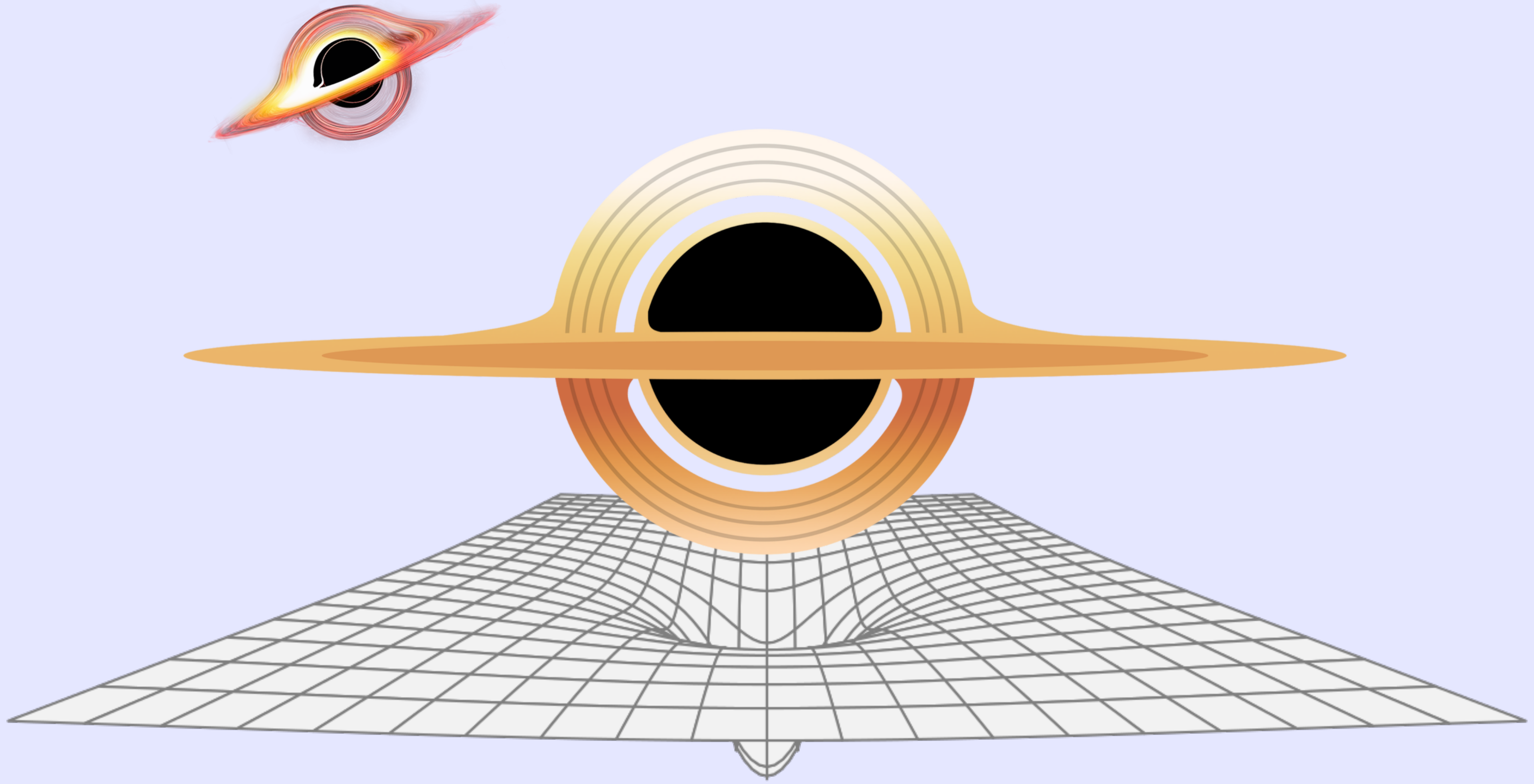
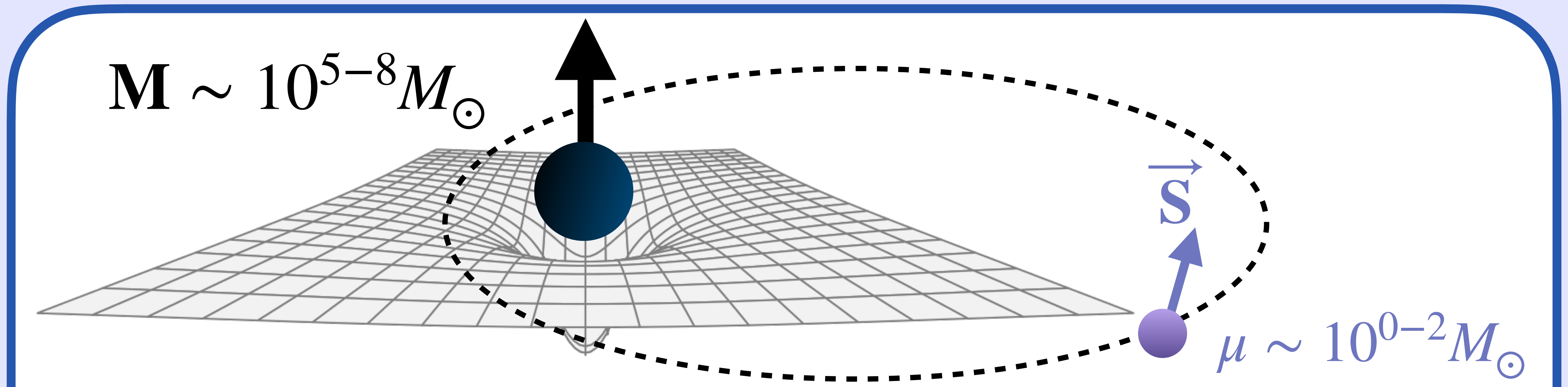
Extreme mass-ratio inspiral of a spinning body into a black hole: Generic trajectory and waveforms

<u>Lisa Drummond</u>

*and **Prof. Scott Hughes***



What about the ***spin*** of the secondary?



Spin-curvature coupling: *How the small body's spin couples to curvature, and how that backreacts on its motion*

Astrophysical black holes have **spin!**

We also need to include the effect of the **spin of the secondary** in EMRI waveform models for LISA

Mathisson-Papapetrou-Dixon equations

Equations describing the motion of a ***spinning test body*** *in curved spacetime*

f_S^α is the **spin-curvature force**

$$\frac{Dp^\alpha}{d\tau} = -\frac{1}{2}R^\alpha_{\beta\gamma\delta}u^\beta S^{\gamma\delta} := f_S^\alpha/\mu \qquad \textbf{(1)}$$

$$\frac{DS^{\alpha\beta}}{d\tau} = p^\alpha u^\beta - p^\beta u^\alpha \qquad \textbf{(2)}$$

$$p_\mu S^{\mu\nu} = 0 \qquad \textbf{(3)}$$

Tulczyjew-Dixon **spin-supplementary condition**

$S^{\mu\nu}$ is the spin tensor of the secondary

$S^\mu = -\frac{1}{2}\epsilon^{\mu\nu}{}_{\alpha\beta}p_\nu S^{\alpha\beta}$ is the spin vector of the secondary

MPD equations follow by requiring **conservation of stress-energy** $\nabla_\mu T^{\mu\nu} = 0$

Mathisson-Papapetrou-Dixon equations

…to leading-order in spin

Because we are studying this system in the very large mass-ratio limit and $S = s\mu^2$ when the spinning mass body is itself a black hole, we can take the ***leading-order-in-spin limit****.*

In this case, the ***MPD equations*** *simplify:*

$$\frac{Du^\alpha}{d\tau} = -\frac{1}{2\mu} R^\alpha_{\beta\gamma\delta} u^\beta S^{\gamma\delta} \qquad \textbf{(1)}$$

Motion of the small body

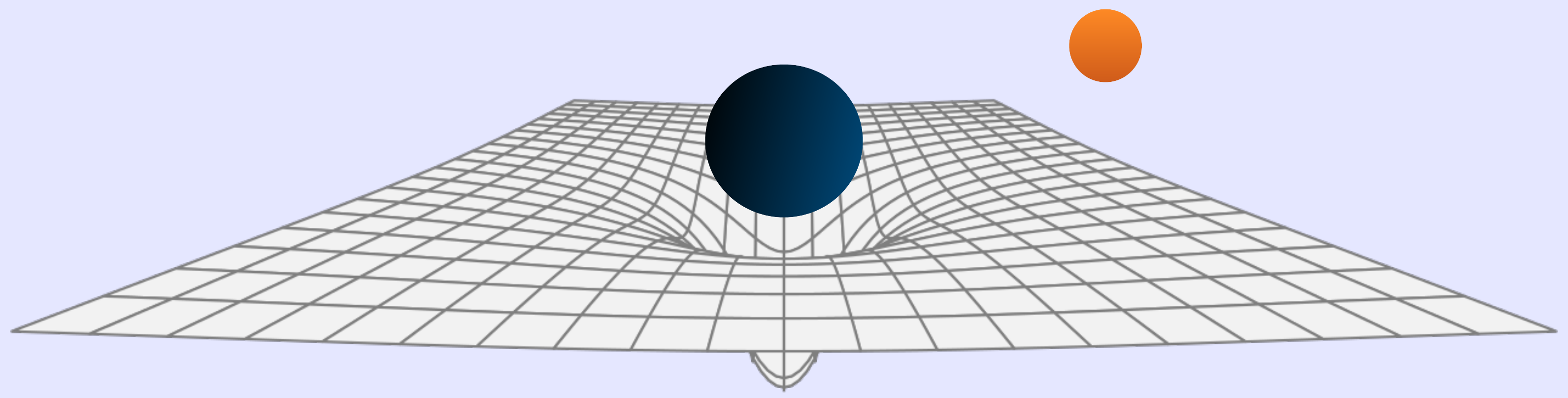
$$\frac{DS^\mu}{d\tau} = 0 \qquad \textbf{(2)}$$

Evolution of spin vector

Kinematics of an orbiting small body

Conservative orbit of a small body around a black hole

Non-spinning body: Geodesic equations



$$\ddot{x}^\alpha = f^\alpha_{geo}\,, \quad f^\alpha_{geo} \equiv -\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau}$$

