Islands and gravity in two dimensions

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2d collaborators:
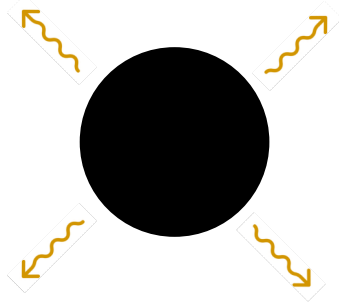
Lars Aalsma, Sergio Aguilar, Friðrik Gautason, Juan Pedraza, Lukas Schneiderbauer, Andy Svesko, Lárus Thorlacius, Manus Visser

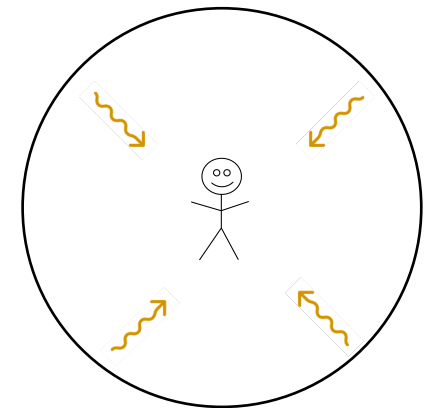Presented at: What's new in Gravity?, August 10th 2022 at Helsingoer

Today's plan

Page curve and island prescription

Semi-classical black holes
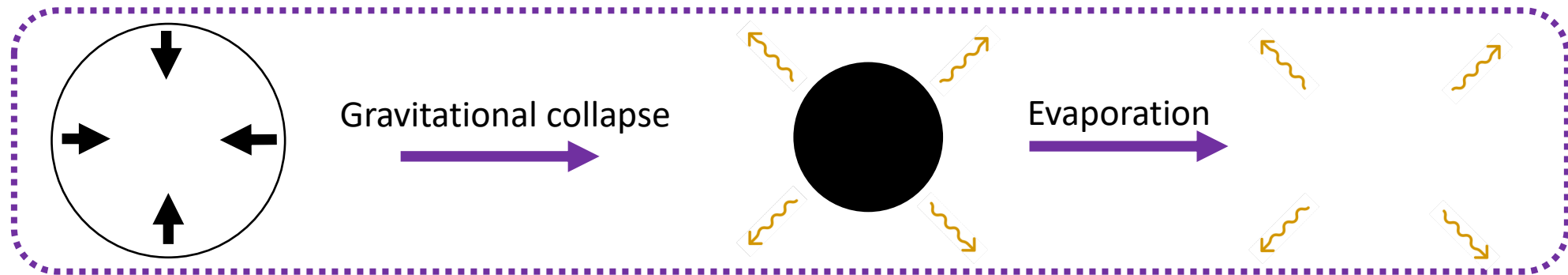
$$G_{\mu\nu} = \langle T_{\mu\nu} \rangle$$

Analytic semi-classical
2d models

Page curve for de Sitter?

