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COMPARING EFFECTIVE-ONE-BODY AND GRAVITATIONAL SELF-FORCE RESULTS FOR BLACK HOLE BINARIES WITH A SPINNING SECONDARY

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OUTLINE

- Motivation
- Quick introduction to the effective-one-body (EOB) formalism
- Building on previous results: gauge-invariant analysis
- Comparison between the EOB model **TEOBResumS** and gravitational self-force (GSF) results for black hole binaries with a spinning secondary
- Modifying the EOB spin-orbit sector (spinning particle info) to improve the agreement with GSF

LOOKING FORWARD

- The next generation of gravitational wave detectors (ET, CE, LISA) will allow us to detect intermediate- and extreme-mass-ratio inspirals
- Numerical relativity simulations become increasingly difficult to be performed…
- EOB models could provide fast and reliable waveforms,
  but they need to be tuned and benchmarked towards exact results
  ➡ comparing to GSF allows to improve EOB
- Already improved quasi-circular nonspinning version of TEOBResumS thanks to 2GSF results (arXiv:2208.02055v2)

THE EFFECTIVE-ONE-BODY FORMALISM

mapping the two-body dynamics in general relativity in the motion of a particle with the reduced mass of the system moving in an effective metric that is the deformation of a Schwarzschild or Kerr black hole

Hamiltonian: found by mapping the “energy levels” of the real problem at a given post-Newtonian (PN) order to the effective ones

Mass ratio $q = \frac{m_1}{m_2}$, $m_1 > m_2$ Symmetric mass ratio $\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}$

THEORETICAL FRAMEWORK

- Hamiltonian: $\hat{H}_{\rm EOB} \equiv \dfrac{H_{\rm EOB}}{\mu} = \dfrac{1}{\nu}\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)}$

$$\hat{H}_{\rm eff} = \boxed{\sqrt{p_{r_*}^2 + A\left(1 + \frac{p_\varphi^2}{r_c^2} + 2\nu(4-3\nu)\frac{p_{r_*}^4}{r_c^2}\right)}} + \boxed{p_\varphi\left(G_S\hat{S} + G_{S_*}\hat{S}_*\right)}$$

orbital + spin-orbit

- Hamiltonian equations of motion complemented by the radiation reaction:

$$\frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} = \Omega$$

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2}\frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi = \hat{\mathcal{F}}_\varphi^\infty + \hat{\mathcal{F}}_\varphi^{\rm H}$$

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2}\frac{\partial \hat{H}_{\rm EOB}}{\partial r}$$

the phase space variables enter the evaluation of the waveform:

$$h_+ - ih_\times = \frac{1}{\mathcal{D}_L}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} h_{\ell m}\,{}_{-2}Y_{\ell m}$$

multipoles

GAUGE-INVARIANT ANALYSIS: Q_ω

- Adiabaticity parameter: $Q_\omega \equiv \frac{\omega^2}{\dot{\omega}}$ $\quad \omega \equiv \omega_{22} = \dot{\phi}_{22}$
- Q_ω >> 1 adiabatic motion
- Phase difference: $\Delta\phi_{(\omega_1,\omega_2)} = \int_{\omega_1}^{\omega_2} Q_\omega \, d\log\omega$

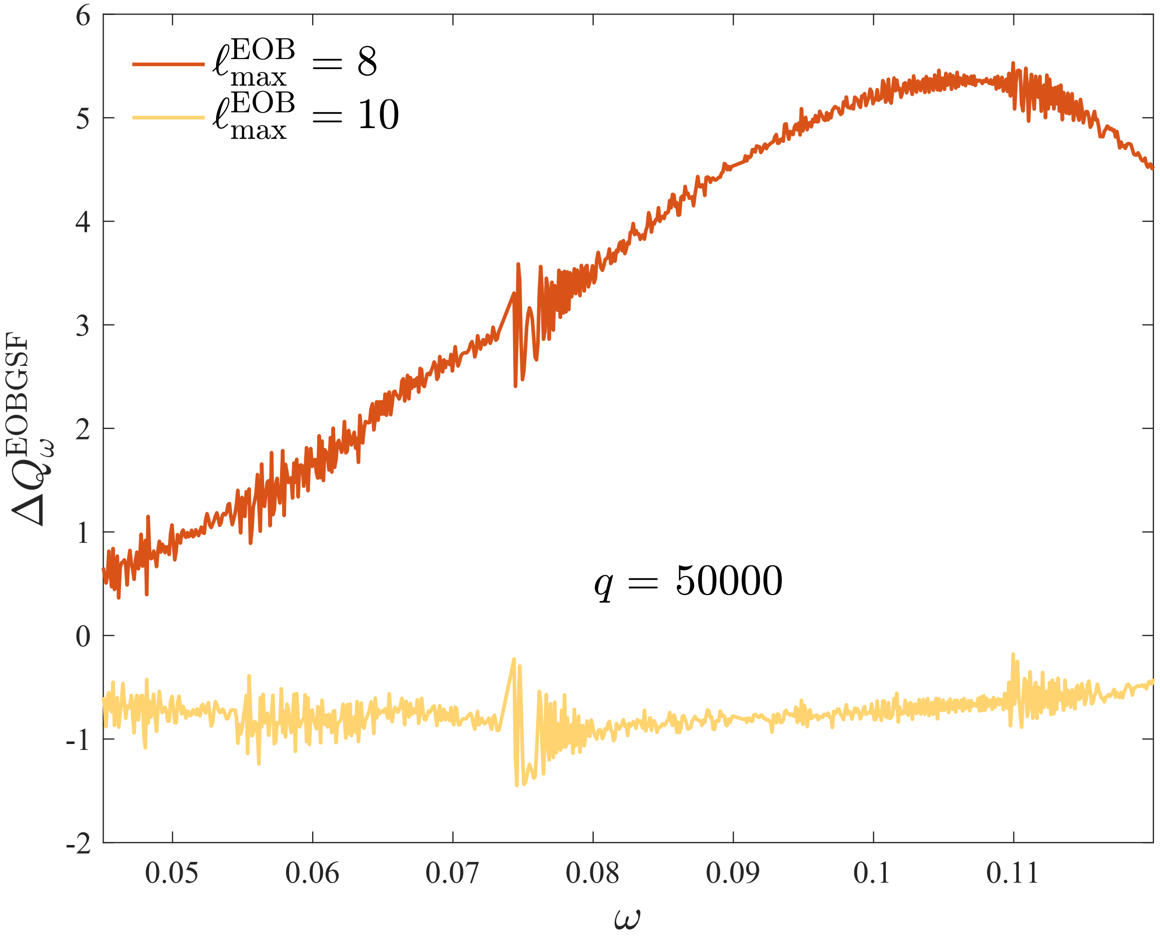
- Expanding in the symmetric mass ratio:

$$Q_\omega(\omega;\nu) = \underbrace{\frac{Q_\omega^{(0)}(\omega)}{\nu}}_{\text{0PA}} + \underbrace{Q_\omega^{(1)}(\omega)}_{\text{1PA}} + \underbrace{\nu Q_\omega^{(2)}(\omega)}_{\text{2PA}} + O(\nu^2)$$

fitting the coefficients for a set of mass ratios at fixed values of the frequency

PREVIOUS WORK: NONSPINNING BINARIES

Some (unpublished) updates
after arXiv:2208.02055v2



- Adding ℓ = 9, 10 to the infinity flux
- Shorter frequency interval:

$$\omega = [0.045, 0.12] \quad f = [0.003, 0.007]\ (Hz)$$

if $m_2 = 10 M_\odot$

- Corresponding to ~1.2 years of EOB evolution, ~1.5 × 10^5 cycles
- Integrated phase differences:

Standard: $\Delta\phi \sim 2.99$
Improved: $\Delta\phi \sim$ -0.74

Q_ω EXPANSION WITH SPIN

- Q_ω can be evaluated analytically for circular orbits (in EOB)
- χ_2 is the dimensionless spin of the smaller black hole
- Memo: $\hat{H}_{\text{eff}} = \boxed{\sqrt{p_{r_*}^2 + A\left(1 + \frac{p_\varphi^2}{r_c^2} + 2\nu(4-3\nu)\frac{p_{r_*}^4}{r_c^2}\right)}} + \boxed{p_\varphi\left(G_S\hat{S} + G_{S_*}\hat{S}_*\right)}$ orbital + spin-orbit

$$A(u;\nu) = 1 - 2u + \nu a_1(u)$$

$$S = S_1 + S_2 \qquad \hat{S} \equiv \frac{S}{M^2} \qquad S_* = \frac{M_2}{M_1}S_1 + \frac{M_1}{M_2}S_2 \qquad \hat{S}_* \equiv \frac{S_*}{M^2}$$

$$G_S = G_S^{(0)} + \nu G_S^{(1)} + \nu^2 G_S^{(2)} \qquad G_{S_*} = G_{S_*}^{(0)} + \nu G_{S_*}^{(1)} + \nu^2 G_{S_*}^{(2)}$$

$$\tilde{G} \equiv G_S\hat{S} + G_{S_*}\hat{S}_* \simeq \chi_2\left(\nu\, G_{S_*}^{(0)} + \nu^2\left(G_S^{(0)} + G_{S_*}^{(1)}\right)\right) + O(\nu^3)$$

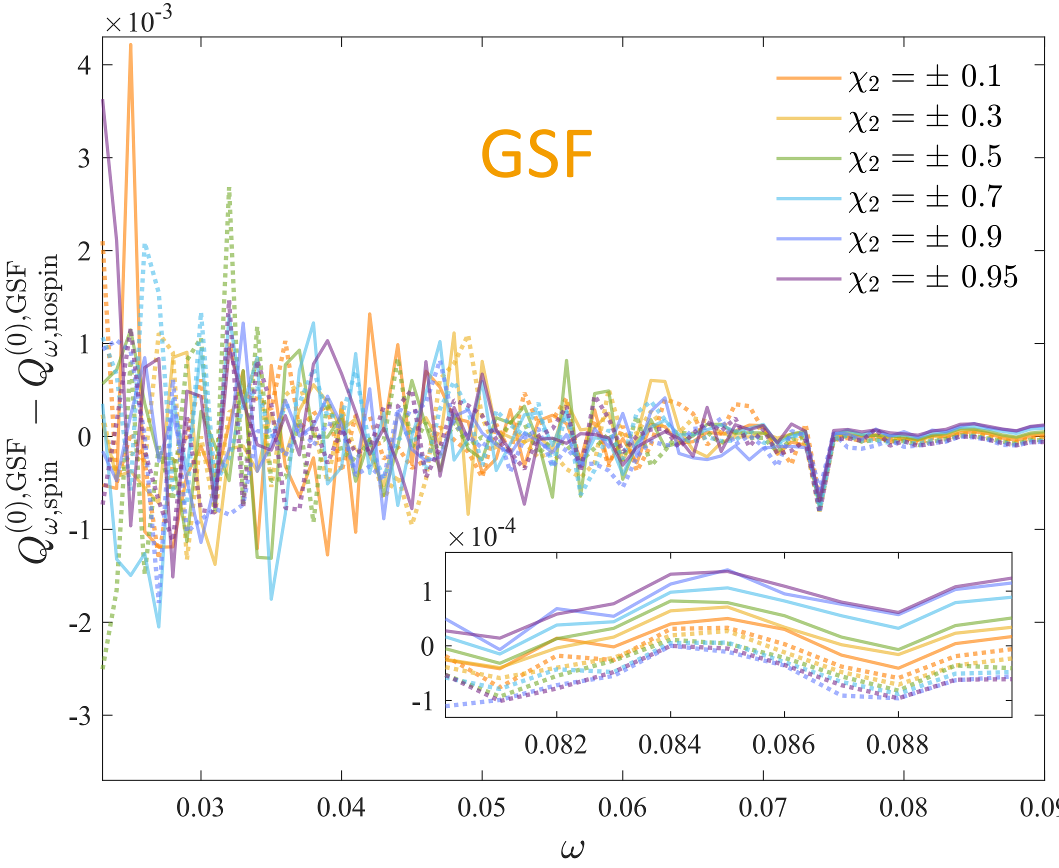
Q_ω EXPANSION WITH SPIN

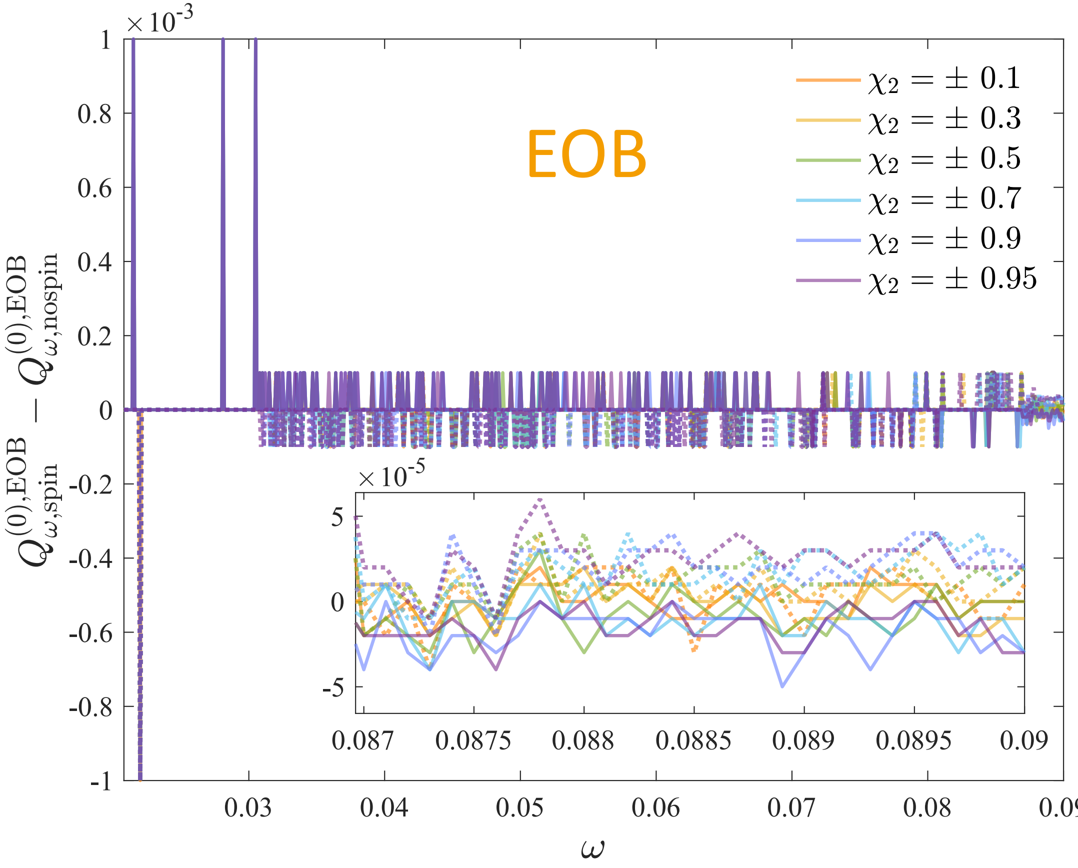
- Expanding the flux up to ν^2:
  $$\mathcal{F} = \mathcal{F}_{1\mathrm{SF}} + \nu\,\mathcal{F}_{2\mathrm{SF}} + \nu^2\,\mathcal{F}_{3\mathrm{SF}} + \chi_2\left(\nu\,\mathcal{F}^{\mathrm{spin}}_{2\mathrm{SF}} + \nu^2\,\mathcal{F}^{\mathrm{spin}}_{3\mathrm{SF}}\right)$$
- Putting everything into Q_ω yields analytical expressions for $Q_\omega^{(0)}$, $Q_\omega^{(1)}$, $Q_\omega^{(2)}$ that show their dependence on the flux, the 1SF term into the potential, and the secondary spin:

$$\begin{cases} Q^{(0)}_\omega = Q^{(0)}_\omega\left(\mathcal{F}_{1\mathrm{SF}}\right) \\ Q^{(1)}_\omega = Q^{(1)}_\omega\left(a_1, \mathcal{F}_{1\mathrm{SF}}, \mathcal{F}_{2\mathrm{SF}}, \chi_2\cdot\mathcal{F}^{\mathrm{spin}}_{2\mathrm{SF}}, \chi_2\right) \\ Q^{(2)}_\omega = Q^{(2)}_\omega\left(a_1, \mathcal{F}_{1\mathrm{SF}}, \mathcal{F}_{2\mathrm{SF}}, \chi_2\cdot\mathcal{F}^{\mathrm{spin}}_{2\mathrm{SF}}, \mathcal{F}_{3\mathrm{SF}}, \chi_2\cdot\mathcal{F}^{\mathrm{spin}}_{3\mathrm{SF}}, \chi_2, \chi_2^2\right) \end{cases}$$

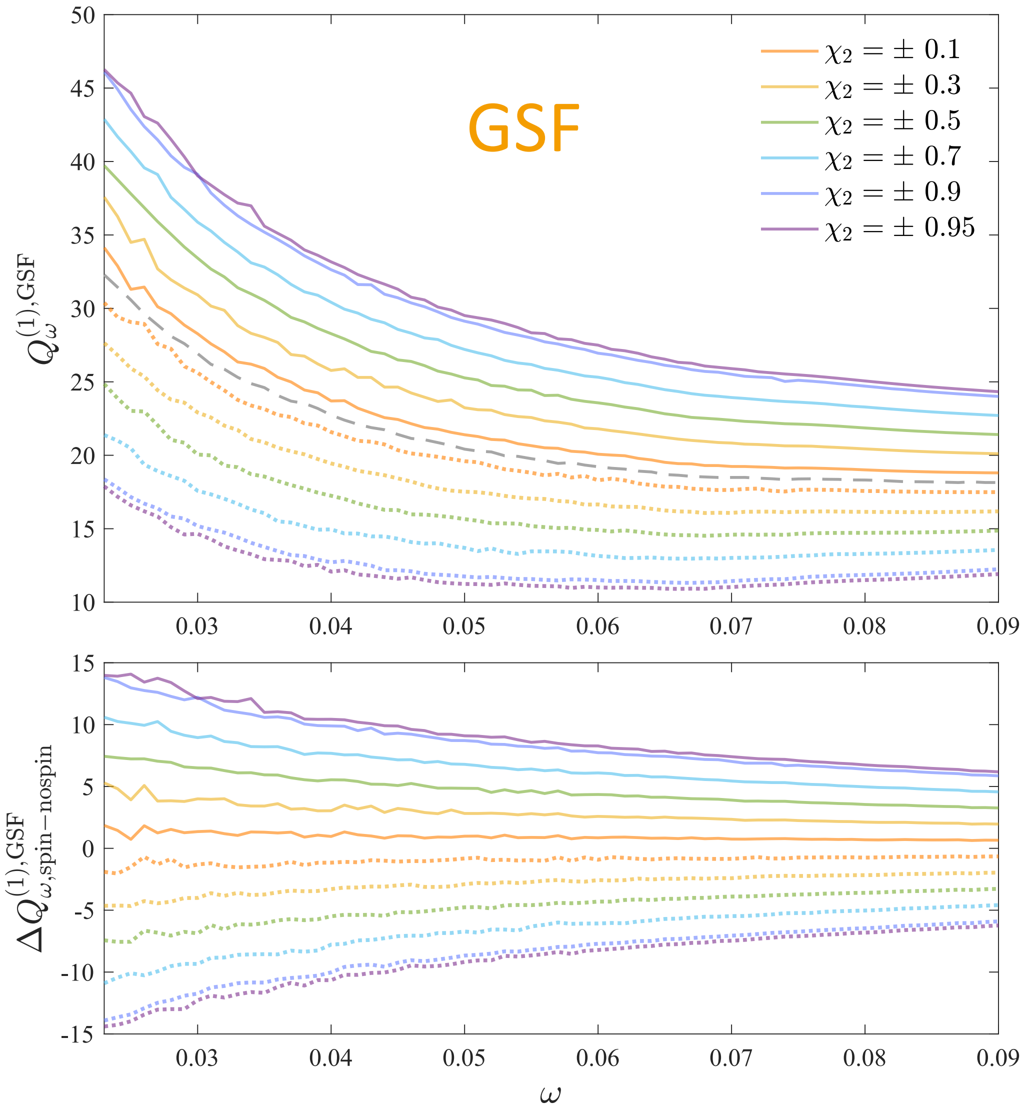
$Q_\omega^{(0)}$

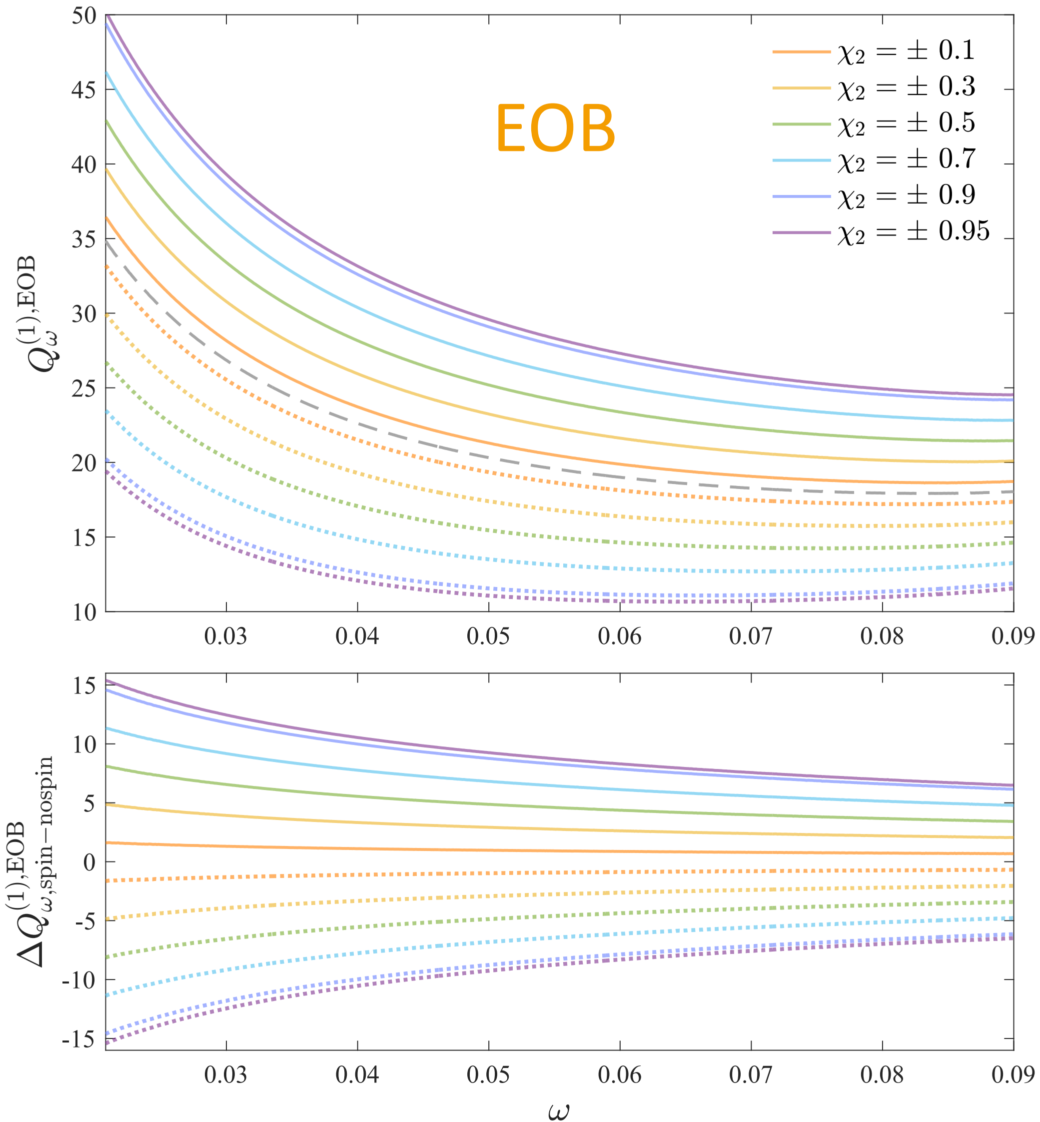
