Tidal deformation of a binary system induced by a Kerr black hole

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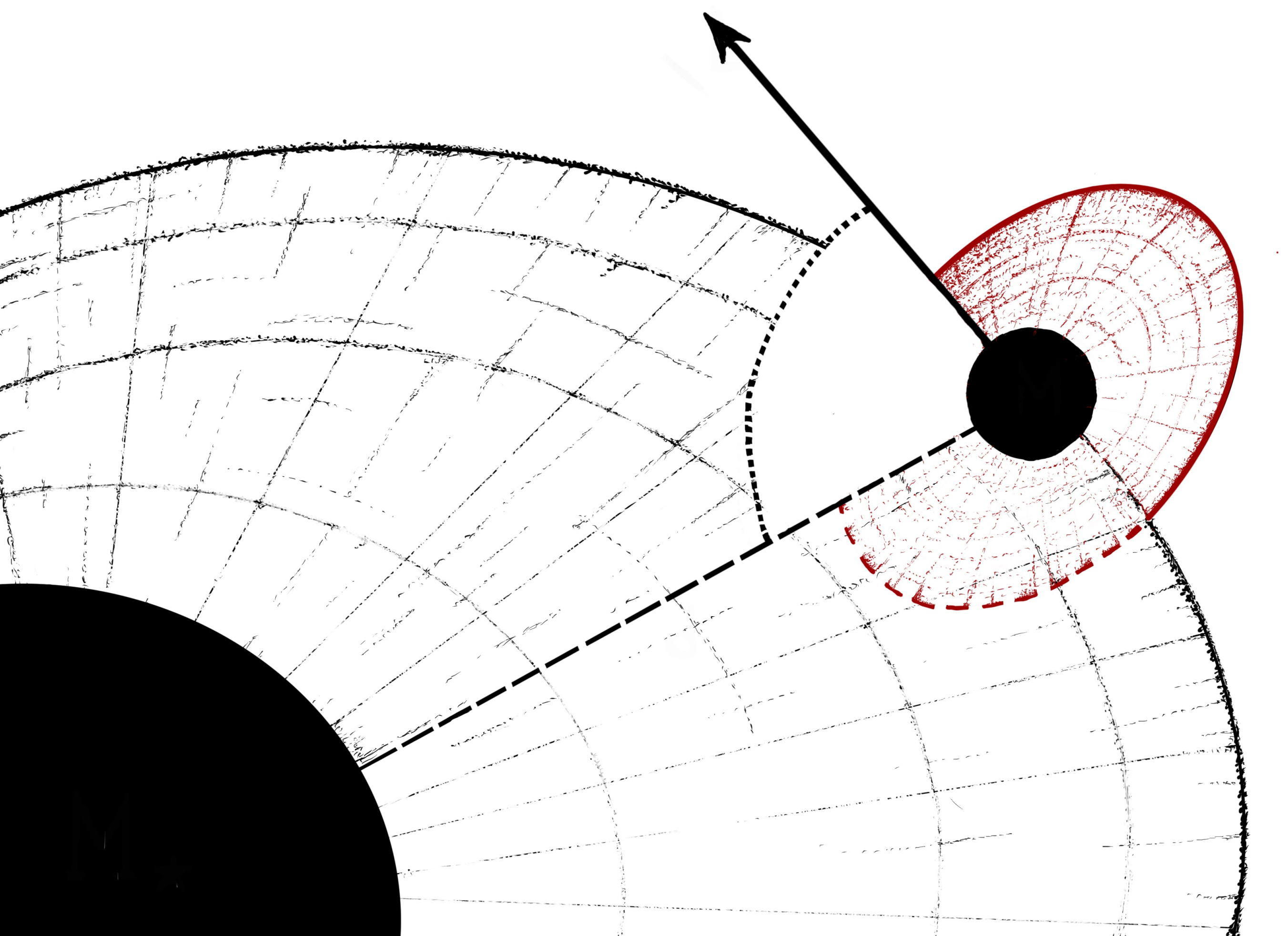
What's new in gravity?

Marienlyst, August 12, 2022



In collaboration with: Camilloni, Harmark, Oliveri, Orselli, Pica

Motivations

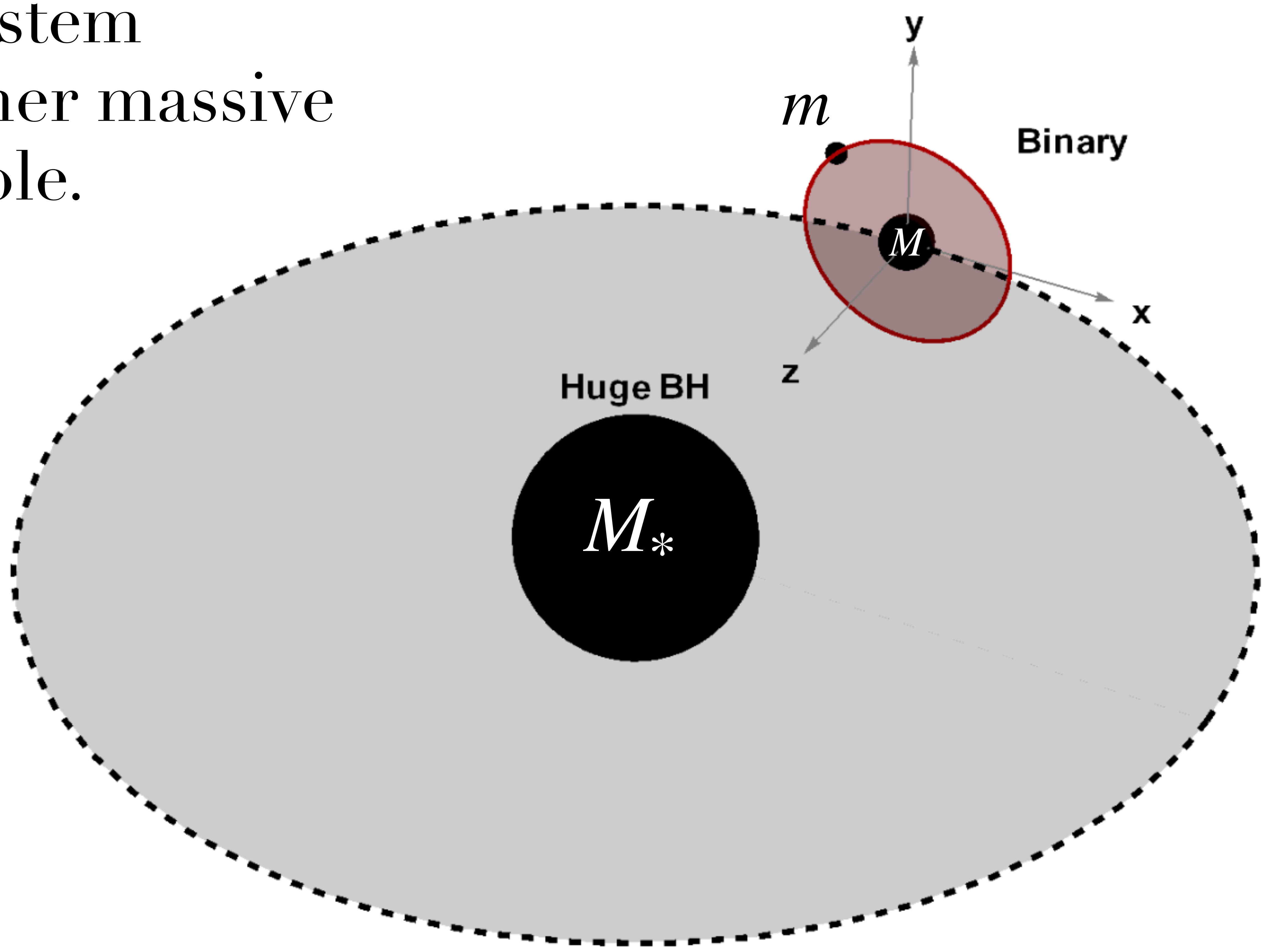
- **Hierarchical three body system**: a binary system under the gravitational interaction of a another massive astrophysical object, a supermassive black hole.

- Hierarchical in scale, a tight inner binary orbited by a tertiary on a wider orbit, forming the outer binary: $m << M << M_*$

- m is a test particle, M is a Schwarzschild black hole, M_* is a Kerr black hole

- In the limit $m \rightarrow 0$, extreme mass ratio inspirals (EMRIs)

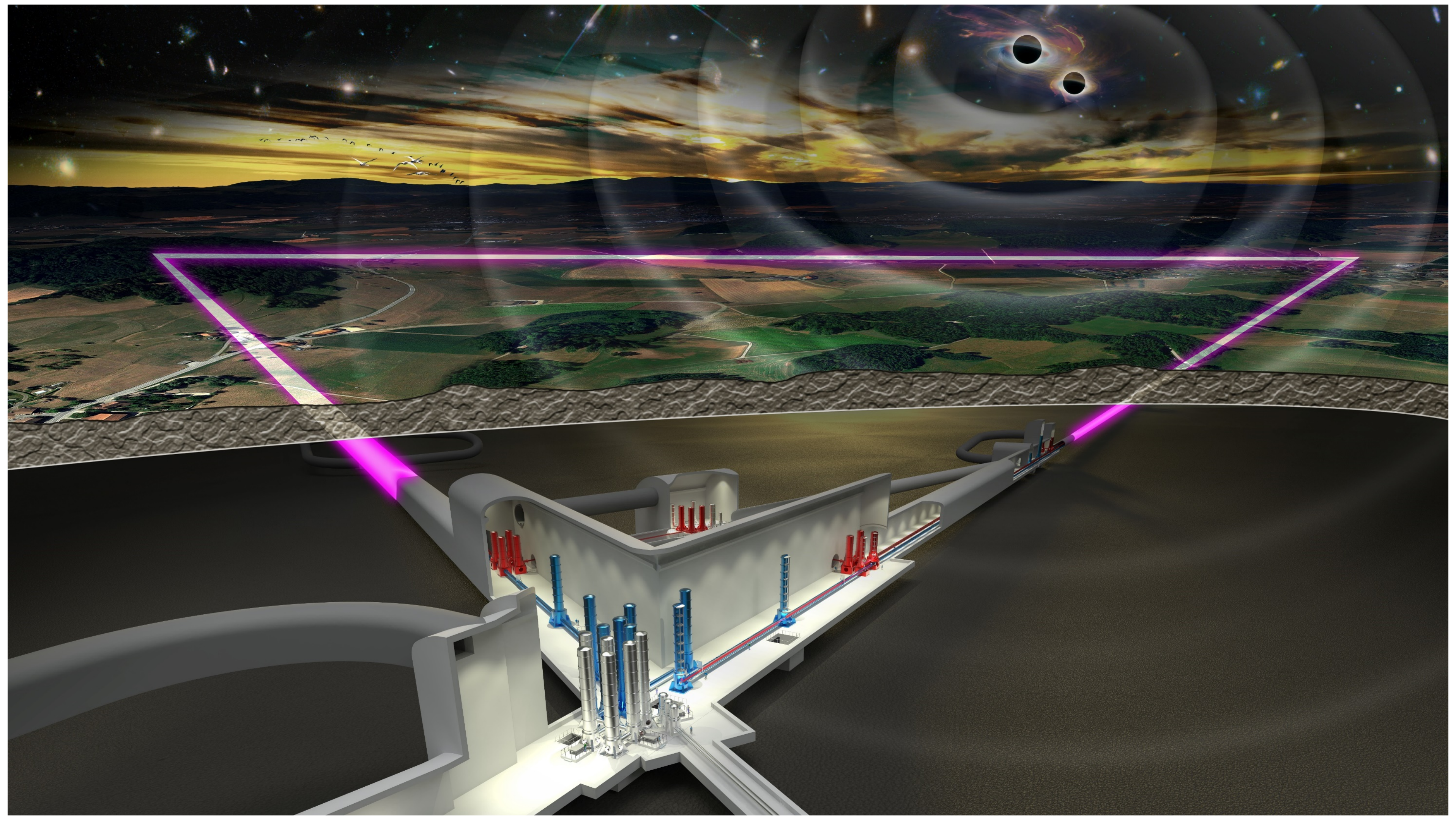
- Why EMRI?



Why EMRI?

Binary system orbital frequency in the inspiral regime (Kepler)

$$\boxed{\omega = \sqrt{G\frac{(m_1 + m_2)}{d^3}}}$$

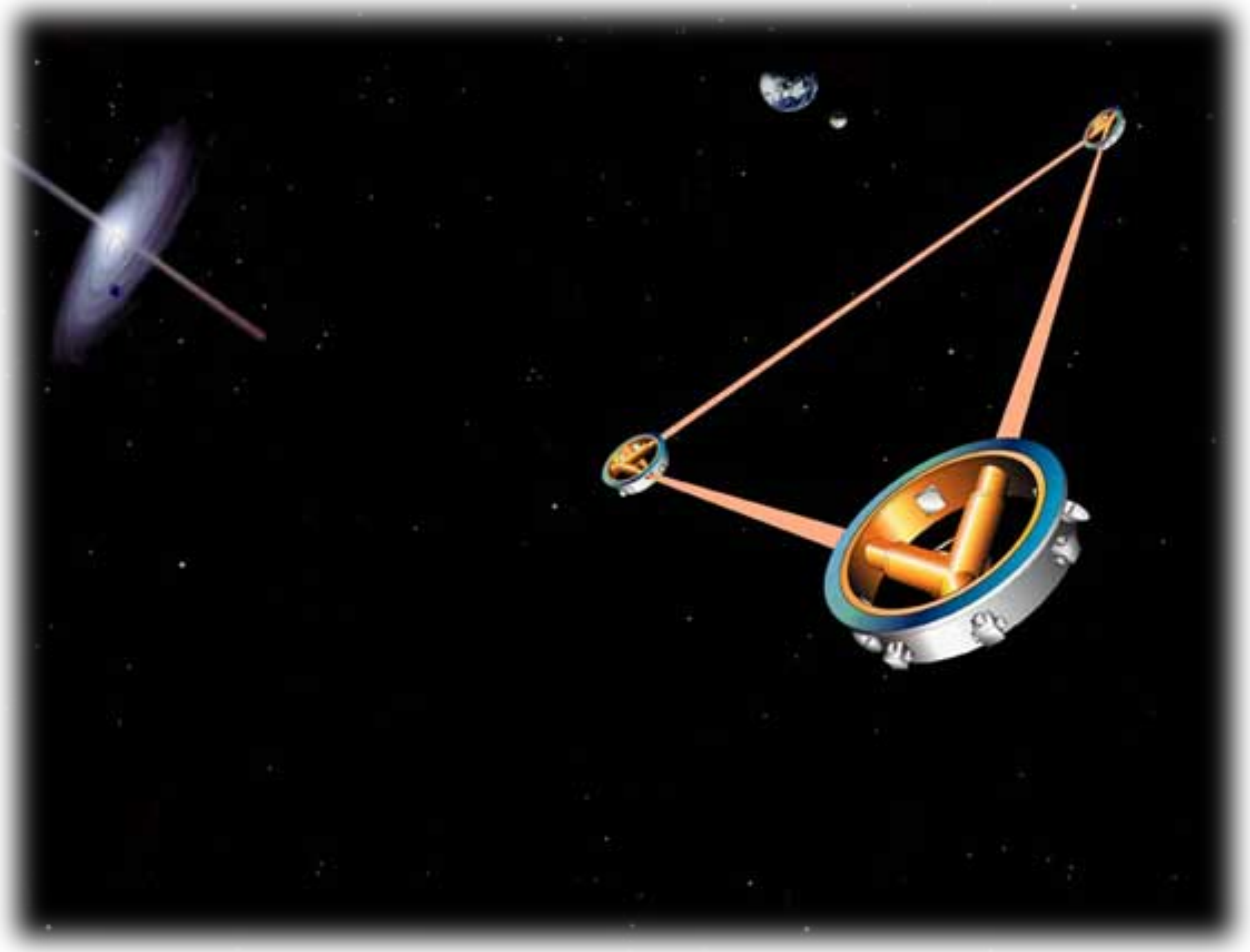


- $\omega \sim 10^{-2} \div 10$ kHz $\longrightarrow$ LIGO-VIRGO

Ex

$$\begin{matrix} m_1 \sim 50\ M_\odot \\ m_2 \sim 50\ M_\odot \end{matrix} \Longrightarrow \qquad \omega \sim 2 \text{ kHZ}$$

- $\omega \sim 10^{-2} \div 10$ Hz : a sensitivity at least 10 times better than the advanced detectors on a large fraction of the detection frequency band, a dramatic improvement in sensitivity in the low frequency range $\longrightarrow$ ET (Einstein Telescope)



- $\omega \sim 0.1 \div 100$ mHz $\longrightarrow$ LISA (Laser Interferometer Space Antenna)

Ex

$$\begin{matrix} m_1 \sim 10^6\ M_\odot \\ m_2 \sim 10\ M_\odot \end{matrix} \Longrightarrow \qquad \omega \sim 10 \text{ mHZ}$$

EMRI dynamics

Coalescence of stellar mass compact objects into SuperMassive Black Holes

$$\boxed{\mu = \frac{m}{M} \sim 10^{-4} \div 10^{-6}}$$

- EMRIs are **primary target** for future GWs observations!

