Gravitational Wave Memory Effects

The 26th Capra Meeting on Radiation Reaction in General Relativity

Kevin Cunningham, kevin.cunningham1@ucdconnect.ie

Supervisor: Dr. Niels Warburton

With: Dr. Chris Kavanagh, Dr. Adam Pound, Dr. Niels Warburton

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Background: Gravitational Wave Memory

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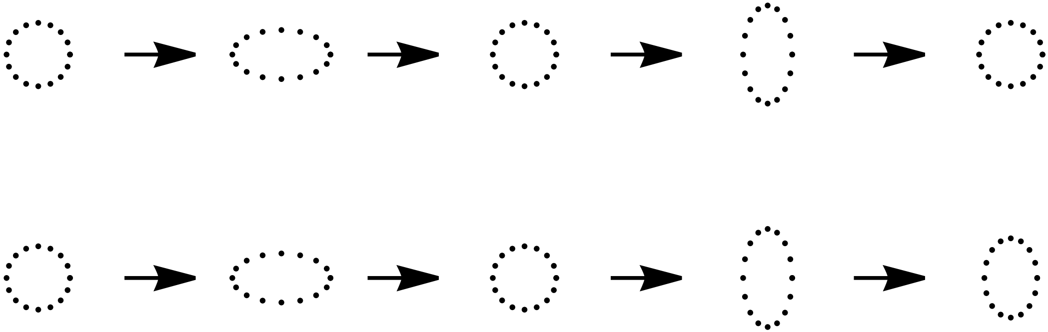
- Gravitational wave memory effects persist after the passing of a gravitational wave.
- In particular, there is a permanent displacement memory.
- First predicted a linear effect for unbound particles (Scattering events, supernovae). [Zel'dovich and Polnarev (1974)].
- Later the non-linear (Christodoulou) memory was calculated for bound orbits. [Christodoulou (1991)].
- Current generation detectors won't be able to detect memory directly: rely on stacking signals to find significant evidence.
- In particular EMRI memory is too small to detect, but does contribute to 2GSF.
- Second order memory captures IMRI behaviour well.

Effect on a ring of test particles



Displacement Memory Calculation

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$$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow \quad \Box \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu},$$

$$\delta h_{ij}^{\mathrm{TT}} = \frac{4}{R} \int_{-\infty}^{u} \mathrm{d}t' \left[\int \mathrm{d}\Omega' \, \frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}t' \mathrm{d}\Omega'} \frac{n_i' n_j'}{1 - \mathbf{n}' \cdot \mathbf{N}} \right]^{\mathrm{TT}}$$

In terms of what a detector sees:

$$h_+ - i h_\times = \sum_{lm} h_{lm}(u, R) \; {}_{-2}Y_{lm}(\Theta, \Phi)$$

$$h_{lm}^{(\mathrm{mem})} = \frac{16\pi}{R} \sqrt{\frac{(l-2)!}{(l+2)!}} \int_{-\infty}^{u} \mathrm{d}t' \int \frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}t' \mathrm{d}\Omega'} {}_0Y_{lm}^*(\Omega') \mathrm{d}\Omega' \qquad \text{[Favata (2009)]}$$

Calculating The Fluxes

$$\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}t\mathrm{d}\Omega} = \frac{R^2}{16\pi} \sum_{l'l''m'm''} \langle \dot{h}_{l'm'} \dot{h}^*_{l''m''} \rangle {}_{-2}Y_{l'm'} {}_{-2}Y^*_{l''m''}$$

PN results:

The PN expansion was calculated for Kerr using MST by Dr. Chris Kavanagh.

For numerical results:

$$-\frac{1}{2}(\ddot{h}_+ - i\ddot{h}_\times) = \lim_{r\to\infty} \psi_4 = \frac{1}{R} \sum_{\ell m} {}_{-2}Z^{\mathrm{Up}}_{\ell m} e^{-im\Omega_\phi u} {}_{-2}S^{am\Omega_\phi}_{\ell m}(\Theta, \Phi),$$

where ${}_{-2}Z^{\mathrm{Up}}_{\ell m}$ is calculated from the Teukolsky Formalism.

