Precision Holography for 5D Super Yang-Mills

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An overview

- The main character: 1/2-BPS Wilson loop (WL) in maximally supersymmetric Yang-Mills (MSYM) on S^5. Its vev can be computed at large N and for all "λ" ('t Hooft coupling) via localisation.
- Our main goal is to compute its vev by means of its holographic dual at large N and up to one-loop in a large "λ" expansion.
- The key words here: precision, holography, non-conformal setting, MSYM.

Outline

- Motivations
- On the field theory side:
  - 5D MSYM on S^5
  - 1/2-BPS WL
- On the string theory side:
  - the dual
  - preparing for the computations
  - the computations
- The matching
- Outlook

Motivations and declaration of intent

- Why precision holography in a non-conformal setting? We want to understand top-down holography beyond the conformal paradigm but still in a mathematically detailed level.
- Why WL operators? These are among the simplest gauge-invariant non-local observables we can study in a gauge theory. Their holographic dual is known and their vev can be computed via localisation.
- I will focus on 2 aspects here:
  - the role and the nature of UV/IR divergences,
  - the role of the dilaton.
- I will underline what is in common and what not to AdS5/CFT4 (AdS4/CFT3). NB. Long tradition of study WL in AdS/CFT!

On the field theory side: 5D MSYM on S^5 (*)

- The Lagrangian [Blau '00][Minahan, Zabzine '15]:

$$\mathscr{L} = -\frac{1}{2g_{\rm YM}^2}\mathrm{Tr}\left(|F|^2 - |D\Phi_I|^2 - \bar{\Psi}D\Gamma\Psi - \frac{1}{2}\left[\Phi_I, \Phi_J\right]^2 + \bar{\Psi}\,\Gamma^I\left[\Phi_I, \Psi\right] \right)$$

- The Lagrangian is obtained via dimensional reduction from 10D SYM on a flat space+minimal coupling of the S^5 + other terms to preserve 16 real supercharges.

- Scalar indices I,J=0,6,...,9. A_μ ($\mu = 1,\ldots,5$), Φ_I, Ψ transform in the adjoint of the gauge group SU(N).

- Ψ have 16 real components and obey $\Gamma_{11}\Psi = \Psi$.

- Euclidean: the above Lagrangian needs to be Wick-rotated ($\phi^0 \to i\phi^0$).

On the field theory side: 5D MSYM on S^5

- The symmetries: The global symmetry supergroup is $SU(4|1,1)$. The R-symmetry group is $SU(1,1) \times SO(2)$. The space transformation group: $SO(6)$.

- The parameters: $\mathscr{R}$ is the radius of the 5-sphere where the theory lives on, N is the rank of the gauge group, g_{YM}^2 is the coupling constant which has dimension [L]. The 't Hooft coupling constant is

$$\xi = \frac{g_{\mathrm{YM}}^2 N}{2\pi\mathscr{R}}$$

- 5D SYM is not renormalizable, at high energies it is UV completed in the (2,0) SCFT [Douglas '10][Lambert, Papageorgakis, Schmidt-Sommerfeld '10]

On the FT side: 1/2-BPS WL on 5D SYM

- The theory can be localised [Pestun '07][Minahan, Zabzine '15][Minahan '15][Gorantis, Minahan, Naseer '17] on a locus where: $A_\mu = 0$ and $\phi_I = 0$ when $I \neq 0$. ϕ_0 is used to construct a Hermitian $N \times N$ matrix M.
- The vev of 1/2-BPS WL [Maldacena '98][Bobev, Bomans, Gautason, Minahan, Nedelin '19] which wraps the equator of S^5 is

$$\langle W \rangle = \left\langle \mathrm{Tr} \left(P e^{i \oint A_\mu dx^\mu + i \oint ds\, \phi^0} \right) \right\rangle$$

For us: $A_\mu = 0$ (5-dimensional gauge field)

- It can be computed from the matrix model:

$$\langle W \rangle = \frac{N}{\xi} \left(e^{\xi} - 1 \right) + \mathcal{O}\left(\frac{1}{N} \right)$$

at large N but for <u>any</u> 't Hooft coupling ξ

On the FT side: 1/2-BPS WL on 5D SYM

- The vev of 1/2-BPS WL which wraps the equator of S^5 at large 't Hooft coupling ξ and large N

$$\langle W \rangle = \frac{N}{\xi} e^{\xi} + \mathcal{O}\left(e^{-\xi}\right) + \mathcal{O}\left(\frac{1}{N}\right)$$

- What are we looking at?

classical leading contribution in the large ξ limit

$$\langle W \rangle = \frac{N}{\xi} e^{\xi} + \mathcal{O}\left(e^{-\xi}\right) + \mathcal{O}\left(\frac{1}{N}\right)$$

This is the subleading contribution in the large ξ limit which we want to compute

On the string theory (ST) side: the dual 10D geom I

- The holographic dual of MSYM on S^5 is a stack of N D4-branes with spherical worldvolume in 10D [Bobev, Bomans, Gautason '18]. The 10D metric is

where FT lives!

$$ds_{10}^2 = \ell_s^2 (N\pi e^{\Phi})^{2/3} \left[\frac{4\left(d\sigma^2 + d\Omega_5^2\right)}{\sinh^2\sigma} + d\theta^2 + \cos^2\theta \, ds_{dS_2}^2 + \frac{\sin^2\theta \, d\phi^2}{1 - \frac{1}{4}\tanh^2\sigma \, \sin^2\theta} \right]$$

and the non-constant dilaton is ($\neq$ AdS case!)

$$e^{\Phi} = \frac{\xi^{3/2}}{N\pi}\left(\coth^2\sigma - \frac{1}{4}\sin^2\theta\right)^{3/4}$$

- The radial direction is $0 \le \sigma < \infty$ (UV is at $\sigma \to 0$ and IR $\sigma \to \infty$)
- The background symmetry: $SO(6) \times SO(1,2) \times SO(2)$

On the ST side: the dual 10 geom II

- The dimensionless parameters here are: N, ξ
- At $\sigma \to \infty$ (IR) (set $r = e^{-\sigma}$): the 5-sphere smoothly shrinks to zero

$$ds^2_{10} \to \ell_s^2 (N\pi e^{\Phi})^{2/3} \left[16\left(dr^2 + r^2 d\Omega_5^2\right) + d\theta^2 + \cos^2\theta \, ds^2_{dS_2} + \frac{\sin^2\theta \, d\phi^2}{1 - \frac{1}{4}\sin^2\theta} \right], \qquad \Phi \to \frac{1}{2}\log\frac{\xi^3}{\pi^2 N^2}$$

- At $\sigma \to 0$ (UV) (set $U = \sinh^{-2}\sigma$): the metric of flat-space D4 brane sol

$$ds^2_{10} \to \xi \ell_s^2 \left[4U^{3/2} d\Omega_5^2 + \frac{dU^2 + U^2 ds^2_{dS_4}}{U^{3/2}} \right] \qquad \Phi \to \frac{1}{2}\log\frac{\xi^3 U^{3/2}}{N^2 \pi^2}$$

On the ST side: the dual 10 geom II (*)

- The gauge potentials are

$$B_2 = \frac{\xi \ell_s^2}{2} \cos^3\theta \, \mathrm{vol}_{dS_2} , \quad C_1 = \frac{iN\pi\xi\ell_s}{2} (N\pi e^{\Phi})^{-4/3} \sin^2\theta \, d\phi , \quad C_3 = - \, iN\pi\ell_s^3 \cos^3\theta \, d\phi \wedge \mathrm{vol}_{dS_2}$$

and so the NSNS and RR fields:

$$H_3 = dB_2 , \quad F_2 = dC_1 , \quad F_4 = dC_3 - H_3 \wedge C_1$$

On the ST side: the dual of 1/2-BPS WL I

- The holographic dual of a circular WL is a fundamental string in this 10D background, whose worldsheet ends on the loop at the boundary [Maldacena '98]

