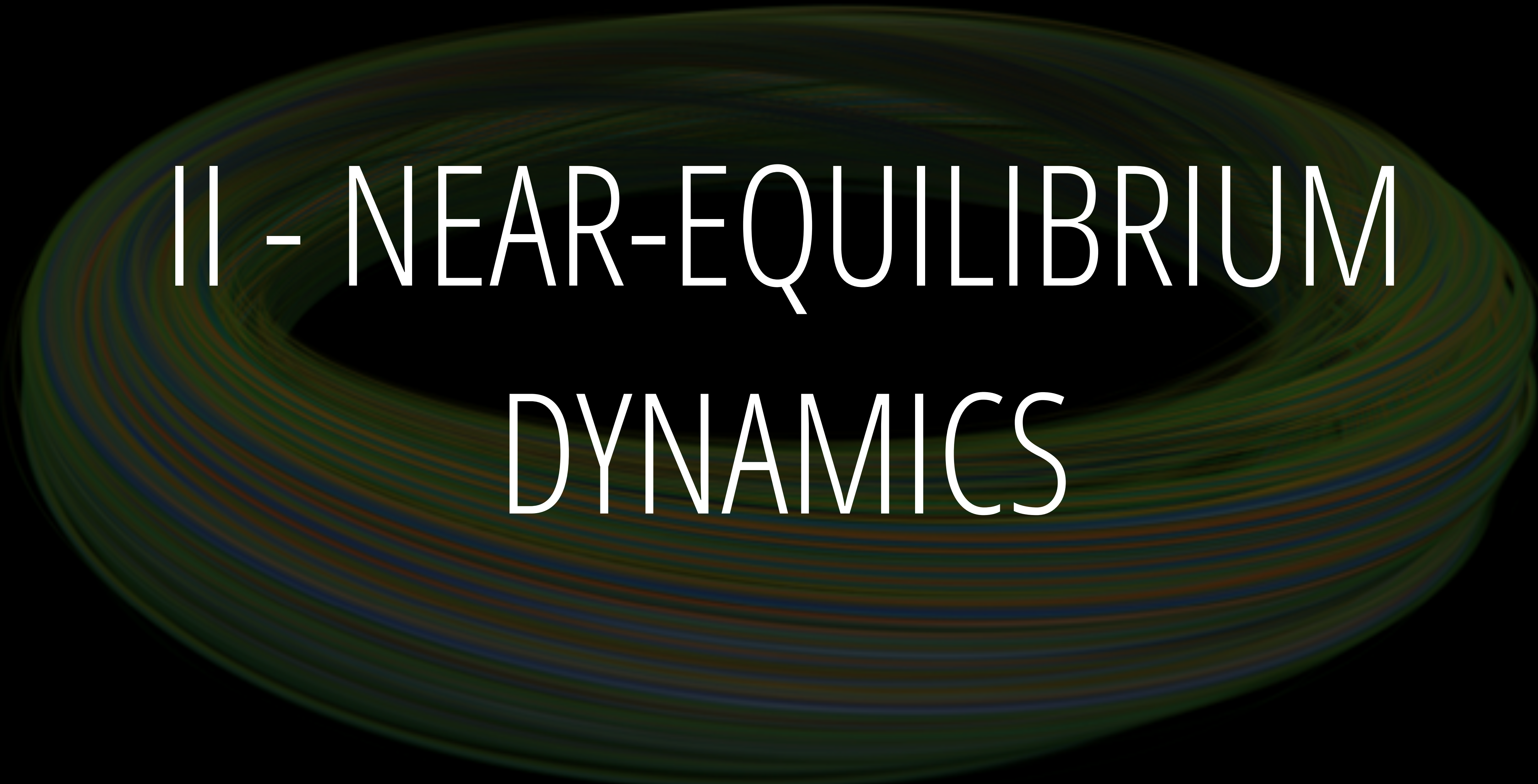
$$\begin{pmatrix} \delta f \\ \delta\tau \\ \delta s \end{pmatrix} = -\frac{T}{2} \begin{pmatrix} \zeta_\perp & \zeta_\times & \tilde{\kappa}_1 \\ \zeta'_\times & \zeta_\| & \tilde{\kappa}_2 \\ \tilde{\kappa}'_1 & \tilde{\kappa}'_2 & r_\| \end{pmatrix} \begin{pmatrix} \Delta^{\mu\nu}\delta_{\mathcal{B}} g_{\mu\nu} \\ h^\mu h^\nu \delta_{\mathcal{B}} g_{\mu\nu} \\ \epsilon^{\mu\nu}\delta_{\mathcal{B}}\xi_{\mu\nu} \end{pmatrix} ,$$

