Comparison of post-Minkowskian and self-force expansions: Scattering in a scalar charge toy model

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Oliver Long

with L. Barack, Z. Bern, E. Herrmann, J. Parra-Martinez, R. Roiban, M. S. Ruf, C.-H. Shen, M. P. Solon, F. Teng, and M. Zeng.

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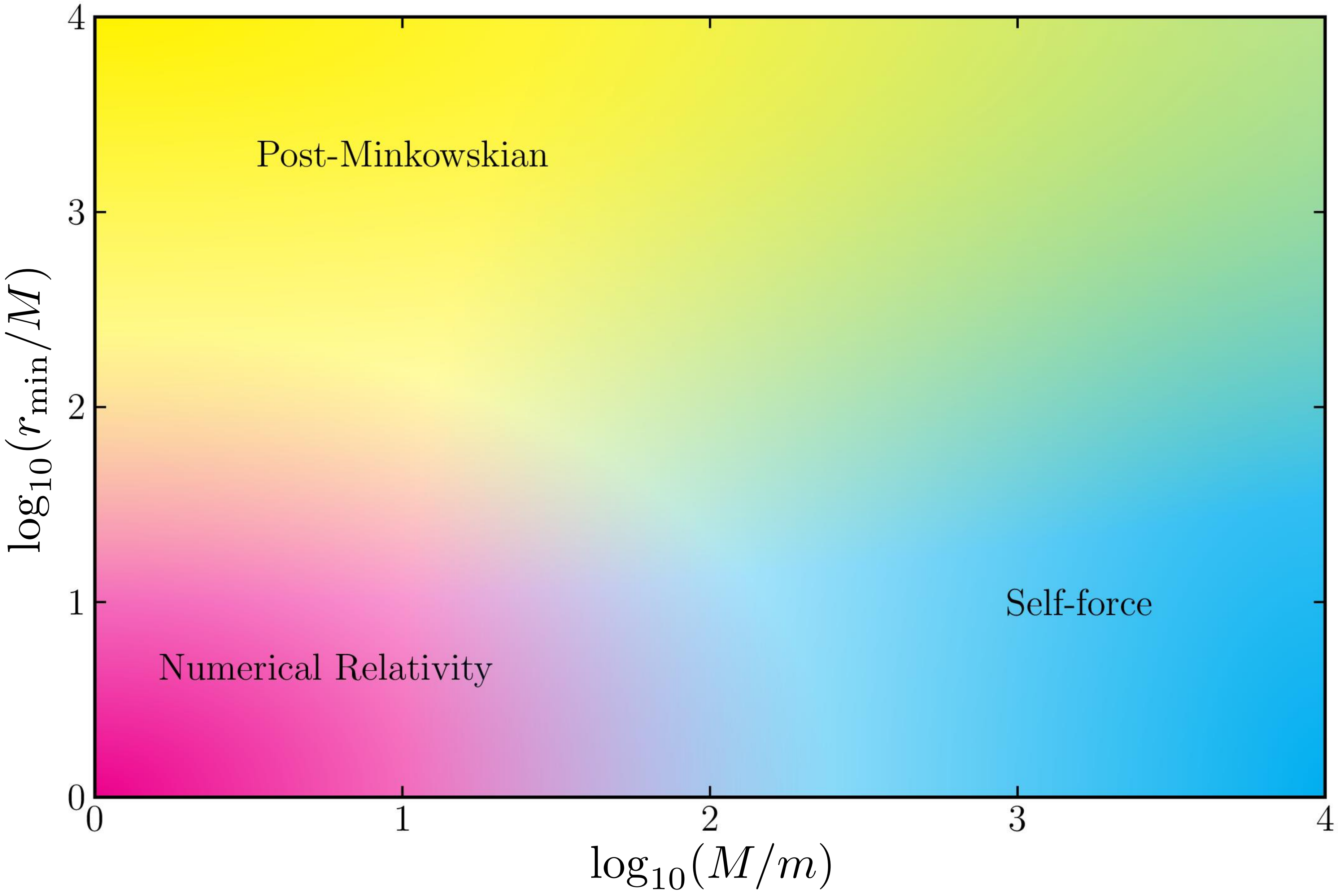
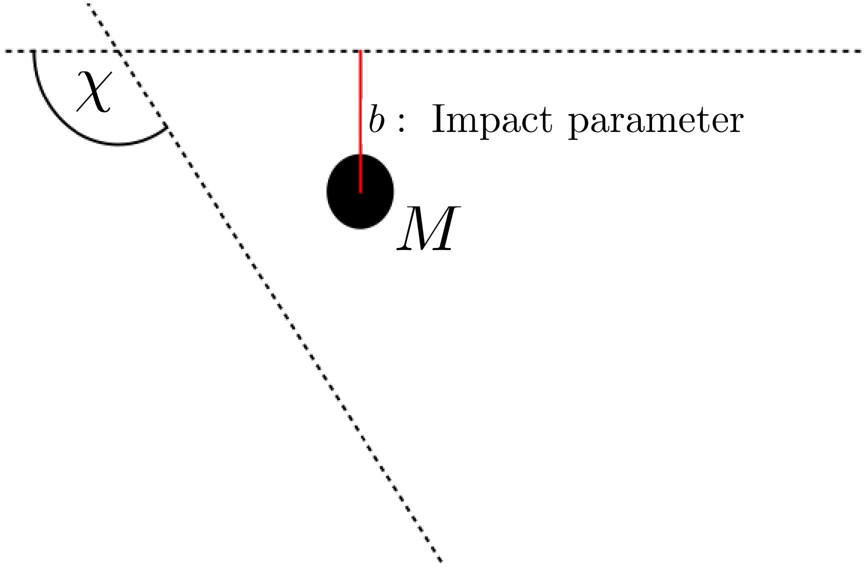
Two-body parameter space for scattering



Self-force correction to the scattering angle

Scattering angle [Barack & OL '22]:

$$\chi = \chi^{(0)} + \eta\delta\chi$$



$$\delta\chi = \sum_{\pm} \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^{\pm}(r) F_t^{\pm} - \mathcal{G}_L^{\pm}(r) F_\varphi^{\pm}\right] dr$$

Functions of geodesics

Can split into conservative and dissipative pieces on outgoing leg:

$$F_\alpha^{\text{cons}}(r, \dot{r}) = -F_\alpha^{\text{cons}}(r, -\dot{r}) \qquad F_\alpha^{\text{diss}}(r, \dot{r}) = F_\alpha^{\text{diss}}(r, -\dot{r}) \qquad \alpha = t, \varphi$$

$$\delta\varphi_{\text{cons}}^{(1)} = \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^{\text{cons}} F_t^{\text{cons}} - \mathcal{G}_L^{\text{cons}} F_\varphi^{\text{cons}}\right] dr \qquad \delta\varphi_{\text{diss}}^{(1)} = \int_{R_{\min}}^{\infty} \left[\mathcal{G}_E^{\text{diss}} F_t^{\text{diss}} - \mathcal{G}_L^{\text{diss}} F_\varphi^{\text{diss}}\right] dr$$

1+1D scalar field evolution scheme

Scalar field obeys the Klein-Gordon equation.

