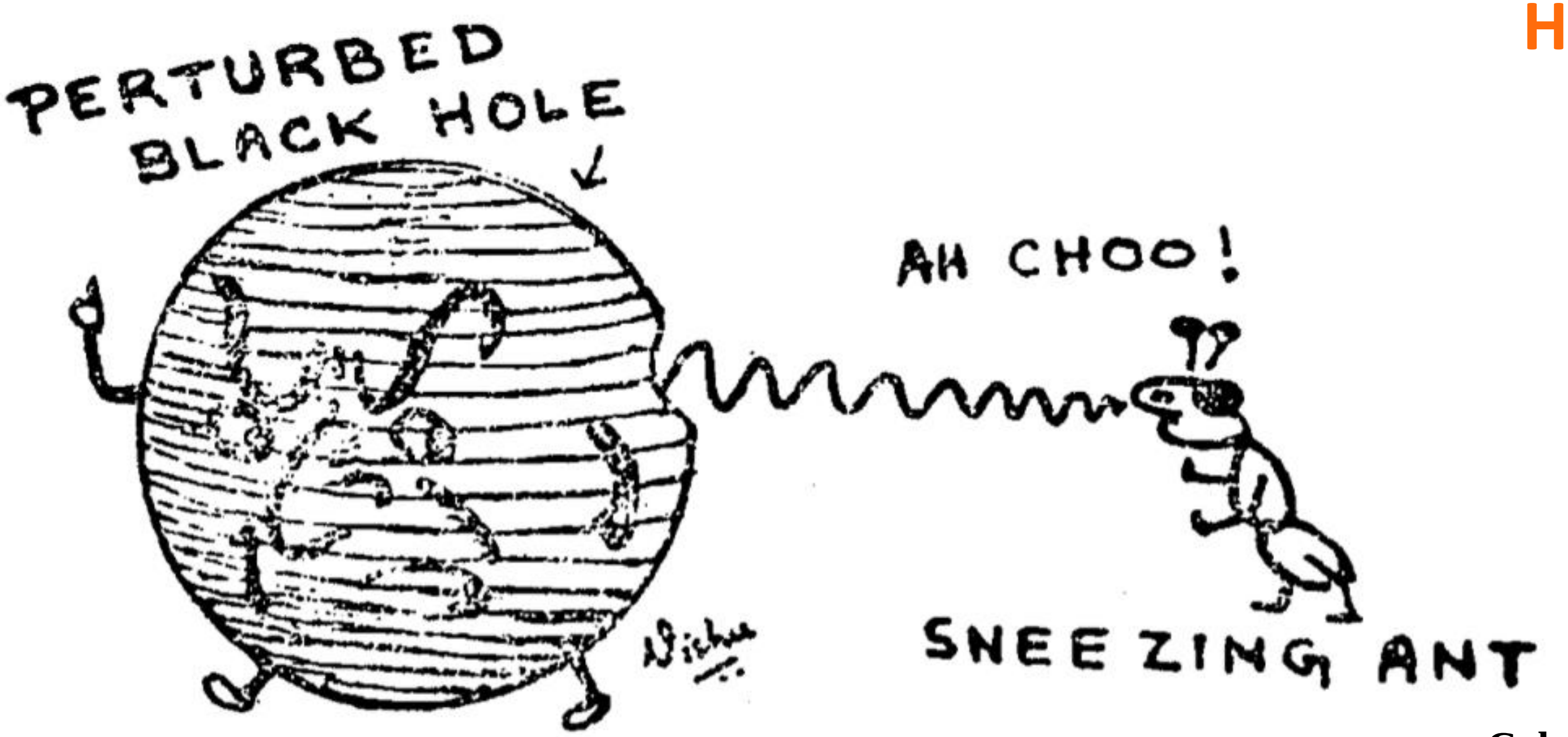
Caltech

The Division of Physics,
Mathematics and Astronomy

Perturbations of Rotating Black Holes in Modified Gravity



(Image credit: C.V. Vishveshwara)

