METRIC RECONSTRUCTION FOR NON-RADIATIVE SPACETIMES

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NBI workshop “What’s new in gravity?” @ Helsingør, Denmark

LUTH l'Observatoire de Paris | PSL Université Paris Cité cnrs

MAIN REFERENCES

Metric reconstruction from celestial multipoles

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Abstract

The most general vacuum solution to Einstein's field equations with no incoming radiation can be constructed perturbatively from two infinite sets of canonical multipole moments, which are found to be exchanged under gravitational electric-magnetic duality at the non-linear level. We demonstrate that in non-radiative regions such spacetimes are completely characterized by a set of conserved celestial charges that consist of the Geroch-Hansen multipole moments, the generalized BMS charges and additional celestial multipoles accounting for subleading memory effects. Transitions among non-radiative regions, induced by radiative processes, are therefore labelled by celestial charges, which are identified in terms of canonical multipole moments of the linearized gravitational field. The dictionary between celestial charges and canonical multipole moments allows to holographically reconstruct the metric in de Donder, Newman-Unti or Bondi gauge outside of sources.

Multipole Expansion of Gravitational Waves: from Harmonic to Bondi coordinates

(or "Monsieur de Donder meets Sir Bondi")

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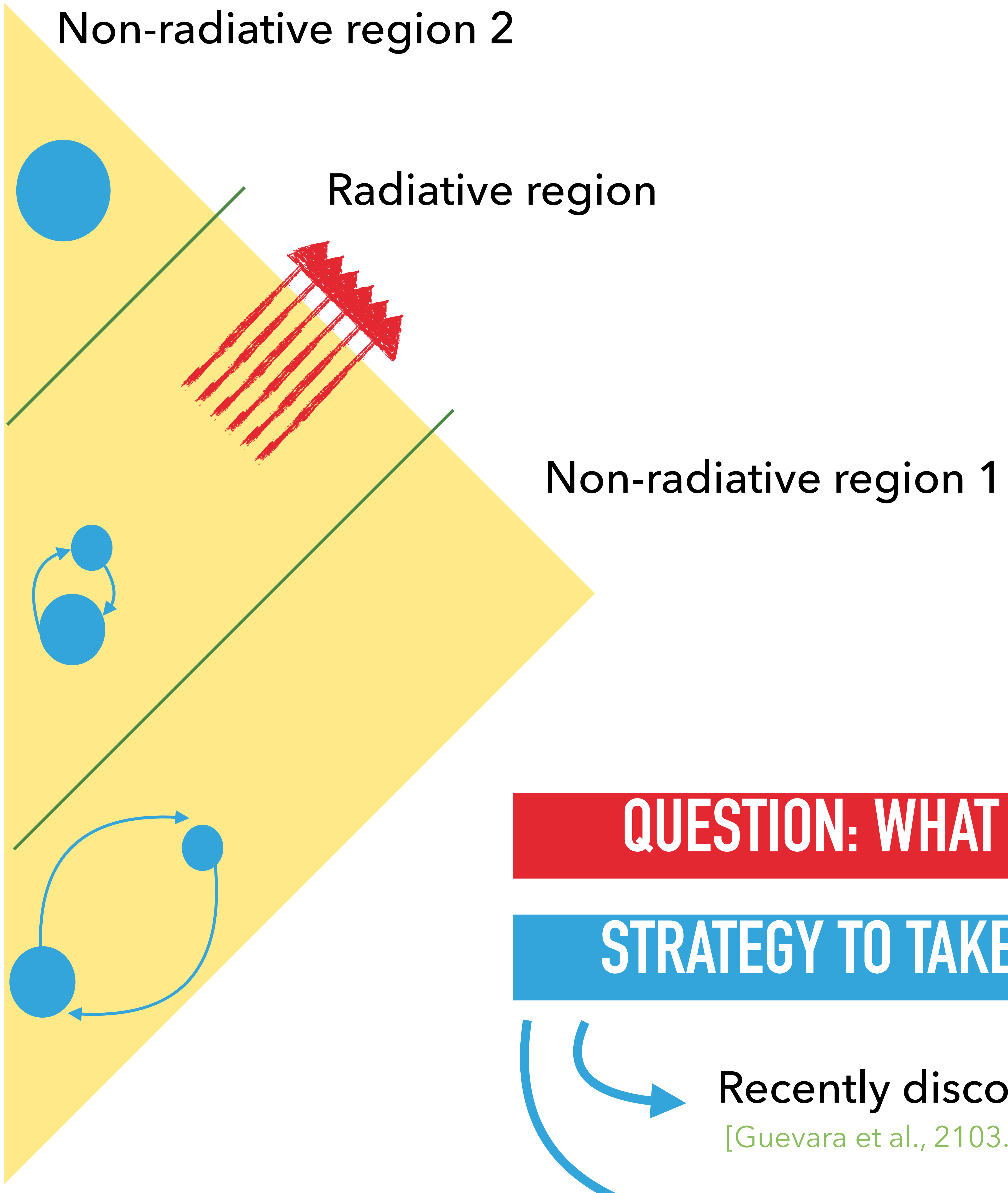
Abstract

We transform the metric of an isolated matter source in the multipolar post-Minkowskian approximation from harmonic (de Donder) coordinates to radiative Newman-Unti (NU) coordinates. To linearized order, we obtain the NU metric as a functional of the mass and current multipole moments of the source, valid all-over the exterior region of the source. Imposing appropriate boundary conditions we recover the generalized Bondi-van der Burg-Metzner-Sachs residual symmetry group. To quadratic order, in the case of the mass-quadrupole interaction, we determine the contributions of gravitational-wave tails in the NU metric, and prove that the expansion of the metric in terms of the radius is regular to all orders. The mass and angular momentum aspects, as well as the Bondi shear, are read off from the metric. They are given by the radiative quadrupole moment including the tail terms.