$$\begin{pmatrix} \ell^\mu \\ m^\mu \end{pmatrix} = -T \begin{pmatrix} \eta_\| & r_\times & \tilde{\eta}_\| & \tilde{r}_\times \\ r'_\times & r_\perp & \tilde{r}'_\times & \tilde{r}_\perp \end{pmatrix} \begin{pmatrix} \Delta^{\mu\sigma} h^\nu \delta_{\mathcal{B}} g_{\sigma\nu} \\ \Delta^{\mu\sigma} h^\nu \delta_{\mathcal{B}} \xi_{\sigma\nu} \\ \epsilon^{\mu\sigma} h^\nu \delta_{\mathcal{B}} g_{\sigma\nu} \\ \epsilon^{\mu\sigma} h^\nu \delta_{\mathcal{B}} \xi_{\sigma\nu} \end{pmatrix}$$

$$t^{\mu\nu} = -\eta_\perp T \Delta^{\rho\langle\mu}\Delta^{\nu\rangle\sigma}\delta_{\mathcal{B}} g_{\rho\sigma} + \tilde{\eta}_\perp T \epsilon^{\rho\langle\mu}\Delta^{\nu\rangle\sigma}\delta_{\mathcal{B}} g_{\rho\sigma}$$

$$\delta_{\mathcal{B}} g_{\mu\nu} = 2\nabla_{(\mu}\left(\frac{u_{\nu)}}{T}\right) \quad , \quad \delta_{\mathcal{B}} b_{\mu\nu} = 2\partial_{[\mu}\left(\frac{\varpi h_{\nu]}}{T}\right) + \frac{u^\sigma}{T} H_{\sigma\mu\nu}$$

II - NEAR-EQUILIBRIUM DYNAMICS



$$\nabla_\mu T^{\mu\nu} = 0$$

Initial equilibrium state:

$$T = T_0 \qquad u^{\mu}_{(0)} = \frac{1}{\sqrt{1-v_0^2}}(1,0,0,v_0)$$

Perturbations around equilibrium:

$$T = T_0 + \delta T$$

$$u^{\mu} = u^{\mu}_{(0)} + \delta u^{\mu}$$

Modes:

$$w(k) = i\frac{(\epsilon + P)\sqrt{1 - v_0^2}}{\eta v_0^2} + \mathcal{O}(k)$$

⇒ Unstable mode

Hiscock and Lindblom, 1985

Kovtun, 2019; Bemfica, Disconzi, Noronha, 2019

Causality is also violated:

$$1 > \lim_{k\to\infty} |Re(\frac{w(k)}{k})| > 0$$

Hiscock and Lindblom, 1985

BDNK PROPOSAL: PICK A SUITABLE HYDRODYNAMIC FRAME THAT SOLVES STABILITY AND CAUSALITY ISSUES

Kovtun, 2019; Bemfica, Disconzi, Noronha, 2019

$$\nabla_\mu T^{\mu\nu} = 0 \quad \nabla_\mu J^{\mu\nu} = 0$$

Equilibrium (ideal order) stress tensor and current: $p(T,\mu)$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} - \mu\rho h^\mu h^\nu$$

$$J^{\mu\nu} = 2\rho u^{[\mu} h^{\nu]}$$

Out of equilibrium:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} - \mu\rho h^\mu h^\nu + T^{\mu\nu}_{(1)}$$

$$J^{\mu\nu} = 2\rho u^{[\mu} h^{\nu]} + J^{\mu\nu}_{(1)}$$

Temperature, velocity, chemical potential and director can be redefined:

$$T \to T + \delta T \qquad \mu \to \mu + \delta\mu$$

$$u^\mu \to u^\mu + \delta u^\mu \qquad h^\mu \to h^\mu + \delta h^\mu$$

General parametrisation:

$$T^{\mu\nu}_{(1)} = \delta\varepsilon u^\mu u^\nu + \delta f \Delta^{\mu\nu} + \delta\tau h^\mu h^\nu + 2\delta\chi h^{(\mu} u^{\nu)} + 2\ell^{(\mu} h^{\nu)} + 2k^{(\mu} u^{\nu)} + t^{\mu\nu}$$

$$J^{\mu\nu}_{(1)} = 2\delta\varrho u^{[\mu} h^{\nu]} + 2m^{[\mu} h^{\nu]} + 2n^{[\mu} u^{\nu]} + s^{\mu\nu}$$

⇒ Fixing frame gives 7 transport coefficients

⇒ General frame gives 28 transport coefficients!