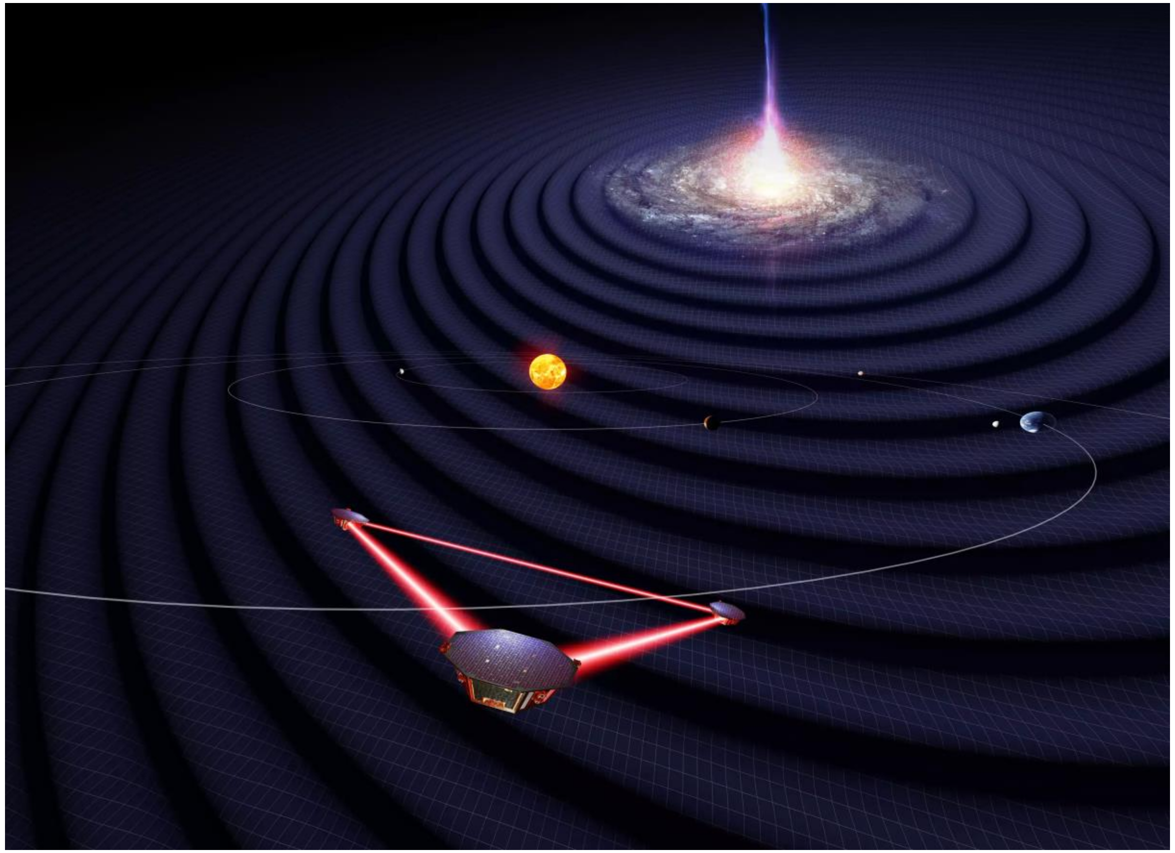
Dongjun Li (Caltech)
The 26th Capra Meeting,
The Niels Bohr Institute,
July 7th, 2023

In collaboration with
Caltech: Colin Weller, Yanbei Chen
Perimeter: Michael LaHaye, Huan Yang
UIUC: Pratik Wagle, Nicolas Yunes
UT Austin: Asad Hussain, Aaron Zimmerman

GR has its limitation

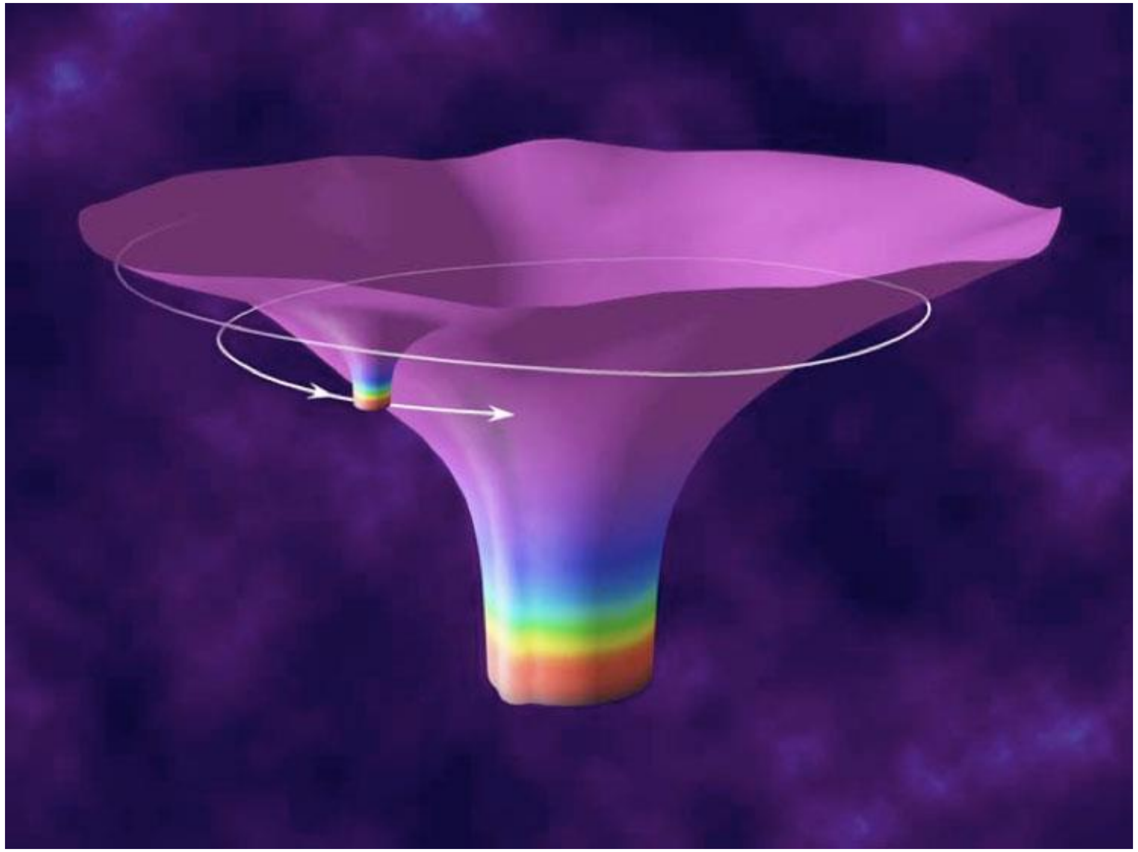
- Incompatibility between GR and QM → Unified theories, e.g., loop gravity, string theory
- Observational anomalies, e.g., matter-antimatter asymmetry

LISA (Image credit: Simon Barke)



