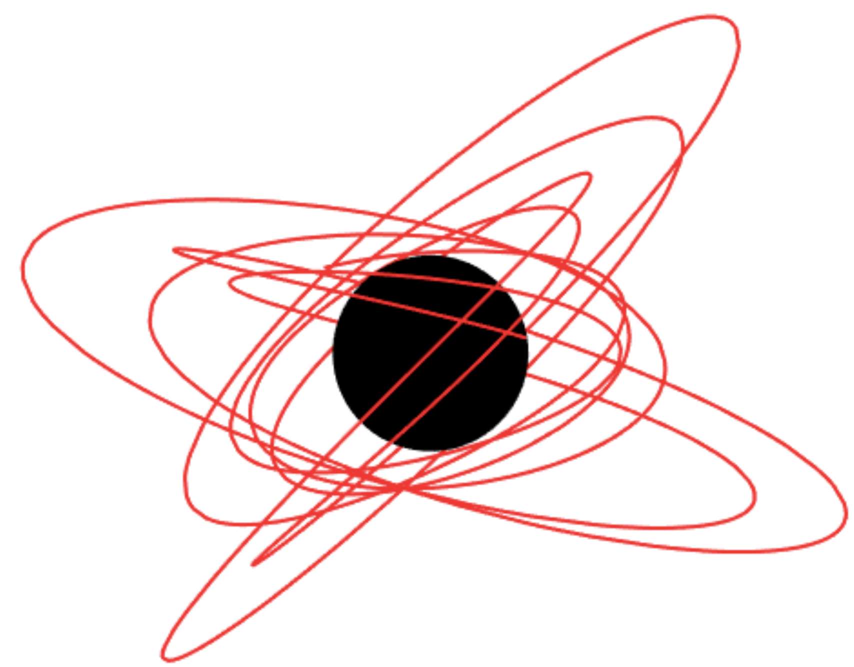
Detecting scalar fields with Extreme Mass Ratio Inspirals

XXVI Capra meeting
6 July 2023

- *S.B+ : arXiv:2212.03888 (PRL accepted)*
- *S.B+ : Phys.Rev.D 106 (2022) 4*
- *Phys. Rev. Lett 125, 141101 (2020)*
- *A.Maselli, SB+, Nature Astron. 6 (2022) 4, 464-470*