Decompose in the time domain:

$$\Phi = \frac{2\pi q}{r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \psi_{\ell m}(t,r) Y_{\ell m}(\theta, \varphi)$$

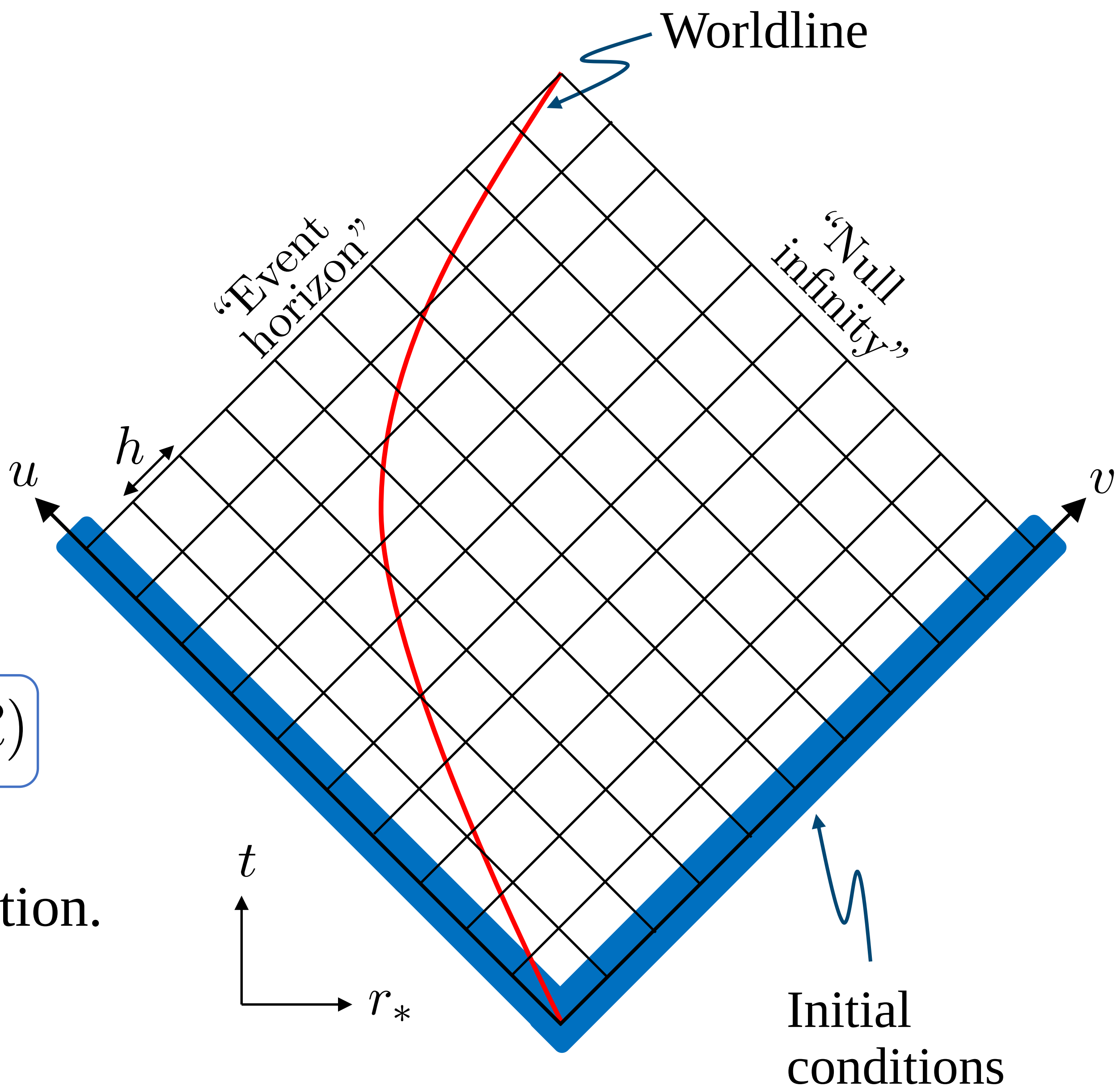
q : Scalar charge

1+1D scalar wave equation:

$$\psi_{,uv} + V(\ell; r)\psi = S_\psi(\ell; x_p^\mu)\, \delta\,(r - R)$$

Evolve finite-difference version of 1+1D equation.

Extract SF via mode-sum regularisation.



Post-processing: Truncation at finite radius

Can only numerically determine the self-force up to a finite radius $R = R_{\rm final}$:

$$\delta\varphi^{(1)} = \int_{R_{\rm min}}^{R_{\rm final}} [\mathcal{G}_E F_t - \mathcal{G}_L F_\varphi]\, dr + \int_{R_{\rm final}}^{\infty} [\mathcal{G}_E F_t - \mathcal{G}_L F_\varphi]\, dr$$