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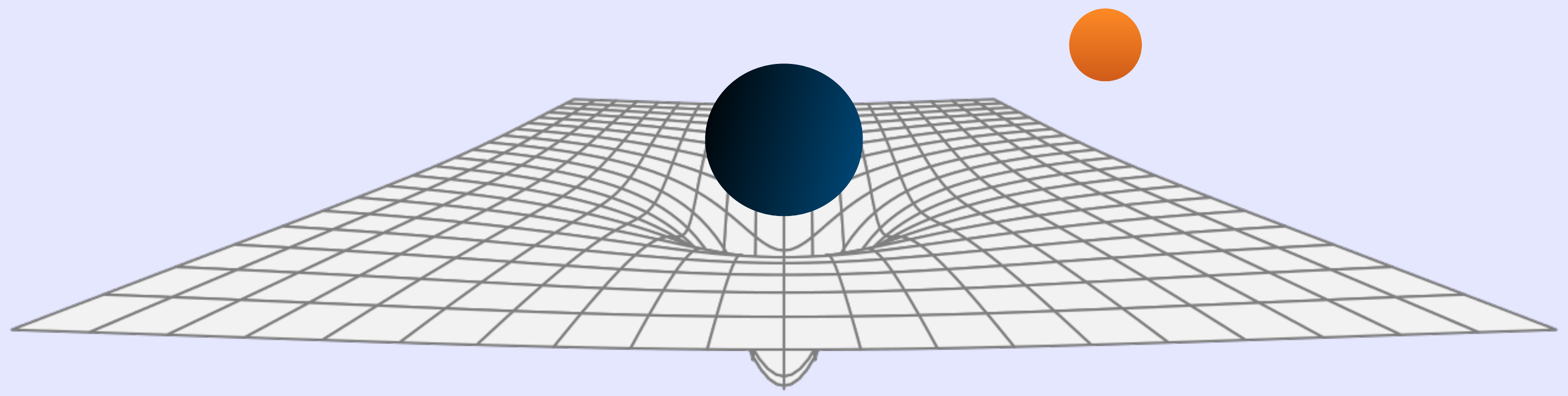
Spinning body: Spin-curvature coupling

- ★ Coupling between curvature and small-body spin leads to **spin-curvature force**
- ★ Pushes the motion of the small body away from the geodesic orbit and causes small body's spin to precess



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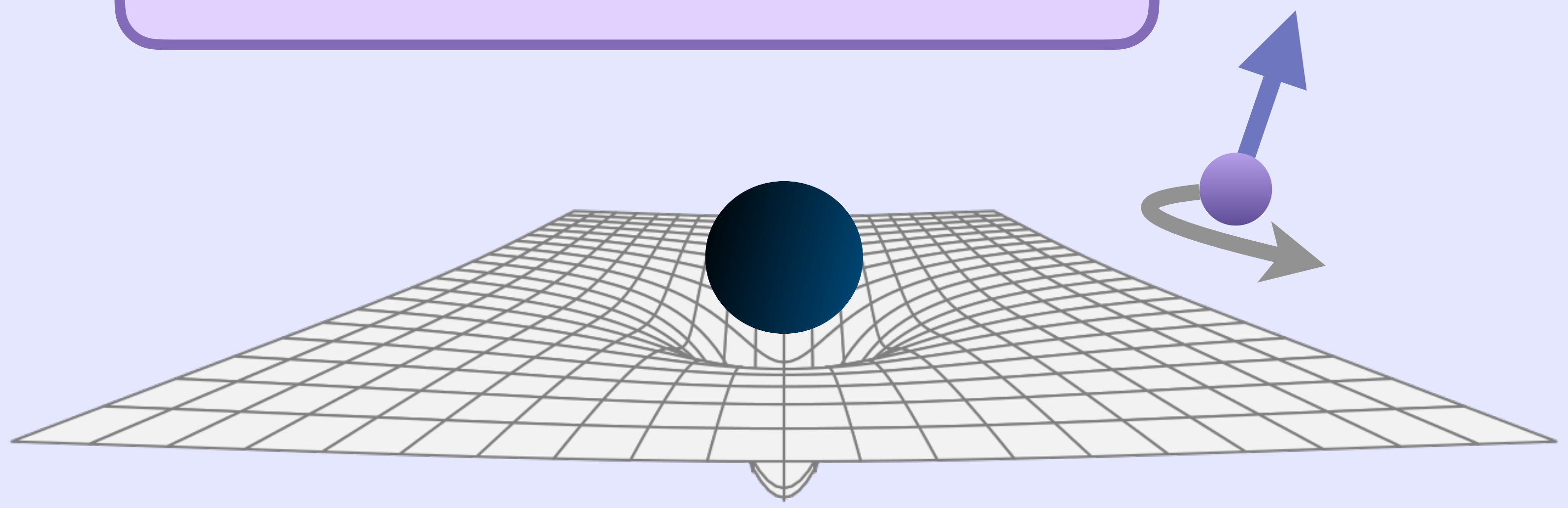

$$\ddot{x}^\alpha = f^\alpha_{geo} + f^\alpha_{SCF}\,, \quad f^\alpha_{SCF} \equiv -\frac{1}{2\mu}R^\alpha{}_{\nu\lambda\sigma}u^\nu S^{\lambda\sigma}$$

M. Mathisson, 1937; A. Papapetrou, 1951; W. G. Dixon, 1970

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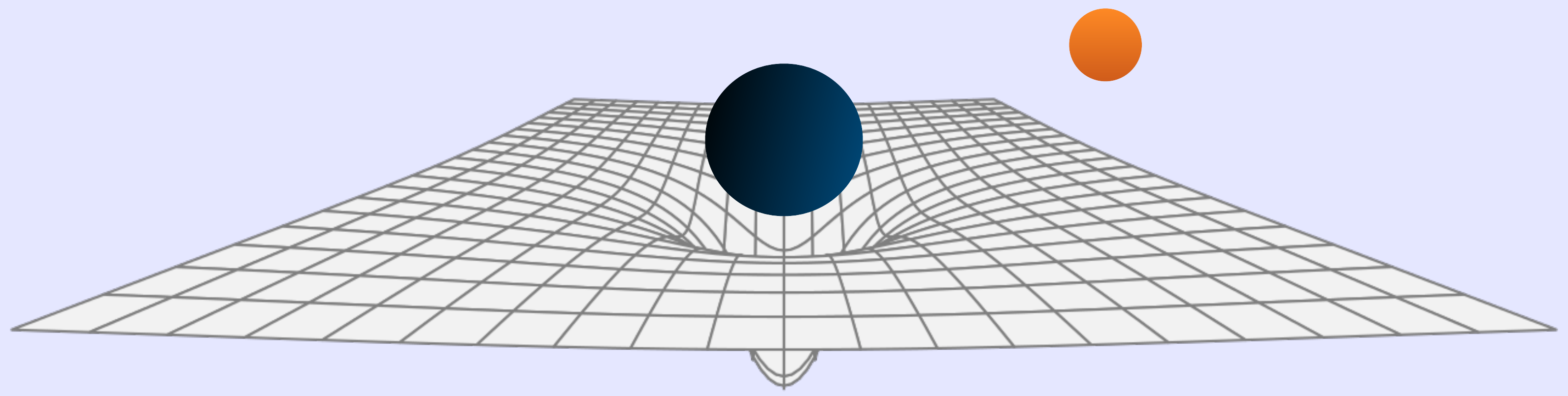

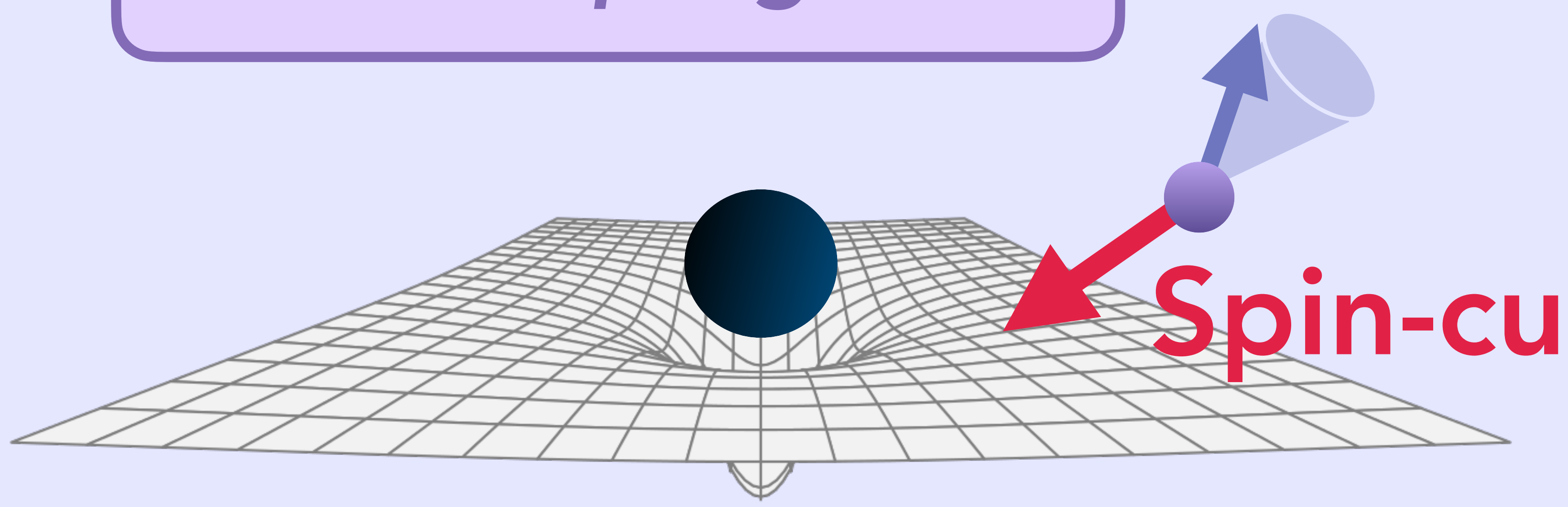

Spin-curvature force f^{α}_{SCF}

$$\ddot{x}^{\alpha} = f^{\alpha}_{geo}\,, \quad f^{\alpha}_{geo} \equiv -\Gamma^{\alpha}_{\beta\gamma}\frac{dx^{\beta}}{d\tau}\frac{dx^{\gamma}}{d\tau}$$

