Gravity as a double copy of gauge theory

Ricardo Monteiro

Queen Mary University of London

NBI mini workshop: What is new in gravity?

Marienlyst, 11 August 2022

Outline

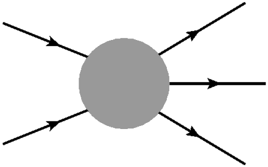
Introduction to the double copy

From scattering amplitudes to classical solutions

Celestial chiral algebras

Superstring amplitudes

Introduction to the double copy



Perturbative gravity is hard!

Feynman rules: expand Einstein-Hilbert Lagrangian $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ [DeWitt '66]

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\
&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],
\end{aligned}$$

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\
&+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})\\
&+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})\\
&-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})\\
&-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})\\
&+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})\\
&\qquad\qquad-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\Big].
\end{aligned}$$

$+$ infinite number of higher-point vertices…

Gravity $\sim$ (Yang-Mills)^2 in Scattering Amplitudes

Asymptotic states

- Yang-Mills theory: gluon $A_\mu = e^{ik\cdot x}\,\epsilon_\mu\,T^a$
  colour index a, polarisation ϵ_μ has $D-2$ dof.

Gravity $\sim$ (Yang-Mills) ^{2} in Scattering Amplitudes

Asymptotic states

- Yang-Mills theory: gluon $A_\mu = e^{ik\cdot x}\,\epsilon_\mu\,T^a$
  colour index a, polarisation ϵ_μ has $D-2$ dof.
- 'Product gravity': state $e^{ik\cdot x}\,\varepsilon_{\mu\nu}$, with $\boxed{\varepsilon_{\mu\nu} = \epsilon_\mu\,\tilde{\epsilon}_\nu}$ or linear comb.
  Contains graviton $h_{\mu\nu}$ + dilaton Φ + B-field $B_{\mu\nu}$, $(D-2)^2$ dof.

Gravity $\sim$ (Yang-Mills)^2 in Scattering Amplitudes

Asymptotic states

- Yang-Mills theory: gluon $A_\mu = e^{ik\cdot x}\,\epsilon_\mu\,T^a$
  colour index a, polarisation ϵ_μ has $D-2$ dof.
- 'Product gravity': state $e^{ik\cdot x}\,\varepsilon_{\mu\nu}$, with $\boxed{\varepsilon_{\mu\nu} = \epsilon_\mu\,\tilde{\epsilon}_\nu}$ or linear comb.
  Contains graviton $h_{\mu\nu}$ + dilaton Φ + B-field $B_{\mu\nu}$, $(D-2)^2$ dof.

Scattering amplitudes

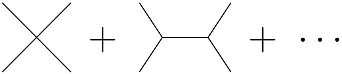
- "Factorisation" of ϵ^μ, $\tilde{\epsilon}^\nu$ preserved by interactions!
- **Double copy** $\boxed{\mathcal{A}_{\text{grav}}(\varepsilon_i^{\mu\nu}) \sim (\text{prop})^{-1}\,\mathcal{A}_{\text{YM}}(\epsilon_i^\mu) \times \mathcal{A}_{\text{YM}}(\tilde{\epsilon}_i^\mu)\,\Big|_{\text{colour stripped}}}$
- First application: supergravity UV behaviour. [Bern, Carrasco, Johansson, Roiban, . . .]

String theory origin

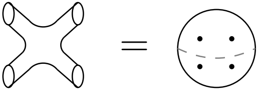
QFT from string theory as $\alpha' = \ell_s^2 \to 0$: alternative to Feynman expansion.

String theory origin

QFT from string theory as $\alpha' = \ell_s^2 \to 0$: alternative to Feynman expansion.



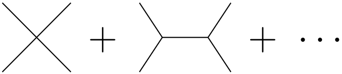
particle scattering
(many Feynman diagrams)



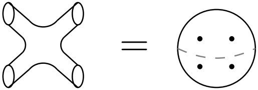
string scattering
(one world-sheet, 2D CFT)

String theory origin

QFT from string theory as $\alpha' = \ell_s^2 \to 0$: alternative to Feynman expansion.



particle scattering
(many Feynman diagrams)



string scattering
(one world-sheet, 2D CFT)

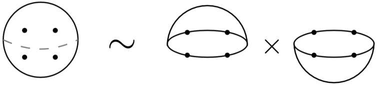
Gravity (closed strings) vs. gauge theory (open strings):

Asymptotic states (vertex operators): $V_{\text{closed}}(\varepsilon^{\mu\nu} = \epsilon^{\mu}\tilde{\epsilon}^{\nu}) \sim V_{\text{open}}(\epsilon^{\mu})\,\bar{V}_{\text{open}}(\tilde{\epsilon}^{\nu})$

Scattering amplitudes:

KLT relations

[Kawai, Lewellen, Tye 86]

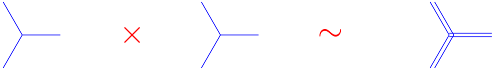


Field theory limit: **Gravity $\sim$ (Yang-Mills)^2** (KLT, BCJ, CHY, . . .)

[Bern, Carrasco, Johansson 08] [Cachazo, He, Yuan 13] [. . .]

Why simpler?

Basic example: 3-pt interactions.



Gauge theory field A^a_μ

3-pt vertex: $\boxed{f^{abc}\, V^{\mu\nu\lambda}\, A^a_\mu(p_1)\, A^b_\nu(p_2)\, A^c_\lambda(p_3)}$

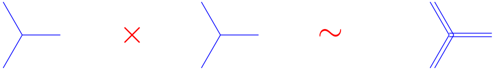
$$V^{\mu\nu\lambda} = (p_1 - p_2)^\lambda \eta^{\mu\nu} + (p_2 - p_3)^\mu \eta^{\nu\lambda} + (p_3 - p_1)^\nu \eta^{\lambda\mu}$$

Gravity field $H_{\mu\mu'} \sim$ graviton + dilaton + B-field ‘fat graviton’

3-pt vertex: $\boxed{V^{\mu\nu\lambda}\, V^{\mu'\nu'\lambda'}\, H_{\mu\mu'}(p_1)\, H_{\nu\nu'}(p_2)\, H_{\lambda\lambda'}(p_3)}$

Why simpler?

