# Spinning Binaries from quantum fields

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Co-financed by the Connecting Europe Facility of the European Union

# Outline

- Why Amplitudes?

- Amplitudes for binaries. Spin.

- Summary / Outlook.

# Classical physics from Quantum field theory?

Progress of Theoretical Physics, Vol. 46, No. 5, November 1971

**Quantum Theory of Gravitation vs. Classical Theory***)

——*Fourth-Order Potential*——

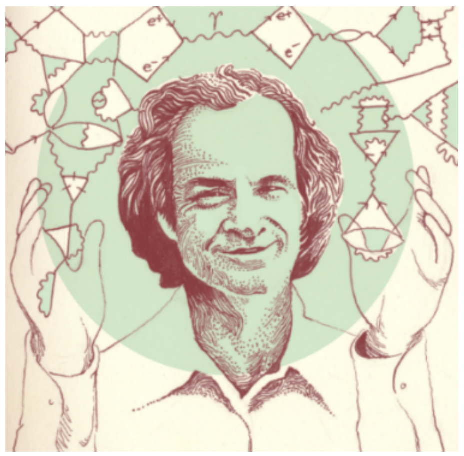
Yoichi IWASAKI

## Trees AND loops?

Here we want to point out that there seems to exist an erroneous belief*),**) that only tree diagrams contribute to the classical process. Contrary to this belief, the quadratic term in $k$ corresponds to fourth-order diagrams each of which contains a closed loop; it is a "radiative correction" term. Since the quantum

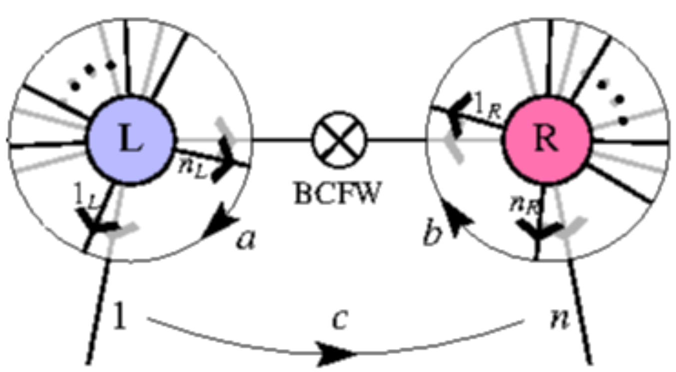
Iwasaki knew it 50 years ago...

# Amplitudes: Efficiency



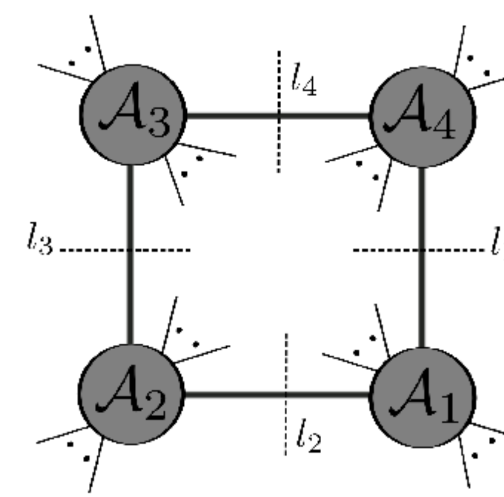
The Amplitudes program:
Avoid summing all Feynman diagrams.

Recycle (use amplitudes as building blocks)




Recursion relations:
big trees from small trees.

Generalized unitarity:
loops from trees.




Cool. But nothing we couldn't have gotten with sufficient time (and sufficient RAM).

But not quite...

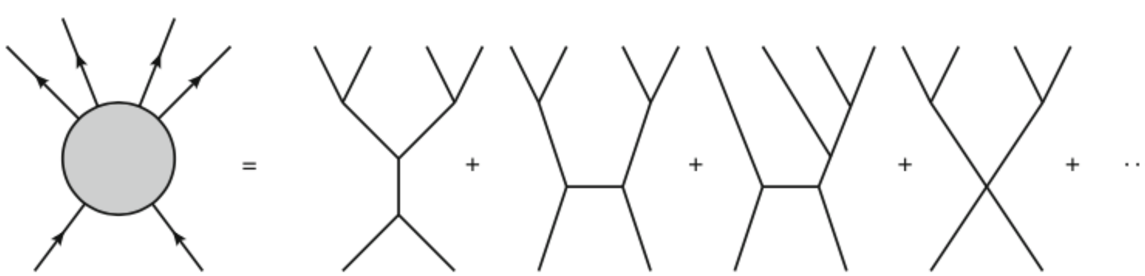
# Amplitudes: Six gluons

**THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON–GLUON FUSION**

Stephen J. PARKE and T.R. TAYLOR

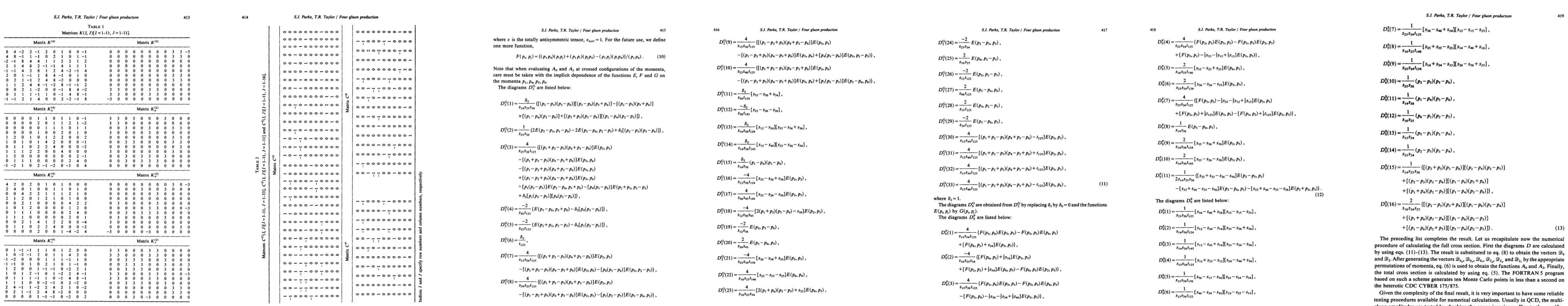
*Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA*

Received 13 September 1985




As the result of the computation of two hundred and forty Feynman diagrams, we obtain