- **Abundance** of sources!

  Stellar-Mass BHs $\longrightarrow 5 \div 10^2\ M_\odot$

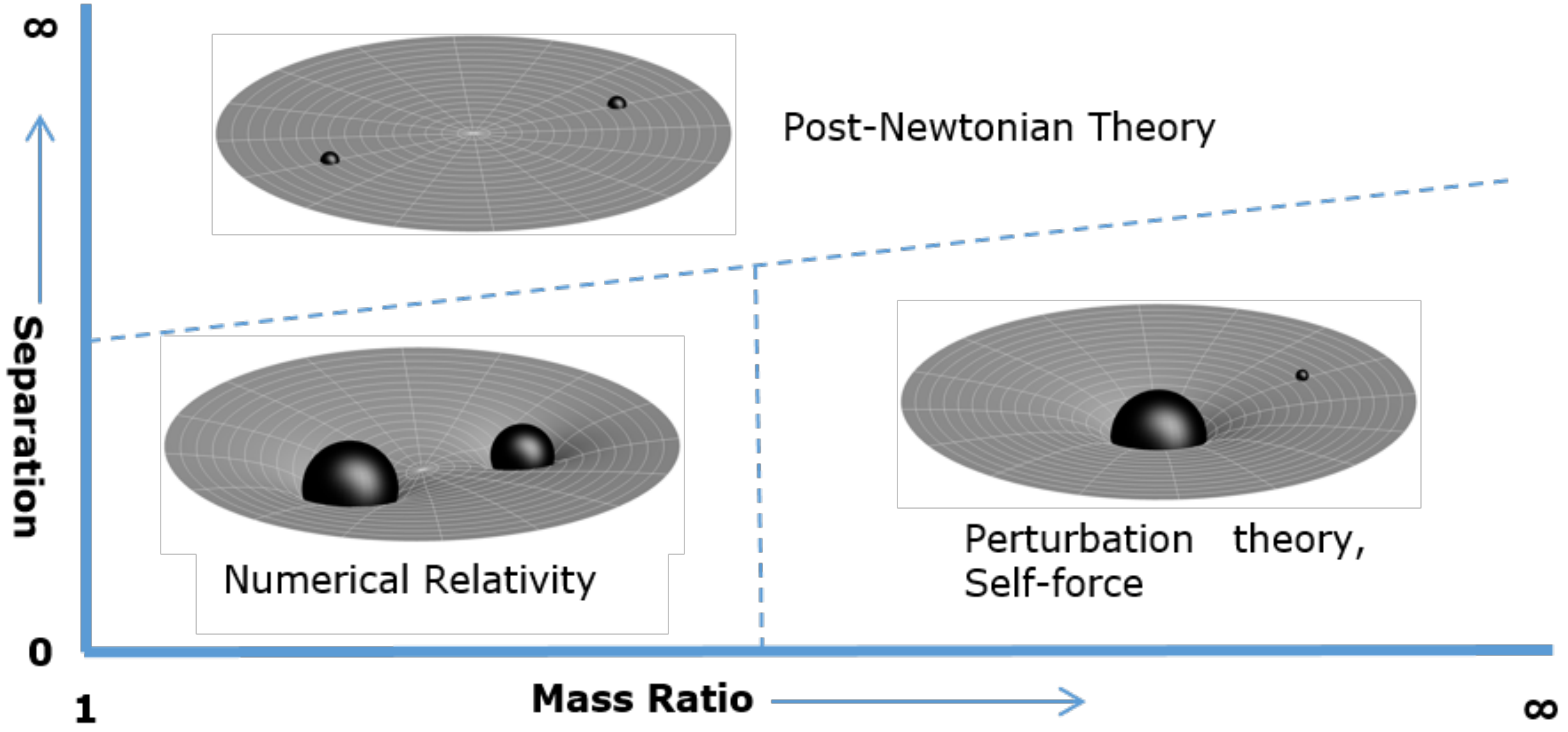
  Intermediate-Mass BHs $\longrightarrow 10^2 \div 10^5\ M_\odot$

  Super Massive BHs $\longrightarrow 10^5 \div 10^9\ M_\odot$

- Difficult to study via Numerical Approaches

- Interesting dynamics!

$\mu = \frac{m}{M} \ll 1 \Longrightarrow$ natural perturbative approaches



SELF-FORCE

Super Massive BH — Stellar-Mass Companion

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \mu\ H_{\mu\nu} + \mathcal{O}(\mu^2) \quad , \quad x^\mu = \bar{x}^\mu + \mu\ \delta x^\mu + \mathcal{O}(\mu)$$

$$\delta\ddot{x}^\mu = \frac{1}{2}(\bar{g}^{\mu\alpha}\dot{\bar{x}}^\beta - 2\bar{g}^{\mu\beta}\dot{\bar{x}}^\alpha - \dot{\bar{x}}^\mu\dot{\bar{x}}^\alpha\dot{\bar{x}}^\beta)\dot{\bar{x}}^\gamma\,\bar{\nabla}_\alpha H_{\beta\gamma} = f^\mu_{\rm sf}$$

$$\Box\,\tilde{H}_{\mu\nu} + 2\bar{R}^{\alpha\ \beta}_{\ \mu\ \nu}\tilde{H}^{\mu\nu} = -\,16\pi\ \mu M \int \frac{\delta^{(4)}(x-\bar{x})}{\sqrt{-\bar{g}}}\dot{\bar{x}}_\mu\dot{\bar{x}}_\nu\ d\tau$$

Outline

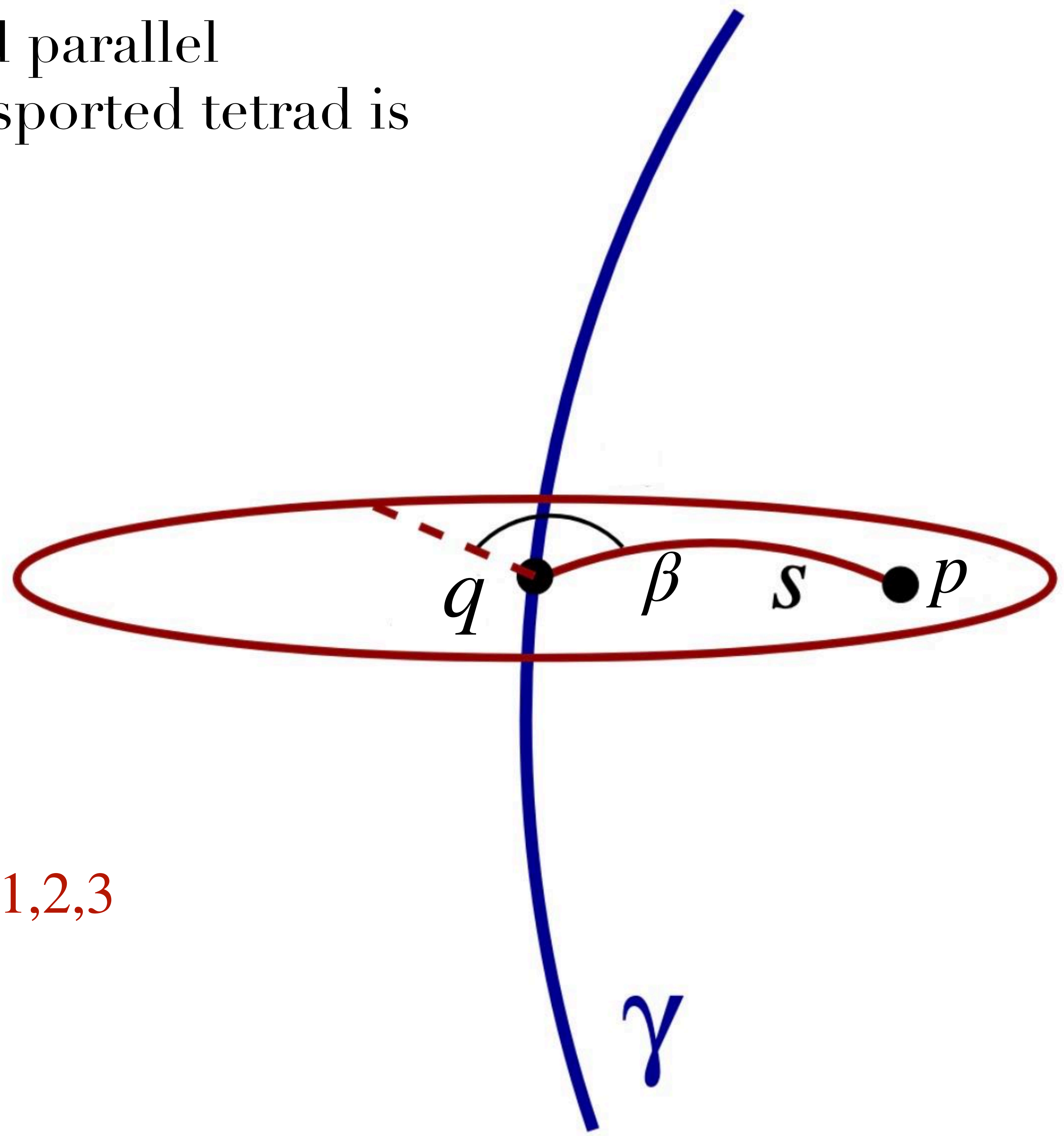
- The EMRI is moving on a geodesic of a Kerr black hole.
- How to construct a metric around a geodesics: Fermi normal coordinates.
- Tidal moments of a Kerr perturberer.
- Secular Hamiltonian.
- Innermost Stable Circular Orbit (ISCO) shifts of conserved quantities.
- Geodesics.
- Conclusions and perspectives.

Fermi normal coordinates

- Consider a vacuum region of spacetime in a neighborhood of a smooth time-like geodesic γ, $x^\mu(\tau)$ with proper time τ.

- The velocity vector $u^\mu = \dfrac{dx^\mu}{d\tau}$ is tangent to the world line. Construct a vectorial basis by adding to u^μ an orthonormal triad $e^\mu_a(\tau)$ of vectors orthogonal to u^μ and parallel transported on the world-line ($a = 1,2,3$). The orthogonal parallel transported tetrad is given by $e^\mu_{(a)} = (u^\mu, e^\mu_1, e^\mu_2, e^\mu_3)$, $(a) = 0,1,2,3$.

- Event p in the neighbourhood of γ. To get the Fermi normal coordinates of p we find the unique space-like geodesic β that intersects γ orthogonally and ends up at p. q is the intersection point of γ and β. The tangent vector v^μ of β at q is orthogonal to the tangent vector u^μ of γ



$$g_{\mu\nu} u^\mu v^\nu = 0 \qquad \text{at } q$$

- The Fermi normal coordinates $\tilde{x}^\mu$ for p are: $\tilde{x}^0 = \tau\ ,\quad \tilde{x}^a = e^a_\mu v^\mu\ ,\ \ a = 1,2,3$

- The geodesic distance between q and p as measured with the space-like curve β is denoted s: $s^2 = \delta_{ab}\tilde{x}^a\tilde{x}^b$

- In terms of Fermi normal coordinates $\tilde{x}^\mu$ the metric close to γ is

$$\tilde{g}_{00} = -1 - \tilde{R}_{0a0b}\tilde{x}^a\tilde{x}^b + \mathcal{O}(s^3)$$
$$\tilde{g}_{0a} = -\frac{2}{3}\tilde{R}_{0bac}\tilde{x}^b\tilde{x}^c + \mathcal{O}(s^3)$$
$$\tilde{g}_{ab} = \delta_{ab} - \frac{1}{3}\tilde{R}_{acbd}\tilde{x}^c\tilde{x}^d + \mathcal{O}(s^3)$$

where the Riemann curvature tensor is evaluated in Riemann normal coordinates on γ.