Basic example: 3-pt interactions.



Gauge theory field A^a_μ

3-pt vertex: $f^{abc}\, V^{\mu\nu\lambda}\, A^a_\mu(p_1)\, A^b_\nu(p_2)\, A^c_\lambda(p_3)$

$$V^{\mu\nu\lambda} = (p_1 - p_2)^\lambda \eta^{\mu\nu} + (p_2 - p_3)^\mu \eta^{\nu\lambda} + (p_3 - p_1)^\nu \eta^{\lambda\mu}$$

Gravity field $H_{\mu\mu'} \sim$ graviton + dilaton + B-field 'fat graviton'

3-pt vertex: $V^{\mu\nu\lambda}\, V^{\mu'\nu'\lambda'}\, H_{\mu\mu'}(p_1)\, H_{\nu\nu'}(p_2)\, H_{\lambda\lambda'}(p_3)$

Simplification: **left-right factorisation**, c.f. $\sim$100 terms in GR 3-pt vertex!

Powerful implementation: colour-kinematics duality. [Bern, Carrasco, Johansson '08] [...]

New directions in (classical) perturbative gravity

Generically, double copy applies in perturbation theory.

- Double-copy-like field theory for gravity.

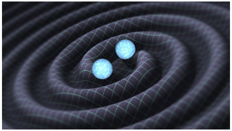
[Bern et al] [Goldberger et al] [Luna et al] [Cheung et al] [Plefka et al] [Borsten et al] [...]

- Gauge-invariant approach: classical physics from scattering amplitudes.

[Neill et al] [Bjerrum-Bohr et al] [Kosower et al] [Di Vecchia et al] [Guevara et al] [Huang et al] [Arkani-Hamed et al] [...]

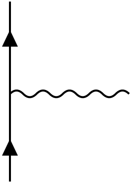
- Beyond Minkowski: "amplitudes" on plane waves / (A)dS.

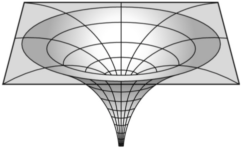
[Farrow et al] [Adamo et al] [Armstrong et al] [Alday et al] [Gomez et al] [...]



- Highlight: new G^3, G^4 (3PM, 4PM) corrections to 2-body potential. [Bern et al]

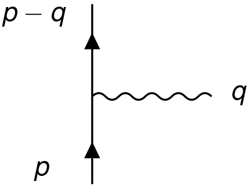
From scattering amplitudes to classical solutions





3-point scattering amplitudes

On-shell 3-pt interaction: massive particle emits gauge boson.



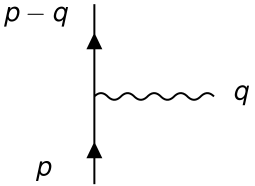
$$p^2 = (p-q)^2 = m^2 \qquad q^2 = 0$$

3-pt amplitudes are the building blocks of modern on-shell methods.

E.g. BCFW recursion.

3-point scattering amplitudes

On-shell 3-pt interaction: massive particle emits gauge boson.



$$p^2 = (p-q)^2 = m^2 \qquad q^2 = 0$$

3-pt amplitudes are the building blocks of modern on-shell methods.

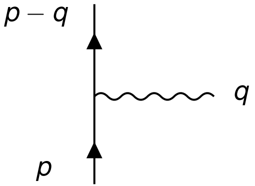
E.g. BCFW recursion.

Lorentzian signature: 3-pt amplitudes supported on complex kinematics.

Split signature (t^1, t^2, x^1, x^2): 3-pt amplitudes supported on real kinematics,
eg, $p_\mu = m\,(0,1,0,0)\ ,\ q_\mu = \hbar\,\omega(1,0,0,1)\,.$

3-point scattering amplitudes

On-shell 3-pt interaction: massive particle emits gauge boson.



$$p^2 = (p-q)^2 = m^2 \qquad q^2 = 0$$

3-pt amplitudes are the building blocks of modern on-shell methods.

E.g. BCFW recursion.

Lorentzian signature: 3-pt amplitudes supported on complex kinematics.

Split signature (t^1, t^2, x^1, x^2): 3-pt amplitudes supported on real kinematics,
eg, $p_\mu = m\,(0,1,0,0)\,,\; q_\mu = \hbar\,\omega(1,0,0,1)\,.$

Classical limit: $q_\mu = \hbar\, k_\mu,\;\; \hbar \to 0.$ KMOC formalism [Kosower, Maybe, O'Connell 18]

Classical fields from 3-pt amplitudes [RM, O'Connell, Peinador, Sergola 20]

What classical objects do 3-pt amplitudes compute?

$\Rightarrow$ Linearised curvature (gravity) and field strength (EM) in split signature.

Classical fields from 3-pt amplitudes [RM, O'Connell, Peinador, Sergola 20]

What classical objects do 3-pt amplitudes compute?

$\Rightarrow$ Linearised curvature (gravity) and field strength (EM) in split signature.

