New SYM phases at finite chemical potential

Óscar Dias

STAG RESEARCH CENTRE

UNIVERSITY OF Southampton

Based on:

OD, Prahar Mitra, Jorge Santos, 2207.07134 & 2210.xx

See also: Bhattacharyya, Minwalla, Papadodimas (2011)
Markeviciute, Santos (2016, 2018)

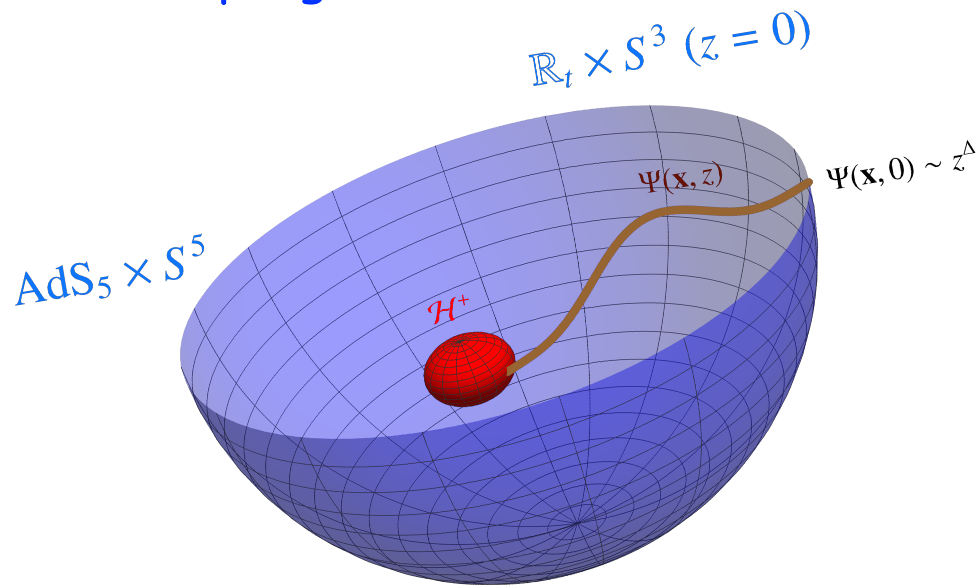
NBI mini workshop: What is new in gravity? Helsingoer, August 2022

→ AdS_5 / CFT_4 duality

Type IIB supergravity theory on **$AdS_5 x S^5$** with radius **L** and **N** units of **$F_{(5)}$** flux on **S^5**

↕

Large **N** and strong t'Hooft coupling $\lambda = g_{\rm YM}^2 N$ limit of $\mathcal{N}=4$ Super Yang-Mills (SYM) theory with gauge group SU(**N**) & YM coupling **g_{YM}**



Thermal states of $\mathcal{N}=4$ **SYM**

with **temperature T**, **chemical potentials μ_j** and energies $O(N^2)$

living on the **Einstein static Universe** $\mathbb{R}_t \times S^3$

↔

Asymptotically **global $AdS_5 x S^5$ BHs** of IIB supergravity

with Hawking **temperature T** and **chemical potentials μ_j**

→ Motivations

- We should find all the BHs and map them into thermal states in the dual SYM
  => identify the **dominant phases** (as saddle points) in the **thermodynamic ensembles.**

- Necessary to **reproduce microscopically** the Bekenstein-Hawking **entropy** of AdS BHs.

  [See Benini's review talk at Strings 2022]

- Contribute to understand a **puzzle of SO(6) gauged supergravity:** its most general SUSY BH known so far — Kunduri-Lucietti-Reall BH — has <u>only</u> **4 independent parameters.**

  However, asymptotically $AdS_5 \times S^5$ BHs are characterized by 6 conserved charges

  with the BPS relation constraint $\mathbf{E = Q_1+Q_2+Q_3+J_1+J_2}$

  => the **most general SUSY BH** should be a **5-parameter solution.**

  From dual CFT perspective, most general SUSY states **also** expected to be characterized by **5 parameters.**

  [Gutowski, Reall '04]
  [Kunduri, Lucietti, Reall '06]

  So, what is the missing gravitational parameter?

→ AIM of this Talk: **identify new thermal phases with a finite chemical potential that can dominate some thermodynamic ensembles.**

→ Known SYM phases

- **The massless bosonic fields of type IIB supergravity:**

  metric tensor g_{ab}, dilaton Φ, axion C, NS-NS antisymmetric 2-tensor $B_{(2)}$, RR 2-form potential $C_{(2)}$, and RR 4-form $C_{(4)}$ with a 5-form field strength $F_{(5)} = dC_{(4)}$ satisfying a **self-duality condition.**

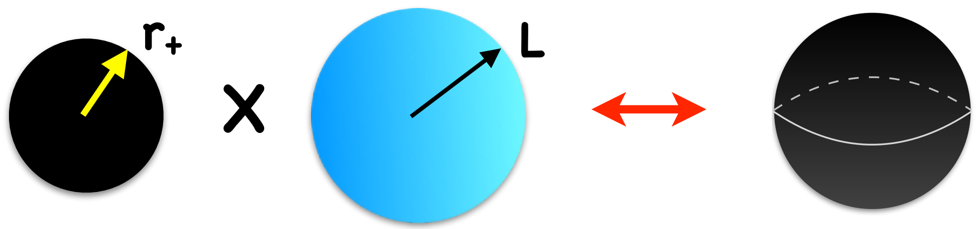
  Fermionic superpartners: complex Weyl gravitino & complex Weyl dilatino.

- **Known solutions** of IIB supergravity with only $\{g_{ab}, F_{(5)}\}$:

  - Global $AdS_5 \times S^5$ **Schwarzschild & its rotating black hole (BH) partners.**

    Everywhere (not only at bdry) the direct product of two base spaces $M_5 \times S^5$ and have horizon topology $S^3 \times S^5$.

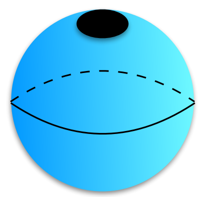
    [Witten '98]



  - Asymptotically globally AdS5 x S^5 BHs that break the SO(6) symmetry of S^5 down to SO(5):

    — **lumpy BHs** with **polar deformations** along the S^5 and

    — **localized BHs on the S^5** (with S^8 horizon topology).



[OD, Santos, Way 1501.06574 & 1605.04911]

➙ Towards finding more SYM phases

- Explore even further the phase space of thermal states (to identify all the relevant saddle points for the thermodynamic partition functions of the theory).

- Useful: **dimensional reduction of IIB along S^5** yields **5d N=8 gauged supergravity**.
  It's believed (not proven) to be a consistent reduction of IIB on **$AdS_5 \times S^5$**.

Gunaydin, Romans, Warner (1986)

- **10d** fields $g_{ab}, \Phi, C, B_{(2)}, C_{(2)}, C_{(4)}$ are equivalently encoded in the **5d field content of gauged N =8 SUGRA:** graviton g_{ab}, **fifteen** SO(6) gauge fields A^{ij}, **twelve** 2-form gauge potentials in the $6+\bar{6}$ representations of SO(6), **42** scalars in the $1+1+20'+10+\overline{10}$ representations of SO(6) & the fermionic superpartners.