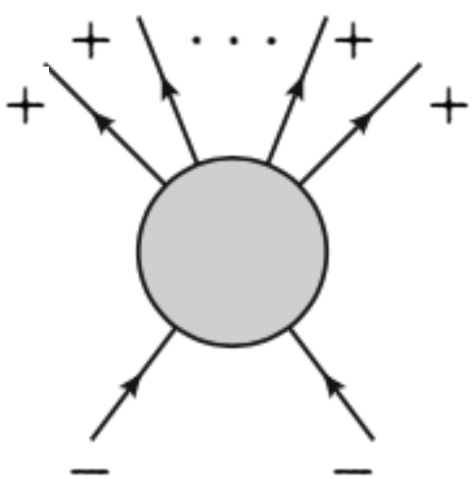
$$A_{\binom{0}{2}}(p_1, p_2, p_3, p_4, p_5, p_6) = (\mathscr{D}^\dagger, \mathscr{D}^\dagger_\rho, \mathscr{D}^\dagger_\sigma, \mathscr{D}^\dagger_\tau)_{\binom{0}{2}} \cdot \begin{pmatrix} K & K_\rho & K_\sigma & K_\tau \\ K_\rho & K & K_\tau & K_\sigma \\ K_\sigma & K_\tau & K & K_\rho \\ K_\tau & K_\sigma & K_\rho & K \end{pmatrix} \cdot \begin{pmatrix} \mathscr{D} \\ \mathscr{D}_\rho \\ \mathscr{D}_\sigma \\ \mathscr{D}_\tau \end{pmatrix}_{\binom{0}{2}}, \qquad (6)$$



# Amplitudes: Simplicity

## Amplitude for *n*-Gluon Scattering

Stephen J. Parke and T. R. Taylor
*Fermi National Accelerator Laboratory, Batavia, Illinois 60510*
(Received 17 March 1986)




Specifying helicities leads to immense simplification

$$|\mathcal{M}_n(+++++\cdots)|^2 = c_n(g,N)[0+O(g^4)],$$
$$|\mathcal{M}_n(-++++\cdots)|^2 = c_n(g,N)[0+O(g^4)],$$
$$|\mathcal{M}_n(--+++\cdots)|^2 = c_n(q,N)[(p_1\cdot p_2)^4 \times \textstyle\sum_P [(p_1\cdot p_2)(p_2\cdot p_3)(p_3\cdot p_4)\cdots(p_n\cdot p_1)]^{-1} + O(N^{-2}) + O(g^2)],$$

The Parke-Taylor formula

$$A_n[1^+\dots i^-\dots j^-\dots n^+] = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Should you care?

You should Kerr!

# Kerr black hole Amplitudes

[Submitted on 18 Sep 2017 (v1), last revised 29 Sep 2017 (this version, v2)]

**Scattering of two spinning black holes in post-Minkowskian gravity, to all orders in spin, and effective-one-body mappings**

Justin Vines

$$S_{\text{int}}[\Psi, h] = \frac{1}{2}\int d^4x\, h_{\mu\nu}\, T^{\mu\nu}[\Psi]$$

For a linearized Kerr black hole

$$T^{\mu\nu}(x) = \int d\tau\, \hat{\mathcal{T}}^{\mu\nu}(p, a, \partial)\, \delta^4(x - z)$$

$$\hat{\mathcal{T}}^{\mu\nu}(p, a, \partial) = m\, \exp(a * \partial)^{(\mu}{}_{\rho}\, u^{\nu)} u^{\rho}$$

[Submitted on 17 Dec 2018 (v1), last revised 9 Sep 2019 (this version, v3)]

**Scattering of Spinning Black Holes from Exponentiated Soft Factors**

Alfredo Guevara, Alexander Ochirov, Justin Vines

The stress energy tensor in momentum space

$$T^{\mu\nu}(-k) = 2\pi\delta(p\cdot k)p^{(\mu} \exp(a * ik)^{\nu)}{}_{\rho} p^{\rho}$$

Matches the simplest three-point amplitude

$$h_{\mu\nu}(k)T^{\mu\nu}(-k) = \frac{1}{2}(2\pi)^2\delta(k^2)\delta(p\cdot k)\lim_{s\to\infty}\langle \mathcal{M}_3^{(s)}\rangle$$

$$h_{\mu\nu} = 2\pi\delta(k^2)\varepsilon_\mu\varepsilon_\nu$$

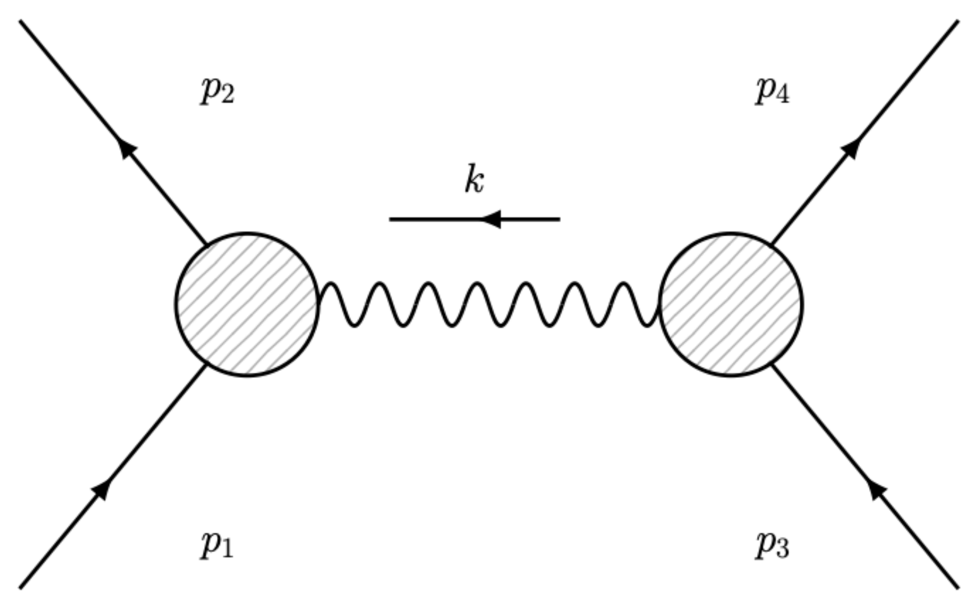
$$\mathcal{M}_3^{(s)}(p_1, p_2, k^+) = \frac{\langle 12\rangle^{2s} x^2}{m^{2s-2}}$$

# Observables from Amplitudes

[Submitted on 17 Dec 2018 (v1), last revised 9 Sep 2019 (this version, v3)]