KMOC formalism: $\langle \mathcal{O} \rangle \equiv {}_{\text{in}}\langle S^\dagger \mathcal{O} S \rangle_{\text{in}}$ $\quad S = 1 + i\,T \quad$ [e.g. $\mathcal{O} = F_{\mu\nu}(x)$]

$$\xrightarrow{\text{calculation}} \quad \langle\, \mathcal{O}(x) \,\rangle_{\text{classical}} = \text{Re} \int d^4k \; e^{-ik\cdot x} \, \delta(k^2)\, \delta(p\cdot k) \; \underbrace{\tilde{\mathcal{O}}(k)}_{\propto \text{ 3-pt amp}}$$

Linearised field is on-shell Fourier transform of amplitude.

Classical fields from 3-pt amplitudes [RM, O'Connell, Peinador, Sergola 20]

What classical objects do 3-pt amplitudes compute?

$\Rightarrow$ Linearised curvature (gravity) and field strength (EM) in split signature.

KMOC formalism: $\langle \mathcal{O} \rangle \equiv {}_{\text{in}}\langle S^\dagger \mathcal{O} S \rangle_{\text{in}}$ $\quad S = 1 + i\,T \quad$ [e.g. $\mathcal{O} = F_{\mu\nu}(x)$]

$$\xrightarrow{\text{calculation}} \quad \langle\, \mathcal{O}(x)\, \rangle_{\text{classical}} = \text{Re} \int d^4k\; e^{-ik\cdot x}\, \delta(k^2)\, \delta(p\cdot k)\, \underbrace{\tilde{\mathcal{O}}(k)}_{\propto \text{ 3-pt amp}}$$

Linearised field is on-shell Fourier transform of amplitude.

EM example: for static particle coupled via $\mathcal{A}^{\pm}_{3,EM}(k) = Q\, p\cdot\varepsilon^{\pm}(k)$, we get

$$\langle\, F_{\mu\nu}(x)\, \rangle_{\text{classical}} = \text{Re} \int d^4k\; e^{-ik\cdot x}\, \delta(k^2)\, \delta(p\cdot k) \sum_{\eta=\pm} k_{[\mu}\epsilon^{\eta}_{\nu]}\, \mathcal{A}^{\eta}_{3,EM}$$

$\rightarrow$ Coulomb solution! (analytically continued to split signature)

Spinorial curvatures from amplitudes

EM: $F_{\mu\nu} \mapsto F_{A\dot{A}B\dot{B}} = f_{AB}\,\varepsilon_{\dot{A}\dot{B}} + \bar{f}_{\dot{A}\dot{B}}\,\varepsilon_{CD}$

(1,3) signature: f_{AB} and $\bar{f}_{\dot{A}\dot{B}}$ are complex conjugates.

(2,2) signature: f_{AB} and $\bar{f}_{\dot{A}\dot{B}}$ are real and independent.

Spinorial curvatures from amplitudes

EM: $F_{\mu\nu} \mapsto F_{A\dot{A}B\dot{B}} = f_{AB}\,\varepsilon_{\dot{A}\dot{B}} + \bar{f}_{\dot{A}\dot{B}}\,\varepsilon_{CD}$

(1,3) signature: f_{AB} and $\bar{f}_{\dot{A}\dot{B}}$ are complex conjugates.

(2,2) signature: f_{AB} and $\bar{f}_{\dot{A}\dot{B}}$ are real and independent.

Vacuum gravity:

$$R_{\mu\nu\rho\lambda} \mapsto R_{A\dot{A}B\dot{B}C\dot{C}D\dot{D}} = C_{ABCD}\,\varepsilon_{\dot{A}\dot{B}}\,\varepsilon_{\dot{C}\dot{D}} + \bar{C}_{\dot{A}\dot{B}\dot{C}\dot{D}}\,\varepsilon_{AB}\,\varepsilon_{CD}$$

Spinorial curvatures from amplitudes

EM: $F_{\mu\nu} \mapsto F_{A\dot{A}B\dot{B}} = f_{AB}\,\varepsilon_{\dot{A}\dot{B}} + \bar{f}_{\dot{A}\dot{B}}\,\varepsilon_{CD}$

(1,3) signature: f_{AB} and $\bar{f}_{\dot{A}\dot{B}}$ are complex conjugates.

(2,2) signature: f_{AB} and $\bar{f}_{\dot{A}\dot{B}}$ are real and independent.

Vacuum gravity:

$$R_{\mu\nu\rho\lambda} \mapsto R_{A\dot{A}B\dot{B}C\dot{C}D\dot{D}} = C_{ABCD}\,\varepsilon_{\dot{A}\dot{B}}\,\varepsilon_{\dot{C}\dot{D}} + \bar{C}_{\dot{A}\dot{B}\dot{C}\dot{D}}\,\varepsilon_{AB}\,\varepsilon_{CD}$$

From 3-pt amplitudes, using $k^2 = 0 :\ k^\mu \mapsto |k\rangle_A\,[k|_{\dot{A}}$, static particle gives

C_{ABCD}	$\lvert k\rangle_A \lvert k\rangle_B \lvert k\rangle_C \lvert k\rangle_D\ \mathcal{A}^{+}_{3,grav}(k)$	Schwarzschild
f_{AB}	$\lvert k\rangle_A \lvert k\rangle_B\ \mathcal{A}^{+}_{3,EM}(k)$	Coulomb
S	$\mathcal{A}^{+}_{3,scalar}(k)$	$1/r$

$$\mathcal{A}^{\pm}_{3,scalar} = (\mathcal{A}^{\pm}_{3,EM})^0 \qquad \mathcal{A}^{\pm}_{3,EM} = (\mathcal{A}^{\pm}_{3,EM})^1 \qquad \mathcal{A}^{\pm}_{3,grav} = (\mathcal{A}^{\pm}_{3,EM})^2$$

Double copy: from amplitudes to classical solutions

Amplitudes

double copy

$$\mathcal{A}^{\pm}_{3,grav} = \frac{1}{\mathcal{A}^{\pm}_{3,scalar}} \left(\mathcal{A}^{\pm}_{3,EM}\right)^2$$

