

$$S_{lm}^{\text{eff}}(t, r) = \lim_{N \rightarrow \infty} \sum_{n=-N}^N S_{lmn}^{\text{eff}}(r) e^{-i\omega_{mn}t}$$