- But IIB with only $g_{ab}, F_{(5)}$ **(relevant for AdS/CFT: source D3's) can be consistently dim reduced** along the S^5 to yield **5d SO(6) gauged supergravity.**

Cvetic-Lü-Pope-Sadrzadeh-Tran [hep-th/0003103]

This is itself **is a consistent truncation of gauged N =8 SUGRA** where we **set** the $1+1+10+\overline{10}$ scalars and the $6+\bar{6}$ 2-form potentials **to zero.** The bosons that survive descend from $\{g_{ab}, F_{(5)}\}$ of IIB:

the **graviton** g_{ab}, the 15 **gauge fields** A^{ij} & the $20'$ **scalars.**

→ $U(1)^3$ gauged supergravity: [OD, Mitra, Santos '22]

Consistent truncation of SO(6) gauged SUGRA down to the Cartan subgroup of SO(6) with associated gauge fields $\{A_{(1)}{}^K\}$ (K=1,2,3): $U(1)^3$ gauged supergravity.

The field content:

Graviton g_{ab} + 2 neutral scalars $\{\varphi_1, \varphi_2\}$ + 3 U(1)'s gauge fields $\{A_{(1)}{}^K\}$,

+ 3 complex scalar fields $\{\Phi_K\}$ minimally coupled to $\{A_{(1)}{}^K\}$ with charge $qL = 2$.

All 5 scalars have mass $m^2L^2 = -2$ => saturate AdS_5 Breitenlöhner-Freedman (BF) bound.

$$S = \frac{1}{16\pi G_5}\int \mathrm{d}x^5\sqrt{-g}\Bigg\{R - V(\varphi_{\tilde{K}},\lambda_K) - \frac{1}{2}\sum_{\tilde{K}=1}^{2}(\nabla\varphi_{\tilde{K}})^2 - \frac{1}{4}\sum_{K=1}^{3}\frac{\left(F^K_{(2)}\right)^2}{X_K^2(\varphi_{\tilde{K}})} - \frac{1}{8}\sum_{K=1}^{3}\left[(D_a\Phi_K)(D^a\Phi_K)^\dagger - \frac{(\nabla\lambda_K)^2}{4(4+\lambda_K)}\right]\Bigg\} - \frac{1}{16\pi G_5}\int F^1_{(2)}\wedge F^2_{(2)}\wedge A^3_{(1)}$$

$$D_a\Phi_K \equiv \partial_a\Phi_K - i\,\frac{2}{L}A^K_a\Phi_K\,, \qquad F^K_{ab} \equiv \partial_a A^K_b - \partial_b A^K_a\,, \qquad \lambda_K \equiv \Phi_K\Phi_K^\dagger$$

$$V(\varphi_{\tilde{K}},\Phi_K) = \frac{1}{2L^2}\left[\sum_{K=1}^{3}X_K^2(\varphi_{\tilde{K}})\lambda_K - \sum_{I\neq J\neq K=1}^{3}\frac{\sqrt{4+\lambda_J}\sqrt{4+\lambda_K}}{X_I(\varphi_{\tilde{K}})}\right]$$

- **EoM of U(1)^3 gauged SUGRA are complicated.**

- Three special cases of **U(1)^3 gauged supergravity** where the **EoM simplify**:

1. Truncation with **three equal charges** => $A^1 = A^2 = A^3 \equiv A,\ \Phi_1 = \Phi_2 = \Phi_3 \equiv \Phi$

$$\varphi_1 = \varphi_2 = 0$$

Liu,Lü,Pope,Vazquez-Poritz '07
Bhattacharyya, Minwalla, Papadodimas (2011)
Markeviciute, Santos (2016, 2018)

2. Truncation with a **single charge** => $A^1_{(1)} = A^2_{(1)} \equiv 0,\ A^3_{(1)} \equiv A_{(1)}$ and $\Phi_1 = \Phi_2 \equiv 0,\ \Phi_3 \equiv \Phi$

$$\varphi_1 \equiv \varphi,\ \varphi_2 = 0$$

This talk

[OD, Mitra, Santos '22]

3. Truncation with **two equal charges** => $A^1 = A^2 \equiv A,\ A^3 \equiv 0$ and $\Phi_1 = \Phi_2 \equiv \Phi,\ \Phi_3 \equiv 0$

$$\varphi_1 \equiv \varphi,\ \varphi_2 = 0$$

This talk

[OD, Mitra, Santos '22]

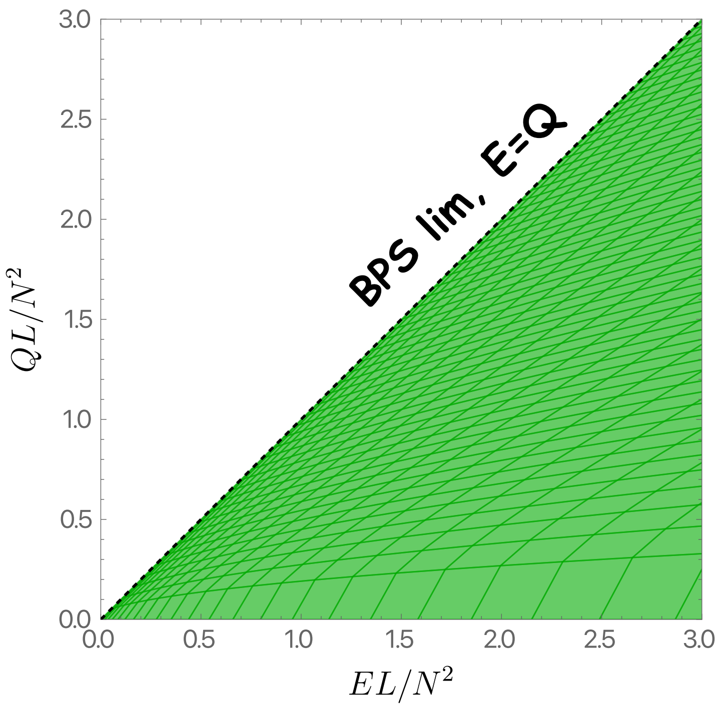
➞ 'Bald' BHs (Cvetic-Lü-Pope 2004): $\Phi_K = 0$ (K=1,2,3) $\Rightarrow$ $\langle \Phi_K \rangle = 0$

- Parameterized by an **energy E** and **three electric charges** $\{Q_1, Q_2, Q_3\}$ <—> $A_{(1)}^K$

  Can be viewed as the "**Reissner-Nordström-AdS$_5$**" (RNAdS) BHs of the theory.

  But, also have non-trivial neutral scalar fields $\{\varphi_1, \varphi_2\}$ supporting them.