Classical black hole spacetime but quantized matter implies Hawking radiation
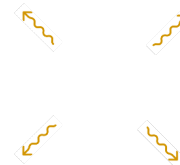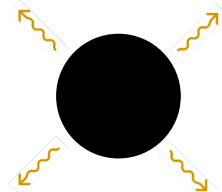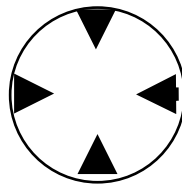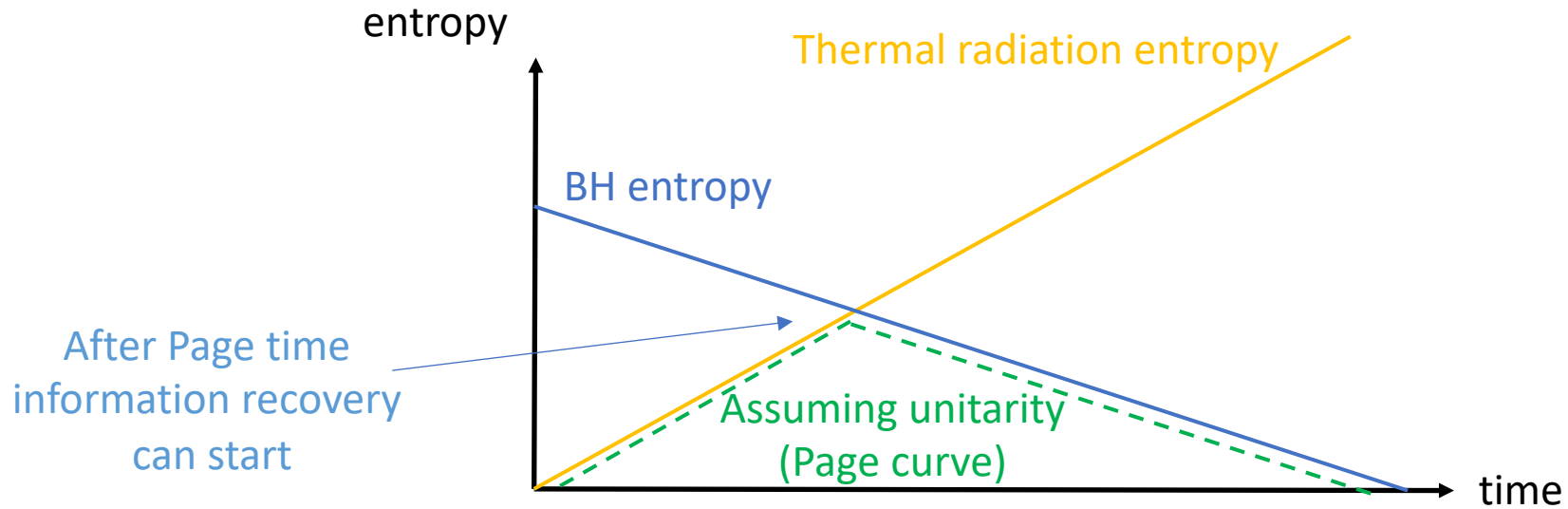
[Hawking '74]

Gravitational collapse

Evaporation

Hawking radiation being thermal leads to a paradox with unitarity

Let us consider Alice collecting Hawking radiation

Entanglement between radiation (Alice) and the black hole as function of time

The name of the game is to find a microscopic computation that reproduces the Page curve or is semi-classical enough?

[Page '93]

The island prescription

The island (or QES) prescription can give you a Page curve
[Ryu, Takayanagi'06][Hubeny, Rangamani, Takayanagi'07][Faulkner, Lewkowycz, Maldacena'09]
[Engelhardt, Wall'15][Penington '19][Almheiri, Engelhardt, Marolf, Maxfield '19]

