Spinning Binaries from quantum fields

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• Why Amplitudes?

• Amplitudes for binaries. Spin.

• Summary / Outlook.

Classical physics from Quantum field theory?

Progress of Theoretical Physics, Vol. 46, No. 5, November 1971

Quantum Theory of Gravitation vs. Classical Theory*

-----Fourth-Order Potential-----

Yoichi IWASAKI

Trees AND loops?

Here we want to point out that there seems to exist an erroneous belief^{*),**)} that only tree diagrams contribute to the classical process. Contrary to this belief, the quadratic term in k corresponds to fourth-order diagrams each of which contains a closed loop; it is a "radiative correction" term. Since the quantum

Iwasaki knew it 50 years ago...

Amplitudes: Efficiency



The Amplitudes program: Avoid summing all Feynman diagrams.

Recycle (use amplitudes as building blocks)





Recursion relations: big trees from small trees.

Generalized unitarity: loops from trees.



Cool. But nothing we couldn't have gotten with sufficient time (and sufficient RAM).

But not quite...

Amplitudes: Six gluons

THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON-GLUON FUSION

Stephen J. PARKE and T.R. TAYLOR

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985



As the result of the computation of two hundred and forty Feynman diagrams, we obtain

$$A_{\binom{0}{2}}(p_{1}, p_{2}, p_{3}, p_{4}, p_{5}, p_{6})$$

$$= (\mathscr{D}^{\dagger}, \mathscr{D}_{\rho}^{\dagger}, \mathscr{D}_{\sigma}^{\dagger}, \mathscr{D}_{\tau}^{\dagger})_{\binom{0}{2}} \cdot \begin{pmatrix} K & K_{\rho} & K_{\sigma} & K_{\tau} \\ K_{\rho} & K & K_{\tau} & K_{\sigma} \\ K_{\sigma} & K_{\tau} & K & K_{\rho} \\ K_{\tau} & K_{\sigma} & K_{\rho} & K \end{pmatrix} \cdot \begin{pmatrix} \mathscr{D} \\ \mathscr{D}_{\rho} \\ \mathscr{D}_{\sigma} \\ \mathscr{D}_{\tau} \end{pmatrix}_{\binom{0}{2}}, \qquad (6)$$

