

Research in theoretical high-energy physics at NBIA

Matthias Wilhelm



NBIA MSc Day 2022

October 12th, 2022



The Niels Bohr
International Academy

VILLUM FONDEN



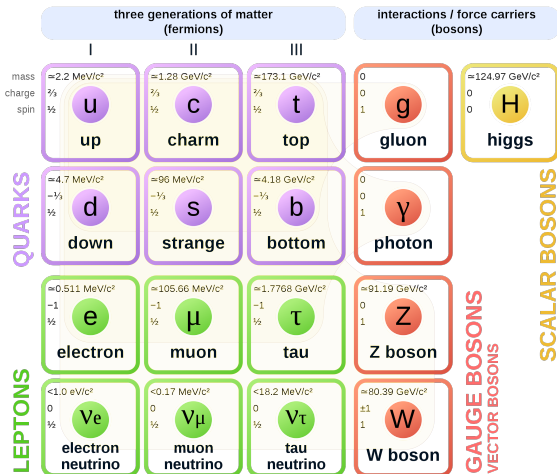
Large Hadron Collider



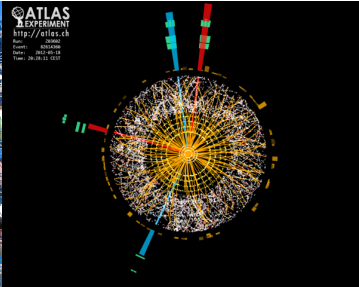
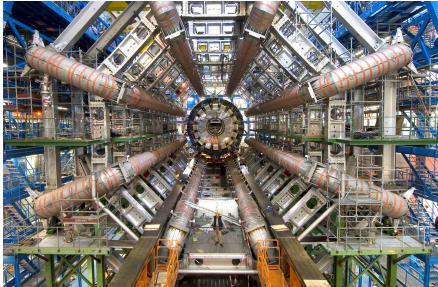
The world's largest machine = most powerful microscope

Standard model of particle physics

Standard Model of Elementary Particles



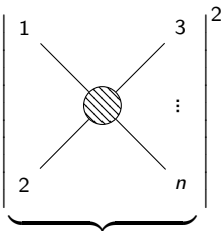
How do we see these particles?



Short lived \Rightarrow Only see decay products in detectors!

Theoretical description

Cross section = probability of two incoming particles to scatter into $n - 2$ outgoing particles:

$$\sigma = \left[\begin{array}{ccc} 1 & & 3 \\ & \diagdown & / \\ & \text{●} & \\ & / & \diagdown \\ 2 & & n \\ & & \vdots \end{array} \right]^2$$


The diagram shows a central shaded circle representing an interaction vertex. Two lines enter from the left, labeled 1 and 2. Two lines exit to the right, labeled 3 and n. A vertical ellipsis (three dots) is placed between lines 3 and n, indicating additional outgoing particles. The entire diagram is enclosed in large square brackets with a superscript 2 to the right. A horizontal curly brace is positioned below the diagram, spanning the width of the interaction region.

Theoretical description

Cross section = probability of two incoming particles to scatter into $n - 2$ outgoing particles:

$$\sigma = \left| \begin{array}{ccc} 1 & & 3 \\ & \circlearrowleft & \\ 2 & & n \\ & & \vdots \end{array} \right|^2$$

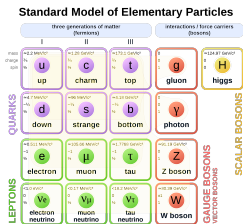
$\underbrace{\hspace{10em}}_{\mathcal{A}}$

Amplitude \mathcal{A} can be calculated using Quantum Field Theory

What is Quantum Field Theory?

Quantum Field Theory

- = Quantum mechanics + special relativity
- describes all known interactions among all known particles except gravity via so-called gauge theories

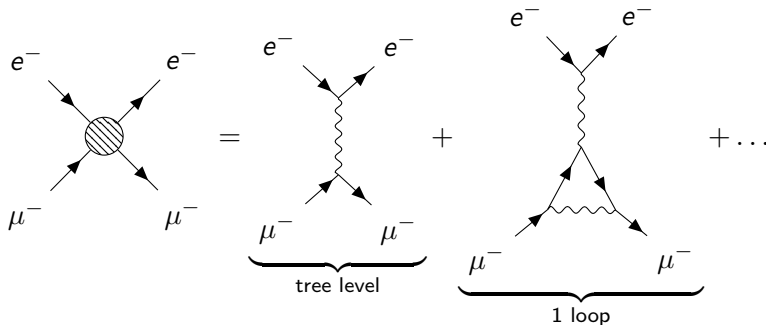
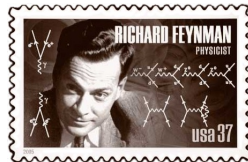


- ▷ Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD)
- describes classical gravity (general relativity) → Emil
- Course “Quantum Field Theory I”

Amplitudes from Quantum Field Theory

Feynman diagrams

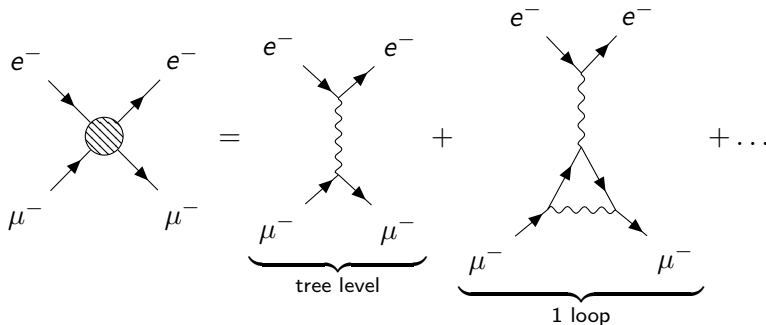
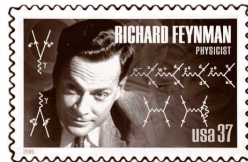
= sums over possible particle histories



Amplitudes from Quantum Field Theory

Feynman diagrams

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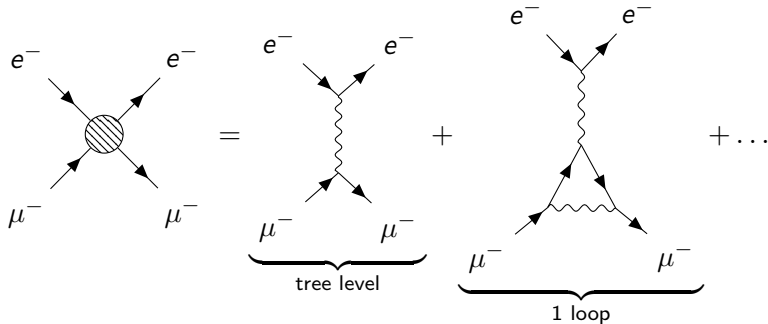
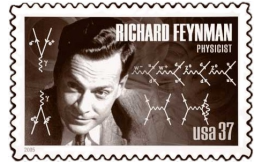


tree level leading order in perturbation theory

Amplitudes from Quantum Field Theory

Feynman diagrams

= sums over possible particle histories



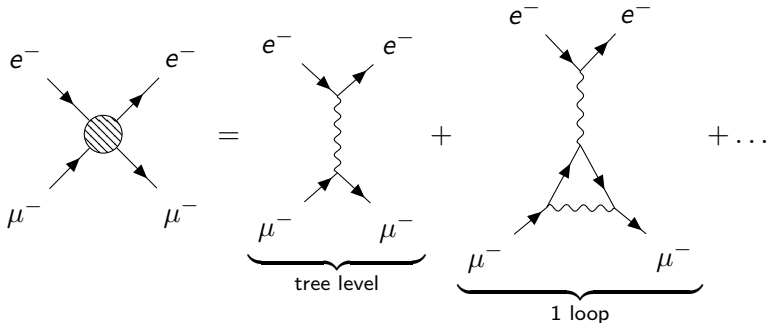
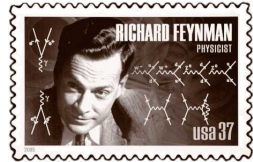
tree level leading order in perturbation theory