- Suppose now the background on γ obeys $R_{\mu\nu} = 0$

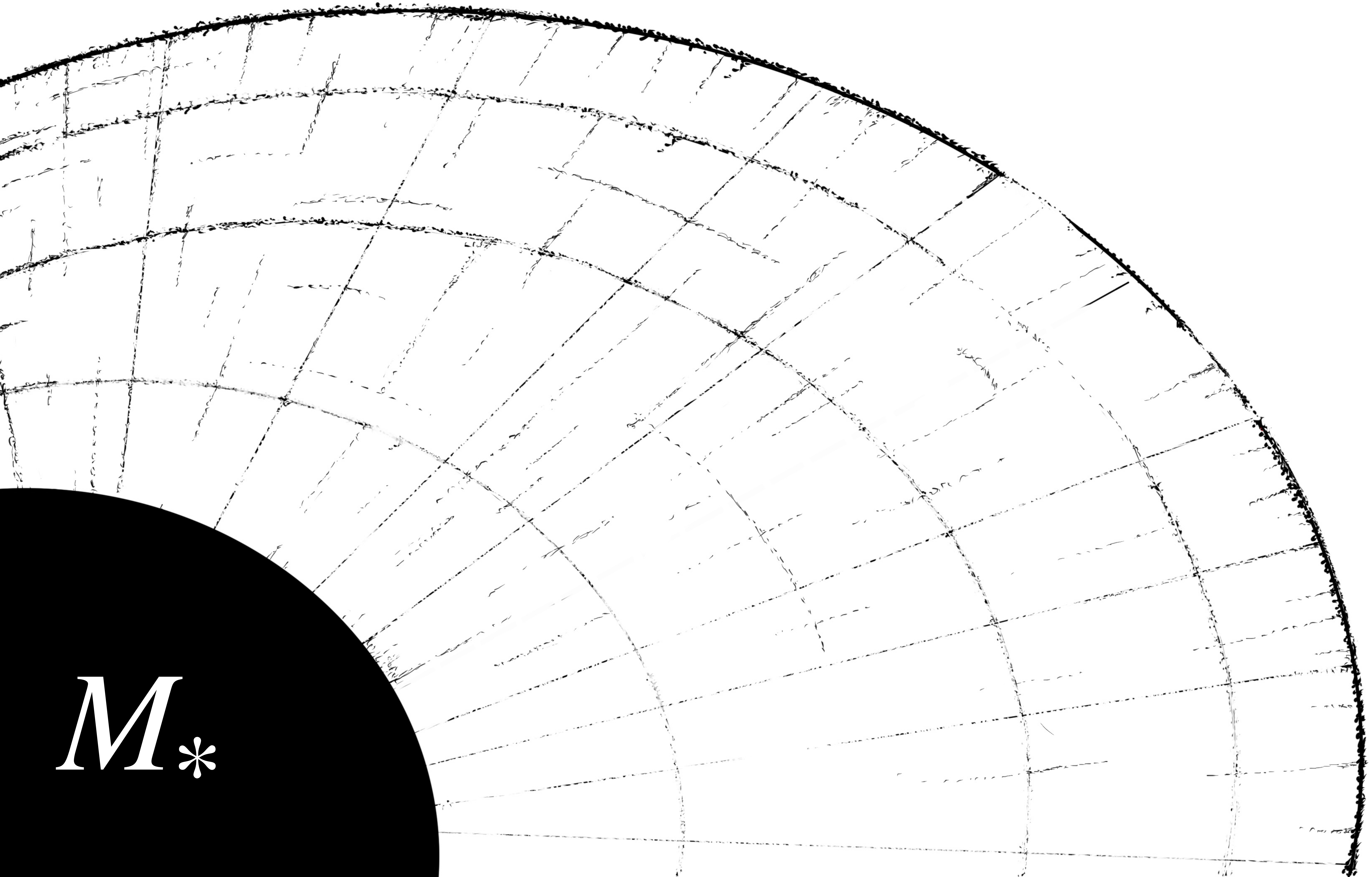
this implies: $\tilde{R}_{acbd} = \delta_{ab}\mathcal{E}_{cd} + \delta_{cd}\mathcal{E}_{ab} - \delta_{ad}\mathcal{E}_{bc} - \delta_{bc}\mathcal{E}_{ad}$

Thorne, Hartle (1985) Poisson (2003)

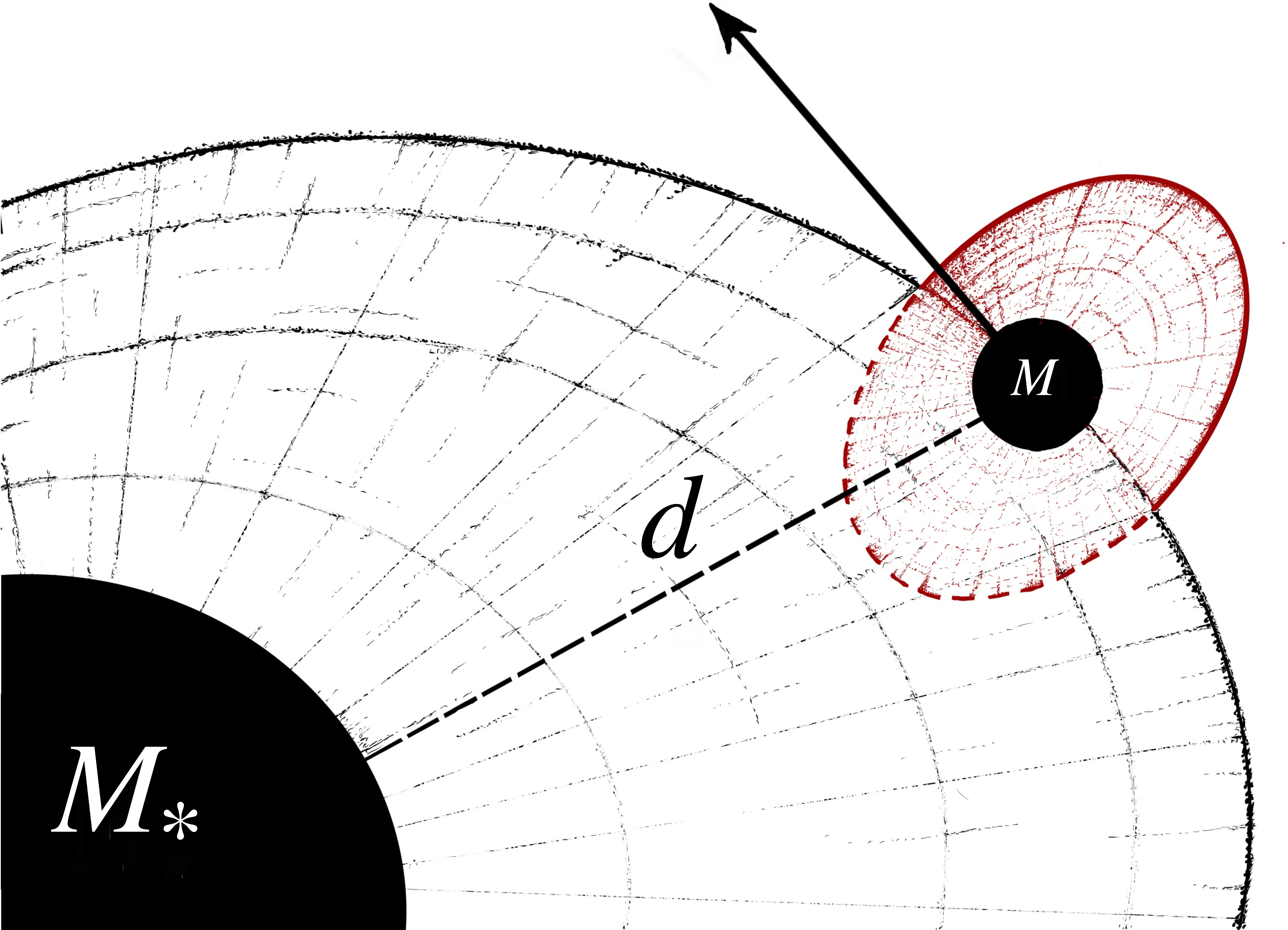
- electric quadrupole moments: $\mathcal{E}_{ab} = \tilde{R}_{0a0b}$
- magnetic quadrupole moments: $\tilde{R}_{0abc} = -\epsilon_{bcd}\mathcal{B}^d{}_a$
- with a further coordinate transformation to the Thorne-Hartle coordinates one can now bring the metric to the form

$$\hat{g}_{00} = -1 - \mathcal{E}_{ab}\hat{x}^a\hat{x}^b + \mathcal{O}(s^3)$$
$$\hat{g}_{0a} = -\frac{2}{3}\epsilon_{abc}\mathcal{B}^b{}_d\hat{x}^c\hat{x}^d + \mathcal{O}(s^3)$$
$$\hat{g}_{ab} = \delta_{ab}(1 - \mathcal{E}_{cd}\hat{x}^c\hat{x}^d) + \mathcal{O}(s^3)$$

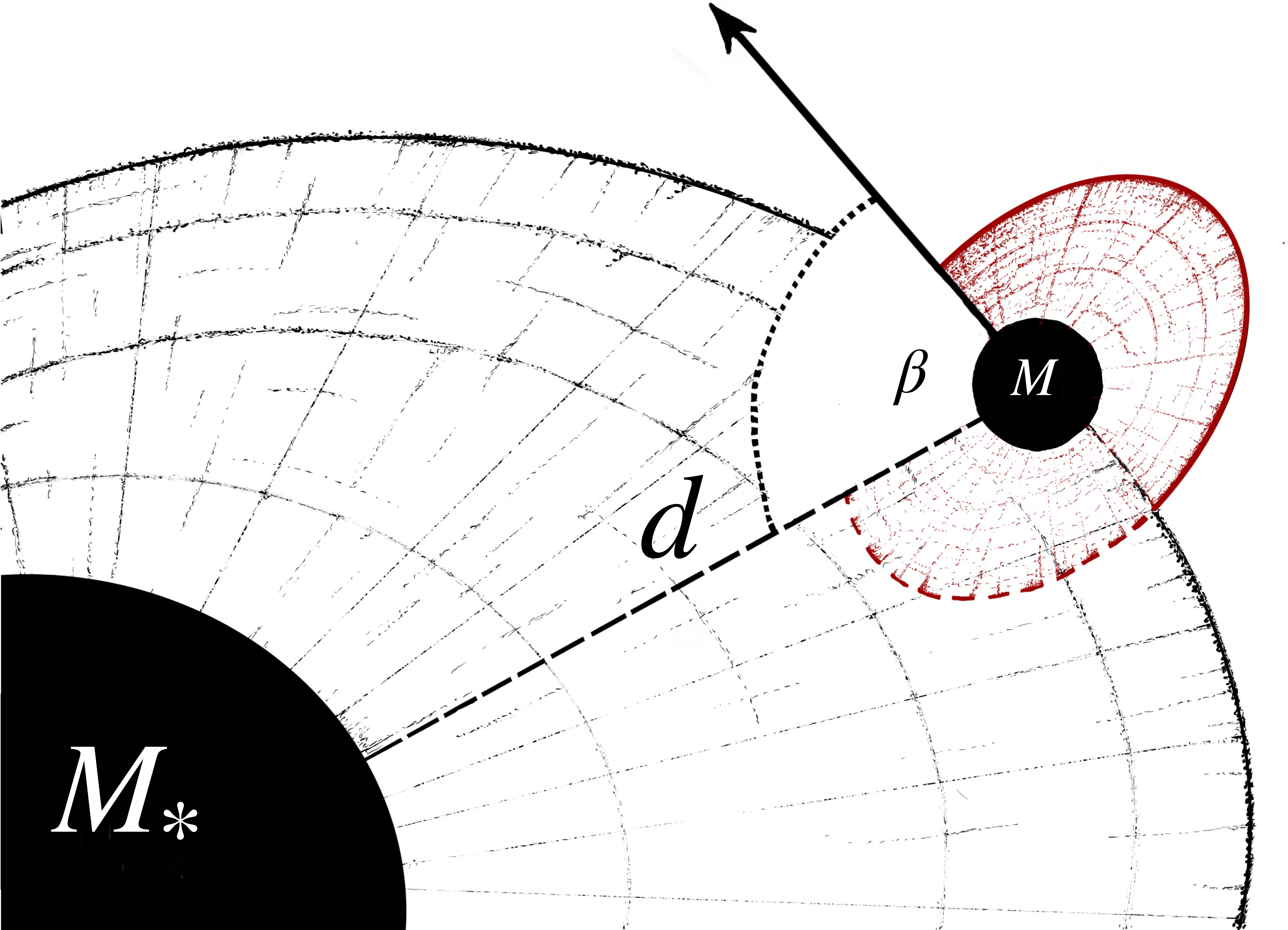
Thorne, Hartle (1985)



M_*



M
d
M_*


β
M
d
M_*

Tidal Scales

- Two different length scales: the mass of the intermediate black hole M and the radius of the curvature generated by the third body evaluated in the position of the binary system $\mathcal{R}$.

- In order to be able to study the physics on the scale of M we need to require that the tidal interaction is weak. This is assured by introducing the **small-tide approximation**:

$$M \ll \mathcal{R}$$

- If d is the distance between the binary system and the big black hole, V is the orbital velocity

$$V \sim \sqrt{\frac{M + M_*}{d}} \quad \text{and} \quad \mathcal{R} \sim \sqrt{\frac{d^3}{M + M_*}} \longrightarrow \frac{M}{\mathcal{R}} \sim \frac{M}{M + M_*} V^3,$$

- $\frac{M}{\mathcal{R}}$ should be a small quantity in order to have a **weak tidal interaction**. To satisfy this condition we will use the **small-hole approximation:**

$$\boxed{M \ll M_*}$$

- In this approximation $\longrightarrow \frac{M}{\mathcal{R}} \ll 1$ independently of the orbital velocity and of the distance between the binary system and the source of the tidal moments.

Tidal Scales

- This allows us to study how the “external spacetime” affects the binary system, not only when $d \to \infty$.

- There is another way to satisfy the condition $M \ll \mathscr{R}$: the **weak-field approximation** the intermediate black hole and the source of the tidal moments can have comparable masses but the orbital velocity must be small. This is true when the binary system is far away from the supermassive black hole $d \to \infty$.

Yang and Casals (2017)

- In the tidal approximation the metric which we use to describe the tidally deformed black hole is expressed as a power expansion of $s \ll \mathscr{R}$, with s being the distance from the black hole M. This metric is computed near the perturbed black hole and it is valid to all orders in $\dfrac{s}{M}$.

- Here we are only interested in the quadrupole order of the perturbed metric, meaning that we will consider only the quadrupole moments $\mathscr{E}_{ab}$ and $\mathscr{B}_{ab}$ which appears at order $\mathscr{O}\left(\dfrac{s}{\mathscr{R}}\right)^2$.

Deformed metric at the EMRI

- Construct a deformed metric for a Schwarzschild black hole of mass M locally in a general (external) spacetime.

- The tidal deformations are sourced by a third body in the external space time: a Kerr black hole of mass $M_* \gg M$.

- The Schwarzschild black hole is approximately parallel-transported along one of the Kerr geodesics and the locally deformed metric is that of a non-rotating black hole corrected with tidal multipole moments incorporating all the informations about the external spacetime.

- The metric of any vacuum spacetime can be constructed in the neighborhood of any geodesic world line and expressed in term of two sets of tidal multiple moments.

Zhang (1986) Poisson, Vlasov (2010)

- Construction in terms of Fermi normal coordinates.

- The velocity vector $u^\alpha := \dfrac{dx^\alpha}{d\tau}$ is tangent to the world line and we construct a vectorial basis by adding to u^α an orthonormal triad $e^\alpha_a(\tau)$ of vectors which we assume to be orthogonal to u^α and parallel transported on the world line. $(a = 1{,}2{,}3)$. The orthogonal parallel transported tetrad is given by $e^\alpha_{(a)} = (u^\alpha, e^\alpha_1, e^\alpha_2, e^\alpha_3)$, $(a) = 0{,}1{,}2{,}3$.

- $e^\alpha_{(a)} = (u^\alpha, e^\alpha_1, e^\alpha_2, e^\alpha_3)$, $(a) = 0{,}1{,}2{,}3$ for Kerr, constructed by Marck.

Marck (1983)

- The tidal moments are defined through the Weyl tensor

$$\boxed{C_{\rho\sigma\mu\nu} \equiv R_{\rho\sigma\mu\nu} - \left(g_{\rho[\mu}R_{\nu]\sigma} - g_{\sigma[\mu}R_{\nu]\rho}\right) + \frac{1}{3}g_{\rho[\mu}g_{\nu]\sigma}R}$$

once it is evaluated on the Kerr geodesic

$$\boxed{\begin{aligned} C_{ab} &\equiv C_{a0b0} = C_{\rho\sigma\mu\nu}\omega^{\rho}{}_{(a)}\omega^{\sigma}{}_{(b)}\omega^{\mu}{}_{(c)}\omega^{\nu}{}_{(d)}e^{(a)}_{a}e^{(b)}_{0}e^{(c)}_{b}e^{(d)}_{0} \\ C_{abc} &\equiv C_{abc0} = \omega^{\rho}{}_{(a)}\omega^{\sigma}{}_{(b)}\omega^{\mu}{}_{(c)}\omega^{\nu}{}_{(d)}e^{(a)}_{a}e^{(b)}_{b}e^{(c)}_{c}e^{(d)}_{0} \end{aligned}}$$

- where $\omega^{(a)}_{\mu}$ is the **Carter tetrad** for the Kerr space-time.
- The symmetries of the Weyl tensor imply that it posses 10 algebraically independent components and these can be encoded in the two symmetric-trace-free (STF) tensors

$$\boxed{\begin{aligned} \mathscr{E}_{ab} &:= \left(C_{a0b0}\right)^{\mathrm{STF}} \\ \mathscr{B}_{ab} &:= \frac{1}{2}\left(\epsilon_{acd}C^{cd}{}_{b0}\right)^{\mathrm{STF}} \end{aligned}}$$

Each STF tensor contains 5 independent components

- We refer to $\mathscr{E}_{ab}$ and $\mathscr{B}_{ab}$ as the **tidal quadrupole moments** associated with the world line γ .