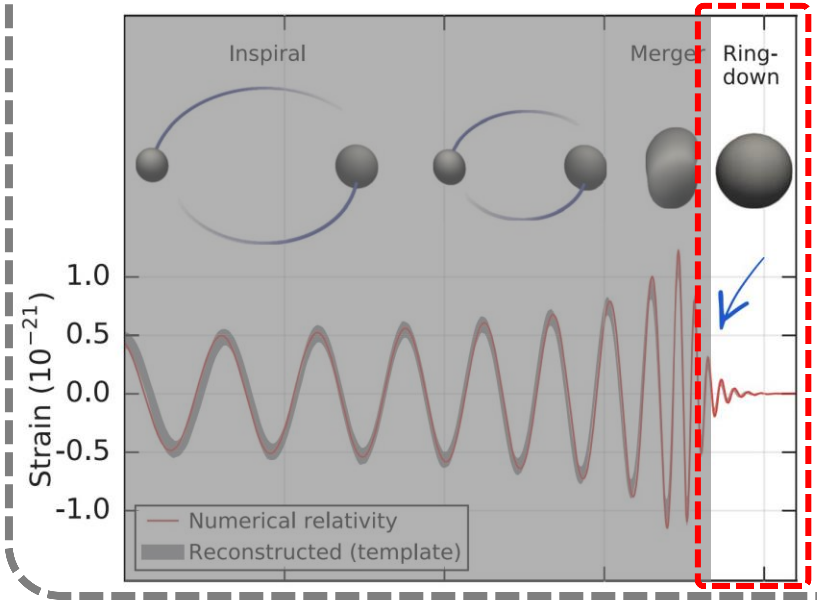
Will be launched ~ 2030s! (Baker et al., 2019)

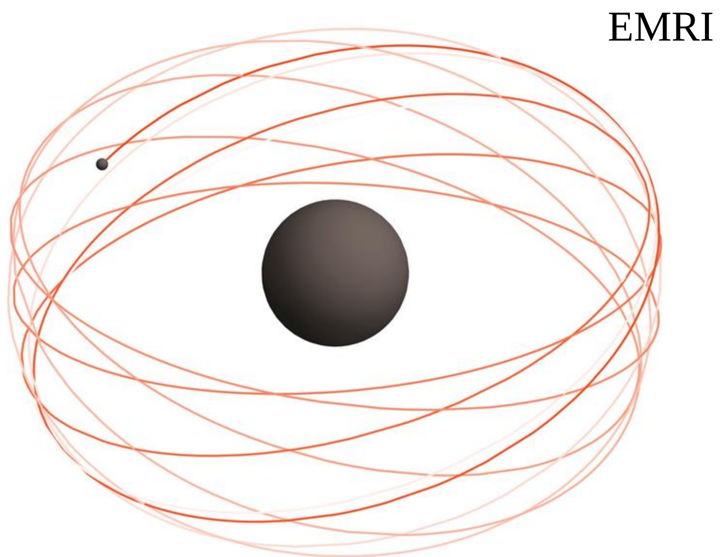
(Image credit: NASA)



Black Hole Perturbation Theory

(Image Credit: Leo C. Stein)





(Image Credit: N. Franchini)

Up to 10 highly coupled PDEs!

Non-rotating BHs

Regge-Wheeler and Zerilli-Moncrief eqns (perturb metric)

- ❑ 2 decoupled separable PDEs (Regge & Wheeler, 1957; Zerilli, 1970)

Also works for slowly rotating BHs

Rotating BHs

Teukolsky eqns (perturb curvature, based on NP formalism)

- ❑ 2 decoupled separable PDEs for Ψ_0 & Ψ_4(Teukolsky, 1973)