Second Order Memory

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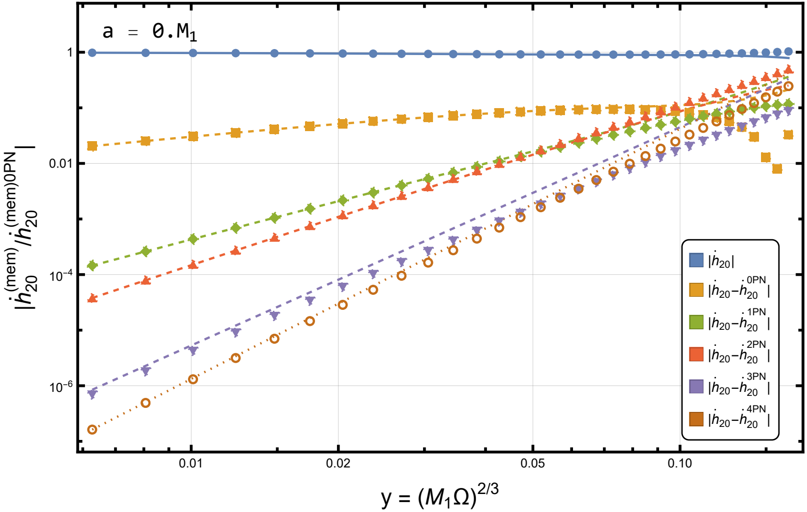
$$Z_{lm} \to \epsilon\, Z_{lm}^{(1)} + \epsilon^2 Z_{lm}^{(2)} + \mathcal{O}(\epsilon^3)$$

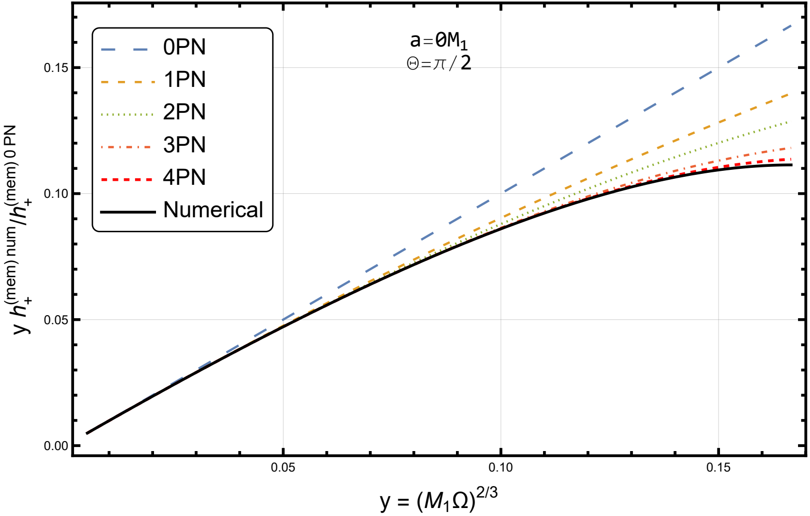
$$Z_{l'm'} Z^*_{l''m''} \to \epsilon^2 Z^{(1)}_{l'm'} Z^{(1)*}_{l''m''} + \epsilon^3 \left(Z^{(1)}_{l'm'} Z^{(2)*}_{l''m''} + Z^{(2)}_{l'm'} Z^{(1)*}_{l''m''} \right) + \mathcal{O}(\epsilon^4)$$

- $Z_{lm}^{(1)}$ comes from the Black Hole Perturbation Toolkit.
- $Z_{lm}^{(2)}$ is the second order Lorenz gauge metric perturbation. [Warburton *et al.* (2021)]