Tidal moments of a Kerr perturberer

Kerr metric in Boyer-Lindquist coordinates $\hat{x}^\mu = (\hat{t}, \hat{r}, \hat{\theta}, \hat{\phi})$ for a black hole of mass M_* and specific angular momentum $a = J_*/M_*$

$$d\hat{s}^2 = -\left(1 - \frac{2M_*\hat{r}}{\Sigma}\right)d\hat{t}^2 - \frac{4M_*\hat{r}}{\Sigma}a\sin^2\hat{\theta}\;d\hat{t}\;d\hat{\phi} + \frac{\mathscr{A}}{\Sigma}\sin^2\hat{\theta}\;d\hat{\phi}^2 + \frac{\Sigma}{\Delta}d\hat{r}^2 + \Sigma d\hat{\theta}^2$$

$$\Sigma = \hat{r}^2 + a^2\cos^2\hat{\theta}, \quad \Delta = \hat{r}^2 - 2M_*\hat{r} + a^2, \quad \mathscr{A} = (\hat{r}^2 + a^2)^2 - a^2\Delta\sin^2\hat{\theta}$$

dimensionless spin parameter:

$$\alpha = \frac{a}{M_*}\,, \qquad 0 \leq \alpha \leq 1$$

the coordinate singularities for the Kerr metric are located at the zeroes of the function Δ

$$\Delta = (\hat{r} - \hat{r}_+)(\hat{r} - \hat{r}_-) \longrightarrow \hat{r}_\pm = M_*\left(1 \pm \sqrt{1 - \alpha^2}\right)$$

Geodesics in a Kerr geometry

- Specified by three constants of motion: the energy per unit mass $\hat{E}$, the angular momentum per unit mass $\hat{L}$ and the Carter constant $\hat{K}$

$$\dot{\hat{t}} = \frac{\mathscr{A}\hat{E} - 2M_*\hat{r}a\hat{L}}{\Delta\Sigma},$$

$$\dot{\hat{r}}^2 = \left[\frac{\hat{E}(\hat{r}^2 + a^2) - a\hat{L}}{\Sigma}\right]^2 - \frac{\Delta}{\Sigma^2}(\hat{r}^2 + \hat{K}),$$

$$\dot{\hat{\theta}}^2 = \frac{1}{\Sigma^2}\left[\hat{K} - a^2\cos\hat{\theta} - \left(a\hat{E}\sin\hat{\theta} - \frac{\hat{L}}{\sin\hat{\theta}}\right)^2\right]$$

$$\dot{\hat{\phi}} = \frac{2aM_*\hat{r}}{\Sigma}\frac{\hat{E}}{\Delta} + \left(1 - \frac{2M_*\hat{r}}{\Sigma}\right)\frac{\hat{L}}{\Delta\sin^2\hat{\theta}}$$

A particular solution is given by **circular time-like geodesics in the Kerr equatorial plane**. Set $\hat{r} = d$, $\dot{\hat{r}} = 0$, $\hat{\theta} = \pi/2$ and $\dot{\hat{\theta}} = 0$

$$\hat{L} = \frac{\sigma\left(d^2 + \alpha\left(\alpha M_*^2 - 2\sigma\sqrt{dM_*^3}\right)\right)}{d^{3/4}\sqrt{\frac{\sqrt{d}(d - 3M_*)}{M_*} + 2\alpha\sqrt{M_*}\sigma}}$$

$$\hat{E} = \frac{d^{3/2} - 2\sqrt{d}M_* + \alpha M_*^{3/2}\sigma}{d^{3/4}\sqrt{d^{3/2} - 3\sqrt{d}M_* + 2\alpha M_*^{3/2}\sigma}}$$

$$\hat{K} = (\alpha M_*\hat{E} - \hat{L})^2$$

Bardeen, Press, Teukolsky (1972)

- **Innermost Stable Circular Orbit (ISCO)** in the Kerr equator

$$\hat{r}^{\sigma}_{\text{isco}} = M_*\left[3 + Z_2 - \sigma\sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}\right], \qquad Z_1 = 1 + (1-\alpha)^{1/3}\left[(1+\alpha)^{1/3} + (1-\alpha)^1/3\right], \quad Z_2 = \sqrt{3\alpha^2 + Z_1^2}$$

- $\sigma = \pm 1$ distinguishes two different ISCOs, one which is co-rotating ($\hat{r}^+_{\text{isco}}$) with respect to the black hole and the other which is counter-rotating ($\hat{r}^-_{\text{isco}}$) .

Tidal moments in Kerr

- The construction of the tidal multipole moments stems from the identification of a local orthonormal tetrad which is designed to be an inertial frame paralleled transported along the Kerr geodesic motion.

Marck (1983)

- The tetrad considerably simplifies for **circular equatorial geodesics** in Kerr $\hat{r} = d$, $\dot{\hat{r}} = 0$, $\hat{\theta} = \pi/2$ and $\dot{\hat{\theta}} = 0$

$$e^{\mu}_{(0)} = \frac{1}{d\sqrt{\Delta}}\left(\hat{E}(d^2 + a^2) - a\hat{L}\right)\delta^{\mu}_{0} + \frac{\left(a\hat{E} - \hat{L}\right)}{d}\delta^{\mu}_{3},$$

$$e^{\mu}_{(3)} = \sqrt{\frac{1}{\hat{K}}}\left(\hat{L} - a\hat{E}\right)\delta^{\mu}_{2},$$

$$e^{\mu}_{(1)} = \tilde{e}^{\mu}_{(1)}\cos\Psi + \tilde{e}^{\mu}_{(2)}\sin\Psi,$$

$$e^{\mu}_{(2)} = \tilde{e}^{\mu}_{(1)}\sin\Psi + \tilde{e}^{\mu}_{(2)}\cos\Psi$$

$$\tilde{e}^{\mu}_{(1)} = \frac{\left(\hat{E}(d^2 + a^2) - a\hat{L}\right)}{\sqrt{\left(d^2 + \hat{K}\right)\Delta}}\delta^{\mu}_{1}, \qquad \hat{K} = \left(a\hat{E} - \hat{L}\right)^2$$

$$\tilde{e}^{\mu}_{(2)} = \sqrt{\frac{\hat{K}}{\left(d^2 + \hat{K}\right)\Delta}}\,\frac{\left(\hat{E}(d^2 + a^2) - a\hat{L}\right)}{d}\delta^{\mu}_{0} + \sqrt{\frac{d^2 + \hat{K}}{\hat{K}}}\,\frac{\left(a\hat{E} - \hat{L}\right)}{d}\delta^{\mu}_{3}$$

- Ψ is an angle depending on the proper time along the Kerr geodesic, which is necessary to introduce in order to ensure that the tetrad $(e^{\mu}{}_{(0)}, e^{\mu}{}_{(1)}, e^{\mu}{}_{(2)}, e^{\mu}{}_{(3)})$ is parallel-transported by the geodesic motion

- This co-moving cartesian frame can be interpreted as a coordinate system centered in the Schwarzschild black hole of mass M.

Weyl tensor in the comoving frame

- Tidal perturbations affect the motion of a test particle around the Schwarzschild black hole. They represent the first order corrections to Extreme Mass Ratio binary systems in the presence of tidal fields. The metric felt by the test particle has the form:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \dots \qquad \text{where } \bar{g}_{\mu\nu} \text{ is the Schwarzschild metric of the mass } M \text{ black-hole}$$

- The metric perturbations $h_{\mu\nu}$ are written in terms of the tidal multipole moments and which represent the tidal deformation sourced by the Kerr black hole of mass M_*.
- Specializing to Kerr geodesics in the equatorial plane $\hat{\theta} = \pi/2$ the explicit expressions for the components of the Weyl tensor considerably simplify

$$C_{11} = \left[1 - 3\left(1 + \frac{\hat{K}}{d^2}\right)\cos^2\Psi\right]\frac{M_*}{d^3},$$

$$C_{22} = \left[1 - 3\left(1 + \frac{\hat{K}}{d^2}\right)\sin^2\Psi\right]\frac{M_*}{d^3},$$

$$C_{12} = -3\left(1 + \frac{\hat{K}}{d^2}\right)\frac{M_*}{d^3}\cos\Psi\sin\Psi,$$

$$C_{33} = \left(1 + 3\frac{\hat{K}}{d^2}\right)\frac{M_*}{d^3}$$

$$C_{121} = -C_{112} = C_{332} = -C_{323} = -\frac{3M_*\sqrt{\hat{K}}}{d^4}\sqrt{1 + \frac{\hat{K}}{d^2}}\cos\Psi,$$

$$C_{221} = -C_{212} = C_{313} = -C_{331} = -\frac{3M_*\sqrt{\hat{K}}}{d^4}\sqrt{1 + \frac{\hat{K}}{d^2}}\sin\Psi,$$

$$\dot{\Psi} = \frac{A}{d^2 + \hat{K}}, \qquad A = \hat{E}\sqrt{\hat{K}} - \frac{a\sqrt{\hat{K}}}{a\hat{E} - \hat{L}}$$

Marck (1983) Alvi (1999)

- The quadrupole **electric** and **magnetic moments** in cartesian coordinates are then defined as

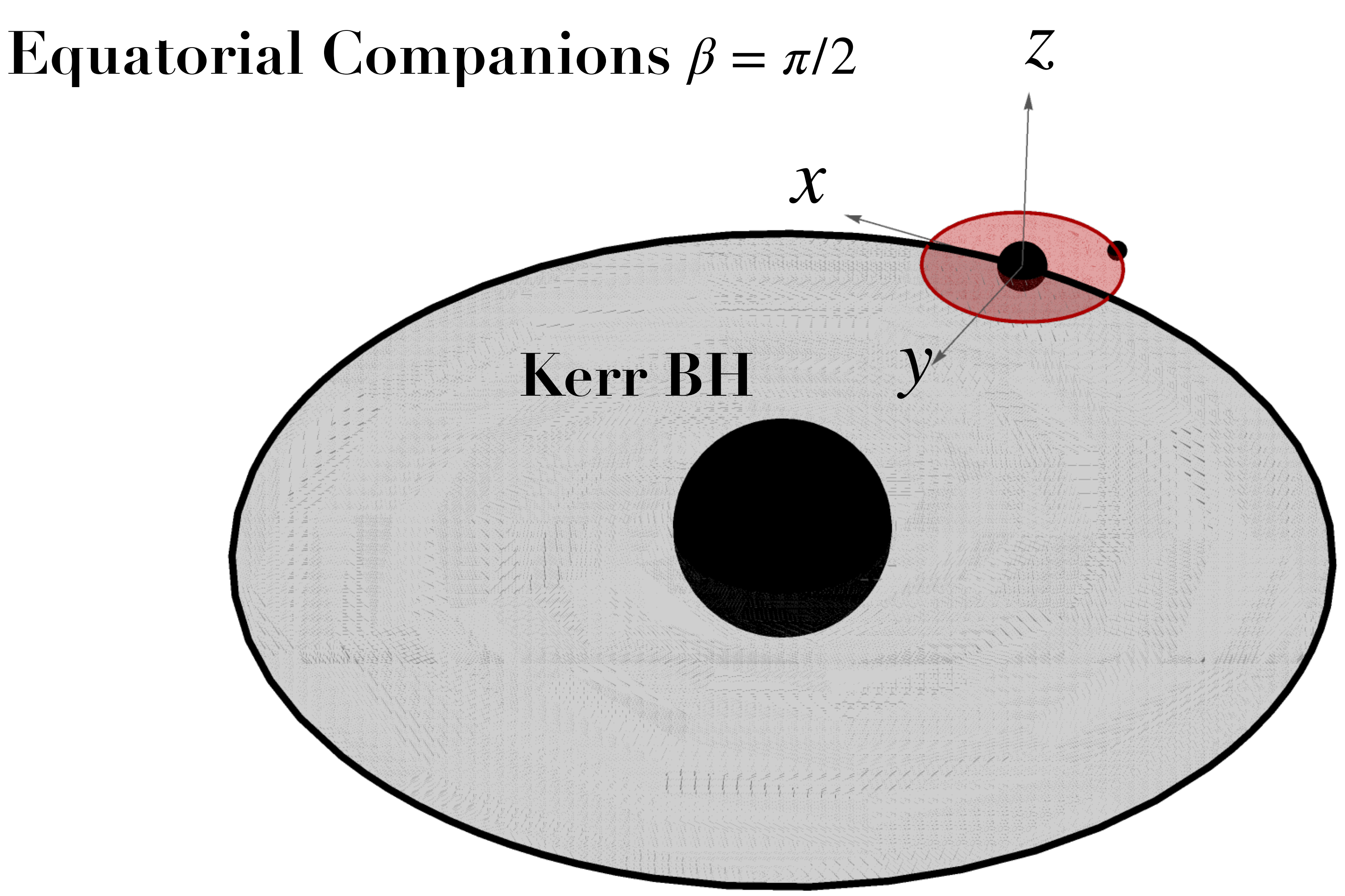
$$\mathscr{E}_{ab} = C_{<ab>}, \quad \mathscr{B}_{ab} = \frac{1}{2}\epsilon_{<a|pq}C^{pq}_{\ |b>},$$

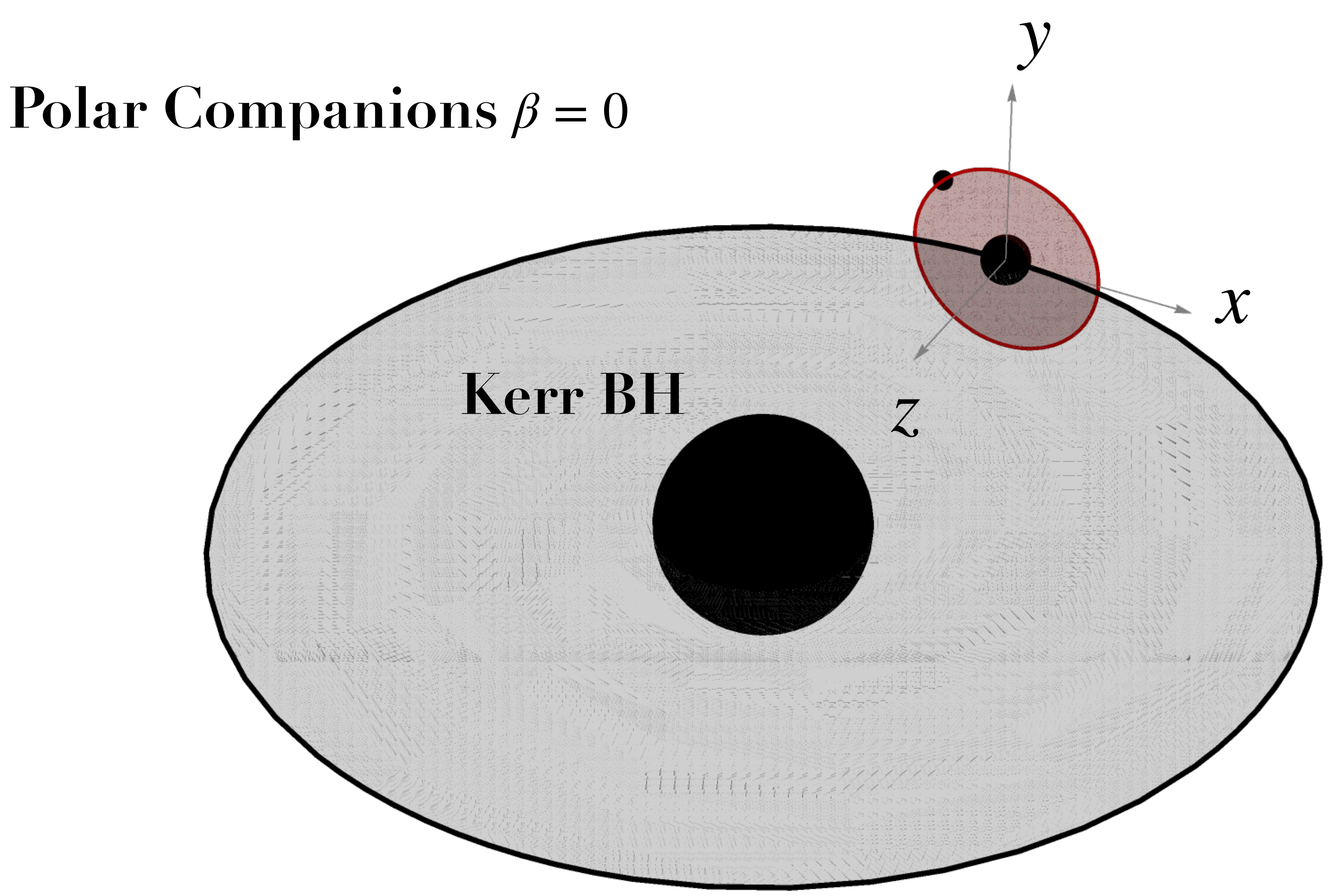
- If one aims to study a binary system around a Schwarzschild black hole influenced by the presence of tidal fields, the problem is more conveniently addressed in spherical coordinates.