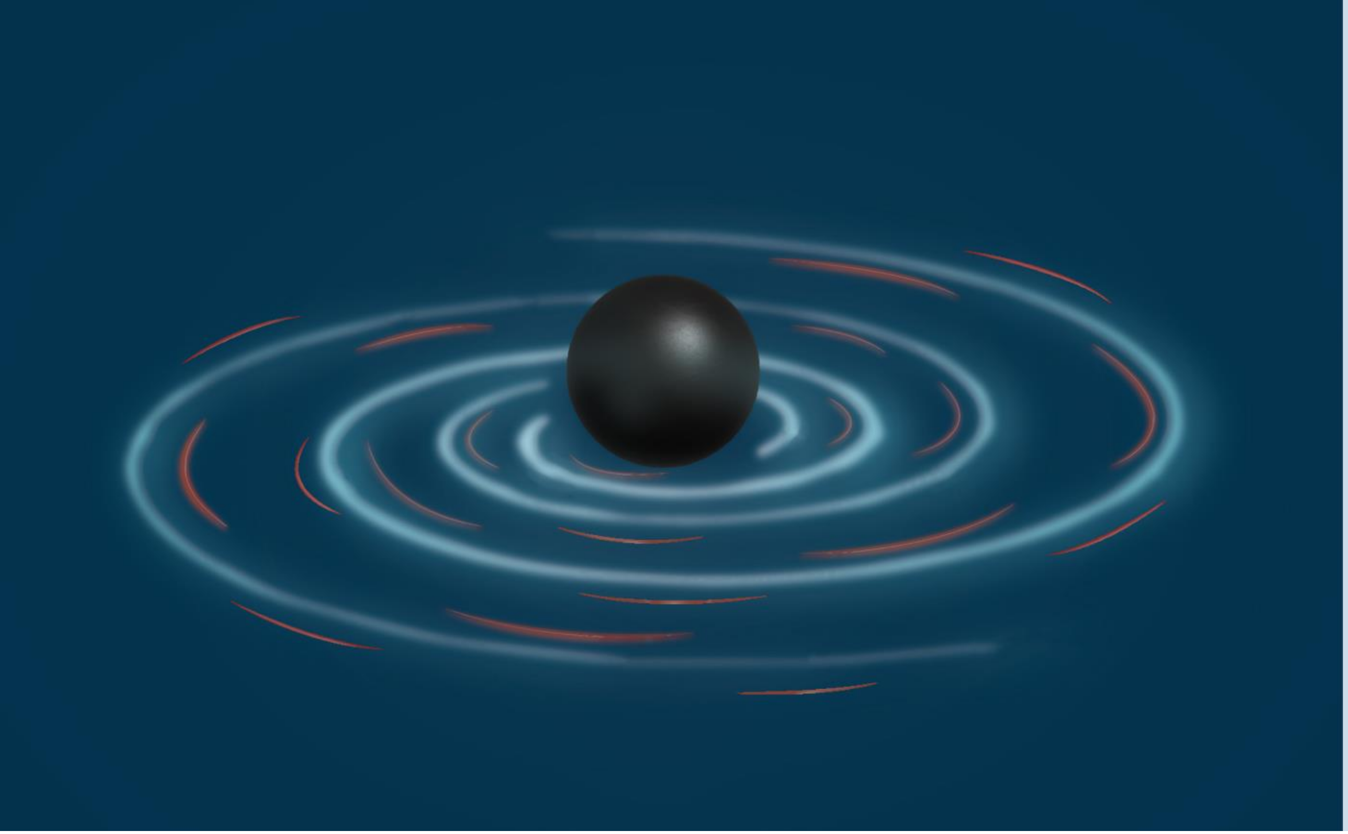
GR

Metric can be reconstructed

Modified Teukolsky eqns (first successful extension)

- ❑ Petrov type D → not necessarily for bGR BHs, e.g, dCS (Yagi et al., 2012)

bGR



An EFT approach:

$\zeta \to$ modified gravity expansion

$\epsilon \to$ GW expansion

Li, Wagle, Chen, & Yunes,
PhysRevX.13.021029 (arXiv 2206.10652)

Background spacetime in GR, type D always!

GWs in GR

$$\Psi = \Psi^{(0,0)}(\mathbf{x}) + \zeta\Psi^{(1,0)}(\mathbf{x}) + \epsilon\Psi^{(0,1)}(\mathbf{x},t) + \zeta\epsilon\Psi^{(1,1)}(\mathbf{x},t)$$

The stationary part of spacetime in bGR, can be algebraically general

Leading correction to GWs due to bGR

- **Modified Teukolsky equation**

$$H_0^{(0,0)}\Psi_0^{(1,1)} = \mathcal{S}_{0,\mathrm{D}}^{(1,1)} + \mathcal{S}_{0,\text{non-D}}^{(1,1)} + \mathcal{S}_{1,\text{non-D}}^{(1,1)} + \mathcal{S}^{(1,1)}$$

$$H_0 = (D - \rho + \cdots)(\mathbf{\Delta} - 4\gamma + \cdots) + \cdots$$

$$H_1 = (D - \rho + \cdots)(\delta - 4\tau - 2\beta) + \cdots$$

Follow Chandrasekhar: rotate away $\Psi_{1,3}^{(1,1)}$

Works for any linear perturbation of a type D spacetime!

- **Theory-independent** source terms (purely geometrical)

$$\mathcal{S}_{0,\mathrm{D}}^{(1,1)} \sim H_0^{(1,0)}\Psi_0^{(0,1)} \sim h^{(1,0)}\Psi_0^{(0,1)}$$ Non-GR type D

$$\mathcal{S}_{i,\text{non-D}}^{(1,1)} \sim H_i^{(0,1)}\Psi_i^{(1,0)} \sim h^{(0,1)}\Psi_i^{(1,0)}$$ Non-type-D

$$\mathcal{S}_{\text{geo}}^{(1,1)} = \mathcal{S}_{0,\mathrm{D}}^{(1,1)} + \mathcal{S}_{0,\text{non-D}}^{(1,1)} + \mathcal{S}_{1,\text{non-D}}^{(1,1)}$$

- **Theory-dependent** source terms (effective $T^{\mu\nu}$)

Extra fields

$$\mathcal{S} = \mathcal{E}_2 S_2 - \mathcal{E}_1 S_1 \sim h^{(0,1)}\vartheta^{(1,0)} + g^{(0,0)}\vartheta^{(1,1)}$$

$$\mathcal{E}_2 = \Psi_2(D - \rho + \cdots)\Psi_2^{-1}\cdots, S_2 = (\delta - 2\beta + 2\bar{\pi})\Phi_{01} + \cdots$$

$h^{(0,1)}$ needs metric reconstruction, but only for GR GWs

$$h_{\mu\nu}^{(0,1)} = (\mathcal{O}_{\mu\nu} + \bar{\mathcal{O}}_{\mu\nu}\mathcal{C})\,\Psi_0^{(0,1)}$$

(Li, Wagle, Chen, & Yunes, arXiv 2206.10652)

An alternative tensorial approach

- **Wald's formalism in GR** (Wald, 1978)

$$H\mathcal{T}\left[h_{cd}\right]=H\left[\psi_s\right]=\mathcal{S}^{ab}\mathcal{E}_{ab}\left[h_{cd}\right]$$