Collaboration with
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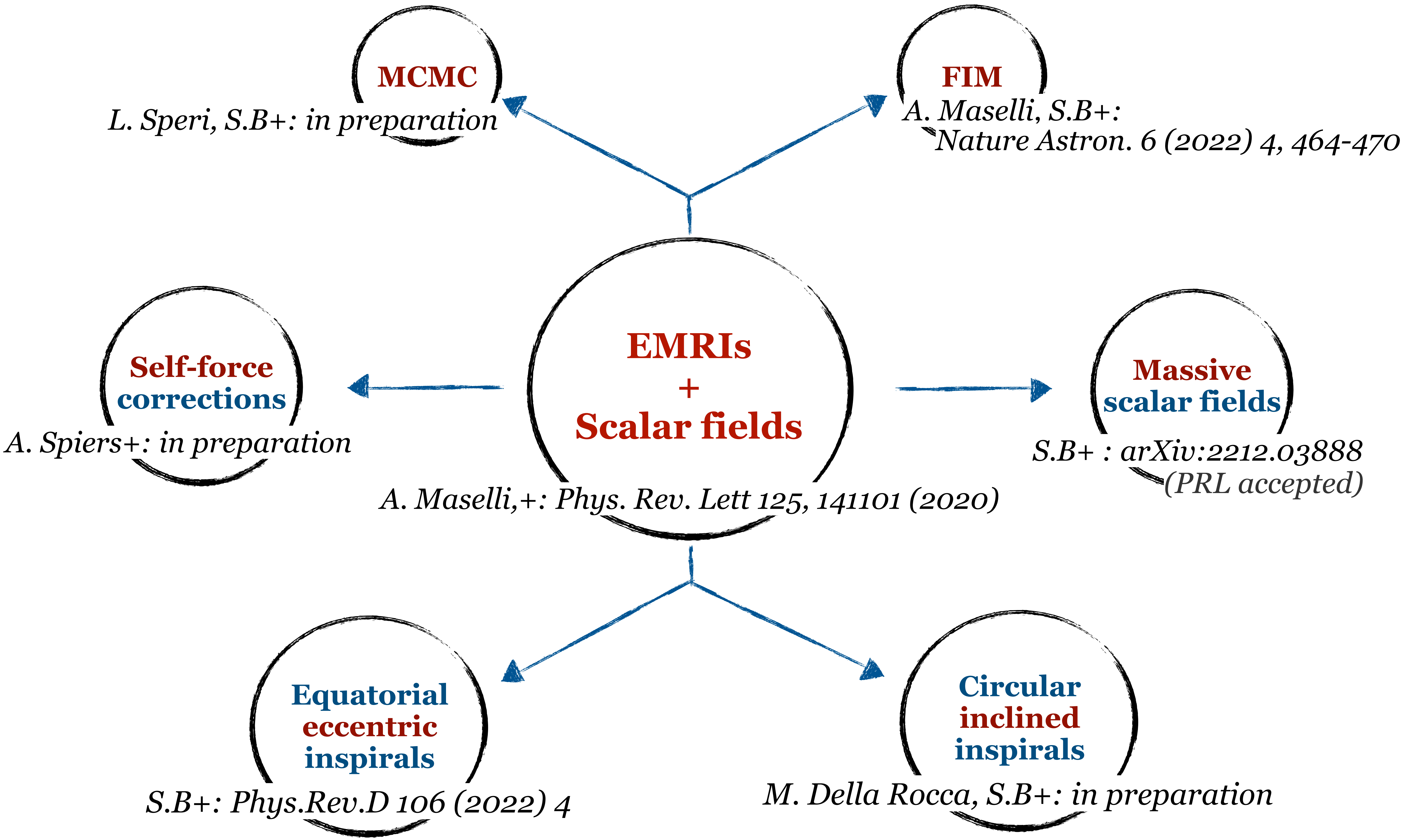
Mindset


MCMC
L. Speri, S.B+: in preparation
FIM
A. Maselli, S.B+:
Nature Astron. 6 (2022) 4, 464-470
Self-force corrections
A. Spiers+: in preparation
EMRIs
+
Scalar fields
A. Maselli,+: Phys. Rev. Lett 125, 141101 (2020)
Massive scalar fields
S.B+ : arXiv:2212.03888
(PRL accepted)
Equatorial eccentric inspirals
S.B+: Phys.Rev.D 106 (2022) 4
Circular inclined inspirals
M. Della Rocca, S.B+: in preparation

Theoretical framework

- Vast class of theories: AGNOSTIC APPROACH

$$S\left[\mathbf{g}, \varphi, \Psi\right] = S_0\left[\mathbf{g}, \varphi\right] + \alpha S_c\left[\mathbf{g}, \varphi\right] + S_m\left[\mathbf{g}, \varphi, \Psi\right]$$

- Leading order in q:

  Decoupled fields equations !

$$\begin{cases} G_{\mu\nu} = T^p_{\mu\nu} = 8\pi m_p \int \dfrac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \dfrac{dy^p_\mu}{d\lambda} \dfrac{dy^p_\nu}{d\lambda} d\lambda \\ \Box \varphi = -4\pi d\, m_p \int \dfrac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda \end{cases}$$