- Convert the tidal moments expressions from cartesian coordinates (x_0, x_1, x_2, x_3) to spherical coordinates (t, r, θ, ϕ). The latter will be later interpreted as the spherical coordinates of a Schwarzschild black hole.

- The conversion from cartesian to spherical allows one to distinguish different orientations for the binary system with respect to the source of the tidal field. Assuming that x_3 is the coordinate along the direction orthogonal to the equatorial plane of Kerr, if $x_3 = r\cos\theta$ we get **equatorial companions** whereas the configuration for which $x_1 = r\cos\theta$ is called **polar companions**

Cardoso, Foschi (2021)





Spherical coordinates

- It is convenient to introduce the **unit radial vector** Ω^a, oriented in the $\hat{r}$-direction and written in Cartesian components.
- In order to study all possible orientations for the binary system around the Kerr black hole, we introduce a rotation angle β:

$$R = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}$$

  - we define the unit radial vector Ω^a as

$$\Omega^a = R\ \Omega^a_* \ , \qquad \Omega^a_* = (\cos\theta, \sin\phi\sin\theta, -\sin\theta\cos\phi)$$

  - Our choice is such that the **polar case** is recovered for $\beta = 0$ ($x_1/r = \cos\theta$) whereas the **equatorial case** is for $\beta = \pi/2$ ($x_3/r = \cos\theta$)
  - A projector onto the space orthogonal to $\hat{r}$ can be defined according to

$$\gamma^{ab} = \eta^{ab} - \Omega^a\Omega^b, \quad a, b = 1{,}2{,}3$$

  - decompose C_{ab} and C_{abc} in terms of irreducible representations of $SO(3)$

$$C_{ab} = \mathscr{E}^q\Omega_a\Omega_b + 2\mathscr{E}^q_{(a}\Omega_{b)} - \frac{1}{2}\mathscr{E}^q\gamma_{ab} + \frac{1}{2}\mathscr{E}_{<ab>}$$

- the scalar and the transverse vector components are defined respectively as

$$\mathscr{E}^q \equiv C_{ab}\Omega^a\Omega^b = -\, C_{ab}\gamma^{ab},$$
$$\mathscr{E}^q_a \equiv C_{cd}\gamma^c_a\Omega^d,$$

- $\mathscr{E}_{<ab>}$ labels the STF tensor in the (θ, ϕ) subspace, defined as

$$\mathscr{E}_{<ab>} \equiv 2\gamma^c_a\gamma^d_b C_{cd} - C_{cd}\gamma^{cd}\gamma_{ab} = 2\gamma^c_a\gamma^d_b C_{cd} + \mathscr{E}^q\gamma_{ab}$$

- Once converted into spherical coordinates, these $\mathscr{E}$-quantities provide the **tidal moments** defining the metric $h_{\mu\nu}$ in coordinates $x^A = (r, \theta, \phi)$.

$$\mathscr{E}^q_a dx^a = \frac{\partial x^a}{\partial x^A}\mathscr{E}^q_a dx^A = \mathscr{E}_\theta(rd\theta) + \mathscr{E}_\phi(rd\phi),$$
$$\mathscr{E}^q_{<ab>}dx^a \otimes dx^b = \frac{\partial x^a}{\partial x^A}\frac{\partial x^b}{\partial x^B}\mathscr{E}^q_{<ab>}dx^A \otimes dx^B = \mathscr{E}_{\theta\theta}(rd\theta)^2 + 2\mathscr{E}_{\theta\phi}r^2 d\theta d\phi + \mathscr{E}_{\phi\phi}(rd\phi)^2,$$

- Similar considerations apply for the case of C_{abc}, leading to the magnetic multipole moments $\mathscr{B}_a$ and $\mathscr{B}_{ab}$.

Tidal deformations on the equatorial plane

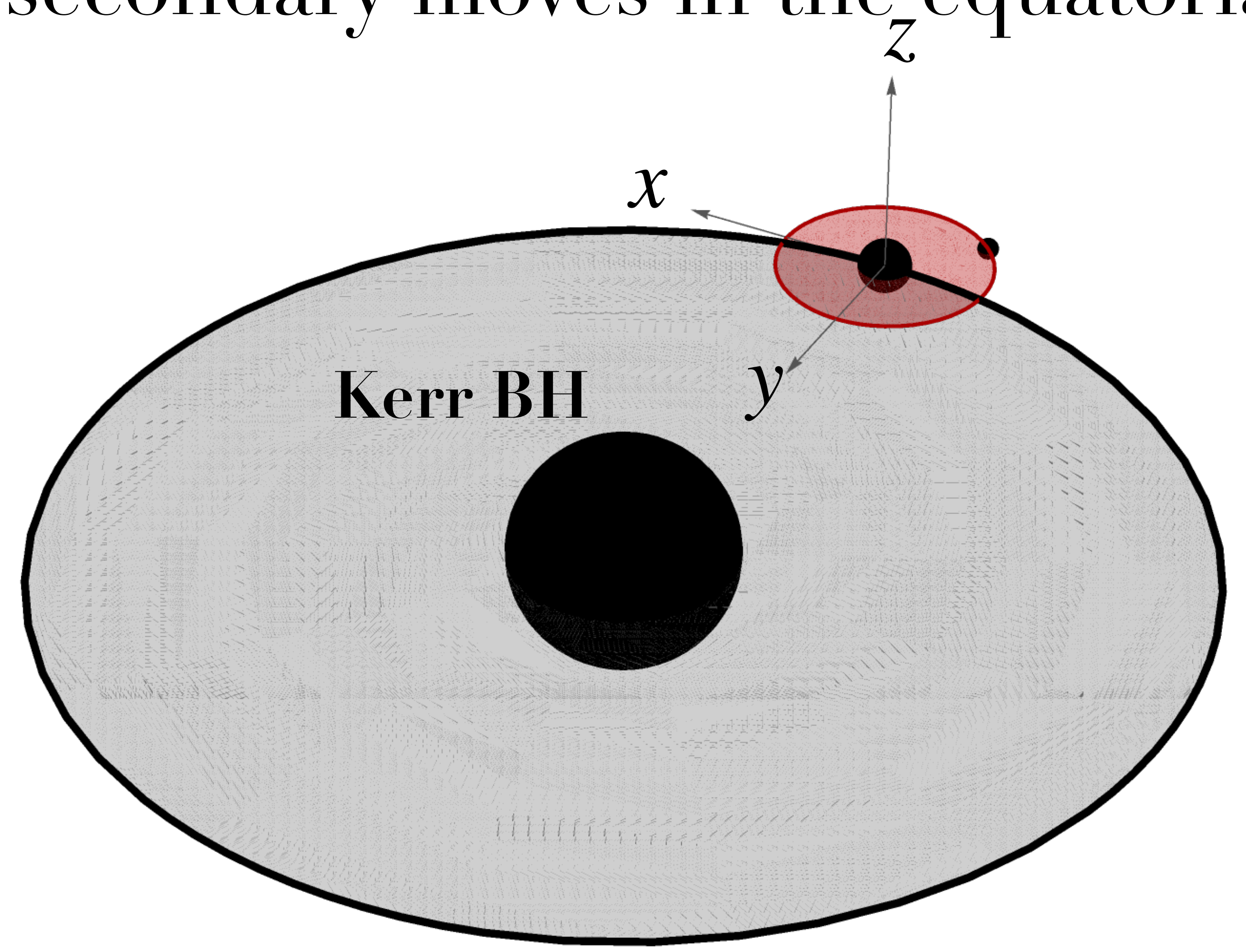
- We now consider the effect of tidal deformation as measured in a local inertial frame which is parallel transported along a circular equatorial geodesic around the Kerr perturber.

- Ψ is a rotation angle introduced by Marck (1983) in order to parallel-transport the tetrad $e^{\mu}_{(a)}$ along the big geodesic. $\dot{\Psi}$ is a constant, $\Psi = const. \times \tau$, where τ is the proper time of the big geodesic around the Kerr black hole.

- In the static approximation we are neglecting the motion of the source of the tidal moments, the timescale associated to the motion of the binary system along the geodesics of the Kerr black hole is much bigger than the one associated to the motion of the test particle around the intermediate black hole. For this reason and because we are interested in the geodesics of the binary system, in the static approximation, we can set $\Psi = 0$.

- For simplicity, we consider a Kerr equatorial geodesic $\hat{\theta} = \pi/2$.

- We analyze the orbital dynamics of a binary system whose secondary moves in the equatorial plane of the Schwarzschild black hole, $\theta = \pi/2$.



Tidal moments for $\theta = \hat{\theta} = \pi/2$

$$\mathcal{E}_q = \frac{M_*}{4d^3}\left(\frac{6q_1\cos 2\beta\cos^2\phi}{q_2} - 3\cos 2\phi + 1\right)$$

$$\mathcal{E}_\theta = -\frac{3M_*}{2d^3}\frac{q_1\sin 2\beta\cos\phi}{q_2}$$

$$\mathcal{E}_\phi = \frac{3M_*}{4d^3}\sin 2\phi\left(1 - \frac{q_1\cos 2\beta}{q_2}\right)$$

$$\mathcal{E}_{\theta\theta} = \frac{3M_*}{4d^3}\left(\frac{q_1\cos 2\beta(\cos 2\phi - 3)}{q_2} - 2\cos^2\phi\right)$$

$$\mathcal{E}_{\theta\phi} = \frac{3M_*}{d^3}\frac{q_1\sin 2\beta\sin\phi}{q_2}$$

$$\mathcal{E}_{\phi\phi} = -\mathcal{E}_{\theta\theta} = -\frac{3M_*}{4d^3}\left(\frac{q_1\cos 2\beta(\cos 2\phi - 3)}{q_2} - 2\cos^2\phi\right)$$

$$\mathcal{B}_\theta = -\frac{3M_*}{4d^3q_2}\left(\sin 2\beta\sin 2\phi\sqrt{(q_1-q_2)(q_1+q_2)}\right)$$

$$\mathcal{B}_\phi = -\frac{3M_*}{2d^3q_2}\left(\cos 2\beta\cos\phi\sqrt{(q_1-q_2)(q_1+q_2)}\right)$$

$$\mathcal{B}_{\theta\theta} = -\frac{3M_*}{d^3q_2}\left(\cos 2\beta\sin\phi\sqrt{(q_1-q_2)(q_1+q_2)}\right)$$

$$\mathcal{B}_{\theta\phi} = -\frac{3M_*}{4d^3q_2}\left(\sin 2\beta(\cos 2\phi - 3)\sqrt{(q_1-q_2)(q_1+q_2)}\right)$$

$$\mathcal{B}_{\phi\phi} = -\mathcal{B}_{\theta\theta} = \frac{3M_*}{d^3q_2}\left(\cos 2\beta\sin\phi\sqrt{(q_1-q_2)(q_1+q_2)}\right)$$

$$q_1 = d^2 - 2\alpha\sqrt{d}M_*^{3/2}\sigma - dM_* + 2\alpha^2M_*^2$$

$$q_2 = 2\alpha\sqrt{d}M_*^{3/2}\sigma + d(d - 3M_*)$$

- Depend on d, α, σ, M_*, parameters in the Kerr spacetime and β which specify the binary system configuration

$\beta \in [0,\pi/2]$, $\sigma = \pm 1$,
$\alpha \in [0,1]$ and $d \in [r_{\mathrm{isco}^+}, \infty]$

Tidal moments for $d \gg M_*$

- In this limit: $$\frac{q_1}{q_2} = 1 + \mathcal{O}\left(\frac{\sqrt{M_*}}{\sqrt{d}}\right) \quad , \quad \frac{\sqrt{(q_1 - q_2)(q_1 + q_2)}}{q_2} = \mathcal{O}\left(\frac{\sqrt{M_*}}{\sqrt{d}}\right)$$

- all the magnetic moments are sub-leading and the tidal effects are only induced by the presence of the electric quadrupole moments

$$\begin{aligned}
\mathscr{E}_q &= \epsilon\ \tilde{\mathscr{E}}_q = \frac{\epsilon}{M^2}(1 - 3\sin^2\beta\cos^2\phi), \\
\mathscr{E}_\theta &= \epsilon\ \tilde{\mathscr{E}}_\theta = -3\frac{\epsilon}{M^2}\cos\beta\sin\beta\cos\phi, \\
\mathscr{E}_\phi &= \epsilon\ \tilde{\mathscr{E}}_\phi = 3\frac{\epsilon}{M^2}\sin^2\beta\cos\phi\sin\phi, \\
\mathscr{E}_{\theta\theta} &= \epsilon\ \tilde{\mathscr{E}}_{\theta\theta} = -\mathscr{E}_{\phi\phi} = -3\frac{\epsilon}{M^2}\left[1 - \sin^2\beta(1 + \sin^2\phi)\right], \\
\mathscr{E}_{\theta\phi} &= \epsilon\ \tilde{\mathscr{E}}_{\theta\phi} = 6\frac{\epsilon}{M^2}\cos\beta\sin\beta\sin\phi, \\
\mathscr{E}_{\phi\phi} &= \epsilon\ \tilde{\mathscr{E}}_{\phi\phi} = 6\frac{\epsilon}{M^2}\cos\beta\sin\beta\sin\phi
\end{aligned}$$

- α disappears in this limit, there is no distinction between a Kerr and a Schwarzschild perturber for $d \gg M_*$

Yang and Casals (2017)

The deformed metric

- The metric for a tidally deformed Schwarzschild black hole will be written in terms of the quadrupole moments.

- It is useful to introduce the dimensionless perturbative parameter

$$\boxed{\epsilon = \frac{M_* M^2}{d^3}}$$

- Under the hierarchical assumption $M \ll M_*$, and noticing that d is naturally measured in units of the Kerr mass M_*, this parameter is automatically small, $\epsilon \ll 1$.

- Make explicit the linear dependence on ϵ, so that $\mathscr{E} \sim \mathcal{O}(\epsilon)$ and $\mathscr{B} \sim \mathcal{O}(\epsilon)$.