Why you don't want to fix the frame:

$$w = i\frac{\sqrt{1-v_0^2}}{v_0^2}\frac{\rho}{\mu r_{||}} + \mathcal{O}(k)$$

⇒ Instability!

Not fixing a frame gives

$$\delta\varepsilon = -\varepsilon_1 \Delta^{\mu\nu}\nabla_\mu u_\nu - \varepsilon_2 h^\mu h^\nu \nabla_\mu u_\nu - \varepsilon_3 u^\mu \nabla_\mu T - \varepsilon_4 u^\mu \nabla_\mu (\mu/T) \quad ,$$

$$\delta f = -f_1 \Delta^{\mu\nu}\nabla_\mu u_\nu - f_2 h^\mu h^\nu \nabla_\mu u_\nu - f_3 u^\mu \nabla_\mu T - f_4 u^\mu \nabla_\mu (\mu/T) \quad ,$$

$$\delta\tau = -\tau_1 \Delta^{\mu\nu}\nabla_\mu u_\nu - \tau_2 h^\mu h^\nu \nabla_\mu u_\nu - \tau_3 u^\mu \nabla_\mu T - \tau_4 u^\mu \nabla_\mu (\mu/T) \quad ,$$

$$\delta\chi = -T\chi_1 u^\mu h^\nu \delta_B g_{\mu\nu} - \chi_2 \nabla_\mu (T\rho h^\mu) \quad ,$$

$$\ell^\mu = -T\ell_1 \Delta^{\mu\sigma} h^\nu \delta_B g_{\nu\sigma} - T\ell_2 \Delta^{\mu\sigma} u^\nu \delta_B b_{\sigma\nu} \quad ,$$

$$k^\mu = -Tk_1 \Delta^{\mu\nu} h^\lambda \delta_B b_{\nu\lambda} - Tk_2 \Delta^{\mu\nu} u^\lambda \delta_B g_{\nu\lambda}$$

$$\delta\varrho = -\varrho_1 \Delta^{\mu\nu}\nabla_\mu u_\nu - \varrho_2 h^\mu h^\nu \nabla_\mu u_\nu - \varrho_3 u^\mu \nabla_\mu T - \varrho_4 u^\mu \nabla_\mu (\mu/T) \quad ,$$

$$m^\mu = -Tm_1 \Delta^{\mu\nu} h^\lambda \delta_B b_{\nu\lambda} - Tm_2 \Delta^{\mu\nu} u^\lambda \delta_B g_{\nu\lambda} \quad ,$$

$$n^\mu = -Tn_1 \Delta^{\mu\sigma} h^\nu \delta_B g_{\nu\sigma} - Tn_2 \Delta^{\mu\sigma} u^\nu \delta_B b_{\sigma\nu} \quad ,$$

$$t^{\mu\nu} = -T\eta_\perp \left(\Delta^{\mu\rho}\Delta^{\nu\sigma} - \frac{1}{2}\Delta^{\mu\nu}\Delta^{\rho\sigma}\right) \delta_B g_{\rho\sigma} \quad ,$$

$$s^{\mu\nu} = -Tr_{||} \Delta^{\mu\rho}\Delta^{\nu\sigma} \delta_B b_{\rho\sigma} \quad ,$$

TAKE CONSTANT MAGNETIC FIELDS

GENERAL ANALYSIS

- Two channels, 11 modes, 7 gapped
- Many inequalities

$$n_2 < 0 \quad , \quad k_2 < 0 \quad , \quad (\chi_1 + \chi_2 \rho T) < 0 \quad , \quad g > 0 \quad , \quad (\varepsilon_4 \varrho_3 - \varepsilon_3 \varrho_4) < 0$$

$$g = T\lambda(T^2 \varrho_3 + \varepsilon_4 - 2\mu\varrho_4) - T^2\chi(\varepsilon_3 - \mu\varrho_3) + \mu\chi(\varepsilon_4 - \mu\varrho_4) - cT^2\varrho_4$$