TABLE Matrices K(I, J)[I =	1 1-11, J = 1-11].	1			S.J. Parke, T.R. Taylor / Four gluon production 415	416 S.J. Parke, T.R. Teylor / Four gluon production	S.J. Parke, T.R. Taylor / Four gluon production	417	418 S.J. Parke, T.R. Taylor / Four glass production	$D_0^5(7) = \frac{1}{s_{22}s_{34}t_{125}} [s_{34} - s_{46} + s_{34}] [s_{12} - s_{15} - s_{25}],$
Matrix K ⁽⁴⁾	Matrix K ⁽²⁾		000-000000		where ε is the totally antisymmetric tensor, $\varepsilon_{syst} = 1$. For the future use, we define one more function,	$D_1^G(9) = \frac{4}{s_{15}s_{96}t_{125}}\{[(p_1 - p_2 + p_3)(p_4 + p_3 - p_6)]E(p_3, p_3)$	$D_2^G(24) = \frac{-2}{s_{23}s_{34}} E(p_2 - p_3, p_3),$		$D_0^{\mu}(4) = \frac{4}{s_{22}s_{34}t_{123}} \{F(p_{2}, p_{3})E(p_{3}, p_{3}) - F(p_{3}, p_{3})E(p_{2}, p_{3})$	$D_0^5(8) = \frac{1}{s_{10}s_{10}t_{10}} [s_{25} + s_{35} - s_{23}][s_{14} - s_{46} + s_{16}],$
4 8 -1 1 -1 0 2 1 0 1 -1 -2 -1 8 4 4 1 1 2 2 1 2			00-00000000		$F(p_{\mu}, p_{j}) = \{(p_{1}p_{4})(p_{j}p_{j}) + (p_{1}p_{j})(p_{j}p_{4}) - (p_{1}p_{j})(p_{j}p_{4})\}/(p_{1}p_{4}). (10)$	$-[(p_1-p_2+p_3)(p_4-p_3+p_6)]E(p_5,p_6)+[p_6(p_3-p_6)]E(p_5,p_2-p_3)],$	$D_2^Q(25) = \frac{2}{s_{10}s_{23}} E(p_0, p_2 - p_3),$		+ [$F(p_3, p_2) - \frac{1}{2}s_{23} - \frac{1}{2}s_{12} + \frac{1}{2}s_{13}$] $E(p_3, p_3)$ },	$D_{n}^{5}(9) = \frac{1}{1 - \frac{1}{1 - \frac{1}{2}}} \left[s_{1n} + s_{1n} - s_{1n} \right] \left[s_{1n} - s_{1n} + s_{1n} \right],$
-1 -1 4 2 8 1 2 4 -2 -1 4		_	0000-00-000	00000000740	Note that when evaluating A_0 and A_2 at crossed configurations of the momenta, core must be taken with the implicit demodence of the functions E . Exact C are	$D_2^{C_1}(10) = \frac{1}{r_{23}r_{40}r_{123}} \{ [(p_1 + p_2 - p_3)(p_4 - p_3 + p_4)] E(p_2, p_4) \}$	$D_{2}^{G}(26) = \frac{-2}{E(p_{2}, p_{1} - p_{1})}$		$D_0^0(5) = \frac{1}{s_{16}s_{25}t_{146}} [s_{25} - s_{23} + s_{25}]E(p_{4s}, p_5),$	525534134
0 2 1 -1 2 4 8 -2 0 0 0 1 1 2 4 4 -1 -2 8 -1 -1 2	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1-16	000-000-000		the momenta p ₁ , p ₄ , p ₃ , p ₄ , p ₄ .	$-[(p_1-p_2+p_3)(p_4-p_3+p_4)]E(p_3,p_4)+[p_1(p_2-p_3)]E(p_3-p_4,p_4)\},$	- 2		$D_0^{\beta}(6) = \frac{2}{s_{36} - s_{26} - s_{26} - s_{25}} E(p_3, p_5),$	$D_0^3(10) = \frac{1}{s_{25}s_{34}} (p_2 - p_3)(p_3 - p_6),$
0 0 2 1 -2 0 0 -1 8 4 -2 0 1 1 1 -1 1 0 -1 4 8 -1 -1 -1 2 1 4 0 0 2 -2 -1 8	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	-	0000000000000	00-7700-00	The singlatus D_2 are noted below.	$D_2^Q(11) = \frac{\delta_2}{s_{36}s_{124}} [s_{33} - s_{56} + s_{56}],$	$D_2^{\omega}(27) = \frac{1}{\pi_{ab}t_{125}} E(p_3 - p_4, p_6)$,		PERSON A PERSON IN IN A REPORT OF	$D_{2}^{5}(11) = \frac{1}{(p_{1} - p_{2})(p_{2} - p_{2})}$
Matrix K ⁽⁴⁾	Matrix K ⁽²⁾	1-1)	0070-000000	Matrix	$D_2^G(1) = \frac{a_2}{s_{14}s_{25}s_{36}} \left\{ \left[(p_2 - p_3)(p_3 - p_4) \right] \left[(p_1 - p_4)(p_3 + p_6) \right] - \left[(p_2 - p_3)(p_3 + p_6) \right] \right\}$	$D_1^Q(12) = \frac{-\delta_2}{[s_{21} - s_{22} - s_{23}]}$	$D_2^G(28) = \frac{2}{s_{13}s_{123}} E(p_3, p_2 - p_3),$		$D_0(t) = \frac{1}{s_{25}s_{36}t_{125}} \left\{ \frac{1}{2} P(p_5, p_2) - \frac{1}{2}s_{25} - \frac{1}{2}s_{12} + \frac{1}{2}s_{15} \right\} E(p_5, p_5)$	314534 CT PD (5) PD (5)
	3 3 0 3 0 0 0 3 0 0 0	r'he	000000000000	00-0-000000	×[$(p_1-p_4)(p_3-p_4)$]+[$(p_2+p_3)(p_3-p_4)$][$(p_1-p_4)(p_2-p_3)$]},	• 5 ₃₄ 5 ₄₆ • • • • • • • • • • • • • • • • • • •	$D_2^{\rm O}(29) = \frac{-2}{-2} E(p_3 - p_4, p_3),$		+ $[F(p_2, p_3) + \frac{1}{4}t_{122}]E(p_3, p_3) - [F(p_5, p_3) + \frac{1}{4}t_{123}]E(p_2, p_3)]$,	$D_0^{S}(12) = \frac{\kappa}{s_{16}s_{25}} (p_6 - p_1)(p_2 - p_5),$
0 0 0 0 0 1 1 1 0 1 1 0 0 0 0 0 0 0 0 0	0 0 3 0 0 3 0 0 3 3 0 3 0 0 0 0 3 0 0 0 0	pu	· · · · · · · · · · · · · · · · · · ·	****	$D_2^G(2) = \frac{1}{s_{24}s_{16}} \left\{ 2E(p_2 - p_3, p_3 - p_6) - 2E(p_3 - p_6, p_2 - p_3) + \delta_2[(p_2 - p_3)(p_3 - p_6)] \right\},$	$D_2^G(13) = \frac{\sigma_2}{s_{14}s_{36}t_{124}} \left[s_{12} - s_{34} \right] \left[s_{33} - s_{36} + s_{36} \right],$	² 34 ² 125		$D_0^{\mu}(8) = \frac{1}{s_{14}s_{36}} E(p_3 - p_4, p_5),$	$D_{2}^{5}(13) = \frac{1}{(p_{2} - p_{1})(p_{2} - p_{4})},$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0 0 0 0 0 0 0 0 0 3 3 0 0 0 3 3 0 0 0 0	Ŧ			$D^{G}(3) = -\frac{4}{((s_{1}+s_{2}-s_{2})(s_{2}+s_{2}-s_{2}))E(s_{2},s_{2})}$	$D_{2}^{G}(14) = \frac{\delta_{2}}{s_{1} \cdot s_{2} \cdot s_{3}} [s_{13} - s_{43}] [s_{23} - s_{24} - s_{34}],$	$D_2(50) = \frac{1}{3_{12} S_{34} t_{125}} \left((p_1 + p_2 - p_5) (p_4 + p_3 - p_6) - t_{125} \right) C(p_2, p_3) ,$		$D_0^T(9) = \frac{2}{s_{14}s_{36}t_{124}} [s_{35} - s_{56} + s_{36}]E(p_2, p_5),$	³ 13 ³ 34
1 1 1 2 2 0 0 4 0 0 0 1 2 0 0 0 0 0 0 0 2 -1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	- = v		· · · · · - , · · · ·	$f_{22}f_{34}f_{32}$ -[(a, +a, -a)]F(a, -a, +a)]F(a, -a)		$D_2^G(31) = \frac{4}{s_{12}s_{44}t_{125}} [(p_1 + p_2 - p_5)(p_4 - p_3 + p_6) + t_{125}]E(p_2, p_6),$		$D_0^F(10) = \frac{2}{(s_{23} - s_{24} - s_{24})} [s_{23} - s_{24} - s_{24}] E(p_1, p_3),$	$\mathcal{D}_0^2(14) = \frac{1}{s_{25}s_{34}} \left((p_2 - p_5)(p_3 - p_4) \right),$
-1 -2 1 0 2 -1 -2 0 -1 0 4	0 0 0 0 0 0 0 0 0 0 0	TANU 1/1-1 dainx	00000-00000		$-[(p_1-p_2+p_3)(p_4+p_3-p_6)]E(p_3, p_3)$	$D_{2}^{-}(15) = \frac{1}{s_{14}s_{26}}(p_{1} - p_{4})(p_{5} - p_{6}),$	$D_2^G(32) = \frac{4}{\delta_{12}\delta_{22}\delta_{12}} [(p_1 - p_2 + p_3)(p_4 + p_3 - p_6) + t_{123}]E(p_3, p_3),$		F14756546	$D_0^{5}(15) = \frac{1}{s_{14}s_{25}s_{36}} \{ [(p_2 + p_5)(p_3 - p_6)][(p_1 - p_4)(p_2 - p_5)] \}$
