Non-relativistic corners of $\mathcal{N} = 4$ super Yang-Mills

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Based on works with T. Harmark, Y. Lei and N. Wintergerst
[arXiv: 2009.03799, 2012.08532, 2111.10149]

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My research interests

Black holes and quantum information

- Holographic complexity (shock waves in dS space...) [SB, Bermar, Chapman]
- Rényi entropies and deformations [SB, Bianchi, Chapman, Galante, 2022]

Non-Lorentzian theories

- Non-relativistic limits of $\mathcal{N} = 4$ SYM [SB, Harmark, Lei]
- Supersymmetric Galilean Electrodynamics [SB, Cederle, Penati, 2022]
- Conformal Carroll scalars [SB, Oling, Sybesma, Søgaard, 2022]

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Motivations

AdS/CFT duality

$\mathcal{N}=4$ SYM with gauge group $\mathrm{SU}(N) \leftrightarrow$ type IIB string theory on $\mathrm{AdS}_5 \times S^5$

Great successes:

- Planar limit $N=\infty$ and integrability [Minahan, Zarembo, 2002][Beisert, Kristjansen, Staudacher, 2003]
- BMN limit [Berenstein, Maldacena, Nastase 2002]
- Supersymmetric localization [Pestun, 2007]

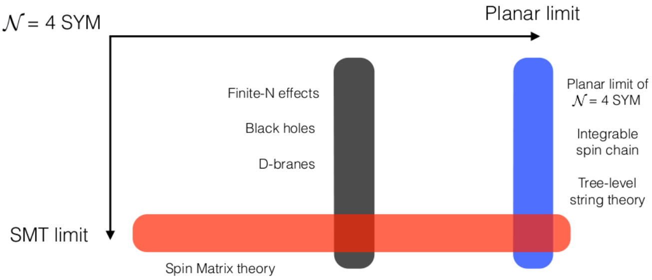
Problem:

- Planar limit: gravity enters as $1/N$ perturbative corrections
  $\Rightarrow$ No access to black holes and D-branes

Spin Matrix Theories (SMT)

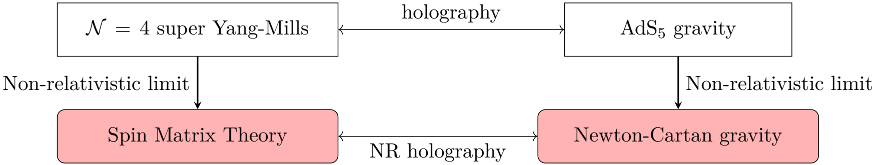
Controlled finite N effects: Spin Matrix Theory limits [Harmark, Orselli, 2014]

- Decoupling limits of $\mathcal{N}=4$ SYM on $\mathbb{R}\times S^3$ $\Rightarrow$ the theory reduces to a subsector with only one-loop contributions of the dilatation operator [Harmark, Orselli, 2006][Harmark, Kristjansson, Orselli, 2006-07]
- Unique extension from $N=\infty$ to finite N $\Rightarrow$ generalization of spin chains



Non-relativistic nature of SMT

- Emergent $\mathrm{U}(1)$ global symmetry $\Rightarrow$ mass conservation
- Bulk duals are non-relativistic string theories [Harmark, Hartong, Obers, 2017][Harmark, Hartong, Menculini, Obers, Yan, 2018][Harmark, Hartong, Menculini, Obers, Oling, 2019]



Decoupling limits

- Grand-canonical ensemble: approach zero-temperature critical points
- Micro-canonical ensemble: zoom-in close to a unitarity (BPS) bound

$$E \geq J = \sum_i (a_i S_i + b_i Q_i) \tag{1}$$

(S_i isometries on S^3, Q_i Cartan generators of $\mathrm{SU}(4)$ R-symmetry)

SMT limit ($\lambda = gN^2$)

$$\lambda \to 0 \,, \qquad \frac{E-J}{\lambda} \text{ finite} \,, \qquad N \text{ fixed} \tag{2}$$