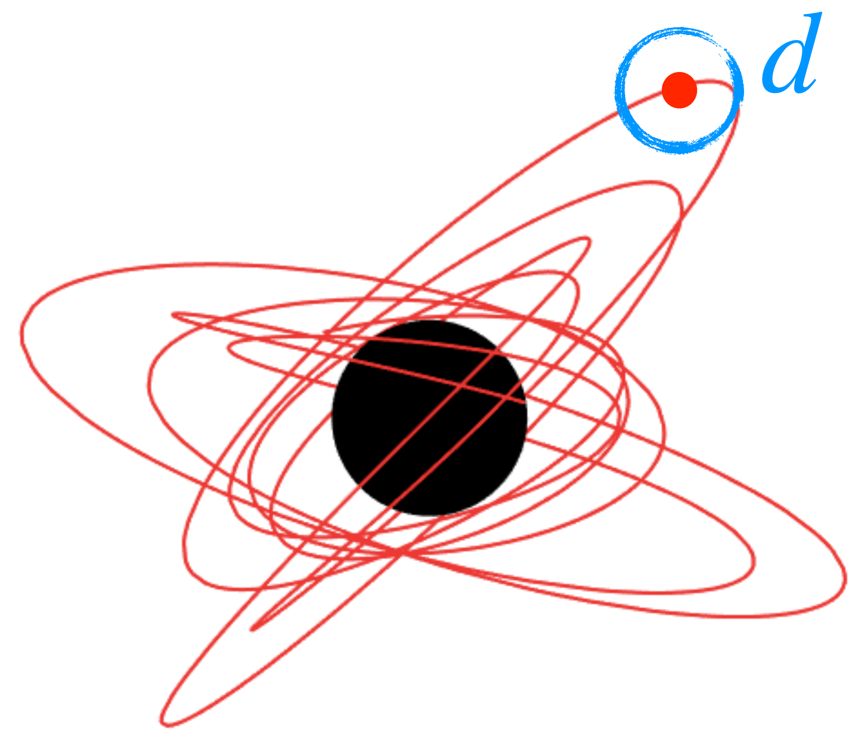
- Teukolsky formalism for the gravitational and scalar perturbations:

$$\boxed{\dot{E}_{GW} = \sum_{i=+,-} [\dot{E}^{(i)}_{\text{grav}} + \dot{E}^{(i)}_{\text{scal}}] = \dot{E}_{grav} + \dot{E}_{scal}} \longrightarrow \dot{E}_{scal} \propto d^2$$

EXTRA emission *simply added* to the gravitational one!