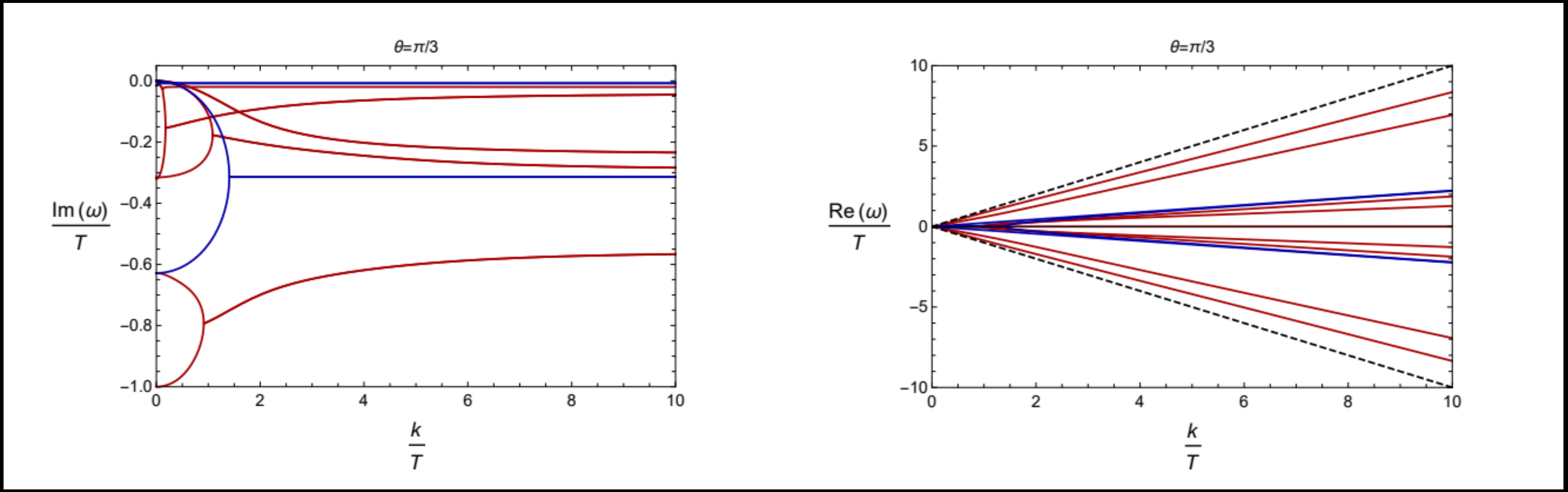
SOLVING CONSTRAINTS:

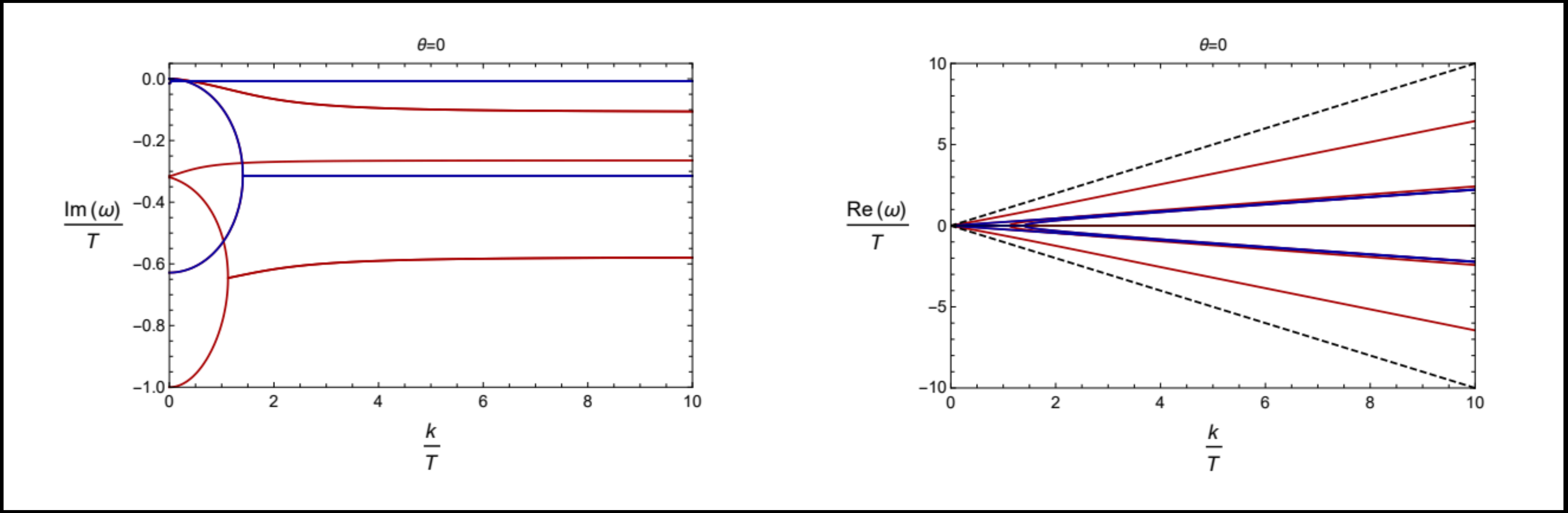
choose holographic equation of state and transport coefficients