- The vev of the circular WL is then given by the string partition function

$$\langle W \rangle = Z_{string}$$

- and at strong coupling ($\xi \gg 1$)

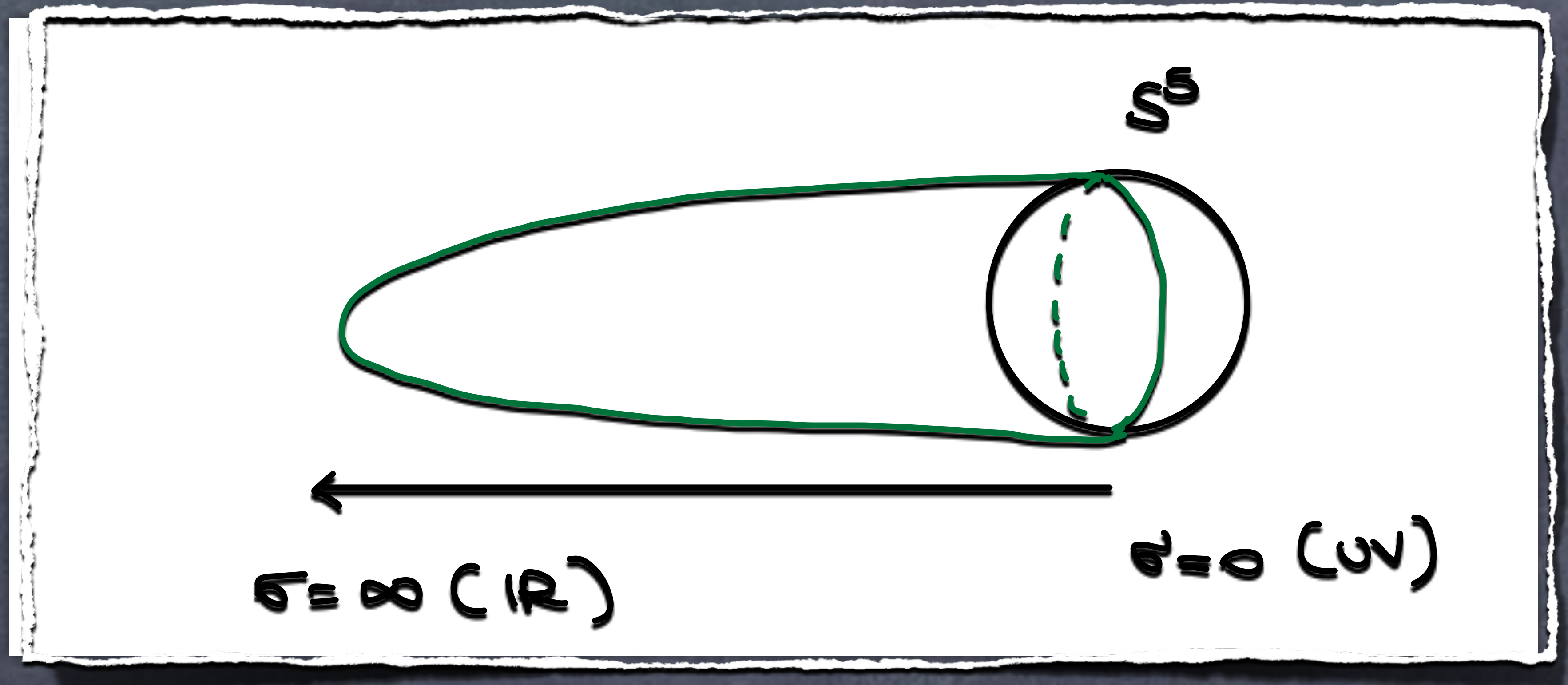
$$\log Z_{string} \approx -S_{classical} \underbrace{- S_{FT} - \Gamma_{\mathbb{K}}} = -S_{classical} \underbrace{- S_{FT} + \log \mathrm{Sdet}^{-1/2}\mathbb{K}}$$

this is what we want to compute!

On the ST side: the dual of 1/2-BPS WL II

- For us: the classical solution is [Bobev, Bomans, Gautason, Minahan, Nedelin '19]

equator of S^5, $\theta = 0$, any fixed point on the internal space



- In static gauge the ws coordinates are (τ, σ): the equator of S^5 is parameterised by τ and σ is the radial coordinate

$$ds_2^2 = e^{2\rho}\left(d\sigma^2 + d\tau^2\right)\,, \quad \sqrt{\gamma} = e^{2\rho} = \frac{4\xi \ell_s^2}{\tanh\sigma \sinh^2\sigma}\,,$$

On the ST side: The classical contribution

- The vev of the circular WL is then given by the string partition function

$$\log Z_{string} \approx -S_{classical} - S_{FT} + \log \mathrm{Sdet}^{-1/2}\mathbb{K} = -S_{classical} - S_{FT} - \Gamma_{\mathbb{K}}$$

- The classical contribution is the area of the minimal ws. The classical regularised action is [Bobev, Bomans, Gautason, Minahan, Nedelin '19]:

$$S_{classical} + S_{ct} = \frac{1}{2\pi\ell_s^2}\int\sqrt{\gamma}\, d\sigma d\tau + S_{ct} = -\xi$$

and it matches the leading contribution of $\langle W \rangle$ at strong coupling.

On the ST side: the one-loop ws string action I

- The one-loop action comprehends two terms:

1. The Fradkin-Tseytlin action [Fradkin, Tseytlin '85, '86]:

$$S_{FT} = \frac{1}{4\pi}\int_M \sqrt{\gamma}\,\Phi\, R^{(2)} + \frac{1}{2\pi}\int_{\partial M} \Phi\, K ds$$

- It is 'classical': in terms of the 'bare' string tension $T = \frac{1}{2\pi\ell_s^2}$, it is of order T^0, but it contributes to the quantum corrections (see also [Chen-Lin, Medina-Rincon, Zarembo '17])
- If the dilaton is constant then we get $S_{FT} = \chi\Phi_0$ (it would contribute as $\sim g_s^{-\chi}$ to the vev).
- It classically violates the Weyl invariance of the ws theory and has UV divergences: not when considered together with the rest of the one-loop term (fluctuations).

On the ST side: the one-loop ws string action II

2. The effective action from the one-loop fluctuations of the string ws

$$\Gamma_{\mathbb{K}} = -\log \int \left[D\zeta D\theta D\bar{\theta}\right] e^{-S_{\mathbb{K}}} = -\log \mathrm{Sdet}^{-1/2}\mathbb{K} \quad \text{with} \quad S_{\mathbb{K}} = \frac{1}{4\pi\ell_s^2}\int \sqrt{\gamma}\left(\zeta^a \mathcal{K}_{ab}\zeta^b + \bar{\theta}^a \mathcal{D}_{ab}\theta^b\right) d^2\sigma\,.$$

- The action $S_{\mathbb{K}}$ can be computed by expanding the Polyakov action at quadratic order and the GS string action (e.g. [Drukker, Gross, Tseytlin '00]). For us: Type IIA.

- These are second order fluctuations around the classical string solution: 8 bosonic fluctuations transverse to the ws ($\zeta^a, a = 1,\ldots,8$) and 8 GS fermionic fluctuations (θ^a).

- The path integral for the fluctuations is Gaussian, and it can be evaluated by means of functional determinants ($\mathrm{Sdet}^{-1/2}\mathbb{K}$).