only depends on the scalar charge d

EMRIs + scalar fields

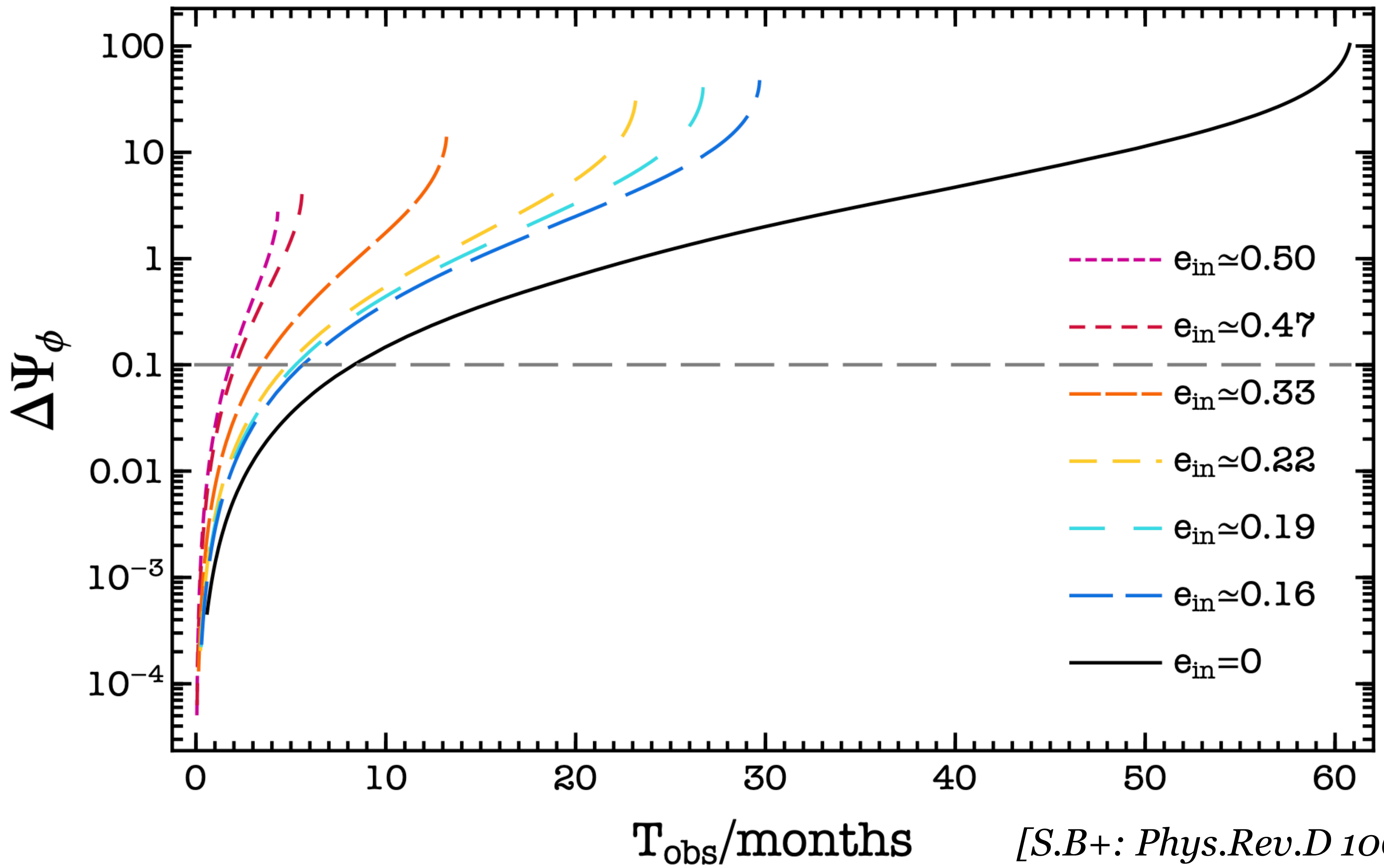


GR + Scalar fields

OUTLINE :

- *Energy emission trough gravitational and scalar waves*
- *<u>Adiabatic orbital evolution</u>* $\longrightarrow$ $\dot{E} = -\dot{E}_{GW}$
- *<u>Imprint</u> on the gravitational waves: dephasing, faithfulness, …*
- *Parameter estimation: FIM, MCMC, …*

Dephasing: equatorial eccentric orbits



[S.B+: Phys.Rev.D 106 (2022) 4]

- $M = 10^6 M_\odot$
  $m_p = 10\ M_\odot$
  $a = 0.9M$
- $r_a = 11M$
  $d = 0.01$