Based on arXiv 2206:12597 with Geoffrey Compère and Ali Seraj

and arXiv 2010:10000 + work in progress with Luc Blanchet, Geoffrey Compère, Guillaume Faye, Ali Seraj

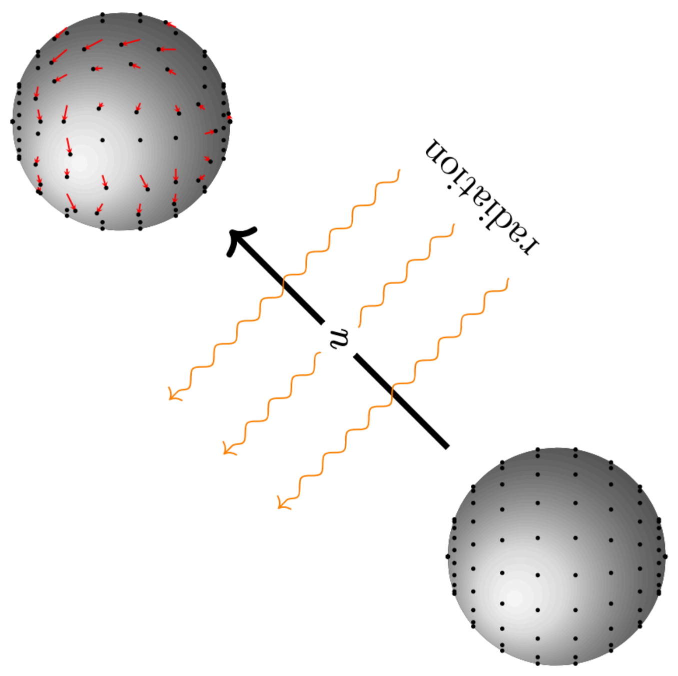
MOTIVATION



Non-radiative regions 1 and 2 differ from each other

Gravitational ``vacua" are degenerate

E.g., supertranslations label gravitational vacua



[Strominger, 1703.05448]

$$\delta_T C_{ab} = -2D_{\langle a}D_{b\rangle}T(\theta,\phi)$$

QUESTION: WHAT COMPLETELY CHARACTERISE NON-RADIATIVE STATES?

STRATEGY TO TAKE: GO DEEPER IN THE INFRARED STRUCTURE OF GRAVITY

Recently discovered $Lw_{1+\infty}$ algebra

[Guevara et al., 2103.03961], [Strominger, 2105.14346], [Freidel-Pranzetti-Raclariu, 2112.15573]

Combining the Bondi-Sachs with multipolar Post-Minkowskian/Post-Newtonian formalisms

[Blanchet, Compère, Faye, RO, Seraj, 2010:10000 & work in progress]

OUTLINE

Part I:

- Einstein's equations - local flux-balance laws;
- BMS & celestial charges.

Part II:

- Celestial charges in the linear theory… ;
- … and in non-radiative regions;
- Physical interpretation.

Part III:

- Connection of celestial charges with $Lw_{1+\infty}$ charges.

PART I

BONDI GAUGE AND METRIC

Bondi coordinates: $\{u, r, \theta^a\}$

Bondi gauge: $g_{rr} = 0 = g_{ra}$ and $\partial_r \det\left(r^{-2} g_{ab}\right) = 0$