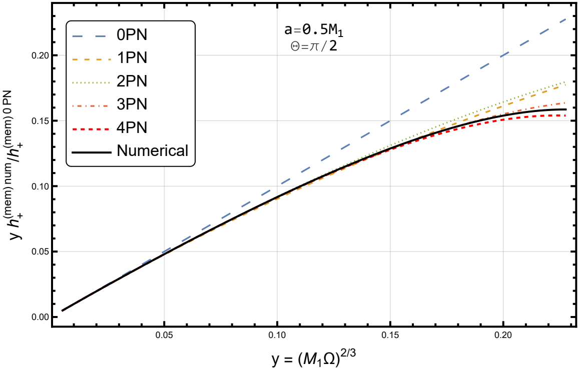
PN Comparison: Schwarzschild

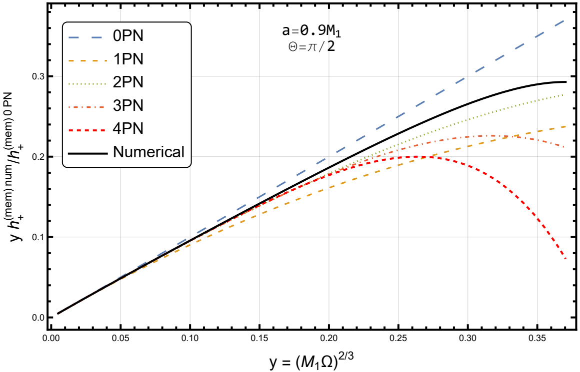


PN Comparison: Adiabatic Schwarzschild Inspiral

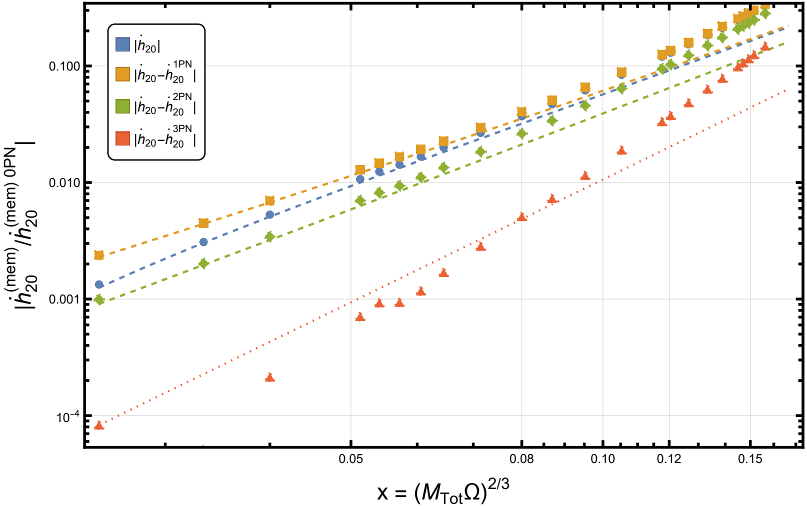


PN Comparison: Adiabatic Kerr Inspiral

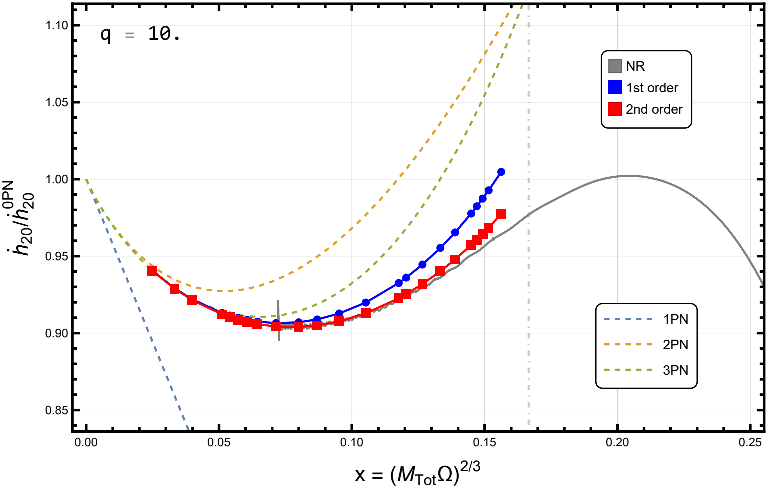




2nd order PN Comparison: Schwarzschild