**Scattering of Spinning Black Holes from Exponentiated Soft Factors**

Alfredo Guevara, Alexander Ochirov, Justin Vines

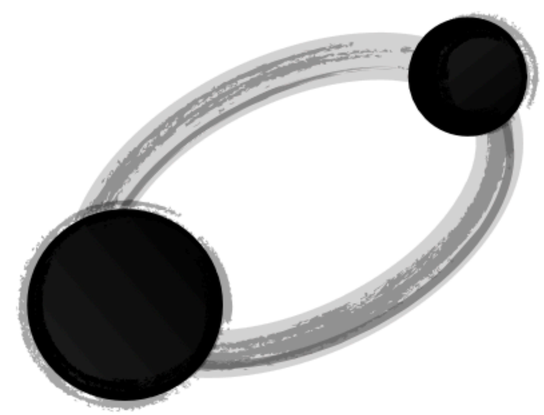



From the 3-point amplitude, computed 1PM scattering angle

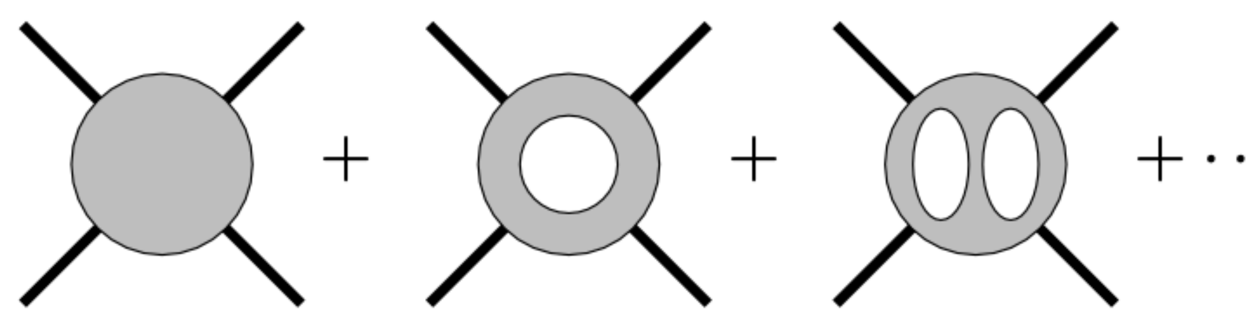
$$\theta = \frac{GE}{v^2}\left[\frac{(1+v)^2}{b+a_a+a_b}+\frac{(1-v)^2}{b-a_a-a_b}\right]$$

(Matches the one computed by Vines)

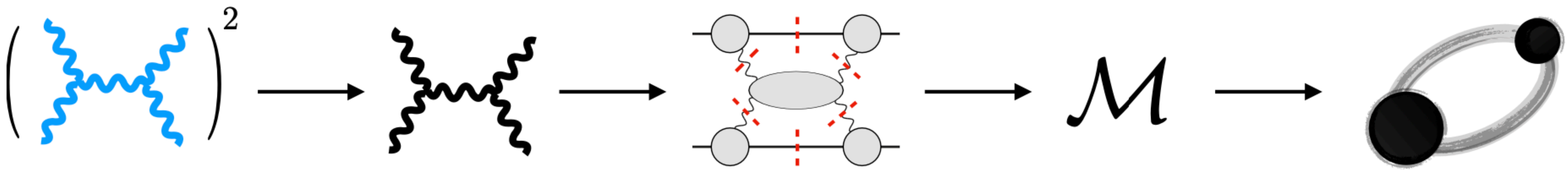
$$\mathcal{M} \longrightarrow$$



$$\mathcal{M} = \mathcal{M}^{(0)} + \mathcal{M}^{(1)} + \mathcal{M}^{(2)} + \cdots =$$



# Pipeline



Gauge trees

Gravity trees. Double copy

Unitarity: Loops from trees

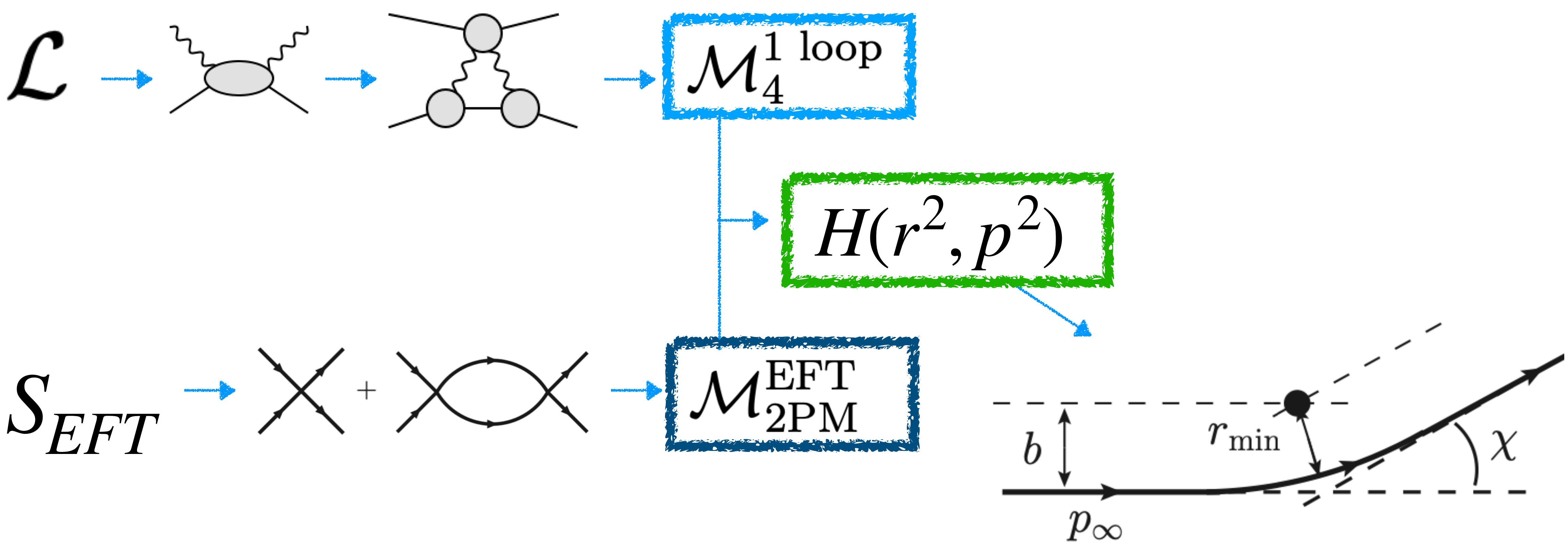
Amplitudes to Observables

# Spin Hamiltonian from Amplitudes

[Submitted on 6 May 2020]