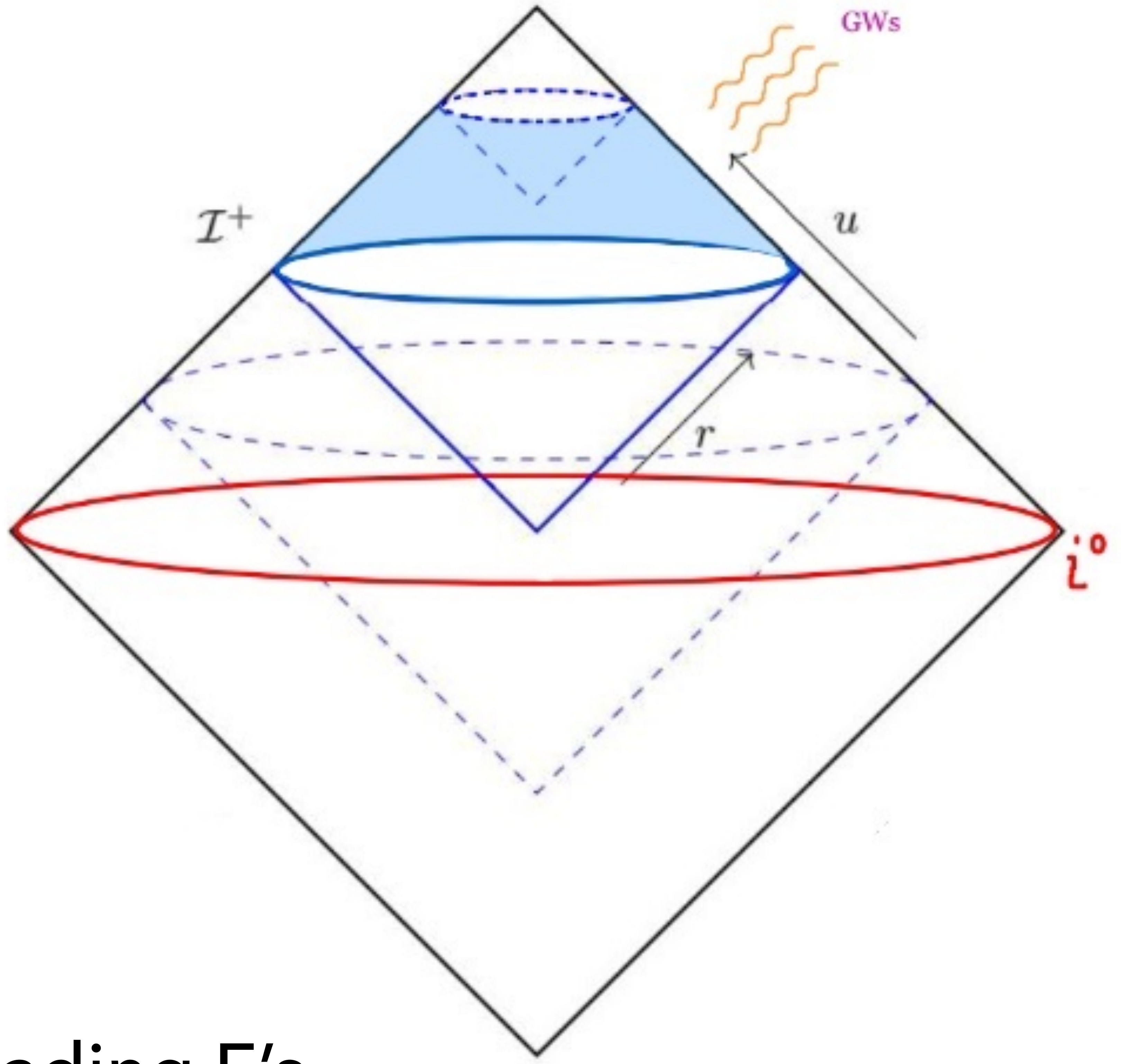
Bondi metric: $ds^2 = -e^{2\beta}\left(F du^2 + 2 du dr\right) + g_{ab}\left(d\theta^a - \frac{U^a}{r^2} du\right)\left(d\theta^b - \frac{U^b}{r^2} du\right)$

Asymptotic expansion: [Bondi-van der Burg-Metzner, 1962], [Sachs, 1962], […], [Grant-Nichols, 2109.03832]

$$F = 1 - \frac{2}{r}\left(m + \frac{1}{8} C_{ab} N^{ab}\right) + \mathcal{O}(r^{-2}) \qquad g_{ab} U^b = \frac{1}{2} D^b C_{ab} + \frac{2}{3r} N_a + \mathcal{O}(r^{-2})$$

$$g_{ab} = r^2 \sqrt{1 + \frac{\mathscr{C}_{cd}\mathscr{C}^{cd}}{2r^2}}\,\gamma_{ab} + r\mathscr{C}_{ab} \qquad \mathscr{C}_{ab} = C_{ab} + \sum_{n=2}^{+\infty} r^{-n} \underset{(n)}{E}_{ab}$$

FOCUS OF THIS TALK!



BONDI FIELDS: mass & angular momentum aspects, shear and sub-leading E's

LOCAL FLUX-BALANCE LAWS

In Bondi gauge, Einstein's equations reduce to a set of algebraic constraints in addition to a countable infinite set of local flux-balance equations on future null infinity:

$$n=0 \quad : \quad \frac{1}{4} D_b D_c N^{bc} = -\mathcal{F}(u) + \partial_u m,$$

$$n=1 \quad : \quad -\frac{u}{2} D_c D_{\langle a} D_{b\rangle} N^{bc} = -\mathcal{F}_a(u) + \partial_u \mathcal{N}_a,$$

$$n=2 \quad : \quad \frac{u^2}{12} \mathrm{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] = -\underset{(2)}{\mathcal{F}_{ab}}(u) + \partial_u \underset{(2)}{\mathcal{E}_{ab}},$$

$$n \geq 3 \quad : \quad \frac{(-u)^n}{6\, n!} \mathcal{D}_{n-3} \cdots \mathcal{D}_0 \, \mathrm{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] = -\underset{(n)}{\mathcal{F}_{ab}}(u) + \partial_u \underset{(n)}{\mathcal{E}_{ab}} .$$

$$N_{ab} = \partial_u C_{ab} \qquad \mathcal{F} \equiv -\frac{1}{8} N_{ab} N^{ab}$$

[Compère-Fiorucci-Ruzziconi, 1810. 00377]

$$\mathcal{N}_a \equiv N_a - \frac{1}{4} C_{ab} D_c C^{bc} - \frac{1}{16} \partial_a (C_{bc} C^{bc}) - u D^b m_{ab}$$

$$\underset{(2)}{\mathcal{E}_{ab}} = \underset{(2)}{E_{ab}} - \frac{u}{2} C_{(a}{}^{c} m_{b)c} - \frac{u}{3} D_{\langle a} \mathcal{N}_{b\rangle} - \frac{u^2}{6} D_{\langle a} D^c m_{b\rangle c}$$

[Grant-Nichols, 2109.03832]
also [Freidel-Pranzetti-Raclariu, 2112.15573]