1 loop next-to-leading order in perturbation theory

Amplitudes from Quantum Field Theory

Feynman diagrams

= sums over possible particle histories



tree level leading order in perturbation theory

1 loop next-to-leading order in perturbation theory

...

Hidden simplicity I: Parke-Taylor amplitude

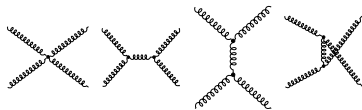
2 gluons \rightarrow 2 gluons: 4 diagrams

2 gluons \rightarrow 3 gluons: 25 diagrams

2 gluons \rightarrow 4 gluons: 220 diagrams

...

2 gluons \rightarrow 8 gluons: > 1 million diagrams



Hidden simplicity I: Parke-Taylor amplitude

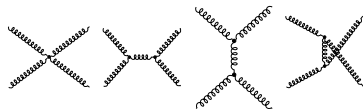
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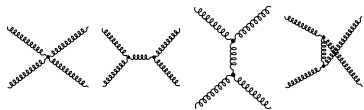
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n -gluon helicity amplitude [Parke-Taylor (1986)] [Mangano, Parke, Xu (1987)]

$$\mathcal{A}_6(1^-, 2^-, 3^+, \dots, 6^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle 61 \rangle}$$

\pm : polarization of the gluon with four-momentum p_i

$$\langle ij \rangle = \sqrt{|s_{ij}|} e^{i\phi_{ij}} \text{ with } s_{ij} = (p_i + p_j)^2$$

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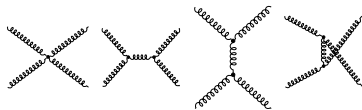
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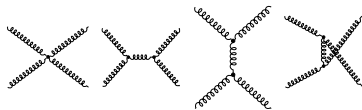
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Next step: Exploit this simplicity!

\Rightarrow Recursion relations \rightarrow all tree-level amplitudes

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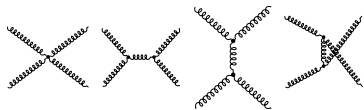
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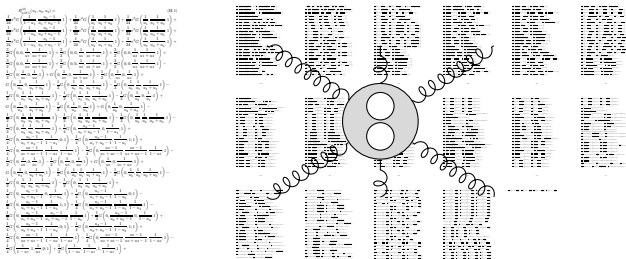
\Rightarrow Recursion relations \rightarrow all tree-level amplitudes

\rightarrow Course “Modern methods in particle scattering”

Hidden simplicity II: Polylogarithms

Two-loop six-gluon remainder function (= non-trivial part of amplitude) in the maximally (super)symmetric gauge theory

[Del Duca, Duhr, Smirnov (2010)]



$$u_1 = \frac{s_{12}s_{45}}{s_{123}s_{345}} \quad u_2 = \frac{s_{23}s_{56}}{s_{234}s_{123}} \quad u_3 = \frac{s_{34}s_{61}}{s_{345}s_{234}}$$

Hidden simplicity II: Polylogarithms

$$18 \text{ pages} = \sum_{i=1}^3 \left(L_4(x_i^+, x_i^-) - \frac{1}{2} \text{Li}_4(1 - 1/u_i) \right) - \frac{1}{8} \left(\sum_{i=1}^3 \text{Li}_2(1 - 1/u_i) \right)^2 + \frac{1}{24} J^4 + \frac{\pi^2}{12} J^2 + \frac{\pi^4}{72}$$

[Gancharov, Spradlin, Vergu, Volovich (2010)]

$$x_i^\pm = u_i x^\pm, \quad x^\pm = \frac{u_1 + u_2 + u_3 - 1 \pm \sqrt{\Delta}}{2u_1 u_2 u_3}, \quad \Delta = (u_1 + u_2 + u_3 - 1)^2 - 4u_1 u_2 u_3$$

$$L_4(x^+, x^-) = \frac{1}{8!!} \log(x^+ x^-)^4 + \sum_{m=0}^3 \frac{(-1)^m}{(2m)!!} \log(x^+ x^-)^m (\ell_{4-m}(x^+) + \ell_{4-m}(x^-))$$

$$\ell_n(x) = \frac{1}{2} (\text{Li}_n(x) - (-1)^n \text{Li}_n(1/x)), \quad J = \sum_{i=1}^3 (\ell_1(x_i^+) - \ell_1(x_i^-))$$

$$\text{Classical polylogarithms } \text{Li}_n(x) = \int_0^x \frac{dt}{t} \text{Li}_{n-1}, \quad \text{Li}_1(x) = -\log(1-x)$$

Hidden simplicity II: Polylogarithms

Exploiting the simplicity:

Bootstrapping

- = ansatz for result from polylogarithms
- + fix coefficients via physical constraints
- ⇒ Avoid Feynman diagrams and Feynman integrals altogether!



Hidden simplicity II: Polylogarithms

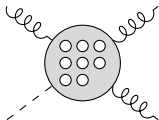
Exploiting the simplicity:

Bootstrapping

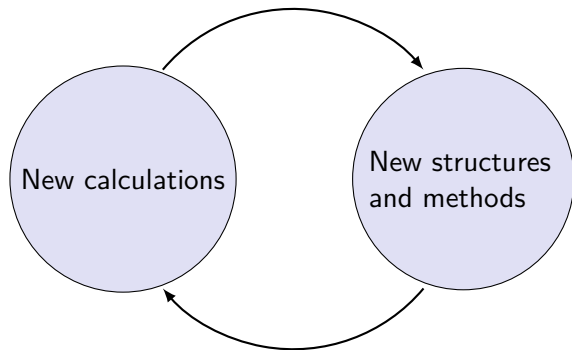
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Higgs \rightarrow 3 gluons (in some approximation) up to 8-loop order!



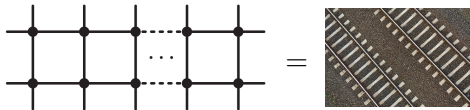
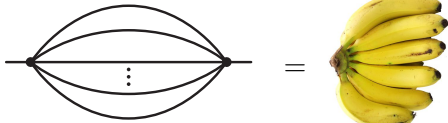
[Dixon, McLeod, MW (2020)], [Dixon, Gurdogan, McLeod, MW (2021)]



⇒ Precision predictions for the LHC to test our understanding of particle physics and to find new physics beyond the standard model of particle physics!