H : Teukolsky operator $\quad$ $\mathcal{E}_{ab}$: Einstein tensor

$\mathcal{T}$: convert metric to Weyl scalars

$\mathcal{S}^{ab}$: convert EE to the Teukolsky equation

- **An extension to bGR theories** (Hussain & Zimmerman, 2022)
  - Apply same $\mathcal{S}^{ab}$ operator to the EE in bGR theories
  - $\mathcal{S}^{ab(0,0)}$ converts EE at $\mathcal{O}(\zeta^1,\epsilon^1)$ to Teukolsky equation at $\mathcal{O}(\zeta^1,\epsilon^1)$ ➡ $\Psi_0^{(1,1)}$ naturally decouples
  - $\mathcal{S}^{ab(0,1)}$ converts EE at $\mathcal{O}(\zeta^1,\epsilon^0)$ to source terms (similar to $\mathcal{S}^{(1,1)}$, $\mathcal{S}^{(1,1)}_{\text{geo}}$), similarly for $\mathcal{S}^{ab(1,0)}$

(Hussain & Zimmerman, arXiv 2206.10653)

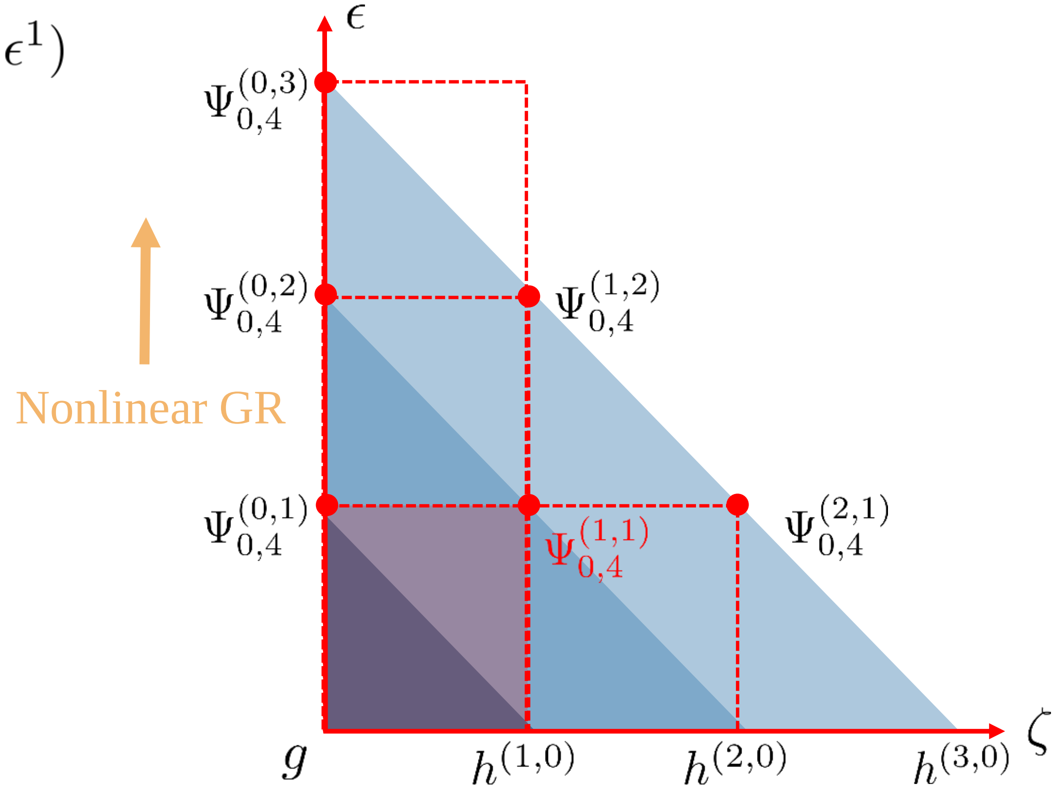
Higher-order (modified) Teukolsky formalism

- Connection to 2nd order Teukolsky equation in GR (Campanelli & Lousto, 1999)
  - Couplings between $h^{(0,1)}$ and $h^{(1,0)}$ → Self-couplings of $h^{(0,1)}$
  - Many pre-developed techniques, e.g., metric reconstruction → Our formalism is very feasible !
- Extension of modified Teukolsky equation beyond $\mathcal{O}(\zeta^1, \epsilon^1)$
  - All the steps can be iterated to higher orders, e.g., rotate away $\Psi_{1,3}^{(m,n)}$ for $m \geq 0,\ n \geq 1$
  - Modified Teukolsky equations at $\mathcal{O}(\zeta^m, \epsilon^n)$:

$$H_0^{(0,0)} \Psi_0^{(m,n)} = \mathcal{S}_{\rm geo}^{(m,n)} + \mathcal{S}^{(m,n)}$$

↓ $\zeta = 0$

Consistent with Campanelli & Lousto, 1999



(Li, Wagle, Chen, & Yunes, arXiv 2206.10652)

Metric Reconstruction (vacuum)

- **Direct reconstruction** (Chandrasekhar, 1983; Loutrel et al. in 2020)
- **CCK-Ori (**Cohen & Kegeles, 1975; Kegeles & Cohen, 1978; Chrzanowski, 1975; Ori, 2003**)**
  - An inversion of Wald's relation: $h_{ab} = \mathrm{Re}\left(\mathcal{S}^\dagger \Phi\right)_{ab}, \; \mathrm{P}^4 \bar{\Phi} = -\Psi_0$
  - Relies on IRG ($l^\mu h_{\mu\nu} = 0, \; h = 0$) or ORG ($n^\mu h_{\mu\nu} = 0, \; h = 0$)
  - Reconstruct NP quantities (Campanelli & Lousto, 1999)