- $Q_\omega^{(0)}$ has no dependence on χ_2, but there is a small (numerical) residual





$Q_{\omega}^{(1)}$

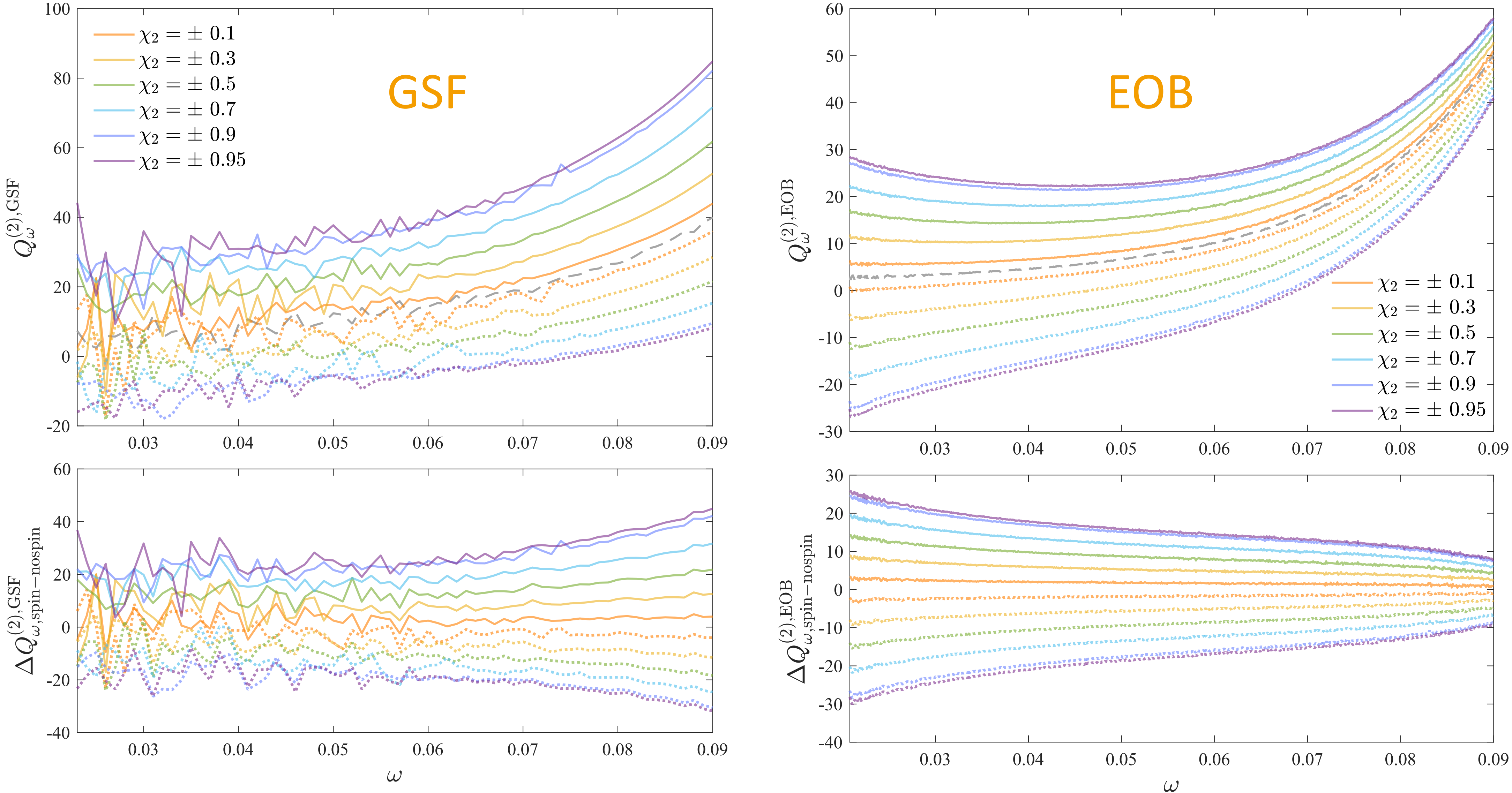

GSF
$\chi_2 = \pm\, 0.1$
$\chi_2 = \pm\, 0.3$
$\chi_2 = \pm\, 0.5$
$\chi_2 = \pm\, 0.7$
$\chi_2 = \pm\, 0.9$
$\chi_2 = \pm\, 0.95$
$Q_{\omega}^{(1),\mathrm{GSF}}$
$\Delta Q_{\omega,\mathrm{spin-nospin}}^{(1),\mathrm{GSF}}$
ω


EOB
$\chi_2 = \pm\, 0.1$
$\chi_2 = \pm\, 0.3$
$\chi_2 = \pm\, 0.5$
$\chi_2 = \pm\, 0.7$
$\chi_2 = \pm\, 0.9$
$\chi_2 = \pm\, 0.95$
$Q_{\omega}^{(1),\mathrm{EOB}}$
$\Delta Q_{\omega,\mathrm{spin-nospin}}^{(1),\mathrm{EOB}}$
ω