Beyond polylogarithms

New functions for collider physics and gravitational waves



Hidden structures and simplicity? How to exploit?

..., Morales, von Hippel, Vergu, Spiering, MW, Zhang,...

Study track: High-Energy Theory and Cosmology

	Block 1	Block 2	Block 3	Block 4
Year 1	Advanced Quantum Mechanics	General Relativity and Cosmology	Quantum Field Theory 1	Fundamentals of High-Energy Astrophysics and Particle Astrophysics
	Elementary Particle Physics	Particle Physics and the Early Universe	Modern Methods for Particle Scattering	<i>Choose one of:</i> Introduction to String Theory* Introduction to Gauge/Gravity Duality** Advanced Topics in QFT & Gravity***

Interested?

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Potential supervisors → Talk to us!



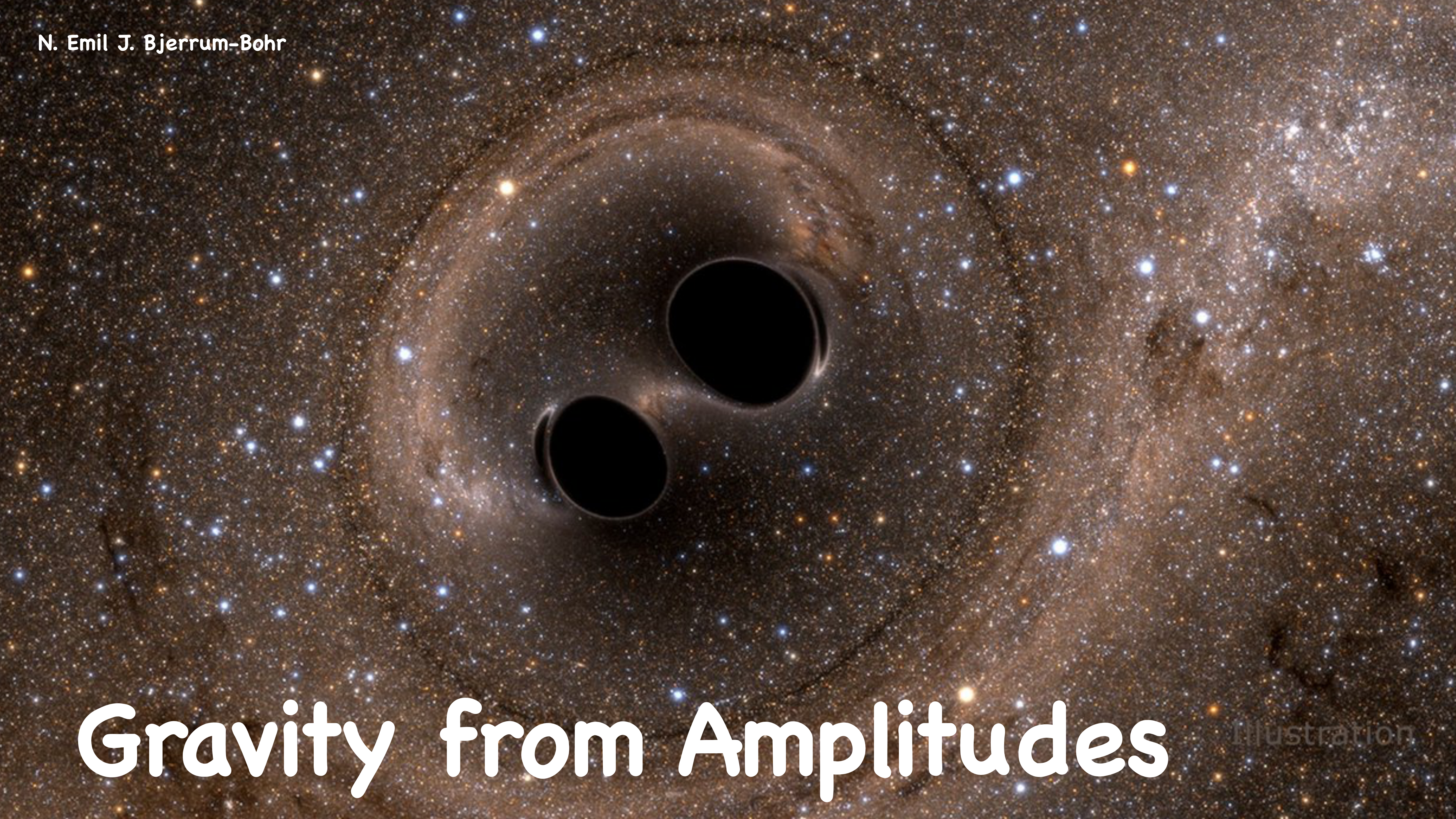
Anne Spiering



Matthias Wilhelm

Gravity from Amplitudes

Illustration



Gravity as a
particle theory?

Einstein's theory presents us with a beautiful theory for gravity.

However geometrical description that does not fit well with a generic (flat space) formulation of quantum mechanics.

Quantum mechanical extension of General Relativity?




- Known since the 1960ties that a particle version of General Relativity can be derived from the Einstein Hilbert Lagrangian (Feynman, DeWitt)
- Expand Einstein-Hilbert Lagrangian :

$$\mathcal{L}_{EH} = \int d^4x \left[\sqrt{-g} R \right]$$

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- Derive vertices as in a particle theory - compute amplitudes as Feynman diagrams!

 (Weinberg) proposed to view the quantization of general relativity from the viewpoint of effective field theory

$$\mathcal{L} = \sqrt{-g} \left[\frac{2R}{\kappa^2} + \mathcal{L}_{\text{matter}} \right]$$



$$\mathcal{L} = \sqrt{-g} \left\{ \frac{2R}{\kappa^2} + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + \dots \right\}$$

Path integral for gravity



vs.

