Modeling the ringdown of binary black hole mergers

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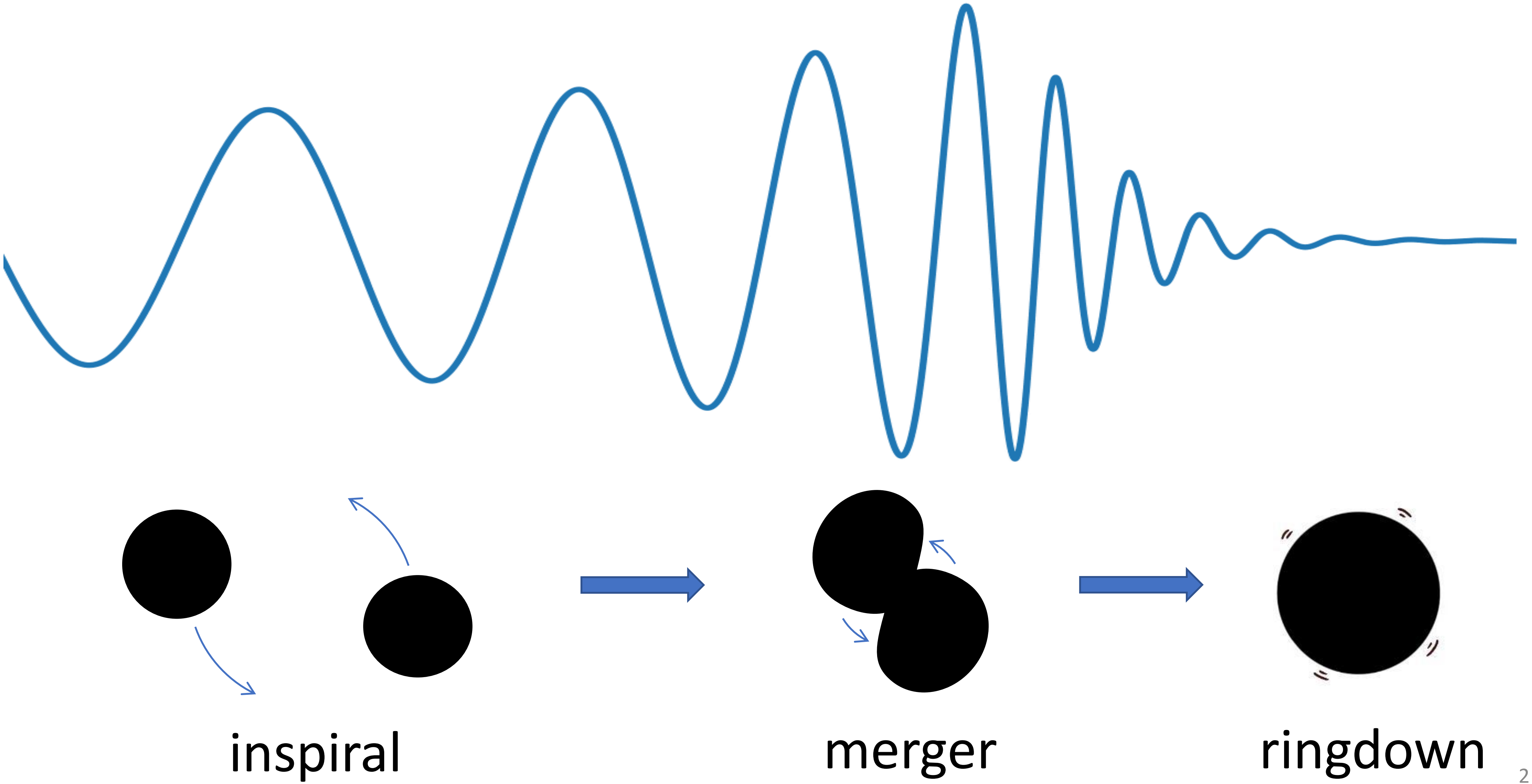
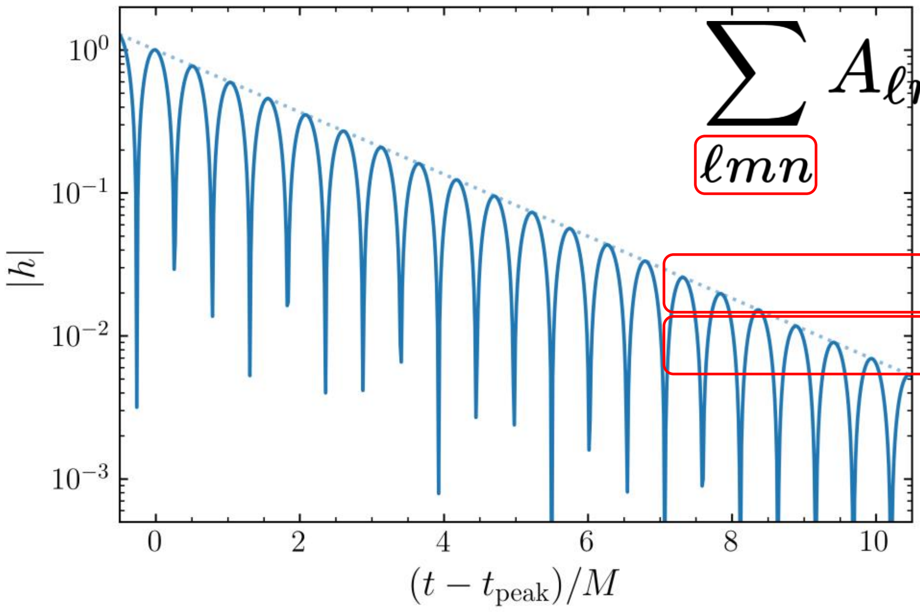
Gravitational waves



