Quantum Critical Membranes and Gravity

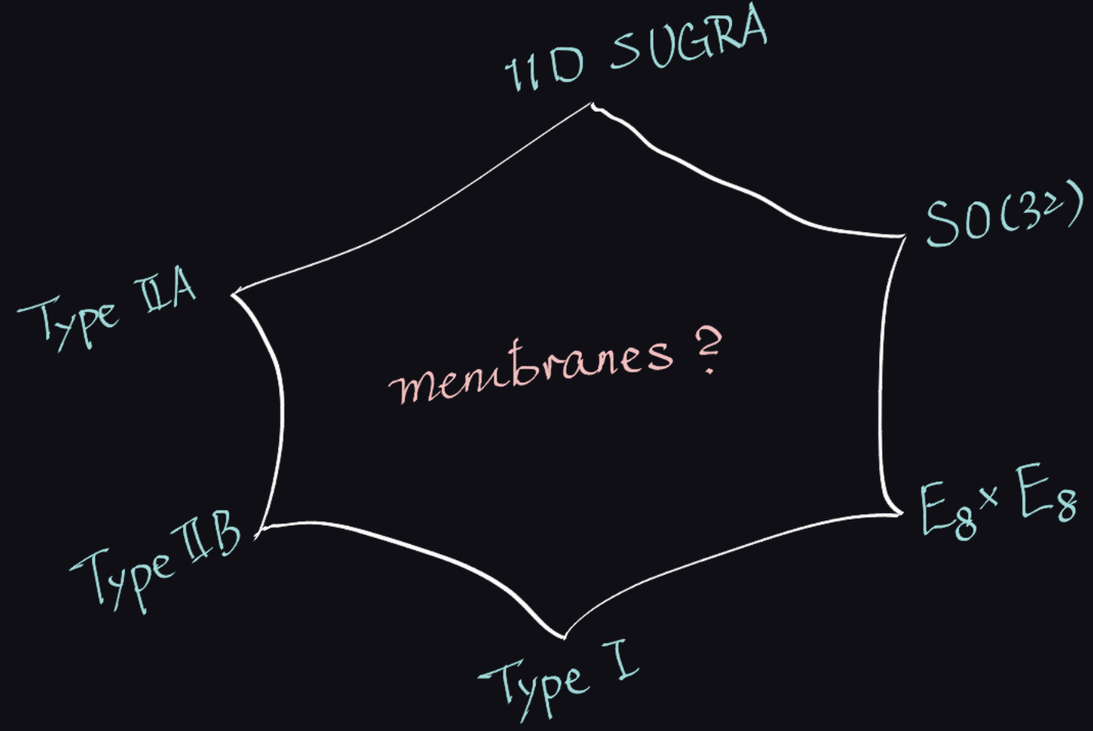
Ziqi Yan

Nordita

"What is new in gravity?" at NBI

2022.08.10

- string theory : a paradigm for quantum gravity
- non-perturbative quantum gravity ?
- nature of string theory : quantum membranes ?



- membrane dynamics: 3D sigma models [Bergshoeff, Sezgin, Townsend, 1987]

$$S_m = -\frac{1}{2g^2}\int d^3\sigma\,\sqrt{-h}\left[h^{\alpha\beta}\,\partial_\alpha X^\mu\,\partial_\beta X^\nu\,G_{\mu\nu}(X) - 1\right] - \frac{1}{g^2}\int A^{(3)}$$

scaling dimension of g: $[g] = -\frac{1}{2}$ $\rightsquigarrow$ nonrenormalizability

light-cone gauge + discretizing 2D space [de Wit, Hoppe, Nicolai, 1988]
[Banks, Fischler, Shenker, Susskind, 1997]

$\rightsquigarrow$ Matrix quantum mechanics [Susskind, 1997]
[Seiberg, 1997] [Sen, 1997]

another perspective: discrete light cone quantization (DLCQ)

- DLCQ of string / M-theory

[Gomis, Ooguri, 2000]

[Oling, Yan, 2022] for a review

A decoupling limit leading to Nonrelativistic String Theory

- no graviton
- instantaneous Newton-like interactions

- Is Matrix theory conjecture correct ?

  Unlikely for the full M-theory

[Calip, 1995] for a review

- Worldvolume gravity ?