List of Spin Matrix Theories with scalars

Spin group	Combination of Cartan charges J
$\mathsf{SU}(2)$	$Q_1 + Q_2$
$\mathsf{SU}(1\|1)$	$\frac{2}{3}S_1 + Q_1 + \frac{2}{3}(Q_2 + Q_3)$
$\mathsf{SU}(1\|2)$	$\frac{1}{2}S_1 + Q_1 + Q_2 + \frac{1}{2}Q_3$
$\mathsf{SU}(2\|3)$	$Q_1 + Q_2 + Q_3$
$\mathsf{SU}(1,1)$	$S_1 + Q_1$
$\mathsf{SU}(1,1\|1)$	$S_1 + Q_1 + \frac{1}{2}(Q_2 + Q_3)$
$\mathsf{SU}(1,1\|2)$	$S_1 + Q_1 + Q_2$
$\mathsf{SU}(1,2\|2)$	$S_1 + S_2 + Q_1$
$\mathrm{PSU}(1,2\|3)$	$S_1 + S_2 + Q_1 + Q_2 + Q_3$

Goal of the talk

Compute the effective Hamiltonian

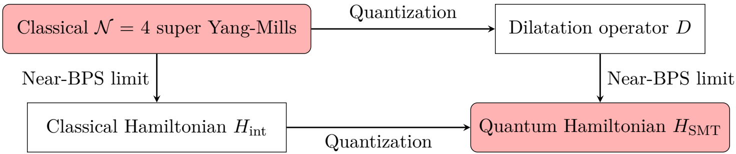
$$\lambda \to 0\,, \qquad H_{\text{int}} = \frac{H-J}{\lambda} \text{ finite}\,, \qquad N \text{ fixed} \tag{3}$$

Focus on SU(1,1|1) sector

Techniques to compute the SMT Hamiltonian

Methods to compute H_{int}:

1. Loop corrections to the dilatation operator, then zoom in towards the BPS bound [Minahan, Zarembo, 2002][Beisert, Kristjansen, Staudacher, 2003]
2. Dimensional reduction along S^3, then quantize [Harmark, Wintergerst, 2019][SB, Harmark, Wintergerst, 2020]



3. Algebraic structure determines quadratic blocks, then build the Hamiltonian based on symmetries [SB, Lei, Harmark, Wintergerst, 2020]
4. Find fermionic generators such that [Beisert, Zwiebel, 2007][SB, Harmark, Lei, 2021]

$$\{\mathcal{Q}, \mathcal{Q}^\dagger\} = H_{\text{int}} \tag{4}$$

SU(1,1|1) SMT Hamiltonian

BPS bound

$$E \geq S_1 + Q_1 + \frac{1}{2}(Q_2 + Q_3) \tag{5}$$

Classical Hamiltonian

$$H_{\text{limit}} = H_0 + \tilde{g}^2 H_{\text{int}} \tag{6}$$

$$H_0 = \sum_{n=0}^{\infty} \left[(n+1)\,\text{tr}\left(\Phi_n^\dagger \Phi_n\right) + \left(n + \frac{3}{2}\right) \text{tr}\left(\psi_n^\dagger \psi_n\right) \right] \tag{7}$$

$$\boxed{H_{\text{int}} = \frac{1}{2N} \sum_{l=1}^{\infty} \frac{1}{l}\,\text{tr}\left(\hat{q}_l^\dagger \hat{q}_l\right) + \frac{1}{2N} \sum_{l=0}^{\infty} \text{tr}\left(F_l^\dagger F_l\right)} \tag{8}$$

Blocks

$$\hat{q}_l^\dagger = \sum_{n=0}^{\infty} \left([\Phi_{n+l}^\dagger, \Phi_n] + \frac{\sqrt{n+1}}{\sqrt{n+l+1}} \{\psi_{n+l}^\dagger, \psi_n\} \right) \tag{9}$$

$$F_l^\dagger = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+l+1}} [\Phi_n, \psi_{n+l}^\dagger] \tag{10}$$