- LHS ~ LINEAR IN THE NEWS TENSOR, SOMETIMES REFERRED TO AS SOFT / MEMORY TERM
- RHS ~ FLUXES + TIME DERIVATIVE OF ``IMPROVED" OR ``DRESSED" BONDI FIELDS:

FLUXES VANISHES WHEN THE NEWS VANISHES; IT CONTAINS INTERACTIONS OF C_{ab}, N_{ab} AND BONDI FIELDS

``IMPROVED" BONDI FIELDS ARE ``DRESSED" WITH U-TERMS

CHARGES FROM THE DRESSED BONDI ASPECTS: POINCARÉ, BMS AND CELESTIAL CHARGES

Let $\hat{n}_L = \mathrm{STF}[n_{i_1} \cdots n_{i_l}]$ be the symmetric and trace-free product of l unit directional vectors n_i

- From the Bondi mass aspect ($n = 0$) and dressed angular momentum aspect ($n = 1$):

$$\mathcal{P}_L = \oint_S m\, \hat{n}_L, \qquad -\mathcal{J}_L = \frac{1}{2} \oint_S \epsilon^{ab} \partial_b \hat{n}_L \mathcal{N}_a, \qquad \mathcal{K}_L = \frac{1}{2} \oint_S \partial^a \hat{n}_L \mathcal{N}_a.$$

[Compère, RO, Seraj, 1912.03164]

The 10 Poincaré charges are recovered for $\ell = \{0,1\}$. BMS charges are defined for $\ell \geq 2$.

- From the dressed Bondi sub-leading field $\underset{(n)}{\mathcal{E}^{ab}}$ ($n \geq 2$), we define the $n \geq 2$ celestial charges:

$$\mathcal{Q}^+_{n,L}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}}\, D_a D_b \hat{n}_L, \qquad \mathcal{Q}^-_{n,L}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}}\, \epsilon_{ac} D_b D^c \hat{n}_L.$$

[Compère, RO, Seraj, 2206.12597]

TWO QUALITATIVE DIFFERENT SETS OF LOCAL FLUX-BALANCE LAWS

Recall:

$$n=0 \quad : \quad \frac{1}{4} D_b D_c N^{bc} = -\mathcal{F}(u) + \partial_u m,$$

$$n=1 \quad : \quad -\frac{u}{2} D_c D_{\langle a} D_{b\rangle} N^{bc} = -\mathcal{F}_a(u) + \partial_u \mathcal{N}_a,$$

$$n=2 \quad : \quad \frac{u^2}{12} \mathrm{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] = -\underset{(2)}{\mathcal{F}_{ab}}(u) + \partial_u \underset{(2)}{\mathcal{E}_{ab}},$$

$$n \geq 3 \quad : \quad \frac{(-u)^n}{6\,n!} \mathcal{D}_{n-3} \cdots \mathcal{D}_0 \, \mathrm{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] = -\underset{(n)}{\mathcal{F}_{ab}}(u) + \partial_u \underset{(n)}{\mathcal{E}_{ab}}.$$

$$\partial_u \mathcal{Q}^+_{n,L}(u) = \oint_S \underset{(n)}{\mathcal{F}^{ab}} D_a D_b \hat{n}_L + \frac{(-u)^n}{6\ n!} \oint_S \hat{n}_L D^{\langle b} D^{a\rangle} \mathcal{D}_{n-3} \cdots \mathcal{D}_0 D_a D_c D_{\langle b} D_{d\rangle} N^{cd},$$

FACT 1:
LHS REMINDS THE MELLIN TRANSFORM:

$$\mathscr{M}_n[f] = \int_0^{+\infty} u^{n-1} f(u)$$

FACT 2:

$$\mathcal{D}_n \equiv -\frac{n+2}{2(n+1)(n+4)} (\Delta + n^2 + 5n + 2)$$

ANNIHILATES THE FIRST $l = n+2$ HARMONIC MODES

MEMORY-LESS FLUX-BALANCE LAWS

$n=0:\ \ell=0$ ENERGY LOSS FORMULA AND $\ell=1$ MOMENTUM LOSS FORMULA
$n=1:\ \ell=1$ ANGULAR AND CENTER-OF-MASS LOSS FORMUALAE

$n=2:\ \varnothing$
$n\geq 3: 2\leq \ell \leq n-1$ NEWMAN-PENROSE CHARGES

MEMORY-FULL FLUX-BALANCE LAWS

$n=0, \ell\geq 2$, DISPLACEMENT MEMORY EFFECT
$n=1, \ell\geq 2$, SPIN AND CENTER-OF-MASS MEMORY EFFECTS

$n\geq 2, \ell \geq n$, SUBLEADING MEMORY EFFECTS

FACT3: IN NON-RADIATIVE REGIONS, THE CELESTIAL CHARGES ARE CONSERVED

PART II

CELESTIAL CHARGES – EXPLICIT EXPRESSIONS IN LINEARISED THEORY (1/3)

