

$$S_{lmn}^{\text{eff}}(r) = \sum_{n'=-\infty}^{\infty} \left[b_{n'-n}^{+}(r) S_{lmn}^{\text{eff},+}(r) + b_{n'-n}^{-}(r) S_{lmn}^{\text{eff},-}(r) \right]$$