I. Tetrad

$$e_a^{\mu(0,1)} = A_a{}^{b(0,1)} e_b^{\mu(0,0)}$$

Orthogonality

$$n^{\mu(0,1)} = \frac{1}{2} h_{nn}^{(0,1)} l^\mu$$

II. Spin coefficients

Commutators

$$\nu^{(0,1)} = -\tfrac{1}{2} \bar{\delta}_{[2,2,-1,-1]} h_{nn}^{(0,1)} + \cdots$$

III. Weyl scalars

Ricci identities

$$\Psi_0^{(0,1)} = \tfrac{1}{2} D_{[-3,1,0,-1]} D_{[-2,2,0,-1]} h_{mm}^{(0,1)}$$

e.g., $\bar{\delta}_{[a,b,c,d]} = \bar{\delta} + a\alpha + b\bar{\beta} + c\pi + d\bar{\tau}$

III. Rotate the tetrad

$$\Psi_{1,3}^{(0,1)} = 0$$

$$a^{(0,1)} = -\bar{\Psi}_3^{(0,1)}/(3\Psi_2)$$
$$b^{(0,1)} = -\Psi_1^{(0,1)}/(3\Psi_2)$$

Metric Reconstruction (non-vacuum)

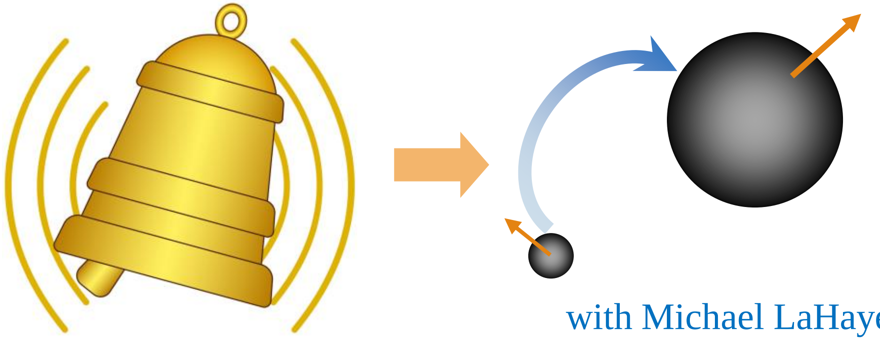
- Necessary for nonlinear effects in both GR (e.g., 2nd order self-force) and bGR (even just EMRIs)
- **Lorenz gauge** (Dolan et al., 2022; Dolan et al., 2023)
  - CCK with an additional gauge transformation

$$\xi^\mu = \zeta^2 \nabla_\nu H^{\mu\nu} - g^{\mu\nu}\nabla_\nu \chi$$

  - (Spin 0 + Spin 1) [gauge in vacuum] + Spin 2 + completion (non-radiative) pieces
  - Free from extended gauge discontinuities; works for point-particle source
- **GHZ** (Green, Holland, & Zimmerman, 2020; Toomani, 2022)
  - CCK with an additional correction tensor

$$h_{ab} = (\mathcal{L}_\xi g)_{ab} + \dot{g}_{ab} + x_{ab} + \mathrm{Re}\left(\mathcal{S}^\dagger \Phi\right)_{ab}$$

  - ξ : transform to the shadowless gauge
  - Regular enough for 2nd order self-force; works for compact source



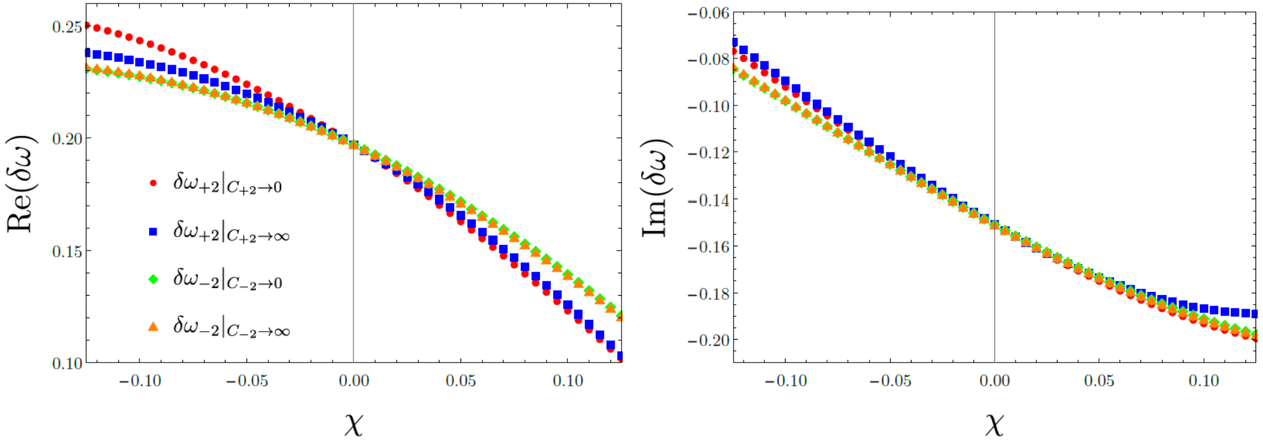
with Michael LaHaye, Colin Weller, Huan Yang