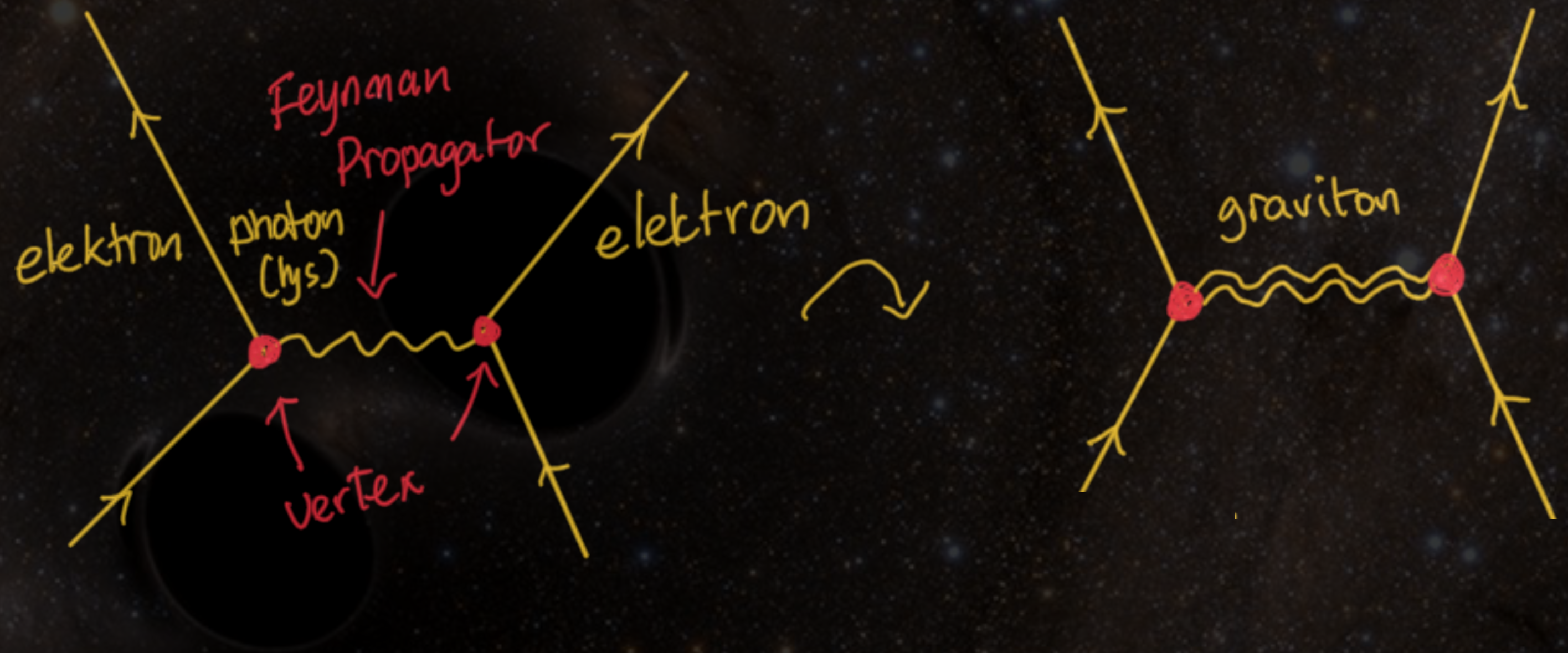


Feynman's integrals

Amplitudes can calculate quantum mechanics via Feynman graphs also when the physics is relativistic

The square of an amplitude gives probability





Probability of particle can be used for prediction eg Higgs particle... and .. gravity.



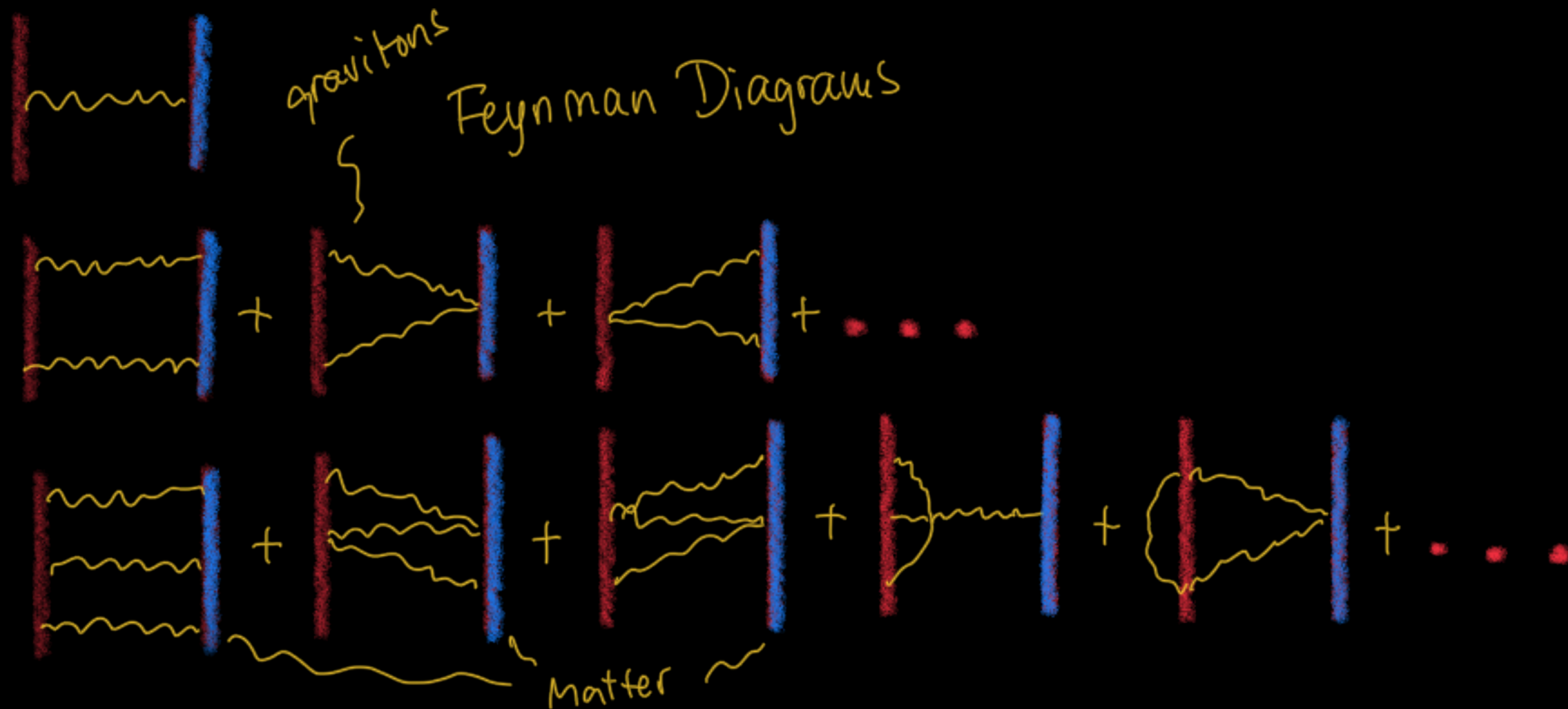
Consistent quantization

-  Working low energy version of quantum gravity

New point of view:

-  General relativity $\hbar \rightarrow 0$ limit of multi-loop expansion
-  Classical pieces comes from loop diagrams!
-  Explanation: contributions appear in loop diagrams feature a **cancellation of the loop diagram \hbar factor**
 -  (mass/ \hbar) expansion.




Quantum mechanical description of black scattering?



- Classical potential from quantum mechanical propagation via quantum / classical correspondence principle.