prescription instructs to extremize and take the minimum of the generalized entropy
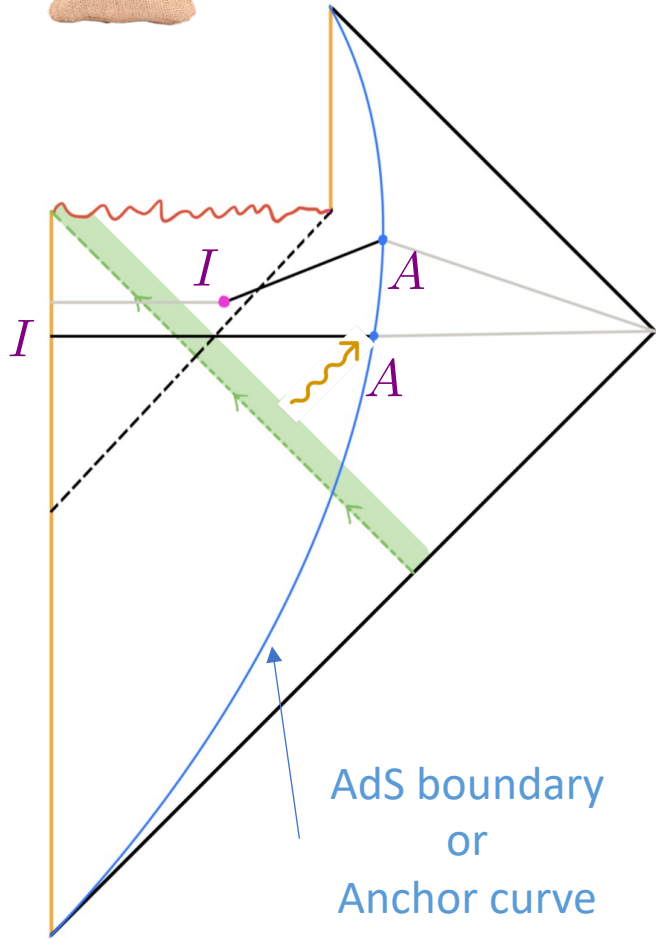
$$S_{\text{gen}} = \frac{\text{Area}(I)}{4G_N} + S_{\text{vN}}[IA]$$
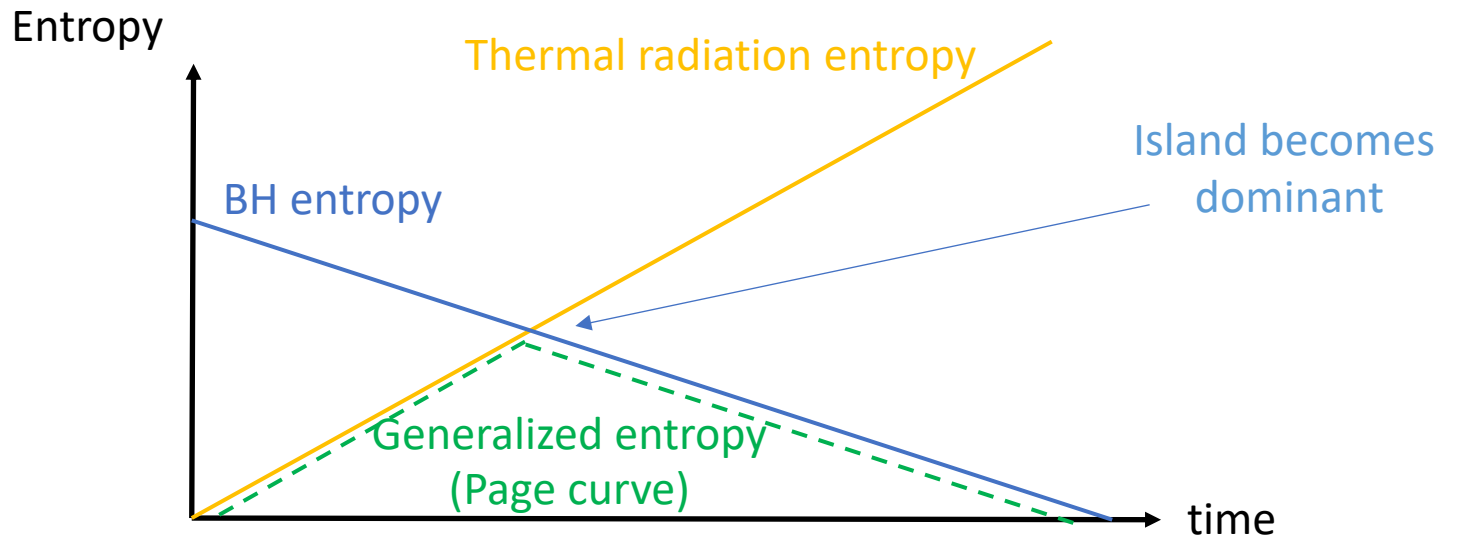
[Gautason, Schneiderbauer, WS, Thorlacius]

Island formula: wait until
Page time for decoding

Scrambling time needed to evade paradoxes
is also reproduced:

$$t_s = \log(S)$$

See e.g. [Susskind, Thorlacius'93][Hayden, Preskill'07][Sekino, Susskind'08]

2d gravity models: classical

Let's consider the analytic working horse: 2d gravity

How to build and motivate a 2d action?