Example: QNMs in Higher-derivative Gravity

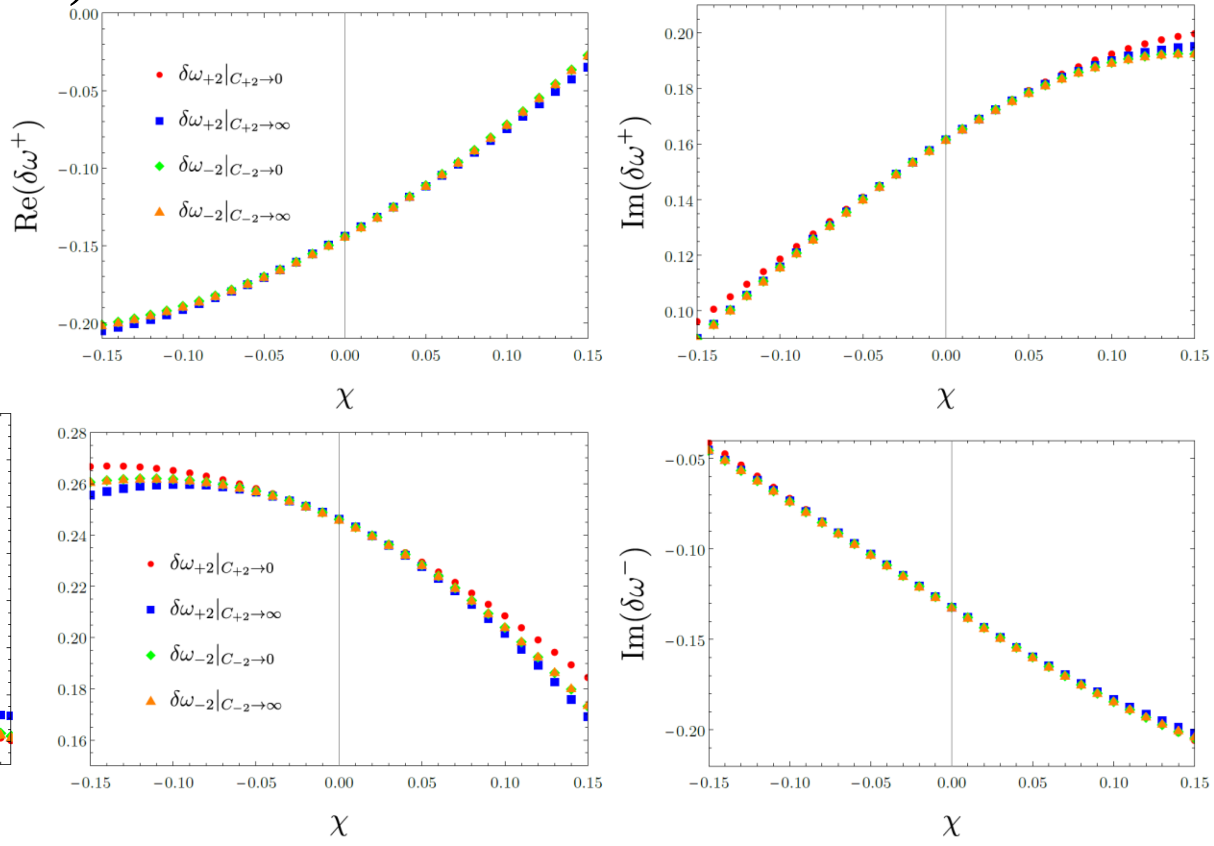
A similar study in dCS:
Wagle, Li, Chen, & Yunes, in prepare

- Up to $\mathcal{O}(\chi^6)$, the results are valid for $\chi \lesssim 0.4$
- Matches well with metric perturbations (mismatch $\lesssim 5\%$)!
- Computed from both $\Psi_{0,4}$'s equations
- **Intriguing:**