On the ST side: the one-loop ws string action III

$$S_{\mathbb{K}} = \frac{1}{4\pi\ell_s^2}\int\sqrt{\gamma}\left(\zeta^a\mathcal{K}_{ab}\zeta^b + \bar{\theta}^a\mathcal{D}_{ab}\theta^b\right)d^2\sigma\,.$$

- The bosonic operators are: $\mathcal{K}_{ab} = \mathrm{diag}(\mathcal{K}_x, \mathcal{K}_x, \mathcal{K}_x, \mathcal{K}_x, \mathcal{K}_y, \mathcal{K}_y, \mathcal{K}_z, \mathcal{K}_z)$

⊥ equator on S^5 S^2 R^2 from (θ,ϕ)

$$\mathcal{K}_a = e^{-2\rho}\tilde{\mathcal{K}}_a\,, \qquad \tilde{\mathcal{K}}_a = -\partial_\sigma^2 - \partial_\tau^2 + E_a$$

"FLAT" OPERATORS

$$E_x = \partial_\sigma^2\rho + (\partial_\sigma\rho)^2 - 1 = \frac{7+8\cosh 2\sigma}{\sinh^2 2\sigma}\,,\quad E_y = \frac{1}{2}\partial_\sigma^2\rho = \frac{1+2\cosh 2\sigma}{\sinh^2 2\sigma}\,,\quad E_z = -\partial_\sigma^2\rho + h^2(\partial_\sigma\rho)^2 - 1 = \frac{3}{\sinh^2 2\sigma}$$

- The x directions: fluctuations transverse to the equator on the 5-sphere. The y directions: fluctuations on 2-sphere. The z directions are the fluctuations in R^2 obtained by combining the θ, ϕ directions.

On the ST side: the one-loop ws string action IV

$$S_{\mathbb{K}} = \frac{1}{4\pi\ell_s^2}\int\sqrt{\gamma}\,\left(\zeta^a\mathscr{K}_{ab}\zeta^b + \bar{\theta}^a\mathscr{D}_{ab}\theta^b\right)d^2\sigma\,.$$

- The fermionic operator is:

$$\mathscr{D} = e^{-3\rho/2}\tilde{\mathscr{D}}e^{\rho/2}, \quad \tilde{\mathscr{D}} = i\tau\cdot\partial + \tau_3 a + v\,, \quad a = \frac{i}{2\cosh\sigma}\,, \quad v = \frac{3i}{2\sinh\sigma}$$

"FLAT" OPERATOR

Here τ are the Pauli matrices.

- The action is computed starting from GS action for type IIA [Cvetic, Lu, Pope, Stelle, '99], and reduced to 8 2D fermions. Here we have a degeneracy, so we ended up with one fermionic operator.

On the ST side: the one-loop ws string action V

$$\log Z_{string} \approx -S_{classical} - S_{FT} + \log \mathrm{Sdet}^{-1/2}\mathbb{K} = -S_{classical} - S_{FT} - \Gamma_{\mathbb{K}}$$

- The one-loop terms continuously talk to each other: they both contribute to the divergences, they both contribute to the cancellation of the Weyl anomaly.

- We want to compute $\Gamma_{\mathbb{K}}$. It is easier to compute functional determinants for differential operators in flat space. We really want to compute $\Gamma_{\tilde{\mathbb{K}}}$, that is for the 'flat' operators.

- Logic here: if the theory is Weyl invariant, then we can strip off the Weyl factor and claim

$$S_{FT} + \Gamma_{\mathbb{K}} = \tilde{S}_{FT} + \Gamma_{\tilde{\mathbb{K}}}$$

On the ST side: the one-loop ws string action VI

- This means that we want to Weyl rescale the ws metric (i.e. remove $e^{2\rho}$)

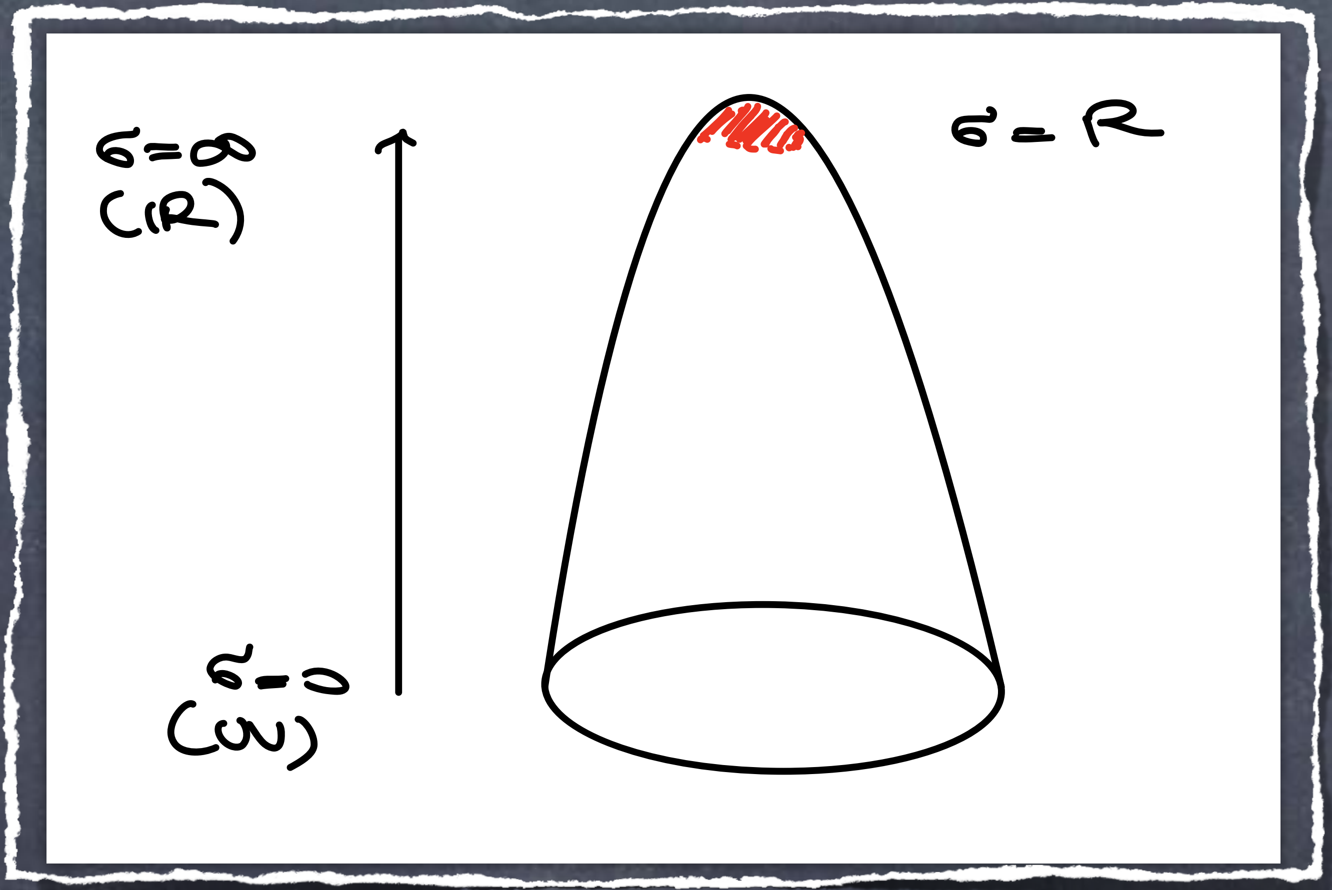
$$ds_2^2 = e^{2\rho}\left(d\sigma^2 + d\tau^2\right)\,, \quad \sqrt{\gamma} = e^{2\rho} = \frac{4\xi\ell_s^2}{\tanh\sigma\sinh^2\sigma}$$

- BUT: $e^{2\rho} \to 16\ell_s^2\xi\, e^{-2\sigma} = 16\ell_s^2\xi\, r^2$ as $\sigma \to \infty$ $(r = e^{-\sigma} \to 0)$, that is the Weyl factor is ill defined at the center of the ws [Cagnazzo, Medina-Rincon, Zarembo '17].

- How do we deal with this? We cut a little disk at the center of the ws, i.e. at $\sigma = R$, where R is an IR cut-off [Cagnazzo, Medina-Rincon, Zarembo '17].