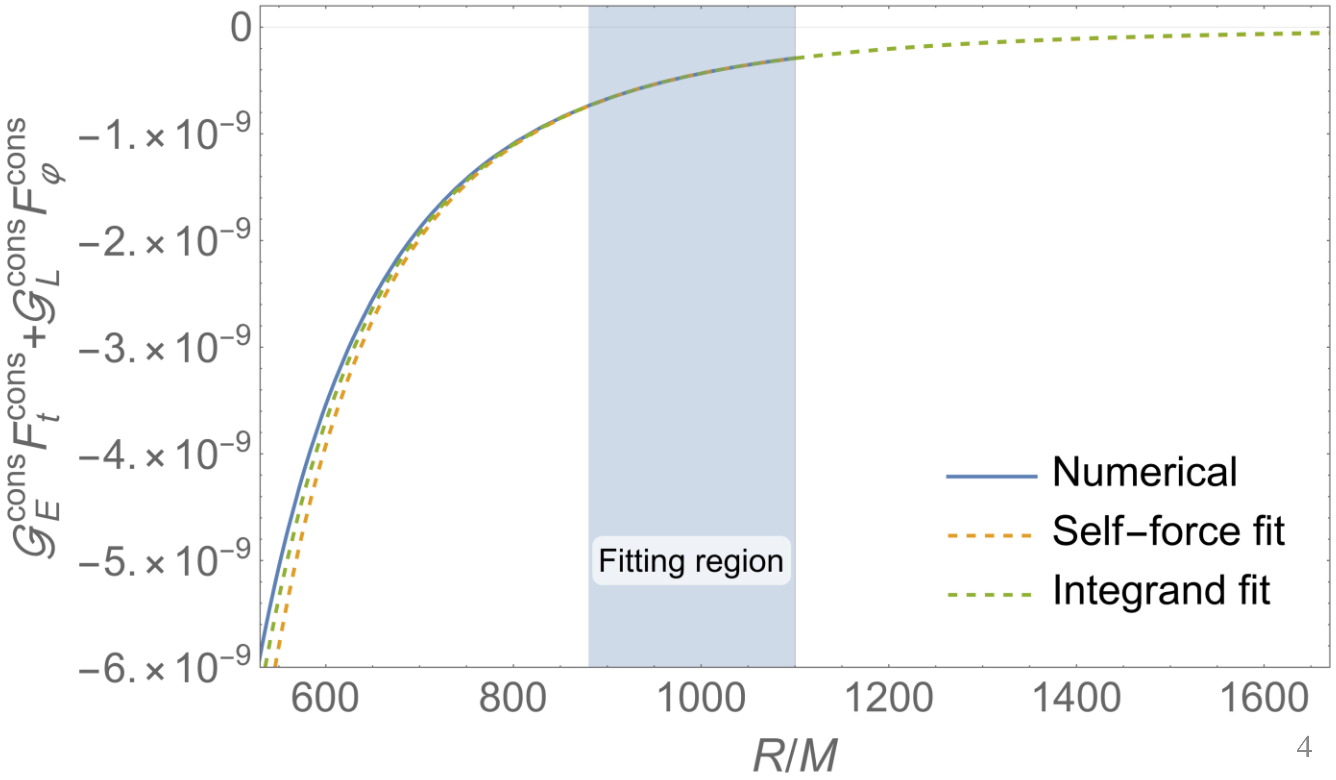
Numerical

Error (~1%)

Form an analytic tail by fitting to the data:

- Fit the self-force data.
- Fit the integrand directly.

Tail contributes an error ~ 0.01%.



Post-processing: Richardson extrapolation

Next dominant error due to finite resolution ~ 0.1%.

Can increase the convergence from quadratic to cubic using Richardson extrapolation.

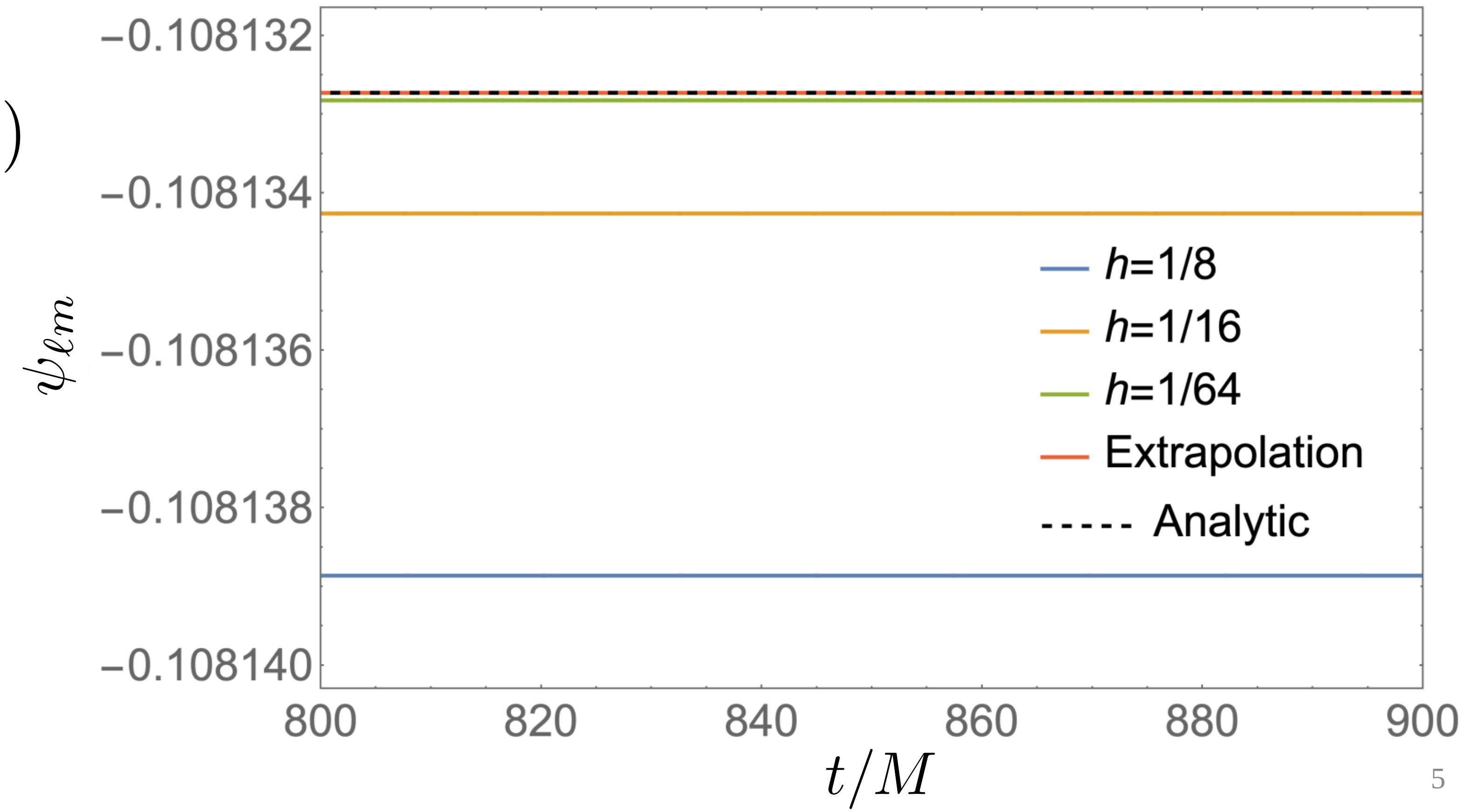
Model:

$$A(h) = A_{\text{exact}} + Ch^n + O\left(h^{n+1}\right)$$

Extrapolation:

$$A_{\text{Extr}} = \frac{t^n A(\frac{h}{t}) - A(h)}{t^n - 1}$$
$$= A_{\text{exact}} + O(h^{n+1})$$

Error in extrapolation < 0.001%.