Quasinormal modes

The ringdown can be modeled by

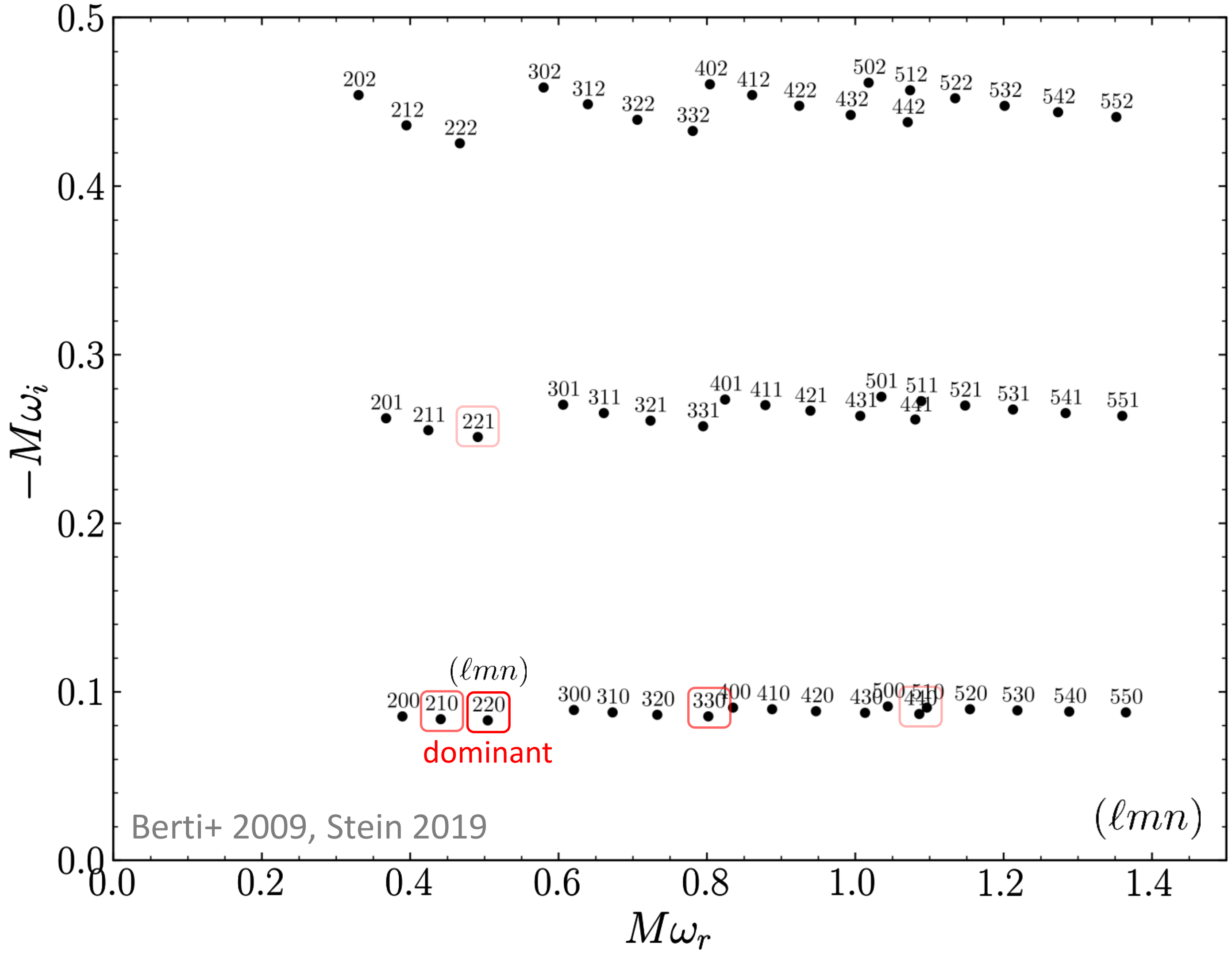


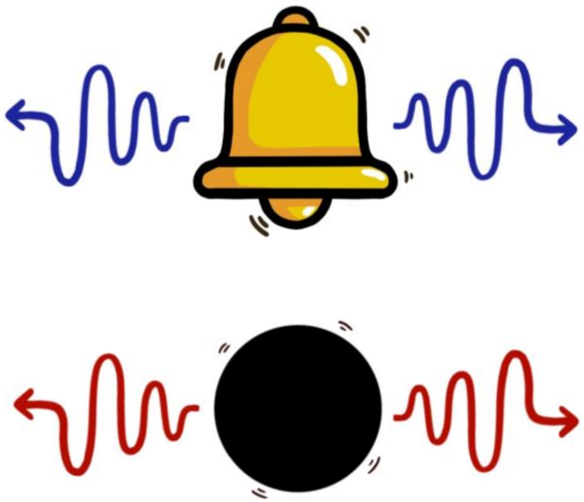
$$\sum_{\ell mn} A_{\ell mn} e^{-i(\tilde{\omega}_{\ell mn} t + \phi_{\ell mn})} \quad \sum_{\ell mn} A_{\ell mn} e^{\omega_{\ell mn,i} t} e^{-i(\omega_{\ell mn,r} t + \phi_{\ell mn})}$$

number

plex) quasinormal-mode frequencies

Quasinormal-mode frequencies





Modeling the ringdown

Depends only on the remnant

$$\sum_{\ell m n} A_{\ell m n} e^{-i(\tilde{\omega}_{\ell m n} t + \phi_{\ell m n})}$$

Depends on initial conditions

Initial conditions:

Mass ratio q, spin of the initial black holes χ_1 and χ_2, …

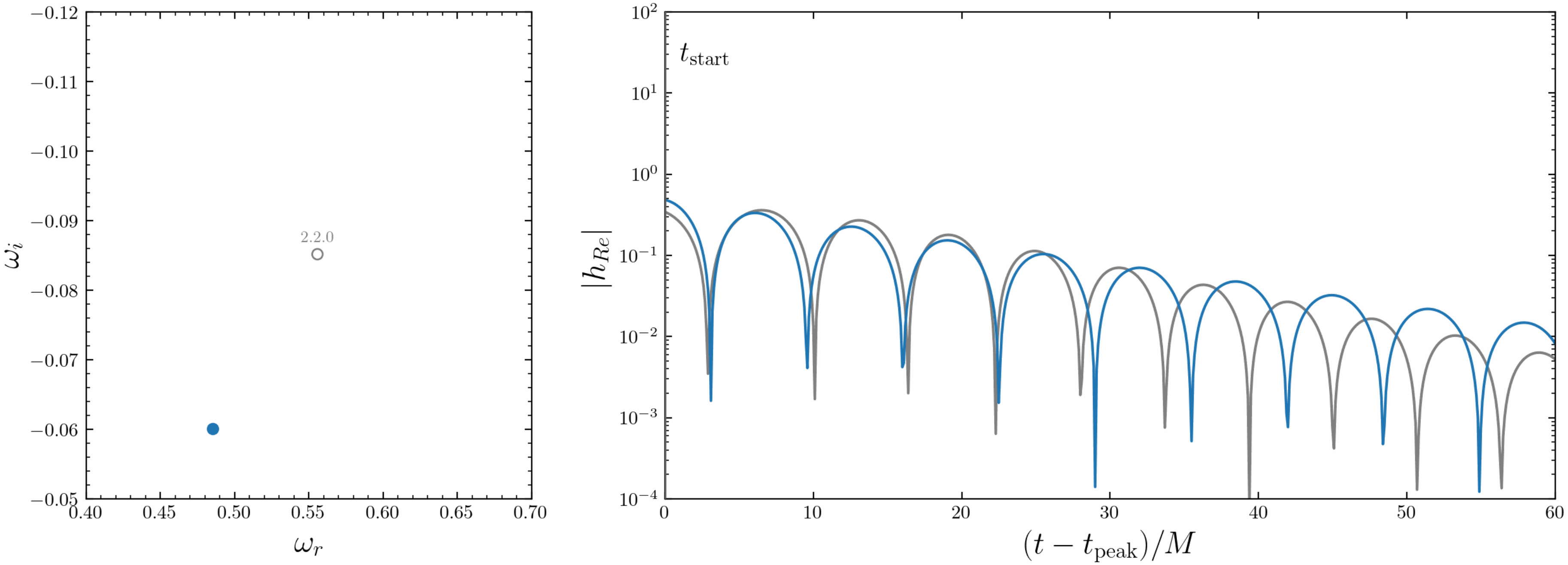
(Models already exist)