Double copy: from amplitudes to classical solutions

Amplitudes

double copy

$$\mathcal{A}^{\pm}_{3,grav} = \frac{1}{\mathcal{A}^{\pm}_{3,scalar}} \left(\mathcal{A}^{\pm}_{3,EM} \right)^2$$

Classical solutions

double copy

$$\text{Schwarzschild} \sim (\text{Coulomb})^2$$

more precisely

$$\text{Schwarzschild} = \text{Minkowski} + (\text{Coulomb})^2$$

Double copy: from amplitudes to classical solutions

Amplitudes

double copy

$$\mathcal{A}^{\pm}_{3,grav} = \frac{1}{\mathcal{A}^{\pm}_{3,scalar}} \left(\mathcal{A}^{\pm}_{3,EM}\right)^2$$

Classical solutions

double copy

Schwarzschild $\sim$ (Coulomb)^2

more precisely

Schwarzschild $=$ Minkowski $+$ (Coulomb)^2

More general 3-pt amplitudes (rotation a, duality θ):

[Arkani-Hamed, O'Connell, Huang,]
[Kol, Emond, Moynihan 19-21]

EM (YM): $\mathcal{A}^{\pm}_{3,EM}(k)\, e^{\mp k\cdot a \pm i\theta} \quad \to \quad f_{AB}(x) = e^{i\theta}\, f^{\text{Coulomb}}_{AB}(x - ia)$

Gravity: $\mathcal{A}^{\pm}_{3,grav}(k)\, e^{\mp k\cdot a \pm i\theta} \quad \to \quad C_{ABCD}(x) = e^{i\theta}\, C^{\text{Schwar.}}_{ABCD}(x - ia)$

[RM, Nagy, O'Connell, Peinador, Sergola 21]
[see also Crawley, Guevara, Miller, Strominger 21]

Double copy: from amplitudes to classical solutions

Amplitudes

double copy

$$\mathcal{A}^{\pm}_{3,grav} = \frac{1}{\mathcal{A}^{\pm}_{3,scalar}} \left(\mathcal{A}^{\pm}_{3,EM}\right)^2$$

Classical solutions

double copy

$$\text{Schwarzschild} \sim (\text{Coulomb})^2$$

more precisely

$$\text{Schwarzschild} = \text{Minkowski} + (\text{Coulomb})^2$$

Double copy: from amplitudes to classical solutions

Amplitudes

double copy

$$\mathcal{A}^{\pm}_{3,grav} = \frac{1}{\mathcal{A}^{\pm}_{3,scalar}} \left(\mathcal{A}^{\pm}_{3,EM}\right)^2$$

Classical solutions

double copy

$$\text{Schwarzschild} \sim (\text{Coulomb})^2$$

more precisely

$$\text{Schwarzschild} = \text{Minkowski} + (\text{Coulomb})^2$$

Direct double copy for classical solutions in position space?

$$C_{ABCD} = f_{(AB} * S^{-1} * f_{CD)}$$

cf. convolutional double copy. [Anastasiou, Borsten, Duff, Hughes, Nagy 14]

Double copy: from amplitudes to classical solutions

Amplitudes

double copy

$$\mathcal{A}^{\pm}_{3,grav} = \frac{1}{\mathcal{A}^{\pm}_{3,scalar}} \left(\mathcal{A}^{\pm}_{3,EM}\right)^2$$

Classical solutions

double copy

$$\text{Schwarzschild} \sim (\text{Coulomb})^2$$

more precisely

$$\text{Schwarzschild} = \text{Minkowski} + (\text{Coulomb})^2$$

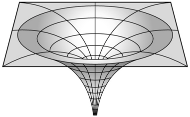
Direct double copy for classical solutions in position space?

$$C_{ABCD} = f_{(AB} * S^{-1} * f_{CD)}$$

cf. convolutional double copy. [Anastasiou, Borsten, Duff, Hughes, Nagy 14]

Simpler map in position space? Beyond linearised?

Yes! (certain algebraically special spacetimes)

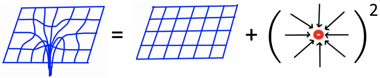


Exact classical double copy

Kerr-Schild double copy [RM, O'Connell, White 14 + Luna 15; . . .]

for stationary multi-Kerr-Schild vacuum spacetimes (Kerr-Taub-NUT):

gravity: $g_{\mu\nu} = \eta_{\mu\nu} + \phi\, k_\mu k_\nu$ gauge theory: $A_\mu = \phi\, k_\mu$ scalar: ϕ

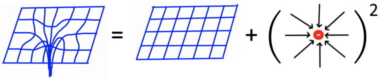


Exact classical double copy

Kerr-Schild double copy [RM, O'Connell, White 14 + Luna 15; ...]

for stationary multi-Kerr-Schild vacuum spacetimes (Kerr-Taub-NUT):

gravity: $g_{\mu\nu} = \eta_{\mu\nu} + \phi\, k_\mu k_\nu$ gauge theory: $A_\mu = \phi\, k_\mu$ scalar: ϕ



E.g. Schwarzschild: $\phi \propto \frac{1}{r}$, $k = dt' + dr$.

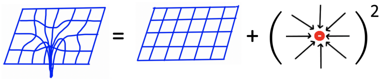
Kerr-Schild linearisation $\leftrightarrow$ Abelianisation.

Exact classical double copy

Kerr-Schild double copy [RM, O'Connell, White 14 + Luna 15; ...]

for stationary multi-Kerr-Schild vacuum spacetimes (Kerr-Taub-NUT):

gravity: $\boxed{g_{\mu\nu} = \eta_{\mu\nu} + \phi\, k_\mu k_\nu}$ gauge theory: $\boxed{A_\mu = \phi\, k_\mu}$ scalar: $\boxed{\phi}$



E.g. Schwarzschild: $\phi \propto \frac{1}{r}$, $k = dt' + dr$.