On the ST side: the one-loop ws string action VII

- Also for the fluctuations: we want to compute the spectra, so we really need a compact manifold, so now we have fluctuations on a cylinder: this is good!
- But, we are changing the topology: from a disk to a cylinder [Cagnazzo, Medina-Rincon, Zarembo '17].



- We need to put the 'central' little disk back!
- Here the fluctuations are free since all the potentials vanish. They contribute only with pure divergences. We will take into account the divergences in $\Gamma_{\tilde{\mathbb{K}}}$.

On the ST side: the one-loop ws string action VIII

- But what about the FT term?

- At the center of the ws, the dilaton is constant:

$$\Phi \to \frac{1}{2} \log \frac{\xi^3}{\pi^2 N^2} \quad \text{as} \quad \sigma \to \infty$$

- Then the FT-action gives

$$\tilde{S}_{FT} = \chi \Phi_{|\sigma=\infty} = -\log \frac{N\pi}{\xi^{3/2}}$$

On the ST side: the one-loop ws string action IX

- Recap: after the Weyl rescaling we expect:

- The FT-action contributes from the small central disk which we cut-off:

$$\tilde{S}_{FT} = \chi\Phi_{|\sigma=\infty} = -\log\frac{N\pi}{\xi^{3/2}}$$

- The fluctuations contribute from the cylinder with a finite and a divergent part:

$$\tilde{\Gamma}_{\mathbb{K}} = \text{finite terms} + \text{divergent terms}$$

On the ST side: Weyl anomaly I

- The classical Weyl rescaling of the FT action is cancelled by the anomaly of the one-loop fluctuations of the string (e.g. [Callan, Thorlacius '89])

$$\langle T_i^i \rangle = \langle T_i^i \rangle_{\mathbb{K}} + (T_i^i)_{FT} = -\frac{1}{2} R^{(2)}$$

- There is a crucial contribution now from the dilaton: The classical Weyl anomaly of the FT-action is proportional to $e^{-2\rho}\partial_\sigma^2\Phi$.

- It does not happen in AdS5/CFT4! [Drukker, Gross, Tseytlin '00], [Forini, VGMP, Griguolo, Seminara, Vescovi '15], [Cagnazzo,Medina-Rincon,Zarembo '17].

On the ST side: Weyl anomaly II

- Then, we are expected to see a logarithmic divergence controlled by χ (e.g. [Drukker, Gross, Tseytlin '00])

$$\frac{1}{2\pi}\int \langle T_i^i \rangle \, \mathrm{vol}_\gamma = -\chi = -1 \,,$$

- This is cancelled against extra contributions due to the 'rotation' of the GS 10D to 2d fermions and to the measure [Alvarez '83][Drukker, Gross, Tseytlin '00].

- We do not include these contributions.

On the ST side: Weyl anomaly III

- In concrete: the one-loop effective action will have a log divergence as

$$\Gamma_{\mathbb{K}} = \chi \log\left(\Lambda e^{-R}\right) + \text{finite terms}$$

$$\langle W \rangle = Z_{string} \approx e^{-S_{cl}} e^{-S_{FT} - \Gamma_{\mathbb{K}}}$$

- This divergence is universal, it is found for fluctuations near any minimal surface with a disk topology in AdS [Drukker, Gross, Tseytlin '00], [Forini, VGMP, Griguolo, Seminara, Vescovi '15], [Giombi,Tseytlin '20].

- We confirm the universality and the topological nature of these divergences in a more general consistent string background.

On the ST side: Computation of $\tilde{\Gamma}_{\mathbb{K}}$

- We use the phase shift method (e.g. [Chen-Lin, Medina-Rincon, Zarembo '17][Cagnazzo, Medina-Rincon, Zarembo '17]) to compute the contribution from the fluctuations with 'flat' operators

$$\tilde{\Gamma}_{\mathbb{K}}(R) = \frac{1}{2}\log\frac{(\det\tilde{\mathcal{K}}_x)^4(\det\tilde{\mathcal{K}}_y)^2(\det\tilde{\mathcal{K}}_z)^2}{(\det\tilde{\mathcal{D}})^8}$$

- And the answer is…

$$\tilde{\Gamma}_{\mathbb{K}}(R) = 2\log\pi + 1\log(\Lambda e^{-R})$$

UV divergence: a large p cut-off

IR cut-off

- NB. $1 = \chi$ and R is the IR cut-off where we cut the small disk.

The matching I

- Recap: We want to reproduce the vev of the circular WL at strong coupling

$$\log\langle \mathscr{W} \rangle \approx \xi + \log \frac{N_{SYM}}{\xi}$$

- Recap: We have computed

$$\log Z \approx -S_{classical} - S_{FT} - \Gamma_{\mathbb{K}} = \xi + \log \frac{N_{SYM}}{\xi^{3/2}\pi} - \log(\Lambda e^{-R_{SYM}})$$

- We have to deal with the cut-offs Λ, R_{SYM}. The only way is to consider a ratio of string partition functions with the same topology [Forini,VGMP,Griguolo,Seminara,Vescovi '15][Faraggi,Pando Zayas,Silva, Trancanelli '16][Forini,Tseytlin,Vescovi '17][Cagnazzo,Medina-Rincon,Zarembo '17][Medina-Rincon,Tseytlin,Zarembo '18], thanks to the fact that the divergences are universal and 'topological'.

The matching II

- But be careful! R should be replaced by a diffeo invariant regulator [Cagnazzo,Medina-Rincon,Zarembo '17]:

$$A = \frac{2\pi}{\ell_s^2} \int_R^\infty e^{2\rho} d\sigma$$

The IR cut-off is translated into the ws area that we cut off when computing the spectra for the fluctuations [Cagnazzo,Medina-Rincon,Zarembo '17]

- Here we have:

$$A = 16\pi\xi\, e^{-2R_{SYM}} .$$

$$\log Z_{SYM}^{string} \approx \xi + \log \frac{4N_{SYM}}{\xi\sqrt{\pi}} - \log(\Lambda\sqrt{A})$$

The matching III

- We are ready. But we divide by what? To keep type IIA set-up, we compute the partition function for a circular string in $AdS_4 \times CP^3$.

- The value of the dual 1/2-BPS WL [Drukker, Trancanelli '09] in ABJM is known via localisation [Kapustin, Willett, Yaakov '09][Marino, Putrov '09][Drukker, Plefka, Young '08][Drukker, Marino, Putrov '10]

$$\langle \mathscr{W} \rangle_{ABJM} \approx \frac{N_{ABJM}}{4\pi\lambda} e^{\pi\sqrt{2\lambda}}, \qquad \lambda \gg 1, \qquad g_s = \left(32\pi^2\lambda^5\right)^{1/4} N_{ABJM}^{-1}$$

where λ is the 't Hooft coupling.

- The string partition function for a circular string in $AdS_4 \times CP^3$ has been computed by with different methods [Kim, Kim, Lee '12][Aguilera-Damia,Faraggi,Pando Zayas, Rathee, Silva '18][Giombi, Tseytlin '20].

The matching IV (*)

- But we need to use the same regularisation scheme to meaningfully consider the ratio. We compute the partition function for a circular string in $AdS_4 \times CP^3$ at one loop in the large λ limit using the phase shift method.

- The classical solution [Drukker,Plefka, Young '08][Chen, Wu '08] wraps the equator of S^3 inside AdS_4 and it is constant on the compact space.

- and the answer is

$$\log Z^{string}_{ABJM} = \pi\sqrt{2\lambda} + \log\frac{N_{ABJM}}{\pi^{3/2}\lambda} - \log(\Lambda\sqrt{A})$$

- NB. This is the holographic dual of ABJM in 3d!