Large quantum numbers (angular impulse enormous) → Classical physics (gravity) (essence of Bohr's correspondence principle)

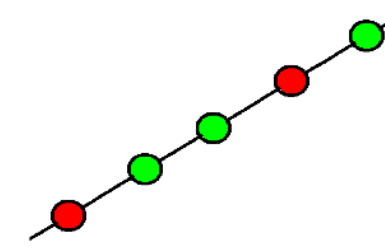
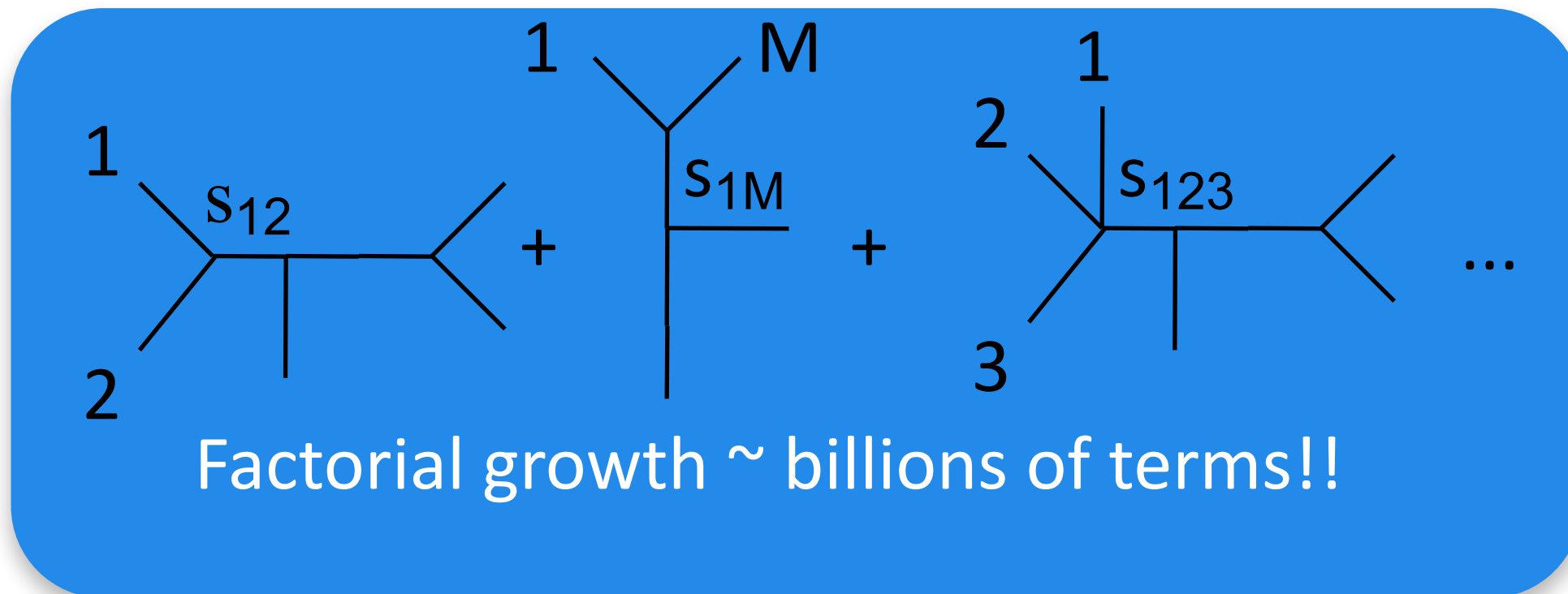
Classical contributions from the Path integral:

-  Novel ways to compute observables in General Relativity
-  Bending of light – a new take on Quantum Gravity and potential quantum corrections in General Relativity?
-  Applications for the physics behind LIGO and observations of gravitational waves

Tools for efficient computation

*(If you are interested in learning
more — check out my **modern
methods for particle scattering
course**) (block 3)*

MHV amplitude (geometric) revolution!



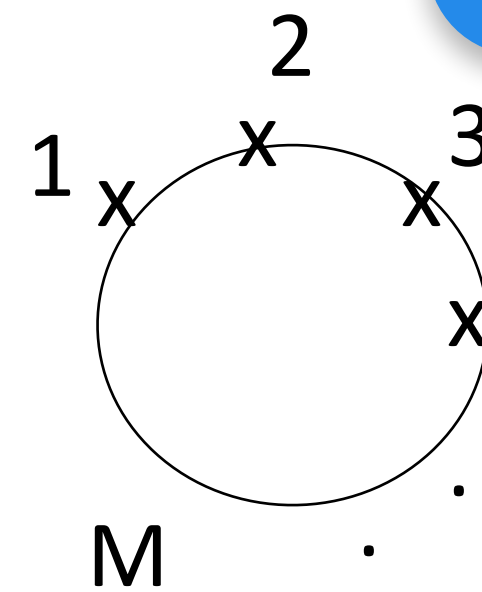
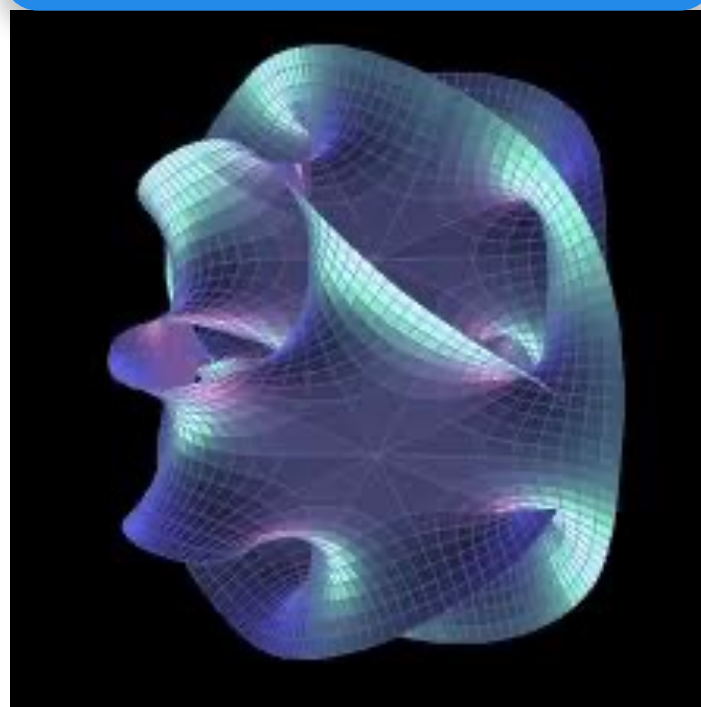
Rich hidden structure

On-shell recursion
MHV only one term!

$$\sim \frac{\langle jk \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle M1 \rangle}$$



String Theory



Inspiration across fields



New relations

It was suggested recently by Cachazo, He and Yuan that one can compute amplitudes via

$$A_n = \int \frac{d^n \sigma}{\text{volSL}(2, \mathbb{C})} \prod'_a \delta \left(\sum_{a \neq b} \frac{k_a \cdot k_b}{z_a - z_b} \right) \left(\frac{\text{Tr}(T^{a_1} T^{a_2} T^{a_3} \dots T^{a_n})}{(z_1 - z_2)(z_2 - z_3) \dots (z_n - z_1)} + \dots \right)^{2-s} (\text{Pf}' \Psi)^s$$