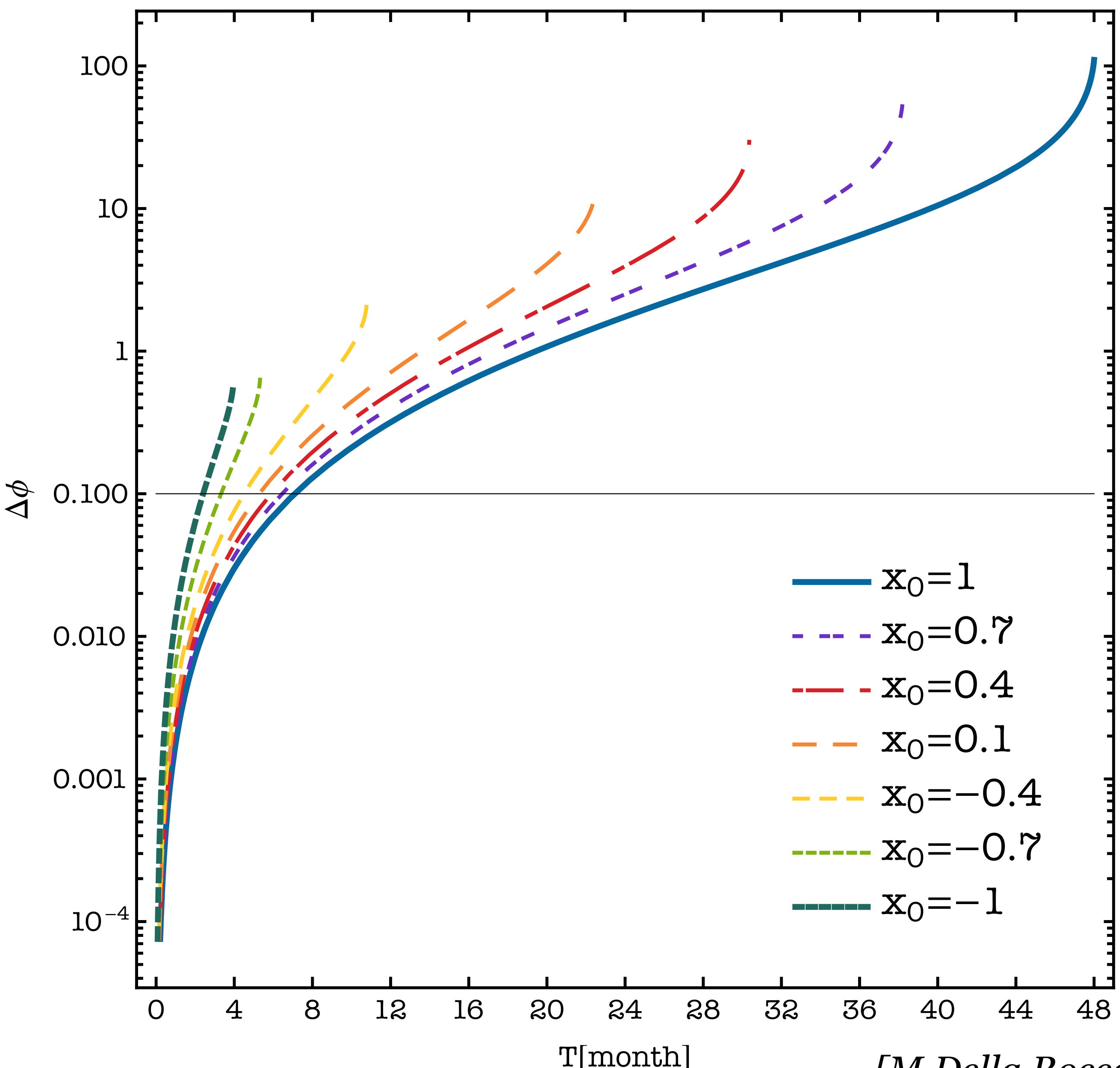
$$r(\chi) = \frac{p}{1 + e\cos\chi}$$

$$\Omega_r(e,p) = \frac{2\pi}{T_r} \qquad \Omega_\phi(e,p) = \frac{\Delta\phi}{T_r}$$

$$\Delta\Psi_\phi = 2\int_0^{T_{obs}} \Delta\Omega_\phi dt$$

- Horizontal dashed line: threshold of phase resolution by LISA of $\Delta\psi_\phi = 0.1$ for $SNR = 30$
- after 3-4 months all the inspirals lead to a dephasing larger then the threshold !
- for a given time of observation, $\Delta\Psi_\phi$ is larger for inspirals with higher e_{in}
- reducing e_{in} , the time it takes for the secondary to reach the plunge grows, leading to larger accumulated dephasings

Dephasing: inclined circular orbits



[M.Della Rocca, SB+, in preparation]

- $M = 10^6 M_\odot$
  $m_p = 10\ M_\odot$
  $a = 0.95M$
- $r_0 = 10M$
  $d = 0.01$

$$\theta_{min} \leq \theta \leq \pi - \theta_{min}$$

$$\theta_{\rm inc} + (\operatorname{sgn} L_z)\theta_{\rm min} = \frac{\pi}{2}$$

$$x = \cos\theta_{inc}$$

- Increasing x_0, the time it takes for the secondary to reach the plunge grows, leading to larger accumulated dephasings
- For a given time of observation, $\Delta\Psi_\phi$ is larger for inspirals with higher x_0
- After 3-4 months all the inspirals lead to a dephasing larger then the threshold !