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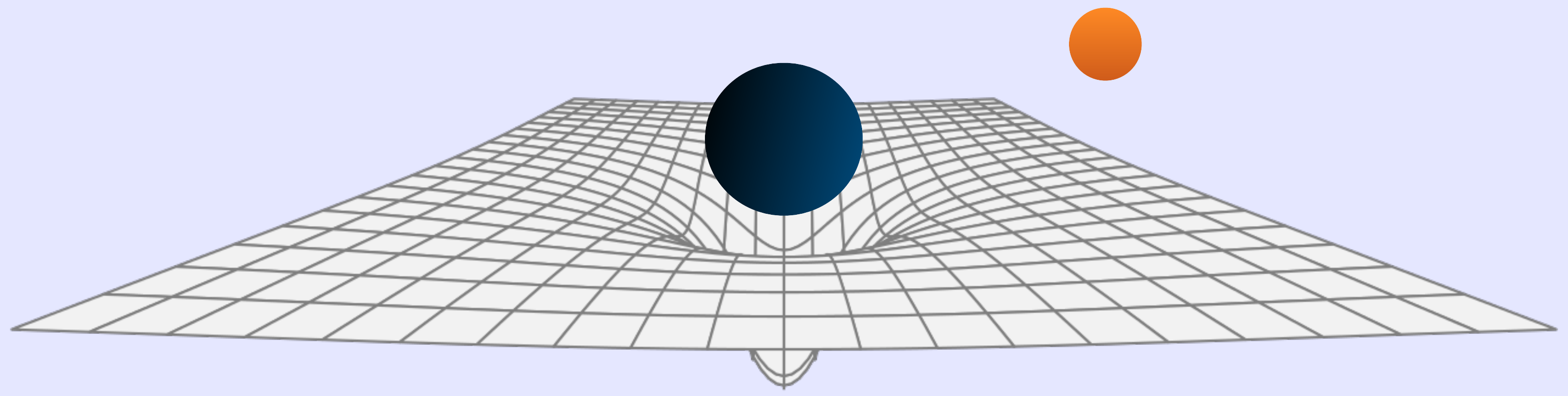
M. Mathisson,1937; A. Papapetrou, 1951; W. G. Dixon, 1970

Radiation due to an orbiting small body

Gravitational radiation emitted due to a small body

Non-spinning body: Point-particle GW fluxes



$$T^{\mu\nu}_{geo} = \int d\tau \left(\frac{\mu u^{\mu}_{geo} u^{\nu}_{geo}}{\sqrt{-g}} \delta^4 \left(x^\rho - z^\rho_{geo}(\tau) \right) \right)$$

*Compute GW radiation using the **Teukolsky equation***

$${}_{-2}\mathcal{O}\ {}_{-2}\Psi = 4\pi\Sigma\mathcal{T}$$

***The source term** $\mathcal{T}$ in the **Teukolsky equation** can be found from the **stress-energy tensor** $T^{\mu\nu}$ describing the small body*

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Compute GW radiation using the ***Teukolsky equation***

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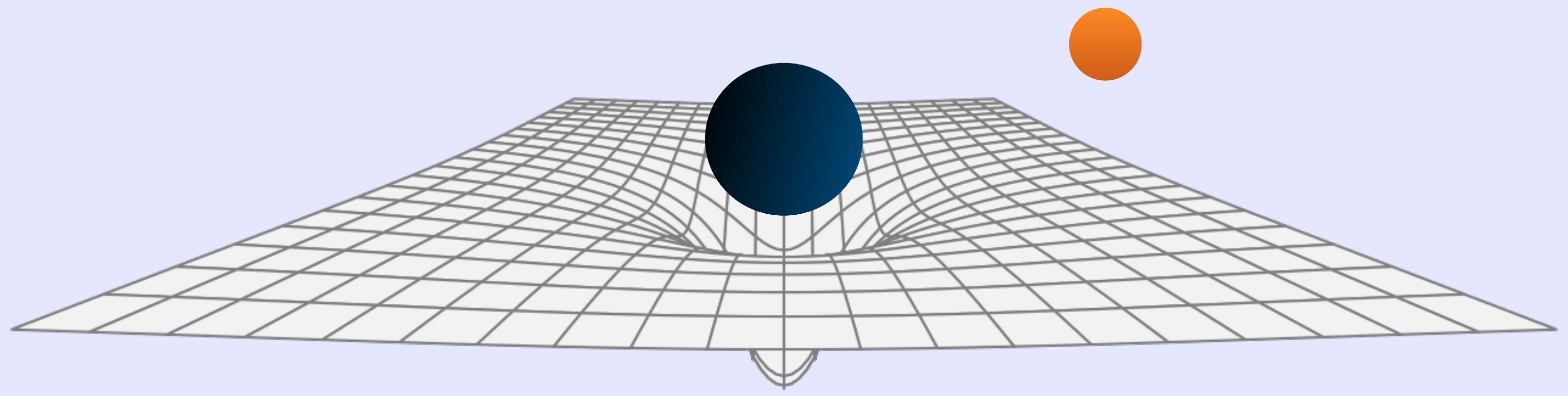
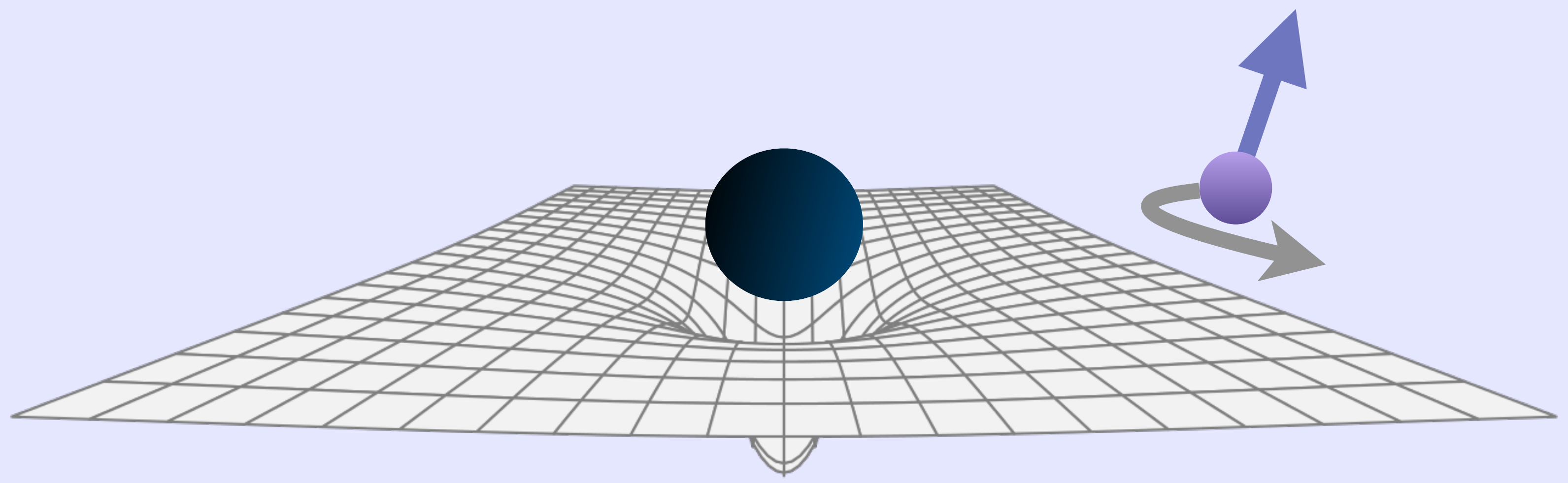


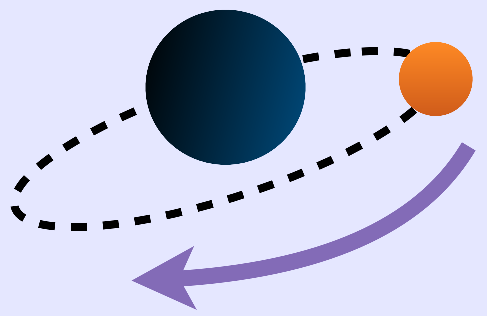


$$T^{\mu\nu}_{geo} = \int d\tau \left(\frac{\mu u^{\mu}_{geo} u^{\nu}_{geo}}{\sqrt{-g}} \delta^4\left(x^\rho - z^\rho_{geo}(\tau)\right)\right)$$

$$T^{\mu\nu}_{spin} = \int d\tau \left(\frac{p^{(\mu}u^{\nu)}}{\sqrt{-g}}\delta^4\left(x^\rho - z^\rho(\tau)\right) - \nabla_\alpha\left(\frac{S^{\alpha(\mu}u^{\nu)}}{\sqrt{-g}}\delta^3\left(x^\rho - z^\rho(\tau)\right)\right)\right)$$

How do we build an **inspiral**?

Orbital *kinematics*



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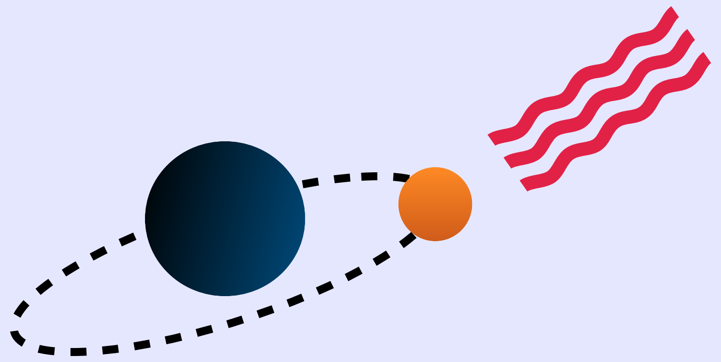
GW *radiation*



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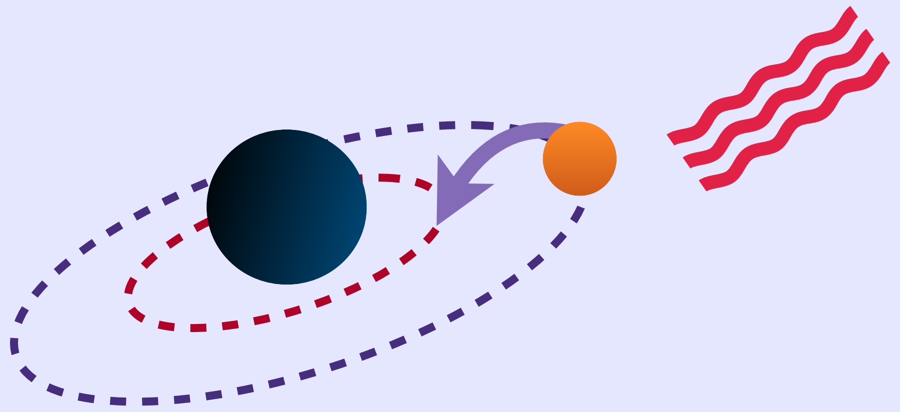
GW-driven *inspiral*