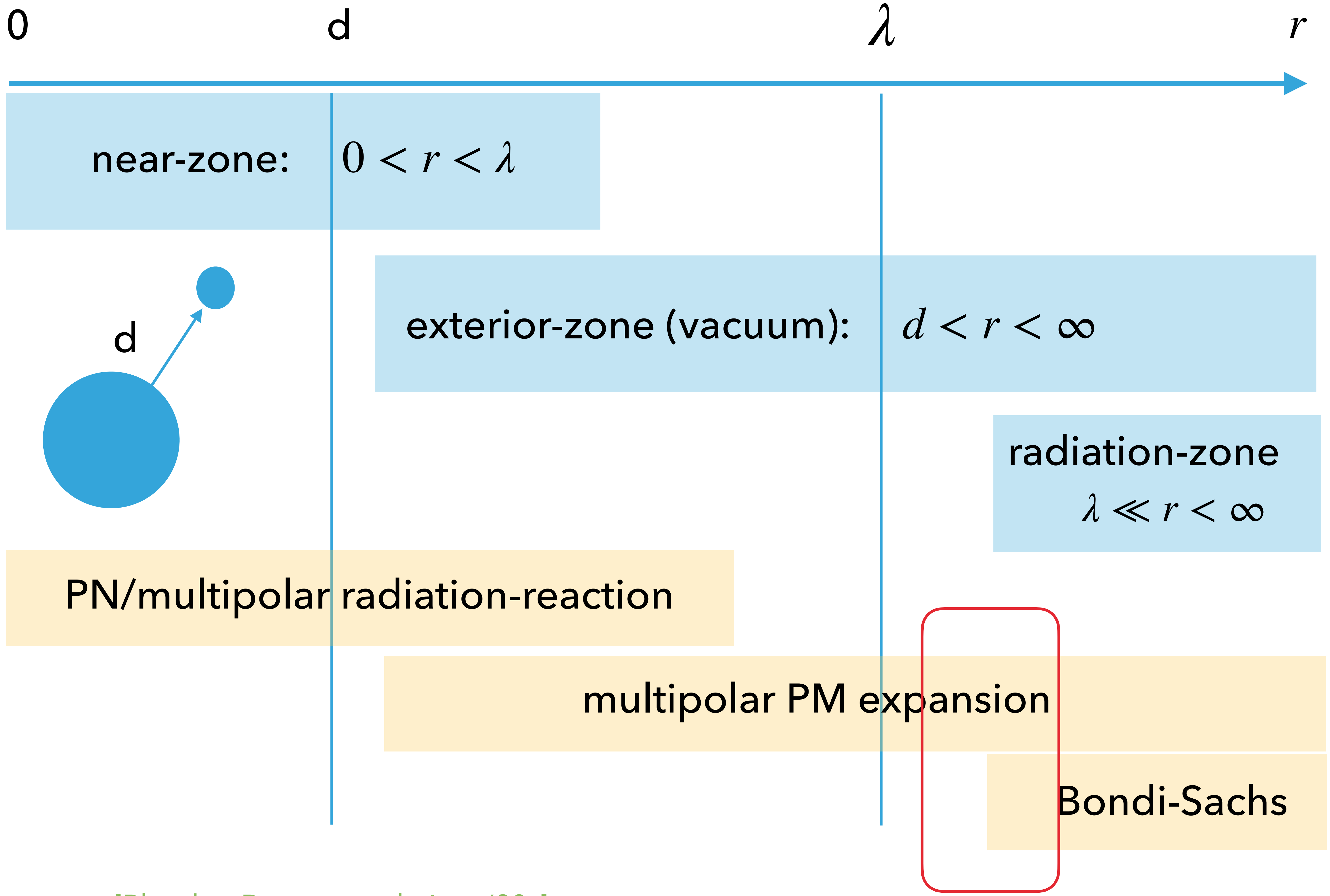
We wish to compute $\mathcal{Q}^{+}_{n,L}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}}\, D_a D_b \hat{n}_L$ in terms of multipole moments.

Explicit expressions in linear theory of the Bondi fields are in [Blanchet, Compère, Faye, RO, Seraj, 2010:10000]

"MONSIEUR DE DONDER MEETS THE SIR BONDI"

MULTIPOLAR POST-MINKOWSKIAN EXPANSION
(IN HARMONIC GAUGE)
TO
BONDI-SACHS EXPANSION
(IN BONDI GAUGE)

- START FROM LINEARISED METRIC IN HARMONIC GAUGE
- IMPOSE RADIATIVE/BONDI GAUGE CONDITIONS
- SOLVE THEM UP TO BMS TRANSFORMATIONS
- READ OFF THE BONDI FIELDS IN TERMS OF MULTIPOLE MOMENTS



[Blanchet-Damour et al, since '80s]

[Blanchet, Compère, Faye, RO, Seraj, 2010:10000]

CELESTIAL CHARGES – EXPLICIT EXPRESSIONS IN LINEARISED THEORY (2/3)

Explicit expressions in linear theory of the Bondi fields are in [Blanchet, Compère, Faye, RO, Seraj, 2010:10000]

$$g_{uu} = -1 - G(\Delta + 2)\dot{f} + 2G\left(\frac{m}{r} + \sum_{n=2}^{+\infty} \frac{1}{r^n} \underset{(n)}{K}\right) + \mathcal{O}(G^2)\,,$$

$$g_{ua} = G\left(\frac{1}{2} D_b C^b{}_a + \frac{2}{3}\frac{N_a}{r} + e^i_a \sum_{n=2}^{+\infty} \frac{1}{r^n} \underset{(n)}{P^i}\right] + \mathcal{O}(G^2)\,,$$

$$g_{ab} = r^2 \left[\gamma_{ab} + 2G D_{\langle a} Y_{b\rangle} + G\left(\frac{C_{ab}}{r} + e^i_{\langle a} e^j_{b\rangle} \sum_{n=2}^{+\infty} \frac{1}{r^n} \underset{(n)}{E^{ij}}\right)\right] + \mathcal{O}(G^2)\,.$$

MAIN ADVANTAGE:

FULL $\frac{1}{r}$ EXPANSION IN BONDI GAUGE AT $\mathcal{O}(G)$

More explicitly, the Bondi mass aspect reads as

$$m = \sum_{\ell=0}^{+\infty} \frac{(\ell+1)(\ell+2)}{2\ell!}\, n_L M_L^{(\ell)} + \mathcal{O}(G)\,,$$

the Bondi angular momentum aspect is

$$N_a = e^i_a \sum_{\ell=1}^{+\infty} \frac{(\ell+1)(\ell+2)}{2(\ell-1)!}\, n_{L-1}\left[M^{(\ell-1)}_{iL-1} + \frac{2\ell}{\ell+1}\varepsilon_{ipq} n_p S^{(\ell-1)}_{qL-1}\right] + \mathcal{O}(G)$$

and finally the Bondi shear is given by

$$C_{ab} = e^i_{\langle a} e^j_{b\rangle} H^{ij}_{\mathrm{TT}} = 4 e^i_{\langle a} e^j_{b\rangle} \perp^{ijkl}_{\mathrm{TT}} \sum_{\ell=2}^{+\infty} \frac{n_{L-2}}{\ell!}\left[M^{(\ell)}_{klL-2} - \frac{2\ell}{\ell+1}\varepsilon_{kpq} n_p S^{(\ell)}_{lqL-2}\right] + \mathcal{O}(G)$$

In particular:

$$\underset{(n)}{E_{ab}} = 4 e^i_{\langle a} e^j_{b\rangle} \frac{n-1}{n+1} \sum_{\ell \geq n} \frac{1}{\ell!} \frac{(\ell+n)!}{2^n n! (\ell-n)!} n_{L-2}\left[M^{(\ell-n)}_{ijL-2} + \frac{2\ell}{\ell+1}\epsilon_{ipq} n_p S^{(\ell-n)}_{jqL-2}\right] + \mathcal{O}(G),$$

MASS MOMENTS

SPIN MOMENTS

[Blanchet, Compère, Faye, RO, Seraj, in progress]

CELESTIAL CHARGES – EXPLICIT EXPRESSIONS IN LINEARISED THEORY (3/3)

We wish to compute $\mathcal{Q}^{+}_{n,L}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}}\, D_a D_b \hat{n}_L$ in terms of multipole moments.

In the linear theory:

$$\mathcal{Q}^{+}_{n,L}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}}\, D_a D_b \hat{n}_L = \begin{cases} \sum_{p=n-l-1}^{n-3} q_{n,\ell,p}\, u^{p+1} M_L^{(\ell-n+p+1)} + b_{n,\ell}\, u^{n-1}\left(1-\frac{u}{n}\partial_u\right) M_L^{(\ell-1)} + \mathcal{O}(G) & 2 \leq \ell \leq n-1 \\ a_{n,\ell}\, M_L^{(\ell-n)} + \sum_{p=0}^{n-3} q_{n,\ell,p}\, u^{p+1} M_L^{(\ell-n+p+1)} + b_{n,\ell}\, u^{n-1}\left(1-\frac{u}{n}\partial_u\right) M_L^{(\ell-1)} + \mathcal{O}(G) & \ell \geq n \end{cases}$$

$\mathcal{Q}^{-}_{n,L}(u)$ same expression with $M_L \rightarrow \frac{2l}{l+1} S_L$

SUMMARY (SO FAR):

- CELESTIAL CHARGES = POINCARÉ + BMS (ALSO DUAL BMS) + $n \geq 2$ CELESTIAL CHARGES
- $n \geq 2$ CELESTIAL CHARGES ARE DIVIDED INTO TWO SETS: $2 \leq \ell \leq n-1$ AND $\ell \geq n$
- WE EXPRESSED THE SET OF CELESTIAL CHARGES IN TERMS OF MULTIPOLE MOMENTS

CELESTIAL CHARGES IN NON-RADIATIVE SPACETIMES

<u>Assuming no-radiation:</u> $$0 = N_{ab} = \dot{C}_{ab} \propto e^k_{\langle a} e^l_{b\rangle} \sum_{\ell=2}^{+\infty} \frac{n_{L-2}}{\ell!} \left[M^{(\ell+1)}_{klL-2} - \frac{2\ell}{\ell+1} \varepsilon_{kpq} n_p S^{(\ell+1)}_{lqL-2} \right] + \mathcal{O}(G)$$

NON-RADIATIVE REGIONS: $M_L^{(l+1)} = 0 = S_L^{(l+1)}$

$$M_L(u) = \sum_{k=0}^{l} M_{L,k} u^k \quad \text{and} \quad S_L(u) = \sum_{k=0}^{l} S_{L,k} u^k$$

The constants $M_{L,k}$ completely characterise non-radiative regions.

One can think of them as initial/final states of a scattering process ~ proportional to positions and velocities of initial/final states.

