Extended-body motion, local symmetries, and Petrov types

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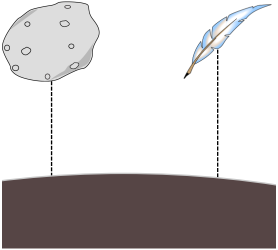
Different things can fall differently



How differently?

Ways to understand extended-body effects

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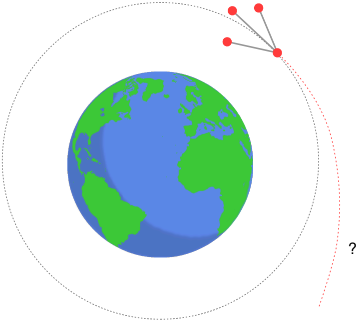
1. Decide on an interesting system, model its internal structure, and solve the resulting equations of motion.

2. Figure out what is *impossible*, regardless of model.

What is allowed by the laws of physics and what is not?

Rocket-free spacecraft



1. Can an appropriate spacecraft maneuver without a rocket?
2. Are some forces or torques impossible?

What is (im)possible?

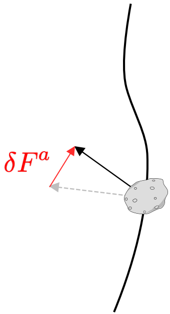
That which is not forbidden is allowed. . .

. . . and things are forbidden mainly by symmetries.

Are all forces and torques possible?

Some possibilities are forbidden by **Killing fields**.

$$\xi^a F_a + \frac{1}{2} \nabla_a \xi_b N^{ab} = 0.$$



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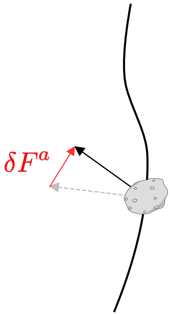

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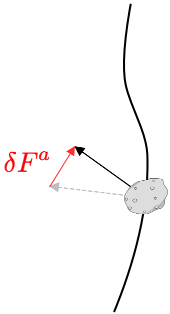

Are these the only constraints?

No! Enlarge your notion of symmetry.

Local symmetries

There are always 10 generalized Killing fields (GKFs) satisfying

$$\mathcal{L}_\xi g_{ab}(z_s) = \nabla_c \mathcal{L}_\xi g_{ab}(z_s) = 0.$$

Force and torque constraints need only a 1-parameter family of GKFs ξ_s^a s.t.

$$\nabla_c \nabla_d \mathcal{L}_{\xi_s} g_{ab}(z_s) = 0 \quad \Leftrightarrow \quad \boxed{\mathcal{L}_{\xi_s} R_{abcd}(z_s) = 0.}$$

Symmetry only necessary at the object's location. Different vector fields can be used at different times!

Are local symmetries related to other symmetries?

There's a local symmetry for every...

1. ... Killing field.
2. ... conformal Killing-Yano tensor $f_{ab} = f_{[ab]}$. These generate approximate Lorentz transformations.

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But there's much more than this. Local symmetries are common!

Constraining extended-body motion

For every local symmetry,

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1. Some torques are impossible.
2. Some forces are possible only with "compensating" torques.
3. Some internal structure can't affect motion.

Some forces possible only at the cost of torques

$$F_a = \underbrace{(\ldots)_a{}^{bc}}_{\text{No control}} N_{bc} + \underbrace{\mathrm{Re} \sum_I J_I \nabla_a \Psi_I}_{\text{Controllable}}$$

How much can the force be controlled independently of torque?

Some internal structure doesn't matter

In GR, up to 10 quadrupole components (5 mass + 5 current) can affect motion [but more in other theories of gravity!]

How many quadrupole *actually* matter?

Local symmetries etc. largely determined by Petrov type.

Petrov type	Quad. comps.	$\dim\{N_{ab}\}$	$\dim\{F_a \vert N_{bc}\}$
N ($\uparrow\uparrow\uparrow\uparrow$)	4	4	0

Local symmetries, motion, and Petrov type

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D ($\uparrow\uparrow, \uparrow\uparrow$)	4–6	4	≤ 2
I ($\uparrow, \uparrow, \uparrow, \uparrow$)	6–10	6	≤ 4

Conclusions

1. Extended-body forces and torques are arbitrary, except where forbidden by symmetries.
2. Constraints need only a very weak **local** notion of symmetry. Killing fields and conformal Killing-Yano tensors are special cases.

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1. Extended-body forces and torques are arbitrary, except where forbidden by symmetries.
2. Constraints need only a very weak **local** notion of symmetry. Killing fields and conformal Killing-Yano tensors are special cases.
3. Consequences of symmetry:
    1. Fewer quadrupole components can affect motion.
    2. Some forces arise only with torques.
    3. Some torques are impossible.