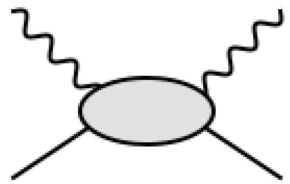
**Spinning Black Hole Binary Dynamics, Scattering Amplitudes and Effective Field Theory**

Zvi Bern, Andres Luna, Radu Roiban, Chia-Hsien Shen, Mao Zeng

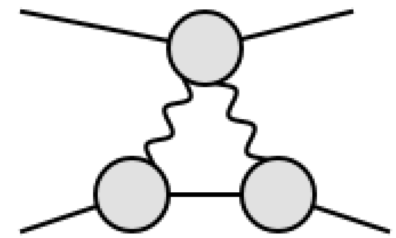


# Full theory

$\mathcal{L}$ →



→



→ $\mathcal{M}_4^{1 \text{ loop}}$

We consider a Lagrangian of rank-s tensor fields, minimally coupled to gravity

$$\mathcal{L}_{\min} = -R(e,\omega) + \frac{1}{2} g^{\mu\nu} \nabla(\omega)_\mu \phi_s \nabla(\omega)_\nu \phi_s$$

$$\phi_s{}^{a_1 a_2 a_3 \ldots a_s}$$

Symmetric traceless tensor field

Covariant derivative

Lorentz Generator

$$\nabla(\omega)_\mu \phi_s \equiv \partial_\mu \phi_s + \frac{i}{2} \omega_{\mu e f} M^{ef} \phi_s$$

The spin tensor is obtained in the classical limit

$$\varepsilon(\boldsymbol{s}, p_1) M^{ab} \varepsilon(\boldsymbol{s}, p_2) = S(p_1, \boldsymbol{S})^{ab}\, \varepsilon(\boldsymbol{s}, p_1) \cdot \varepsilon(\boldsymbol{s}, p_2) + \mathcal{O}(q^0)$$

# Effective theory

$S_{EFT}$ $\longrightarrow$  $\longrightarrow$ $\mathcal{M}^{\mathrm{EFT}}_{\mathrm{2PM}}$

An EFT of non-relativistic fields...

$$S = \int_{\boldsymbol{k}} \sum_{a=1,2} \xi_a^\dagger(-\boldsymbol{k}) \left( i\partial_t - \sqrt{\boldsymbol{k}^2 + m_i^2} \right) \xi_a(\boldsymbol{k}) - \int_{\boldsymbol{k},\boldsymbol{k}'} \xi_1^\dagger(\boldsymbol{k}')\xi_2^\dagger(-\boldsymbol{k}') \hat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\boldsymbol{S}}_a) \xi_1(\boldsymbol{k})\xi_2(-\boldsymbol{k})$$

...generalized to describe spinning fields. The potential...

$$\hat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\boldsymbol{S}}_i) = \sum_A \hat{V}^A(\boldsymbol{k}', \boldsymbol{k}) \hat{\mathbb{O}}^A$$

...contains long range interactions...

$$\hat{V}^A(\boldsymbol{k}', \boldsymbol{k}) = \frac{4\pi G}{\boldsymbol{q}^2} d_1^A \left(\boldsymbol{p}^2\right) + \frac{2\pi^2 G^2}{|\boldsymbol{q}|} d_2^A \left(\boldsymbol{p}^2\right) + \mathcal{O}(G^3)$$

$$\hat{\mathbb{O}}^{(0)} = \mathbb{I}\,, \qquad \hat{\mathbb{O}}^{(1,1)} = \boldsymbol{L}_q \cdot \hat{\boldsymbol{S}}_1\,, \qquad \hat{\mathbb{O}}^{(1,2)} = \boldsymbol{L}_q \cdot \hat{\boldsymbol{S}}_2\,,$$

$$\hat{\mathbb{O}}^{(2,1)} = \boldsymbol{q} \cdot \hat{\boldsymbol{S}}_1\, \boldsymbol{q} \cdot \hat{\boldsymbol{S}}_2\,, \qquad \hat{\mathbb{O}}^{(2,2)} = \boldsymbol{q}^2\, \hat{\boldsymbol{S}}_1 \cdot \hat{\boldsymbol{S}}_2\,, \qquad \hat{\mathbb{O}}^{(2,3)} = \boldsymbol{q}^2\, \boldsymbol{k} \cdot \hat{\boldsymbol{S}}_1\, \boldsymbol{k} \cdot \hat{\boldsymbol{S}}_2$$

Organized by classical spin operators

# Hamiltonian

$$H = H^{(0)}(r^2,p^2) + H^{(1,i)}(r^2,p^2)\boldsymbol{L}\cdot\boldsymbol{S}_i$$

$$+ H^{(2,1)}(r^2,p^2)\boldsymbol{r}\cdot\boldsymbol{S}_1\boldsymbol{r}\cdot\boldsymbol{S}_2 + H^{(2,2)}(r^2,p^2)\boldsymbol{S}_1\cdot\boldsymbol{S}_2 + H^{(2,3)}(r^2,p^2)\boldsymbol{p}\cdot\boldsymbol{S}_1\boldsymbol{p}\cdot\boldsymbol{S}_2 + \dots\,,$$