- To keep track of the various order in ϵ we can rescale the tidal moments so as to make explicit such a dependence and define

$$\mathscr{E} = \epsilon\, \tilde{\mathscr{E}}, \quad \mathscr{B} = \epsilon\, \tilde{\mathscr{B}}$$

EMRI +External Tidal Fields

- EMRI PERTURBED BY THE PRESENCE OF A THIRD MASSIVE BODY

$$ds^2 \simeq -\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)+\epsilon\, h_{\mu\nu}dx^\mu dx^\nu+\mathcal{O}(\epsilon^2)$$

- At the leading order the Tidal Environment specified by **QUADRUPOLE TIDAL MOMENTS**

Poisson and Vlasov (2009)

$$\boxed{\begin{aligned}
&h_{tt}=-r^2\left(1-\frac{2M}{r}\right)^2\mathcal{E}^q\\
&h_{tr}=\left(1-\frac{2M}{r}\right)h_{rr}=-r^2\left(1-\frac{2M}{r}\right)\mathcal{E}^q\\
&h_{tA}=\left(1-\frac{2M}{r}\right)h_{rA}=-\frac{2}{3}r^3\left(1-\frac{2M}{r}\right)\left(\mathcal{E}^q_A-\mathcal{B}^q_A\right)\\
&h_{AB}=-\frac{1}{3}r^4\left[\left(1-\frac{2M^2}{r^2}\right)\mathcal{E}^q_{AB}-\left(1-3\frac{2M^2}{r^2}\right)\mathcal{B}^q_{AB}\right]
\end{aligned}}$$

$\longrightarrow$ Tidal fields induced by the external SuperMassive BH!

$$\mathcal{E}^q\,,\ \mathcal{E}^q_\theta\,,\ \mathcal{E}^q_\phi\,,\ \mathcal{E}^q_{\theta\theta}\,,\ \mathcal{E}^q_{\theta\phi}\,,\ \mathcal{E}^q_{\phi\phi}$$

$$\mathcal{B}^q_\theta\,,\ \mathcal{B}^q_\phi\,,\ \mathcal{B}^q_{\theta\theta}\,,\ \mathcal{B}^q_{\theta\phi}\,,\ \mathcal{B}^q_{\phi\phi}$$

Tidal ISCO shift

- The tidal fields generated by the outer body deform the orbits of the unperturbed Schwarzschild metric.

- If $\bar{x}^\mu(\tau)$ solves the geodesic equation in the unperturbed Schwarzschild geometry

$$\ddot{\bar{x}}^\mu = -\Gamma^{(0)\mu}_{\nu\rho}|_{\bar{x}}\dot{\bar{x}}^\nu\dot{\bar{x}}^\rho \quad \bar{g}_{\mu\nu}|_{\bar{x}}\dot{\bar{x}}^\mu\dot{\bar{x}}^\nu = -1$$

the effect of the tidal deformation will reflect in a deviation from the unperturbed curve $\bar{x}^\mu$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon^2) \qquad\qquad x^\mu(\tau) = \bar{x}^\mu(\tau) + \epsilon\ y^\mu(\tau)$$

- In general the tidal moments depend on ϕ $\longrightarrow$ circular orbits are deformed into elliptic orbits.

- This can also be seen by solving the geodesics equations where the solution for the y^r component can be written as $y^r_{mean} + y^r_{(\phi)}$, where y^r_{mean} does not depend of ϕ while $y^r_{(\phi)}$ depends on it.

- We want to construct the Hamiltonian per unit mass squared H of the EMRI + tidal interaction system.

- The radial correction enters the Hamiltonian only with terms of order $\mathcal{O}\left(\epsilon^2\right)$, as a consequence it is possible to replace the true trajectory in the perturbed spacetime with the "mean" circular trajectory.

- Define:

$$\langle \mathscr{A} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \mathscr{A}\,|_\gamma \; d\phi$$

  where γ is the mean circular orbit on $G = g + \epsilon\ h$.

- This averaging procedure allows one to consider the **secular dynamics** of bound orbits in the tidally deformed spacetime.

- We are interested in computing the shift in the ISCO quantities due to the tidal field.

- In a hierarchical three body system the tidal field generated by the third body modifies the ISCO frequency, energy, angular momentum and radius of a Schwarzschild EMRI

- Consider the Hamiltonian per unit mass squared H of the EMRI + tidal interaction system

$$\boxed{H = \frac{1}{2} p^\mu p^\nu \langle G_{\mu\nu} \rangle = \frac{1}{2} p^\mu p^\nu \left(g_{\mu\nu} + \epsilon\ \langle h_{\mu\nu} \rangle \right) + \mathcal{O}(\epsilon^2)}$$

Yang and Casals (2017)

- To compute $\langle h_{\mu\nu} \rangle$ the average of the equatorial tidal moments is needed

$$\langle \mathscr{E}_q \rangle = \frac{\epsilon}{r_0^2} \left(1 + 3\frac{q_1}{q_2}\cos 2\beta\right)$$

$$\langle \mathscr{E}_\theta \rangle = \langle \mathscr{E}_\phi \rangle = 0$$

$$\langle \mathscr{E}_{\theta\theta} \rangle = -\langle \mathscr{E}_{\phi\phi} \rangle = -3\frac{\epsilon}{r_0^2}\left(1 + 3\frac{q_1}{q_2}\cos 2\beta\right)$$

$$\langle \mathscr{E}_{\theta\phi} \rangle = 0$$

$$\langle \mathscr{B}_\theta \rangle = \langle \mathscr{B}_\phi \rangle = 0$$

$$\langle \mathscr{B}_{\theta\theta} \rangle = \langle \mathscr{B}_{\phi\phi} \rangle = 0$$

$$\langle \mathscr{B}_{\theta\phi} \rangle = 3\frac{\epsilon}{r_0^2}\frac{\sqrt{(q_1 - q_2)(q_1 + q_2)}}{q_2}\sin 2\beta$$

- The secular dynamics of every bound geodesic is captured by a mean circular orbit.

- It is possible to consider an averaged total momentum p^μ by setting $p^r = p^\theta = 0$

$$p^\mu = \left(\frac{E}{(1 - r_0/r) - \epsilon\,\langle h_{tt}\rangle}, 0, 0, \frac{L}{r^2 + \epsilon\,\langle h_{\phi\phi}\rangle}\right)$$

$$\langle h_{tt} \rangle = -\epsilon\,\frac{r^2}{r_0^2}\left(1 - \frac{r_0}{r}\right)^2\left(1 + 3\frac{q_1}{q_2}\cos 2\beta\right)$$

$$\langle h_{\phi\phi} \rangle = -\epsilon\,\frac{r^4}{r_0^2}\left(1 - \frac{r_0^2}{2r^2}\right)\left(1 + 3\frac{q_1}{q_2}\cos 2\beta\right)$$

- the secular Hamiltonian becomes

$$H = \frac{L^2}{r^2} - \frac{E^2}{1 - \frac{r_0}{r}} - 8\eta \, \frac{r^2}{r_0^2} \left[\frac{L^2}{r^2} \left(1 - \frac{r_0^2}{2r^2} \right) + E^2 \right]$$

- where the effective perturbative parameter η is

$$\eta = -\frac{1}{8}\epsilon \left(1 + 3\frac{q_1}{q_2} \cos 2\beta \right)$$

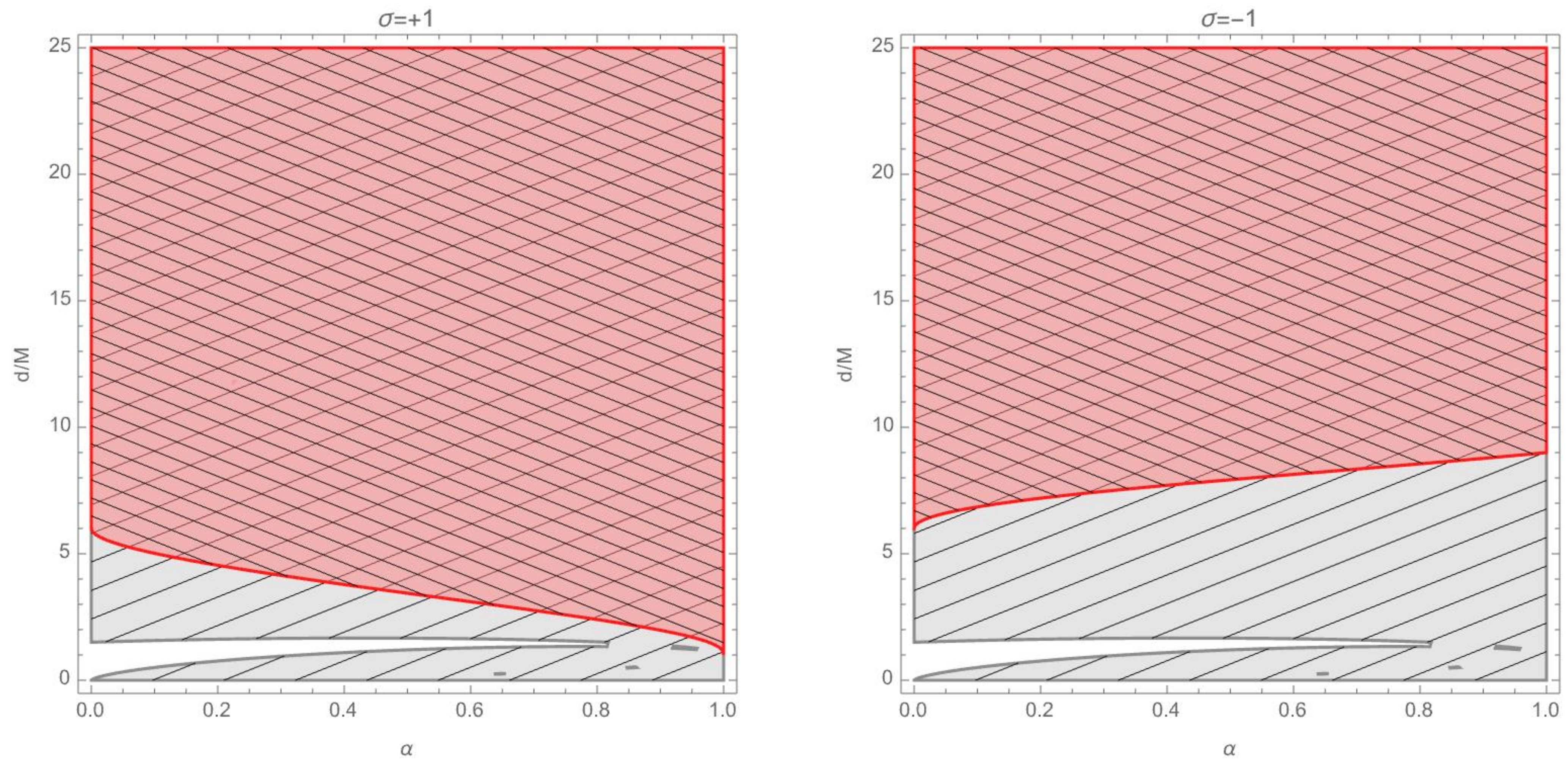
$$q_1 = d^2 - 2\alpha\sqrt{d}M_*^{3/2}\sigma - dM_* + 2\alpha^2 M_*^2$$

$$q_2 = 2\alpha\sqrt{d}M_*^{3/2}\sigma + d(d - 3M_*)$$

- Notice that all the informations regarding how the binary system is oriented, as well as where it is located with respect to the Kerr perturber, are contained in η .

- In particular, it is readily verified that a particular orientation β^* for the binary system exists for which $\eta = 0$, and thus no corrections due to tidal moments are induced in the secular dynamics

$$\cos 2\beta^* = -\frac{1}{3}\frac{q_2}{q_1}$$



- The red region represents the physical region where $d > \hat{r}^{\sigma}_{\text{isco}}$, with $\sigma = +1$ in the left panel and $\sigma = -1$ in the right panel. The oblique line region shows where the quantity $1 > -\frac{1}{3}\frac{q_2}{q_1} > -1$. This shows that the angle β^* can always be defined in the entire physical region of Kerr circular geodesics.

- The ISCO can be obtained upon demanding the Hamiltonian H to be on-shell, the orbit to be circular and that the radial perturbations become stationary

$$\boxed{H = -\frac{1}{2}\,, \quad \frac{\partial H}{\partial r} = 0\,, \quad \frac{\partial^2 H}{\partial r^2} = 0}$$

- Using these conditions and expanding at the first order in ϵ, it is possible to determine the secular shift caused by the tidal perturbations to the energy, angular momentum and radius of the Schwarzschild

$$\boxed{\begin{aligned} r &= r_{\rm isco} + \eta\ r_1 + \mathcal{O}(\eta^2) \\ E &= E_{\rm isco} + \eta\ E_1 + \mathcal{O}(\eta^2) \\ L &= L_{\rm isco} + \eta\ L_1 + \mathcal{O}(\eta^2) \end{aligned}}$$

- at the leading order one finds $r_{\rm isco}$, $E_{\rm isco}$, $E_{\rm isco}$, namely the values for the radius, the energy and the angular momentum of the ISCO for an unperturbed Schwarzschild black hole

$$\boxed{r_{\rm isco} = 3r_0\,, \quad E_{\rm isco} = \frac{\sqrt{8}}{3}\,, \quad L_{\rm isco} = \sqrt{3}\ r_0}$$

- At the first order in η one determines the corrections to the three quantities

$$\boxed{r_1 = 1536\ r_0\,, \quad E_1 = -\frac{152\sqrt{2}}{3}\,, \quad L_1 = -174\sqrt{3}\ r_0}$$

Yang and Casals (2017)

- The **effective secular perturbative parameter η is new!** It allows to specify not only the orientation of the binary system and its location with respect to a spinning tidal perturber.

$$\eta = -\frac{1}{8}\epsilon\left(1 + 3\frac{q_1}{q_2}\cos 2\beta\right)$$

$$q_1 = d^2 - 2\alpha\sqrt{d}M_*^{3/2}\sigma - dM_* + 2\alpha^2 M_*^2$$
$$q_2 = 2\alpha\sqrt{d}M_*^{3/2}\sigma + d(d - 3M_*)$$

- It is also possible to compute the shift in the ISCO orbital frequency. In general for quasi-circular orbits the orbital frequency can be determined by means of the ratio

$$\Omega = \frac{p^\phi}{p^t} \qquad \text{expanding} \qquad \longrightarrow \qquad \Omega = \Omega_{\rm isco} + \eta\ \Omega_1 + \mathcal{O}\left(\eta^2\right)$$

$$\boxed{r_0\Omega_{\rm isco} = \frac{1}{3\sqrt{6}}\ , \quad r_0\Omega_1 = -\sqrt{\frac{2}{3}}\frac{491}{3}}$$

- Shift induced by the tidal fields on the orbital frequency of the ISCO.

- In the case in which the binary system is located asymptotically away from the Kerr perturber, $d \gg M_*$ every information about the black hole spin parameter α and the σ parameter are lost.

- These results also follow by the geodesic approach.