Scattering angle correction: PM expansion

Expansion around flat space:

$$\delta\chi^{\mathrm{PM}} = \sum_{i=0}^{\infty} \delta\chi_i \left(\frac{M}{b}\right)^i$$

2PM [Gralla & Lobo '22]:

$$\delta\chi_2^{\mathrm{cons}} = -\frac{\pi}{4}\left(\frac{M}{b}\right)^2$$

$$\delta\chi_2^{\mathrm{diss}} = 0$$

$v:$ Velocity at infinity

$b:$ Impact parameter

3PM:

$$\delta\chi_3^{\mathrm{cons}} = -\frac{4\left(3-v^2\right)}{3v^2\sqrt{1-v^2}}\left(\frac{M}{b}\right)^3$$

$$\delta\chi_3^{\mathrm{diss}} = \frac{2\left(v^2+1\right)^2}{3v^3\sqrt{1-v^2}}\left(\frac{M}{b}\right)^3$$

LO

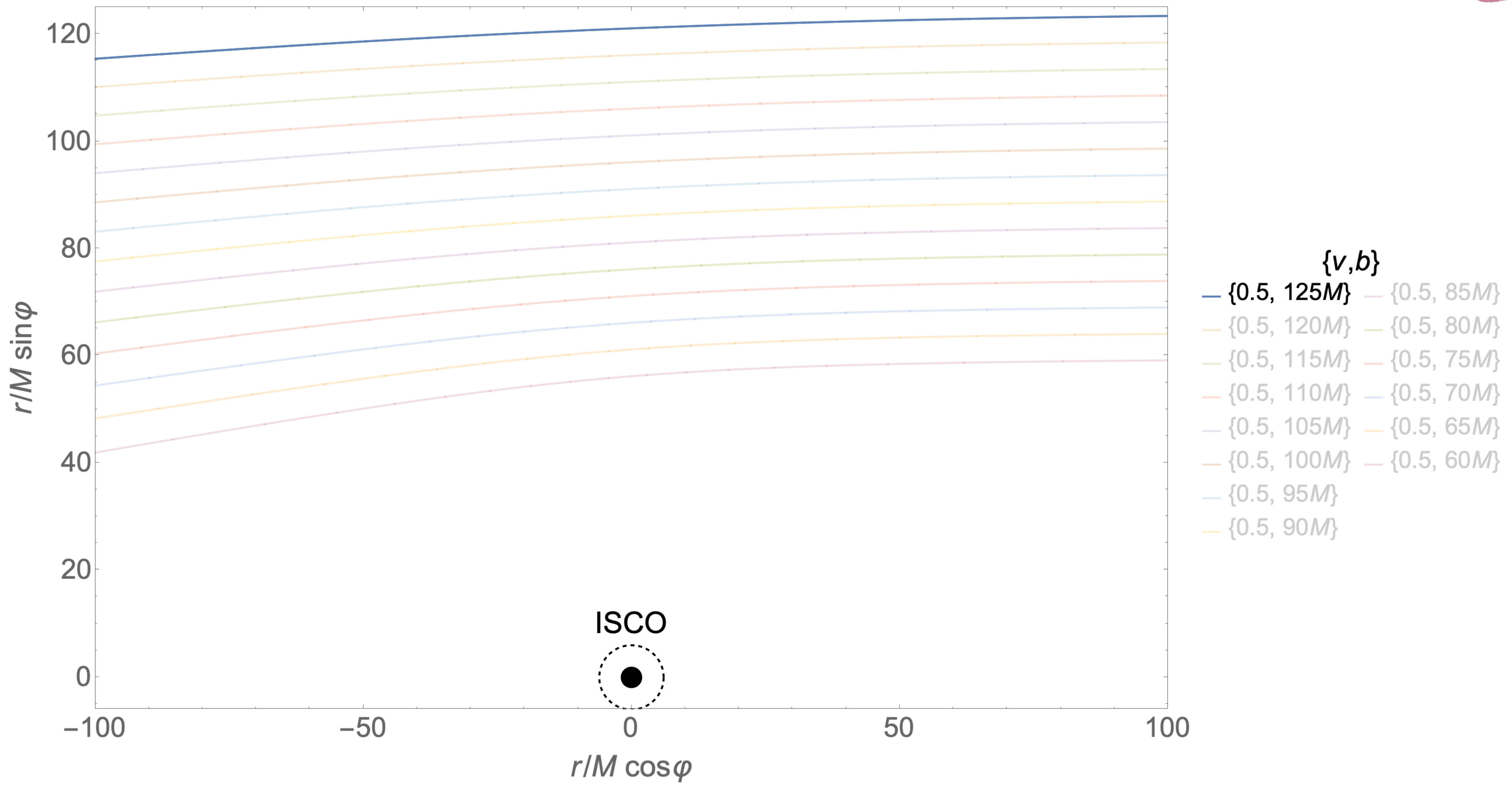
NLO

4PM dissipative:

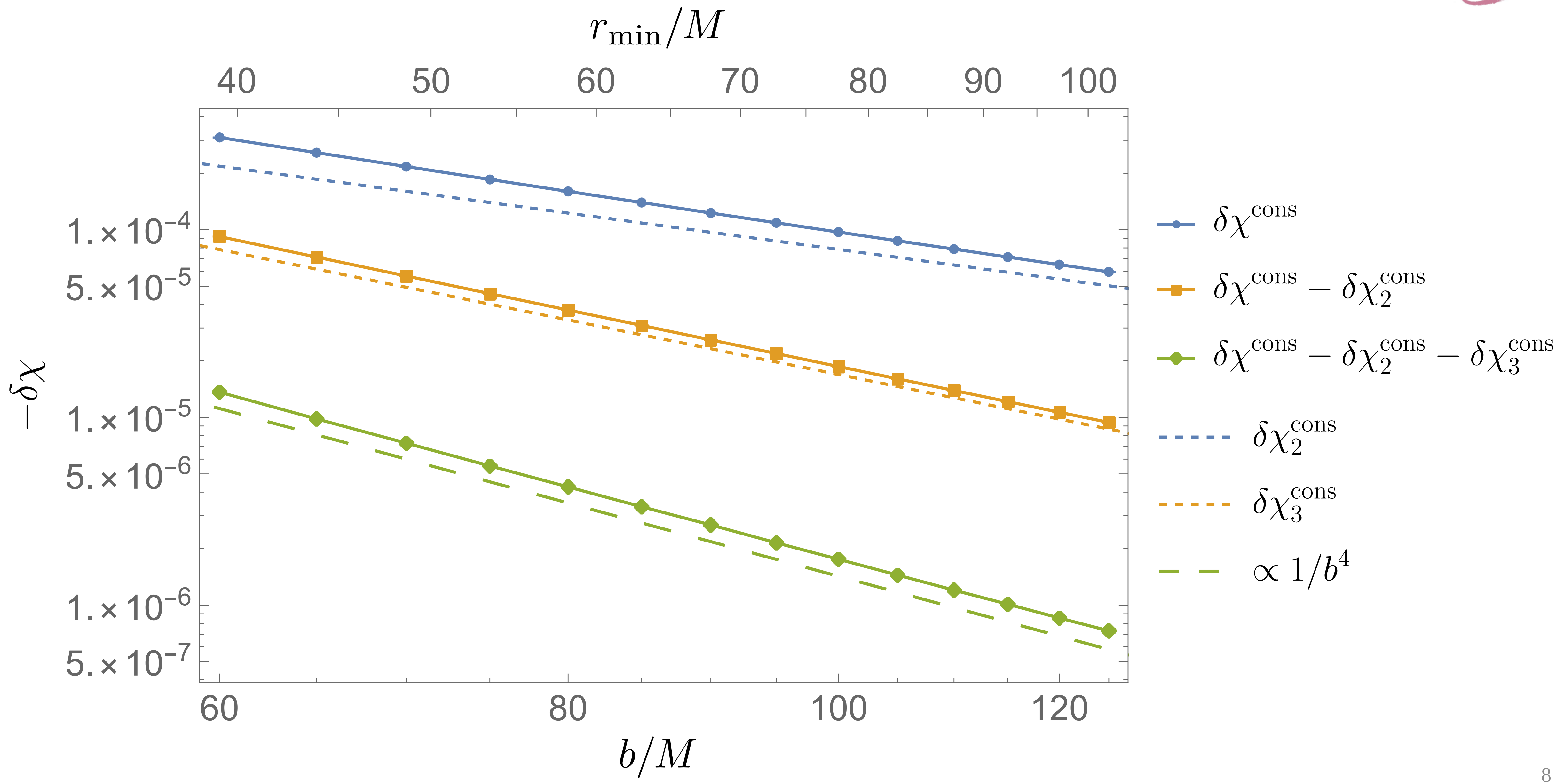
$$\delta\chi_4^{\mathrm{diss}} = \left(r_1 + r_2 \,\mathrm{arcsech}\left(\sqrt{1-v^2}\right) + r_3 \log\left[\frac{1}{2}\left(\frac{1}{\sqrt{1-v^2}}+1\right)\right]\right)\left(\frac{M}{b}\right)^4$$

r_i = rational coefficients

Sample orbits



Conservative: $v = 0.5$



Extraction of high-order conservative PM results

PM expansion with free parameters:

$$\delta\varphi^{\rm PM}_{\rm cons} = \frac{a_2}{b^2} + \frac{a_3}{b^3} + \frac{a_4}{b^4} + \frac{a_5}{b^5} + \dots$$

Up to 3PM can fit value or use **analytic value**.

a_2	a_3	a_4	a_5
-1.0886	$-$	$-$	$-$
-0.7535	-21.77	$-$	$-$
-0.7899	-16.17	-206.5	$-$
-0.7803	-18.49	-25.0	-4620
$\mathbf{-0.785398}$	-19.18	$-$	$-$
$\mathbf{-0.785398}$	-16.93	-176.2	$-$
$\mathbf{-0.785398}$	-17.20	-131.1	-1793
$\mathbf{-0.785398}$	$\mathbf{-16.9356}$	-175.9	$-$
$\mathbf{-0.785398}$	$\mathbf{-16.9356}$	-174.4	-107
$< 1\%$	$\sim 1\%$	~ -175	$< 0(?)$

Scattering angle correction: 4PM conservative

$$\delta\varphi_4^{\text{cons}} = \Bigg(r_1 + r_2 \operatorname{arccosh}\left(\frac{1}{\sqrt{1-v^2}}\right) + r_3 \operatorname{arccosh}\left(\frac{1}{\sqrt{1-v^2}}\right)^2 + r_4 \mathsf{E}\left(-\frac{v^2+2\sqrt{1-v^2}-2}{v^2}\right)^2$$

$$+ r_5 \mathsf{K}\left(-\frac{v^2+2\sqrt{1-v^2}-2}{v^2}\right) \mathsf{E}\left(-\frac{v^2+2\sqrt{1-v^2}-2}{v^2}\right) + r_6 \mathsf{K}\left(-\frac{v^2+2\sqrt{1-v^2}-2}{v^2}\right)^2$$

$$+ r_7 \log\left(\frac{v}{2\sqrt{1-v^2}}\right) + r_8 \log\left(\frac{v}{2\sqrt{1-v^2}}\right) \operatorname{arccosh}\left(\frac{1}{\sqrt{1-v^2}}\right)$$

$$+ r_9 \log\left(\frac{v}{2\sqrt{1-v^2}}\right) \log\left(\frac{1}{2}\left(\frac{1}{\sqrt{1-v^2}}+1\right)\right) + r_{10} \log\left(\frac{1}{2}\left(\frac{1}{\sqrt{1-v^2}}+1\right)\right)$$

$$+ r_{11} \log^2\left(\frac{1}{2}\left(\frac{1}{\sqrt{1-v^2}}+1\right)\right) + r_{12}\,\boxed{\alpha} + r_{13}\frac{\boxed{\beta}}{v^2} + r_4 \log(b)\Bigg)\left(\frac{M}{b}\right)^4$$

Elliptic integrals

$r_i = \text{rational coefficients}$

Free coefficients

Log term

Extraction of high-order conservative PM results

Subtract known analytic parts of conservative 4PM:

$$\Delta_4(v) := (\delta\chi_4^{\text{cons}} - \delta\chi_4^{\text{known}})b^4 = \boxed{\frac{3}{8}\pi M^4\left[c_2 + c_1(5 - 4/v^2)\right]} + \mathcal{O}(1/b)$$