- Single charge truncation

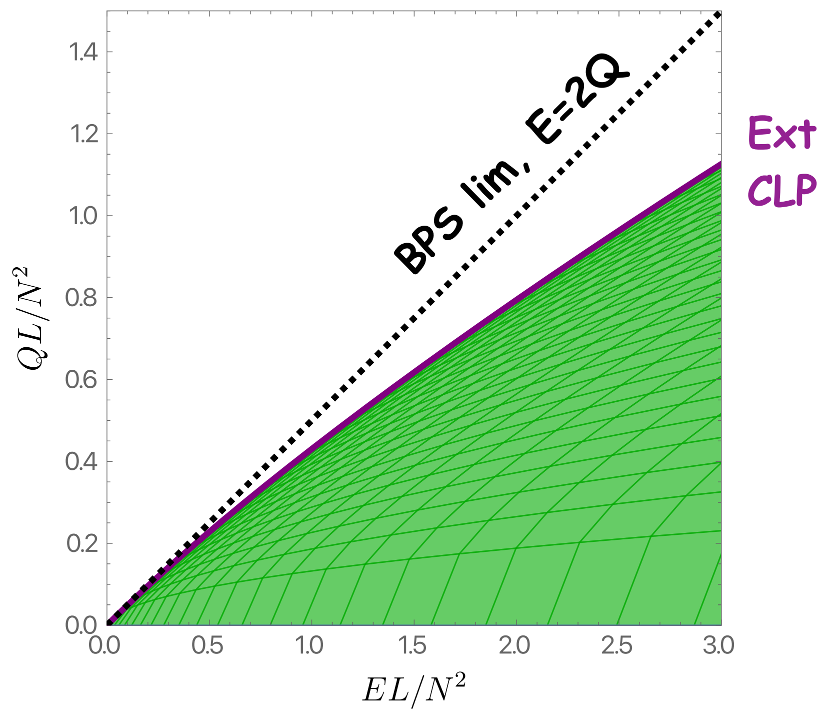


Smooth CLP: $E > Q$

CLP are singular (S=0) at $E = Q$

- Two equal charge truncation

  (equal 3-charge truncation is similar)



Smooth CLP: $E > Q_{\text{ext}} < 2Q$

CLP are singular (S=0) at $E = Q_{\text{ext}}$

➙ Truncation of **U(1)³ gauged supergravity** with a <u>single</u> charge

$$A^1_{(1)} = A^2_{(1)} \equiv 0, A^3_{(1)} \equiv A_{(1)} \text{ and } \Phi_1 = \Phi_2 \equiv 0, \Phi_3 \equiv \Phi$$

$$\varphi_1 \equiv \varphi,\ \varphi_2 = 0$$

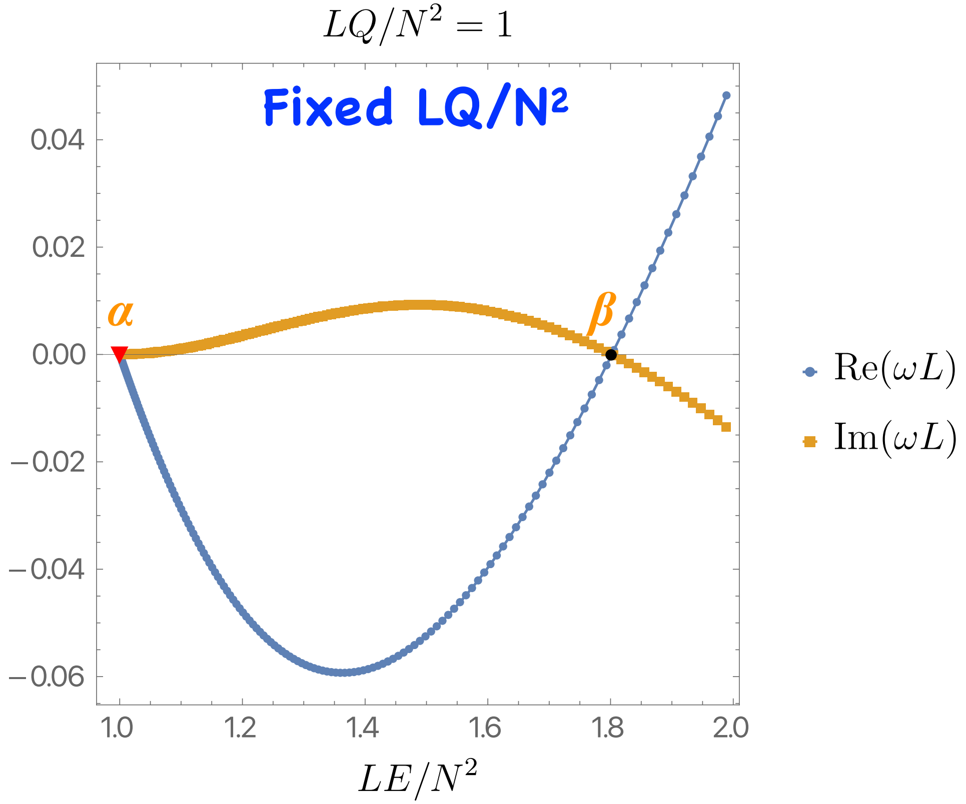
➞ Truncation of $U(1)^3$ gauged supergravity with a single charge

- **Scalar condensation instability of 'Bald' CLP BHs ($\Phi = 0$):**

  Perturb CLP BH with charged scalar field $\Phi_3 = \Phi$: $\Phi(t,r) = \widehat{\Phi}_\omega(r)\, e^{-i\omega t}$

  & solve the associated "Klein-Gordon" eqn with appropriate BCs.

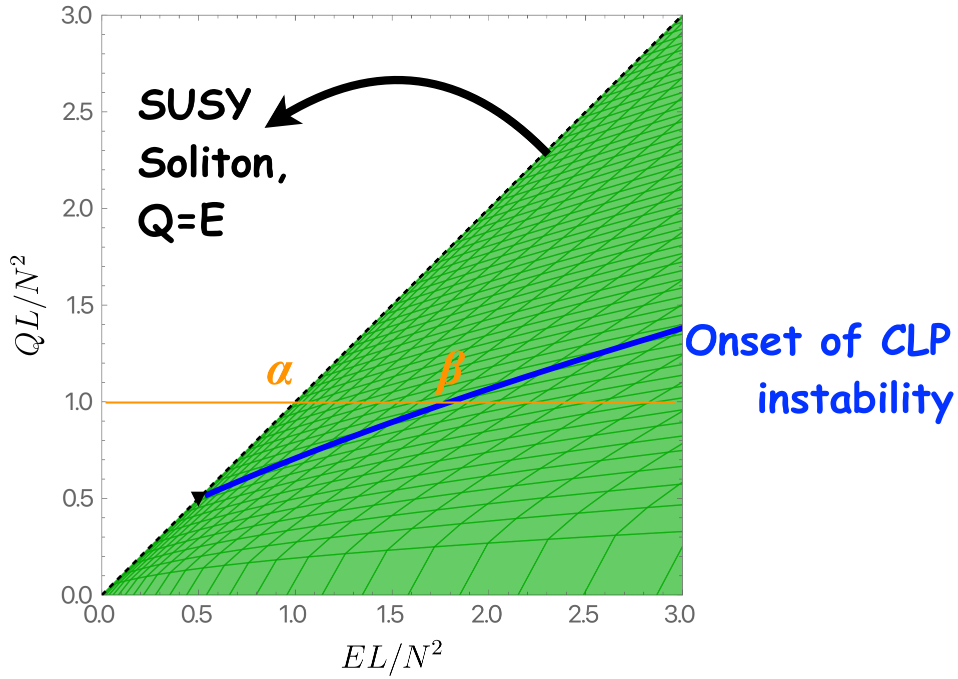
  => Unstable when Im (ωL) >0



- We can also solve directly for the **onset of the instability**, $Q_0(E)$, where $\omega = 0$. **CLP unstable above onset.**

- There is also a **supersymmetric Soliton** with **Q=E** and $\langle\Phi\rangle \neq 0$ (analytical soln)

  [Liu, Lü, Pope, Vazquez-Poritz '07]
  [OD, Mitra, Santos '22]



- **CLP BHs are linearly unstable to condensation of charged scalar Φ.**

  => beyond leading order in perturbation theory,

  there should exist **hairy BHs** with charged hair Φ

  bifurcating (in a 2nd order phase transition) from the **onset of the CLP instability,**

  in a phase diagram of static solutions.