  Results from Ψ_0's equation are less accurate

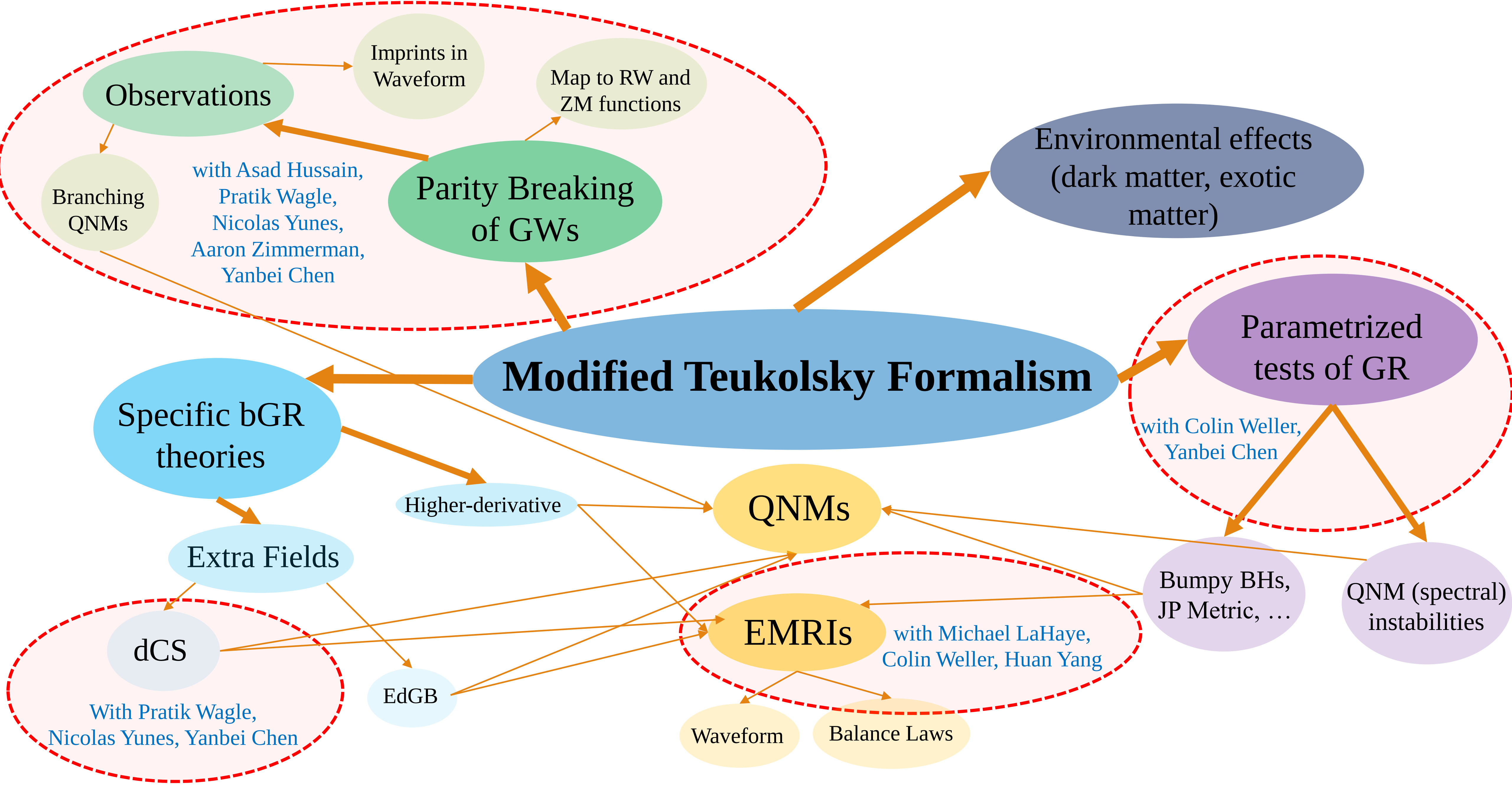


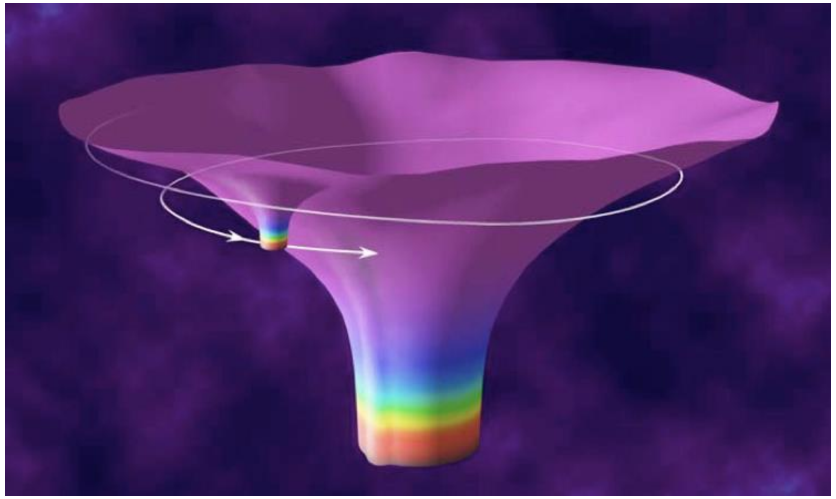
Parity violating

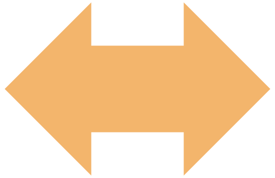


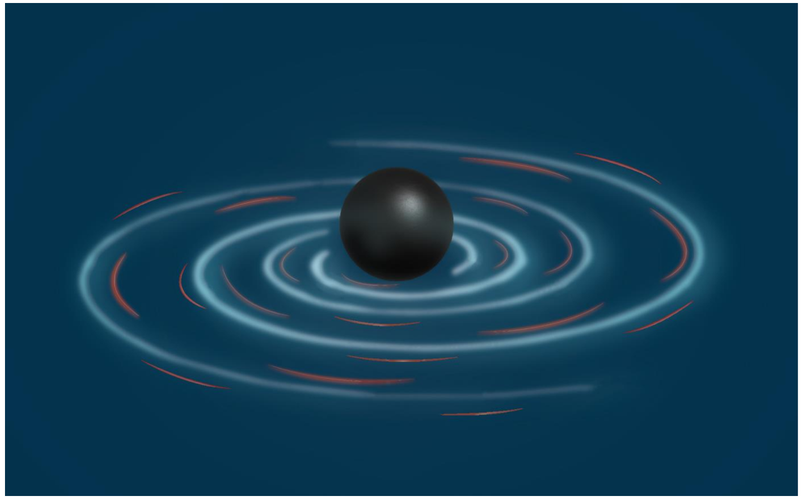
Parity preserving

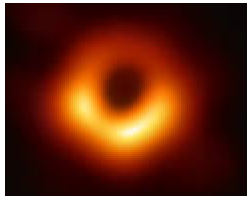
(Cano, et al., 2023)


Observations
Imprints in Waveform
Map to RW and ZM functions
Branching QNMs
with Asad Hussain, Pratik Wagle, Nicolas Yunes, Aaron Zimmerman, Yanbei Chen
Parity Breaking of GWs
Environmental effects (dark matter, exotic matter)
Modified Teukolsky Formalism
Parametrized tests of GR
with Colin Weller, Yanbei Chen
Specific bGR theories
Higher-derivative
QNMs
Extra Fields
Bumpy BHs, JP Metric, …
QNM (spectral) instabilities
dCS
EMRIs
with Michael LaHaye, Colin Weller, Huan Yang
EdGB
With Pratik Wagle, Nicolas Yunes, Yanbei Chen
Waveform
Balance Laws







Thank yu!