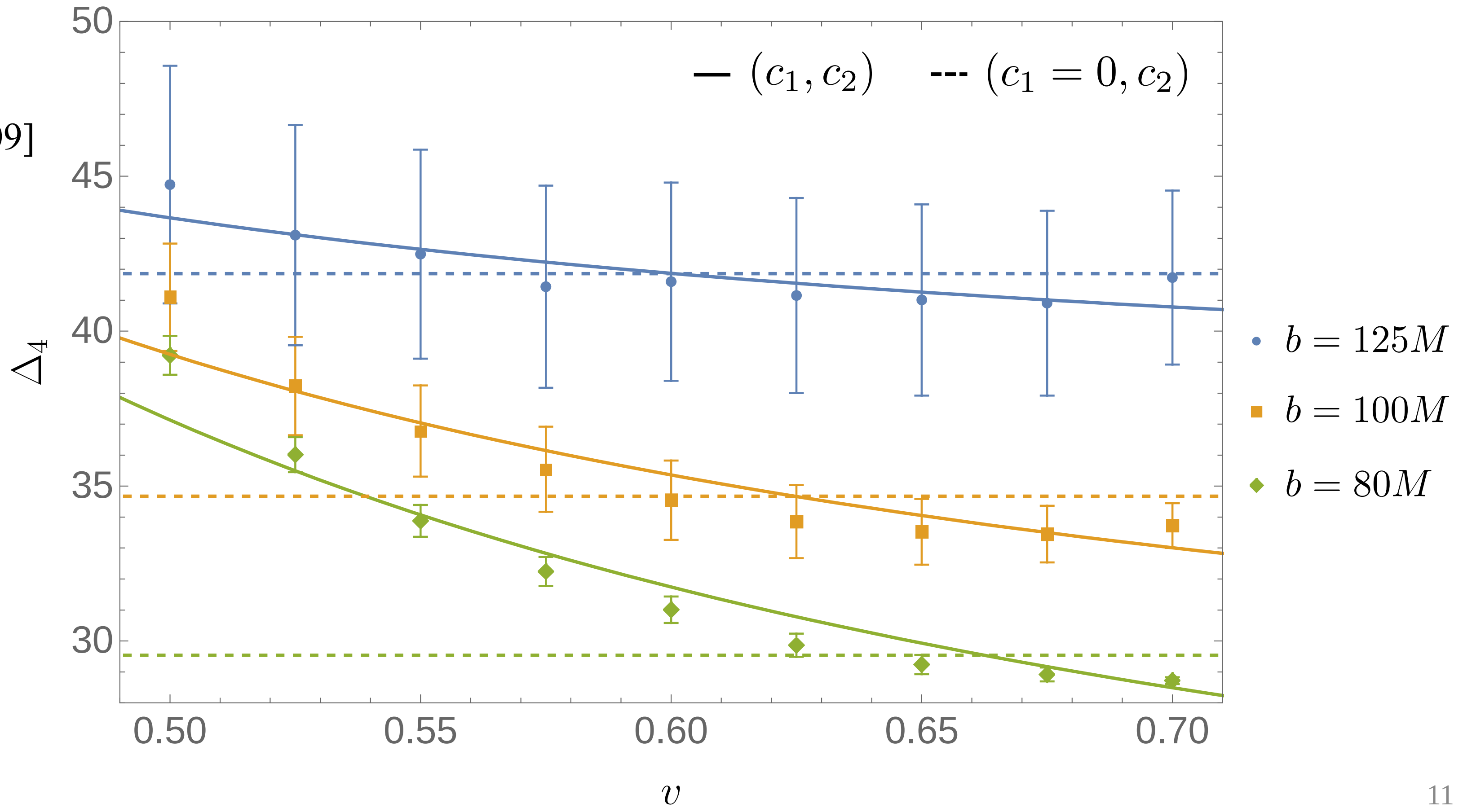
c_1 & c_2 are Love numbers.

Expect $c_1 = 0$ [Binnington & Poisson '09]

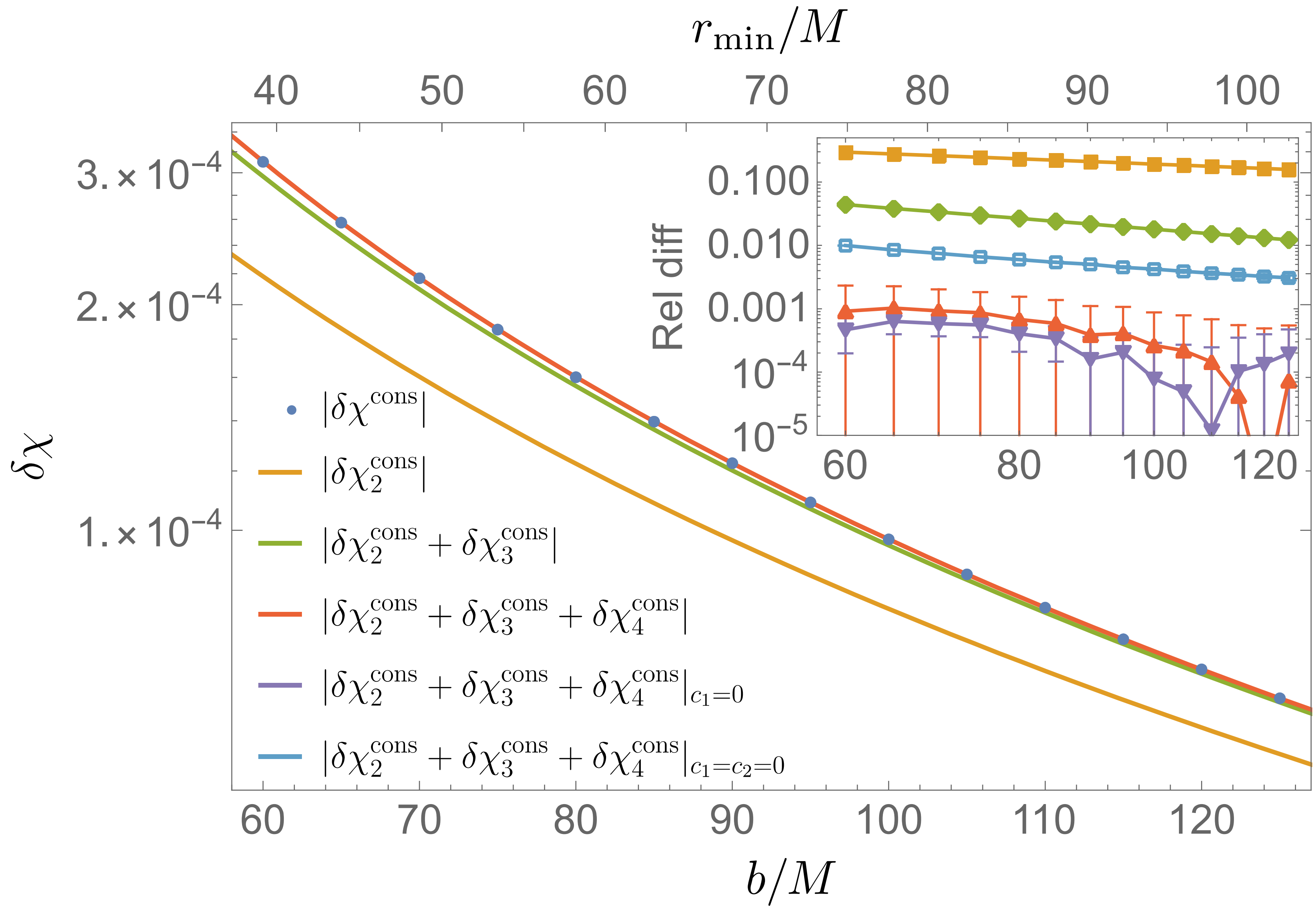
b/M	c_1	c_2
80	0.94	−21.2
100	0.68	−25.9
125	0.31(±0.38)	−33.6
80	**0**	−25.1
100	**0**	−29.4
125	**0**	−35.5