$\mathcal{M}_4^{1\text{ loop}}$

$H(r^2,p^2)$

$\mathcal{M}_{2\text{PM}}^{\text{EFT}}$

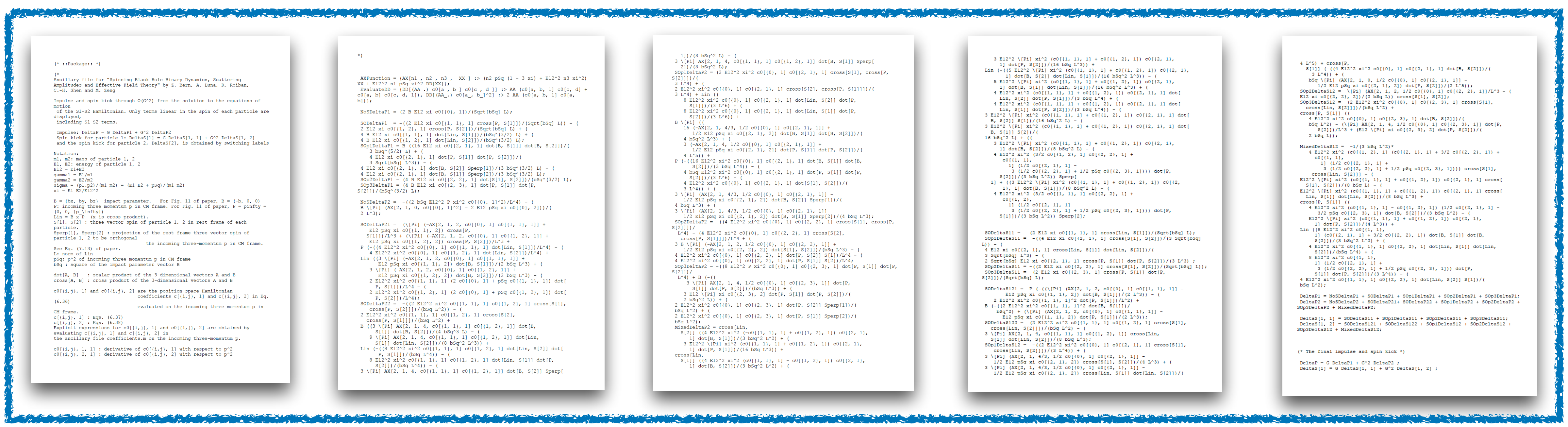
Use Hamilton's equations of motion

$$\dot{\boldsymbol{r}} = \frac{\partial H}{\partial \boldsymbol{p}}\,, \qquad \dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{r}}\,, \qquad \dot{\boldsymbol{S}}_a = -\boldsymbol{S}_a \times \frac{\partial H}{\partial \boldsymbol{S}_a}\,, \qquad a = 1,2$$

To compute change in observables

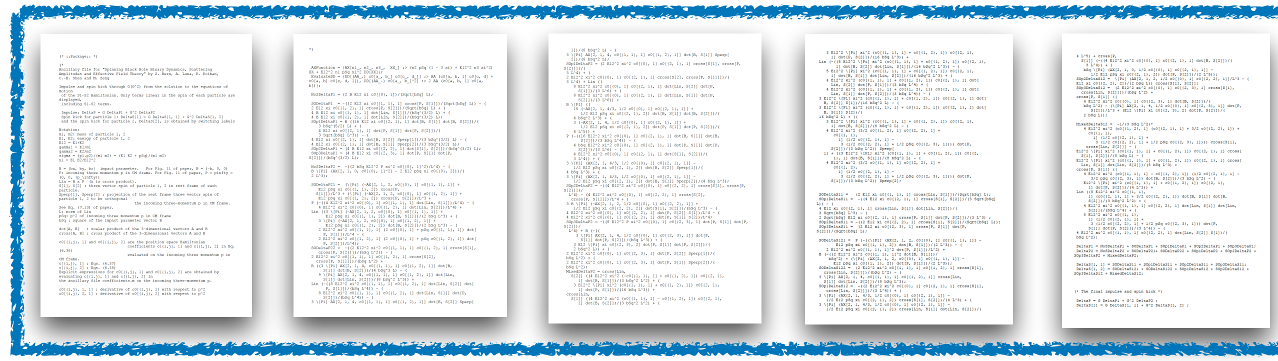
$$\Delta O_n = \int_{-\infty}^{\infty} dt\, \frac{dO_n}{dt}$$

$$\Delta\boldsymbol{p} \qquad \Delta\boldsymbol{S}_i$$

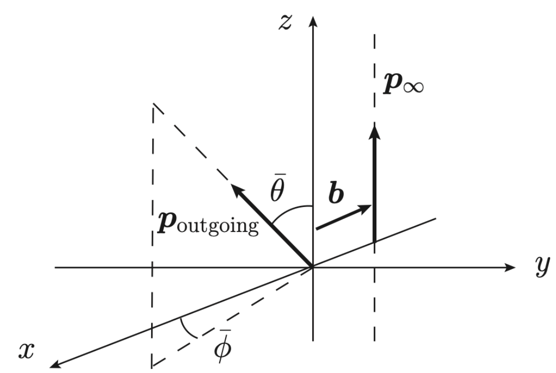


# Observables

$$\mathcal{M}_4^{1\text{ loop}} \rightarrow H(r^2, p^2) \rightarrow \begin{matrix}\Delta \boldsymbol{p} \\ \Delta \boldsymbol{S}_i\end{matrix}$$



$$\mathcal{M}_4^{1\text{ loop}} \rightarrow$$




$$\Delta \boldsymbol{p}_\perp = -\{\boldsymbol{P}_\perp, \chi\} - \frac{1}{2}\{\chi, \{\boldsymbol{P}_\perp, \chi\}\} - \mathcal{D}_{SL}(\chi, \{\boldsymbol{P}_\perp, \chi\}) + \frac{1}{2}\{\boldsymbol{P}_\perp, \mathcal{D}_{SL}(\chi, \chi)\}$$

$$\Delta \boldsymbol{S}_i = -\{\boldsymbol{S}_i, \chi\} - \frac{1}{2}\{\chi, \{\boldsymbol{S}_i, \chi\}\} - \mathcal{D}_{SL}(\chi, \{\boldsymbol{S}_i, \chi\}) + \frac{1}{2}\{\boldsymbol{S}_i, \mathcal{D}_{SL}(\chi, \chi)\}$$

$$\{\boldsymbol{P}_\perp, f\} \equiv -\nabla_{\boldsymbol{b}} f$$

$$\{S_a^i, f\} \equiv \epsilon^{ijk} \frac{\partial f}{\partial S_a^j} S_a^k$$

$$\mathcal{D}_{SL}(f, g) \equiv -\sum_{a=1,2} \epsilon^{ijk} S_a^k \frac{\partial f}{\partial S_a^i} \frac{\partial g}{\partial L^j}$$

The Eikonal phase

$$\chi_2 = \frac{1}{4m_1 m_2\sqrt{\sigma^2 - 1}} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{M}^{\triangle+\triangledown}(\boldsymbol{q})$$

# Higher powers in Spin: Lagrangian

## Effective description of spin-induced multipoles

[Submitted on 20 Jan 2015 (v1), last revised 5 Oct 2015 (this version, v3)]

**Spinning gravitating objects in the effective field theory in the post-Newtonian scheme**

Michele Levi, Jan Steinhoff

$$L_{\rm SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} + \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

[Submitted on 19 Feb 2021]

**Quadratic-in-Spin Hamiltonian at $\mathcal{O}(G^2)$ from Scattering Amplitudes**

Dimitrios Kosmopoulos, Andres Luna

[Submitted on 11 Mar 2022 (v1), last revised 29 Dec 2022 (this version, v3)]