[Blanchet-Schaefer, CQG 1993]

THE CELESTIAL CHARGES FOR NON-RADIATIVE REGIONS = $$\begin{cases} 0 & 2 \le \ell \le n-1 \\ a_{n,\ell}(\ell - n)!\, M_{L,\ell-n} + \mathcal{O}(G) & \ell \ge n \end{cases}$$

CELESTIAL CHARGES - PHYSICAL INTERPRETATION

NON-RADIATIVE REGIONS: $M_L(u) = \sum_{k=0}^{l} M_{L,k}\, u^k \Longrightarrow \mathcal{Q}^+_{n,L} = a_{n,\ell}\,(\ell - n)!\, M_{L,\ell-n} + \mathcal{O}(G)$

BMS charges	non-stationary features	Geroch-Hansen

$$M_L(u) = M_{L,\ell}\, u^\ell + M_{L,\ell-1}\, u^{\ell-1} + M_{L,\ell-2}\, u^{\ell-2} + \ldots + M_{L,1}\, u + M_{L,0}$$

$$\{g_{uu}, \frac{1}{r} g_{ra}, \frac{1}{r^2} g_{ab}\} \propto \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \ldots + \frac{1}{r^\ell} + \frac{1}{r^{\ell+1}}$$

Linear displ. memory $M_{L,\ell}$ Spin memory $S_{L,\ell-1}$ CoM memory $M_{L,\ell-1}$	Subleading memory effects

NON-RADIATIVE REGIONS
=
GEROCH-HANSEN ($k = 0$)
+
BMS CHARGES ($k = \ell, k = \ell - 1$)
+
CELESTIAL CHARGES ($1 \leq k \leq \ell - 2$)

$$\mathcal{Q}^+_{n,L}\Big|_i^f \propto M_{L,\ell-n}\Big|_i^f$$

$$C_{ab} = 4 e^i_{\langle a} e^j_{b\rangle} \sum_{\ell=2}^{+\infty} n_{L-2}\left[M_{ijL-2,\ell} - \frac{2\ell}{\ell+1}\varepsilon_{ipq} n_p S_{jqL-2,\ell}\right]$$

k →, L ↓

$M_{\emptyset,0} \sim \mathcal{E}$			
$M_{i,0} \sim \mathcal{K}_i$	$M_{i,1} \sim \mathcal{P}_i$		
$M_{ij,0} \sim \mathcal{Q}^+_{2,ij}$	$M_{ij,1} \sim \mathcal{K}_{ij}$	$M_{ij,2} \sim \mathcal{P}_{ij}$	
$M_{ijk,0} \sim \mathcal{Q}^+_{3,ijk}$	$M_{ijk,1} \sim \mathcal{Q}^+_{2,ijk}$	$M_{ijk,2} \sim \mathcal{K}_{ijk}$	$M_{ijk,3} \sim \mathcal{P}_{ijk}$

PART III

CONNECTION WITH CELESTIAL HOLOGRAPHY (1/3)

Celestial charges defined earlier $\mathcal{Q}^{\pm}_{n,L}$ are (proportional to) gravitational multipole moments

The $\mathcal{Q}^{\pm}_{n,L}$ are also proportional to the (real part of the) $Lw_{1+\infty}$ charges proposed in [Freidel-Pranzetti-Raclariu, 2112.15573]

[Freidel-Pranzetti, 2109.06342]

$$\partial_u q_{a_1\cdots a_s} = D_{\langle a_1} q_{a_2\cdots a_s\rangle} + \frac{s+1}{2} C_{\langle a_1 a_2} q_{a_3\ldots a_s\rangle}. \qquad s \geq 3$$

DERIVED FROM SYMMETRY ARGUMENTS

RE-ORGANIZATION OF ASYMPTOTIC DATA AS PRIMARIES OF THE HOMOGENEOUS SUBGROUP OF WEYL-BMS

$$\delta_{(Y,W)} O_{(\Delta,s)} = (\mathcal{L}_Y + (\Delta - s)W) O_{(\Delta,s)}$$

Primary fields $s = \{-2, -1, 0, +1, +2\}$
(also proportional to Weyl scalars)

evolution equations

$$\begin{aligned}
q_{-2}^{ab} &\equiv \frac{1}{2}\partial_u N^{ab},\\
q_{-1}^{a} &\equiv \frac{1}{2} D_b N^{ab},\\
q &\equiv \frac{1}{2}\gamma^{ab} m_{ab} + \frac{1}{8} C_{ab} N^{ab} = m + \frac{1}{8} C_{ab} N^{ab},\\
\tilde{q} &\equiv \frac{1}{2}\epsilon^{ab} m_{ab} + \frac{1}{8} C_{ab}\tilde{N}^{ab} = \frac{1}{4} D_a D_b \tilde{C}^{ab} + \frac{1}{8} C_{ab}\tilde{N}^{ab}\\
q_a &\equiv N_a,\\
q_{ab} &\equiv 3\left(\underset{(2)}{\mathcal{E}_{ab}} - \frac{1}{16} C_{ab} C_{cd} C^{cd}\right).
\end{aligned}$$

$$\begin{aligned}
\partial_u q_{-1}^{a} &= D_b q_{-2}^{ab},\\
\partial_u \tilde{q} &= \frac{1}{2} D_a \tilde{q}_{-1}^{a} + \frac{1}{4} C_{ab} \tilde{q}_{-2}^{ab}.\\
\partial_u q &= \frac{1}{2} D_a q_{-1}^{a} + \frac{1}{4} C_{ab} q_{-2}^{ab},\\
\partial_u q_a &= \partial_a q + \tilde{\partial}_a \tilde{q} + C_{ab} q_{-1}^{b},\\
\partial_u q_{ab} &= D_{\langle a} q_{b\rangle} + \frac{3}{2}(C_{ab} q + \tilde{C}_{ab}\tilde{q}).
\end{aligned}$$

[Freidel-Pranzetti-RO-Speziale, 2104.05793]