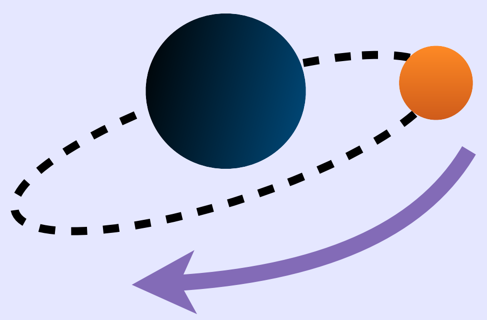


Kinematics of an orbiting small body

GW Radiation due to an orbiting small body

Inspiral of an orbiting small body

Orbital kinematics



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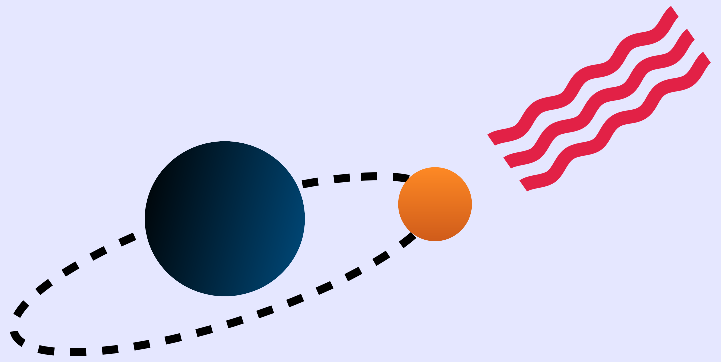
GW radiation



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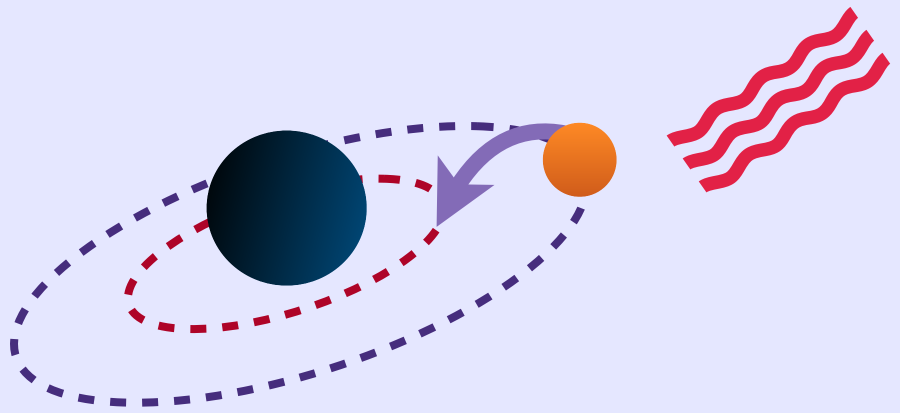
GW-driven inspiral



Kinematics of an orbiting small body

GW Radiation due to an orbiting small body

Spinning body: MPD equations

+

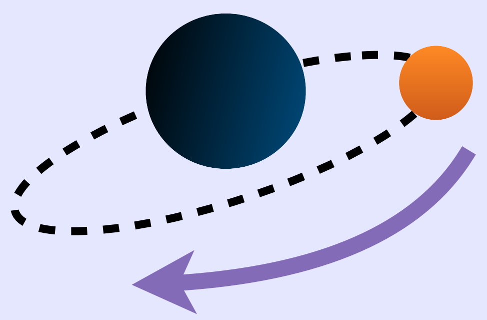
Spinning body: Spinning-particle GW fluxes

Skoupý, Lukes-Gerakopoulos, LVD & Hughes, 2023, **arXiv:**2303.16798

Generic orbit, any spin orientation

Inspiral of an orbiting small body

Orbital kinematics



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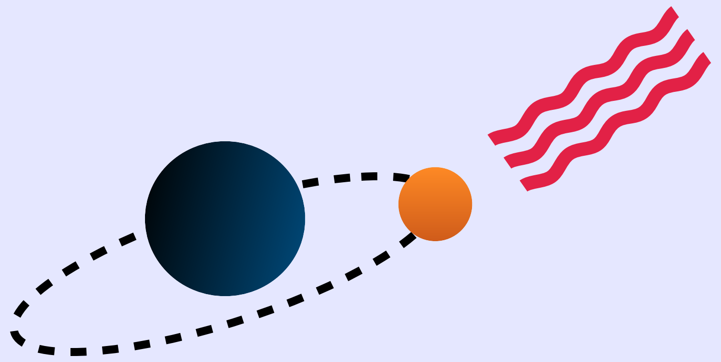

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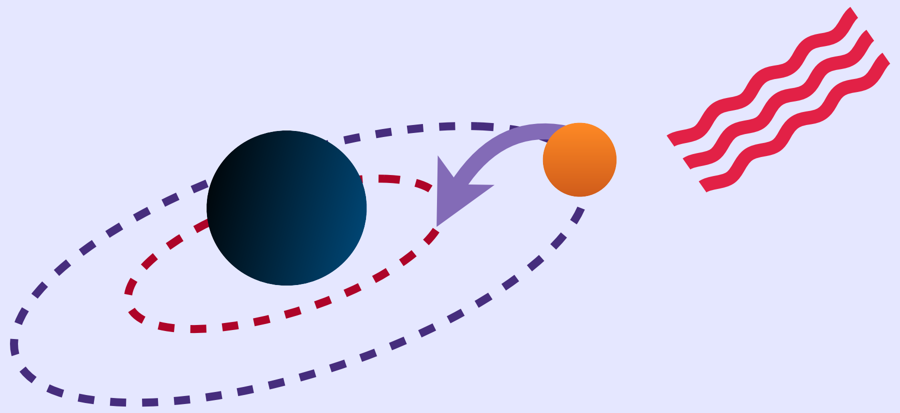
GW-driven inspiral



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Generic orbit, any spin orientation

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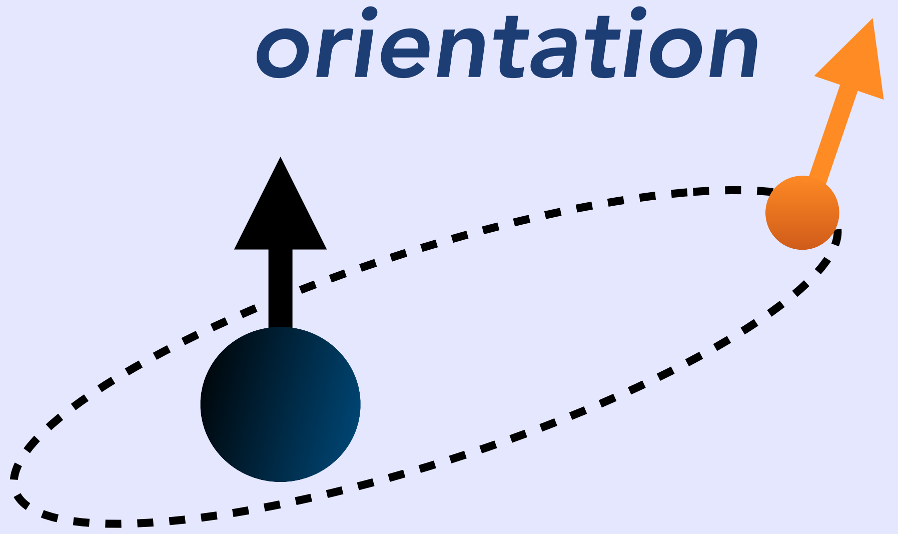
Generic inspiral: work in progress

Generic inspiral with any spin orientation



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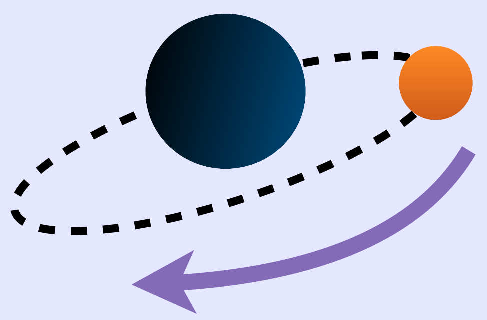

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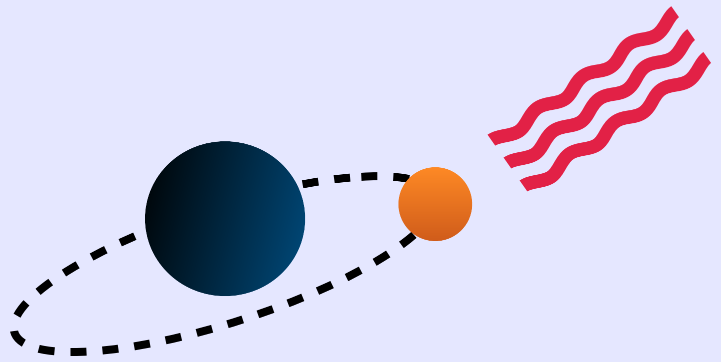
GW radiation



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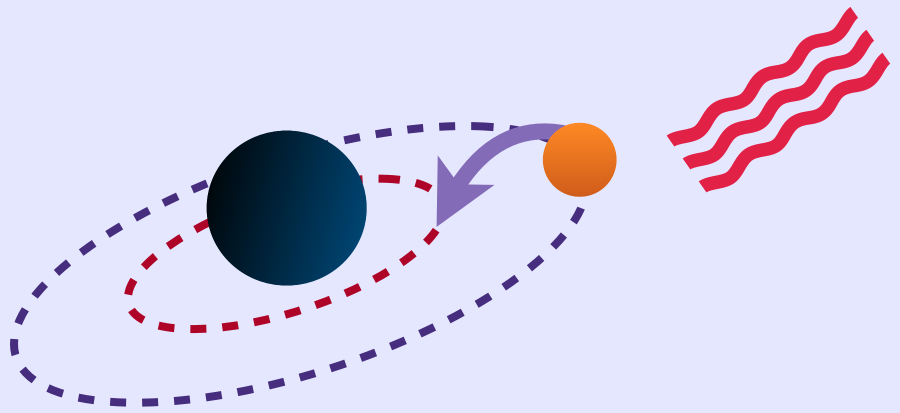
GW-driven inspiral



Kinematics of an orbiting small body

GW Radiation due to an orbiting small body

Spinning body: Spin-curvature coupling

+

Non-spinning body: Point-particle GW fluxes

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This work; LVD in collaboration with Hughes, Hanselman, Becker & Lynch

Generic inspiral with arbitrary spin orientation; but using ***point-particle*** *GW fluxes*

Overview of our model

Motivation: To incorporate **some** spinning secondary effects into generic Kerr waveforms.

- We **do not include** the correction to the GW fluxes due to the spin of the secondary (see the next talk by **Viktor Skoupý**).

- Therefore, our work is an intermediate step on the path towards waveforms which **fully** incorporate all spinning secondary effects.