Fixed

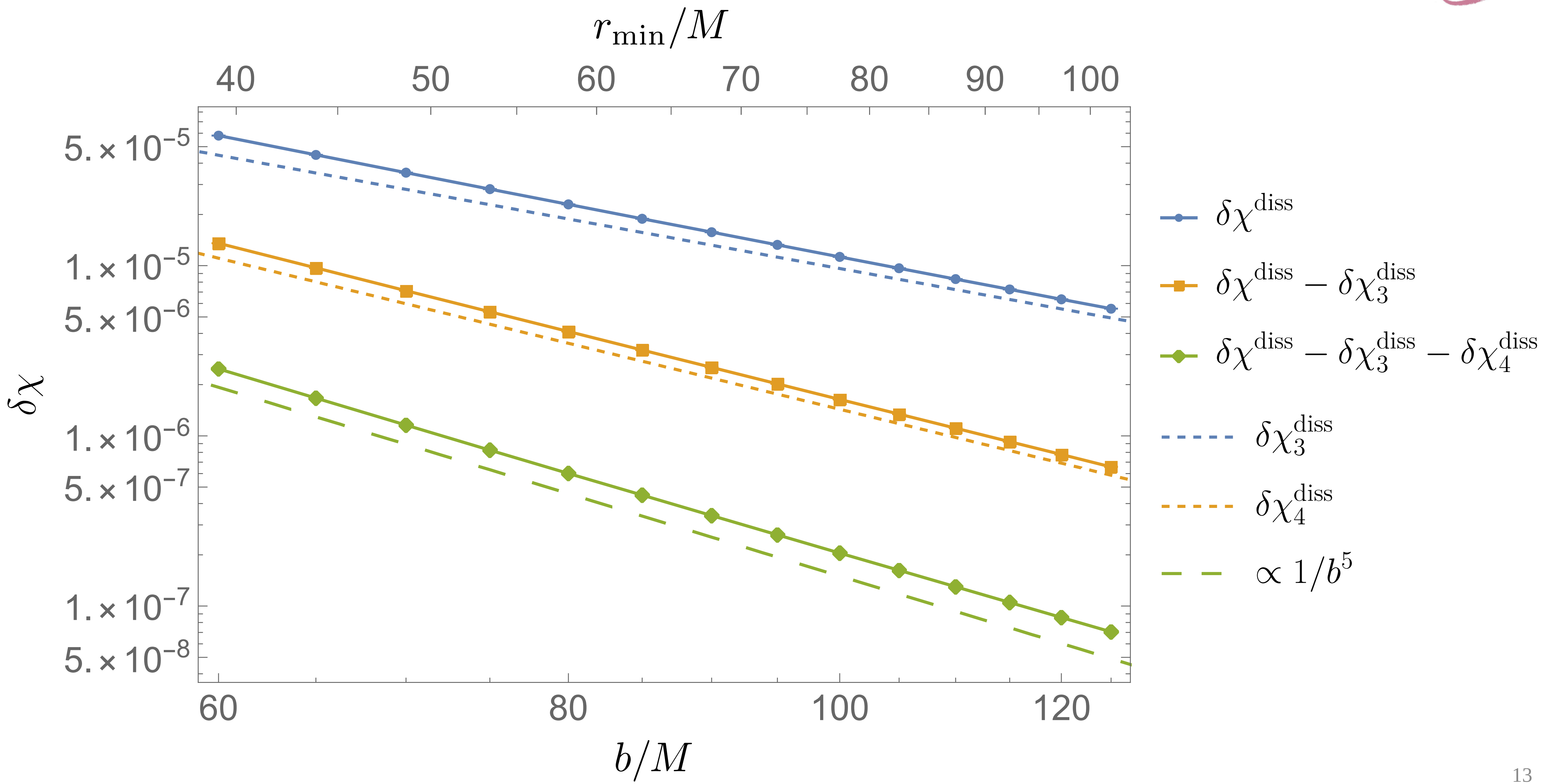
(Fitting error)



PM comparison: Conservative $v = 0.5$



Dissipative: $v = 0.5$



Extraction of high-order dissipative PM results

PM expansion with free parameters:

$$\delta\varphi^{\mathrm{PM}}_{\mathrm{diss}} = \frac{\alpha_3}{b^3} + \frac{\alpha_4}{b^4} + \frac{\alpha_5}{b^5} + \frac{\alpha_6}{b^6} + \ldots$$

Up to 4PM can fit value or use **analytic value**.

α_3	α_4	α_5	α_6
11.19	–	–	–
9.44	188	–	–
9.64	142	1900	–
9.61	154	920	26615
9.6225	169	–	–
9.6225	147	1720	–
9.6225	149	1321	15859
9.6225	**143.344**	1965	–
9.6225	**143.344**	2248	−20216
$< 1\%$	$\sim 1\%$	$\sim 2000(?)$	???

Summary and future work

Compared numerical scalar self-force results with analytic PM results.