Exciting new framework for amplitudes

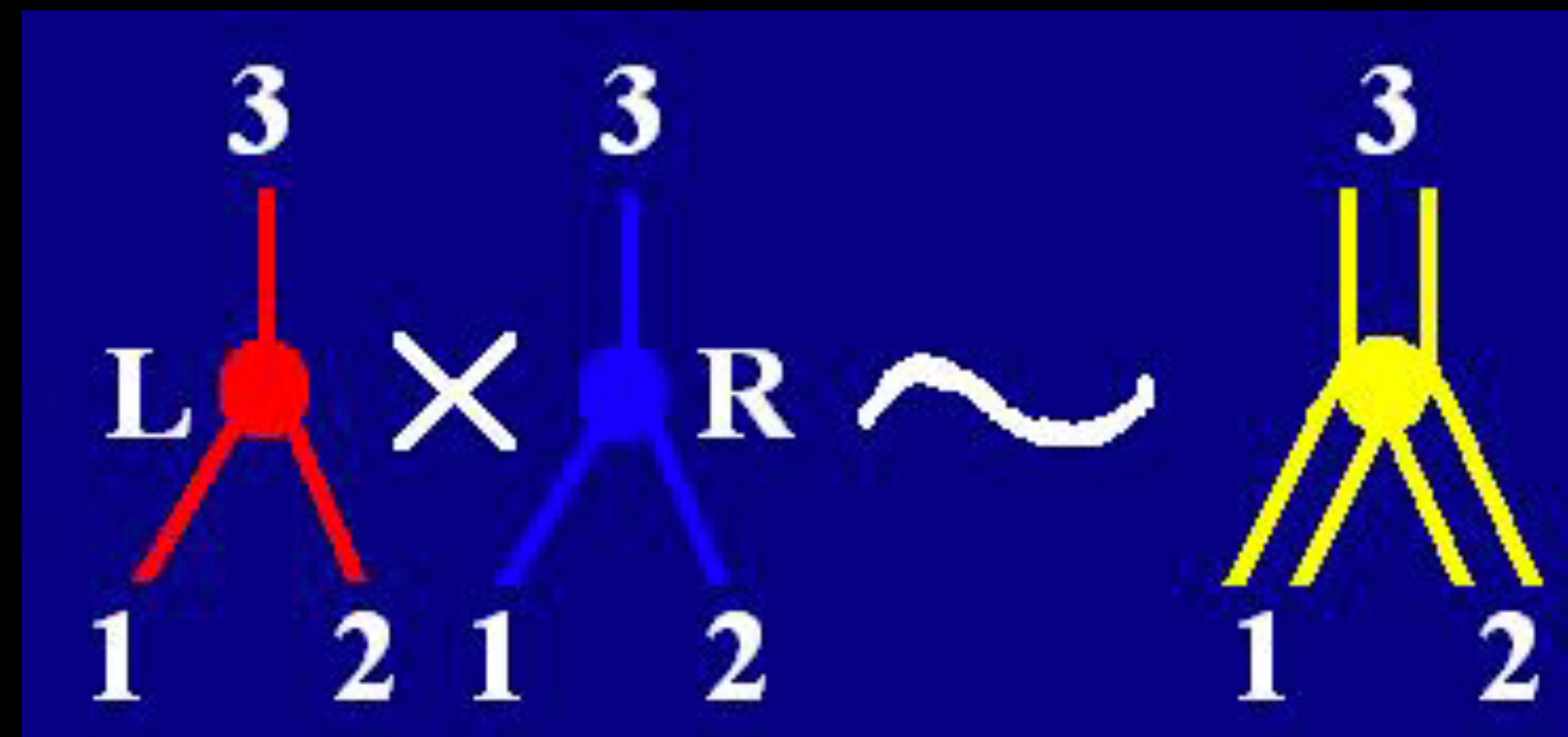
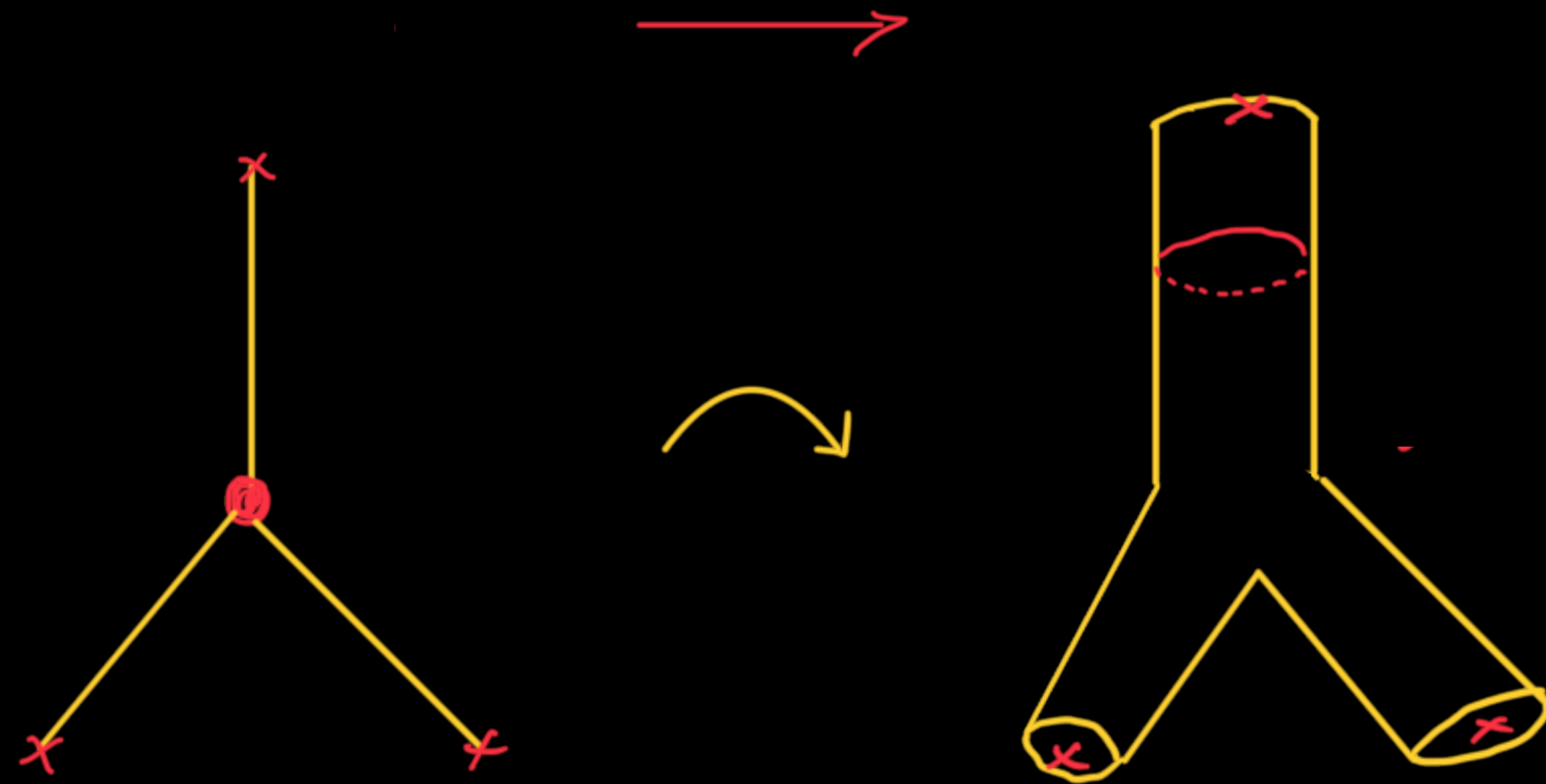
Color trace

Algebraic solutions

Pfaffian
(dependent on polarisations and momenta)

Applications of String theory

- Gives naturally a quantum gravity
- Important principle: gravity is like the product of something simpler (double-copy in particle physics)



Quantum gravity from effective field theory

PRL 114, 061301 (2015)