Osculating geodesic (OG) framework

Geodesic orbit of a small body around a black hole

Parameterized by orbital elements **{p, e, x}**

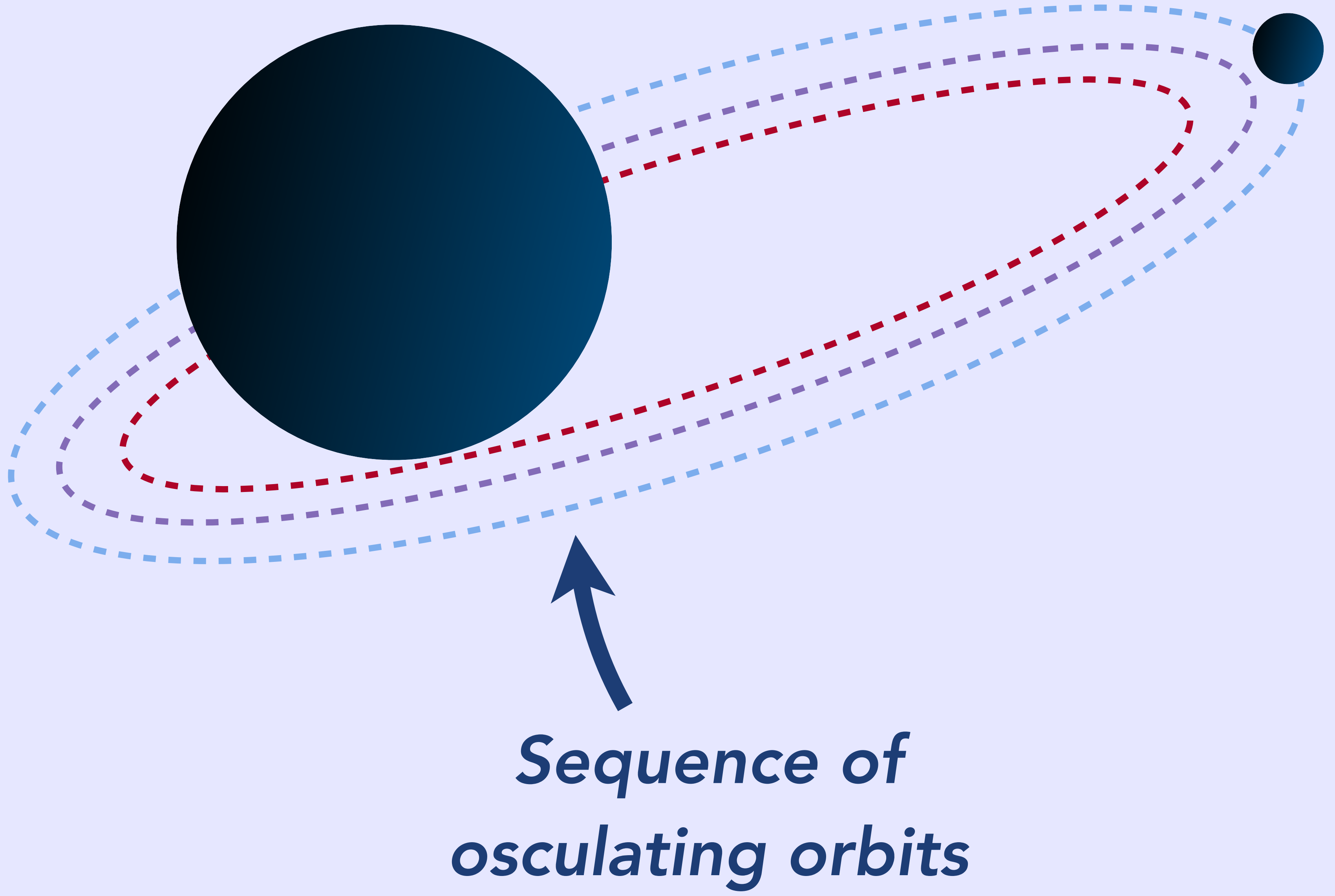
Orbit evolves due to perturbing force

Inspiral of small body into a black hole

Parameterized by orbital elements **{p(t), e(t), x(t)}**

We use an osculating geodesic framework

Stitch together a sequence of **osculating orbits** to construct an **inspiral**



$$\vec{P} = \{p, e, x\}$$

$$\vec{q} = \{q_r, q_z, q_s\}$$

Geodesic:

$$\dot{P}_i = 0$$

$$\dot{q}_i = \Upsilon_i^{(0)}(\vec{P})$$

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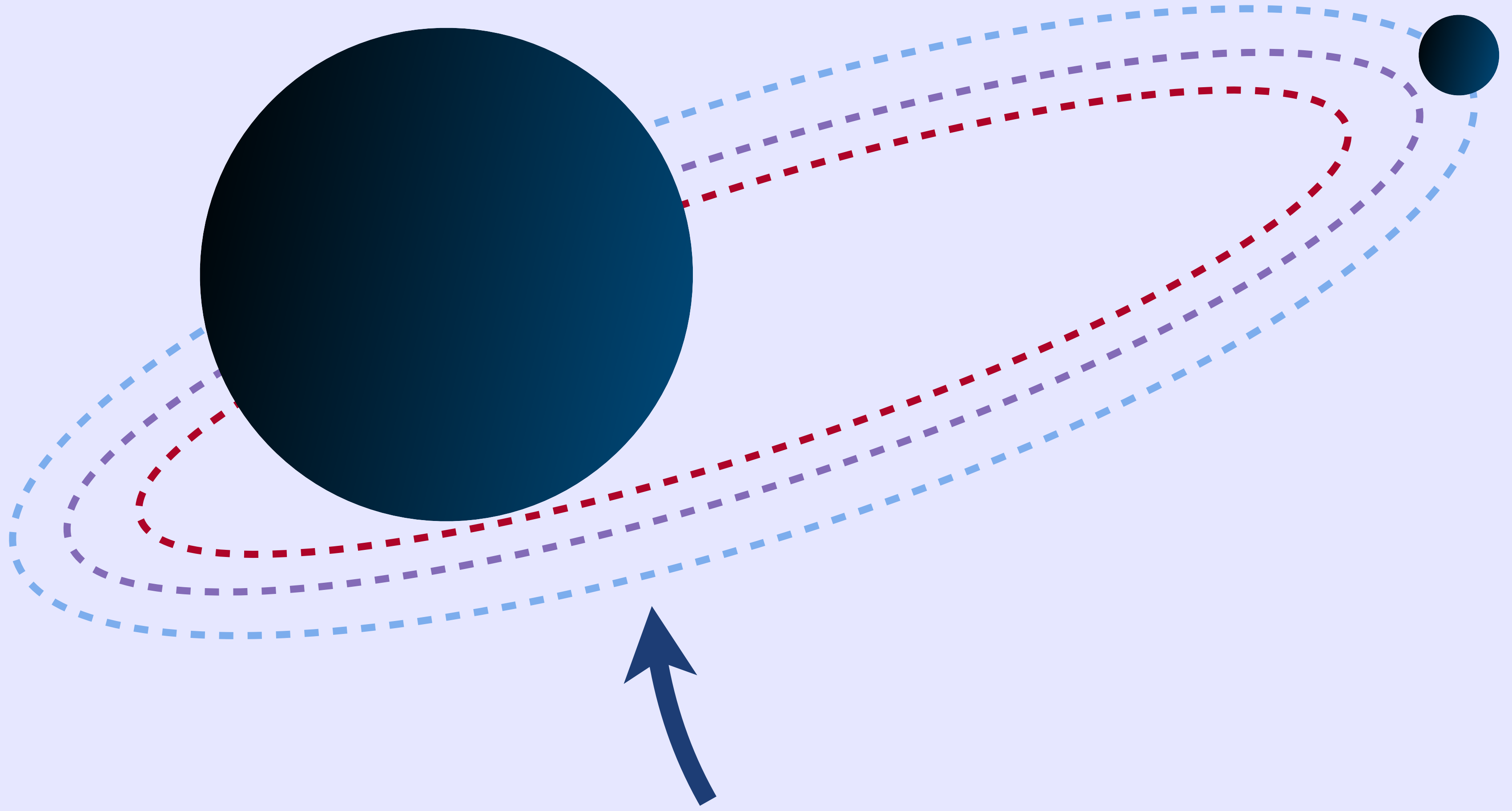


Sequence of osculating orbits

$$\vec{P} = \{p, e, x\}$$

$$\vec{q} = \{q_r, q_z, q_s\}$$

Post-geodesic:

$$\dot{P}_i = F_i(\vec{P}, \vec{q})$$

$$\dot{q}_i = \Upsilon_i^{(0)}(\vec{P}) + f_i(\vec{P}, \vec{q})$$

Osculating geodesic (OG) framework

Geodesic orbit of a small body around a black hole

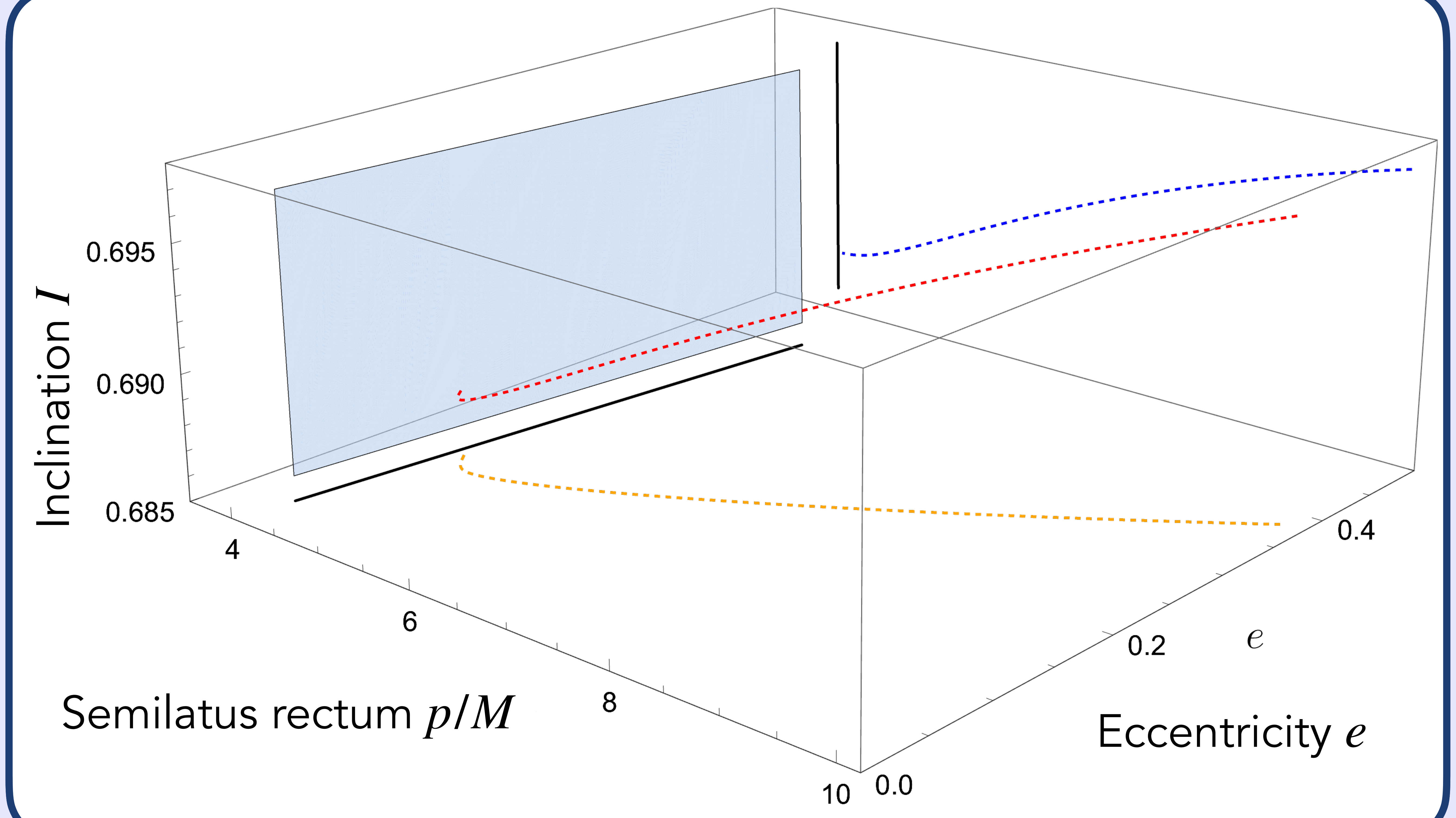
Orbit evolves due to perturbing force

Adiabatic fluxes

Inspiral of small body into a black hole

We use an osculating geodesic framework

Stitch together *a sequence of* ***osculating orbits*** *to construct an* ***inspiral***



Osculating geodesic (OG) framework

Geodesic orbit of a small body around a black hole

Orbit evolves due to perturbing force

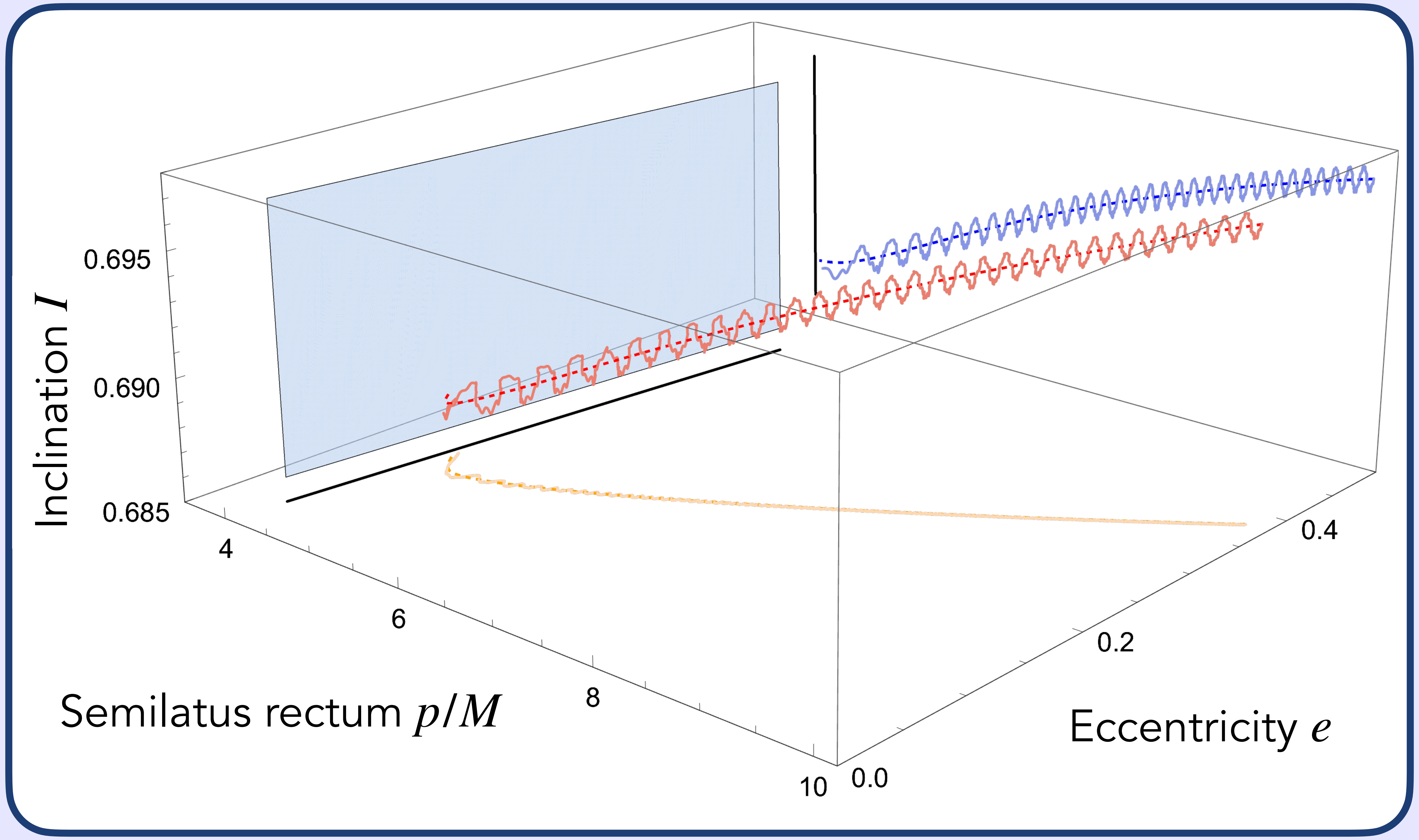
→

Adiabatic fluxes + spin-curvature force f^{α}_{SCF}

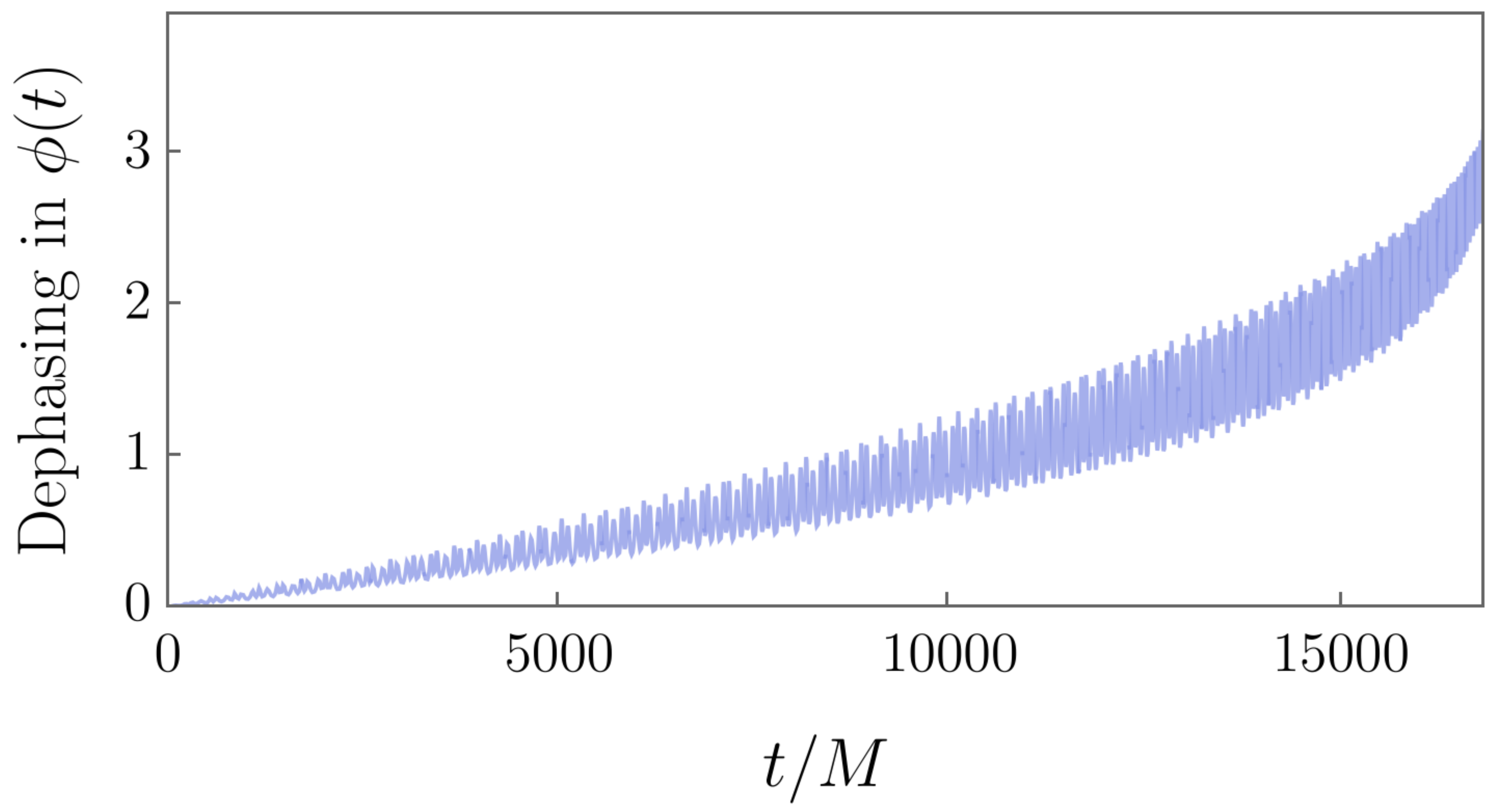
Inspiral of small body into a black hole

We use an osculating geodesic framework

Stitch together** a sequence of **osculating orbits** to construct an **inspiral



NIT: Near-Identity Transformation



Dephasing between the spinning- and non-spinning-body trajectory.