Good agreement for both conservative and dissipative:

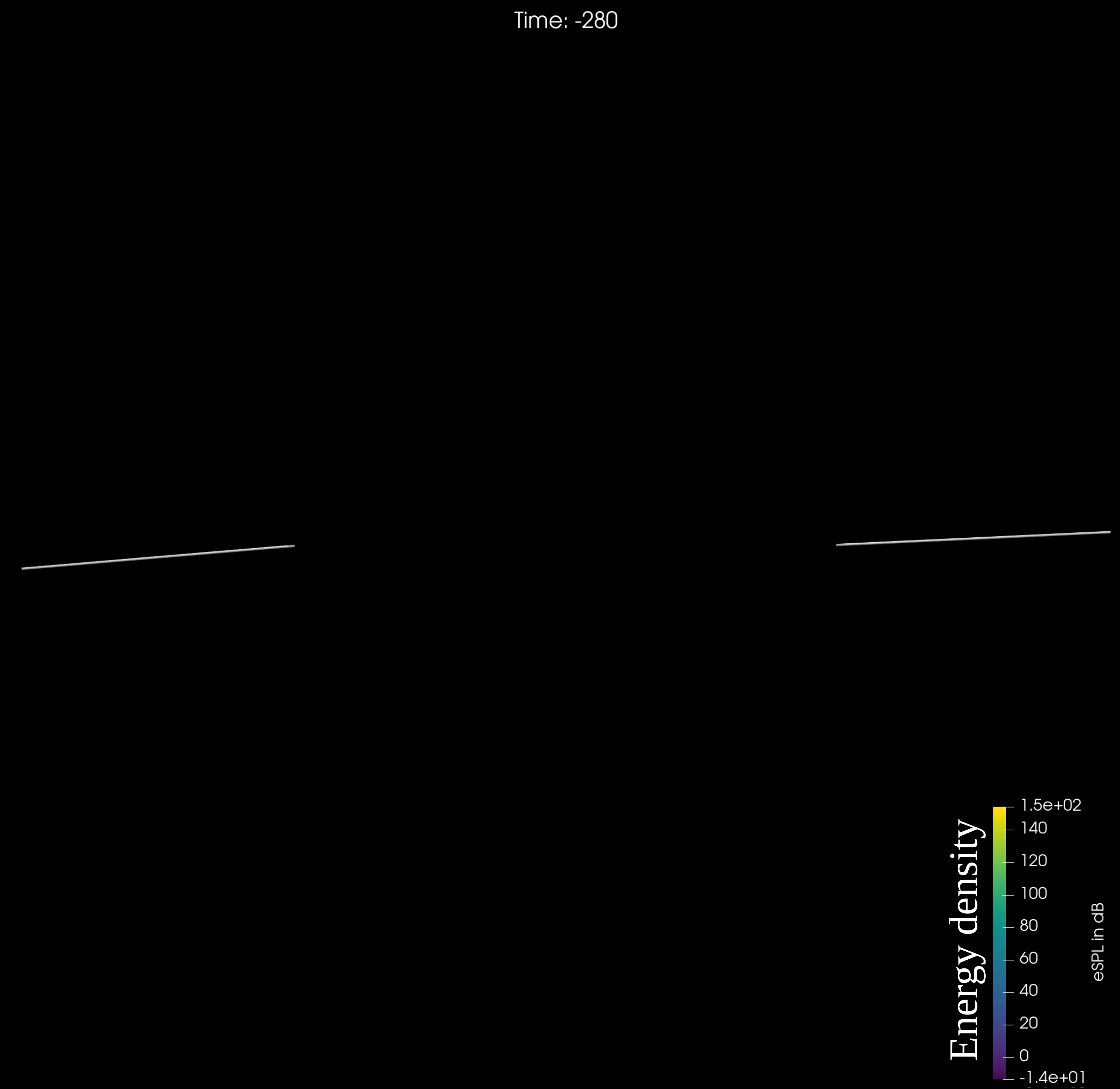
- ~ 10% for LO.
- ~ 1% for NLO.

Have investigated extractions of higher-order components from numerics:

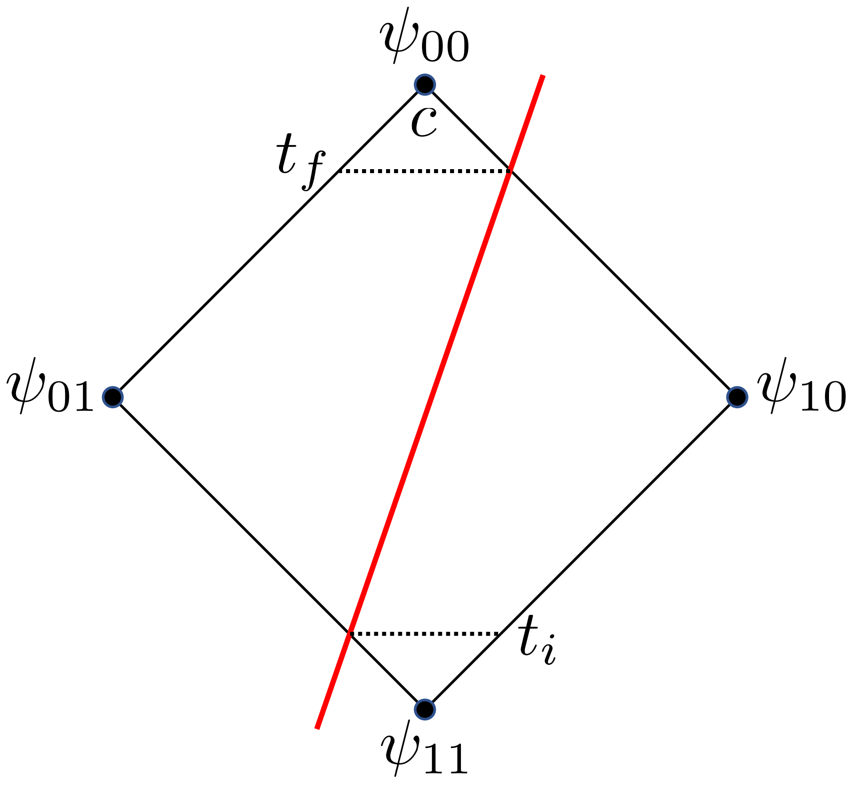
- Fixed free parameters in 4PM conservative: agreement with numerics to < 0.5%.
- Extraction of 5PM dissipative coefficient.

Future work:

- Hybrid FD/TD model for more accurate calculations (with **C. Whittall**).
- 1+1D with hyperboloidal slicing and compactification (with **R. Panosso Macedo**).
- Calculate the gravitational self-force correction to the scattering angle.
- Compare to PM/NR/EOB in the gravitational case.


Time: -280
Energy density
eSPL in dB
1.5e+02
140
120
100
80
60
40
20
0
-1.4e+01

Scalar field evolution scheme



Finite-difference version of scalar wave equation:

$$\psi_{00} = -\psi_{11} + (\psi_{01} + \psi_{10})\left(1 - \frac{1}{2}h^2 V\right) + Z + \mathcal{O}(h^4)$$

$Z = 0$ if particle does not enter cell.