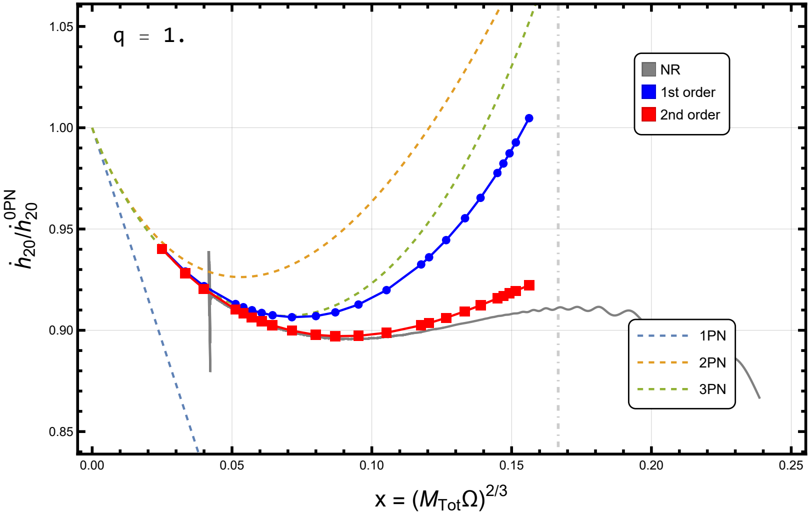
NR Comparison (SXS:BBH:1107)



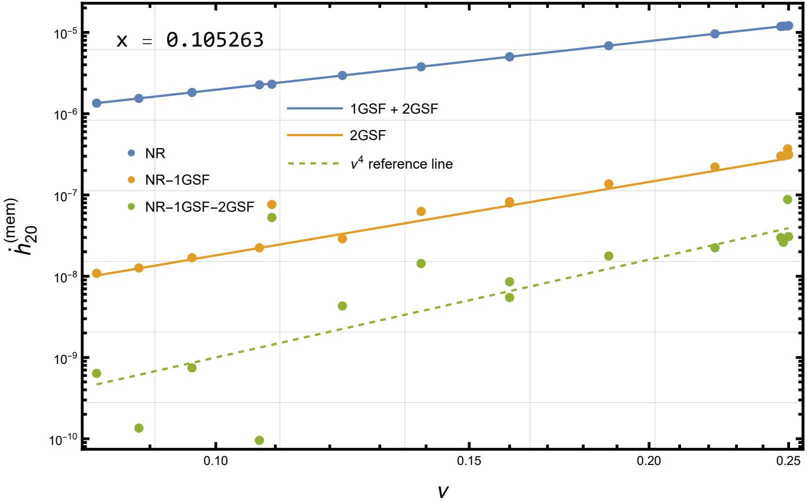
The `waveform.memory.J_E` function of the `SXS python library` extracts GW memory from SXS simulations using BMS flux balance laws.

[Mitman *et al.* (2020), (2023)]

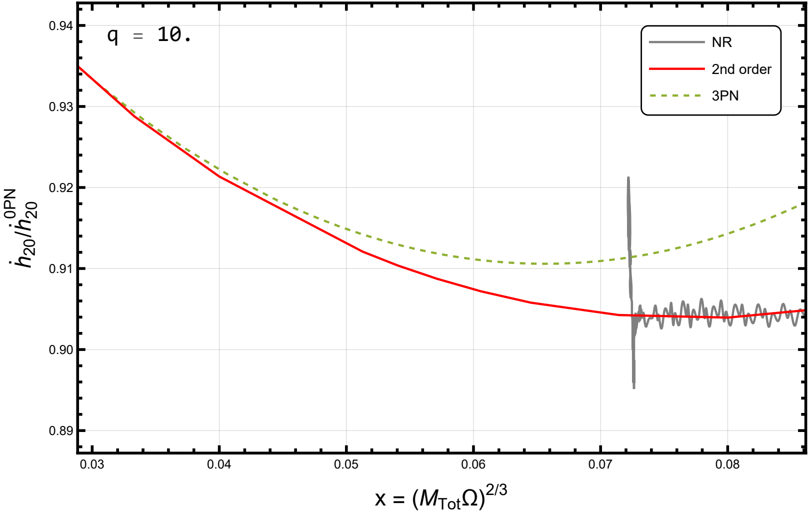
NR Comparison (SXS:BBH:1132)