WEYL-BMS GROUP

$$(\mathrm{Diff}(S) \ltimes \mathrm{Weyl}) \ltimes \mathbb{R}^S$$

For $s \geq 0$, the dressed complex charges of helicity s read as [Freidel-Pranzetti-Raclariu, 2112.15573]

$$Q_s(\tau) \equiv \oint_S \tau_{-s}(x^a)\hat{q}_s(u, x^a), \qquad \hat{q}_s(u, x^a) \equiv \sum_{n=0}^{s} \frac{(-u)^{s-n}}{(s-n)!} \eth^{s-n} q_n \, .$$

They obey the $Lw_{1+\infty}$ algebra

$$\{Q_s(\tau), Q_{s'}(\tau')\}^{\text{lin}} = (s'+1)Q_{s+s'-1}(\tau'\eth\tau) - (s+1)Q_{s+s'-1}(\tau\eth\tau')$$

We define the following real dressed charges

$$Q^R_s(\tau) = \oint_S \tau^{a_1\cdots a_s}(x^a)\left(q_{a_1\cdots a_s} + \sum_{n=1}^{s-1} \frac{(-u)^{s-n}}{(s-n)!} D_{a_{n+1}} \cdots D_{a_s} q_{a_1\cdots a_n} + \frac{(-u)^s}{s!} D_{a_1} \cdots D_{a_{s-1}} D_b q_{0a_s}{}^b \right).$$

[Compère, RO, Seraj, 2206.12597]

such that

$$Q_s(\tau) = \frac{1}{2}\left(Q^R_s(\tau) + iQ^R_s(\tilde{\tau})\right) \qquad \tilde{\tau}^{a_1\cdots a_s} = \epsilon^{a_1}{}_b \, \tau^{ba_2\cdots a_s}$$

CONNECTION WITH CELESTIAL HOLOGRAPHY (3/3)

It is helpful to decompose Q_s^R in its two polarisations:

$$\mathcal{Q}^{R+}_{s,L} = Q^R_s(\tau = \tau^{a_1\ldots a_s}_{+,L}), \qquad \mathcal{Q}^{R-}_{s,L} = Q^R_s(\tau = \tau^{a_1\ldots a_s}_{-,L}).$$

Two polarisation of Q_s^R

$$\tau^{a_1\ldots a_s}_{+,L} \equiv D^{\langle a_1}\ldots D^{a_s\rangle}\hat{n}_L, \qquad \tau^{a_1\ldots a_s}_{-,L} \equiv \widetilde{D}^{\langle a_1}D^{a_2}\ldots D^{a_s\rangle}\hat{n}_L,$$

For $s = n = 0,1$ and $\ell \geq n$, we get the BMS charges:

$$\mathcal{Q}^{R+}_{0,L} = 2\mathcal{P}_L = 2\mathcal{Q}^+_{0,L}, \qquad \mathcal{Q}^{R-}_{0,L} = 2\mathcal{P}^-_L = 2\mathcal{Q}^-_{0,L},$$

$$\mathcal{Q}^{R+}_{1,L} = 2\mathcal{K}_L = 2\mathcal{Q}^+_{1,L}, \qquad \mathcal{Q}^{R-}_{1,L} = -2\mathcal{J}_L = 2\mathcal{Q}^-_{1,L},$$

For $s = n \geq 2$ and $\ell \geq n$, we get the additional celestial charges:

in non-radiative regions!

$$\mathcal{Q}^{R\pm}_{n,L} = \frac{(n+1)!}{2(n-1)}\mathcal{Q}^{\pm}_{n,L} + \text{non-linear terms} = \frac{(\ell+n)!(\ell+2)(\ell+1)}{2^n(2\ell+1)!!}M^{\pm}_{L,\ell-n} + \text{non-linear terms}$$

REMARKS AND FUTURE PERSPECTIVES

1. Gravitational EM duality:

$$^\star R_{\alpha\beta\mu\nu}(M_L^+(u), M_L^-(u); x^\mu) = R_{\alpha\beta\mu\nu}(M_L^-(u), -M_L^+(u); x^\mu)$$

(Symmetry of the space of solutions)

$$\lim_{u\to\pm\infty}\mathcal{P}_L^- = 0 \quad \leftrightarrow \quad S_{L,\ell} = 0, \quad \forall L,\ \ell \geq 0.$$

(Restriction of the space of solutions: no Taub-NUT)

2. Computation of the algebra for non-radiative multipole moments:

$$\{\mathrm{Re}\, Q_s(\tau), \mathrm{Re}\, Q_{s'}(\tau')\}^{\text{lin}} = \frac{1}{2}\mathrm{Re}\,\{Q_s(\tau), Q_{s'}(\tau')\}^{\text{lin}} + \frac{1}{2}\mathrm{Re}\,\{Q_s(\tau), Q^*_{s'}(\tau')\}^{\text{lin}}$$

$\propto M_{L,\ell-n}$

[Freidel-Pranzetti-Raclariu, 2112.15573]

?

The $Lw_{1+\infty}$ algebra

$$\{Q_s(\tau), Q_{s'}(\tau')\}^{\text{lin}} = (s'+1)Q_{s+s'-1}(\tau'\eth\tau) - (s+1)Q_{s+s'-1}(\tau\eth\tau')$$

3. Celestial charges at the quadratic order in G ?

[Blanchet, Compère, Faye, RO, Seraj, 2010:10000 & work in progress]

Include tails and memory (semi-hereditary and hereditary) contributions to the celestial charges;

4. The NP charges vanish at the linear order! What are their expressions at the quadratic order?

THANK YOU FOR YOUR ATTENTION!

QUESTIONS?