- Solve EoM (5 coupled nonlinear ODEs) either **numerically** (at full nonlinear level) or within **perturbation theory** (matched asymp expansion) with boundary conditions:

— **Regularity at horizon** (hairy BH) or at origin (supersymmetric soliton)

— **Asymp global AdS_5** BCs for metric: bdry is Einstein Static Universe

$$\mathrm{d}s^2_{\partial\mathcal{M}} = -\mathrm{d}t^2 + L^2\,\mathrm{d}\Omega_3^2$$

Leave the **homogeneous chemical potential** (source of U(1) gauge field) free :

$$\lim_{z\to 0} A^K = \mu_K\,\mathrm{d}t$$

(z is FG coordinate with bdry at z=0)

NO sources for the charged & neutral scalar fields (all saturate the BF bound).

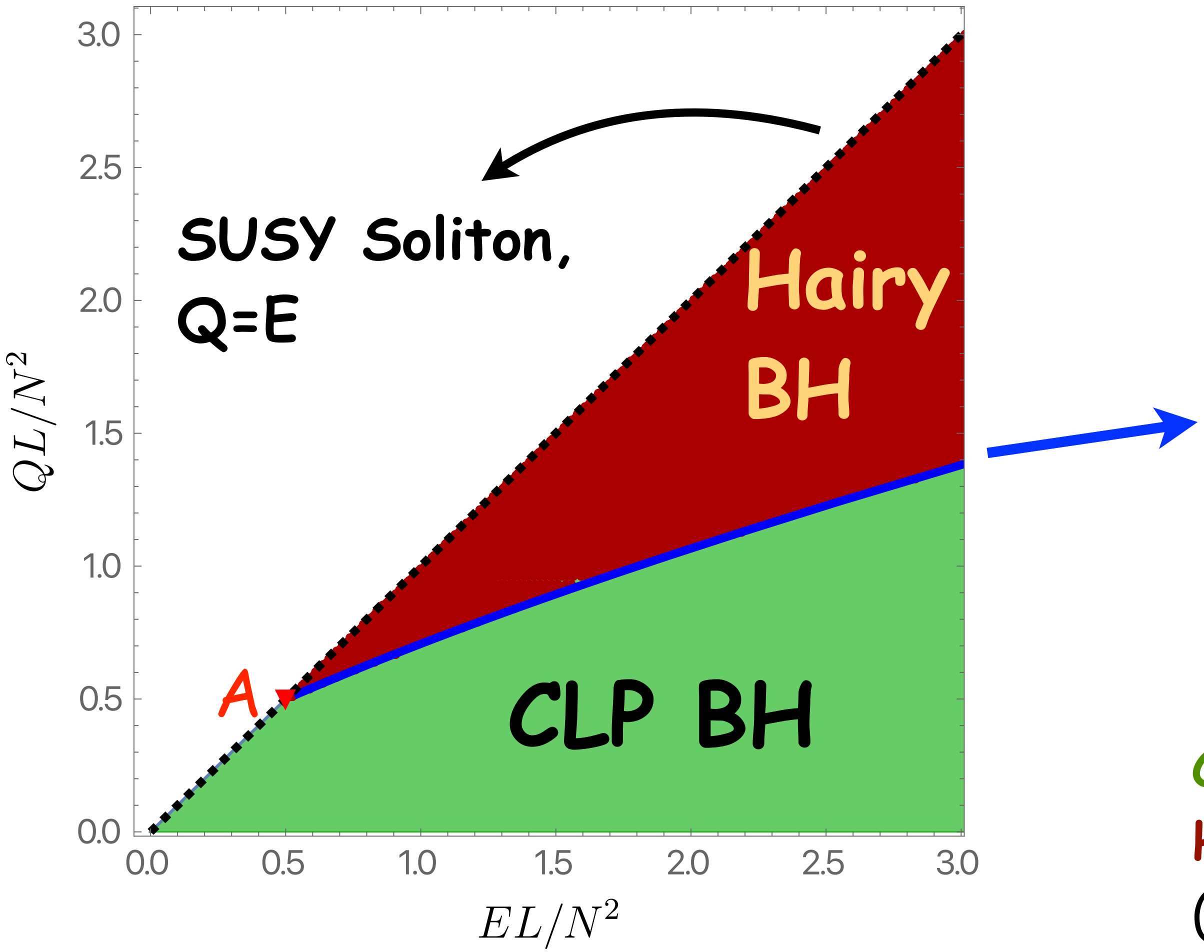
$$\varphi_{\tilde K} \underset{z\to 0}{\longrightarrow} V_{\tilde K}\, z^2 \log z + \langle \mathcal{O}_{\varphi_{\tilde K}} \rangle z^2 + \dots \qquad \Rightarrow \qquad \varphi_{\tilde K} \underset{z\to 0}{\longrightarrow} \langle \mathcal{O}_{\varphi_{\tilde K}} \rangle z^2 + \dots$$

$$\Phi_K \underset{z\to 0}{\longrightarrow} V_K\, z^2 \log z + \langle \mathcal{O}_{\Phi_K} \rangle z^2 + \dots \qquad\qquad \Phi_K \underset{z\to 0}{\longrightarrow} \langle \mathcal{O}_{\Phi_K} \rangle z^2 + \dots$$

- Thermodynamic quantities computed using **Holographic Renormalization**

→ Microcononical phase diagram (of truncation with a single charge)

Fix energy E and charge Q: dominant phase is the one with highest entropy S



Onset of CLP instability

CLP BHs exist for $0 \leqslant Q < E$

Hairy BHs exist for Onset<Q<E

(Both Singular — S=0 — in BPS lim)

- **Hairy BHs always have higher S than CLP BH with same E,Q**
- Hairy BH **temperature approaches T L=1/π (& μ=1)** in the **singular** (S=0) **BPS lim !!**

  This is **also min T that CLP can reach** (at A) in the singular BPS limit.

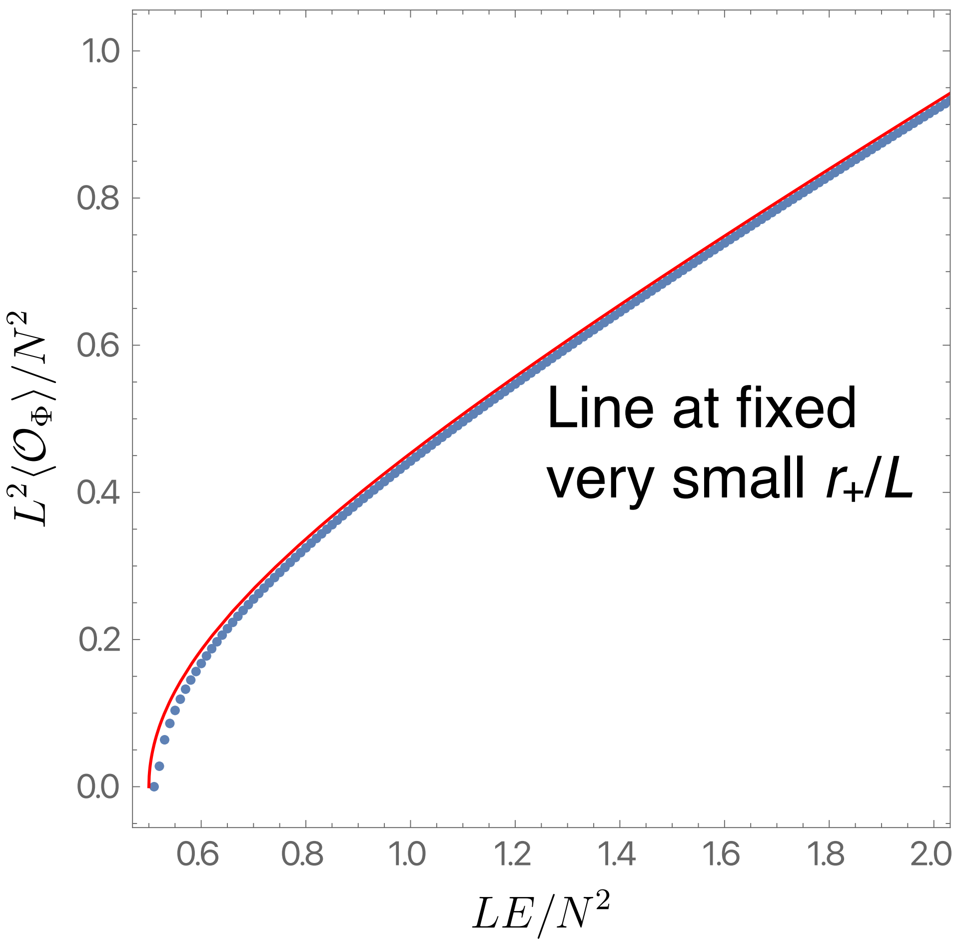
➙ Microcononical phase diagram (of truncation with a <u>single</u> charge)