NR Comparison



BHPT as a bridge between PN and NR



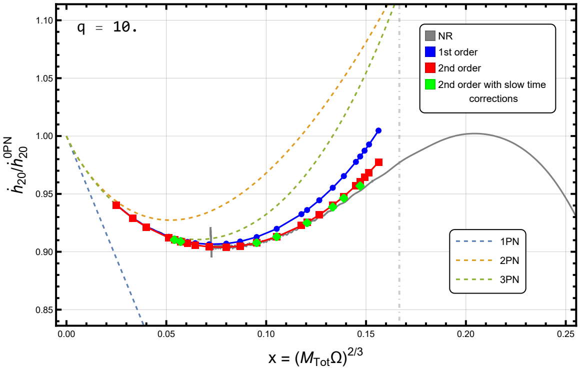
Conclusions

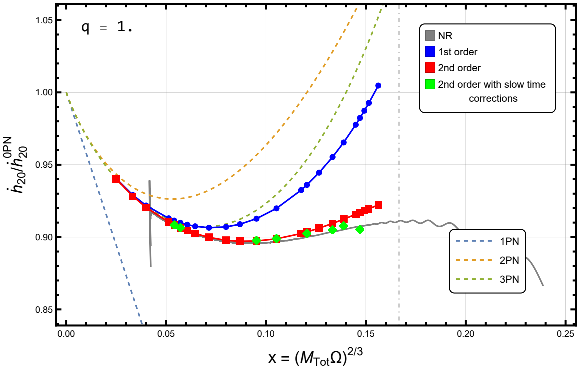
- We have calculated the displacement memory for Kerr using both PN and 1st order in mass ratio BHPT and found good agreement.
- We have calculated the displacement memory for Schwarzschild to 2nd order in the mass ratio and found good agreement with both PN and NR.
- This puts us in a good place to model memory for IMRIs and to bridge the gap between PN and NR.

Questions?

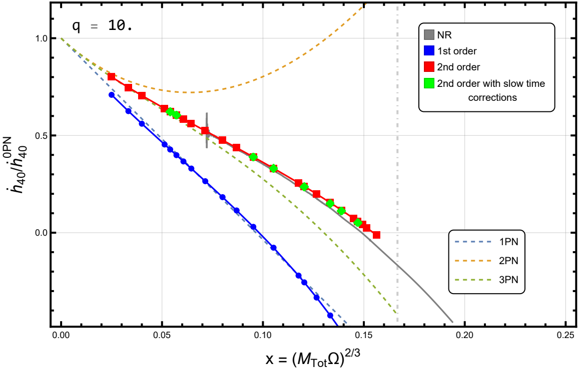
Thanks for Listening!
Any questions?

Bonus Plots!

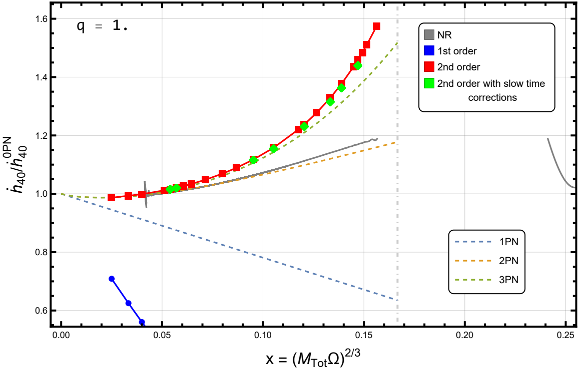

q = 10.
NR
1st order
2nd order
2nd order with slow time corrections
1PN
2PN
3PN
$\dot{h}_{20}/\dot{h}_{20}^{0PN}$
$x = (M_{Tot}\Omega)^{2/3}$


q = 1.
NR
1st order
2nd order
2nd order with slow time corrections
1PN
2PN
3PN
$\dot{h}_{20}/\dot{h}_{20}^{0PN}$
$x = (M_{Tot}\Omega)^{2/3}$

Bonus Plots!