The matching VI

- Recap: on the string side, we computed up tp one-loop:

$$\log Z^{string}_{SYM} = \xi + \log \frac{4N_{SYM}}{\xi\sqrt{\pi}} - \log(\Lambda\sqrt{A}) \qquad \log Z^{string}_{ABJM} = \pi\sqrt{2\lambda} + \log \frac{N_{ABJM}}{\pi^{3/2}\lambda} - \log(\Lambda\sqrt{A})$$

- To be compared with the vevs up to one-loop:

$$\langle W \rangle_{SYM} = \frac{N_{SYM}}{\xi} e^{\xi} \qquad \langle W \rangle_{ABJM} = \frac{N_{ABJM}}{4\pi\lambda} e^{\pi\sqrt{2\lambda}}$$

- Then it is clear:

$$\frac{\langle W \rangle_{SYM}}{\langle W \rangle_{ABJM}} = \frac{Z^{string}_{SYM}}{Z^{string}_{ABJM}}\,.$$

Summary and ...

- We have computed the holographic dual of the vev of a 1/2-BPS circular WL in MSYM on a S^5 at one-loop in ξ (the 't Hooft coupling).

- In doing this, we have faced a few issues: the role of the dilaton in the Weyl anomaly as well as in the finite result, IR and UV divergences.

- By means of a ratio with a the string partition function for a circular string in $AdS_4 \times CP^3$, we find an agreement with the field theory results.

... outlook I (work in progress with Friðrik Freyr Gautason and Pieter Bomans)

- We need more examples of precision tests in non-conformal settings.
- A possibility: MSYM on S^3. The 1/2-BPS WL is known at any ξ [Bobev, Bomans, Gautason, Minahan, Nedelin '19]:

$$\log\langle W\rangle = \log\frac{3}{\xi^3}\left(\xi\cosh\xi - \sinh\xi\right) \approx \xi - 2\log\xi + \log\frac{3}{2} + \ldots\,, \qquad \xi\to\infty\,.$$

where ξ is related to the 't Hooft coupling, N is the rank of the gauge group, and $\mathscr{R}$ is the radius of the 3-sphere:

$$\xi^3 = 6\pi^2\lambda\,, \qquad \lambda = Ng^2_{YM}\mathscr{R}$$

- The holographic dual is a fundamental string (with the proper boundary conditions) in a 10D background realised by D2 spherical branes [Bobev, Bomans, Gautason '18] [Bobev, Bomans, Gautason, Minahan, Nedelin '19].

- The classical contribution was matched in [Bobev, Bomans, Gautason, Minahan, Nedelin '19].

- We want to reproduce the one-loop contributions from the corresponding string partition function:

$$\log\langle W\rangle = \log\frac{3}{\xi^3}\left(\xi\cosh\xi - \sinh\xi\right) \approx \xi - 2\log\xi + \log\frac{3}{2} + \dots\,, \qquad \xi\to\infty$$

- So far: We have checked the Weyl anomaly, preliminarily computed the contributions from the fluctuations, however here the problem is that the dilaton is IR and UV divergent.

… outlook II (work in progress with Friðrik Freyr Gautason and Konstantin Zarembo)

- We need more examples of precision tests in non-conformal settings.

- Another possibility: $\mathcal{N}=2^*$ SYM on S^4. This is a mass deformation of $\mathcal{N}=4$ MSYM in 4D.

- The vev of 1/2-BPS WL can be computed via localisation at large N [Pestun '07]. It depends on the 't Hooft coupling λ and on the (dimensionless) mass parameter $M\mathcal{R}$, where $\mathcal{R}$ is the radius of the 4-sphere and M is mass deformation.

- At large λ, at leading (classical) and next-to-leading order (one-loop), the vev for the circular WL was computed from the matrix model for any $M\mathcal{R}$ [Buchel, Russo, Zarembo '13] [Chen-Lin, Gordon, Zarembo '13].

- The holographic dual of $\mathcal{N} = 2*$ SYM on S^4 is numerically known [Bobev, Elvang, Freedman, Pufu '13] [Bobev, Gautason, van Muiden '18]. It is a generalisation of the holographic dual of $\mathcal{N} = 2*$ SYM on R^4 [Pilch, Warner '00].

- The classical vev was holographically reproduced for any mass parameter [Bobev, Gautason, van Muiden '18].

- The one-loop vev was holographically reproduced in the large mass limit ('decompactification' limit) [Chen-Lin, Medina-Rincon, Zarembo '17]. The geometry here is given by the PW background.

- We want to compute the one-loop vev Γ_1 for small and large mass parameter from the string side:

$$\log\langle W\rangle \approx \sqrt{\lambda(1+M^2\mathcal{R}^2)} - \Gamma_1$$

where

$$\Gamma_1 = \begin{cases} a_0 M\mathcal{R} + a_1 \log(M\mathcal{R}) + a_2\,, & M\mathcal{R}\to\infty \\ b_0 M^2\mathcal{R}^2\,, & M\mathcal{R}\to 0 \end{cases}$$

- N.B. The small mass limit corresponds to correction to $\mathcal{N}=4$ SYM (holographically dual to $AdS_5\times S^5$), while the large mass limit corresponds to correction to $\mathcal{N}^*=2$ on flat space (holographically dual to the PW background).

Thanks!

Bonus track: The 2d geometry of the dual of 1/2-BPS WL

- The 2d Ricci scalar is $$R^{(2)} = \frac{\tanh\sigma}{4\ell_s^2\xi}\left(\mathrm{sech}^2\sigma - 4\right)$$

$$R^{(2)} \to -\frac{3}{4\ell_s^2\xi\sqrt{U}} \quad \text{as} \quad \sigma \to 0 \quad (U \to \infty)$$

$$R^{(2)} \to -\frac{1}{\ell_s^2\xi} \quad \text{as} \quad \sigma \to \infty \quad (r = e^{-\sigma} \to 0)$$

- To be compared with the corresponding solution in AdS5 x S5, where the worldsheet metric is AdS2 [Drukker, Gross, Tseytlin '00].

Bonus track: the one-loop ws string action I

- The ws action for the fluctuations is $S_{bosons} + S_{fermions}$
- In static gauge, the bosonic action can be written in terms of the 8 fluctuations transverse to the ws [Forini, VGMP, Griguolo, Seminara, Vescovi '15]:

$$\mathscr{L}_{\text{transv}} = \sqrt{\gamma}\left(\gamma^{ij}\mathrm{D}_i\zeta^a\mathrm{D}_j\zeta_a - M_{ab}\zeta^a\zeta^b\right)$$

Here a=1,...,8, and i,j=curved ws indices. The transverse fluctuations are defined as

$$\zeta^{\hat{\mu}} = \zeta^\mu E_\mu^{\hat{\mu}} = N_a^{\hat{\mu}}\zeta^a$$

Here $\hat{\mu}$ are 10D flat indices, μ are 10D curved indices, $N_a^{\hat{\mu}}$ are 8 orthonormal vector fields orthogonal to the ws, $E_\mu^{\hat{\mu}}$ are the vierbein.

- For our classical configuration: $\mathrm{D}_i = \partial_i$.

Bonus track: the one-loop ws string action II

- The mass term is constructed from the 10D Riemann tensor and from the extrinsic curvature. For our classical solution: $K^{\mu}_{ij} = 0$, then

$$M_{ab} = R_{\hat{\mu}\,\hat{\lambda},\,\hat{\nu}\,\hat{\kappa}} E^{\hat{\lambda}}_{\mu}\,\partial_i X^{\mu}\,E^{\hat{\kappa}}_{\nu}\,\partial^i X^{\nu}\,N^{\hat{\mu}}_{a} N^{\hat{\nu}}_{b}$$

- Explicitly the final transverse bosonic action reads as:

$$\mathscr{L}_{\text{transv}} = -\,\zeta^a \delta^{ij}\partial_i\partial_j\zeta_a - e^{2\rho}M_{aa}\zeta^a\zeta^a \equiv \zeta^a \tilde{\mathscr{K}}_a \zeta^a\,,$$

where

$$E_x = -\,e^{2\rho}M_x = \partial^2_\sigma\rho + (\partial_\sigma\rho)^2 - 1 = \frac{7 + 8\cosh 2\sigma}{\sinh^2 2\sigma}, \quad E_y = -\,e^{2\rho}M_y = \frac{1}{2}\partial^2_\sigma\rho = \frac{1 + 2\cosh 2\sigma}{\sinh^2 2\sigma}, \quad E_z = -\,e^{2\rho}M_y = \frac{1 - 3h^2}{2}\partial^2_\sigma\rho + h^2(\partial_\sigma\rho)^2 - h^2 = \frac{1 + 2h^2 + 2(1 - h^2)\cosh 2\sigma}{\sinh^2 2\sigma}$$

- The x directions: fluctuations transverse to the equator on the 5-sphere. The y directions: fluctuations on 2-sphere. The z directions are the fluctuations in R^2 obtained by combining the θ, ϕ directions.