$Q_{\omega}^{(1)}$

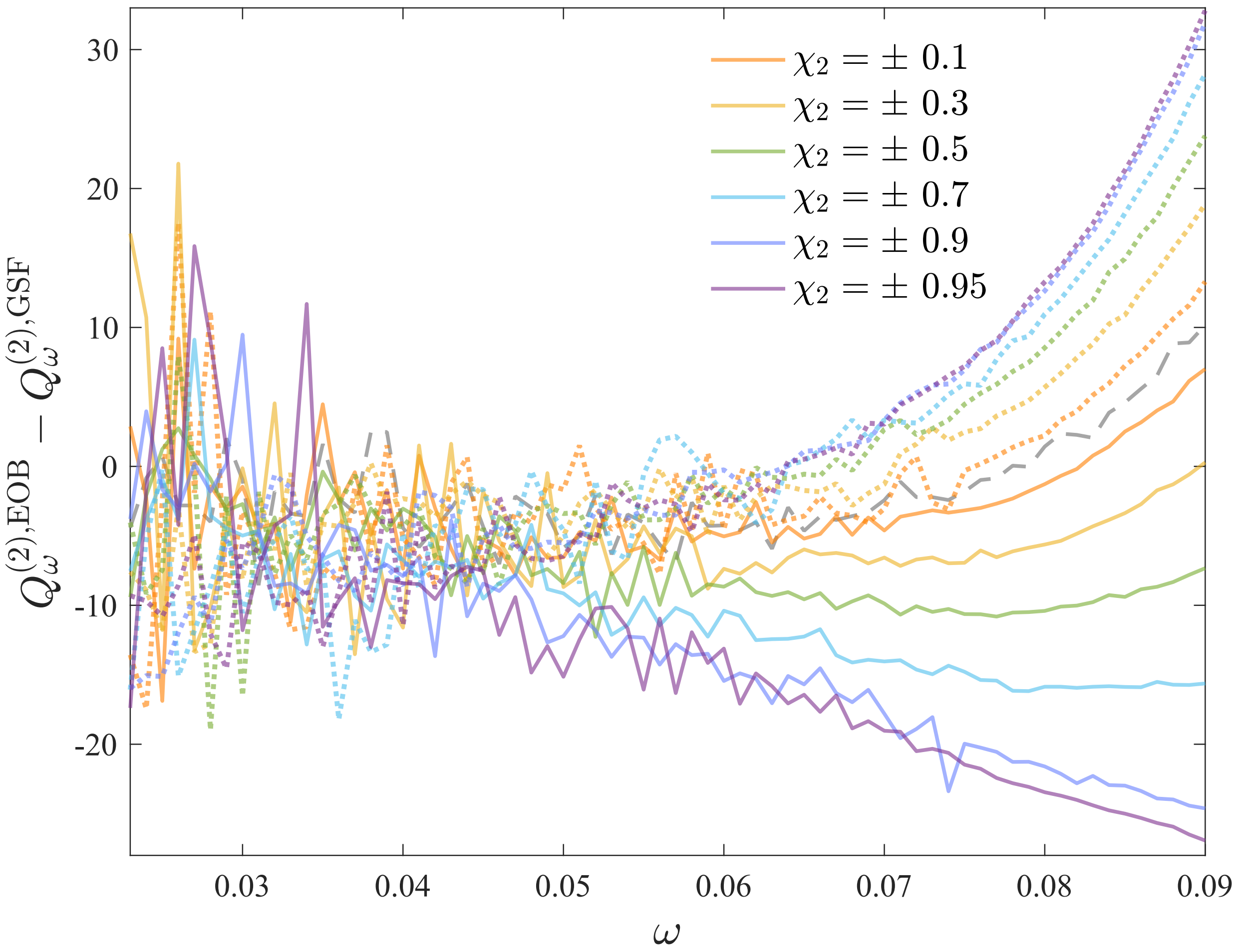
- Perfect symmetry between positive and negative values of the secondary spin since $Q_{\omega}^{(1)}$ is linear in χ_2
- From the physical point of view: the fact that positive χ_2 values have a larger $Q_{\omega}^{(1)}$ means the evolution is more adiabatic…
  A positive spin-orbit coupling yields a repulsive pull that delays the plunge (Nature behaving in the same way on many different scales!)

$Q_{\omega}{}^{(2)}$



GSF
EOB
$\chi_2 = \pm\,0.1$
$\chi_2 = \pm\,0.3$
$\chi_2 = \pm\,0.5$
$\chi_2 = \pm\,0.7$
$\chi_2 = \pm\,0.9$
$\chi_2 = \pm\,0.95$
$Q_{\omega}^{(2),\mathrm{GSF}}$
$Q_{\omega}^{(2),\mathrm{EOB}}$
$\Delta Q_{\omega,\mathrm{spin-nospin}}^{(2),\mathrm{GSF}}$
$\Delta Q_{\omega,\mathrm{spin-nospin}}^{(2),\mathrm{EOB}}$
ω
0.03
0.04
0.05
0.06
0.07
0.08
0.09

COMPARING $Q_\omega^{(2)}$: EOB - GSF



- Asymmetry due to nonlinear χ_2-dependence of $Q_{\omega}^{(2)}$
- The EOB $(\chi_2)^2$-dependence is smaller than the GSF one (less asymmetry in EOB $Q_{\omega}^{(2)}$) ➡ the EOB $Q_{\omega}^{(2)}$ is more/less adiabatic than the GSF one for negative/positive spins

INSIGHT INTO THE SPIN-ORBIT SECTOR

- Current version of the gyro-gravitomagnetic functions:

$$G_S = G_S^0 \hat{G}_S, \qquad \boxed{G_S^0 = 2uu_c^2}$$

$$G_{S_*} = G_{S_*}^0 \hat{G}_{S_*}, \qquad \boxed{G_{S_*}^0 = (3/2)u_c^2}$$

leading-order test-mass expressions are factorized out and the rest is (inverse) resummed

$$\hat{G}_S = \frac{1}{1 + c_{10}u_c + c_{20}u_c^2 + \cancel{c_{30}}u_c^3 + c_{02}p_{r*}^2 + c_{12}u_c p_{r*}^2 + c_{04}p_{r*}^4}$$

$$\hat{G}_{S_*} = \frac{1}{1 + c_{10}^* u_c + c_{20}^* u_c^2 + c_{30}^* u_c^3 + c_{40}^* u_c^4 + c_{02}^* p_{r*}^2 + c_{12}^* u_c p_{r*}^2 + c_{04}^* p_{r*}^4}$$

- All c_{ij}/c^*_{ij} coefficients depend on ν except from c30* and c40* (test-mass terms coming from the expansion of the exact G_{S*} of a spinning particle on Schwarzschild)

DIFFERENT CHOICE FOR G_{S*}: ANTI-DJS GAUGE

Different version (obtained in "anti-Damour-Jaranowski-Schäfer" gauge):