3D Quantum Gravity

- nonrenormalizable

it's okay : finite degrees of freedom ⇝ quantum mechanics

- no fixed definition of time

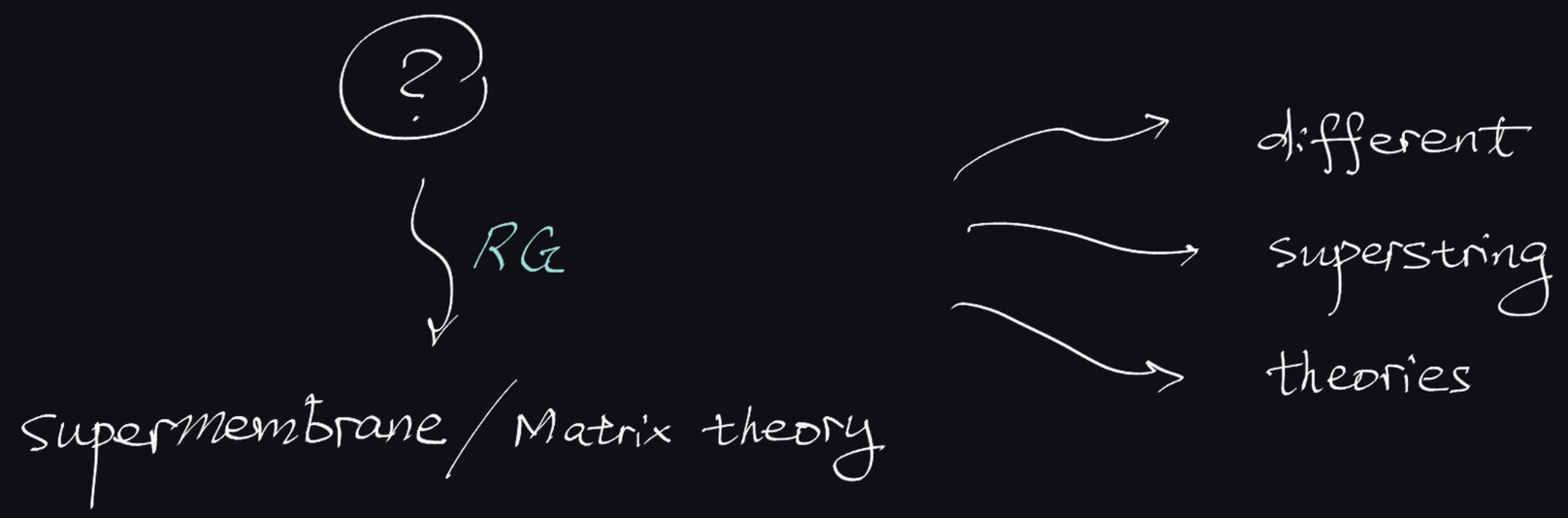
no complete understanding

- UV completion of quantum membranes ?

first quantization of strings : conformality on worldsheet

Quantum Gravity : ensemble of sigma models in all backgrounds

RG away from the conformal fixed point : string field theory ?



[Hořava, 2008]

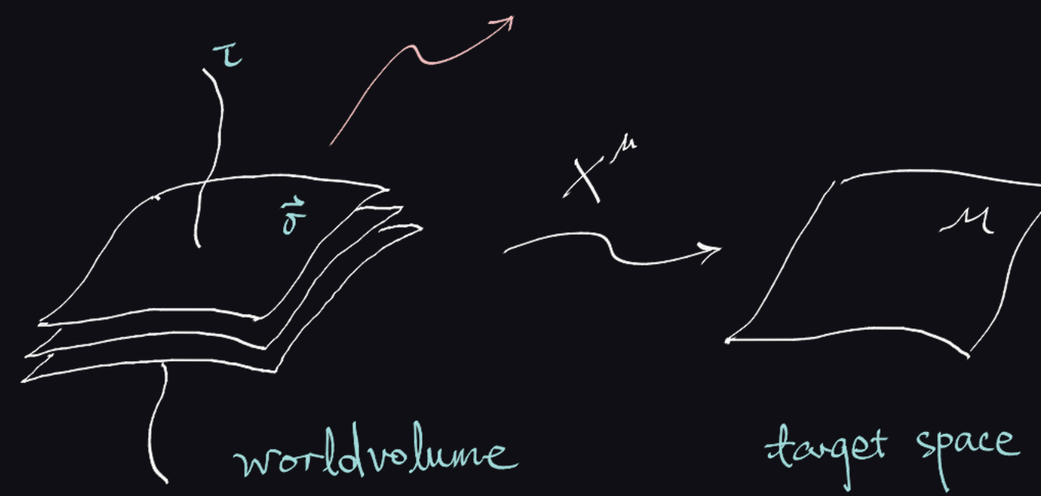
- Membranes at Quantum Criticality

Aristotelian worldvolume: no boost included!

anisotropic worldvolume

Lifshitz scaling $\begin{cases} \tau \to b\tau \\ \vec{\sigma} \to b^{1/2}\vec{\sigma} \end{cases}$

perturbative expansion w.r.t. genera of Riemann surfaces?