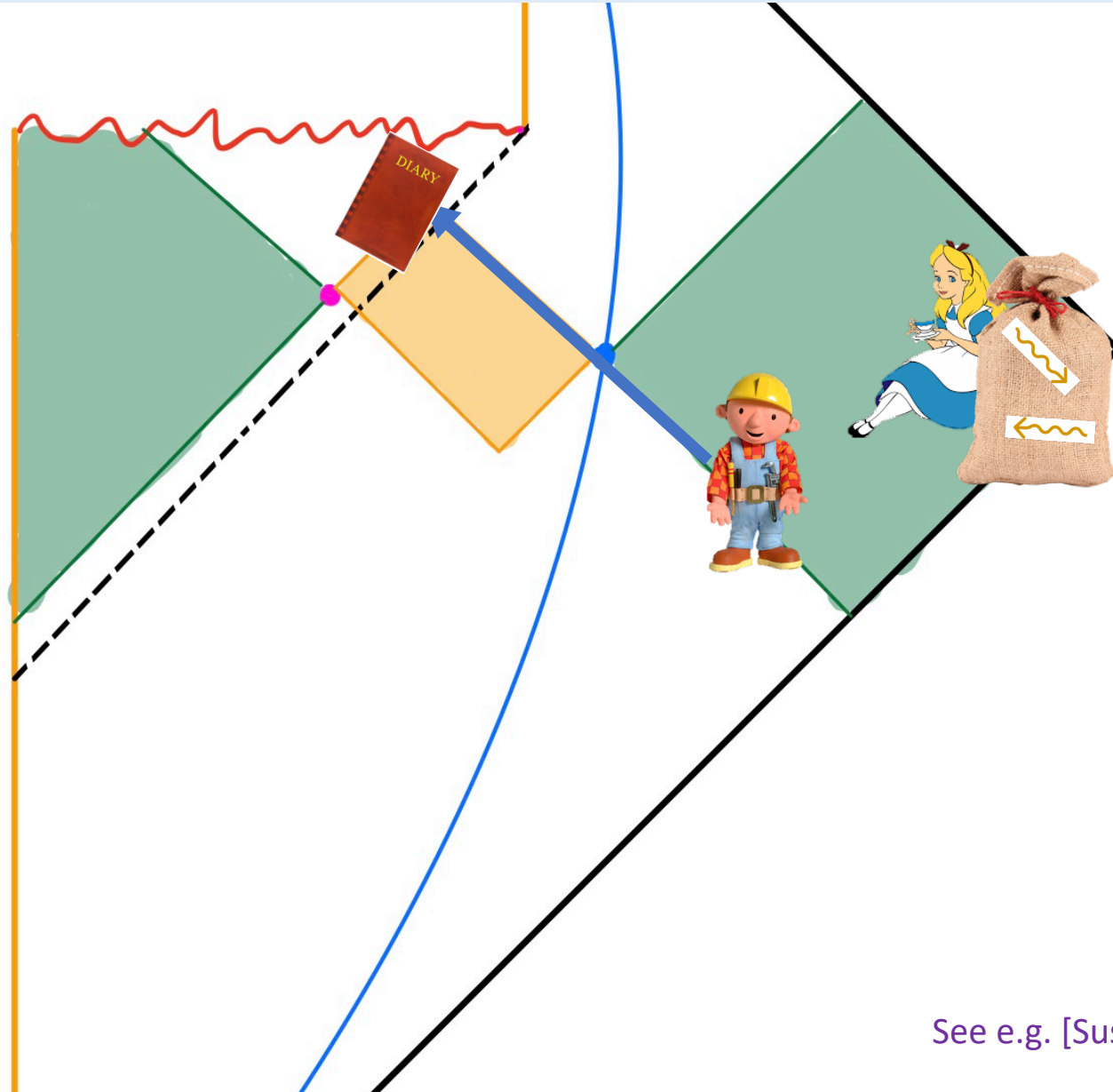A higher dimensional pedigree helps constraining models (spherical reduction, consistent truncation, radial modes)

Two often used models are JT and CGHS

$$S_{\text{JT}} = \frac{1}{2\pi} \int d^2x \sqrt{-g}\, e^{-2\phi} \left(R - 2\Lambda \right)$$

$$S_{\text{CGHS}} = \frac{1}{2\pi} \int d^2x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\nabla\phi)^2 + 4\lambda^2 \right)$$

[Teitelboim '83][Jackiw '85]

[Callan, Giddings, Harvey, Strominger '92]

Both models admit black hole solutions

Dilaton is transverse two-sphere; encodes entropy but also singularities

Add CFT as avatar for radiation → Quantize $\quad G_{\mu\nu} = \langle T_{\mu\nu} \rangle$

2d gravity models: quantized CFT mater

Let's make a bit more insightful how 2d dilaton gravity can provide useful semi-classical models