4 2 0 2 0 1 0 1 0 0 0	0 0 0 0 0 0 0 0 0 3 0 -3	- n ₁₀		11	$+[(p_1-p_2+p_3)(p_4-p_3+p_6)]E(p_5,p_6)$	$D_2^G(16) = \frac{-4}{s_{12}s_{34}s_{124}} [s_{33} - s_{34} + s_{34}]E(p_2, p_2),$	$D_{2}^{G}(33) = \frac{4}{100000000000000000000000000000000000$	11)	$D_{6}^{*}(11) = \frac{1}{2s_{16}s_{23}s_{36}} \{ (s_{23} + s_{35} - s_{26} - s_{56}) E(p_2 - p_5, p_5) \}$	+ $[(p_2 - p_5)(p_3 - p_4)][(p_1 - p_4)(p_3 + p_6)]$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0 0 0 0 3 0 0 3 0 0 0 0 0 0 0 0 0 0 0 0	166-			$\begin{bmatrix} b \\ c \\$	$D_2^Q(17) = \frac{4}{s_{-}s_{-}s_{-}} [s_{23} - s_{26} - s_{36}]E(p_3, p_3),$	3153ml125		$-[s_{23}+s_{26}-s_{33}-s_{34}]E(p_3-p_6, p_3)-[s_{23}+s_{34}-s_{33}-s_{26}]E(p_2+p_5, p_3)].$ (12)	+[$(p_1+p_4)(p_2-p_3)$][$(p_1-p_4)(p_3-p_5)$]},
0 0 2 1 0 0 0 0 4 2 2 1 0 1 2 0 0 0 0 1 2 0	0 3 0 0 0 0 3 0 0 0 0 0 0 0 0 0 3 3 0 0 0 -3	5	000000000		$D_{2}^{Q}(4) = \frac{-2}{(E(n-n,n+n)-\delta(n(n-n)))}$	-4 (a) -1 (b) 10()	The diagrams D_0^G are obtained from D_2^G by replacing δ_2 by $\delta_0 = 0$ and the function	ins	The diagrams D ₀ ³ are listed below:	$D_0^{S}(16) = \frac{2}{s_{16}s_{14}s_{25}} \{ [(p_2 - p_5)(p_3 + p_4)][(p_1 - p_6)(p_3 - p_4)] \}$
0 1 1 1 0 0 0 0 2 4 0 1 1 1 0 0 0 0 0 2 0 1 0 0 2 1 4 1 2 2 0 0 -4	0 0 3 0 3 3 3 0 0 0 0 0 0 0 0 0 0 0 0 0	3				$D_2^{-}(13) = \frac{1}{s_{12}s_{19}s_{43}} \left[2(p_1 + p_2)(p_3 - p_6) - s_{36} \right] E(p_2, p_3) ,$	$E(p_a, p_j)$ by $G(p_a, p_j)$. The diagrams D_b^{μ} are listed below:		$D_{0}^{2}(1) = \frac{1}{s_{23}s_{36}t_{123}} [s_{34} - s_{46} + s_{36}][s_{12} - s_{15} - s_{25}],$	+ $[(p_1 + p_4)(p_3 - p_4)][(p_1 - p_6)(p_2 - p_5)]$
0 1 1 0 2 2 4 0 0 0 -2 0 0 0 0 2 0 0 1 -4 -2 4	0 0 0 0 0 0 0 0 0 0 3 0 -3 0 0 0 0 -3 0 0 0 0 0 0	96.12			$\begin{bmatrix} 5 \\ D_{2}^{*}(5) = \frac{1}{s_{25}t_{123}} \left[E(p_{2} + p_{5}, p_{2} - p_{5}) - \delta_{3}[p_{1}(p_{2} - p_{5})] \right],$	$D_2^G(19) = \frac{-2}{s_{12}s_{36}} E(p_2, p_3 - p_6)$,	$D_0^p(1) = -\frac{4}{4} \{F(p_5, p_6)E(p_3, p_3) - F(p_5, p_3)E(p_6, p_5)\}$		$D_0^5(2) = \frac{1}{s_{14}s_{56}t_{124}} [s_{12} - s_{24} - s_{14}] [s_{35} - s_{56} + s_{36}],$	+ $[(p_1 - p_6)(p_2 + p_5)][(p_3 - p_4)(p_2 - p_5)]]$. (13)
Matrix K ⁽⁴⁾	Matrix K ⁽²⁾	5 8	00-0-000000	00-1700-000	$D_2^Q(6) = \frac{\delta_2}{t_{125}},$	$D_2^G(20) = \frac{2}{t_{p_1}t_{p_1}} E(p_2 - p_{t_1}, p_3),$	$^{2}_{33} ^{5}_{34} ^{4}_{123}$ + { $F(p_{4}, p_{3}) + s_{34} \} E(p_{5}, p_{5}) \} ,$		$D_{6}^{5}(3) = \frac{1}{s_{-5} - s_{-5}} [s_{13} - s_{43} + s_{14}] [s_{23} - s_{36} - s_{36}],$	The preceding list completes the result. Let us recapitulate now the numerical procedure of calculating the full cross section. First the diagrams D are calculated
0 1 -1 -1 1 1 0 1 2 0 0 1 0 -2 -1 2 0 1 1 4 2 0 -1 -2 0 0 0 1 1 1 -1 1 0	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Math			$D_{1}^{Q}(7) = \frac{4}{J_{1} + J_{2} + J_{2}} \{ [(p_{1} + p_{2} - p_{3})(p_{4} + p_{3} - p_{4})] E(p_{3}, p_{3}) \}$	$D_2^G(21) = \frac{-4}{(s_{28} - s_{36} + s_{23})E(p_3, p_3)},$	$D_0^F(2) = \frac{-4}{1 - 1 - 1} \{ [F(p_6, p_2) + \frac{1}{2} s_{16}] E(p_3, p_3) \}$		$D_{2}^{S}(4) = -\frac{1}{1} [s_{11} + s_{22} - s_{22}][s_{22} - s_{22} + s_{22}].$	by using eqs. (11)-(13). The result is substituted to eq. (8) to obtain the vectors \mathcal{D}_0 and \mathcal{D}_1 . After generating the vectors \mathcal{D}_0 , \mathcal{D}_1 , \mathcal{D}_2 , \mathcal{D}_3 , and \mathcal{D}_3 by the appropriate
-1 -1 0 1 0 2 1 0 1 -1 0 1 2 0 0 1 -1 -1 0 -2 2 1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		00	• • - • , • • • • • • •	$ = \frac{1}{2} \left[(p_1 + p_2 - p_1)(p_4 - p_3 + p_4)]E(p_3, p_4) - [p_4(p_3 - p_4)]E(p_3, p_2 - p_3) \right], $	*20*3**13* =0	+ $[F(p_{2s}, p_{3}) + \frac{1}{2}s_{3s}]E(p_{4s}, p_{3}) - F(p_{4s}, p_{3})E(p_{2s}, p_{3})\},$		515344125 - 55 - 568-54 - 48 - 5363	permutations of momenta, eq. (6) is used to obtain the functions A_0 and A_2 . Finally, the total cross section is calculated by using a_0 (5) The EOPTRAN sectors
0 1 1 1 -1 1 0 -1 4 8 -1 1 1 1 0 0 -2 -1 0 2 -2 0	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		• • - - - • • • • • • • • • • • • • • • • • •	0000-00-00	$D_{2}^{G}(8) = \frac{4}{t_{1}, t_{2}, t_{3}} [[(p_{1} + p_{2} - p_{3})(p_{4} + p_{3} - p_{4})]E(p_{2}, p_{3})$	$D_2^{r}(22) = \frac{1}{s_{16}s_{25}t_{166}} \left[s_{23} - s_{55} - s_{25} \right] k \left(p_0, p_0 \right),$	$D_{0}^{F}(3) = \frac{4}{(p_{2}, p_{4})E(p_{2}, p_{3}) - F(p_{2}, p_{3})E(p_{4}, p_{3})}$		$D_0^{S}(5) = \frac{1}{x_{15}x_{24}t_{156}} [x_{26} - x_{15} - x_{16}][x_{23} - x_{24} - x_{34}],$	based on such a scheme generates ten Monte Carlo points in less than a second on the heterorie CDC CYDER 125/875
2 4 -1 1 -2 2 4 2 1 0 -2 0 2 1 -1 2 4 8 -2 0 0 0 0 0 0 0 1 -1 -1 0 -2 0 2	0 0 0 0 0 0 0 0 3 3 0 9 0 0 0 0 0 0 0 3 3 0 9 0 0 3 0 -3 0 0 0 0 0 0				$ \frac{1}{\frac{2}{3}} \frac{1}{2} \frac{1}{$	$D_1^G(23) = \frac{4}{s_{16}s_{25}s_{14}} [2(p_1 + p_k)(p_2 - p_3) + s_{23}]E(p_4, p_3) ,$	$s_{15}s_{34}s_{125}$ -[$F(p_{3}, p_{4}) - \frac{1}{3}s_{34} - \frac{1}{3}s_{44} + \frac{1}{3}s_{46}]E(p_{5}, p_{5})$ },		$D_0^5(6) = \frac{1}{s_{13}s_{34}t_{123}}[s_{46} - s_{34} - s_{36}][s_{13} - s_{25} - s_{15}],$	Given the complexity of the final result, it is very important to have some reliable
		1	1	r	1.5					testing procedures available for numerical calculations. Usually in QCD, the multi- gluon amplitudes are tested by checking the gauge invariance. Due to the specifics