Non-spinning progenitors: Kamaretsos+ 1112.3077, London+ 1404.3197, …

Spinning progenitors: Meidam+ 1406.3201 , Baibhav+ 1710.02156, London 1801.08208, Zertuche+ 2110.15922, …

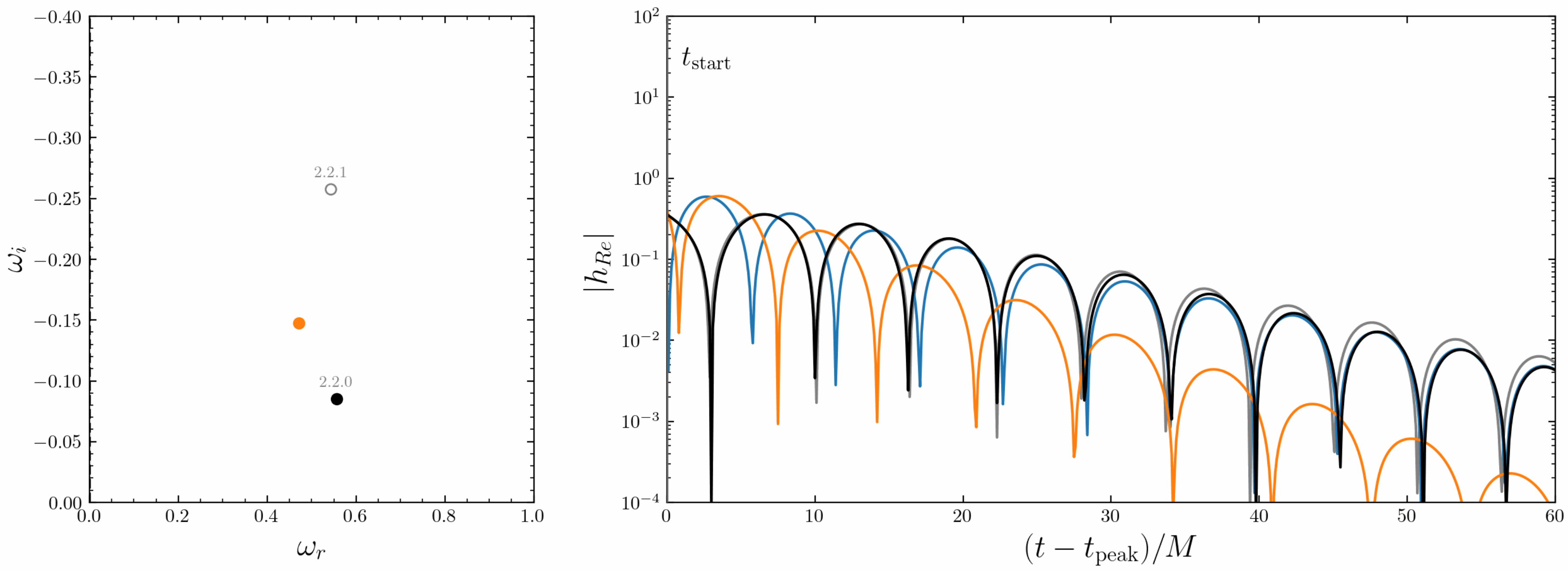
What modes exist in the ringdown?

$$\sum_{\ell mn} A_{\ell mn} e^{-i(\tilde{\omega}_{\ell mn} t + \phi_{\ell mn})}$$



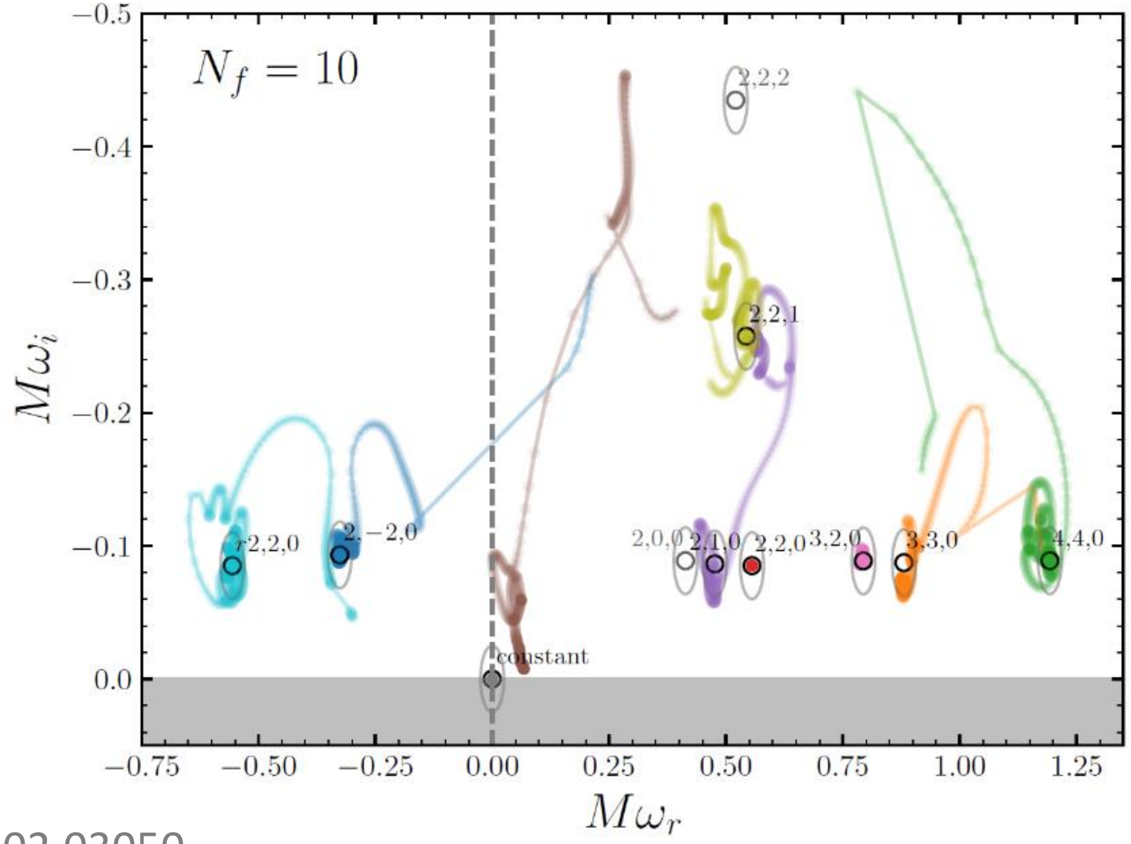
What modes exist in the ringdown?

$$\sum_{\ell mn} A_{\ell mn} e^{-i(\tilde{\omega}_{\ell mn} t + \phi_{\ell mn})}$$



What modes exist in the ringdown?