Bonus track: the one-loop ws string action III

- The fermionic action is computed starting from GS action for type IIA [Cvetic, Lu, Pope, Stelle, '99]

$$S_{\text{fermions}} = -\frac{1}{2\pi\ell_s^2}\int\left\{i\bar{\theta}P^{ij}\Gamma_i D_j\theta - \frac{i}{8}\bar{\theta}P^{ij}\Gamma_{11}\Gamma_i^{\ \mu\nu}H_{j\mu\nu}\theta + \frac{i}{8}e^{\Phi}\bar{\theta}P^{ij}\Gamma_i(-\Gamma_{11}\not{F}_2 + \not{F}_4)\Gamma_j\theta\right\},$$

where Γ_μ are the 10D Gamma matrices, the projector is $P^{ij} = \sqrt{\gamma}\gamma^{ij} - i\epsilon^{ij}\Gamma_{11}$, the covariant derivative is $D_j = \partial_j + \frac{1}{4}\partial_j X^\mu \omega_\mu^{\hat{\mu}\hat{\nu}}\Gamma_{\hat{\mu}\hat{\nu}}$. We fix the κ-symmetry with the projector $\mathscr{P} \equiv \frac{(\mathbf{1} - i\Gamma_{\hat{\sigma}\hat{\tau}}\Gamma_{11})}{2}$.

- Explicitly the kinetic term is

$$\not{D} = e^{-\rho}\left(\Gamma_{\hat{\sigma}}\partial_\sigma + \Gamma_{\hat{\tau}}\partial_\tau + \frac{1}{2}\partial_\sigma\rho\Gamma_{\hat{\sigma}}\right).$$

- Explicitly the flux terms are $F_2 = dC_1 = \frac{hN\pi i\ell_s}{\xi}\tanh^2\sigma\,\text{vol}_{\text{R}_z^2}, \quad F_4 = dC_3 = -3N\pi\ell_s^3\,\text{vol}_{\text{R}_z^2}\wedge\text{vol}_{\text{R}_y^2} \quad H_3 = 0$

Bonus track: Weyl anomaly

- The Weyl anomaly is closely related to the log divergences in the partition function

$$\langle T^i_i \rangle_{\mathbb{K}} = \frac{2\pi}{\sqrt{\gamma}} \frac{\delta \Gamma_{\mathbb{K}}}{\delta \rho}$$

- The RHS can be expressed in terms of the so called Seeley-De Witt coefficients $b_2(F), b_2(B)$

$$\delta \log \det \mathscr{K} = -2a_2(\delta\rho \,|\, \mathscr{K})\,, \qquad \delta \log \det \mathscr{D}^2 = -2a_2(\delta\rho \,|\, \mathscr{D}^2)\,,$$

$$a_2(f \,|\, \mathscr{O}) = \frac{1}{4\pi} \int \sqrt{\gamma}\, f\, b_2(\mathscr{O}) + \; boundary \;\; terms$$

Here f is a test function, $\mathscr{K}$ is a bosonic operator, $\mathscr{D}$ is the fermionic operator.

- The Seeley-De Witt coefficients control the logarithmic divergences of the bosonic and fermionic fluctuations

$$\langle T^i_i \rangle_{\mathbb{K}} = \frac{1}{4} \mathrm{Tr}\, b_2(\mathscr{D}^2) - \frac{1}{2} \mathrm{Tr}\, b_2(\mathscr{K})$$

- The bosonic and fermionic contribution proportional to $R^{(2)}$ is in AdS5/CFT4 [Drukker, Gross, Tseytlin '00]

$$8\frac{R^{(2)}}{6}+\frac{R^{(2)}}{12}\mathrm{Tr}(\mathbf{I}) = 8\frac{R^{(2)}}{6}+8\frac{4R^{(2)}}{12} = 4\,R^{(2)}$$

As in AdS5/CFT4 we have 8 2d fermions but these are GS fermions [Drukker, Gross, Tseytlin '00]

On the field theory side: 5D MSYM on S^5 (*)

- The Lagrangian [Blau '00][Minahan, Zabzine '15]:

$$\mathscr{L} = -\frac{1}{2g_{\rm YM}^2}\mathrm{Tr}\left(\frac{1}{2}F_{MN}F^{MN} - \bar{\Psi}\Gamma D\Psi + \frac{1}{2\mathscr{R}}\Psi\Gamma^{089}\Psi + \frac{4}{\mathscr{R}^2}\phi^A\phi_A + \frac{3}{\mathscr{R}^2}\phi_i\phi^i + \frac{2i}{3\mathscr{R}}\left[\phi^A,\phi^B\right]\phi^C\varepsilon_{ABC} - K_m K^m\right)$$

- The Lagrangian is obtained via dimensional reduction from 10D SYM on a flat space+minimal coupling of the S^5 + other terms to preserve 16 real supercharges.
- Euclidean: the above Lagrangian needs to be Wick-rotated ($\phi^0 \to i\phi^0$).
- M,N=0, ... 9 are Lorentz indices, and they split into spacetime indices on S^5 and scalar indices I,J=0,6,...,9. I,J are further broken to i,j=6,7 and A,B=0,8,9.
- Ψ have 16 real components and obey $\Gamma_{11}\Psi = \Psi$. K_m are auxiliary fields.

On the FT side: The matrix model I (*)

- The theory can be localised [Pestun '07][Minahan, Zabzine '15][Minahan '15][Gorantis, Minahan, Naseer '17] on a locus where: $A_\mu = 0$ and $\phi_I = 0$ when $I \neq 0$. ϕ_0 is used to construct a Hermitian $N \times N$ matrix M.
- The large N matrix-model partition function is (up to non-perturbative corrections): [Minahan, Zabzine '15][Minahan '15][Gorantis, Minahan, Naseer '17]

$$Z = \frac{1}{N!}\int \prod_{i=1}^{N} d\mu_i \, e^{-S_{eff}} \quad \text{with} \quad S_{eff} = \frac{2\pi^2 N}{\xi}\sum_{i=1}^{N}\mu_i^2 - \sum_{j\neq i}^{N}\sum_{i=1}^{N}\log|\sinh(\pi(\mu_i - \mu_j))|$$

where μ_i are the eigenvalues of the Hermitian $N \times N$ matrix M

On the FT side: The matrix model II (*)

- The saddle point equation:

$$N\frac{2\pi}{\xi}\mu_i = \sum_{j\neq i}\coth\pi(\mu_i-\mu_j)\,, \qquad\qquad i,j=1,\ldots,N\,.$$

- We introduce an eigenvalue distribution $\rho(\mu) := \frac{1}{N}\sum_{i=1}^{N}\delta(\mu-\mu_i)$
- We take the large N continuum limit: the saddle point eq is