Amplitudes: Simplicity

Amplitude for *n*-Gluon Scattering

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Specifying helicities leads to immense simplification

$$\begin{aligned} |\mathcal{M}_{n}(+++++\cdots)|^{2} &= c_{n}(g,N)[0+O(g^{4})], \\ |\mathcal{M}_{n}(-++++\cdots)|^{2} &= c_{n}(g,N)[0+O(g^{4})], \\ |\mathcal{M}_{n}(--+++\cdots)|^{2} &= c_{n}(q,N)[(p_{1}\cdot p_{2})^{4} \\ &\times \sum_{P}[(p_{1}\cdot p_{2})(p_{2}\cdot p_{3})(p_{3}\cdot p_{4})\cdots(p_{n}\cdot p_{1})]^{-1} + O(N^{-2}) + O(g^{2})], \end{aligned}$$

The Parke-Taylor formula

$$A_n[1^+ \dots i^- \dots j^- \dots n^+] = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

Should you care?

You should Kerr!

Kerr black hole Amplitudes

[Submitted on 18 Sep 2017 (v1), last revised 29 Sep 2017 (this version, v2)]

Scattering of two spinning black holes in post-Minkowskian gravity, to all orders in spin, and effective-one-body mappings

$$\mathcal{S}_{
m int}[\Psi,h] = rac{1}{2}\int d^4x \; h_{\mu
u} \; T^{\mu
u}[\Psi]$$

n

Justin Vines

For a linearized Kerr black hole

$$T^{\mu\nu}(x) = \int d\tau \ \hat{\mathcal{T}}^{\mu\nu}(p, a, \partial) \ \delta^4(x - z)$$
$$\hat{\mathcal{T}}^{\mu\nu}(p, a, \partial) = m \ \exp(a * \partial)^{(\mu}{}_{\rho} \ u^{\nu)} u^{\rho}$$

[Submitted on 17 Dec 2018 (v1), last revised 9 Sep 2019 (this version, v3)]

Scattering of Spinning Black Holes from Exponentiated Soft Factors

Alfredo Guevara, Alexander Ochirov, Justin Vines

The stress energy tensor in momentum space $T^{\mu\nu}(-k) = 2\pi \delta(p\cdot k) p^{(\mu} \exp(a*ik)^{\nu)}{}_{\rho} p^{\rho}$

Matches the simplest three-point amplitude

$$h_{\mu\nu}(k)T^{\mu\nu}(-k) = \frac{1}{2}(2\pi)^2\delta(k^2)\delta(p\cdot k)\lim_{s\to\infty}\langle\mathcal{M}_3^{(s)}\rangle$$

$$h_{\mu\nu} = 2\pi\delta(k^2)\varepsilon_{\mu}\varepsilon_{\nu_{2}} \qquad \qquad \mathcal{M}_{3}^{(s)}(p_1, p_2, k^+) = \frac{\langle 12\rangle^{2s}x^2}{m^{2s-2}}$$

Observables from Amplitudes

[Submitted on 17 Dec 2018 (v1), last revised 9 Sep 2019 (this version, v3)]

Scattering of Spinning Black Holes from Exponentiated Soft Factors

Alfredo Guevara, Alexander Ochirov, Justin Vines



From the 3-point amplitude, computed 1PM scattering angle

$$\theta = \frac{GE}{v^2} \left[\frac{(1+v)^2}{b+a_a+a_b} + \frac{(1-v)^2}{b-a_a-a_b} \right]$$

(Matches the one computed by Vines)



Pipeline



Gauge trees Gravity trees. Double copy Unitarity: Loops from trees

Amplitudes to Observables

Spin Hamiltonian from Amplitudes

[Submitted on 6 May 2020]

Spinning Black Hole Binary Dynamics Scattering Amplitudes and Effective Field Theory

Zvi Bern, Andres Luna, Radu Roiban, Chia-Hsien Shen, Mao Zeng





We consider a Lagrangian of rank-s tensor fields, minimally coupled to gravity

$$\mathcal{L}_{\min} = -R(e,\omega) + \frac{1}{2}g^{\mu\nu}\nabla(\omega)_{\mu}\phi_{s}\nabla(\omega)_{\nu}\phi_{s}$$

$$\phi_s{}^{a_1a_2a_3...a_s}$$

Symmetric traceless tensor field

Covariant derivative

Lorentz Generator

$$\nabla(\omega)_{\mu}\phi_{s} \equiv \partial_{\mu}\phi_{s} + \frac{i}{2}\omega_{\mu ef}M^{ef}\phi_{s}$$

The spin tensor is obtained in the classical limit $\varepsilon(s, p_1) M^{ab} \varepsilon(s, p_2) = S(p_1, S)^{ab} \varepsilon(s, p_1) \cdot \varepsilon(s, p_2) + O(q^0)$

Effective theory



An EFT of non-relativistic fields...

$$S = \int_{\boldsymbol{k}} \sum_{a=1,2} \xi_a^{\dagger}(-\boldsymbol{k}) \left(i\partial_t - \sqrt{\boldsymbol{k}^2 + m_i^2} \right) \xi_a(\boldsymbol{k}) - \int_{\boldsymbol{k},\boldsymbol{k}'} \xi_1^{\dagger}(\boldsymbol{k}') \xi_2^{\dagger}(-\boldsymbol{k}') \widehat{V}(\boldsymbol{k}',\boldsymbol{k},\hat{\boldsymbol{S}}_a) \xi_1(\boldsymbol{k}) \xi_2(-\boldsymbol{k})$$

...generalized to describe spinning fields. The potential...

$$\hat{V}(oldsymbol{k}',oldsymbol{k},\hat{oldsymbol{S}}_i) = \sum_A \hat{V}^A(oldsymbol{k}',oldsymbol{k}) \, \hat{\mathbb{O}}^A$$

...contains long range interactions...

$$\hat{V}^{A}(\boldsymbol{k}',\boldsymbol{k}) = \frac{4\pi G}{\boldsymbol{q}^{2}} d_{1}^{A}\left(\boldsymbol{p}^{2}\right) + \frac{2\pi^{2}G^{2}}{\left|\boldsymbol{q}\right|} d_{2}^{A}\left(\boldsymbol{p}^{2}\right) + \mathcal{O}(G^{3})$$

 $egin{aligned} \hat{\mathbb{O}}^{(0)} &= \mathbb{I}\,, & \hat{\mathbb{O}}^{(1,1)} &= oldsymbol{L}_q \cdot \hat{oldsymbol{S}}_1\,, & \hat{\mathbb{O}}^{(1,2)} &= oldsymbol{L}_q \cdot \hat{oldsymbol{S}}_2\,, & \ \hat{\mathbb{O}}^{(2,1)} &= oldsymbol{q} \cdot \hat{oldsymbol{S}}_1\,oldsymbol{q} \cdot \hat{oldsymbol{S}}_2\,, & \hat{\mathbb{O}}^{(2,2)} &= oldsymbol{q}^2\,\hat{oldsymbol{S}}_1\cdot\hat{oldsymbol{S}}_2\,, & \hat{\mathbb{O}}^{(2,3)} &= oldsymbol{q}^2\,oldsymbol{k} \cdot \hat{oldsymbol{S}}_1\,oldsymbol{k} \cdot \hat{oldsymbol{S}}_2\,, & \ \hat{\mathbb{O}}^{(2,2)} &= oldsymbol{q}^2\,\hat{oldsymbol{S}}_1\cdot\hat{oldsymbol{S}}_2\,, & \ \hat{\mathbb{O}}^{(2,3)} &= oldsymbol{q}^2\,oldsymbol{k} \cdot \hat{oldsymbol{S}}_1\,oldsymbol{k} \cdot \hat{oldsymbol{S}}_2\,, & \ \hat{\mathbb{O}}^{(2,2)} &= oldsymbol{q}^2\,oldsymbol{k} \cdot \hat{oldsymbol{S}}_2\,, & \ \hat{\mathbb{O}}^{(2,3)} &= oldsymbol{k}^2\,oldsymbol{k} \cdot \hat{oldsymbol{S}}_2\,, & \ \hat{\mathbb{O}}^{(2,3)} &= oldsymbol{k}^2\,oldsymbol{k} \cdot \hat{oldsymbol{S}}_2\,, & \ \hat{\mathbb{O}}^{(2,3)} &= oldsymbol{k}^2\,oldsymbol{k} \cdot \hat{oldsymbol{S}}_2\,, & \ \hat{\mathbb{O}}^{(2,3)} &= oldsymbol{k}^2\,oldsym$