PHYSICAL REVIEW LETTERS

week ending
13 FEBRUARY 2015



Bending of Light in Quantum Gravity

N. E. J. Bjerrum-Bohr,^{1,*} John F. Donoghue,^{2,†} Barry R. Holstein,^{2,‡} Ludovic Planté,^{3,§} and Pierre Vanhove^{3,4,¶}

¹*Niels Bohr International Academy and Discovery Center, The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

²*Department of Physics-LGRT, University of Massachusetts, Amherst, Massachusetts 01003, USA*

³*CEA, DSM, Institut de Physique Théorique, IPhT, CNRS, MPPU, URA2306, Saclay, F-91191 Gif-sur-Yvette, France*

⁴*Institut des Hautes Études Scientifiques, Bures-sur-Yvette, F-91440, France*

(Received 31 October 2014; revised manuscript received 18 November 2014; published 12 February 2015)

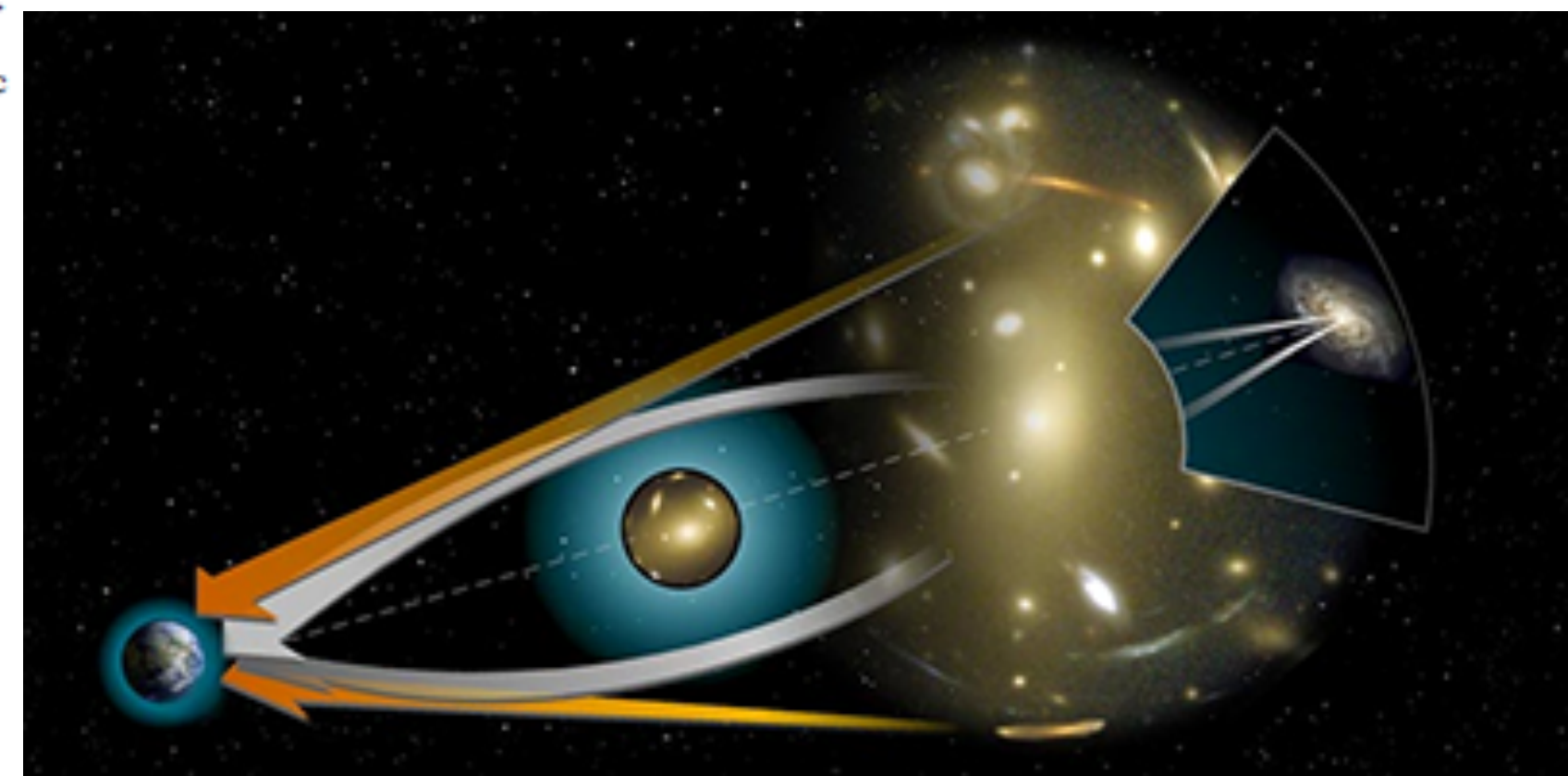
We consider the scattering of lightlike matter in the presence of a heavy scalar object (such as the Sun or a Schwarzschild black hole). By treating general relativity as an effective field theory we directly compute the nonanalytic components of the one-loop gravitational amplitude for the scattering of massless scalars or photons from an external massive scalar field. These results allow a semiclassical computation of the bending angle for light rays grazing the Sun, including long-range \hbar contributions. We discuss implications of this computation, in particular, the violation of some classical formulations of the equivalence principle.

DOI: 10.1103/PhysRevLett.114.061301

PACS numbers: 04.60.-m, 04.62.+v, 04.80.Cc

Using only a few
computational
tricks!

Reproduces Einstein's
result plus quantum
effects in particle theory!



Precision physics and the experiment LIGO



Research visions

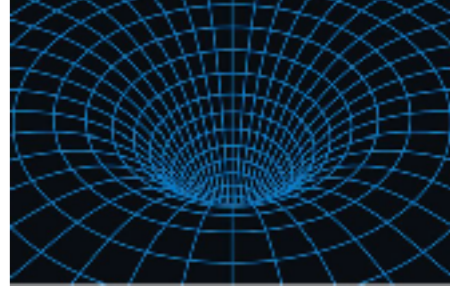
Still many things we **do not know**..

Many deep mysteries in quantum gravity research.

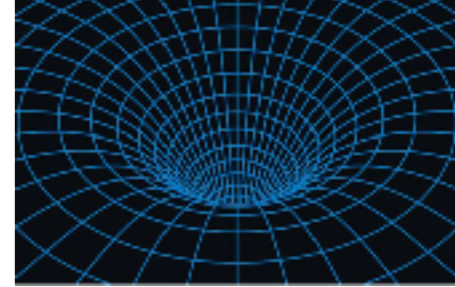
Developments no one would have believed of a few years ago.

Results now extend into for black-hole scattering and precision measurements of the gravitational attraction.

(Exact) Black holes and the double copy



(Exact) Black holes and the double copy



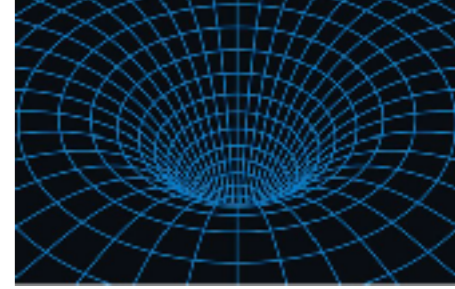
BCJ

double copy

$$\frac{1}{g^{n-2}} \mathcal{A}_n = \sum_{\text{diags. } i} \frac{n_i c_i}{\prod_{\alpha_i} s_{\alpha_i}}$$
$$\frac{-i}{(\kappa/2)^{n-2}} \mathcal{M}_n = \sum_{\text{diags. } i} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

...a relation between
scattering amplitudes...