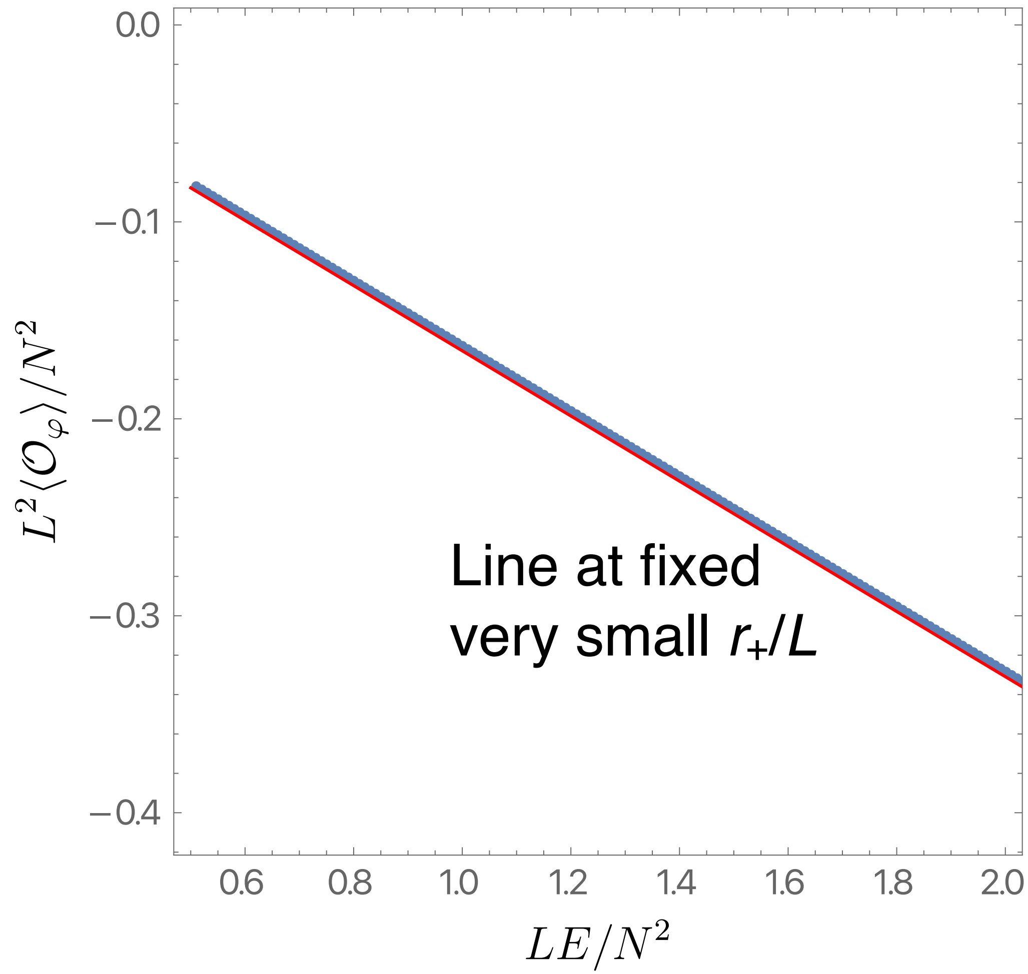
Limit E—>Q of Hairy BHs ? Singular limit (S=0) coincides with soliton at E=Q.

Charged and neutral scalar field **VEVs** for the line of Hairy BH solutions closest to the supersymmetric bound (blue disks)

Solid red lines are analytical results for for the SUSY soliton.

[Liu,Lü,Pope,Vazquez-Poritz '07]
[OD, Mitra, Santos '22]





→ Truncation of **$U(1)^3$ gauged supergravity** with two equal charges

$$A^1 = A^2 \equiv A, A^3 \equiv 0 \text{ and } \Phi_1 = \Phi_2 \equiv \Phi, \Phi_3 \equiv 0$$

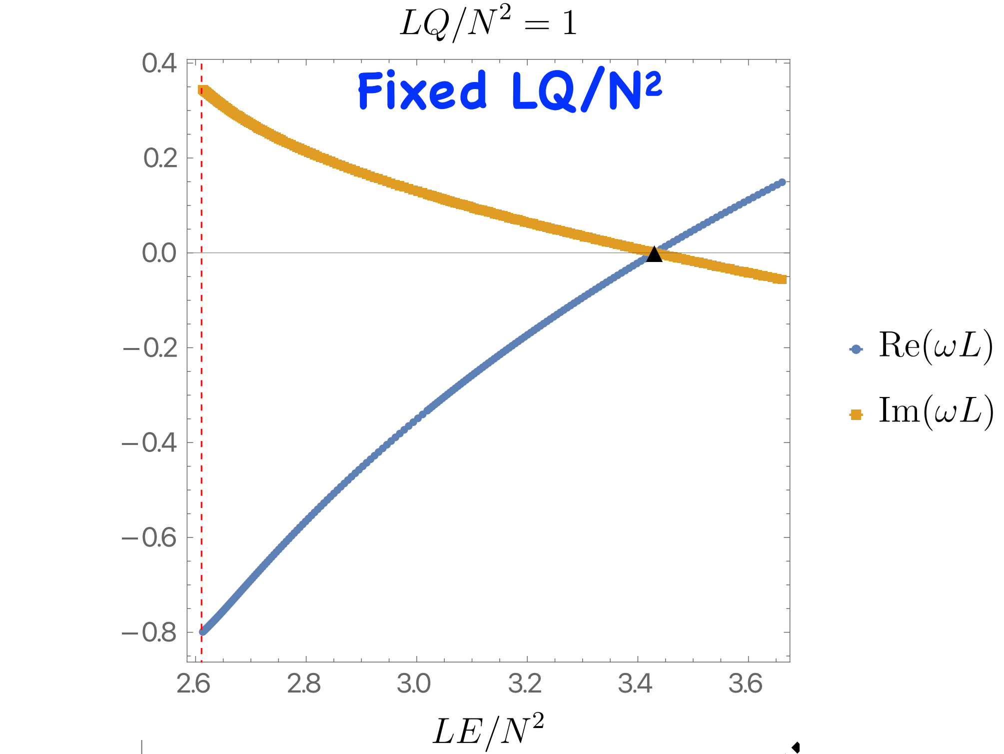
$$\varphi_1 \equiv \varphi, \; \varphi_2 = 0$$

Physics of equal three charge truncation is similar
(General arbitrary charge case should be similar)

➞ Truncation of $U(1)^3$ gauged supergravity with two equal charges

- **Scalar condensation instability of 'Bald' CLP BHs ($\Phi = 0$):**
  Perturb CLP BH with charged scalar field $\Phi_{1,2} = \Phi$: $\Phi(t,r) = \widehat{\Phi}_\omega(r)\, e^{-i\omega t}$
  & solve the associated "Klein-Gordon" eqn in the CLP BH with appropriate BCs.