Organized by classical spin operators

Hamiltonian

 $H = H^{(0)}(r^2, p^2) + H^{(1,i)}(r^2, p^2) \boldsymbol{L} \cdot \boldsymbol{S}_i$ + $H^{(2,1)}(r^2, p^2) \boldsymbol{r} \cdot \boldsymbol{S}_1 \boldsymbol{r} \cdot \boldsymbol{S}_2$ + $H^{(2,2)}(r^2, p^2) \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$ + $H^{(2,3)}(r^2, p^2) \boldsymbol{p} \cdot \boldsymbol{S}_1 \boldsymbol{p} \cdot \boldsymbol{S}_2$ + ..., $\mathcal{M}_4^{1 \ ext{loop}}$ Use Hamilton's equations of motion $\dot{oldsymbol{S}}_a = -oldsymbol{S}_a imes rac{\partial H}{\partial oldsymbol{S}_a} \ , \quad a=1,2$ $\dot{\boldsymbol{r}} = \frac{\partial H}{\partial \boldsymbol{p}},$ $\dot{oldsymbol{p}}=-rac{\partial H}{\partialoldsymbol{r}}\,,$ $H(r^2, p^2)$ $\Delta O_n = \int_{-\infty}^{\infty} dt \, \frac{dO_n}{dt}$ To compute change in observables 1])/(8 b5q*2 t) - (
3 /[0] a5(2, 1, 4, c0[{1, 1}, 1] c0[{1, 2}, 1]] dot[8, S[1]] Sperp[
2])/(8 b5q*2 t);
SopiDeltaP2 = (2 E12*2 xi*2 c0[{0}, 1] c0[{2, 1}, 1] cross[S[1], cross[$\begin{array}{l} 4^{-1} \mathcal{G}_{2} & = \exp\{[j_{1}, \\ \mathcal{G}_{2} & = \exp\{[j_{1}, \\ \mathcal{G}_{2} & \in \{1, 2\}, \\ \mathcal{G$ Ancillary file for "Spinning Black Hole Binary Dynamics, Scattering Amplitudes and Effective Field Theory" by Z. Bern, A. Luna, R. Roiban, C.-E. Shen and M. Zong ulse and spin kick through $O(G^2)$ from the solution to the essentiant of (11) c0[{2, 1}, 1] dot[protection. [12]... air cu([0], i] cu([r, 3], i] cross[2[1], cucalF, 5[1]]... (bdg 172)... 4 Eir2 Air2 cu([0], i] cu([2, 3], i] dot[B, 5[2]])/(bdg 172) - ([2]AX[2], i, i] cu([2, 3], i] dot[B, 5[2]])/(5[2])/(12) + (Ei2 \[[2]AX[2], i, i] cu([2], 3], i]] dot[F, 5[2]])/(12) + (Ei2 \[[2]AX[2], i] cu([2], 3], i] dot[F, 5[2]])/(2 bdg 4)); 1 = (2 B E12 xi c0[(0], 1])/(Sart[bSa] L); [Pi] xi^2 (c0[(1, 1], 1] + c0[(1, 2], 1]) c0[(2, 1], 1] dot
] \$(1))/(16 b8q^2 L) - (
[Pi] xi^2 (c0[(1, 1], 1] + c0[(1, 2], 1]) c0[(2, 1], 1] dot -(12 El2 xi c0[(1, 1), 1] cross[P, S[1]))/(Sqrt[bSq] L)) -(1, 2), 1] cross[P, S[1])/(Sqrt[bSq] L) + (0[(1, 1), 1] dot[Lin, S[1])/(bSq'(3/2) L) + (0[(1, 2), 1] dot[Lin, S[1])/(bSq'(3/2) L) + (0[(1, 2), 1] dot[Lin, S[2])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[1]]) dot[B, S[2])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[1]]) dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]])/(bSq'(3/2) L)) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - = 0[(16) ff[2] to(0[(2, 1), 1] dot[B, S[2]]) - \\= 0[(([Pi] ((
15 (-AX[2, 1, 4/3, 1/2 c0[{0}, 1] c0[{2, 1}, 1]] +
1/2 E12 pSq xi c0[{2, 1}, 2]) dot[B, S[1]] dot[B, S[2]])/(e 1: $DeltaS[1] = G DeltaS[1, 1] + G^2 DeltaS[1, 2]$ 4 bSq^2 L^3) + (
3 (-AK[2, 1, 4, 1/2 c0[{0}, 1] c0[{2, 1}, 1]] +
1/2 E12 pSq xi c0[{2, 1}, 2]) dot[P, S[1]] dot[P, S[2]])/(+ ({ (%2) x12 (c0[(1, 1), 1] + c0[(1, 2), 1]) c0[(2, 1), (%, \$[2]])/(% b\$q^2 L) - (x1^2 (3/2 c0[(1, 2), 1] c0[(2, 2), 1] + . 1). medDeltaSi2 = -1/(3 bSg L^2)*
4 El2^2 x1^2 (c0[(1, 2), 1] (c0[(2, 1), 1] + 3/2 c0[(2, 2), 1]) + D) = ((2, 1), 1] dot[P, S[1]] dot[P, S[2]])/(123)) ((1, 1), 1] dot[P, S[2]] Speep[1]/(3 bdq*(3/2) L) - ((1, 1), 1] dot[P, S[1]] Speep[2]/(3 bdq*(3/2) L) - ((4 B El2 xi c0[(2, 1), 1] dot[P, S[1]])/(Bdq*(3/2) L); (4 B El2 xi c0[(2, 1), 1] dot[P, S[1]])/(Bdq*(3/2) L); 0([1, 1), 1] [1/2 c0[[2, 1], 1] + 3 (1/2 c0[[2, 2], 1] + 1/2 pSq c0[[2, 3], 1]))) cross[S[1], 3 (1/2 co[i2, 2], 1] + 1/2 pSq co[i2, 3], 1]))) dot[P, S[2]))(1) bSq 1/2)) Sperp[1] + ((3 E12^2 \[P1] xi^2 (co[i1, 1], 1] + co[i1, 2], 1]) co[i2, 1], 1] dot[B, S[1]])/(0 bSq^2 1) - (4 E12^2 xi^2 (3/2 co[i4, 1], 1] co[i2, 2], 1] + 1), 1] + c0[(1, 2), 1]) c0[(2, 1), 1] cross 4^6) - (c0[{0}, 1] c0[{2, 1}, 1] dot[S[1], S[2]])/((n1 m2) = (E1 E2 + pSq)/(m1 m2) , 5[_]])(0 Log L) - ((Pi) xi^2 (c0[[1, 1], 1] + c0[[1, 2], 1]) c0[[2, 1], 1] cross[5[1]] dot[Lin, 5[2]])/(0 b5q L^3) + P. 5[1]] () {
 1, 4/3, 1/2 c0[{0}, 1] c0[{2, 1}, 1]] sq xi c0[{2, 1}, 2]) dot[B, S[2]] Sperp[1])/(- (bx, by, bz) impact parameter. For Fig. 11 of paper, B = (-b, 0, 0) incoming three momentum p in CM frame. For Fig. 11 of paper, P = pinfty = (1/2 c0[(2, 1], 1] = (1/2 c0[(2, 2], 1] + 1/2 pSg c0[(2, 3], 1]))) dot[P,)/(3 bSg L^2)) Sperp[2]; (AX(2, 1, 4/3, 1/2 c0[{0}, 1] c0[{2, 1}, 1]] -E12 pSq xi c0[{2, 1}, 2]) dot[B, S[1]] Sperp[2])/(4 bSt LaP2 = -((4 E12^2 xi^2 c0)(0, 1] c0[42, 2], 1] cross1 , [p_iinfty]) B x P (x is cross product). $\epsilon(21$: three vector spin of particle 1, 2 in rest frame of each ue.
], Sperp[2] : projection of the rest frame three vector spin of
le 1, 2 to be orthogonal [11]// ^4) - (4 E12^2 xi^2 c0[{0}, 1] c0[{2, 2}, 1] cross[S[2], cross[P, S[1]])/[^4 + (eltaSii = (2 Ei2 xi c0[{1, 1}, 1] cross[Lin, S[1]])/(Sqrt[bSq] L); iDeltaSii = -((4 Ei2 xi c0[{2, 1}, 1] cross[S[1], S[2]])/(3 Sqrt[bSq] 0[(1, 1), 1] + 3/2 c0[(2, 2), 1]) dot[8, 5[1]] dot[8, the incoming three-momentum p in CM f S[2]])/(3 bSg^2 L-2) + (E12^2 xi^2 cO[(1, 1), 1] cO[(2, 2), 1] dot[Lin, S[1]] dot[Lin, S[2]])/(bSg L^4) + (i; 2]) croad[v, S[2]]//L⁻³ + [00, 1] c0[1, 1], 1] dot[Lin, S[1]])/L⁴], 1] c0[1, 2], 1] dot[Lin, S[2]])/L⁴] + 1, 2, c0[0, 1] c0[1, 1], 1] + , 1], 2]) dot[B, S[1]])/(2 hSq L³) + (incoming three momentum p in CM fram of the impact parameter vector B dot[A, B] : scalar product of the 3-dimensional vectors A and B cross[A, B] : cross product of the 3-dimensional vectors A and B 2, CO((0), 1 CO((1, 2), 1)] + , 2), 2) dot[8, S[2]])/(2 bSq L^3) - (1), 1] (2 cO[(0), 1] + pSq cO[(1, 1), 1]) dot[+ B (-([\[E1] XX[2, 1, 4, 1/2 c0[[0], 1] c0[[2, 3], 1]] dot[P, S[1]] dot[P, S[2]])/(bSg L^3)) + (12 \[P1] xi_c0[(2, 3], 2] dot[P, S[1]] dot[P, S[2]])/(, \$[2]])/(3 L^4)) - (
(1, 1), 1] c0[(2, 2), 1] dot[Lin, 5[2]] 5[1])/(//igtriesd[ii] Histoll = F (-(1[F1] (MX[2, 1, 2, c0[(0), 1] c0[(1, 1), 1]) -Ei:phg at c0[(1, 1), 2]) doc[B, 5[1])/(2 ''3)) - (Hir'n xt' 2 c0[(1, 1), 1]'2 doc[B, 5[1])/(2 ''3)) - ((12 Eir'n xt' 2 c0[(1, 1), 1]'2 doc[B, 5[1])/(2 ''3))); Histoll 2 (X2(2, 1, 2, c0[(0, 1] c0[(1, 1), 1]) -Eir phy at c0[(1, 1), 2]) doc[B, 5[1])/(2 ''3)); Histoll 2 (X2(2, 1, 2, c0[(0, 1] c0[(1, 1), 1]) -Eir phy at c0[(1, 1), 2]) doc[B, 5[1])/(2 ''3)); Histoll 2 (X2(2, 1, 2, c0[(1, 1), 1])); Histoll 2 (X2(2, 1), 2, c0[(1, 1), 1]) (d(1, 2, 1)); Histoll 2 (X2(2, 1), 2, c0[(1, 1), 1])); Histoll 2 (X2(2, 1), 2, c0[(1, 1), 2]); Histoll 2 (X2(2, 1), 2]); Histoll 2 (X2(2, 1), 2]); Histoll 2 (X2(2, 1), 2]); Histoll 2 (X2 0[(i,j), 1] and c0[(i,j), 2] are the position space Hamiltonian coefficients c(i,j), 1] and c(i,j), 2] in Fe DeltaPi = NoSDeltaPi + SODeltaPi + SOpiDeltaPi + SOpiDeltaPi + SOpiDeltaPi DeltaP2 = NoSDeltaP2 + SODeltaP21 + SODeltaP22 + SOpiDeltaP2 + SOpi 812^2 xi^2 c0[{1, 1}, 1] c0[{1, 2}, 1] cross[S[1])) + {
 c0[{0}, 1] c0[{2, 3}, 1] dot[P, S[2]] Sperp[1])/ { ^2 c0[{0}, 1] c0[{2, 3}, 1] dot[P, \$[1]] Sperp[2])/ 2 c0([i,]), 1] c0([i, 2), 1] cross[8[2], S([j]))/(5g/c2) + AX[2, i, 4, c0([1, 1], 1] c0([1, 2], 1]] dot[8, dot[8, 52])/(4 b5q^2 3 L, -AX[2, i, 4, c0([1, 1], 1] c0([1, 2], 1]] dot[Lin, dot[Lin, 3])/(6 b5q² 2 N) + 12²² xi² c0([1, 1], 1] c0([1, 2], 1] dot[Lin, 5[2]] dot[lin/(5)]/(5g/c2) + A (2) (1)/(5g/c2) + (1)/(5 DeltaS[1, 1] = SODeltaS11 + SOpiDeltaS11 + SOpiDeltaS11; DeltaS[1, 2] = SODeltaS121 + SODeltaS122 + SOpiDeltaS12 + SOpiDeltaS12 + SOpiDeltaS12 + MixedDeltaS122 :ef2 = cross[Lin, ((4 E12* xi*2 (-0(1, 1), 1) + c0((1, 2), 1)) c0((2, 1)) dc(5, s(1))(/(5 bcq*2 L*2) + ('2 (1) Li*2 ((1) (2 bcq L*2) + ('2 (1) Li*2 ((1) (2 bcq L*3)) + (b(1, 2))((16 bcq L*3)) + 1 : Eqs. (6.38) opressions for co[(i,j), 1] and co[(i,j), 2] are obtained by c[(i,j), 1] and c[(i,j), 2] in ary file coefficients.m on the incoming three-momentum p. cross[Lin, S[2]]])/(bSg L^2) - (\[Pi] AX[2, i, 4, co[(i, i), i] co[(i, 2), i]] cross[Lin, S[1]] dot[Lin, S[2]])/(0 bSg L^3); S[1]Blata3i2 = -((2 E12^2 xi^2 co](0), i] co[(2, 1), i] cross[S[1] ([i,j], 1, 1] : derivative of c0[(i,j], 1] with respect to p^2
[(i,j], 2, 1] : derivative of c0[(i,j], 2] with respect to p^2 (* The final impulse and spin kick *) 1] c0[(2, 1), 1]] -roas(S[1], S[2]))/(4 L^3) + (store_imat, s[1] ((4 %12^2 xi^2 (c0[{1, 1}, 1] - c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[B, S[2]])/(3 bSq^2 L^2) + (), 11 c0[(1, 2), 111 dot[8, S[211 Spec