GW150914-like simulation, $\ell m = 22$ harmonic waveform

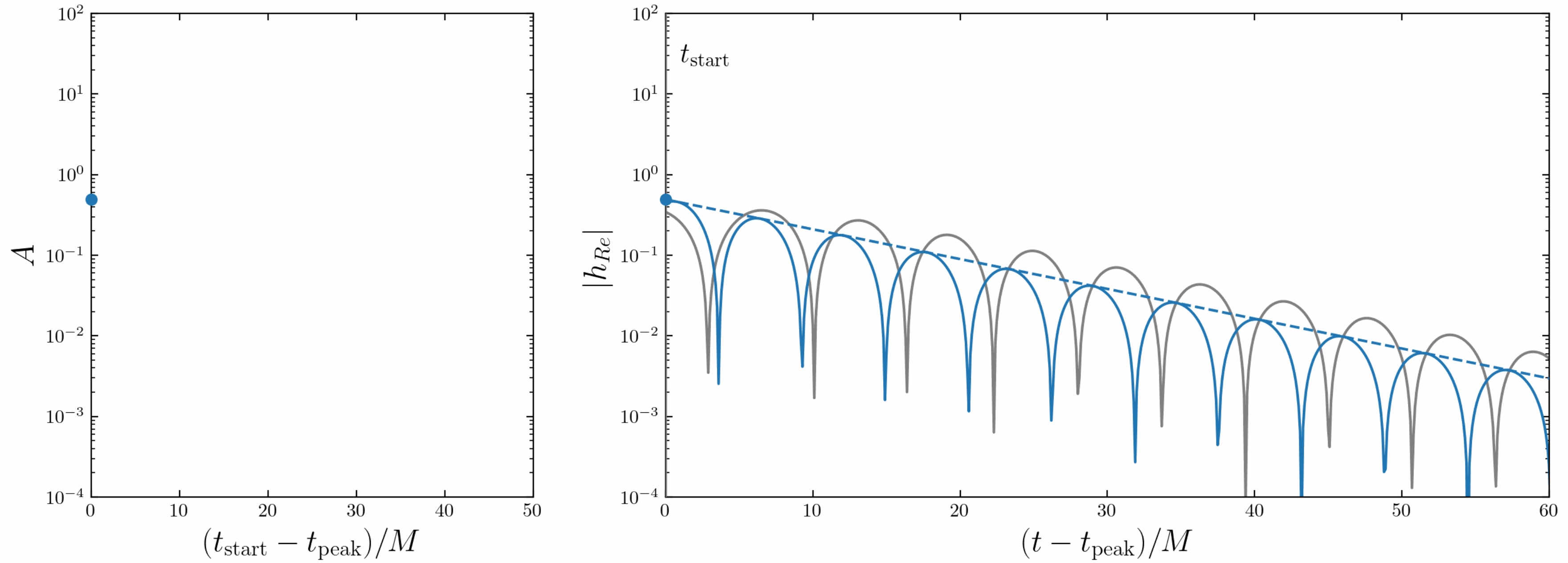


See also: Baibhav, Cheung+ 2302.03050

What are the amplitude of the modes?

$$A e^{-i(\omega t + \phi)}$$

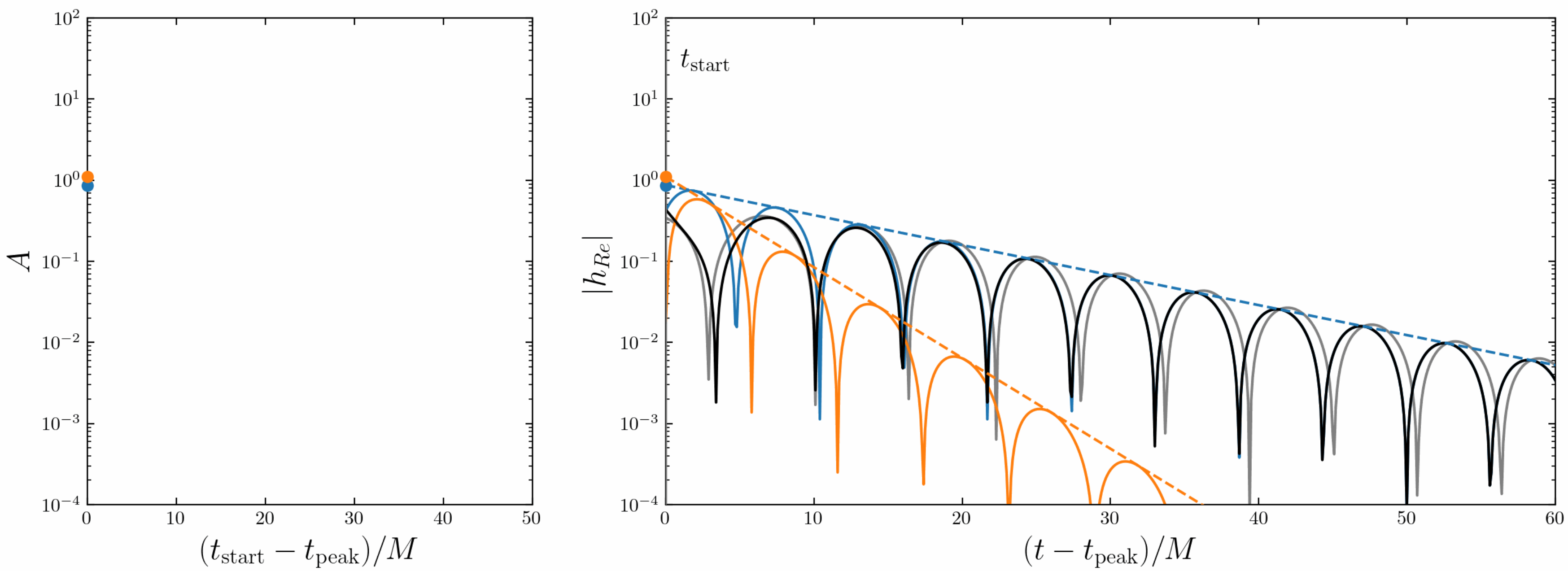
constant in time



What are the amplitude of the modes?