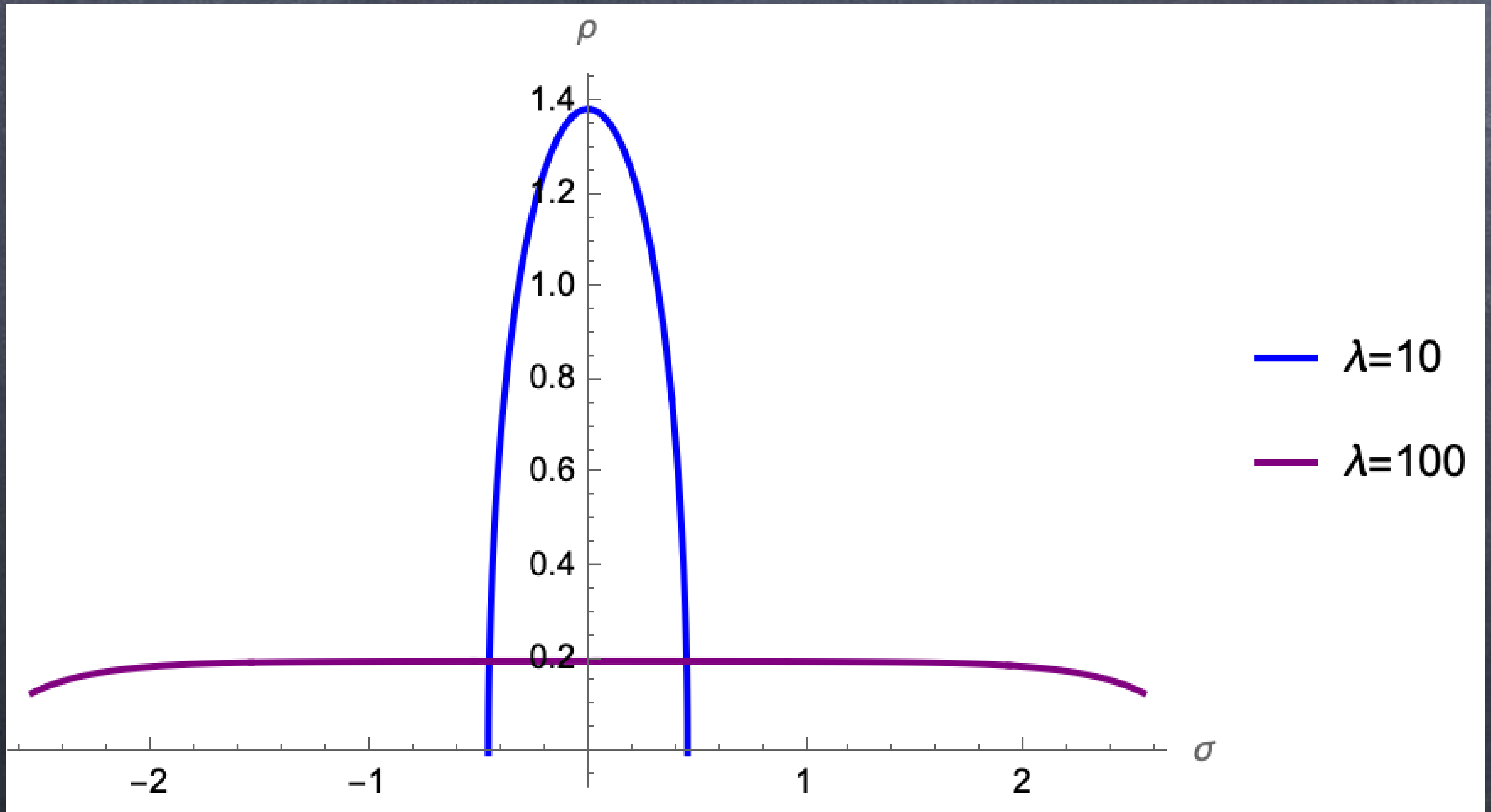
$$\frac{2\pi}{\xi}\mu = \mathrm{PV}\int_{-b}^{b}\rho(\mu')\,\coth(\mu-\mu')d\mu'$$

NB. it is an eq for ρ and b (+normalization condition)

On the FT side: The matrix model III (*)

- The matrix-model partition function as well as the corresponding integral eq are well-known since they appear in the matrix formulation of Chern-Simons theories on S^3 [Kim, Kim '12].

- Solutions [Marino '04]: $$\rho(\mu) = \frac{2}{\xi}\arctan\left(\frac{\sqrt{e^{\xi} - \cosh^2(\pi\mu)}}{\cosh(\pi\mu)}\right)$$ with $$b = \frac{1}{\pi}\text{arccosh}\,(e^{\xi/2})$$



On the FT side: 1/2-BPS WL on 5D SYM

- The vev of 1/2-BPS WL [Maldacena '98][Bobev, Bomans, Gautason, Minahan, Nedelin '19] which wraps the equator of S^5 is

$$\langle W \rangle = \left\langle \mathrm{Tr}\left(P e^{i\oint A_\mu dx^\mu + i\oint ds\, \phi^0} \right) \right\rangle$$

For us: $A_\mu = 0$ (5-dimensional gauge field)

- Using the density of eigenvalues ρ and taking the continuum limit:

$$\langle W \rangle = N \int_{-b}^{b} \rho(\mu) e^{2\pi\mu} d\mu + \mathcal{O}\left(\frac{1}{N}\right) = \frac{N}{\xi}\left(e^{\xi} - 1\right) + \mathcal{O}\left(\frac{1}{N}\right)$$

at large N but for <u>any</u> 't Hooft coupling ξ

On the ST side: Weyl anomaly I (*)

- The Weyl anomaly is closely related to the log divergences in the partition function

$$\langle T^i_i \rangle_{\mathbb{K}} = \frac{2\pi}{\sqrt{\gamma}} \frac{\delta \Gamma_{\mathbb{K}}}{\delta \rho}$$

The trace of the quantum energy-momentum tensor is defined by the variation of the effective action w.r.t. the Weyl factor ρ.

- The RHS can be expressed in terms of the so called Seeley-De Witt coefficients $b_2(F), b_2(B)$
- The Seeley-De Witt coefficients control the logarithmic divergences of the bosonic and fermionic fluctuations

$$\langle T^i_i \rangle_{\mathbb{K}} = -\frac{1}{2}\left(b_2(\mathscr{D}^2) + b_2(\mathscr{K})\right)$$

On the ST side: Weyl anomaly II (*)

- The 8 bosons contribute with [Gilkey '95]

$$b_2(\mathscr{K}) = \mathrm{Tr}\left(\frac{R^{(2)}}{6} - e^{-2\rho}E\right)$$

- The 8 2d fermions contribute with [Gilkey '95]

$$b_2(\mathscr{D}^2) = \frac{R^{(2)}}{12}\mathrm{Tr}(\mathbf{I}) - \frac{1}{4}\mathrm{Tr}\left(\tau^i \mathscr{M}_F \tau_i \mathscr{M}_F\right) - \frac{1}{4}\mathrm{Tr}\left(\tau^i \mathscr{M}_F^\dagger \tau_i \mathscr{M}_F^\dagger\right) .$$

where the fermionic 'mass' is related to (rescaled) $a^2 - v^2$ in $\mathscr{D}$, and τ are the Pauli matrices.