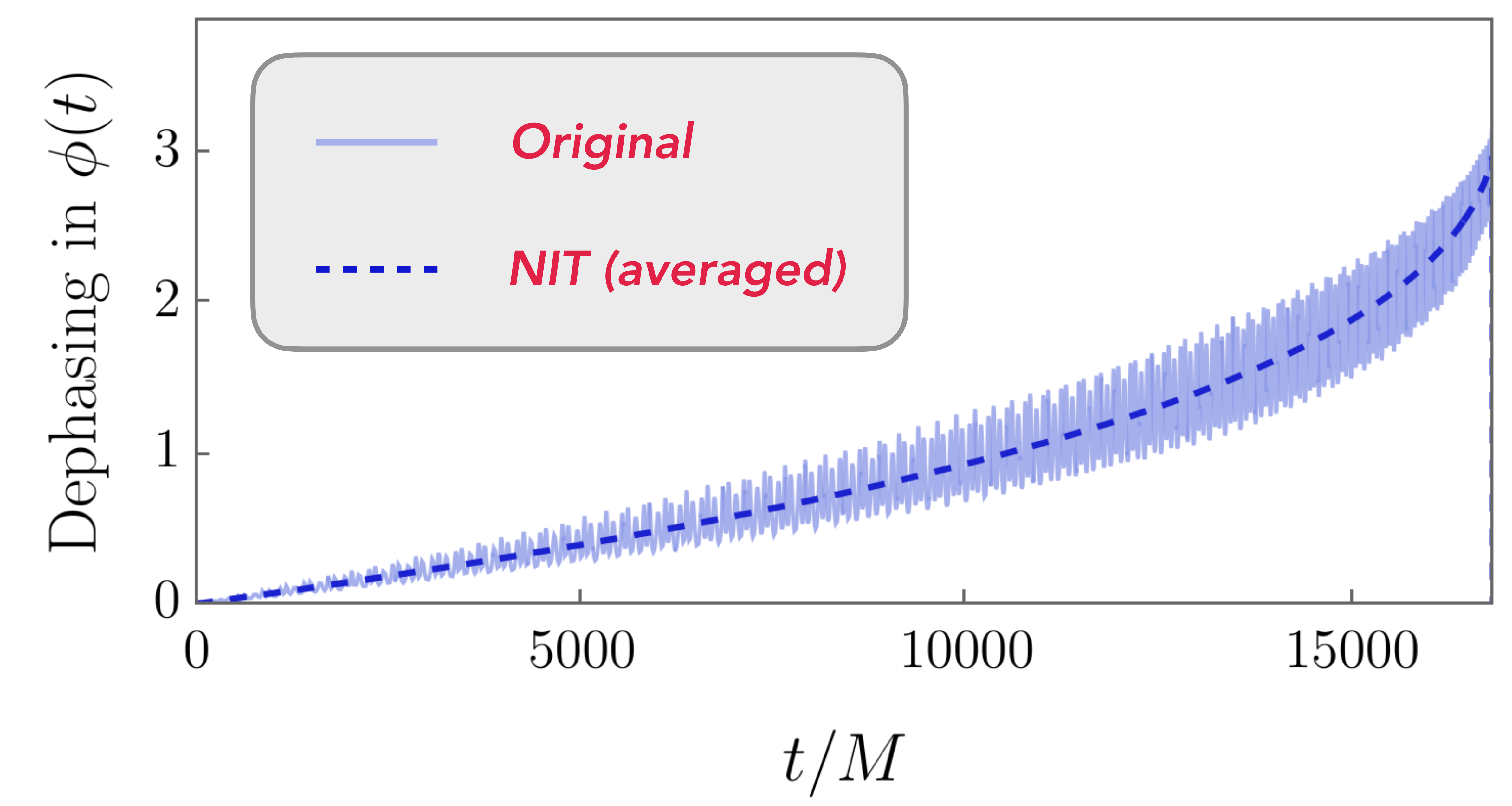
$$p_0 = 10M,\ e_0 = 0.2,\ x_0 = 0.7,\ a = 0.7M$$

Notice all the **short timescale oscillations** which our integrator must resolve!

↓

We are only interested in the averaged behavior on **long timescales**

NIT: Near-Identity Transformation



Dephasing between the spinning- and non-spinning-body trajectory.

$$p_0 = 10M,\ e_0 = 0.2,\ x_0 = 0.7,\ a = 0.7M$$

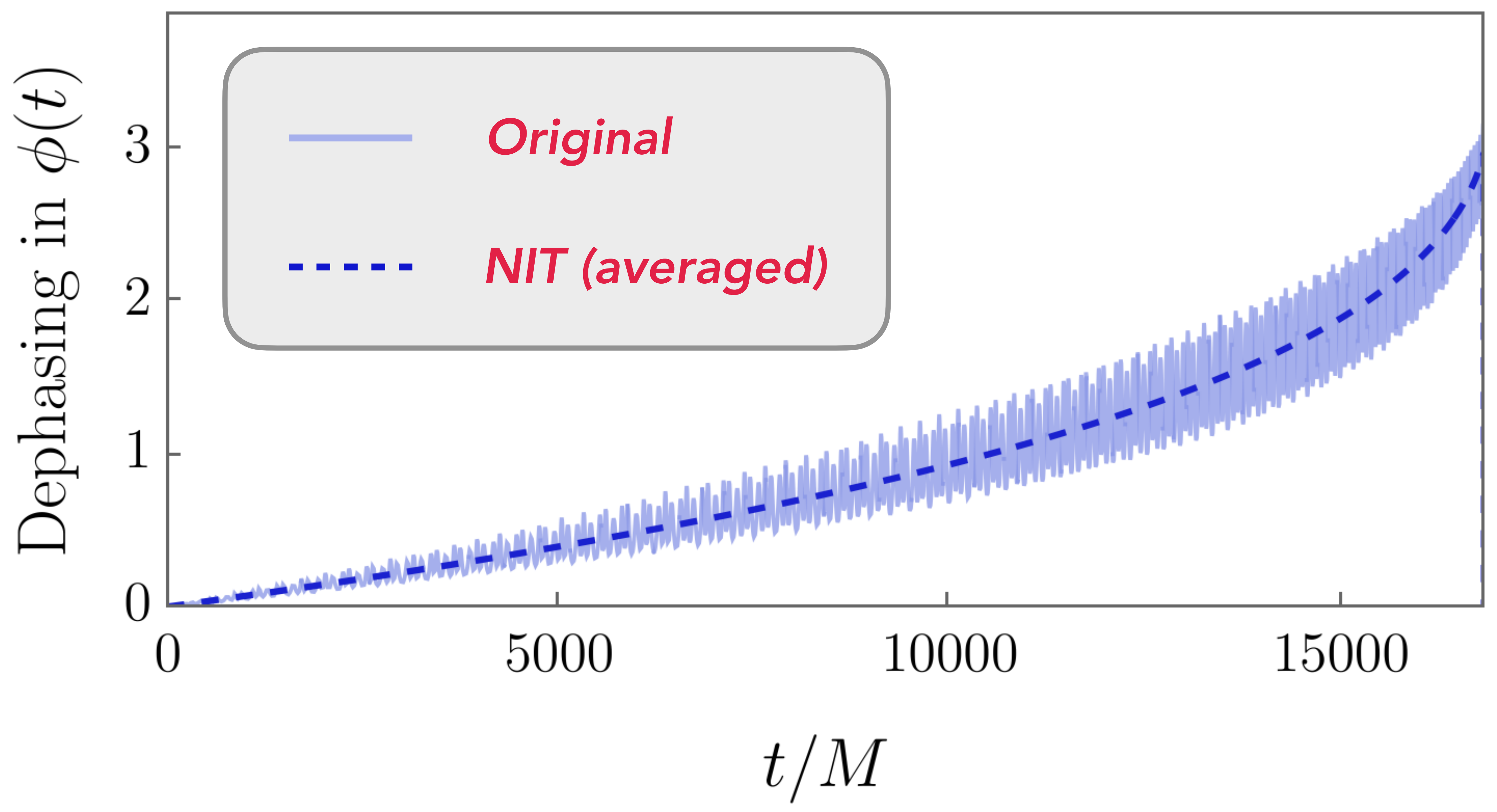
We use a generic **NIT (Near-Identity Transformation)** to isolate the long timescale evolution (as in Philip Lynch's talk on Tuesday)

$$P_i \to \tilde{P}_i$$
$$q_i \to \tilde{q}_i$$

$$\dot{P}_i = \tilde{F}_i(\tilde{P}_i)$$
$$\dot{q}_i = \Upsilon_i^{(0)}(\vec{P}) + \tilde{f}_i(\tilde{P}_i)$$

[Work in collaboration with Philip Lynch]

NIT: Near-Identity Transformation



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↓

★ Allows a **2-5 order of magnitude** speed up in computation

★ NIT averaged phases are the input we use for **generating waveforms**

[Work in collaboration with Philip Lynch]

Why use OG + NIT formulation for spinning secondary effects?

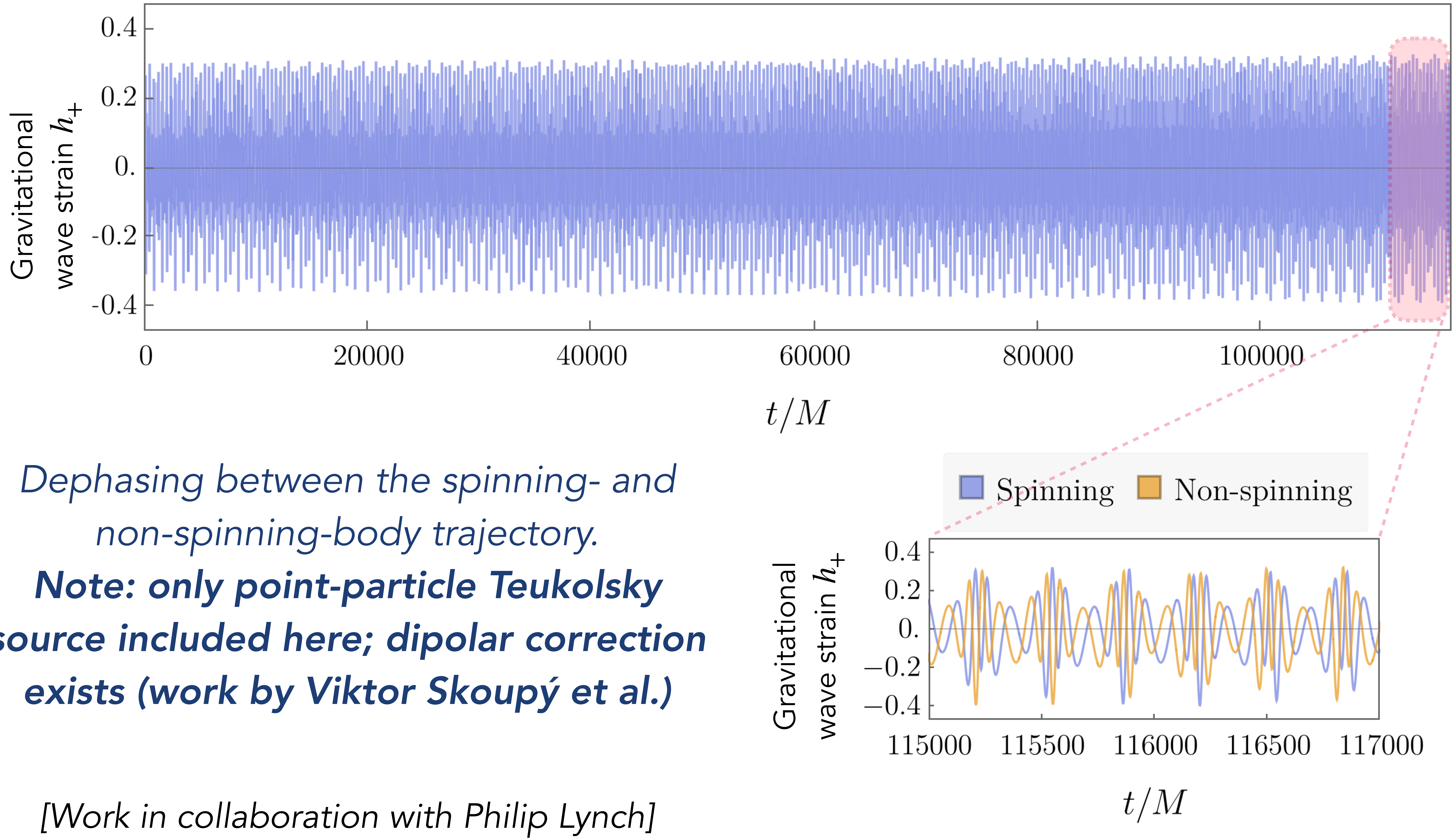
- Will **interface immediately** with NIT set-up for self-force (as described by in talk by Philip Lynch); same parameterization and framework