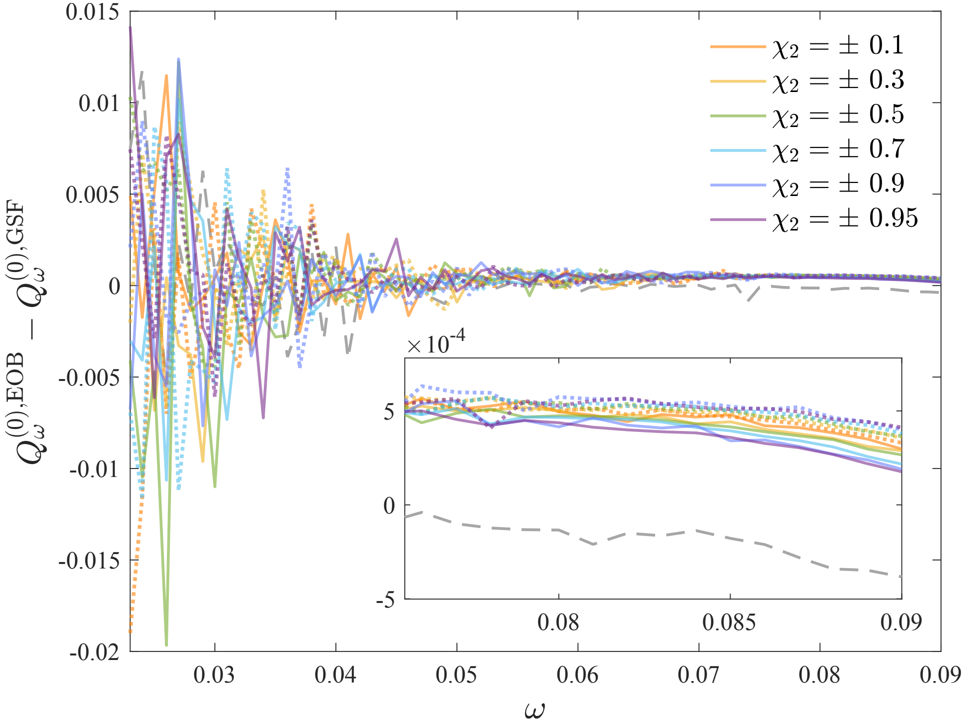
$$G_{S_*}^K = \frac{1}{(r_c^K)^2}\left\{ \frac{\sqrt{A^K}}{\sqrt{Q^K}}\left[1-\frac{(r_c^K)'}{\sqrt{B^K}}\right]+ \right.$$
$$\left. + \frac{r_c^K}{2\left(1+\sqrt{Q^K}\right)}\frac{(A^K)'}{\sqrt{A^K B^K}}\right\}, \qquad (15)$$

(see arXiv:1911.10818v2)

- The factored-out G^0_{S*} is now formally equal as the complete spinning-particle expression BUT all the Kerr functions (r_c, A, B, Q) are replaced with the EOB ν-dependent ones
- The residual functions are again resummed with their inverse Taylor representation

WORK IN PROGRESS…

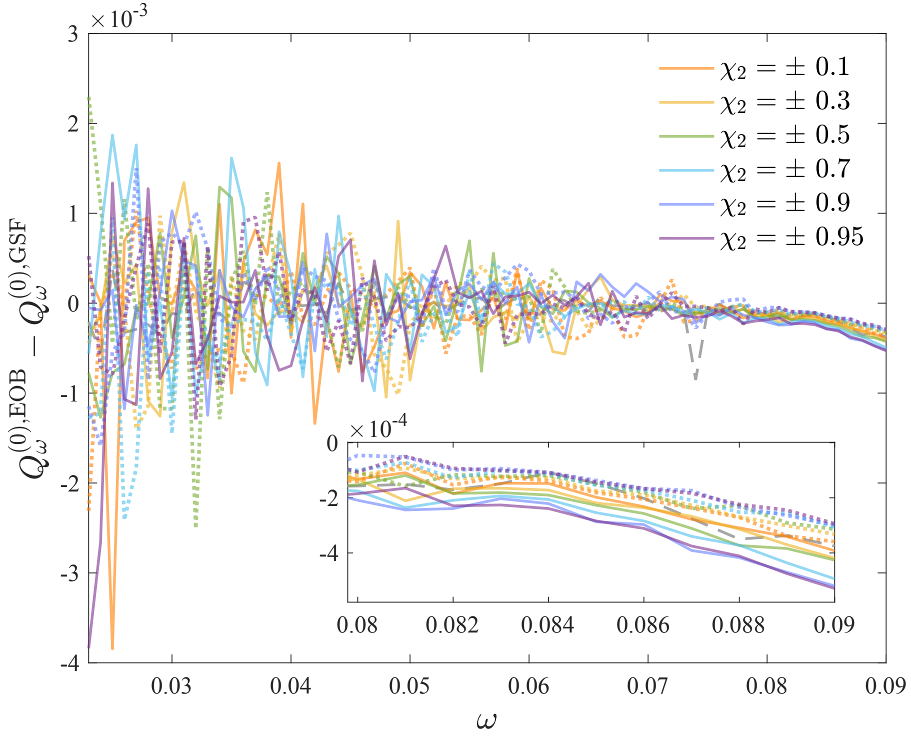
χ_2	$\Delta\phi^{\mathrm{EOBGSF}}_{\mathrm{DJS}}$	$\Delta\phi^{\mathrm{EOBGSF}}_{\mathrm{antiDJS}}$
0.5	−0.1037	−0.0886
−0.5	0.0477	0.0338
0.9	−0.1434	−0.0955
−0.9	0.1137	0.0648



Time-domain accumulated dephasings for q = 500 I got some time ago, before realising the spinning branch of the code was running without 21 contribution in the horizon flux

$\Delta Q_{\omega}^{(0)}$ (EOB-GSF) was different between spin and nonspin!

“THE BLANKET IS SHORT”



adding the 21 contribution into the horizon flux allows for a consistent result in $Q_\omega^{(0)}$ …

χ_2	$\Delta\phi^{\mathrm{EOBGSF}}_{\mathrm{DJS}}$	$\Delta\phi^{\mathrm{EOBGSF}}_{\mathrm{antiDJS}}$
0.5	0.18843	0.20739
−0.5	0.34057	0.32179
0.9	0.1499	0.19796
−0.9	0.42221	0.37256

… but changes all the dephasings! The new G_{S*} seems to improve only negative spins

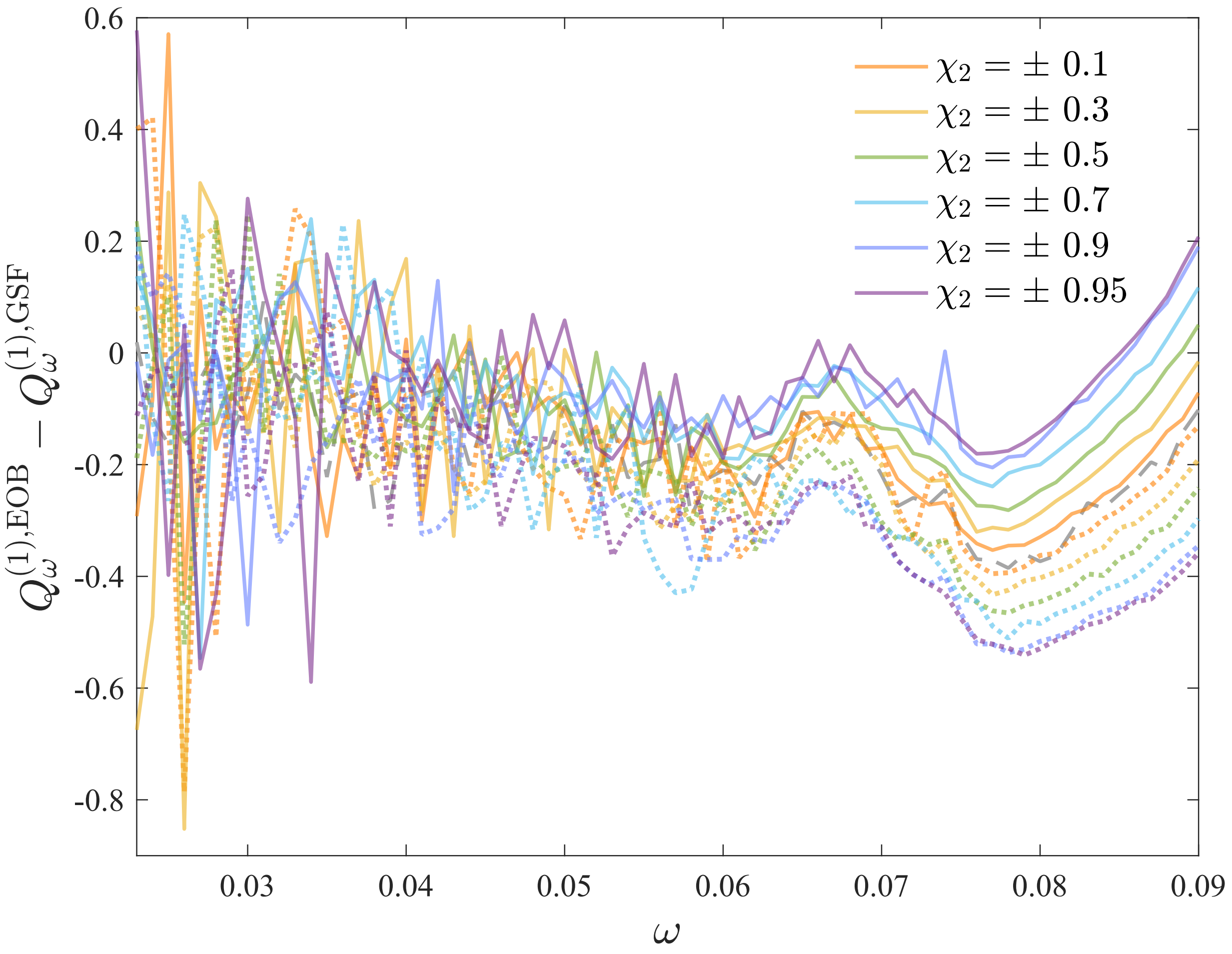
Will probably need to improve the horizon flux for spinning binaries!