EMRI + External Tidal Field, Geodesics

HIERARCHICAL THREE-BODY PROBLEM!

$$\boxed{M_* \gg M \gg m}\qquad \boxed{\epsilon = \frac{M_* M^2}{d^3} \ll 1}$$

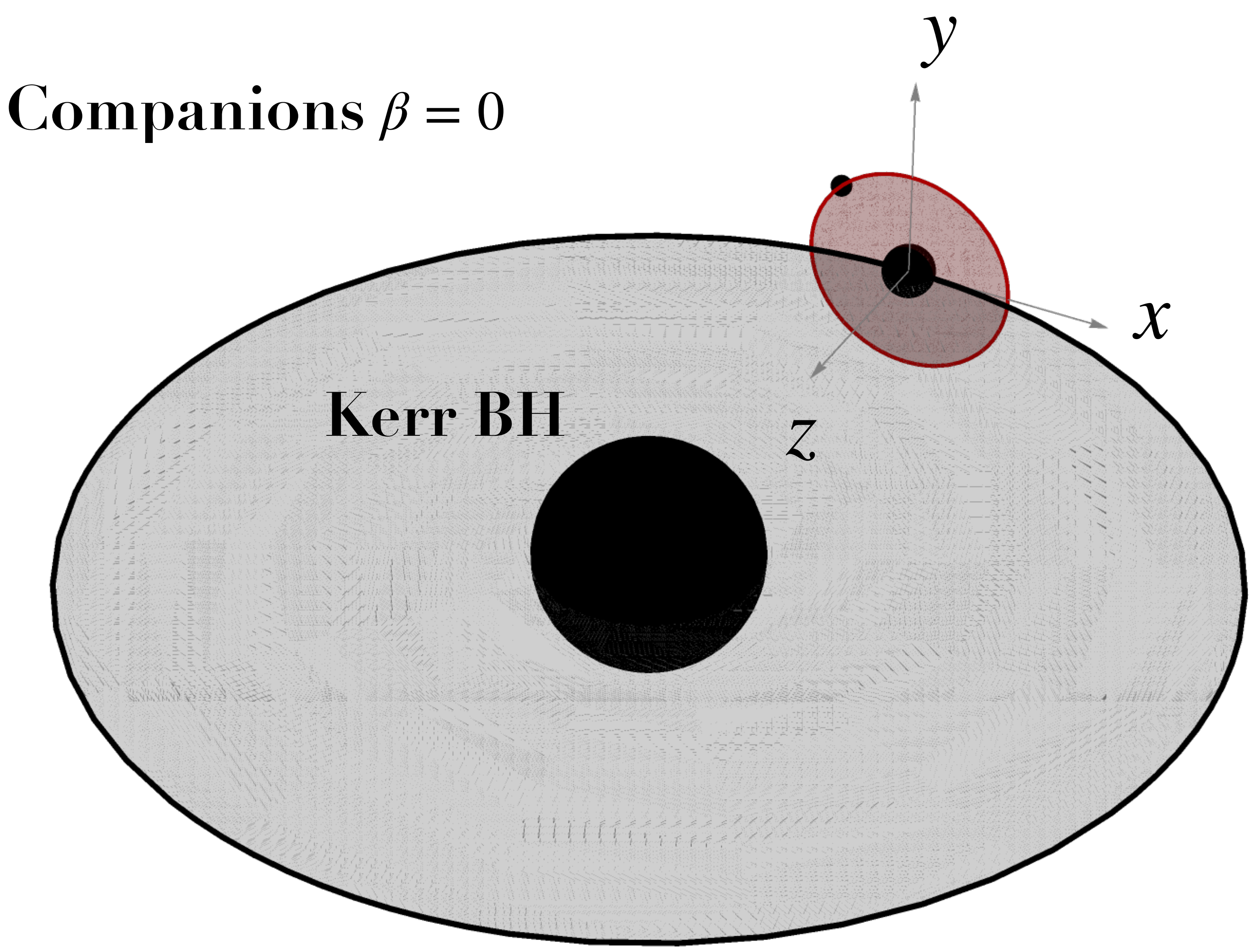
Study the EMRI motion in the tidal environment $\quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon^2)$

$$\begin{cases} x^\mu(\tau) = \bar{x}^\mu(\tau) + \epsilon y^\mu(\tau) \\ \ddot{y}^\mu = -2\bar{\Gamma}^\mu_{\nu\rho}\,|_{\bar{x}}\,\dot{\bar{x}}^\nu \dot{y}^\rho - \left(y^\sigma (\partial_\sigma \bar{\Gamma}^\mu_{\nu\rho})\,|_{\bar{x}} + \Gamma^{(h)\mu}_{\nu\rho}\,|_{\bar{x}} \right) \dot{\bar{x}}^\nu \dot{\bar{x}}^\rho \end{cases}$$

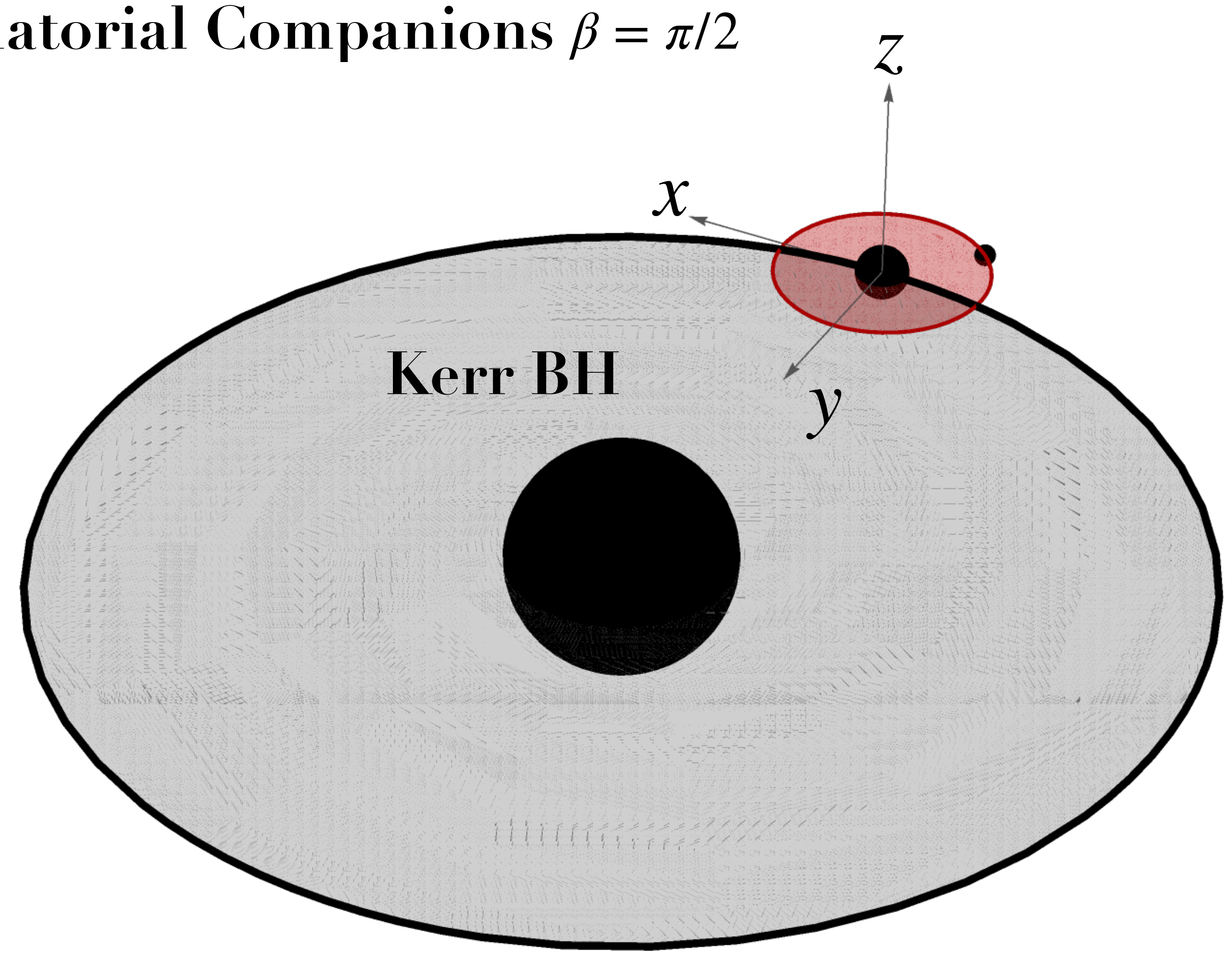
RICHER **PHENOMELOGY!**

- Tidal environment depends on the **orientation of the EMRI**!
- Tidal environment depends on **where the EMRI is located**!

Polar Companions $\beta = 0$



Equatorial Companions $\beta = \pi/2$



EMRI + External Tidal Field, Geodesics

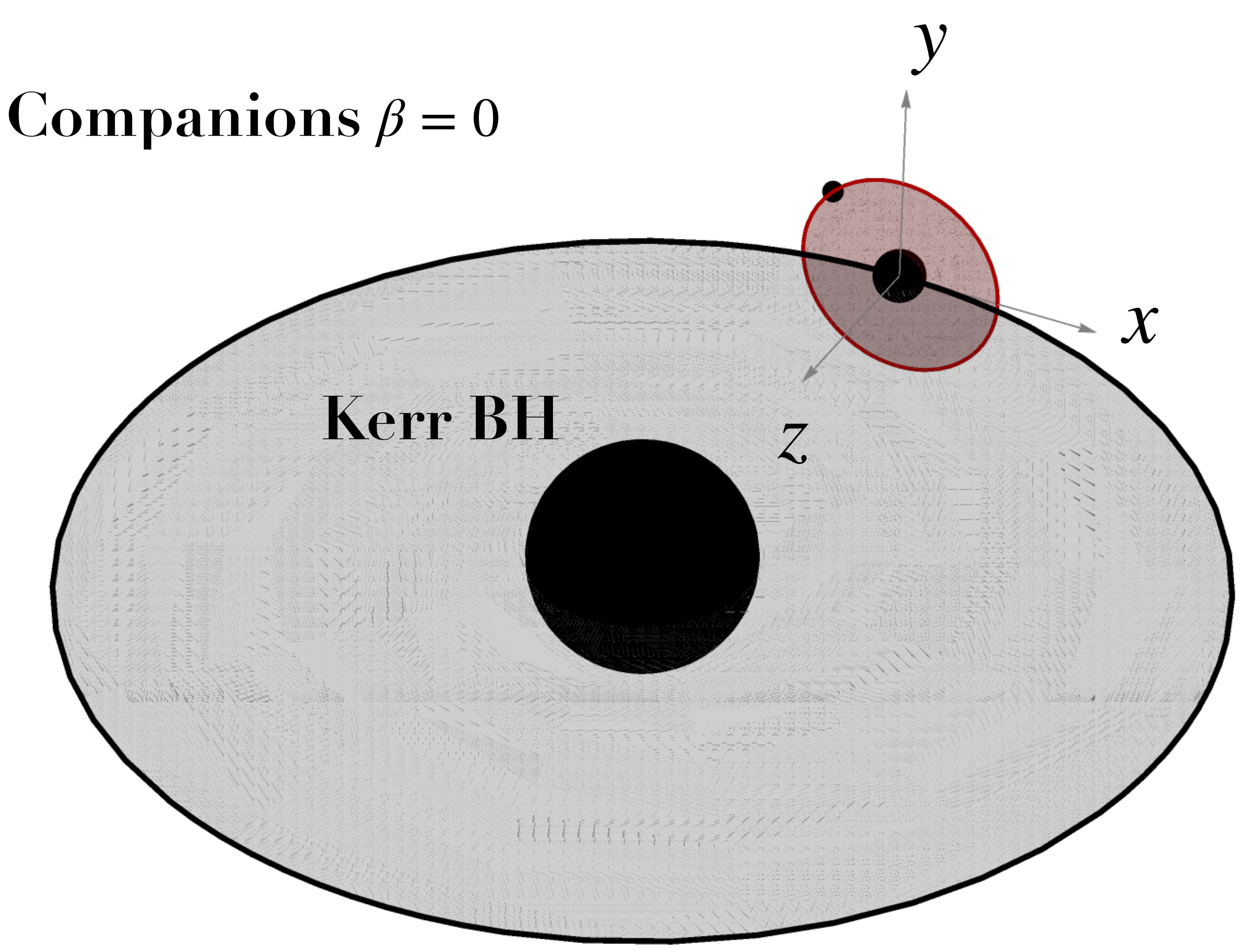
HIERARCHICAL THREE-BODY PROBLEM!

$$\boxed{M_* \gg M \gg m}$$

$$\boxed{\epsilon = \frac{M_* M^2}{d^3} \ll 1}$$

RICHER PHENOMELOGY!

- Tidal environment depends on the orientations of the EMRI!
- Tidal environment depends on where the EMRI is located!



Orientation: **POLAR CONFIGURATIONS,**

Tidal Moments:

$$\mathcal{E}^q = -\epsilon \frac{1 + 3\cos(2\theta)}{2M^2}$$

$$\mathcal{E}^q_A = -\epsilon \frac{3}{2} \frac{\sin(2\theta)}{M^2} \delta^\theta{}_A \,, \quad \mathcal{B}^q_A = 0$$

$$\mathcal{E}^q_{AB} = -\epsilon 3 \frac{\sin^2\theta}{M^2} (\delta^\theta{}_A \delta^\theta{}_B - \delta^\phi{}_A \delta^\phi{}_B) \,, \quad \mathcal{B}^q_{AB} = 0$$

Location:

Yang and Cassals (2017)

Cardoso and Foschi (2021)

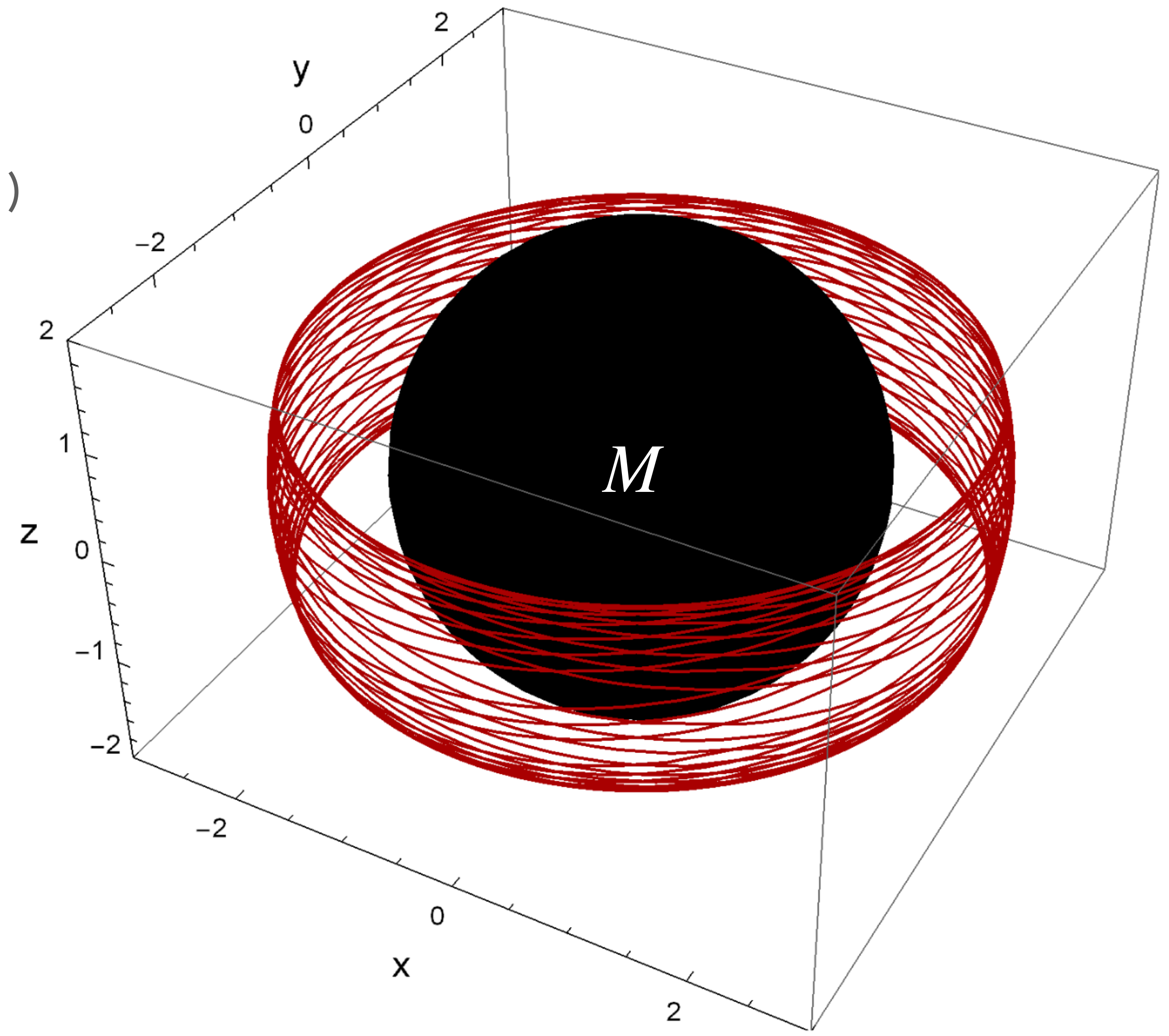
ISCO shift:

$$r_{\text{ISCO}} = 6M(1 + \epsilon\ 256) + \mathcal{O}(\epsilon^2)$$

$$\theta_{\text{ISCO}} = \frac{\pi}{2}(1 - \epsilon\ c_1 \sin\phi) + \mathcal{O}(\epsilon^2)$$

$$E_{\text{ISCO}} = \frac{2\sqrt{2}}{3}(1 + \epsilon\ 38) + \mathcal{O}(\epsilon^2)$$

$$L_{\text{ISCO}} = 2\sqrt{2}M(1 + \epsilon\ 87) + \mathcal{O}(\epsilon^2)$$



EMRI + External Tidal Field, Geodesics

HIERARCHICAL THREE-BODY PROBLEM!