Kerr-Schild linearisation $\leftrightarrow$ Abelianisation.

Weyl double copy [Luna, RM, Nicholson, O'Connell 18; Godazgar, Godazgar, RM, Peinador, Pope 20; ...]

for 4D spacetimes of algebraic type D and (non-twisting) type N:

$$\boxed{C_{ABCD} = \frac{1}{S}\, f_{(AB}\, f_{CD)}}$$

Unconvoluted! [RM, Nagy, O'Connell, Peinador, Sergola 20]
Twistorial interpretation. [White 20 + Chacon, Nagy 21; Guevara 21]

Beyond vacuum solutions

$$\boxed{(\text{YM})^2 \ \sim\ \text{Einstein } g_{\mu\nu} + \text{dilaton } \Phi + \text{B-field } B_{\mu\nu}}$$

e.g. in 4D, $\varepsilon_\mu^{\eta_L}\varepsilon_\nu^{\eta_R} \mapsto \{\, \varepsilon_\mu^+\varepsilon_\nu^+ \,,\ \varepsilon_\mu^-\varepsilon_\nu^- \,,\ \varepsilon_{(\mu}^+\varepsilon_{\nu)}^- \,,\ \varepsilon_{[\mu}^+\varepsilon_{\nu]}^- \,\}$.

Beyond vacuum solutions

$$\boxed{(\text{YM})^2 \;\sim\; \text{Einstein } g_{\mu\nu} + \text{dilaton } \Phi + \text{B-field } B_{\mu\nu}}$$

e.g. in 4D, $\varepsilon_\mu^{\eta_L}\varepsilon_\nu^{\eta_R} \mapsto \{\, \varepsilon_\mu^+\varepsilon_\nu^+ \,,\; \varepsilon_\mu^-\varepsilon_\nu^- \,,\; \varepsilon_{(\mu}^+\varepsilon_{\nu)}^- \,,\; \varepsilon_{[\mu}^+\varepsilon_{\nu]}^- \,\}$.

Exact map

Double field theory Kerr-Schild-type ansatz. [Lee 18 +Cho 19; Kim,Lee,RM,Nicholson,Peinador 19]

$\Rightarrow$ Exact classical double copy with dilaton and B-field,

e.g. $(\text{Coulomb})^2 \sim$ JNW (graviton + dilaton) [Janis, Newman, Winicour '68] .

Beyond vacuum solutions

$$\boxed{\text{(YM)}^2 \;\sim\; \text{Einstein } g_{\mu\nu} + \text{dilaton } \Phi + \text{B-field } B_{\mu\nu}}$$

e.g. in 4D, $\varepsilon_\mu^{\eta_L}\varepsilon_\nu^{\eta_R} \mapsto \{\, \varepsilon_\mu^+\varepsilon_\nu^+ \,,\; \varepsilon_\mu^-\varepsilon_\nu^- \,,\; \varepsilon_{(\mu}^+\varepsilon_{\nu)}^- \,,\; \varepsilon_{[\mu}^+\varepsilon_{\nu]}^- \,\}$.

Exact map

Double field theory Kerr-Schild-type ansatz. [Lee 18 +Cho 19; Kim,Lee,RM,Nicholson,Peinador 19]

$\Rightarrow$ Exact classical double copy with dilaton and B-field,

e.g. $(\text{Coulomb})^2 \sim$ JNW (graviton + dilaton) [Janis, Newman, Winicour '68] .

Linearised map

Amplitudes: $e^{\eta_L(-k\cdot a_L + i\theta_L)}\mathcal{A}^{\eta_L}_{\text{Coulomb}} \times e^{\eta_R(-k\cdot a_R + i\theta_R)}\mathcal{A}^{\eta_R}_{\text{Coulomb}}$

Beyond vacuum solutions

$$\boxed{(\text{YM})^2 \ \sim \ \text{Einstein } g_{\mu\nu} + \text{dilaton } \Phi + \text{B-field } B_{\mu\nu}}$$

e.g. in 4D, $\varepsilon_\mu^{\eta_L}\varepsilon_\nu^{\eta_R} \mapsto \{ \varepsilon_\mu^+\varepsilon_\nu^+ \,,\ \varepsilon_\mu^-\varepsilon_\nu^- \,,\ \varepsilon_{(\mu}^+\varepsilon_{\nu)}^- \,,\ \varepsilon_{[\mu}^+\varepsilon_{\nu]}^- \}$.

Exact map

Double field theory Kerr-Schild-type ansatz. [Lee 18 +Cho 19; Kim,Lee,RM,Nicholson,Peinador 19]

$\Rightarrow$ Exact classical double copy with dilaton and B-field,
e.g. $(\text{Coulomb})^2 \sim$ JNW (graviton + dilaton) [Janis, Newman, Winicour '68] .