CONCLUSIONS AND FUTURE WORK

- Nonspinning EOB model good for q = 50 000
- Spinning secondary: good EOB/GSF agreement in $Q_{\omega}^{(0)}$ and $Q_{\omega}^{(1)}$, but adding more spinning-particle analytical information into the conservative sector doesn't help if we don't improve the flux as well
- On a different note: could choose better integration scheme + will consider some speed-up technique at some point (ML?)
- Everything EMRI evolution needs: eccentricity, precession, environment, resonances… most effects are easy to be incorporated in EOB, but we can only improve by interfacing with others!

BACKSLIDES

COMPARING $Q_\omega^{(1)}$ (EOB - GSF)

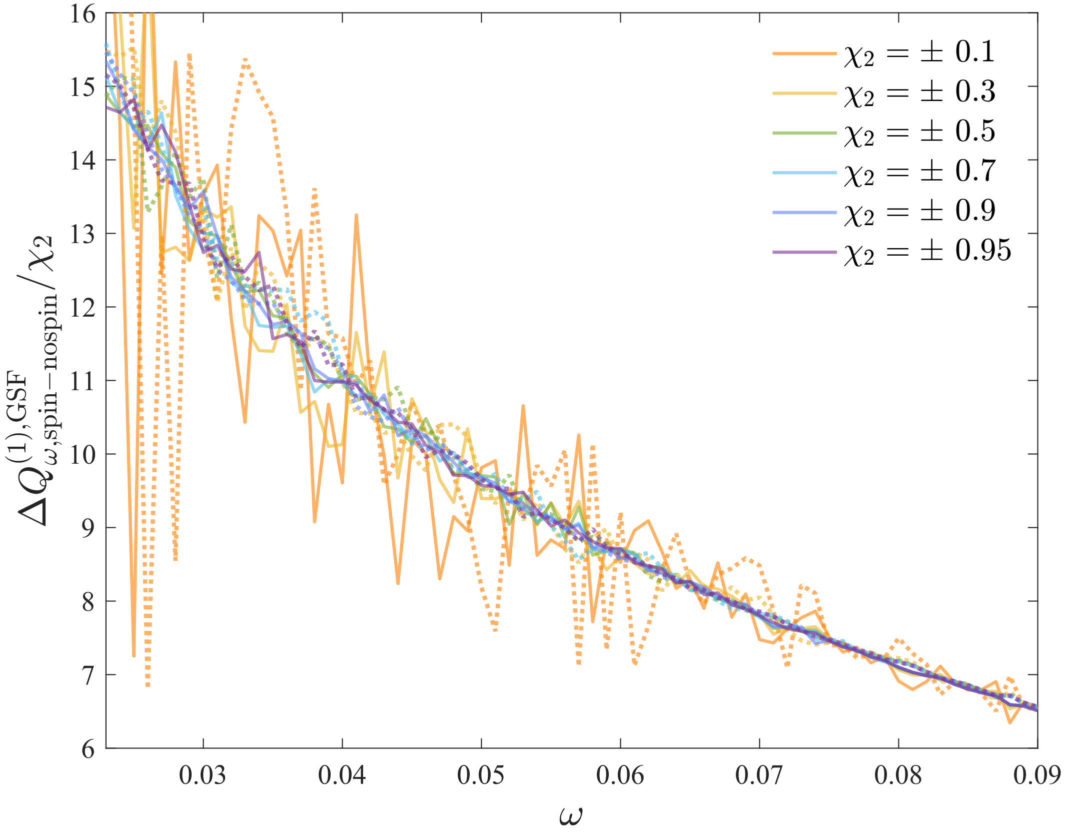


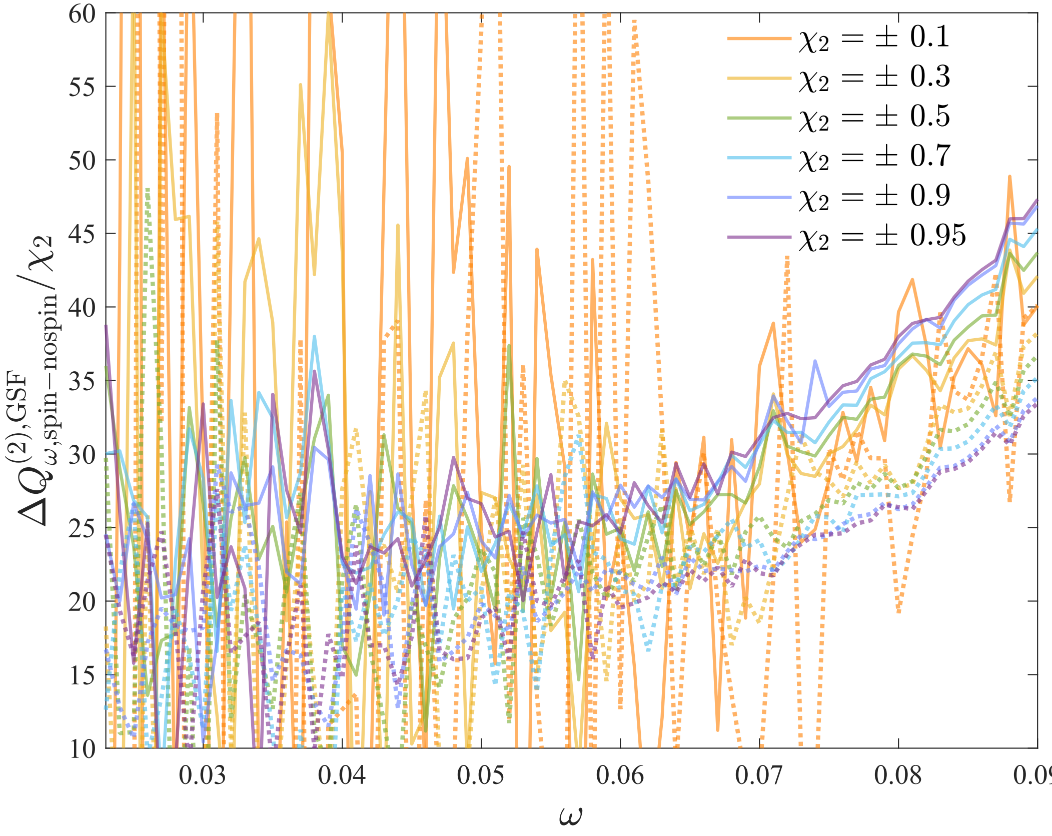
- The difference is mostly negative towards the end of the evolution (EOB contribution less adiabatic than the GSF one)