Properties of the effective Hamiltonian

$$\boxed{H_{\text{int}} = \frac{1}{2N}\sum_{l=1}^{\infty}\frac{1}{l}\,\text{tr}\left(\hat{q}_l^\dagger \hat{q}_l\right) + \frac{1}{2N}\sum_{l=0}^{\infty}\text{tr}\left(F_l^\dagger F_l\right)} \tag{11}$$

- Positive definite
- Measure of the distance from the saturation of the BPS bound
- Admits a (semi)local superfield formulation

Semi-local formulation of the $SU(1,1|1)$ sector

Surviving fields of the $SU(1,1|1)$ near-BPS limit

$$\Phi(t,x) = \sum_{n=0}^{\infty} \Phi_n(t) e^{i(n+\frac{1}{2})x} \,, \qquad \psi(t,x) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}} \psi_n(t) e^{i(n+1)x} \tag{12}$$

Equal-time (anti)commutators

$$[\Phi(t,x), \Phi(t,x')] = 0 \ , \quad [\Phi(t,x), -i\Phi^\dagger(t,x')] = iS_{\frac{1}{2}}(x-x') \tag{13}$$

$$\{\psi(t,x), \psi(t,x')\} = 0 \ , \quad \{\psi(t,x), \partial_{x'}\psi^\dagger(t,x')\} = iS_1(x-x') \tag{14}$$

$$S_j(x) = \sum_{n=0}^{\infty} e^{i(n+j)x} \tag{15}$$

Superfield formulation of the $\mathrm{SU}(1,1|1)$ sector

- (Anti)chiral fermionic superfield

$$\Psi(t,x,\theta,\theta^\dagger) = \psi + \theta\Phi + \frac{i}{2}\theta\theta^\dagger\partial_x\psi\,, \quad \Psi^\dagger(t,x,\theta,\theta^\dagger) = \psi^\dagger + \theta^\dagger\Phi^\dagger - \frac{i}{2}\theta\theta^\dagger\partial_x\psi^\dagger$$

- (Anti)chiral bosonic superfield:

$$\mathcal{A}(t,x,\theta,\theta^\dagger) = A(t,x) + \theta\lambda(t,x) + \tfrac{i}{2}\theta\theta^\dagger\partial_x A(t,x)\,,$$
$$\mathcal{A}^\dagger(t,x,\theta,\theta^\dagger) = A^\dagger(t,x) - \theta^\dagger\lambda^\dagger(t,x) - \tfrac{i}{2}\theta\theta^\dagger\partial_x A^\dagger$$

A, λ residual gauge field and gaugino

Action:

$$S = \int dtdx \int d\theta^\dagger d\theta \ \mathrm{tr}\left(i\Psi^\dagger(\mathcal{D}_0 - \mathcal{D}_x)\Psi + \mathcal{A}^\dagger\mathcal{A}\right) \qquad (16)$$

with $\mathcal{D}_0 \equiv \partial_0\,, \mathcal{D}_x \equiv \partial_x - ig_0\mathcal{A} - ig_0\mathcal{A}^\dagger$.

Conclusions

- Non-relativistic theories from near-BPS limits of $\mathcal{N} = 4$ SYM
- Positivity of the interactions
- Block structure
- Semi-local interpretation as QFT
- Superfield formulation

Future developments

- Local formulation of SU(1,2) theories as 2+1 dimensional QFTs
- PSU(1,2|3) SMT limit [SB, Harmark, Lei, in progress]
- Relation to black holes in the PSU(1,2|3) sector [Gutowski, Reall, 2004]
- Holographic investigations: TNC and SNC strings [Bergshoeff, Gomis, Gürsoy, Harmark, Hartong, Oling, Rosseel, Simsek, Yan, Zinnato, . . .]

Thank you!