Conformal gauge is convenient in 2d: $ds^2 = -e^{2\rho}dx^+ dx^-$

Let's turn to the semi-classical Einstein equations.

$$G_{\mu\nu} = \langle T_{\mu\nu} \rangle$$
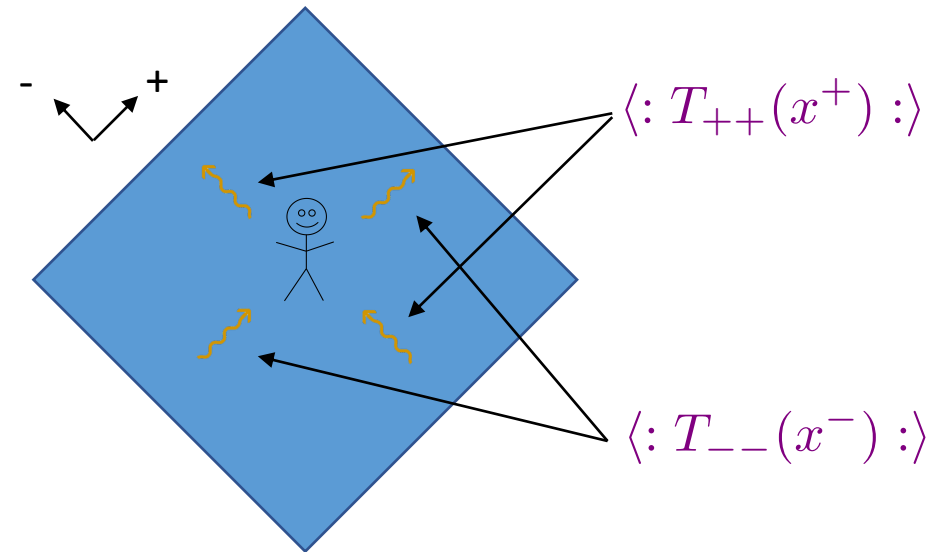
2d conformal anomaly is quite restrictive

$$\langle T^{\mu}{}_{\mu} \rangle = \frac{c}{24} R \qquad \Leftrightarrow \qquad \langle T_{+-} \rangle = -\frac{c}{12} \partial_+ \partial_- \rho$$

Combining with the continuity equation one establishes

$$\langle T_{\pm\pm}(x^{\pm}) \rangle = -\frac{c}{12} \left(\partial_+ \rho \partial_+ \rho - \partial_+^2 \rho \right) + \langle : T_{\pm\pm}(x^{\pm}) : \rangle$$

The normal ordered EMT satisfies the anomalous transformation

[Christensen, Fulling '77]

$\langle : T_{++}(x^+) : \rangle$

$\langle : T_{--}(x^-) : \rangle$

2d gravity models: semi-classical

$$\langle T^\mu{}_\mu \rangle = \frac{c}{24} R \qquad \Leftrightarrow \qquad S = -\frac{c}{96\pi} \int dx^2 \sqrt{-g(x)} \int d\tilde{x}^2 \sqrt{-g(\tilde{x})} R(x) G(x;\tilde{x}) R(\tilde{x})$$

Polyakov term is the way to incorporate quantization 2d CFT

Just add to the models

$$S_{\mathrm{JT}} = \frac{1}{2\pi} \int d^2x \sqrt{-g}\, e^{-2\phi} \left(R - 2\Lambda \right)$$

$$S_{\mathrm{CGHS}} = \frac{1}{2\pi} \int d^2x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\nabla\phi)^2 + 4\lambda^2 \right)$$

Enables description of evaporating black holes!

The cosmological horizon of de Sitter

de Sitter space can describe stages of the universe where positive vacuum energy dominates

enigmatic feature is the cosmological horizon due to expanding space

[Gibbons, Hawking '77]