	weak field ($T/\sqrt{\mathcal{B}} \gg 1$)	strong field ($T/\sqrt{\mathcal{B}} \ll 1$)
ε	$\frac{N_c^2}{2\pi^2}\left(74.1 \times T^4\right)$	$\frac{N_c^2}{2\pi^2}\left(5.62 \times \mathcal{B}^2\right)$
p	$\frac{N_c^2}{2\pi^2}\left(25.3 \times T^4\right)$	$\frac{N_c^2}{2\pi^2}\left(5.32 \times \mathcal{B}^2\right)$
s	$\frac{N_c^2}{2\pi^2}\left(99.4 \times T^3\right)$	$\frac{N_c^2}{2\pi^2}\left(7.41 \times \mathcal{B}\, T\right)$
μ	$\frac{N_c^2}{2\pi^2}\left(10.9 \times \mathcal{B}\right)$	$\frac{N_c^2}{2\pi^2}\left(2.88 \times \mathcal{B}\right)$

	weak field ($T/\sqrt{\mathcal{B}} \gg 1$)	strong field ($T/\sqrt{\mathcal{B}} \ll 1$)
$\eta_\perp$	$\frac{s}{4\pi}$	$\frac{s}{4\pi}$
$\eta_\parallel$	$1.00 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(21.32 \times \frac{T^2}{\mathcal{B}}\right)$
$\zeta_\perp$	$0.33 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(16.34 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$\zeta_\parallel$	$1.33 \times \frac{s}{4\pi}$	$\frac{s}{4\pi}\left(65.37 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$\zeta_\times$	$-0.66 \times \frac{s}{4\pi}$	$-\frac{s}{4\pi}\left(32.69 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$r_\perp$	$\frac{\mathcal{B}}{\mu}\left(3.37 \times \frac{1}{T}\right)$	$\frac{\sqrt{\mathcal{B}}}{\mu}\left(4.7 \times \frac{T^3}{\mathcal{B}^{3/2}}\right)$
$r_\parallel$	$\frac{\mathcal{B}}{\mu}\left(3.37 \times \frac{1}{T}\right)$	$\frac{\sqrt{\mathcal{B}}}{\mu}\left(62.3 \times \frac{T}{\sqrt{\mathcal{B}}}\right)$

Grozdanov & Poovuttikul 2017.


θ=π/3
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
Im (ω) / T
0
2
4
6
8
10
k / T
θ=π/3
10
5
0
−5
−10
Re (ω) / T
0
2
4
6
8
10
k / T


θ=0
0.0
-0.2
-0.4
-0.6
-0.8
−1.0
Im (ω) / T
0
2
4
6
8
10
k / T
θ=0
10
5
0
−5
−10
Re (ω) / T
0
2
4
6
8
10
k / T

OUTLOOK

(1) There is a choice of frame that leads to causality and stability
(2) To understand how useful the model is requires simulations, but looks promising Pandya & Pretorius 2021, Pandya & Most & Pretorius, 2022
(3) Additional elements would be useful: baryon current, accreting matter, parity-violation
(4) Can nonlinear stability and causality be proven?
(5) Can this solve the issues of variability in simulations for EHT?
(6) Dynamical gravity?