GW template: Faithfulness

Waveform quadrupolar approximation:

$$h_{ij}^{TT} = \frac{2}{D}\left(P_{i\ell}P_{jm} - \frac{1}{2}P_{ij}P_{\ell m}\right)\ddot{I}_{\ell m}$$

$$I_{ij} = \int d^3x T^{tt}(t,x^i)x^i x^j = m_p x^i x^j$$

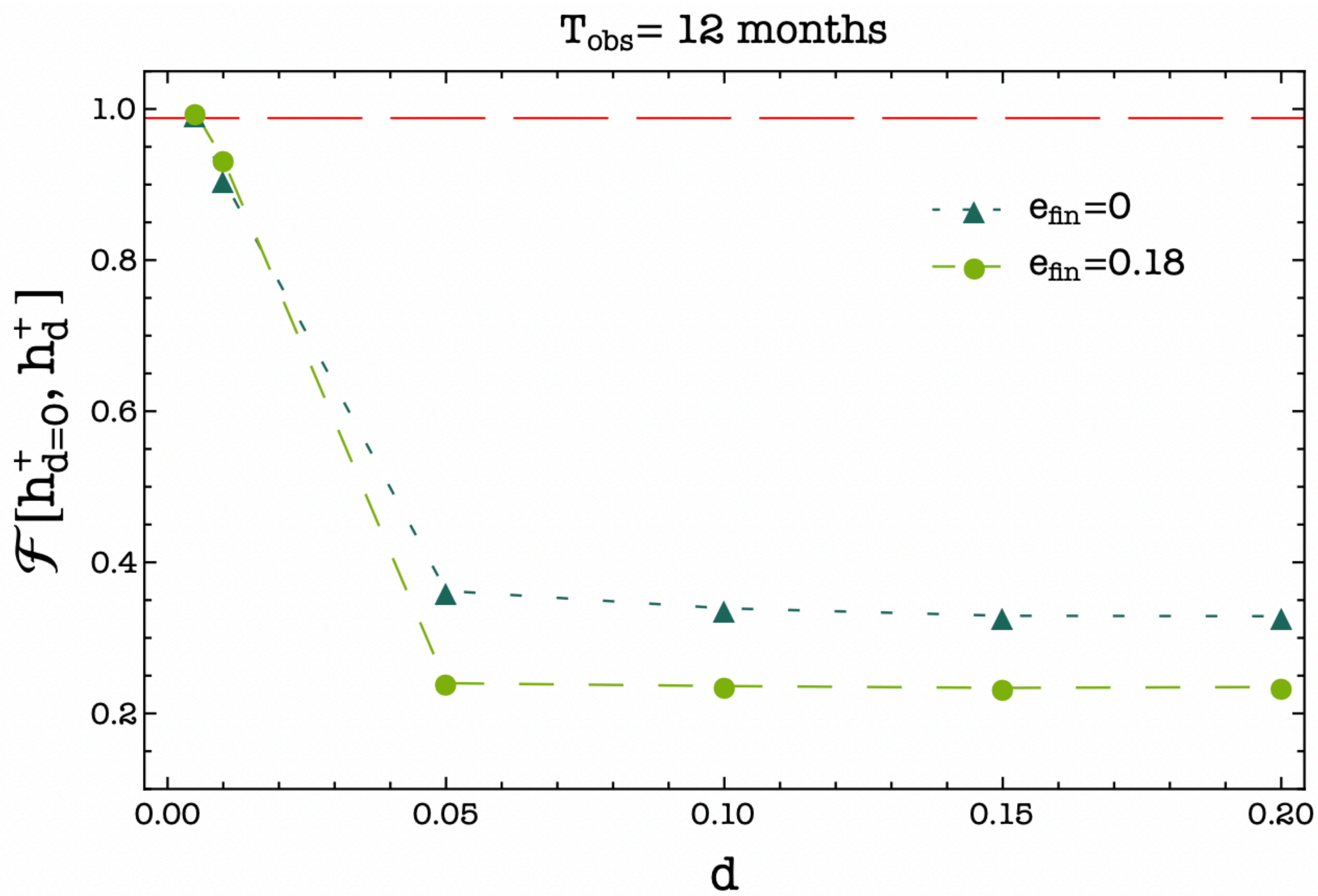
Estimate how much two signals differ:

$$\mathcal{F}[h_1,h_2] = \max_{\{t_c,\phi_c\}} \frac{\langle h_1|h_2\rangle}{\sqrt{\langle h_1|h_1\rangle\langle h_2|h_2\rangle}}$$