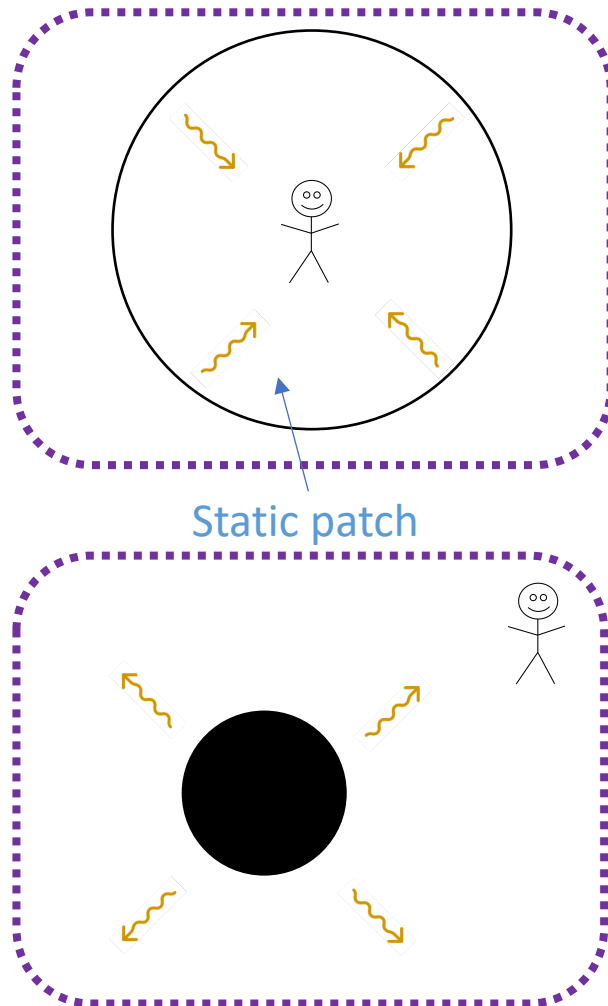Static patch

Some similarities with BH horizon

⭐ Surface of no return

⭐ $T_{\mathrm{dS}} = \dfrac{1}{2\pi\ell}$

⭐ $S_{\mathrm{dS}} = \dfrac{\mathrm{Area}}{4G_N}$

Some differences with BH horizon

⭐ Observer dependent

⭐ $\delta E = -T\delta S$

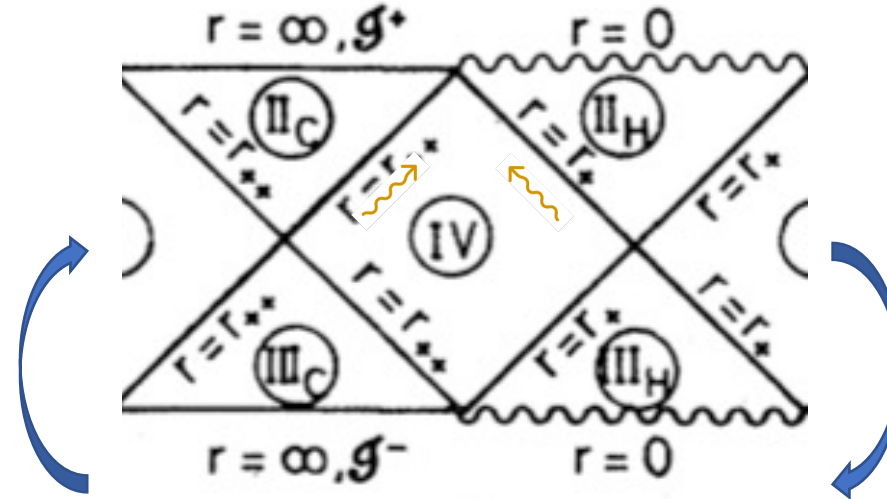Adding energy to static patch makes horizon shrink and vice versa

Black holes in de Sitter

Figure from [Gibbons, Hawking'77]