Observables

$$\mathcal{M}_4^{1 \text{ loop}} \rightarrow H(r^2, p^2) \rightarrow \frac{\Delta p}{\Delta S_i}$$

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$$\Delta \boldsymbol{p}_{\perp} = -\{\boldsymbol{P}_{\perp}, \boldsymbol{\chi}\} - \frac{1}{2} \{\chi, \{\boldsymbol{P}_{\perp}, \chi\}\} - \mathcal{D}_{SL} (\chi, \{\boldsymbol{P}_{\perp}, \chi\}) + \frac{1}{2} \{\boldsymbol{P}_{\perp}, \mathcal{D}_{SL} (\chi, \chi)\}$$
$$\Delta \boldsymbol{S}_{i} = -\{\boldsymbol{S}_{i}, \chi\} - \frac{1}{2} \{\chi, \{\boldsymbol{S}_{i}, \chi\}\} - \mathcal{D}_{SL} (\chi, \{\boldsymbol{S}_{i}, \chi\}) + \frac{1}{2} \{\boldsymbol{S}_{i}, \mathcal{D}_{SL} (\chi, \chi)\}$$

 $\{P_{\perp}, f\} \equiv -\nabla_{b}f$ $\{S_{a}^{i}, f\} \equiv \epsilon^{ijk} \frac{\partial f}{\partial S_{a}^{j}} S_{a}^{k}$ $\mathcal{D}_{SL}(f, g) \equiv -\sum_{a=1,2} \epsilon^{ijk} S_{a}^{k} \frac{\partial f}{\partial S_{a}^{i}} \frac{\partial g}{\partial L^{j}}$

The Eikonal phase

$$\chi_2 = \frac{1}{4m_1m_2\sqrt{\sigma^2 - 1}} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{M}^{\Delta + \nabla}(\boldsymbol{q})$$

Higher powers in Spin: Lagrangian

Effective description of spin-induced multipoles

[Submitted on 20 Jan 2015 (v1), last revised 5 Oct 2015 (this version, v3)]