- Another formulation of spinning-body trajectories; another **independent cross-check**

- May be useful to consider contribution from conservative and dissipative spinning-body sectors **individually**

Equatorial waveform *with initial eccentricity of* $e = 0.5$

Waveforms for **equatorial** orbits



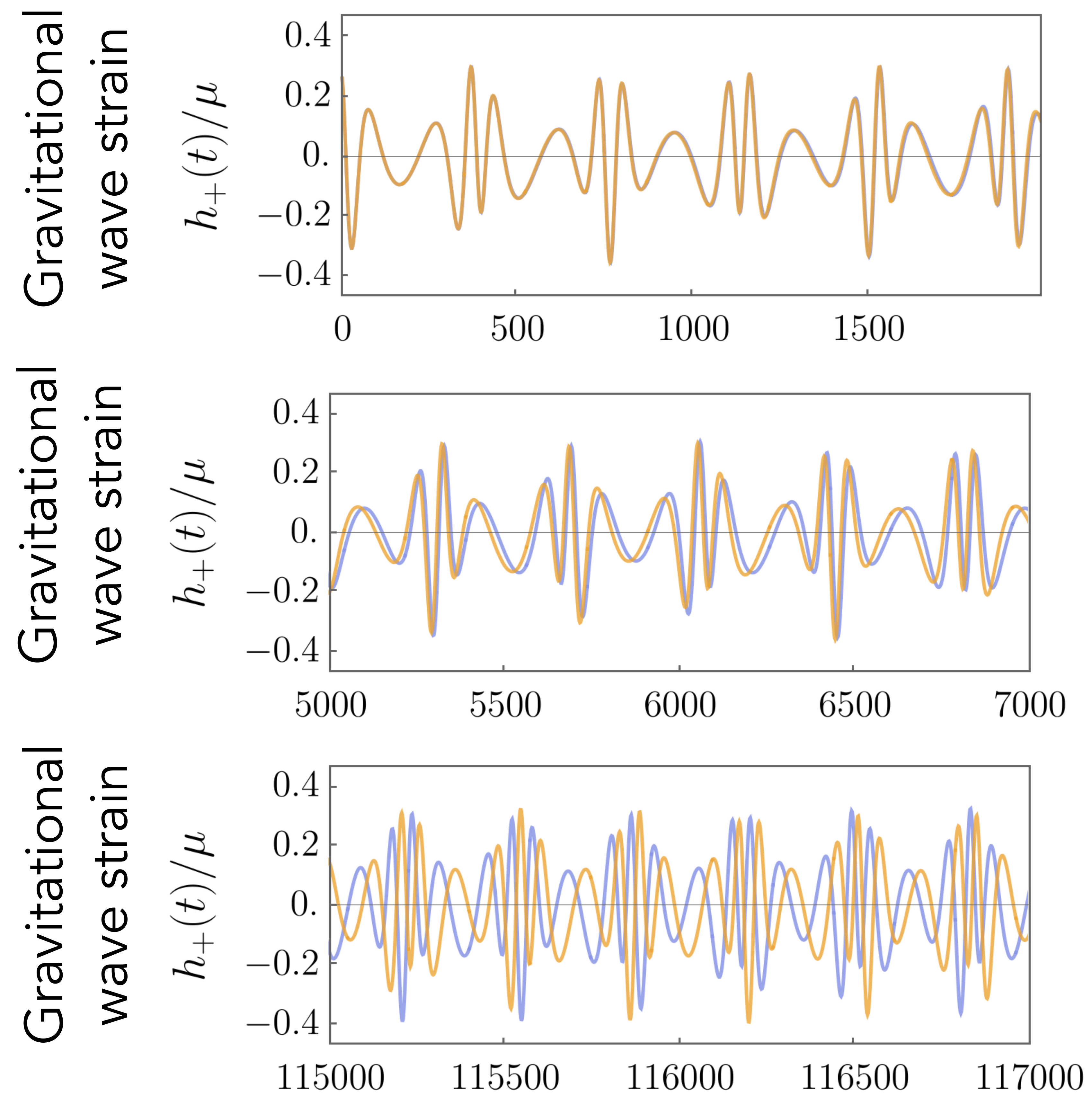
Dephasing between the spinning- and non-spinning-body trajectory.

Note: only point-particle Teukolsky source included here; dipolar correction exists (work by Viktor Skoupý et al.)

[Work in collaboration with Philip Lynch]

Waveforms for **generic** orbits

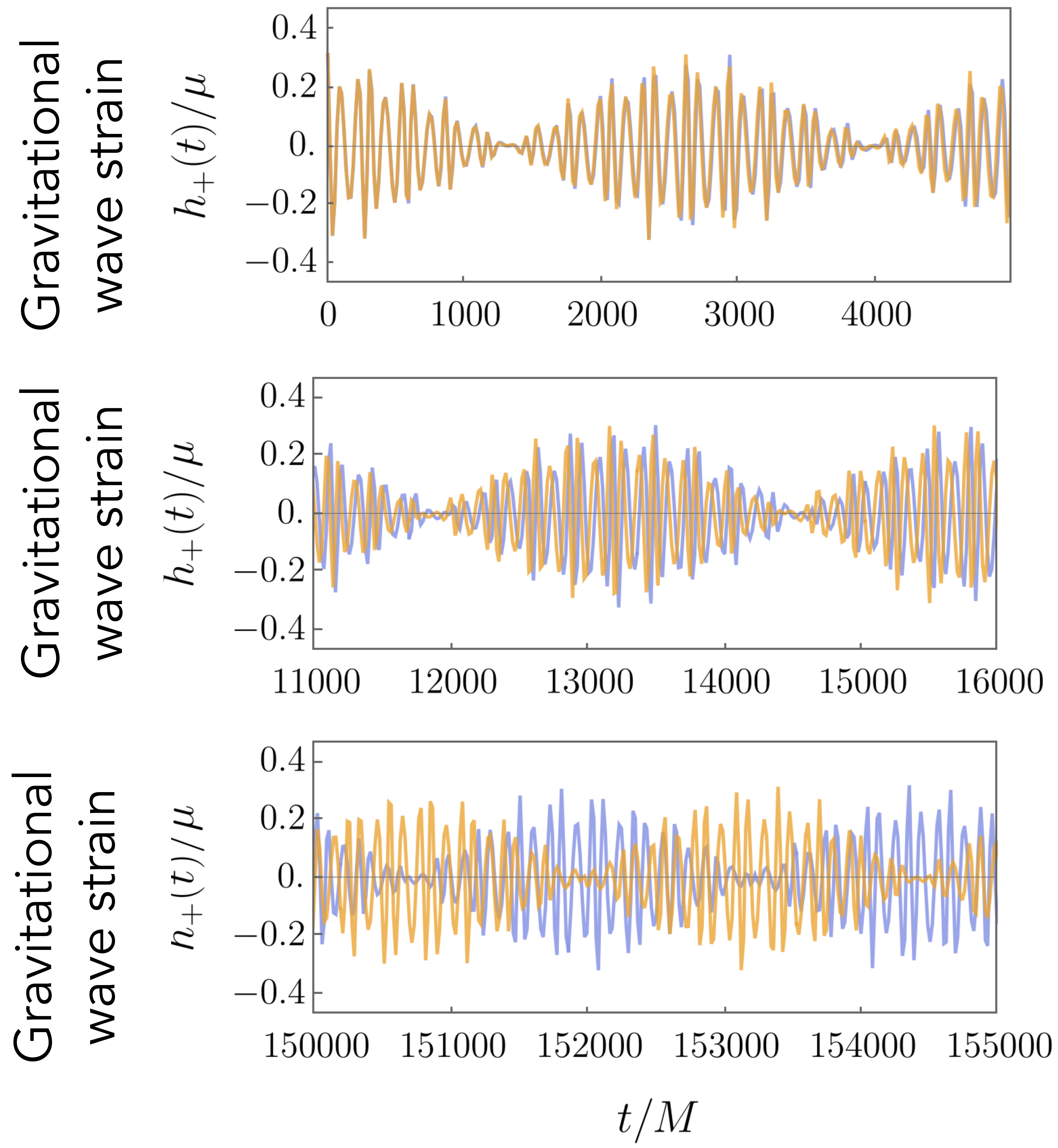
Equatorial waveform *with initial eccentricity of* $e = 0.5$



Spinning

Non-spinning

Generic waveform *with initial eccentricity of* $e = 0.2$



Some caveats

- **Not** a fully self-consistent model; only includes **some** of the spinning secondary effects. Everything is sufficiently modular that additional elements can be included straightforwardly.

Missing post-adiabatic elements: the GW fluxes due to the dipole term in the stress-energy tensor, first- and second- order self-force.

- Dephasings due to secondary spin contributions and self-force contributions **could cancel** each another in certain regions of parameter space

- Dephasings alone are **not sufficient** to determine detectability; need a full Bayesian parameter estimation analysis

Conclusions and some open questions...

★ We have computed **fully generic** spinning-body inspirals and waveforms with arbitrary spin alignment (**arXiv:2305.08919**) — but there are **missing elements**

★ **Natural next steps** are to include more of the missing elements: e.g., include the GW backreaction due to the dipole term in the stress-energy tensor into our framework. (Conduct a careful comparison with the calculations of Skoupý & Lukes-Gerakopoulos.)

★ **Detectability** of small-body spin for generic orbital configurations? Size of dephasing is not sufficient to assess detectability, need to do full Bayesian infererence study e.g., with ***few* (Fast EMRI waveforms)**

★ **Carter-like constant evolution** for spinning secondary?

Thank you!