

$$S_{lm}^{\text{eff},\pm}(t,r)=\sum_{n=-\infty}^{\infty}S_{lmn}^{\text{eff},\pm}(r)e^{-i\omega_{mn}t}$$