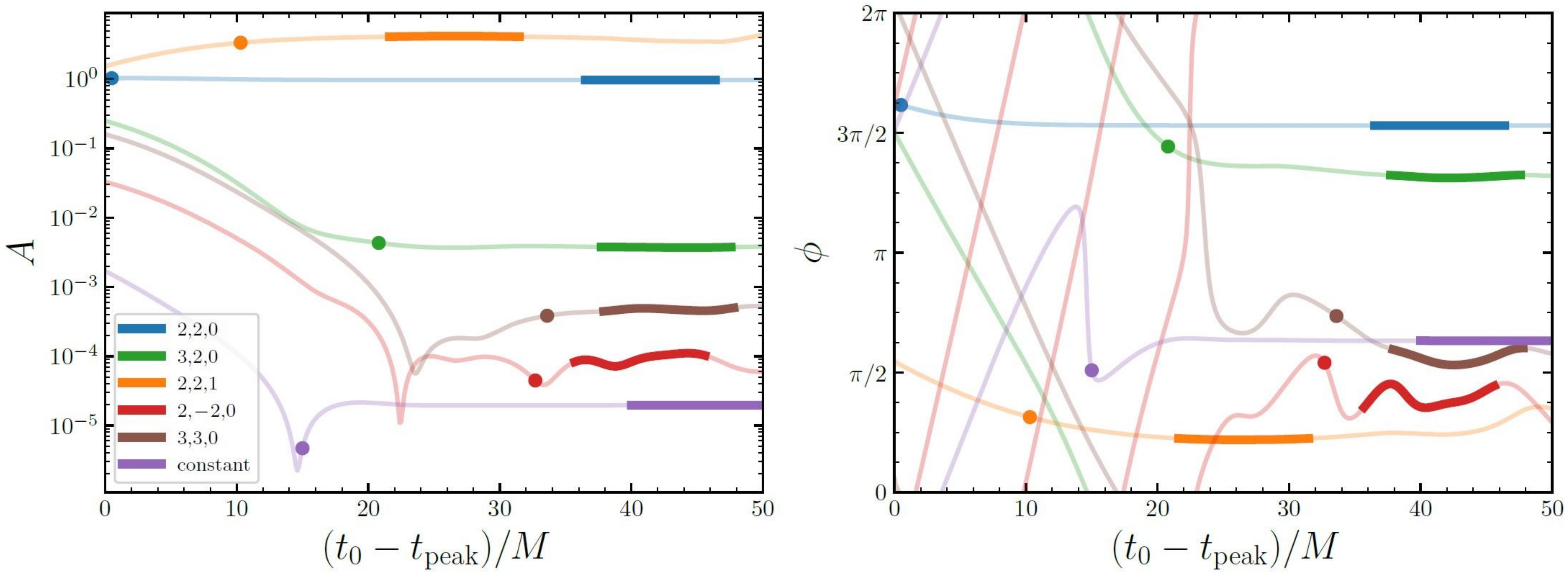
$$\boxed{A}e^{-i(\omega t+\phi)}$$

constant in time



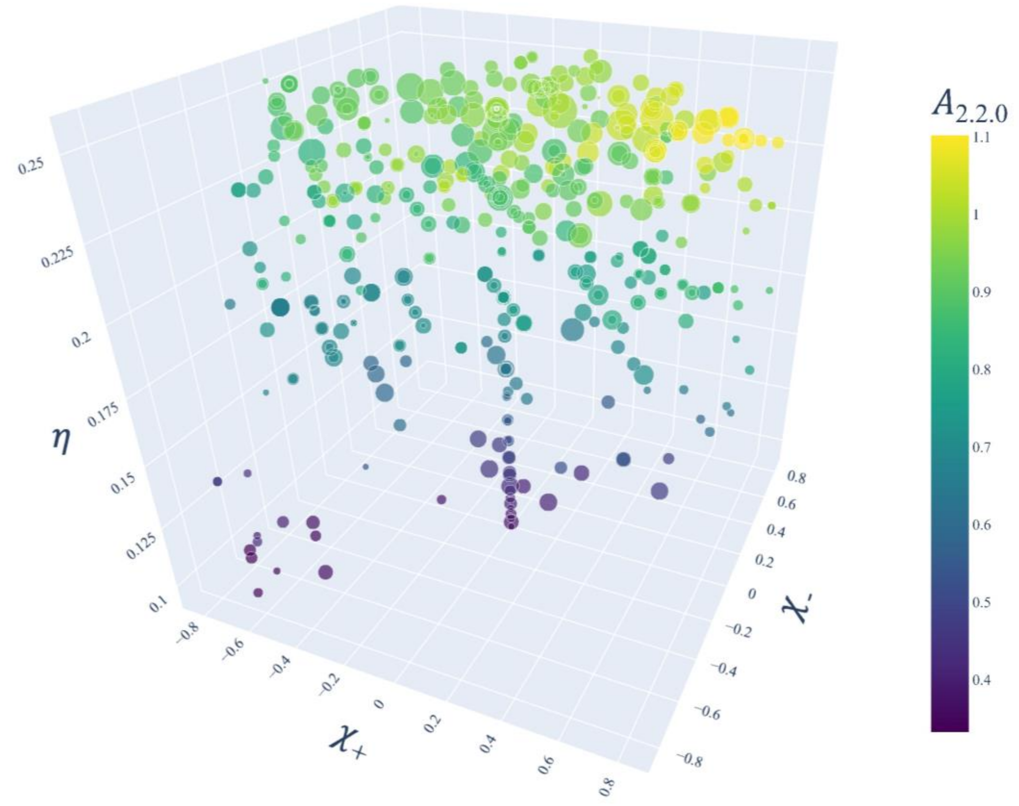
Extracting A and ϕ

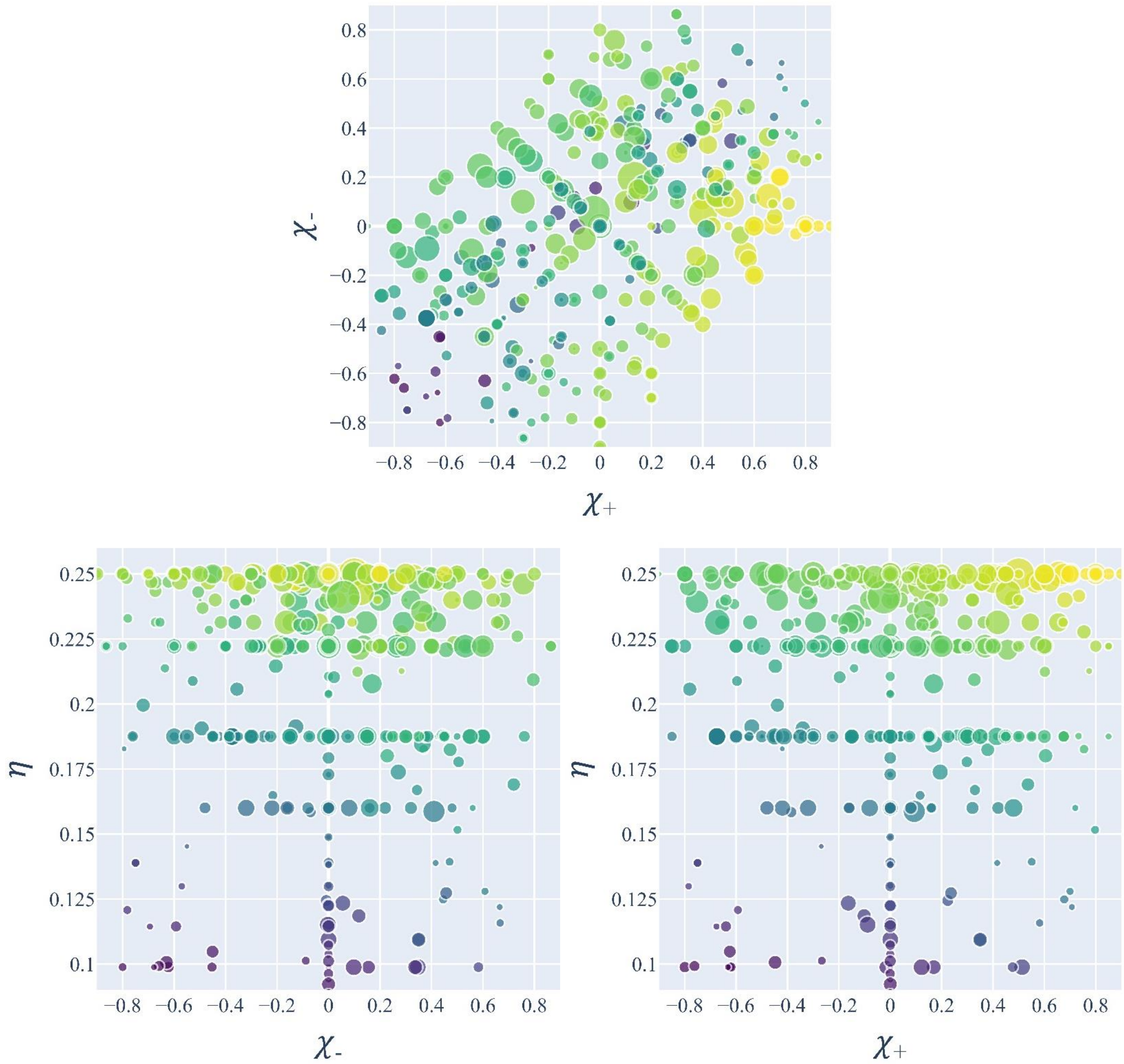
Bolded lines: "flat" region



(Not a new idea: Lim+ 1901.05902)