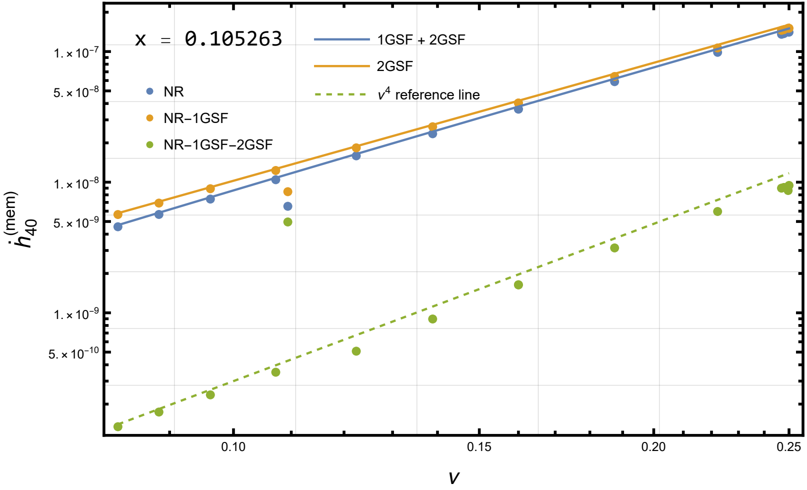


q = 10.
NR
1st order
2nd order
2nd order with slow time corrections
1PN
2PN
3PN
$\dot{h}_{40}/\dot{h}_{40}^{0PN}$
$x = (M_{Tot}\Omega)^{2/3}$



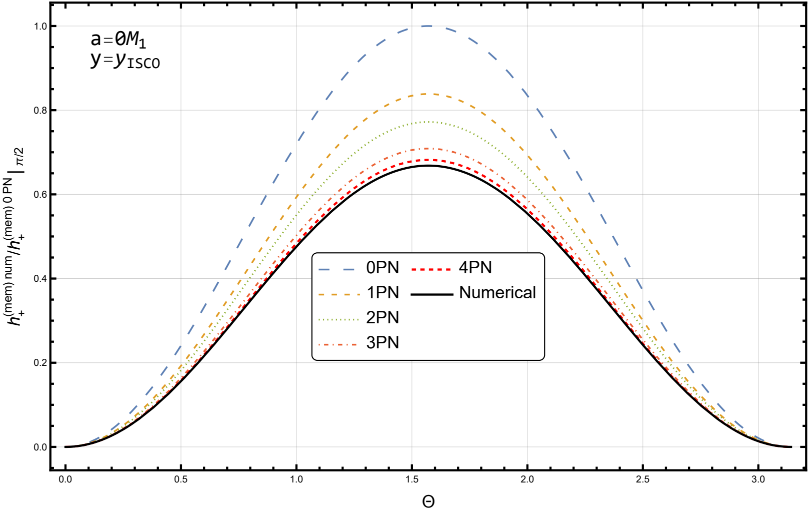
q = 1.
NR
1st order
2nd order
2nd order with slow time corrections
1PN
2PN
3PN
$\dot{h}_{40}/\dot{h}_{40}^{0PN}$
$x = (M_{Tot}\Omega)^{2/3}$

Bonus Plots!