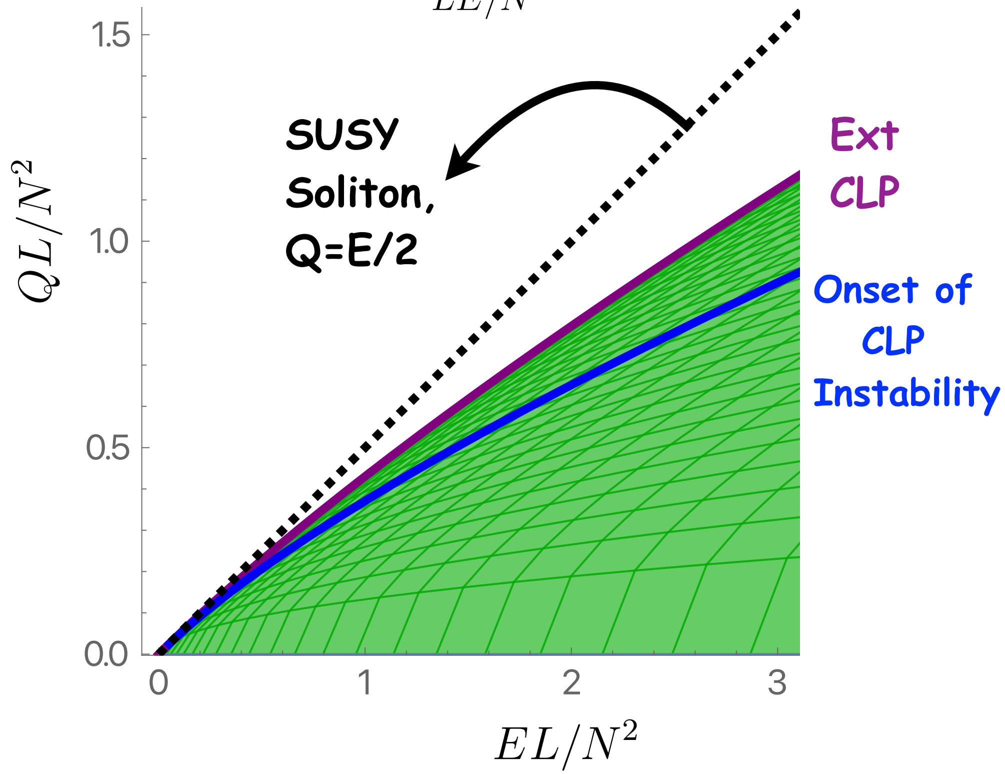
  => Unstable when Im (ωL) >0



- We can also solve directly for the **onset of instability**, $Q_o(E)$, where $\omega = 0$.
  CLP unstable above onset.

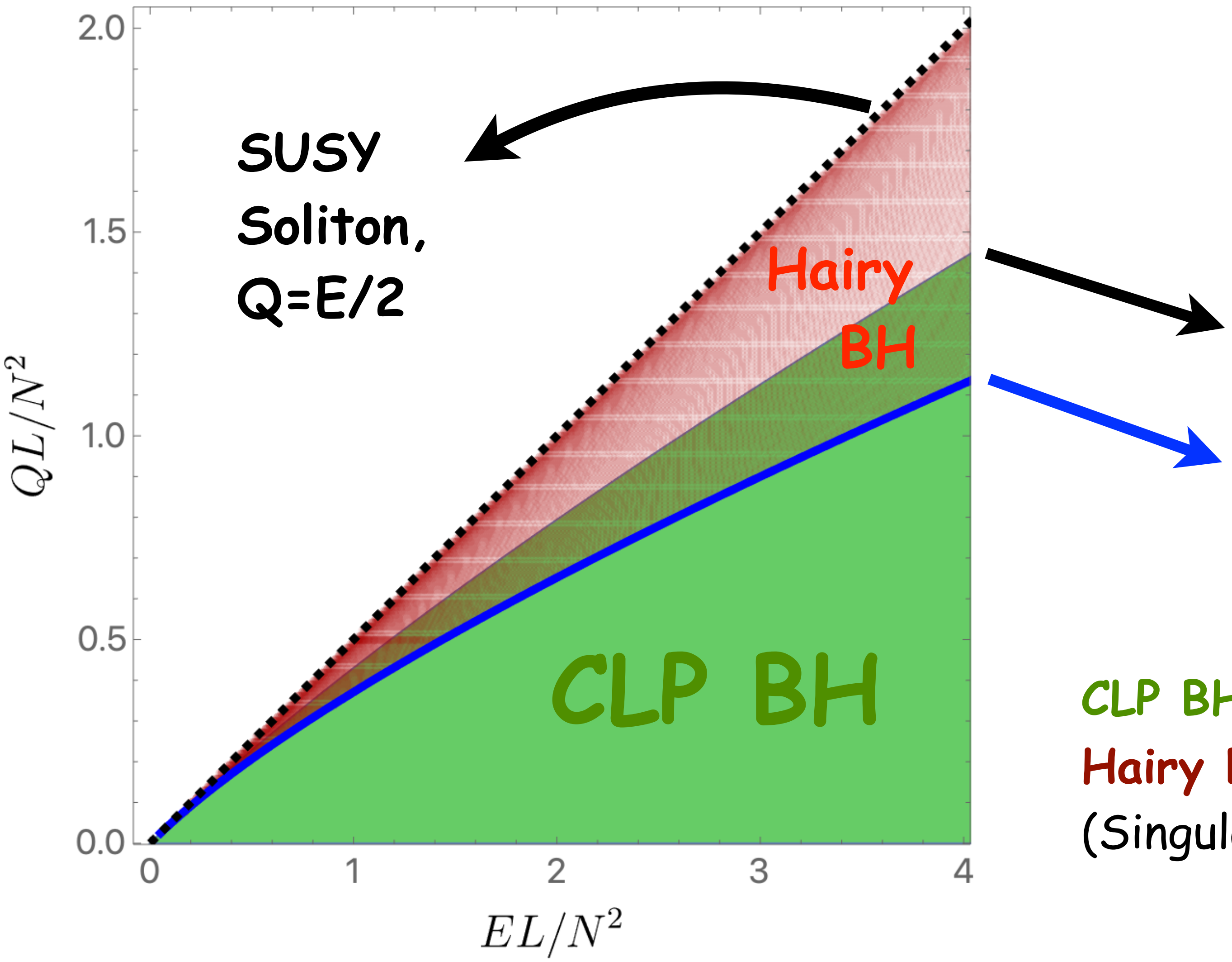
- There is also a **supersymmetric Soliton** with **Q=E/2** & $\langle\Phi\rangle \neq 0$ (numerical soln)

  [Liu, Lü, Pope, Vazquez-Poritz '07]
  [OD, Mitra, Santos '22]