Performing a consistent truncation of a black hole in de Sitter
and considering the near-horizon region in the Near-Nariai limit
A model found to be governed by JT!
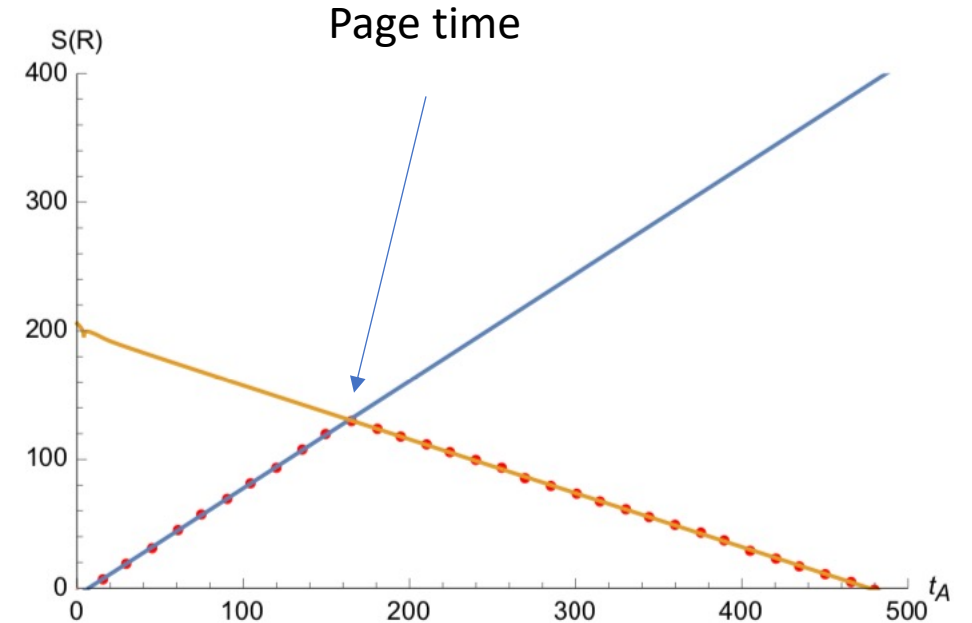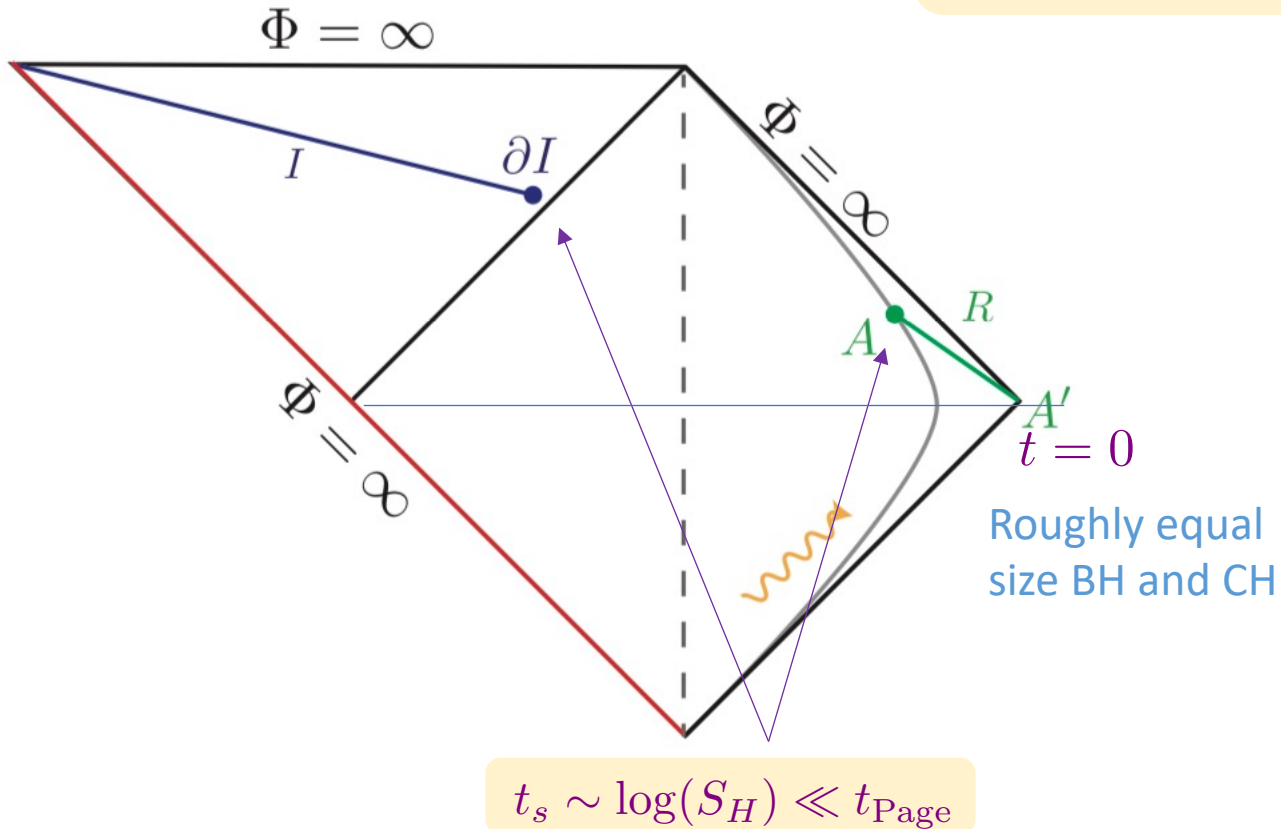
see e.g. [Maldacena, Turiaci, Yang '19]