Linearised map

Amplitudes: $e^{\eta_L(-k\cdot a_L + i\theta_L)}\mathcal{A}^{\eta_L}_{\text{Coulomb}} \times e^{\eta_R(-k\cdot a_R + i\theta_R)}\mathcal{A}^{\eta_R}_{\text{Coulomb}}$

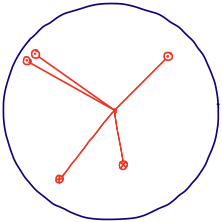
Riemann-Cartan curvature: [RM, Nagy, O'Connell, Peinador, Sergola 21]

$$\mathcal{R}_{A\dot{A}B\dot{B}C\dot{C}D\dot{D}} = C_{ABCD}\,\epsilon_{\dot{A}\dot{B}}\,\epsilon_{\dot{C}\dot{D}} + \bar{C}_{\dot{A}\dot{B}\dot{C}\dot{D}}\,\epsilon_{AB}\,\epsilon_{CD} + \Phi_{AB\dot{C}\dot{D}}\,\epsilon_{\dot{A}\dot{B}}\,\epsilon_{CD} + \bar{\Phi}_{\dot{A}\dot{B}CD}\,\epsilon_{AB}\,\epsilon_{\dot{C}\dot{D}}$$

$$(\eta_L,\eta_R) = (+,+) \qquad C_{ABCD}(x) = e^{i(\theta_L+\theta_R)}\,C^{\text{JNW}}_{ABCD}(x - i(a_L + a_R))$$

$$(\eta_L,\eta_R) = (+,-) \qquad \Phi_{AB\dot{C}\dot{D}}(x) = e^{i(\theta_L-\theta_R)}\,\Phi^{\text{JNW}}_{AB\dot{C}\dot{D}}(x - i(a_L - a_R))$$

Chiral algebras in celestial holography



Prologue: self-dual theories and double copy

Self-dual Yang-Mills ($*F = iF$) & gravity: simplest double copy setting.

Light-cone gauge: $ds^2 = -du dv + dw d\bar{w}$

$$S_{\text{SDYM}}(\Psi, \bar{\Psi}) = \int d^4x \ \text{tr}\Big(\bar{\Psi}\big(\Box\Psi + i[\partial_u\Psi, \partial_w\Psi]\big)\Big)$$

$$S_{\text{SDG}}(\phi, \bar{\phi}) = \int d^4x \ \bar{\phi}\big(\Box\phi + \{\partial_u\phi, \partial_w\phi\}\big) \qquad \{f, g\} = \partial_u f\, \partial_w g - \partial_w f\, \partial_u g$$

Prologue: self-dual theories and double copy

Self-dual Yang-Mills ($*F = iF$) & gravity: simplest double copy setting.

Light-cone gauge: $ds^2 = -du dv + dw d\bar{w}$

$$S_{\text{SDYM}}(\Psi, \bar{\Psi}) = \int d^4x \ \text{tr}\Big(\bar{\Psi}\big(\Box\Psi + i[\partial_u\Psi, \partial_w\Psi]\big)\Big)$$

$$S_{\text{SDG}}(\phi, \bar{\phi}) = \int d^4x \ \bar{\phi}\big(\Box\phi + \{\partial_u\phi, \partial_w\phi\}\big) \qquad \{f, g\} = \partial_u f\, \partial_w g - \partial_w f\, \partial_u g$$

BCJ colour-kinematics duality [Bern, Carrasco, Johansson 08] manifest! [Monteiro, O'Connell 11]

- propagator $(-+)$: $\frac{i}{k^2}\,\delta^{ab}$
- cubic vertex $(-++)$: $\boxed{V_{\text{SDYM}} = X(k_1, k_2)\, f^{a_1 a_2 a_3}\,, \quad V_{\text{SDG}} = X(k_1, k_2)^2}$

$$L_k = \{e^{i\,k\cdot x}, \cdot\} = ie^{i\,k\cdot x}(k_u\partial_w - k_w\partial_u) \quad \text{area-preserving diffeos}$$

$$[L_{k_1}, L_{k_2}] = X(k_1, k_2)\, L_{k_1+k_2}\,, \qquad X(k_1, k_2) = k_{1z}k_{2u} - k_{1u}k_{2z}$$

Prologue: self-dual theories and double copy

Self-dual Yang-Mills ($*F = iF$) & gravity: simplest double copy setting.

Light-cone gauge: $ds^2 = -du dv + dw d\bar{w}$

$$S_{\text{SDYM}}(\Psi, \bar{\Psi}) = \int d^4x \;\; \text{tr}\Big(\bar{\Psi}\big(\Box\Psi + i[\partial_u\Psi, \partial_w\Psi]\big)\Big)$$

$$S_{\text{SDG}}(\phi, \bar{\phi}) = \int d^4x \;\; \bar{\phi}\big(\Box\phi + \{\partial_u\phi, \partial_w\phi\}\big) \qquad \{f, g\} = \partial_u f\, \partial_w g - \partial_w f\, \partial_u g$$

BCJ colour-kinematics duality [Bern, Carrasco, Johansson 08] manifest! [Monteiro, O'Connell 11]

- propagator $(-+)$: $\frac{i}{k^2}\,\delta^{ab}$
- cubic vertex $(-++)$: $\boxed{V_{\text{SDYM}} = X(k_1, k_2)\, f^{a_1 a_2 a_3}\,, \quad V_{\text{SDG}} = X(k_1, k_2)^2}$

$$L_k = \{e^{i\,k\cdot x}, \cdot\} = ie^{i\,k\cdot x}(k_u\partial_w - k_w\partial_u) \quad \text{area-preserving diffeos}$$

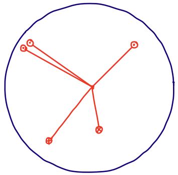
$$[L_{k_1}, L_{k_2}] = X(k_1, k_2)\, L_{k_1+k_2}\,, \qquad X(k_1, k_2) = k_{1z}k_{2u} - k_{1u}k_{2z}$$

Tree amp's $(-+\cdots+)$ vanish. One-loop amp's $(+\cdots+)$ finite.

Classically integrable. One-loop exact. Well-defined sectors of full theories.

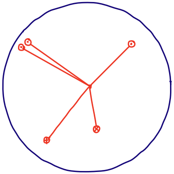
Celestial chiral OPEs

See amplitude as a 2D CFT correlator on celestial sphere. [Strominger et al]



Celestial chiral OPEs

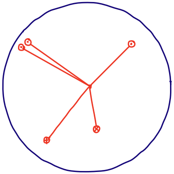
See amplitude as a 2D CFT correlator on celestial sphere. [Strominger et al]
(Double copy [Casali, Puhm, Pasterski, Nagy, Campiglia, Adamo, Kol, Godazgar^2, RM, Peinador, Pope, ...].)