On the ST side: Weyl anomaly III (*)

- The bosonic and fermionic contribution proportional to $R^{(2)}$ is in AdS5/CFT4 [Drukker, Gross, Tseytlin '00]

- The bosonic and fermionic 'mass' contribution to the logarithmic divergences is:

$$\mathrm{Tr}\,\left(e^{-2\rho}E\right)+\frac{1}{4}\mathrm{Tr}\left(\tau^{i}\mathscr{M}_{F}\tau_{i}\mathscr{M}_{F}\right)+\frac{1}{4}\mathrm{Tr}\left(\tau^{i}\mathscr{M}_{F}^{\dagger}\tau_{i}\mathscr{M}_{F}^{\dagger}\right)=R^{(2)}+\underbrace{2\,e^{-2\rho}\,\partial_{\sigma}^{2}\Phi}_{!!}$$

It does not happen in AdS5/CFT4! [Drukker, Gross, Tseytlin '00], [Forini, VGMP, Griguolo, Seminara, Vescovi '15], [Cagnazzo,Medina-Rincon,Zarembo '17]

- Indeed the classical Weyl anomaly of the FT-action is

$$(T_{i}^{i})_{FT}=-\,\partial_{i}X^{\mu}\partial^{i}X^{\nu}\,\nabla_{\mu}\nabla_{\nu}\Phi$$

Comments on the Giombi-Tseytlin's proposal I (*)

- Proposal to match the strong coupling expansion of circular WL in $\mathcal{N}=4$ SYM and ABJM (for a string with a disk topology):

$$\langle W\rangle = Z_{string} \approx \frac{1}{g_s}\sqrt{\frac{T}{2\pi}}e^{2\pi T}e^{\bar{\Gamma}_1}$$

- where T is the effective string tension: $e^{-S_{classical}} = e^{\text{vol}(AdS_2)T}$;

$$\text{AdS}_5: \quad T = \frac{\sqrt{\lambda}}{2\pi}, \qquad \text{AdS}_4: \quad T = \frac{\sqrt{2\lambda}}{2}$$

- where $\bar{\Gamma}_1$ is the ratio of the one-loop det's computed by means of the heat kernel

$$\text{AdS}_5: \ \bar{\Gamma}_1 = \frac{1}{2}\log(2\pi), \qquad \text{AdS}_4: \ \bar{\Gamma}_1 = 0$$

Comments on the Giombi-Tseytlin's proposal II (*)

- How does this compare with us? Let's read the effective tension from the classical action: $e^{-S_{classical}} = e^{\xi}$ then $\xi \sim T$.
- What is the term $\sqrt{T}$? This is nothing but our $\sqrt{A}$!
- What is the term g_s^{-1}? This is nothing but our FT-term.
- Our finite term $\bar{\Gamma}_1$ is different, but we are employing a different method with a different regularisation scheme (the matching has been 'adjusted' for the heat kernel method)
- We confirm the GT-proposal for the first time outside a conformal setting.

On the ST side: Computation of $\tilde{\Gamma}_{\mathbb{K}}$ I (*)

- We use the phase shift method (e.g. [Chen-Lin, Medina-Rincon, Zarembo '17][Cagnazzo, Medina-Rincon, Zarembo '17]) to compute the contribution from the fluctuations with 'flat' operators

$$\tilde{\Gamma}_{\mathbb{K}}(R) = \frac{1}{2}\log\frac{(\det\tilde{\mathscr{K}}_x)^4(\det\tilde{\mathscr{K}}_y)^2(\det\tilde{\mathscr{K}}_z)^2}{(\det\tilde{\mathscr{D}})^8}$$

- This amounts to solve a one-dimensional Schrödinger problem once we have Fourier-expanded w.r.t. $\tau \to i\omega$. For example for the bosonic operators:

$$\tilde{\mathscr{K}}_a\eta_\omega(\sigma) = \left(-\partial_\sigma^2 + \omega^2 + E_a(\sigma)\right)\eta_\omega(\sigma) = \lambda\,\eta_\omega(\sigma)$$

- The solutions behave as waves at large σ (since the potentials vanish there). Then the dispersion relation is $\lambda = \omega^2 + p^2$. The effect of the 'scattering' is only to shift the waves:

$$\eta_\omega \to C\sin(p\sigma + \delta(\omega, p))\,.$$

On the ST side: Computation of $\tilde{\Gamma}_{\mathbb{K}}$ II (*)

- The goal is to compute the phase shift $\delta(\omega, p)$ for all the operators.
- It is a Schrödinger problem with Dirichlet boundary conditions:

$$\eta_\omega(\sigma = 0) = 0\,, \qquad \eta_\omega(\sigma = R) = 0$$

where R is a IR cut-off. This gives the quantisation condition and the distribution of eigenvalues:

$$pR + \delta(\omega, p) = \pi k\,, \qquad \rho = \frac{dk}{dp} = \frac{1}{\pi}\left(R + \frac{d\delta(\omega, p)}{dp}\right)$$

- The functional determinant is then

$$\log \det \tilde{\mathscr{K}} = \sum_\omega \int_0^\infty \frac{dp}{\pi}\left(R + \frac{d\delta(\omega, p)}{dp}\right)\log(p^2 + \omega^2)$$

On the ST side: Computation of $\tilde{\Gamma}_{\mathbb{K}}$ III (*)

- After integrating by parts over p, transforming the sum over Matsubara frequencies as a contour integral (it picks up poles $\omega = \pm ip$) we have (e.g. [Chen-Li, Medina-Rincon, Zarembo '17][Cagnazzo, Medina-Rincon, Zarembo '17])

$$\tilde{\Gamma}_{\mathbb{K}}(R) = -\int_0^{\infty} dp \left[\coth(\pi p)(4\delta_x + 2\delta_y + 2\delta_z) - \tanh(\pi p)(4\delta_+ + 4\delta_-)\right] - \underbrace{R}_{\text{IR cut-off}},$$

- We can compute the bosonic phase shifts $\delta_x, \delta_y, \delta_z$ analytically. We have to compute the fermionic phase shifts δ_+, δ_- numerically.

- And the answer is...

$$\tilde{\Gamma}_{\mathbb{K}}(R) = 2\log\pi + 1\log(\Lambda \underbrace{e^{-R}}_{\text{IR cut-off}})$$

UV divergence: a large p cut-off

The matching IV (*)

- But we need to use the same regularisation scheme to meaningfully consider the ratio.
- We compute the partition function for a circular string in $AdS_4 \times CP^3$ at one loop in the large λ limit using the phase shift method.
- The classical solution [Drukker,Plefka, Young '08][Chen, Wu '08] wraps the equator of S^3 inside AdS_4 and it is constant on the compact space.
- The ws metric is $$ds^2 = e^{2\rho}\left(d\sigma^2 + d\tau^2\right), \qquad e^{2\rho} = \frac{\pi\sqrt{2\lambda}\,\ell_s^2}{\sinh^2\sigma}$$
- The (regularised) classical action:

$$S_{classical} = -\pi\sqrt{2\lambda}$$

The matching V (*)

- The dilaton is constant and the FT term gives:

$$S_{FT} = \chi \log g_s = -\log \frac{N_{ABJM}}{\sqrt{\pi}(2\lambda)^{5/4}},$$

- The contributions from the fluctuations with the phase shift method is:

$$\Gamma_{AdS_4} = 2\log\pi + \log(\Lambda e^{-R_{ABJM}})$$

- The diffeo invariant IR regulator is $A_{ABJM} = 4\pi^2\sqrt{2\lambda}\, e^{-2R_{ABJM}}$

- Collecting all together:

$$\log Z^{string}_{ABJM} \approx \pi\sqrt{2\lambda} + \log\frac{N_{ABJM}}{\pi^{3/2}\lambda} - \log(\Lambda\sqrt{A})$$

… outlook III (*)

- We need more observables. Is there an analogue of the 'latitude' 1/4-BPS WL of $\mathcal{N} = 4$ SYM in 4d? See [Mezei, Pufu, Wang '18] (the holographic dual is a fundamental string which ends on a circle at the boundary of AdS_5 and on a latitude (at angle θ_0) on a $S^2 \subset S^5$ [Drukker, Fiol '05] [Drukker '06] [Drukker, Giombi, Ricci, Trancanelli '07])

- We have the free energy of MSYM on a S^5 at any ξ in the planar limit. The leading order at strong coupling was computed and matched in [Bobev, Bomans, Gautason, Minahan, Nedelin '19]. It would be interesting to holographically compute the next-to-leading order corrections in ξ.

- 5D SYM is supposed to flow to (2,0) theory in 6D [Douglas '10][Lambert, Papageorgakis, Schmidt-Sommerfeld '10] in the UV. There the WL should correspond to a BPS surface operator. On the holographic side: M2-brane in AdS_7. Classically we checked, but what about at one-loop?