Can we collect radiation from the cosmological horizon
and decode stuff before backreaction will interfere?

island in the full model

The island formula for in the Unruh-de Sitter vacuum:
Extremize and take minimal saddle

$$S_{\text{gen}} = \frac{\text{Area}(I)}{4G_N} + S_{\text{vN}}[IA]$$

$\Phi = \infty$

I

∂I

$\Phi = \infty$

R

A

A'

$t = 0$

$\Phi = \infty$

Roughly equal
size BH and CH

$t_s \sim \log(S_H) \ll t_{\text{Page}}$

Page time

$S(R)$

400

300

200

100

0

0   100   200   300   400   500 t_A

[Aalsma, WS'21]

Dilaton is avatar of transverse volume; effects of backreaction
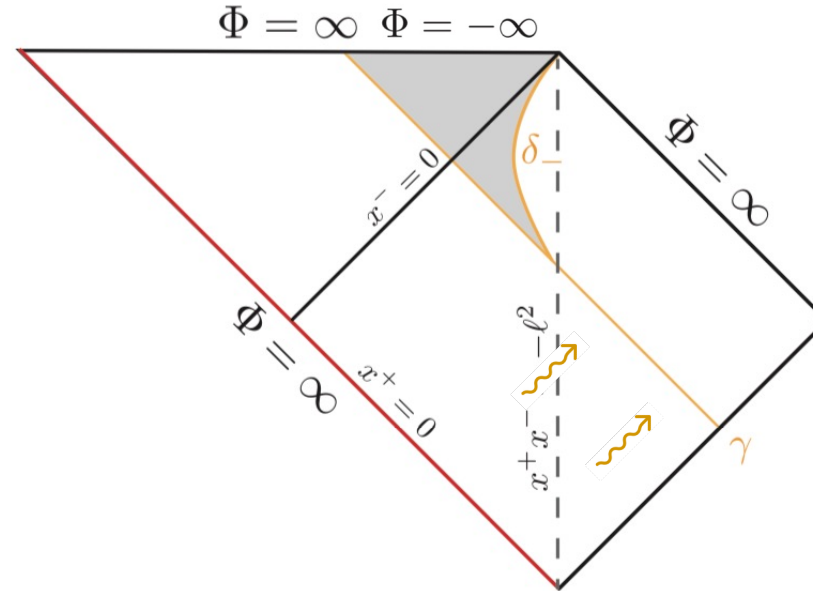
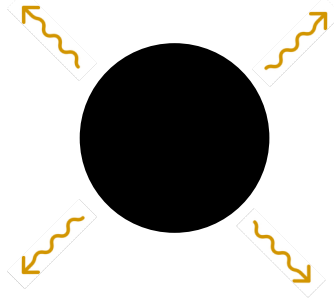Roughly equal size BH and CH

The time scale after which the trapped surface forms long before Page time

quantum singularity theorem implies that singularity forms

Thus, after trapped time scale a singularity is unavoidable   [Wall'10]

Summary

Page curve and island prescription

2d Gravity is a great tool for analytically studying semi-classical corrections

Island prescription is a great tool to provide new vantage point to address questions in semi-classical gravity

Semi-classical black holes

EMMY AND THE BLACK HOLE THAT STOLE HER SOCK

ZHAO-HE WATSE SYBESMA   DIEGO COHEN-MALDONADO

Emmy
and the
black hole
that stole her sock

By Zhao-He Watse Sybesma
Illustrated by Diego Cohen-Maldonado
Translated by Elly Dutton

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