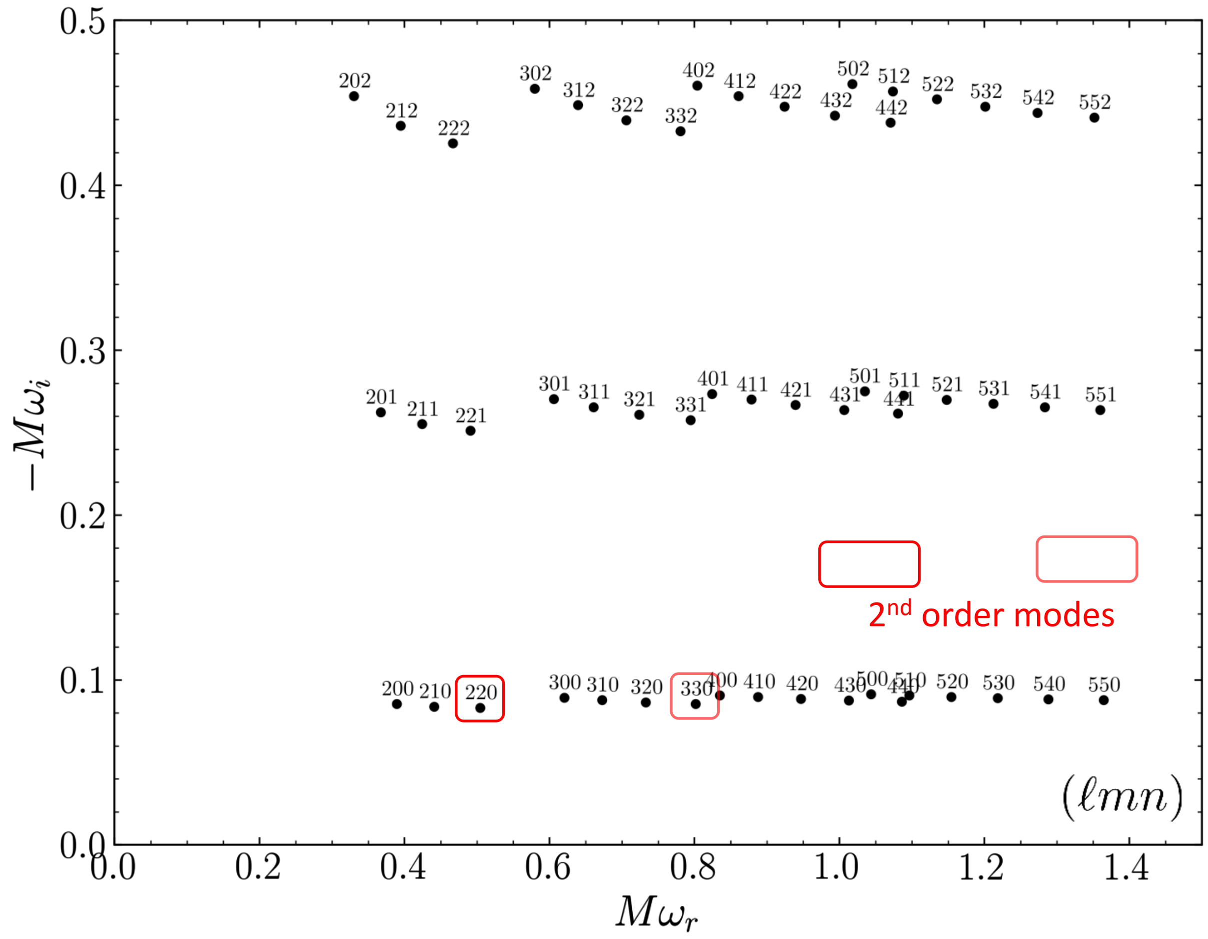
$A_{lmn}(\eta, \chi_+, \chi_-)$





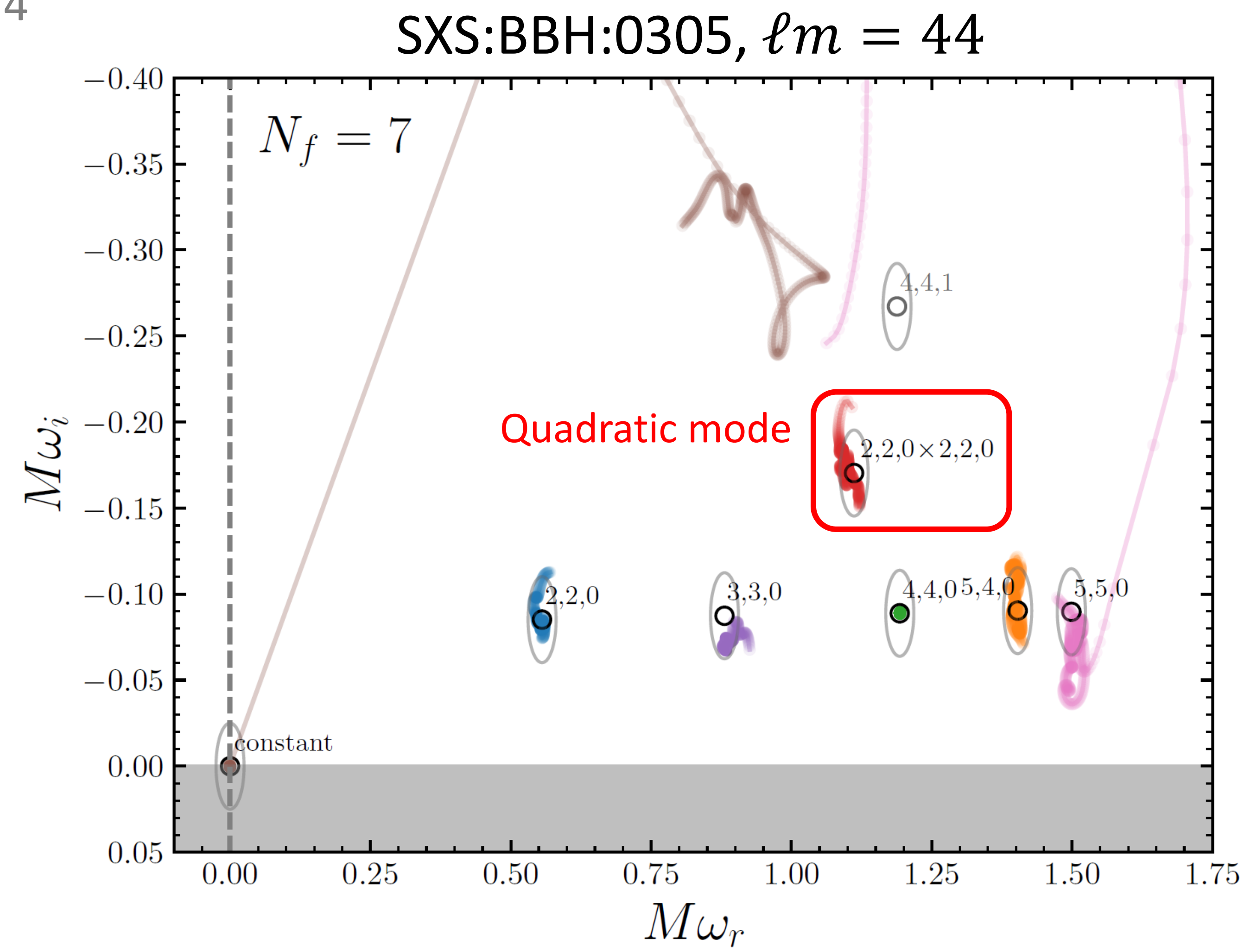
$$\tilde{A}_{2,2,0} = 3.995 + 1.371\chi_- + 17.25\eta\chi_+ - 11.48\eta\chi_- + 2.346\chi_+\chi_- - 5.001\eta^3 - 101.3\eta^2\chi_+ + 24\eta^2\chi_- + 5.95\eta\chi_+^2 - 23.72\eta\chi_+\chi_- + 167.7\eta^3\chi_+ - 19.68\eta^2\chi_+^2 + 56.16\eta^2\chi_+\chi_- - 0.3504\eta\chi_+^3 - 0.2335\chi_+^4$$