**Binary Dynamics Through the Fifth Power of Spin at $\mathcal{O}(G^2)$**

Zvi Bern, Dimitrios Kosmopoulos, Andrés Luna, Radu Roiban, Fei Teng

### Covariantization of Levi-Steinhoff

$$\begin{aligned}\mathcal{L}_C = & \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{\mathrm{ES}^{2n}}}{m^{2n}} \nabla_{f_{2n}} \ldots \nabla_{f_3} R_{a f_1 b f_2} \\ & \times \nabla^a \phi_s \mathbb{S}^{(f_1} \mathbb{S}^{f_2} \ldots \mathbb{S}^{f_{2n})} \nabla^b \phi_s \\ & - \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{\mathrm{BS}^{2n+1}}}{m^{2n+1}} \nabla_{f_{2n+1}} \ldots \nabla_{f_3} \widetilde{R}_{(a|f_1|b)f_2} \\ & \times \nabla^a \phi_s \mathbb{S}^{(f_1} \mathbb{S}^{f_2} \ldots \mathbb{S}^{f_{2n+1})} \nabla^b \phi_s \,,\end{aligned}$$

### More EFT operators

$$\begin{aligned}\mathcal{L}_H = & -\sum_{n=1}^{\infty} \frac{(-1)^n (2n-1)}{(2n)!(2n+1)} \frac{H_{2n}}{m^{2n-2}} \nabla_{f_{2n}} \ldots \nabla_{f_3} R^{(a}{}_{f_1}{}^{b)}{}_{f_2} \\ & \times \phi_s M_a{}^{(f_1} M_b{}^{f_2} \mathbb{S}^{f_3} \ldots \mathbb{S}^{f_{2n})} \phi_s \\ & + \sum_{n=1}^{\infty} \frac{(-1)^n n}{(2n+1)!(n+1)} \frac{H_{2n+1}}{m^{2n-1}} \nabla_{f_{2n+1}} \ldots \nabla_{f_3} \widetilde{R}^{(a}{}_{f_1}{}^{b)}{}_{f_2} \\ & \times \phi_s M_a{}^{(f_1} M_b{}^{f_2} \mathbb{S}^{f_3} \ldots \mathbb{S}^{f_{2n+1})} \phi_s \,.\end{aligned}$$

# Higher powers in spin: Amplitudes

$$\mathcal{M}^{\triangle+\nabla} = \frac{2\pi^2 G^2 \varepsilon_1\cdot\varepsilon_4 \varepsilon_2\cdot\varepsilon_3}{\sqrt{-q^2}} \sum_n \sum_i \alpha^{(n,i)} \mathcal{O}^{(n,i)} .$$

$$\alpha^{(2,i)} = \frac{m_1^2 m_2^2}{16(-1+\sigma^2)^2}(\gamma^{(2,i)} m_1 + \delta^{(2,i)} m_2) ,$$

$$\alpha^{(3,i)} = \frac{m_1^2 m_2^2 \sigma}{8(-1+\sigma^2)^2}(\gamma^{(3,i)} m_1 + 2\delta^{(3,i)} m_2) ,$$

| | $i$ | | $i$ | | $i$ | |
|---|---|---|---|---|---|---|
| $\mathcal{O}^{(2,i)}$ | 1 | $\mathcal{E}_1^2$ | 2 | $q^2(u_2\cdot a_1)^2$ | 3 | $(q\cdot a_1)^2$ |
| $\mathcal{O}^{(4,i)}$ | 1 | $\mathcal{E}_1^4$ | 2 | $q^2(u_2\cdot a_1)^2\mathcal{E}_1^2$ | 3 | $q^4(u_2\cdot a_1)^4$ |
| | 4 | $(q\cdot a_1)^2\mathcal{E}_1^2$ | 5 | $q^2(q\cdot a_1)^2(u_2\cdot a_1)^2$ | 6 | $(q\cdot a_1)^4$ |

| | |
|---|---|
| $Z_{2,1} = C_2 + 1$ | $Z_{2,2} = C_2 - 1$ |
| $Z_{3,1} = 3C_2 + C_3$ | $Z_{3,2} = C_2 - C_3$ |
| $Z_{4,1} = 3C_2^2 + 4C_3 + C_4$<br>$Z_{4,2} = 3C_2^2 + C_4$ | $Z_{4,3} = C_2^2 - C_4$<br>$Z_{4,4} = 3C_2^2 - 4C_3 + C_4$ |
| $Z_{5,1} = 10C_2C_3 + 5C_4 + C_5$ | $Z_{5,2} = 2C_2C_3 - C_4 - C_5$<br>$Z_{5,3} = 2C_2C_3 - 3C_4 + C_5$ |

| $i$ | $\gamma^{(2,i)}$ | $i$ | $\gamma^{(2,i)}$ |
|---|---|---|---|
| 1 | $7 + 23C_2 - Z_{2,1}\sigma^2(102 - 95\sigma^2)$ | 3 | $12Z_{2,2}(\sigma^2-1)^2(5\sigma^2-1)$ |
| 2 | $5 - 11C_2 + 5Z_{2,1}\sigma^2(6 - 7\sigma^2)$ | | |
| $i$ | $\gamma^{(3,i)}$ | $i$ | $\gamma^{(3,i)}$ |
| 1 | $Z_{3,1}(5 - 9\sigma^2)$ | 3 | $4Z_{3,2}(\sigma^2-1)(5\sigma^2-3)$ |
| 2 | $Z_{3,1}(7\sigma^2 - 3)$ | | |
| $i$ | $\gamma^{(4,i)}$ | $i$ | $\gamma^{(4,i)}$ |
| 1 | $44C_3 + 59Z_{4,2} - Z_{4,1}\sigma^2(250-239\sigma^2)$ | 4 | $12Z_{4,3}(1-\sigma^2)(23-102\sigma^2+95\sigma^4)$ |
| 2 | $72C_3 - 78Z_{4,2} + Z_{4,1}\sigma^2(276-294\sigma^2)$ | 5 | $12Z_{4,3}(\sigma^2-1)(11-30\sigma^2+35\sigma^4)$ |
| 3 | $28C_3 - 9Z_{4,2} + 7Z_{4,1}\sigma^2(2-3\sigma^2)$ | 6 | $24Z_{4,4}(\sigma^2-1)^3(5\sigma^2-1)$ |
| $i$ | $\gamma^{(5,i)}$ | $i$ | $\gamma^{(5,i)}$ |
| 1 | $Z_{5,1}(7 - 13\sigma^2)$ | 4 | $12Z_{5,2}(\sigma^2-1)(9\sigma^2-5)$ |
| 2 | $2Z_{5,1}(11\sigma^2 - 5)$ | 5 | $12Z_{5,2}(\sigma^2-1)(3-7\sigma^2)$ |
| 3 | $Z_{5,1}(3\sigma^2 - 1)$ | 6 | $8Z_{5,3}(\sigma^2-1)^2(3-5\sigma^2)$ |

What do to do with this amplitude?