(Exact) Black holes and the double copy



BCJ double copy

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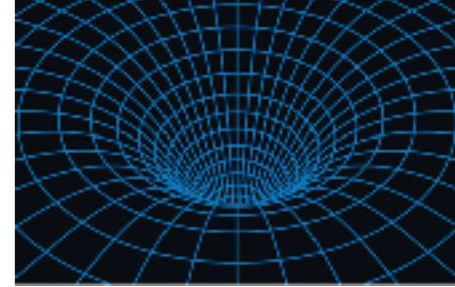
...a relation between
scattering amplitudes...

Kerr-Schild double copy

$$A_a^\mu = c_a \phi k^\mu$$
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$\equiv \eta_{\mu\nu} + k_\mu k_\nu \phi$$

...a relation between
classical fields...

(Exact) Black holes and the double copy



BCJ double copy

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...a relation between scattering amplitudes...

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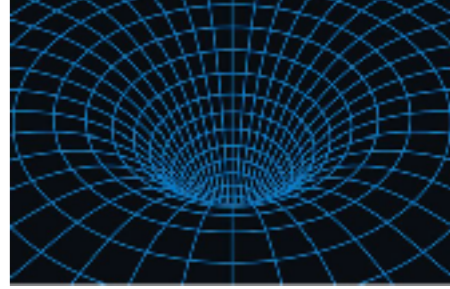
...a relation between classical fields...

Weyl double copy

$$C_{ABCD} = \frac{1}{S} f_{(AB} f_{CD)}$$

...a relation between spinors...

(Exact) Black holes and the double copy



BCJ double copy

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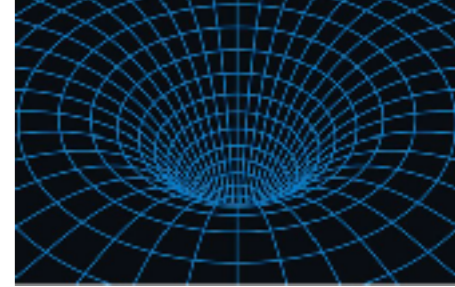
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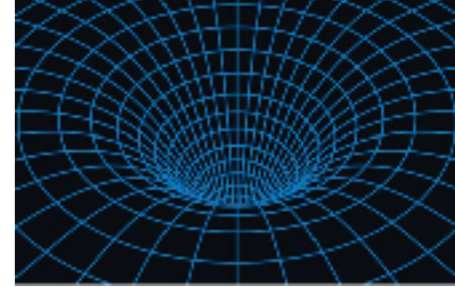
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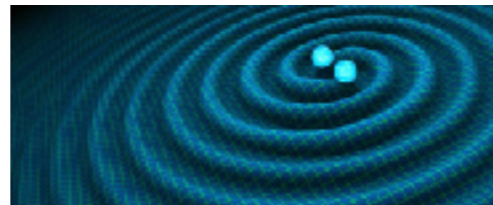
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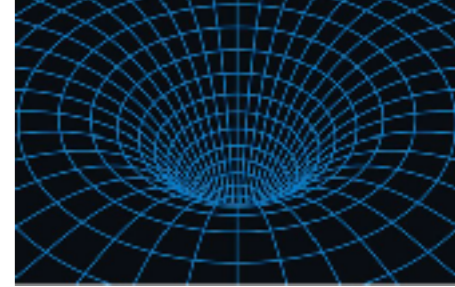
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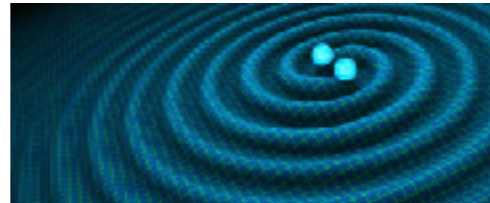
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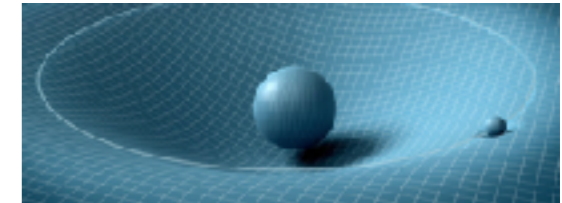
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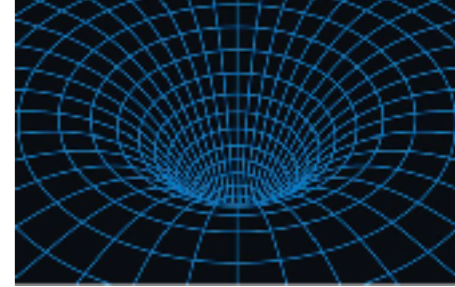
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(test bodies)



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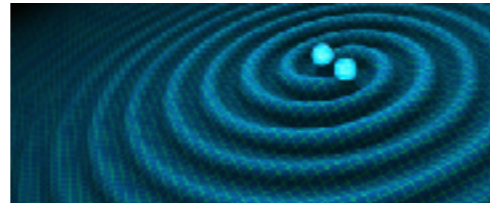
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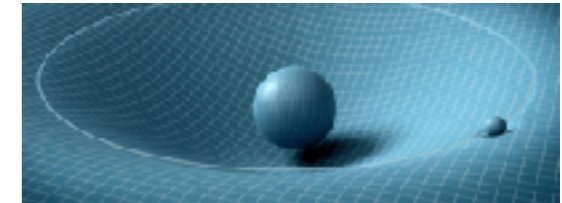
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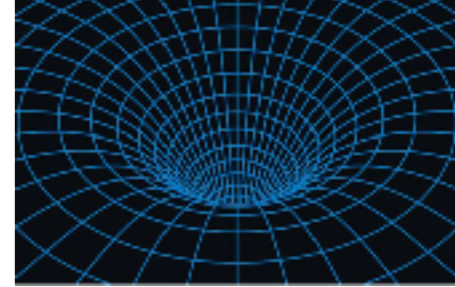
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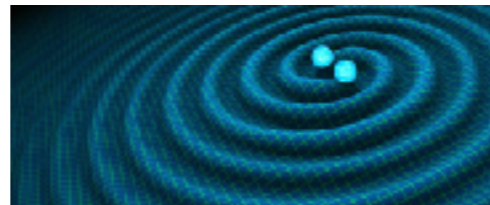
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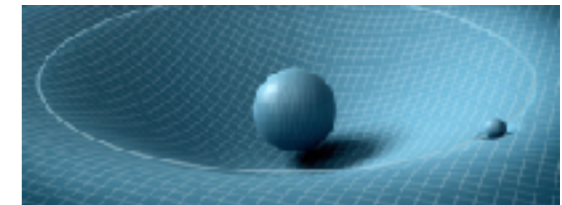
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Open questions:

Can we get exact dynamical results beyond geodesics?

How deep does the rabbit (black) hole go?

(or How fundamental is the classical double copy?)