$$S = \frac{1}{2g^2}\int d\tau\, d^2\vec{\sigma}^{\,i}\left(\partial_\tau X^\mu\, \partial_\tau X_\mu - \partial_i^2 X^\mu\, \partial_j^2 X_\mu - c^2\,\partial_i X^\mu\, \partial_i X_\mu + \text{interactions}\right)$$

renormalizable 3D sigma model! $\omega^2 \sim k^4 + c^2 k^2$

- 3D sigma models at $z=2$ Lifshitz point

  - $O(N)$ nonlinear sigma model [Anagnostopoulos, Farakos, Pasipoularides, Tsapalis, 2010] [Griffin, Grosvenor, Hořava, Ryu, Wen, ZY]

$$S = -\frac{1}{2g^2}\int d\tau\, d^2\vec{x}\, \Big\{ \left(\partial_\tau X^I \partial_\tau X^J - \partial_i^2 X^I\, \partial_j^2 X^I - c^2\, \partial_i X^I \partial_i X^J\right) G_{IJ}$$

$$- \eta_1 \left(\partial_i X^I \partial_i X^J G_{IJ}\right)^2 - \eta_2 \left(\partial_i X^I \partial_j X^J G_{IJ}\right)^2 \Big\}$$

applications in condensed matter { Rokhsar-Kivelson quantum dimer models for spin liquid; quantum spherical models w/ competing interactions

  - general bosonic sigma model [ZY, 2021]

$$S = -\frac{1}{2g^2}\int d\tau\, d^2\vec{x}\, \Big\{ \partial_\tau X^I \partial_\tau X^J G_{IJ} - \partial_i^2 X^I\, \partial_j^2 X^J H_{IJ} - c^2\, \partial_i X^I \partial_i X^J \widetilde{G}_{IJ}$$

$$- \partial_i X^I \partial_i X^J \partial_j X^k \partial_j X^l M_{IJkl} \Big\}$$

[Hořava, Melby-Thompson, Randall]

- real interest: quantum critical supermembranes

- simplification from $\mathcal{N}=1$ supersymmetry spacetime bimetricity !?

$$S = \frac{1}{2g^2}\int d\tau\, d^2\theta^i\, d^2\theta \left(\bar{D}^\alpha Y^I\, D_\alpha Y^J\, G_{IJ} - \partial_i Y^I\, \partial_i Y^J\, H_{IJ} \right)$$

$$Y^I = X^I + \bar{\theta}\,\psi^I + \tfrac{1}{2}\theta^2\, B^I \qquad\qquad D = \frac{\partial}{\partial\bar{\theta}} + \not{\partial}\theta$$

supersymmetry transformations

$$\delta_\epsilon X^I = \bar{\epsilon}\,\psi^I$$

$$\delta_\epsilon \psi^I = (-\not{\partial} X^I + B^I)\,\epsilon \qquad\qquad \text{just no boost}$$

$$\delta_\epsilon B^I = \bar{\epsilon}\not{\partial}\psi^I$$

[ZY, to appear]

- beta-functions (assume that G & H are non-degenerate)

$$\beta^G_{IJ} = \frac{g^2}{8\pi}\left[-H^{KL}R^M{}_{KL(I}G_{J)M} + \frac{1}{2}G_{M(I}\nabla_{J)}\left(H^{KL}S^M{}_{KL}\right)\right]$$

$$-\frac{g^2}{16\pi}\,\mathrm{tr}\left[\mathcal{H}^{-1}\left(\mathcal{H}+\tilde{\mathcal{H}}\right)^{-1}\nabla_{(I}\mathcal{H}\,\mathcal{H}^{-1}\nabla_{J)}\mathcal{H}\right] + \mathcal{O}(g^4)$$

$$\beta^H_{IJ} = \frac{g^2}{8\pi}\left(\Sigma_{IJ} + \frac{1}{2}H^{KL}H_{M(I}\Delta_{J)}S^M{}_{KL}\right) + \mathcal{O}(g^4)$$

Riemann tensor $\begin{cases} R^I{}_{JKL} & \text{for } G_{IJ} \\ \Sigma^I{}_{JKL} & \text{for } H_{IJ} \end{cases}$ $\qquad \mathcal{H} = G^{-1}H$

$\nabla_I G_{JK} = \Delta_I H_{JK} = 0$ $\qquad S^I{}_{Jk}$: difference between connections

What are next ?

- coupling to worldvolume supergravity

  w/ preferred worldvolume time ⇝ Hořava-Lifshitz gravity

  $z=2$ scaling ⇝ renormalizable in 3D

1. one-loop eff. action from heat kernel method

   [Grosvenor, Melby-Thompson, Yan, 2021]

2. BRST symmetries and unitarity ?

3. perturbative expansion w.r.t. 2D spatial topologies ?

4. RG flow towards the relativistic supermembrane ?

and more !

Thank you