Celestial chiral OPEs

See amplitude as a 2D CFT correlator on celestial sphere. [Strominger et al]
(Double copy [Casali, Puhm, Pasterski, Nagy, Campiglia, Adamo, Kol, Godazgar^2, RM, Peinador, Pope, ...].)

Massless kinematics:



$$k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}} , \quad \lambda = (1, z) , \quad \tilde{\lambda} = (k_u, k_w) , \quad z = \frac{k_{\bar{w}}}{k_u} = \frac{k_v}{k_w} ,$$

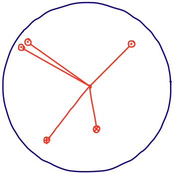
$$X(k_1, k_2) = k_{1z}k_{2u} - k_{1u}k_{2z} = \epsilon^{\dot{A}\dot{B}}\, \tilde{\lambda}_{1\,\dot{A}}\, \tilde{\lambda}_{2\,\dot{B}} = [12] ,$$

$$s_{12} = (k_1 + k_2)^2 = (z_1 - z_2)\, X(k_1, k_2) .$$

Celestial chiral OPEs

See amplitude as a 2D CFT correlator on celestial sphere. [Strominger et al]
(Double copy [Casali, Puhm, Pasterski, Nagy, Campiglia, Adamo, Kol, Godazgar², RM, Peinador, Pope, ...].)

Massless kinematics:



$$k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}\,, \quad \lambda = (1, z)\,, \quad \tilde{\lambda} = (k_u, k_w)\,, \quad z = \frac{k_{\bar{w}}}{k_u} = \frac{k_v}{k_w}\,,$$

$$X(k_1, k_2) = k_{1z} k_{2u} - k_{1u} k_{2z} = \epsilon^{\dot{A}\dot{B}}\, \tilde{\lambda}_{1\,\dot{A}}\, \tilde{\lambda}_{2\,\dot{B}} = [12]\,,$$

$$s_{12} = (k_1 + k_2)^2 = (z_1 - z_2)\, X(k_1, k_2)\,.$$

Collinear limit $z_1 \to z_2$ of amplitudes translates into

SDYM & YM:

$$\mathcal{O}_+^{a_1}(k_1)\, \mathcal{O}_+^{a_2}(k_2) \sim \frac{X(k_1, k_2)\, f^{a_1 a_2 a_3}}{s_{12}}\, \mathcal{O}_+^{a_3}(k_1 + k_2) = \frac{f^{a_1 a_2 a_3}}{z_1 - z_2}\, \mathcal{O}_+^{a_3}(k_1 + k_2)$$

SDG & GR:

$$\mathcal{O}_+(k_1)\, \mathcal{O}_+(k_2) \sim \frac{X(k_1, k_2)^2}{s_{12}}\, \mathcal{O}_+(k_1 + k_2) = \frac{X(k_1, k_2)}{z_1 - z_2}\, \mathcal{O}_+(k_1 + k_2)$$

$\omega_{1+\infty}$ from soft expansion in SDG

$$\mathcal{O}_+(k_1)\,\mathcal{O}_+(k_2) \sim \frac{X(k_1,k_2)}{z_1 - z_2}\,\mathcal{O}_+(k_1+k_2) \quad \longleftrightarrow \quad [L_{k_1}, L_{k_2}] = X(k_1,k_2)\,L_{k_1+k_2}$$

Recall kinematics: $\quad k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}\,, \quad \lambda = (1, z)\,, \quad \tilde{\lambda} = (k_u, k_w)\,.$

Soft expansion is $\tilde{\lambda} \to 0$. At fixed z,

$$L_k = \{e^{i\,k\cdot x}, \cdot\} = \sum_{a,b=0}^{\infty} \frac{(ik_u)^a}{a!}\,\frac{(ik_w)^b}{b!}\,\ell_{a,b} \qquad \ell_{a,b} = \{(u + z\bar{w})^a (w + zv)^b,\, \cdot\}$$

$\omega_{1+\infty}$ from soft expansion in SDG

$$\mathcal{O}_+(k_1)\,\mathcal{O}_+(k_2) \sim \frac{X(k_1,k_2)}{z_1 - z_2}\,\mathcal{O}_+(k_1+k_2) \quad \longleftrightarrow \quad [L_{k_1}, L_{k_2}] = X(k_1,k_2)\,L_{k_1+k_2}$$

Recall kinematics: $\quad k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}\,, \quad \lambda = (1, z)\,, \quad \tilde{\lambda} = (k_u, k_w)\,.$

Soft expansion is $\tilde{\lambda} \to 0$. At fixed z,

$$L_k = \{e^{i\,k\cdot x}, \cdot\} = \sum_{a,b=0}^{\infty} \frac{(ik_u)^a}{a!}\,\frac{(ik_w)^b}{b!}\,\ell_{a,b} \qquad \ell_{a,b} = \{(u+z\bar{w})^a(w+zv)^b, \cdot\}$$

$$[L_{k_1}, L_{k_2}] = X(k_1,k_2)\,L_{k_1+k_2} \quad \leftrightarrow \quad [\ell_{a,b}\,,\,\ell_{c,d}] = (ad - bc)\,\ell_{a+c-1,b+d-1}$$

"wedge subalgebra of Lie algebra $\omega_{1+\infty}$"

[Strominger 21; Adamo, Mason, Sharma 21; . . .]