→ Microcononical phase diagram (of truncation with two equal charges)

Fix energy E and charge Q: dominant phase is the one with highest entropy S



Physics of equal three charge truncation is similar

Bhattacharyya, Minwalla, Papadodimas (2011)
Markeviciute Santos (2016, 2018)

CLP BHs exist for $0<Q<Q_{ext}$

Hairy BHs exist for Onset $<Q<$ E/2
(Singular — S=0 — in BPS lim)

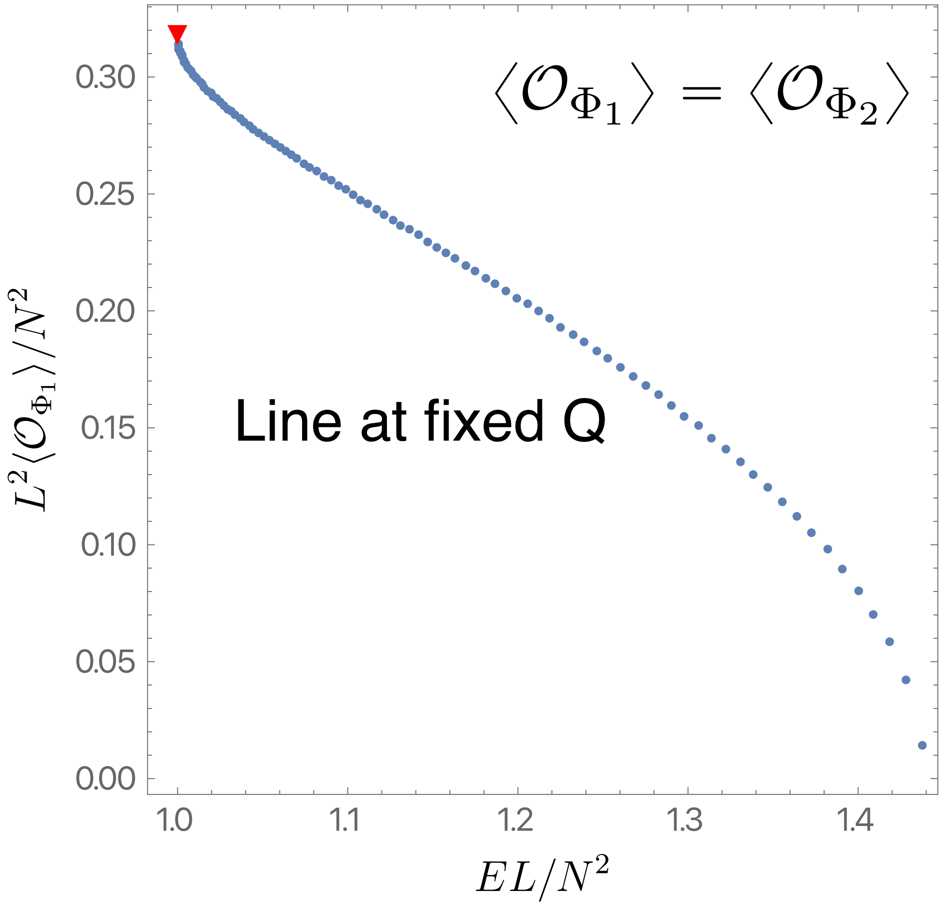
- **Hairy BHs always have higher entropy S than CLP BH with same E,Q**
- Hairy BH **temperature approaches T L=1/(2π) & μ=1** in the **singular** (S=0) **BPS lim !!**
  This is also min T that CLP can reach in the singular extremal limit.

➛ Microcononical phase diagram (of truncation with <u>two equal</u> charges)

Limit E—>2Q of Hairy BHs ? Singular limit (S=0) coincides with soliton at E=2Q that we constructed numerically.

Charged scalar **VEVs** for the line of Hairy BH at fixed Q L/N^2=0.5 (blue disks).

Red triangle is the SUSY soliton result at E=2Q=1.



- **Soliton** ∃ for <u>**any**</u> E = 2Q ≥ 0!

Unlike when **Q_1=Q_2=Q_3**,
where **soliton** ∃ for **E < Emax**

Liu, Lü, Pope, Vazquez-Poritz '07
Bhattacharyya, Minwalla, Papadodimas (2011)
Markeviciute Santos (2016, 2018)

- The **Q_1=Q_2=Q_3** behaviour is **believed** to be **generic**.

[Liu, Lü, Pope, Vazquez-Poritz '07]
[OD, Mitra, Santos '22]

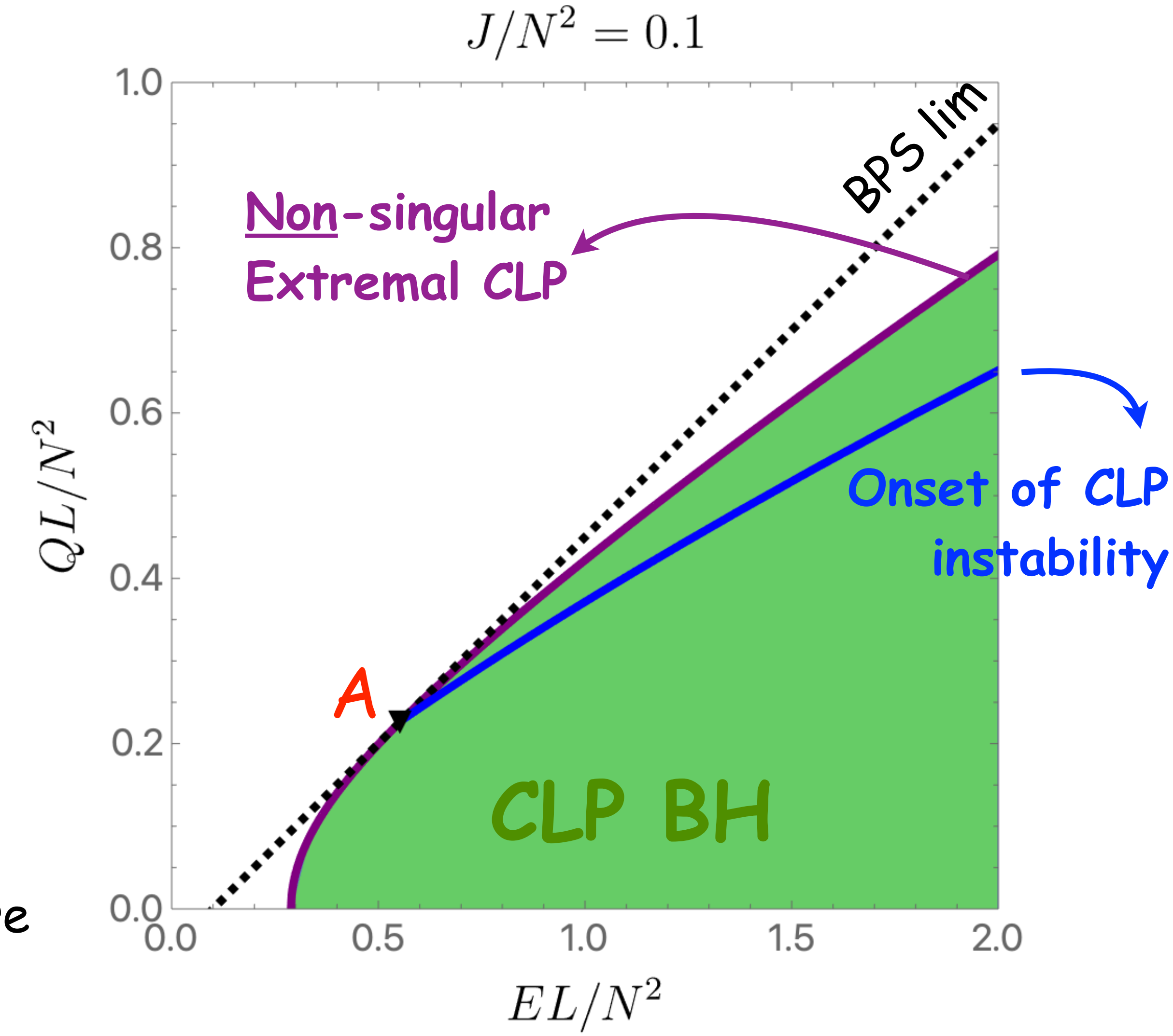
➜ What happens when we add rotation?

