

$$S_{lm}^{\text{eff}}(t,r)=S_{lm}^{\text{eff},+}(t,r)\Theta^{+}(t,r)+S_{lm}^{\text{eff},-}(t,r)\Theta^{-}(t,r)$$