Spinning gravitating objects in the effective field theory in the post-Newtonian scheme

Michele Levi, Jan Steinhoff

$$L_{\rm SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} + \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

[Submitted on 19 Feb 2021]

Quadratic-in-Spin Hamiltonian at $\mathcal{O}(G^2)$ from Scattering Amplitudes

Dimitrios Kosmopoulos, Andres Luna

Covariantization of Levi-Steinhoff

$$\mathcal{L}_{C} = \sum_{n=1}^{\infty} \frac{(-1)^{n}}{(2n)!} \frac{C_{\mathrm{ES}^{2n}}}{m^{2n}} \nabla_{f_{2n}} \dots \nabla_{f_{3}} R_{af_{1}bf_{2}}$$

$$\times \nabla^{a} \phi_{s} \mathbb{S}^{(f_{1}} \mathbb{S}^{f_{2}} \dots \mathbb{S}^{f_{2n}}) \nabla^{b} \phi_{s}$$

$$- \sum_{n=1}^{\infty} \frac{(-1)^{n}}{(2n+1)!} \frac{C_{\mathrm{BS}^{2n+1}}}{m^{2n+1}} \nabla_{f_{2n+1}} \dots \nabla_{f_{3}} \widetilde{R}_{(a|f_{1}|b)f_{2}}$$

$$\times \nabla^{a} \phi_{s} \mathbb{S}^{(f_{1}} \mathbb{S}^{f_{2}} \dots \mathbb{S}^{f_{2n+1}}) \nabla^{b} \phi_{s} ,$$

[Submitted on 11 Mar 2022 (v1), last revised 29 Dec 2022 (this version, v3)]

Binary Dynamics Through the Fifth Power of Spin at $\mathcal{O}(G^2)$

Zvi Bern, Dimitrios Kosmopoulos, Andrés Luna, Radu Roiban, Fei Teng

More EFT operators

$$\begin{aligned} \mathcal{L}_{H} &= -\sum_{n=1}^{\infty} \frac{(-1)^{n} (2n-1)}{(2n)! (2n+1)} \frac{H_{2n}}{m^{2n-2}} \nabla_{f_{2n}} \dots \nabla_{f_{3}} R^{(a}{}_{f_{1}}{}^{b)}{}_{f_{2}} \\ &\times \phi_{s} M_{a}{}^{(f_{1}} M_{b}{}^{f_{2}} \mathbb{S}^{f_{3}} \dots \mathbb{S}^{f_{2n}}) \phi_{s} \\ &+ \sum_{n=1}^{\infty} \frac{(-1)^{n} n}{(2n+1)! (n+1)} \frac{H_{2n+1}}{m^{2n-1}} \nabla_{f_{2n+1}} \dots \nabla_{f_{3}} \widetilde{R}^{(a}{}_{f_{1}}{}^{b)}{}_{f_{2}} \\ &\times \phi_{s} M_{a}{}^{(f_{1}} M_{b}{}^{f_{2}} \mathbb{S}^{f_{3}} \dots \mathbb{S}^{f_{2n+1}}) \phi_{s} \,. \end{aligned}$$

Higher powers in spin: Amplitudes

$$\mathcal{M}^{\triangle+\bigtriangledown} = \frac{2\pi^2 G^2 \varepsilon_1 \cdot \varepsilon_4 \varepsilon_2 \cdot \varepsilon_3}{\sqrt{-q^2}} \sum_n \sum_i \alpha^{(n,i)} \mathcal{O}^{(n,i)} \,.$$



	i		i		i	
$\mathcal{O}^{(2,i)}$	1	\mathcal{E}_1^2	2	$q^2(u_2\!\cdot\!a_1)^2$	3	$(q\!\cdot\!a_1)^2$
$\mathcal{O}^{(4,i)}$	1	\mathcal{E}_1^4	2	$q^2(u_2\!\cdot\! a_1)^2\mathcal{E}_1^2$	3	$q^4(u_2\!\cdot\!a_1)^4$
C	4	$(q{\cdot}a_1)^2\mathcal{E}_1^2$	5	$q^2 (q \!\cdot\! a_1)^2 (u_2 \!\cdot\! a_1)^2$	6	$(q \cdot a_1)^4$

$Z_{2,1} = C_2 + 1$	$Z_{2,2} = C_2 - 1$
$Z_{3,1} = 3C_2 + C_3$	$Z_{3,2} = C_2 - C_3$
$\begin{aligned} Z_{4,1} &= 3C_2^2 + 4C_3 + C_4 \\ Z_{4,2} &= 3C_2^2 + C_4 \end{aligned}$	$Z_{4,3} = C_2^2 - C_4$ $Z_{4,4} = 3C_2^2 - 4C_3 + C_4$
$Z_{5,1} = 10C_2C_3 + 5C_4 + C_5$	$ \begin{vmatrix} Z_{5,2} = 2C_2C_3 - C_4 - C_5 \\ Z_{5,3} = 2C_2C_3 - 3C_4 + C_5 \end{vmatrix} $

	i	$\gamma^{(2,i)}$	i	$\gamma^{(2,i)}$
	1 2	$7 + 23C_2 - Z_{2,1}\sigma^2(102 - 95\sigma^2) 5 - 11C_2 + 5Z_{2,1}\sigma^2(6 - 7\sigma^2)$	3	$12Z_{2,2}(\sigma^2-1)^2(5\sigma^2-1)$
	i	$\gamma^{(3,i)}$	i	$\gamma^{(3,i)}$
	1 2	$egin{array}{llllllllllllllllllllllllllllllllllll$	3	$4Z_{3,2}(\sigma^2 - 1)(5\sigma^2 - 3)$
	i	$\gamma^{(4,i)}$	i	$\gamma^{(4,i)}$
	1	$44C_3 + 59Z_{4,2} - Z_{4,1}\sigma^2(250 - 239\sigma^2)$	4	$12Z_{4,3}(1-\sigma^2)(23-102\sigma^2+95\sigma^4)$
	2	$72C_3 - 78Z_{4,2} + Z_{4,1}\sigma^2(276 - 294\sigma^2)$	5	$12Z_{4,3}(\sigma^2-1)(11-30\sigma^2+35\sigma^4)$
	3	$28C_3 - 9Z_{4,2} + 7Z_{4,1}\sigma^2(2 - 3\sigma^2)$	6	$24Z_{4,4}(\sigma^2-1)^3(5\sigma^2-1)$
-	i	$\gamma^{(5,i)}$	i	$\gamma^{(5,i)}$
	1	$Z_{5,1}(7-13\sigma^2)$	4	$12Z_{5,2}(\sigma^2-1)(9\sigma^2-5)$
	2	$2Z_{5,1}(11\sigma^2-5)$	5	$12Z_{5,2}(\sigma^2-1)(3-7\sigma^2)$
	3	$Z_{5,1}(3\sigma^2 - 1)$	6	$8Z_{5,3}(\sigma^2-1)^2(3-5\sigma^2)$

What do to do with this amplitude?

Conjecture

[Submitted on 11 Mar 2022 (v1), last revised 29 Dec 2022 (this version, v3)]

Binary Dynamics Through the Fifth Power of Spin at $\mathcal{O}(G^2)$

Zvi Bern, Dimitrios Kosmopoulos, Andrés Luna, Radu Roiban, Fei Teng

We conjectured that the scattering amplitude of two Kerr black holes is the one that realises the symmetry

$$a_i^\mu \to a_i^\mu + \xi_i q^\mu / q^2, \quad i = 1, 2$$

Computed observables up to S^5

Why do we trust these results?

Checks against GR!