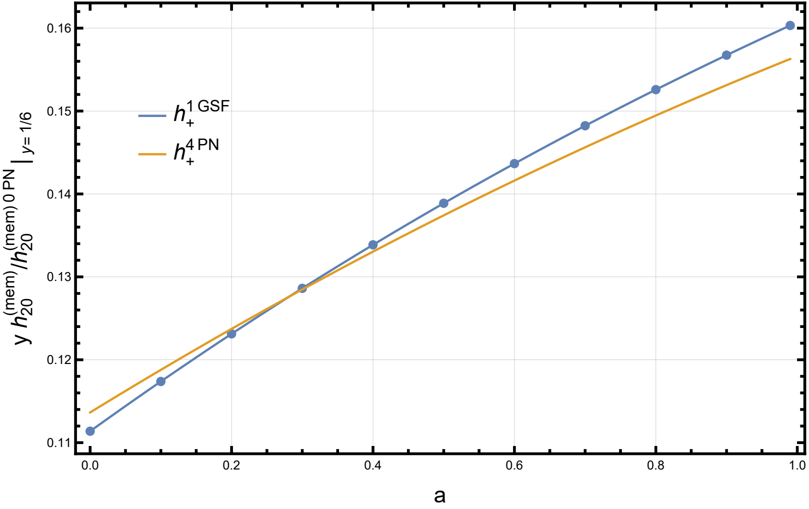
x = 0.105263
1GSF + 2GSF
2GSF
v^4 reference line
NR
NR−1GSF
NR−1GSF−2GSF
1. × 10^−7
5. × 10^−8
1. × 10^−8
5. × 10^−9
1. × 10^−9
5. × 10^−10
0.10
0.15
0.20
0.25
$\dot{h}_{40}^{(\mathrm{mem})}$
ν

Bonus plots!



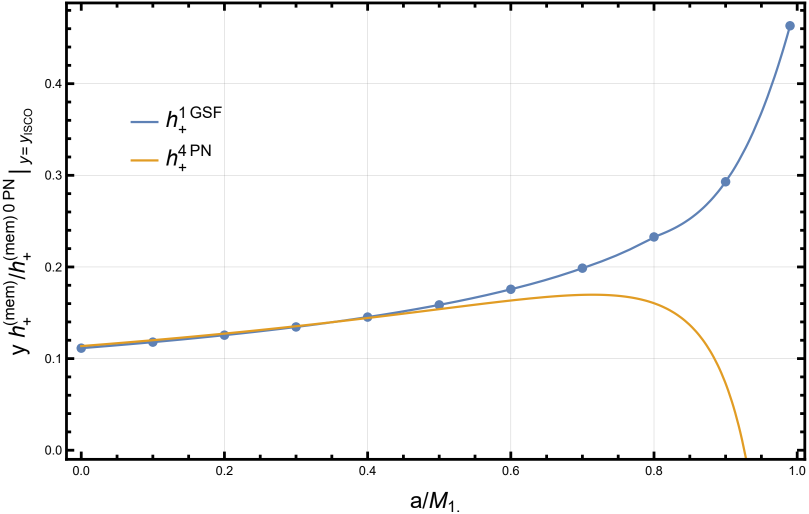
a=0M_1
y=y_{ISCO}
0PN
1PN
2PN
3PN
4PN
Numerical
$h_+^{(mem)\,num}/h_+^{(mem)\,0\,PN}|_{\pi/2}$
Θ
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Bonus plots!



$y\, h_{20}^{(\mathrm{mem})}/h_{20}^{(\mathrm{mem})\,0\,\mathrm{PN}}\big|_{y=1/6}$
0.16
0.15
0.14
0.13
0.12
0.11
$h_+^{1\,\mathrm{GSF}}$
$h_+^{4\,\mathrm{PN}}$
0.0
0.2
0.4
0.6
0.8
1.0
a

Bonus plots!



$h_+^{1\,\mathrm{GSF}}$
$h_+^{4\,\mathrm{PN}}$
$y\, h_+^{(\mathrm{mem})}/h_+^{(\mathrm{mem})\,0\,\mathrm{PN}}|_{y=y_{\mathrm{ISCO}}}$
0.0
0.1
0.2
0.3
0.4
0.0
0.2
0.4
0.6
0.8
1.0
a/M_1.