A

GSF: SPIN - NONSPIN

- As expected, $Q_{\omega}^{(1)}$ depends linearly on χ_2, $Q_{\omega}^{(2)}$ doesn't

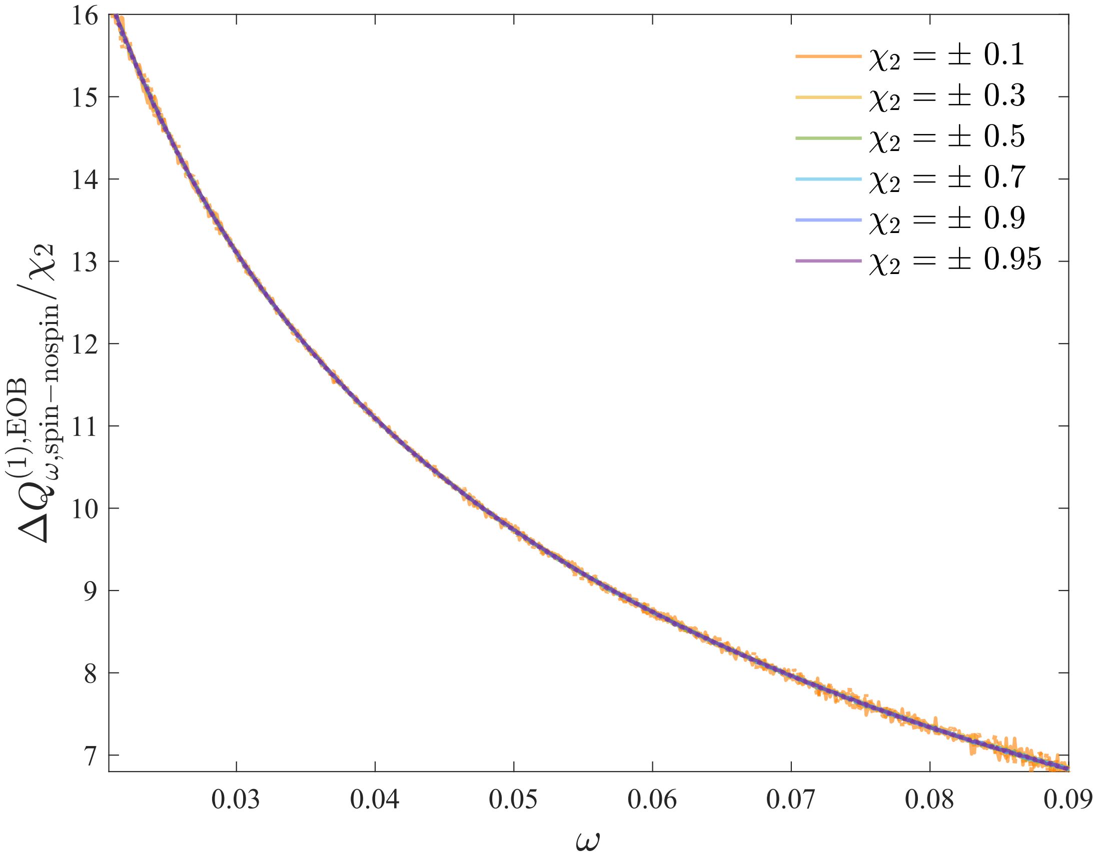


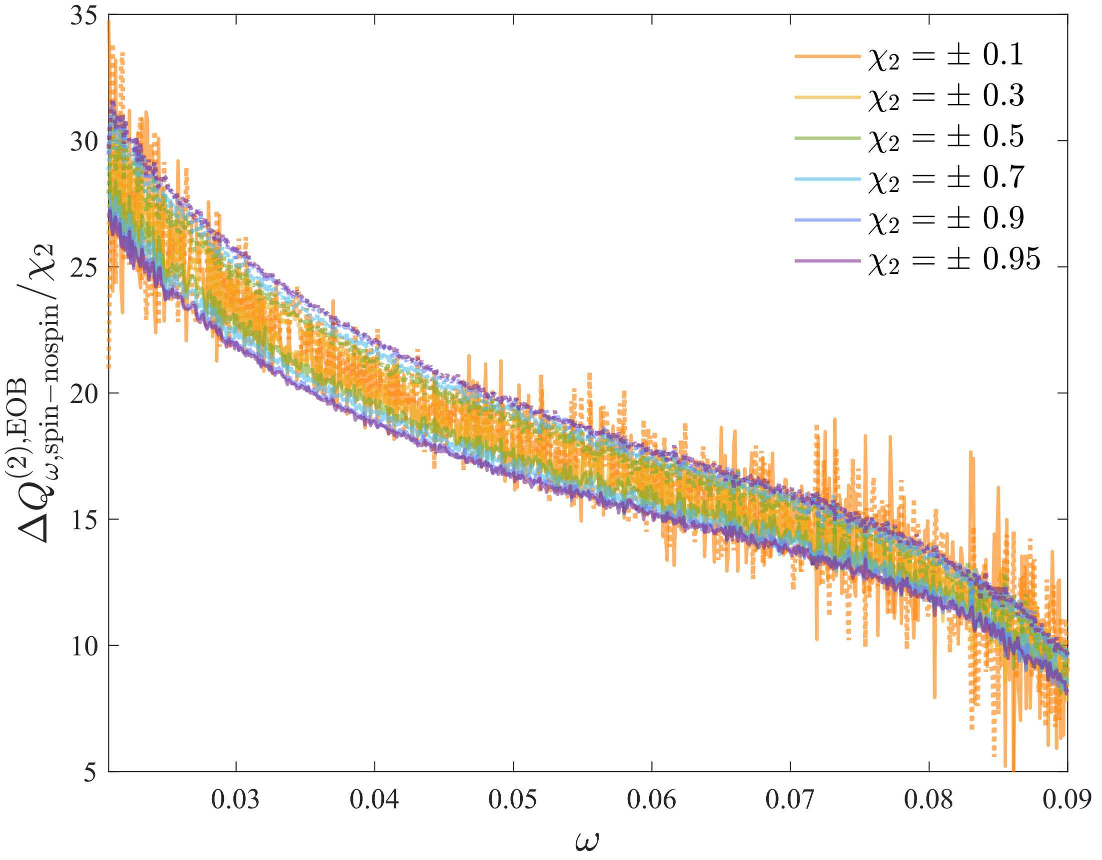


B

EOB: SPIN - NONSPIN

- As expected, $Q_{\boldsymbol{\omega}}^{(1)}$ depends linearly on χ_2, $Q_{\boldsymbol{\omega}}^{(2)}$ doesn't





C