Example: **$U(1)^3$ gauged supergravity** with two equal charges

➔ Microcononical phase diagram (of truncation with two equal Q's): adding rotation

- When $J_1=J_2 = J$, we can keep co-homogeneity one (10 coupled nonlinear ODEs)
- **BPS relation** is now $E = 2Q + 2J$.
- Work at **finite temperature** and approach $T \to 0$: **want to find novel SUSY BHs!**

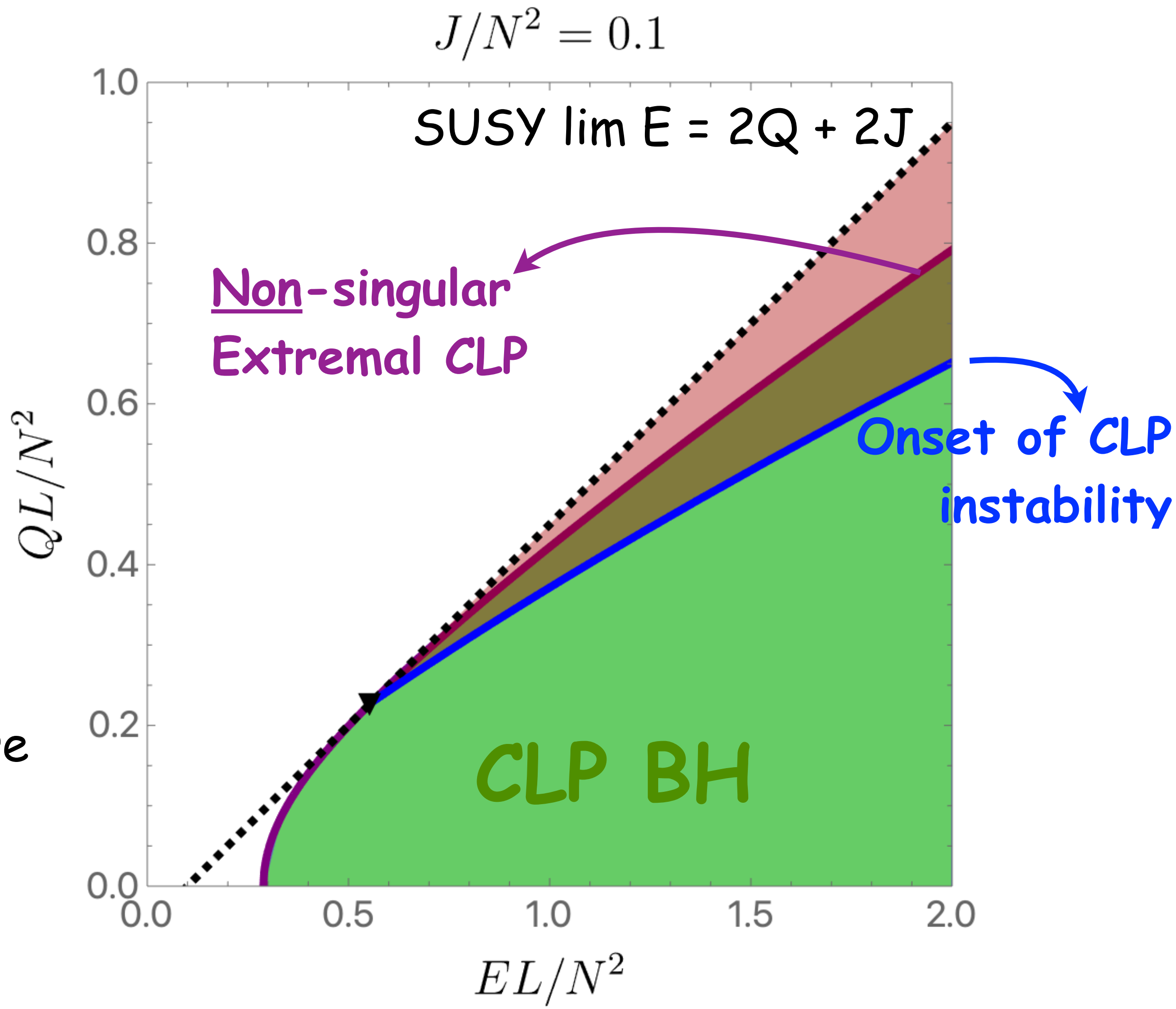
- **∃ CLP also with $J_1=J_2 = J$**
- **Smooth CLP**: $E \geq Q_{\rm ext}(E,J)$.
- CLP are **non**-singular and extremal at $E = Q_{\rm ext}(E,J)$.
- At fixed J, ∃ a single point A where **extremal CLP** is SUSY: the (1-parameter) Gutowski, Reall '04 BH.



(Kunduri-Lucietti-Reall BH: arbitrary $Q_{1,2,3}$,$J_{1,2}$)

➜ Microcononical phase diagram (of truncation with two equal Q's): **adding rotation**

- Work at **finite temperature** and approach **T** → **0**: want to find novel SUSY BHs!



- At fixed J, ∃ a single point where **extremal CLP** is SUSY: the Gutowski, Reall '04 BH.
- **Hairy BHs condense with Φ_1=Φ_2** (as in the **static** case) & **have higher entropy S than CLP for given {E,Q,J} !**
- **Hairy BHs** have a **non**-singular lim (where **T → 0, μ →1**) in the BPS lim: **novel SUSY BHs (this time with hair)! => can be missing grav parameter!**

→ Conclusions & Future work

→ **Our hairy BHS** with **$O(N^2)$ entropy** are **dominant** in the **microcanonical ensemble**
=> should be important for the **microstate counting** of the **entropy** of **SUSY BHs:**

[See Benini's review talk at Strings 2022]

- **Static** hairy BHs do not have a (smooth) BPS limit with $O(N^2)$ entropy.
- However, **rotating** hairy BHs **have** smooth BPS limit with $O(N^2)$ entropy.
- Microstate counting should compute the entropy of bald and hairy BHs!

→ **Future work:**

- **Complete Rotation study.**
- In the absence of charged scalar hair ($\Phi_K=0$), all SUSY BHs are **1/16-BPS**. However, recent computations of a twisted SYM index in the so-called Macdonald limit ($Q_3+J_2=0$) suggest an $O(N^2)$ entropy for 1/8-BPS states

  [Choi, Kim, Kim, Nahmgoong, 1810.12067]

  It is unclear what solutions this index corresponds to in gauged supergravity.
  Can we have 1/8-BPS hairy BHs ?
- **Classification scheme for SUSY BHs ?**
- **Generalizes to AdS_4xS^7 ?**