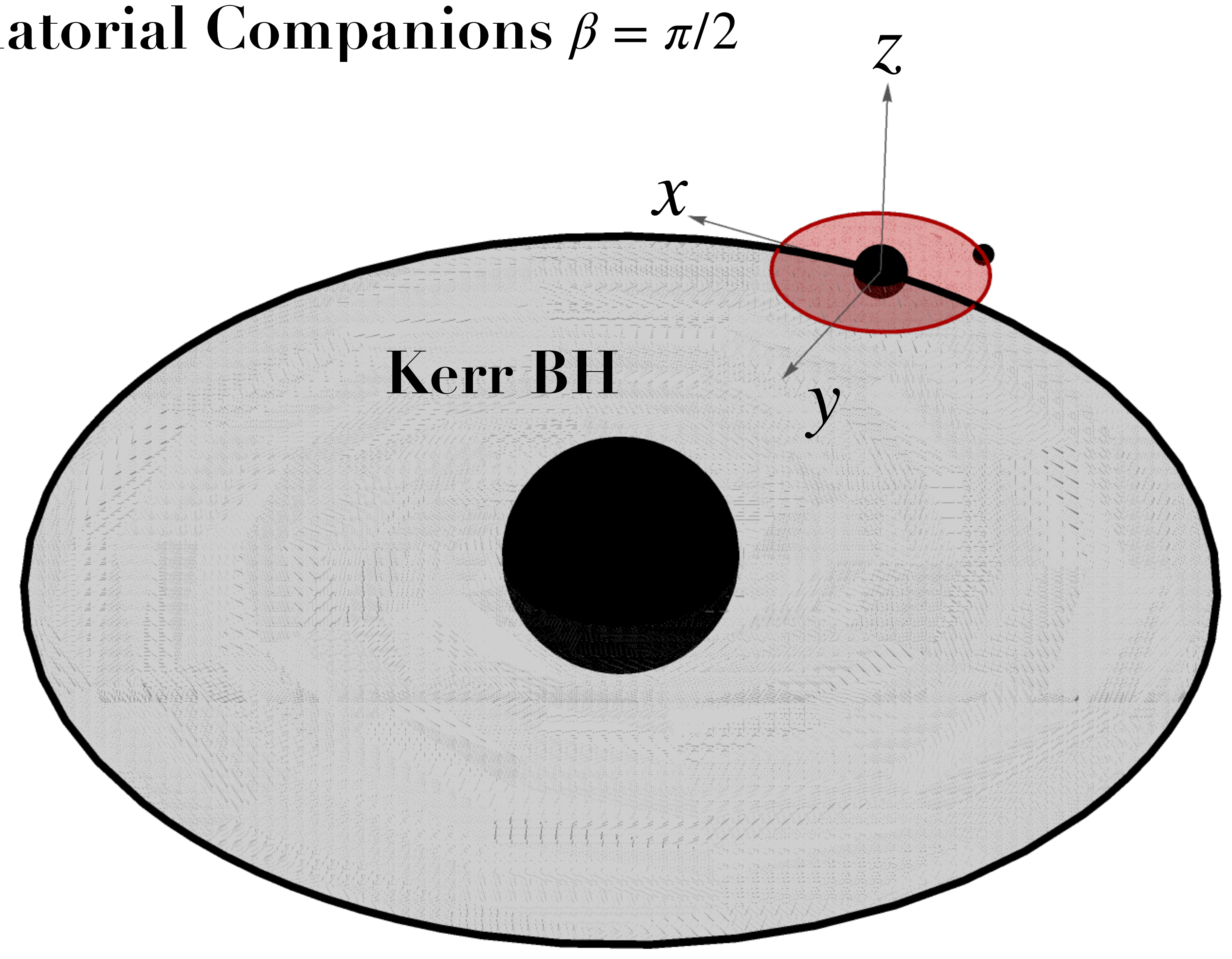
$$\boxed{M_* \gg M \gg m}$$

$$\boxed{\epsilon = \frac{M_* M^2}{d^3} \ll 1}$$

RICHER PHENOMELOGY!

- Tidal environment depends on the orientations of the EMRI!
- Tidal environment depends on where the EMRI is located!

Equatorial Companions $\beta = \pi/2$



Orientation: **EQUATORIAL CONFIGURATIONS,** **Location:**

Tidal Moments:

$$\mathcal{E}^q = -\epsilon \frac{1 + 3\cos(2\phi)}{M^2}$$

$$\mathcal{E}^q_A = -\epsilon \frac{3}{2} \frac{\sin(2\phi)}{M^2} \delta^{\phi}{}_A\,, \quad \mathcal{B}^q_A = 0$$

$$\mathcal{E}^q_{AB} = 3\epsilon \frac{\sin^2\phi}{M^2} (\delta^{\theta}{}_A \delta^{\theta}{}_B - \delta^{\phi}{}_A \delta^{\phi}{}_B)\,, \quad \mathcal{B}^q_{AB} = 0$$

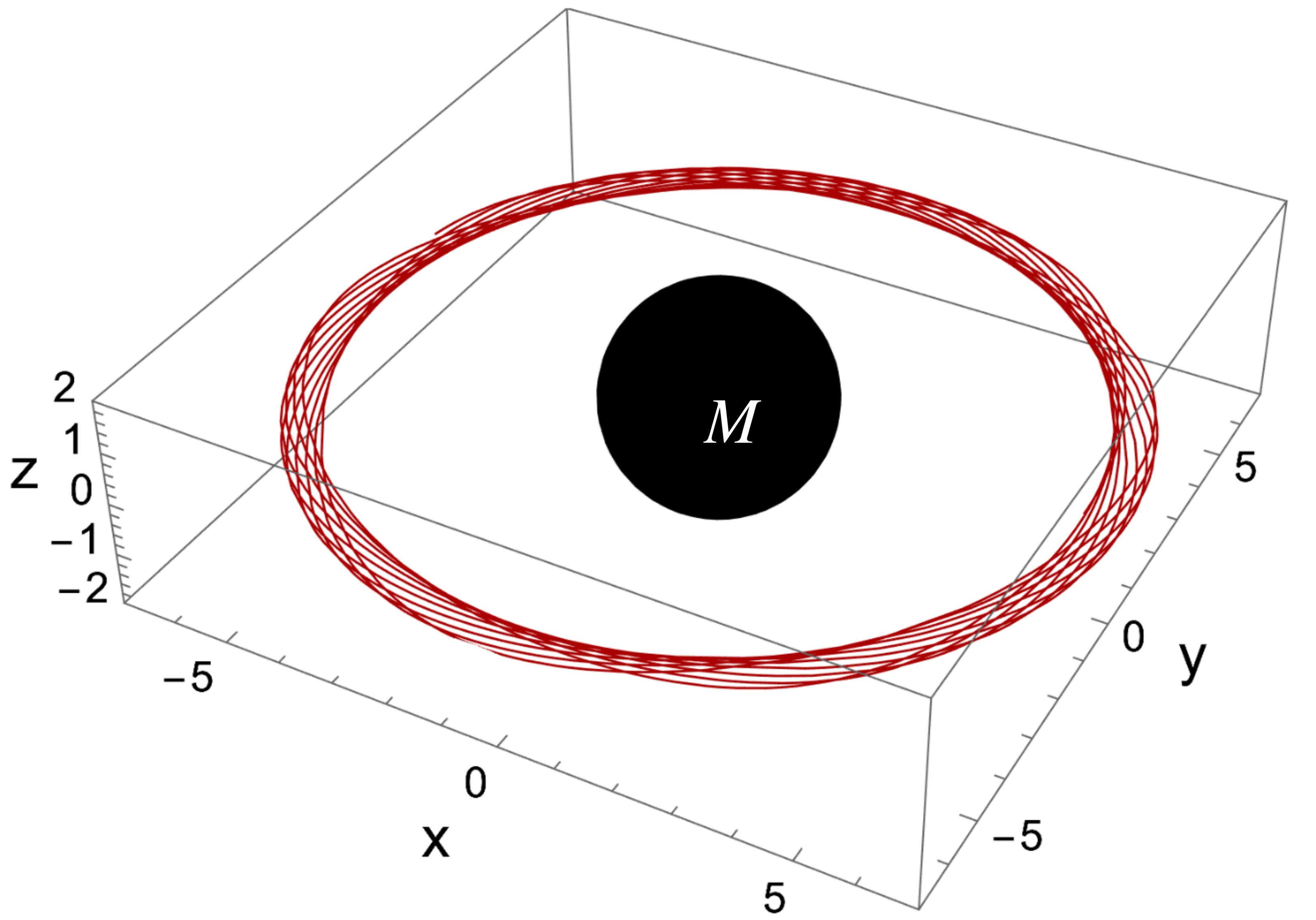
ISCO now eccentric!

$$r_{\rm ISCO} = 6M \left[1 + \epsilon \left(128 - 57\cos\phi\right)\right] + \mathcal{O}(\epsilon^2)$$

$$\theta_{\rm ISCO} = \frac{\pi}{2}$$

$$E_{\rm ISCO} = \frac{2\sqrt{2}}{3}(1 - \epsilon\ 19) + \mathcal{O}(\epsilon^2)$$

$$L_{\rm ISCO} = 2\sqrt{2}M(1 - \epsilon\ \frac{87}{2}) + \mathcal{O}(\epsilon^2)$$

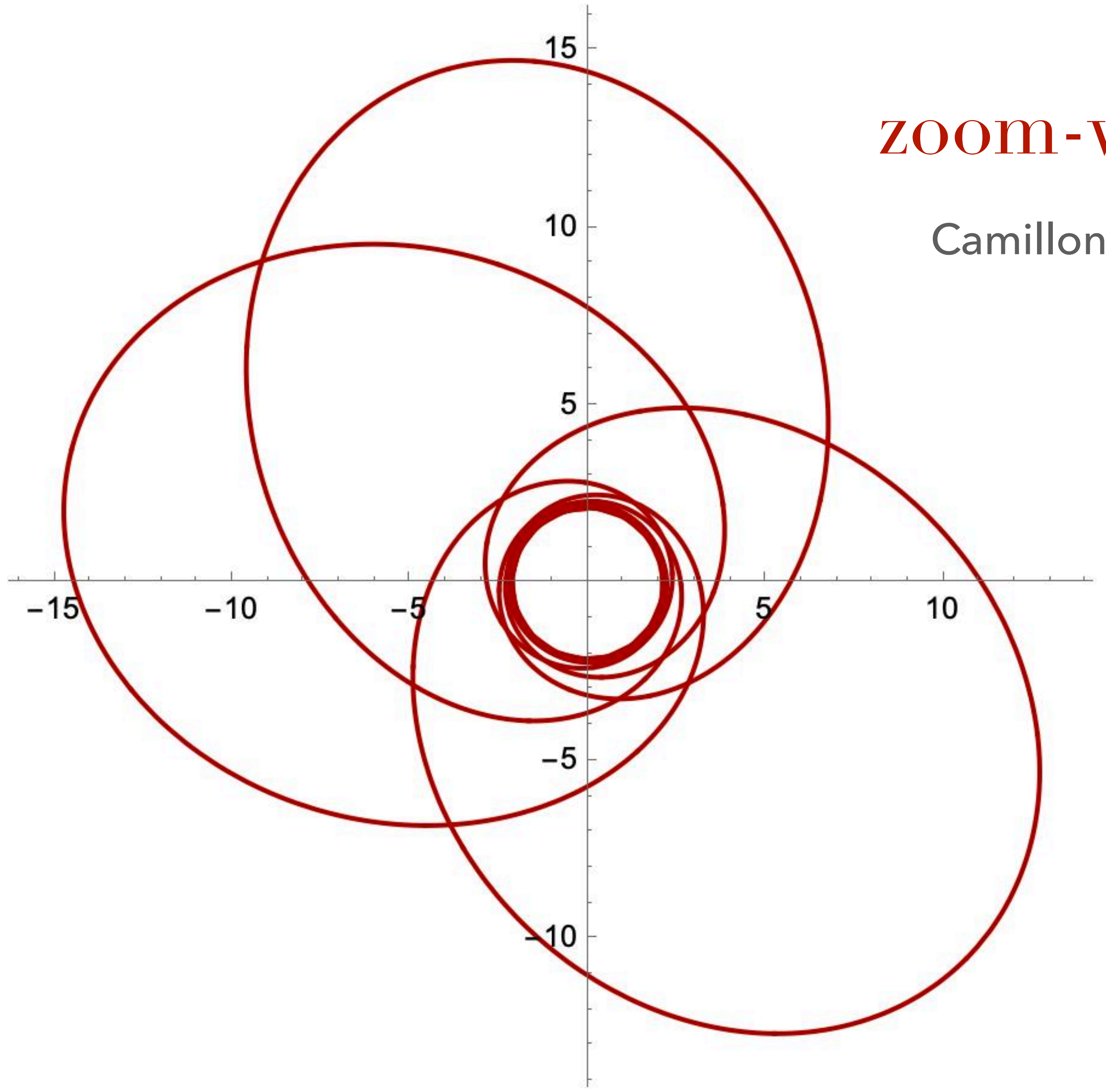


Summary

- We computed the ISCO shifts induced by the tidal moments of a Kerr black hole on an EMRI
- ISCO shifts of the energy, angular momentum, radius and angular velocity, for any value of the distance, the spin of the Kerr black hole and the inclination of the orbit of the test particle around the Schwarzschild black hole

Perspectives

- It is possible to compute the frequencies of the motion through the action angle variables.
- Non circular orbits, eccentricity, resonances!
- Corrections to the orbital precession.
- ISCO $\longrightarrow$ ISO $\longrightarrow$ $\Omega_r = 0$.



zoom-whirl orbit

Camilloni, Pica (2022)

Thank you for the attention!