Inner product:

$$\langle h_1|h_2\rangle = 4\Re\int_{f_{min}}^{f_{\max}} \frac{\tilde{h}_1(f)\tilde{h}_2^\star(f)}{S_n(f)}df$$

LISA power spectral density



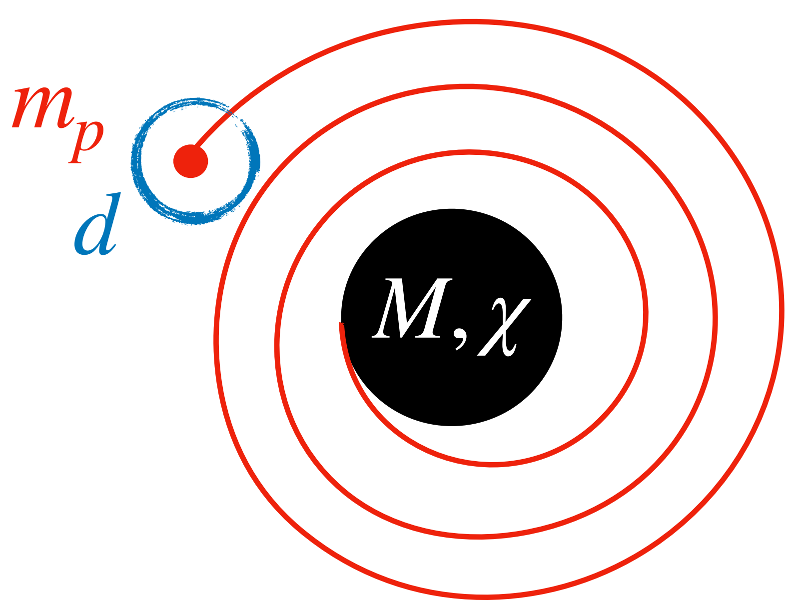
- Red line: threshold under which the signals are significantly different - $\boxed{\mathcal{F} \lesssim 0.988 \text{ for } SNR = 30}$
- After 1year $\mathcal{F}$ is always smaller than the threshold for scalar charges as small as $d = 0.01$
- For the eccentric inspirals the distinguishability increases, leading to a smaller $\mathcal{F}$

FIM: Fisher Information Matrix analysis

- Inject parameters to generate the waveform $\vec{\theta} = \left(\ln M, \ln m_p, \chi, \ln D, \theta_s, \phi_s, \theta_1, \phi_1, r_0, \Phi_0, d \right)$
- Fisher Information Matrix analysis

$$\Gamma_{ij} = \left\langle \frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right\rangle_{\theta=\hat{\theta}} \longrightarrow \mathbf{\Sigma} = \mathbf{\Gamma}^{-1} \longrightarrow \sigma_i = \Sigma_{ii}^{1/2} \quad , \quad c_{\theta_i \theta_j} = \Sigma_{ij}^{1/2} / (\sigma_{\theta_i} \sigma_{\theta_j})$$