# Conjecture

[Submitted on 11 Mar 2022 (v1), last revised 29 Dec 2022 (this version, v3)]

**Binary Dynamics Through the Fifth Power of Spin at $\mathcal{O}(G^2)$**

Zvi Bern, Dimitrios Kosmopoulos, Andrés Luna, Radu Roiban, Fei Teng

We conjectured that the scattering amplitude of two Kerr black holes is the one that realises the symmetry

$$a_i^\mu \to a_i^\mu + \xi_i q^\mu / q^2, \quad i = 1, 2$$

Computed observables up to S^5

# Why do we trust these results?

Checks against GR!

# Checks: Quadratic-in-spin Hamiltonians

[Submitted on 14 Jul 2016 (v1), last revised 22 Sep 2021 (this version, v2)]

**Complete conservative dynamics for inspiralling compact binaries with spins at the fourth post-Newtonian order**

Michèle Levi, Jan Steinhoff

NNLO post-Newtonian ✔

[Submitted on 27 Jan 2016 (v1), last revised 28 Jun 2021 (this version, v3)]

**Canonical Hamiltonian for an extended test body in curved spacetime: To quadratic order in spin**

Justin Vines, Daniela Kunst, Jan Steinhoff, Tanja Hinderer

Test-body limit ✔

To compare with (overlapping parts of) them, we may compute Amplitudes from the Hamiltonian using EFT.

# Checks: Higher powers in spin (angles)

*[Submitted on 16 Sep 2019]*

**Test black holes, scattering amplitudes and perturbations of Kerr spacetime**

Nils Siemonsen, Justin Vines

Consider a generalization of MPD equations ✔

$$\frac{\mathrm{D}}{\mathrm{d}\tau}p_\mu + \frac{1}{2}R_{\mu\nu\kappa\lambda}\dot{z}^\nu S^{\kappa\lambda} = \frac{p\cdot\dot{z}}{2}\frac{\mathrm{D}}{\mathrm{D}z^\mu}\log\mathcal{M}^2,$$

$$\frac{\mathrm{D}}{\mathrm{d}\tau}S^{\mu\nu} - 2p^{[\mu}\dot{z}^{\nu]} = p\cdot\dot{z}\left(p^{[\mu}\frac{\partial}{\partial p_{\nu]}} + 2S^{[\mu}{}_\rho\frac{\partial}{\partial S_{\nu]\rho}}\right)\log\mathcal{M}^2,$$

With a dynamical mass function

$$\mathcal{M}^2_{\mathrm{GOV}} = m^2 + 2m^2u^\mu u^\nu\sigma^{\rho_1}\sigma^{\rho_2}\left(-\frac{1}{2!}R_{\mu\rho_1\nu\rho_2} + \frac{1}{3!}{}^*R_{\mu\rho_1\nu\rho_2;\rho_3}\sigma^{\rho_3} + \frac{1}{4!}R_{\mu\rho_1\nu\rho_2;\rho_3\rho_4}\sigma^{\rho_3}\sigma^{\rho_4}\right) + \mathcal{O}(\sigma^5)$$

Angles agree!

*[Submitted on 25 Nov 2022]*

**Completing the Fifth PN Precision Frontier via the EFT of Spinning Gravitating Objects**

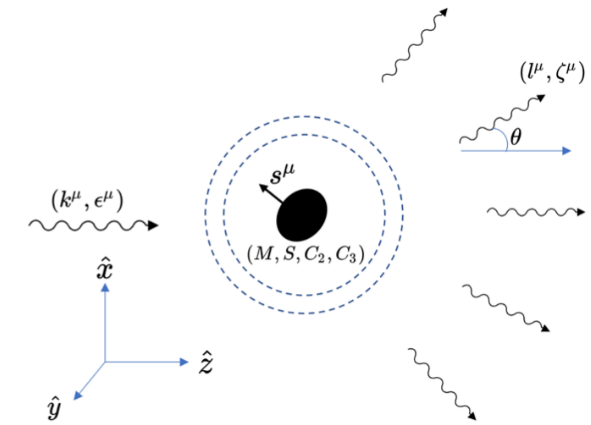
Michèle Levi, Zhewei Yin ✔

# Checks: Generic bodies (Neutron Stars)

*[Submitted on 5 Aug 2022 (v1), last revised 30 Aug 2022 (this version, v2)]*

**Scattering of gravitational waves off spinning compact objects with an effective worldline theory**

M. V. S. Saketh, Justin Vines




$$\mathcal{M}_{++} = GM\omega \frac{\cos^4(\theta/2)}{\sin^2(\theta/2)} \Big( \exp[a\cdot(k+l-2w_{\rm S})] + \frac{C_2-1}{2}[(k-w_{\rm S})\cdot a]^2 + [(l-w_{\rm S})\cdot a]^2$$
$$+ \frac{C_2-1}{2}[(k-w_{\rm S})\cdot a][(l-w_{\rm S})\cdot a][(k+l-2w_{\rm S})\cdot a] - (C_2-1)^2[(k-w_{\rm S})\cdot a][(l-w_{\rm S})\cdot a](w_{\rm S}\cdot a)$$
$$+ \frac{C_3-1}{6}\{[(k-w_{\rm S})\cdot a]^3 + [(l-w_{\rm S})\cdot a]^3\}\Big).$$

Compton Amplitude and scattering angles agree

$$\frac{\theta^{\rm NLO}_{\rm S_1^3}}{\Gamma} = \tilde{v}\tilde{a}_1^3 \Bigg[ -\frac{4}{\tilde{b}} C_{\rm 1BS^3} + \frac{\pi}{\tilde{b}^2}\Bigg( \left(\frac{15\nu}{4}\tilde{v}^2 + \left(3\nu + 6 + \left(-\frac{3\nu}{2} - \frac{27}{4}\right)\tilde{v}^2\right) C_{\rm 1ES^2}\right.$$
$$+ \left(-6 + \left(\frac{27\nu}{4} - \frac{33}{4}\right)\tilde{v}^2\right) C_{\rm 1BS^3}$$
$$+ \frac{\nu}{q}\left(\frac{15}{4}\tilde{v}^2 + \left(3 - \frac{3}{2}\tilde{v}^2\right) C_{\rm 1ES^2} + \frac{27}{4}\tilde{v}^2 C_{\rm 1BS^3}\right)\Bigg)\Bigg],$$