Quasinormal-mode frequencies



Quadratic modes

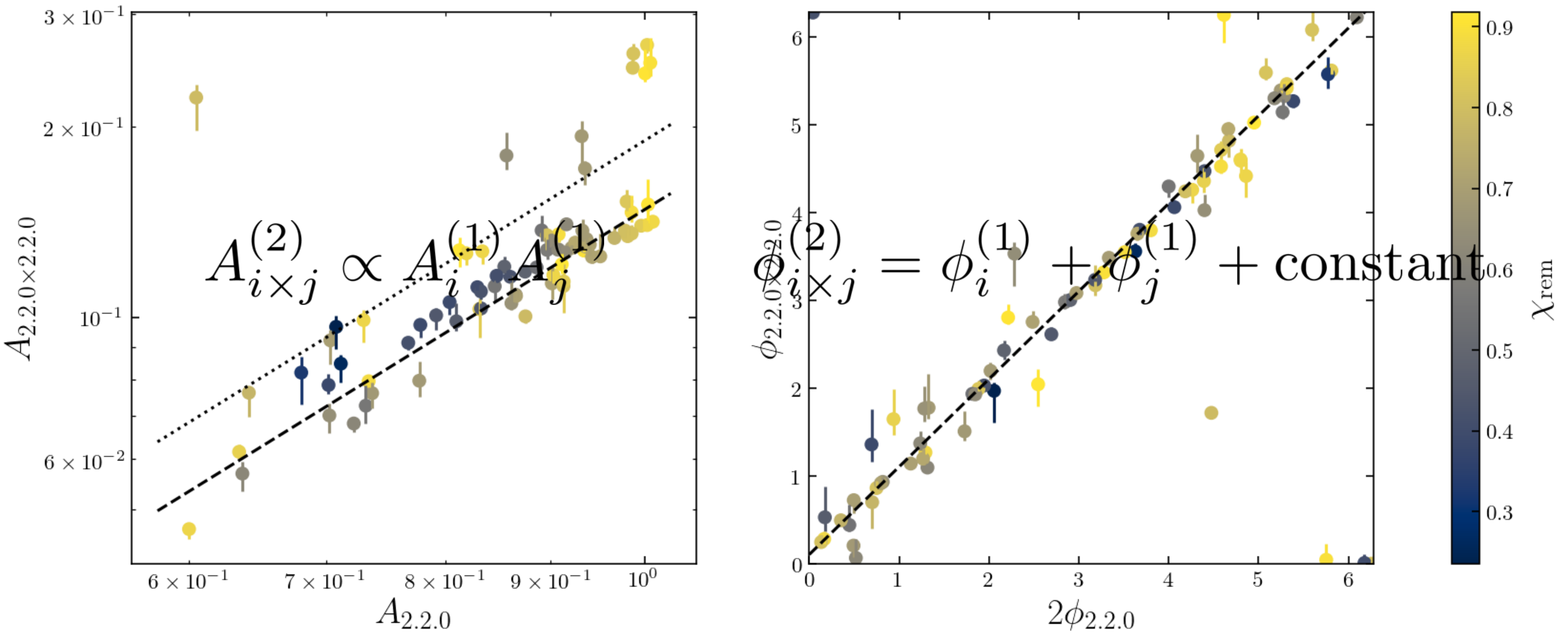
Cheung+ 2208.07374



Also found by: Ma+ 2207.10870, Mitman+ 2208.07380; Tentative evidence: London+ 1404.3197
Other nonlinearities: Sberna+ 2112.11168, Zertuche+ 2110.15922

Quadratic amplitude dependence

Cheung+ 2208.07374



$$A^{(2)}_{i\times j} \propto A^{(1)}_i A^{(1)}_j \qquad \phi^{(2)}_{i\times j} = \phi^{(1)}_i + \phi^{(1)}_j + \text{constant}$$

Dotted line: Extremal Kerr (Kehagias+ 2023)