$\omega_{1+\infty}$ from soft expansion in SDG

$$\mathcal{O}_+(k_1)\,\mathcal{O}_+(k_2) \sim \frac{X(k_1,k_2)}{z_1 - z_2}\,\mathcal{O}_+(k_1+k_2) \quad \longleftrightarrow \quad [L_{k_1}, L_{k_2}] = X(k_1,k_2)\,L_{k_1+k_2}$$

Recall kinematics: $k_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}\,, \quad \lambda = (1,z)\,, \quad \tilde{\lambda} = (k_u, k_w)\,.$

Soft expansion is $\tilde{\lambda} \to 0$. At fixed z,

$$L_k = \{e^{i\,k\cdot x}, \cdot\} = \sum_{a,b=0}^{\infty} \frac{(ik_u)^a}{a!}\,\frac{(ik_w)^b}{b!}\,\ell_{a,b} \qquad \ell_{a,b} = \{(u+z\bar{w})^a (w+zv)^b, \cdot\}$$

$$[L_{k_1}, L_{k_2}] = X(k_1,k_2)\,L_{k_1+k_2} \quad \leftrightarrow \quad [\ell_{a,b}\,,\,\ell_{c,d}] = (ad-bc)\,\ell_{a+c-1,b+d-1}$$

“wedge subalgebra of Lie algebra $\omega_{1+\infty}$”

[Strominger 21; Adamo, Mason, Sharma 21; ...]

Can consider Moyal deformation of SDG (MSDG): [related to Mago et al 21; Ren et al 22]

$$[L_{k_1}, L_{k_2}]^M = X^M(k_1,k_2)\,L_{k_1+k_2} \quad \leftrightarrow \quad \text{resembles } W_{1+\infty} \text{ algebra}$$

Integrability in chiral sector

SDYM and (M)SDG are classically integrable $\leftrightarrow$ tree amplitudes vanish.

Integrability in chiral sector

SDYM and (M)SDG are classically integrable $\leftrightarrow$ tree amplitudes vanish.

Consider celestial OPE coefficients ${C_{IJ}}^K$:

SDYM: $f^{a_1 a_2 a_3}$, SDG: $X(k_1, k_2)$, MSDG: $X^M(k_1, k_2)$.

$$\mathcal{O}_I(k_1)\,\mathcal{O}_J(k_2) \sim \frac{{C_{IJ}}^K}{z_1 - z_2}\,\mathcal{O}_K(k_1 + k_2) = \frac{X(k_1, k_2)\,{C_{IJ}}^K}{s_{12}}\,\mathcal{O}_K(k_1 + k_2)$$

Integrability in chiral sector

SDYM and (M)SDG are classically integrable $\leftrightarrow$ tree amplitudes vanish.

Consider celestial OPE coefficients $C_{IJ}{}^{K}$:

SDYM: $f^{a_1 a_2 a_3}$, SDG: $X(k_1, k_2)$, MSDG: $X^M(k_1, k_2)$.

$$\mathcal{O}_I(k_1)\,\mathcal{O}_J(k_2) \sim \frac{C_{IJ}{}^{K}}{z_1 - z_2}\,\mathcal{O}_K(k_1 + k_2) = \frac{X(k_1, k_2)\,C_{IJ}{}^{K}}{s_{12}}\,\mathcal{O}_K(k_1 + k_2)$$

If chiral OPEs fully determine theory, then double copy ensures integrability (similar to Ward conjecture).

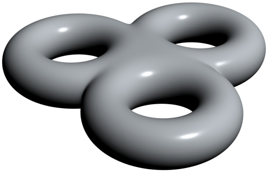
KLT amplitude formula:

$$\mathcal{A} = \sum_{\text{permutations}} \vec{A}_{\text{left}} \cdot S_{\text{KLT}} \cdot \vec{A}_{\text{right}}$$

$\vec{A}$ are ordered amp's with vertices $X(k_1, k_2)$ and $C_{IJ}{}^{K}$. $\vec{A}_{\text{left}} = 0 \Rightarrow \mathcal{A} = 0$.

Non-vanishing quantities? Form factors. [Boels, Isermann, RM, O'Connell 11] [Costello, Paquette 22]

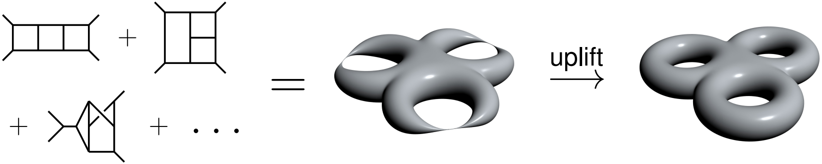
Application to String Amplitudes



Importing field theory into string theory

[Geyer, RM, Stark-Muchão 21]

Supergravity loop integrand $\longrightarrow$ Superstring moduli-space integrand.



(i) Take sugra g-loop integrand obtained from BCJ double copy, (superYM)^2.

(ii) Translate it into Riemann sphere with g nodes cf. ambitwistor string.

(iii) Uplift from nodal sphere to genus g guided by modular invariance.

New conjecture for 3-loop 4-pt type II superstring amplitude.

17 years after 2 loops [D'Hoker, Phong; Berkovits 05], 40 years after 1 loop [Green, Schwarz, Brink 82].

Conclusion

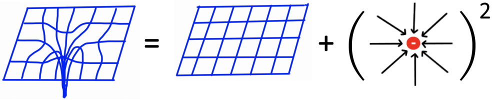
Conclusion

- Double copy (DC) is **useful** and **insightful**, from LIGO to superstrings.

  Pervasive in perturbative gauge theory, gravity, string theory.

- DC of amplitudes $\rightarrow$ DC of classical solutions.

  DC of exact classical solutions possible for some classes.



- Role of self-dual kinematic algebra in celestial holography, integrability.

- **Much more to explore**