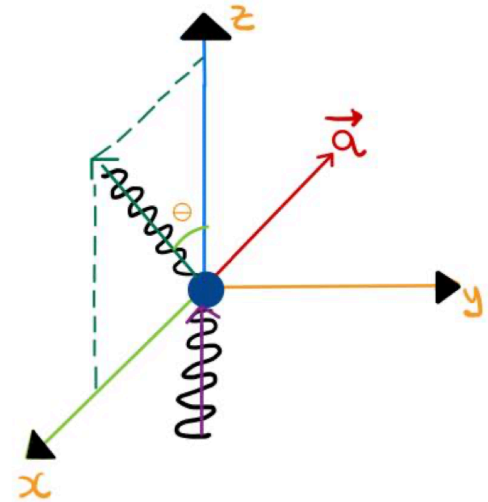
$$\frac{\theta^{\rm NLO}_{\rm S_1^2 S_2}}{\Gamma} = \tilde{v}\tilde{a}_1^2\tilde{a}_2 \Bigg[ -\frac{12}{\tilde{b}} C_{\rm 1ES^2} + \frac{\pi}{\tilde{b}^2}\Bigg( 6\nu - 12 + \tilde{v}^2\left(\frac{27\nu}{8} - \frac{99}{8}\right)$$
$$+ \left(-3\nu + \left(\frac{39\nu}{8} - \frac{207}{8}\right)\tilde{v}^2 - 21\right) C_{\rm 1ES^2}$$
$$+ \frac{\nu}{q}\left(6 + \frac{27}{8}\tilde{v}^2 + \left(-3 + \frac{39}{8}\tilde{v}^2\right) C_{\rm 1ES^2}\right)\Bigg)\Bigg].$$

*[Submitted on 31 Oct 2022]*

**From the EFT of Spinning Gravitating Objects to Poincaré and Gauge Invariance**

Michèle Levi, Roger Morales, Zhewei Yin

# Checks: BHPT




[Submitted on 15 Dec 2022 (v1), last revised 21 Feb 2023 (this version, v2)]

**Scattering in Black Hole Backgrounds and Higher-Spin Amplitudes: Part II**

Yilber Fabian Bautista, Alfredo Guevara, Chris Kavanagh, Justin Vines

Matching solutions of the Teukolsky equation

$$f'^{\rm QFT}_{lm}(\gamma) = f'^{\rm BHPT}_{lm}(\gamma)$$

$$f'^{\rm QFT}_{lm}(\gamma) = \int d\Omega' \, {}_{-2}Y^*_{lm}(\theta,\phi')\langle A_4(\gamma,\theta,\phi')\rangle$$

$$\langle A_4^S\rangle = \langle A_4^0\rangle \times \left(e^{(2w+k_3-k_2)\cdot a} + P_\xi(k_2\cdot a, -k_3\cdot a, w\cdot a)\right)_{2S}$$

A most general form of the amplitude compatible with crossing symmetry, locality, unitarity.

$$f'^{(\rm BHPT)}_{lm}(\gamma) = \sum_{m'} D^{l*}_{m'm}(\gamma)\, f_{lm'}$$

$$f_{lm} = \frac{2\pi}{i\omega}\sum_{P=\pm 1}\left(e^{2i\delta^P_{lm}} - 1\right)$$

$$e^{2i\delta^P_{lm}} = (-1)^{l+1}\frac{\mathcal{C}_{lm} + 12iM\omega P}{16\omega^4}\frac{B^{\rm ref}_{lm\omega}}{B^{\rm inc}_{lm\omega}}$$

Existing solutions in BHPT literature

Agreement up to S^4
(but not at S^5)

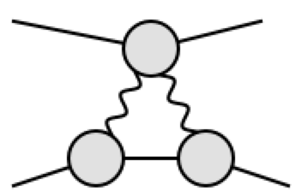
# Summary

EFT

$\mathcal{M}_{2\text{PM}}^{\text{EFT}}$

Amplitudes

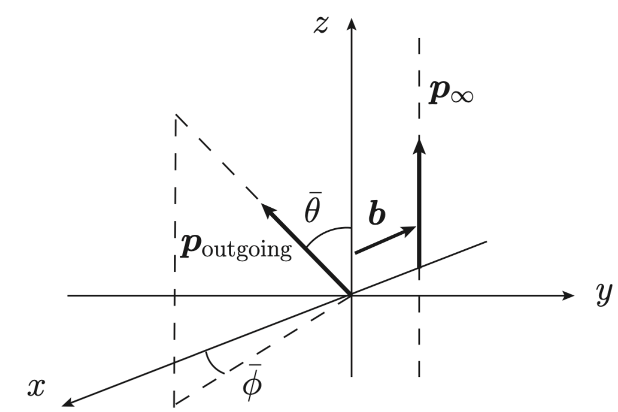
$\mathcal{M}_4^{1 \text{ loop}}$



$\mathcal{L}$

$H(r^2, p^2)$

Observables




$$\Delta\mathcal{O} = e^{-i\chi^{\mathcal{D}}}[\mathcal{O}, e^{i\chi^{\mathcal{D}}}]$$

Eikonal

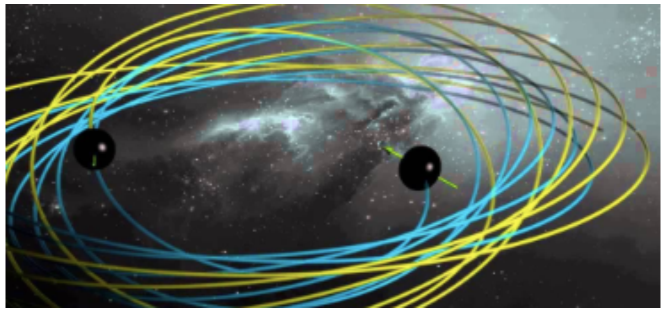
$$a_i^\mu \to a_i^\mu + \xi_i q^\mu / q^2, \quad i = 1, 2$$

| Spin | |
|---|---|
| $a^4$ | ✔ |
| $a^5$ | Working on it |

- A framework to translate Amplitudes into Observables for black holes (and generic)
- Combine Amplitudes, Effective Theory, Eikonal.
- Extensive GR checks in PN, self-force, BHPT.
- Agreement up to S^4 BH, and S^3 generic.

# Outlook

- Higher spin. Understand black hole S^5 discrepancy with BHPT. And beyond?



- More loops.
  Improve amplitudes (bootstrap?),
  Improve observables from amplitudes