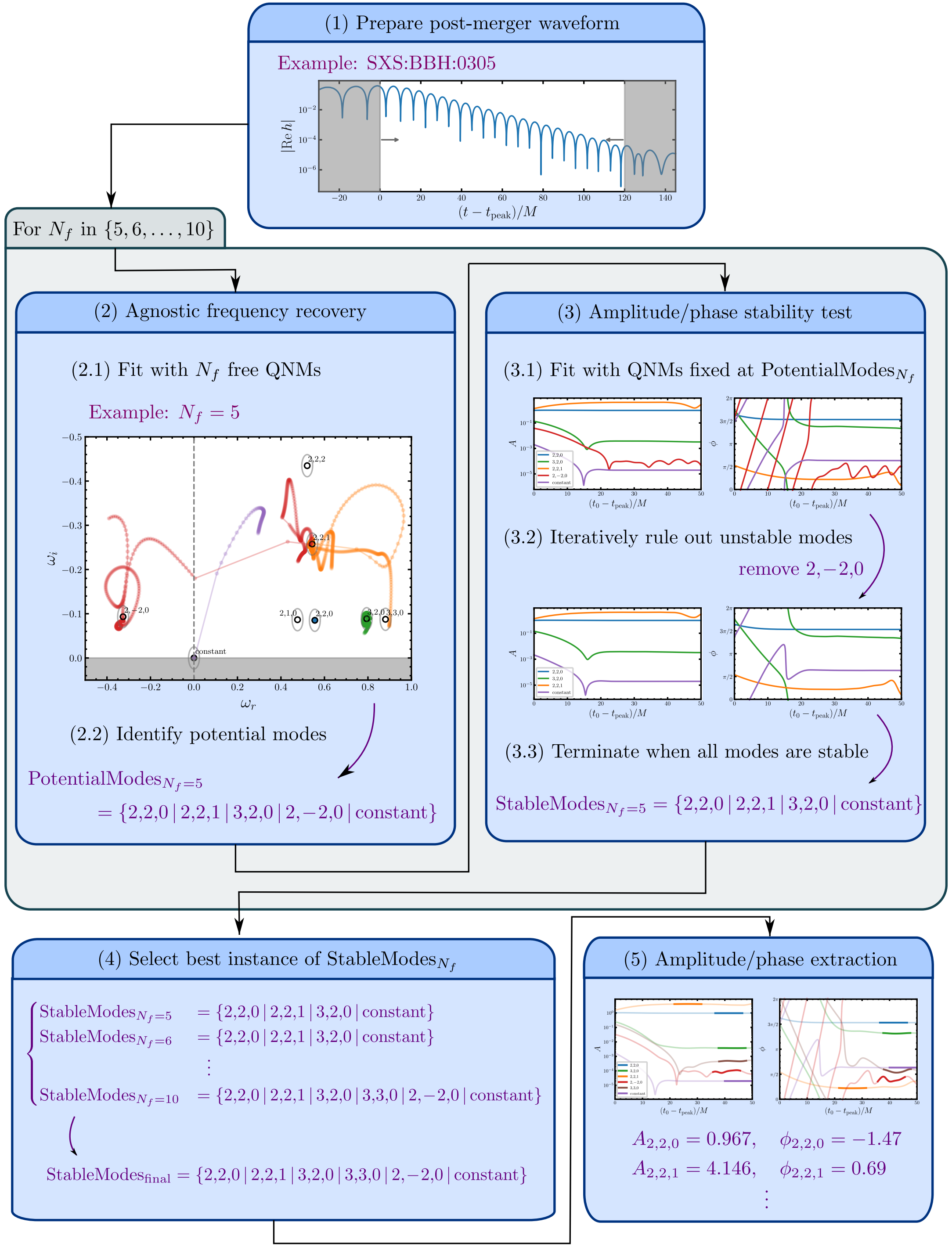
Dashed lines: Schwarzschild (Nakano+ 2007 & Lorenzo Pierini)

Takeaways

- A frequency-agnostic fit could help identify QNMs in the ringdown
- Fitting the ringdown over a varying time window can help test the robustness of our results
- Nonlinear QNMs are important for modeling
- A model of the amplitudes and phases of QNMs could be useful
  - for waveform modeling
  - for consistency checks (e.g. Forteza+ 2205.14910)

Thank you!

Back-up slides


(1) Prepare post-merger waveform
Example: SXS:BBH:0305
$|\mathrm{Re}\, h|$
$(t - t_{\mathrm{peak}})/M$
For N_f in $\{5, 6, \ldots, 10\}$
(2) Agnostic frequency recovery
(2.1) Fit with N_f free QNMs
Example: $N_f = 5$
ω_i
ω_r
2,2,2
2,2,1
2,−2,0
2,1,0
2,2,0
3,2,0
3,3,0
constant
(2.2) Identify potential modes
$\mathrm{PotentialModes}_{N_f=5}$
$= \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, 2{,}{-2}{,}0 \,|\, \mathrm{constant}\}$
(3) Amplitude/phase stability test
(3.1) Fit with QNMs fixed at $\mathrm{PotentialModes}_{N_f}$
$(t_0 - t_{\mathrm{peak}})/M$
(3.2) Iteratively rule out unstable modes
remove 2,−2,0
(3.3) Terminate when all modes are stable
$\mathrm{StableModes}_{N_f=5} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, \mathrm{constant}\}$
(4) Select best instance of $\mathrm{StableModes}_{N_f}$
$\mathrm{StableModes}_{N_f=5} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, \mathrm{constant}\}$
$\mathrm{StableModes}_{N_f=6} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, \mathrm{constant}\}$
⋮
$\mathrm{StableModes}_{N_f=10} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, 3{,}3{,}0 \,|\, 2{,}{-2}{,}0 \,|\, \mathrm{constant}\}$
$\mathrm{StableModes}_{\mathrm{final}} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, 3{,}3{,}0 \,|\, 2{,}{-2}{,}0 \,|\, \mathrm{constant}\}$
(5) Amplitude/phase extraction
$A_{2,2,0} = 0.967, \quad \phi_{2,2,0} = -1.47$
$A_{2,2,1} = 4.146, \quad \phi_{2,2,1} = 0.69$
⋮

Black Hole Perturbation theory (2nd order)

Same Teukolsky operator

$$\mathcal{T}[\Psi_4^{(2)}] = \mathcal{S}_4^{(2)}$$

$$\mathcal{S}_4^{(2)} \propto \left(\Psi_4^{(1)}\right)^2 = (A^{(1)})^2 e^{-i(2\omega^{(1)}t+2\phi^{(1)})}$$

$$\Psi_4^{(2)} = \Psi_{4,\mathrm{homogeneous}}^{(2)} + \Psi_{4,\mathrm{particular}}^{(2)}$$

$$\Psi_{4,\mathrm{homogeneous}}^{(2)} \sim \Psi_4^{(1)}$$

$$\Psi_{4,\mathrm{particular}}^{(2)} = A^{(2)} e^{-i(\omega^{(2)}t+\phi^{(2)})}$$

2nd order quasinormal mode

$$\omega^{(2)} = 2\omega^{(1)}$$

$$A^{(2)} \propto (A^{(1)})^2$$

$$\phi^{(2)} = 2\phi^{(1)} + \mathrm{constant}$$

general

$$\omega_{i\times j}^{(2)} = \omega_i^{(1)} + \omega_j^{(1)}$$

$$A_{i\times j}^{(2)} \propto A_i^{(1)} A_j^{(1)}$$

$$\phi_{i\times j}^{(2)} = \phi_i^{(1)} + \phi_j^{(1)} + \mathrm{constant}$$

Ioka+ 2007, Nakano+ 2007, Pazos+ 2010, London+ 2014

Why do we care about nonlinearities?

- Nonlinearities are important for modeling the ringdown
- New way to test general relativity
    - Do nonlinear modes exist in detected gravitational waves?
        - Require next generation detectors
    - If they are identified, do their amplitudes / phases follow the expected relationships?
- Some exciting new work:
    - Kehagias+ 2301.09345 (Kerr/CFT correspondence)
    - Guerreiro+ 2306.09974 (probing the quantum nature of gravity)
    - Khera+ 2306.11142 (nonlinearities at the BH horizon)
    - Many more!

$$A_{i\times j} = c_{A,i\times j} A_i A_j$$

$$\phi_{i\times j} = \phi_i + \phi_j + c_{\phi,i\times j}$$

might depend on the theory of gravity