Checks: Quadratic-in-spin Hamiltonians

[Submitted on 14 Jul 2016 (v1), last revised 22 Sep 2021 (this version, v2)]

Complete conservative dynamics for inspiralling compact binaries with spins at the fourth post-Newtonian order

Michèle Levi, Jan Steinhoff

[Submitted on 27 Jan 2016 (v1), last revised 28 Jun 2021 (this version, v3)]

Canonical Hamiltonian for an extended test body in curved spacetime: To quadratic order in spin

Justin Vines, Daniela Kunst, Jan Steinhoff, Tanja Hinderer

To compare with (overlapping parts of) them, we may compute Amplitudes from the Hamiltonian using EFT.



NNLO post-Newtonian



Test-body limit

Checks: Higher powers in spin (angles)

[Submitted on 16 Sep 2019]

Test black holes, scattering amplitudes and perturbations of Kerr spacetime

Nils Siemonsen, Justin Vines

Consider a generalization of MPD equations

$$\begin{split} &\frac{\mathrm{D}}{\mathrm{d}\tau}p_{\mu} + \frac{1}{2}R_{\mu\nu\kappa\lambda}\dot{z}^{\nu}S^{\kappa\lambda} = \frac{p\cdot\dot{z}}{2}\frac{\mathrm{D}}{\mathrm{D}z^{\mu}}\log\mathcal{M}^{2},\\ &\frac{\mathrm{D}}{\mathrm{d}\tau}S^{\mu\nu} - 2p^{[\mu}\dot{z}^{\nu]} = p\cdot\dot{z}\bigg(p^{[\mu}\frac{\partial}{\partial p_{\nu]}} + 2S^{[\mu}{}_{\rho}\frac{\partial}{\partial S_{\nu]\rho}}\bigg)\log\mathcal{M}^{2}, \end{split}$$

With a dynamical mass function

$$\mathcal{M}_{\rm GOV}^2 = m^2 + 2m^2 u^{\mu} u^{\nu} \sigma^{\rho_1} \sigma^{\rho_2} \left(-\frac{1}{2!} R_{\mu\rho_1\nu\rho_2} + \frac{1}{3!} R_{\mu\rho_1\nu\rho_2;\rho_3} \sigma^{\rho_3} + \frac{1}{4!} R_{\mu\rho_1\nu\rho_2;\rho_3\rho_4} \sigma^{\rho_3} \sigma^{\rho_4} \right) + \mathcal{O}(\sigma^5)$$

Angles agree!

[Submitted on 25 Nov 2022]

Completing the Fifth PN Precision Frontier via the EFT of Spinning Gravitating Objects

Michèle Levi, Zhewei Yin



Checks: Generic bodies (Neutron Stars)

[Submitted on 5 Aug 2022 (v1), last revised 30 Aug 2022 (this version, v2)]

Scattering of gravitational waves off spinning compact objects with an effective worldline theory

M. V. S. Saketh, Justin Vines

$$\begin{aligned} \mathcal{M}_{++} &= GM\omega \frac{\cos^4(\theta/2)}{\sin^2(\theta/2)} \bigg(\exp[a \cdot (k+l-2w_{\rm S})] + \frac{C_2 - 1}{2} [(k-w_{\rm S}) \cdot a]^2 + [(l-w_{\rm S}) \cdot a]^2 \\ &+ \frac{C_2 - 1}{2} [(k-w_{\rm S}) \cdot a] [(l-w_{\rm S}) \cdot a] [(k+l-2w_{\rm S}) \cdot a] - (C_2 - 1)^2 [(k-w_{\rm S}) \cdot a] [(l-w_{\rm S}) \cdot a] (w_{\rm S} \cdot a) \\ &+ \frac{C_3 - 1}{6} \{ [(k-w_{\rm S}) \cdot a]^3 + [(l-w_{\rm S}) \cdot a]^3 \} \bigg). \end{aligned}$$

$$\begin{split} \frac{\theta_{\rm S_1^3}^{\rm NLO}}{\Gamma} &= \tilde{v}\tilde{a}_1^3 \left[-\frac{4}{\tilde{b}}C_{1\rm BS^3} + \frac{\pi}{\tilde{b}^2} \left(\frac{15\nu}{4}\tilde{v}^2 + \left(3\nu + 6 + \left(-\frac{3\nu}{2} - \frac{27}{4} \right)\tilde{v}^2 \right) C_{1\rm ES^2} \right. \\ &+ \left(-6 + \left(\frac{27\nu}{4} - \frac{33}{4} \right)\tilde{v}^2 \right) C_{1\rm BS^3} \right. \\ &+ \frac{\nu}{q} \left(\frac{15}{4}\tilde{v}^2 + \left(3 - \frac{3}{2}\tilde{v}^2 \right) C_{1\rm ES^2} + \frac{27}{4}\tilde{v}^2 C_{1\rm BS^3} \right) \right) \bigg], \end{split}$$

$$\begin{split} \frac{\theta_{\mathrm{S}_{1}^{2}S_{2}}^{\mathrm{NLO}}}{\Gamma} &= \tilde{v}\tilde{a}_{1}^{2}\tilde{a}_{2}\left[-\frac{12}{\tilde{b}}C_{1\mathrm{ES}^{2}} + \frac{\pi}{\tilde{b}^{2}}\left(6\nu - 12 + \tilde{v}^{2}\left(\frac{27\nu}{8} - \frac{99}{8}\right)\right) \\ &+ \left(-3\nu + \left(\frac{39\nu}{8} - \frac{207}{8}\right)\tilde{v}^{2} - 21\right)C_{1\mathrm{ES}^{2}} \\ &+ \frac{\nu}{q}\left(6 + \frac{27}{8}\tilde{v}^{2} + \left(-3 + \frac{39}{8}\tilde{v}^{2}\right)C_{1\mathrm{ES}^{2}}\right)\right) \end{split}$$

Compton Amplitude and scattering angles agree

 (k^{μ},ϵ^{μ})

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[Submitted on 31 Oct 2022]

From the EFT of Spinning Gravitating Objects to Poincaré and Gauge Invariance

Michèle Levi, Roger Morales, Zhewei Yin

Checks: BHPT

[Submitted on 15 Dec 2022 (v1), last revised 21 Feb 2023 (this version, v2)]

Scattering in Black Hole Backgrounds and Higher-Spin Amplitudes: Part II

Yilber Fabian Bautista, Alfredo Guevara, Chris Kavanagh, Justin Vines 🎿

Matching solutions of the Teukolsky equation

$$f_{lm}^{\prime \rm QFT}(\gamma) = f_{lm}^{\prime \rm BHPT}(\gamma)$$

$$f_{lm}^{'\text{QFT}}(\gamma) = \int d\Omega'_{-2} Y_{lm}^*(\theta, \phi') \langle A_4(\gamma, \theta, \phi') \rangle$$
$$\langle A_4^S \rangle = \langle A_4^0 \rangle \times \left(e^{(2w+k_3-k_2)\cdot a} + P_{\xi}(k_2 \cdot a, -k_3 \cdot a, w \cdot a) \right)_{2S}$$

A most general form of the amplitude compatible with crossing symmetry, locality, unitarity.

Agreement up to S^4 (but not at S^5)

$$f_{lm}^{\prime(\text{BHPT})}(\gamma) = \sum_{m'} D_{m'm}^{l*}(\gamma) f_{lm'}$$
$$f_{lm} = \frac{2\pi}{i\omega} \sum_{P=\pm 1} \left(e^{2i\delta_{lm}^P} - 1 \right)$$
$$e^{2i\delta_{lm}^P} = (-1)^{l+1} \frac{\mathcal{C}_{lm} + 12iM\omega P}{16\omega^4} \frac{B_{lm\omega}^{\text{ref}}}{B_{lm\omega}^{\text{inc}}}$$

Existing solutions in BHPT literature





Outlook

• Higher spin. Understand black hole S^5 discrepancy with BHPT. And beyond?



• More loops. Improve amplitudes (bootstrap?), Improve observables from amplitudes