- 1 year of observation before the plunge
- Equatorial circular inspiral



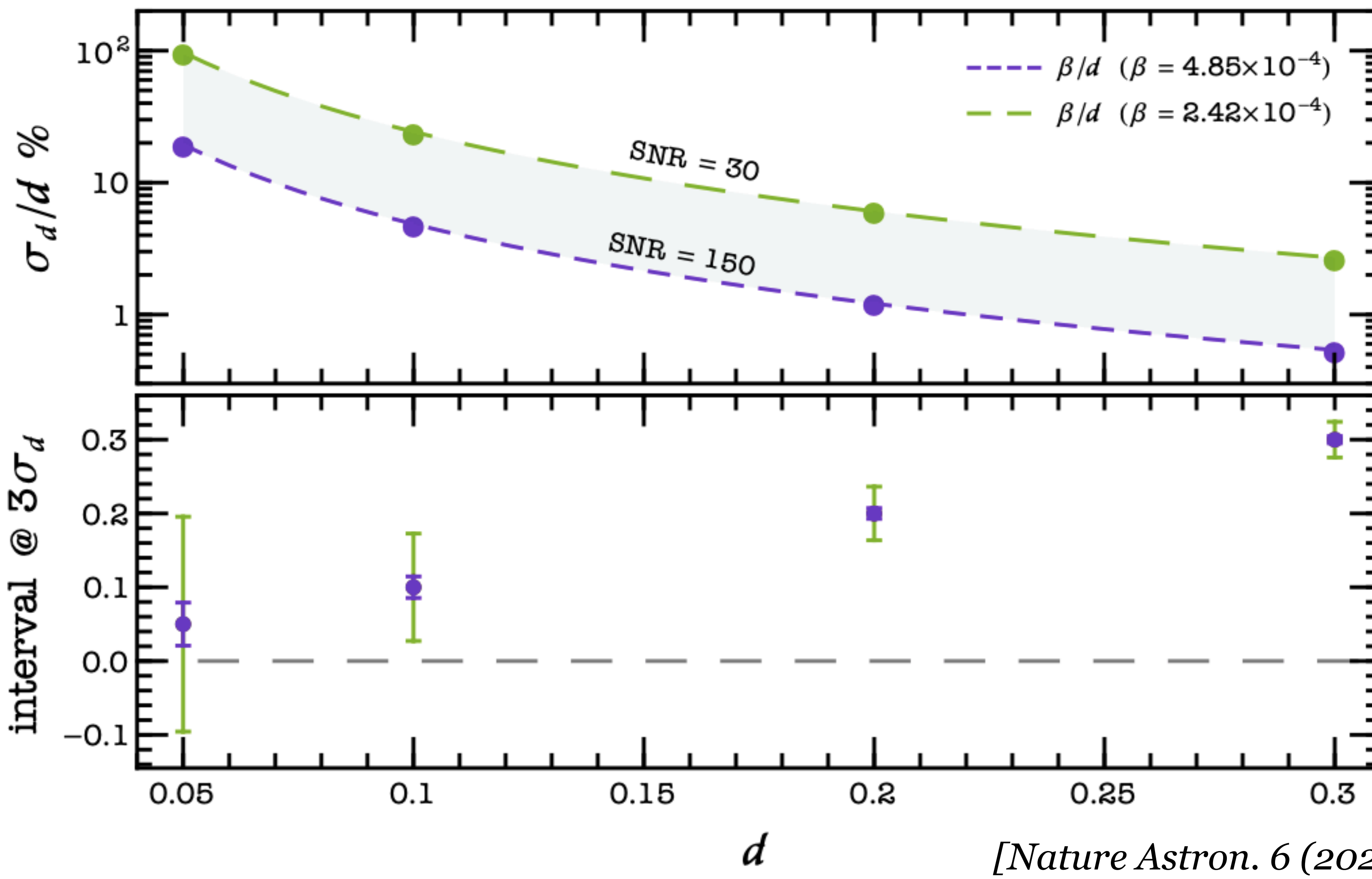
— Primary :

- $M/M_\odot = 10^6$
- $\chi = 0.9$

— Secondary :

- $m_p/M_\odot = 10$
- $d = (0.05,0.3)$

FIM: Relative error for the scalar charge



[Nature Astron. 6 (2022) 4, 464-470]

- Top: relative error on the scalar charge
- Bottom: $3-\sigma$ interval around the true values of the scalar charge

LISA potentially able to measure scalar charges with % error !

What about massive scalar fields?

Ultra-light scalar fields: energy emission

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi}\left(R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\mu_s^2\varphi^2\right) + \alpha S_c\left[\mathbf{g},\varphi\right] + S_m\left[\mathbf{g},\varphi,\Psi\right]$$

$$\left(\Box - \mu_s^2\right)\varphi = -4\pi d m_{\rm p}\int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}}d\lambda$$

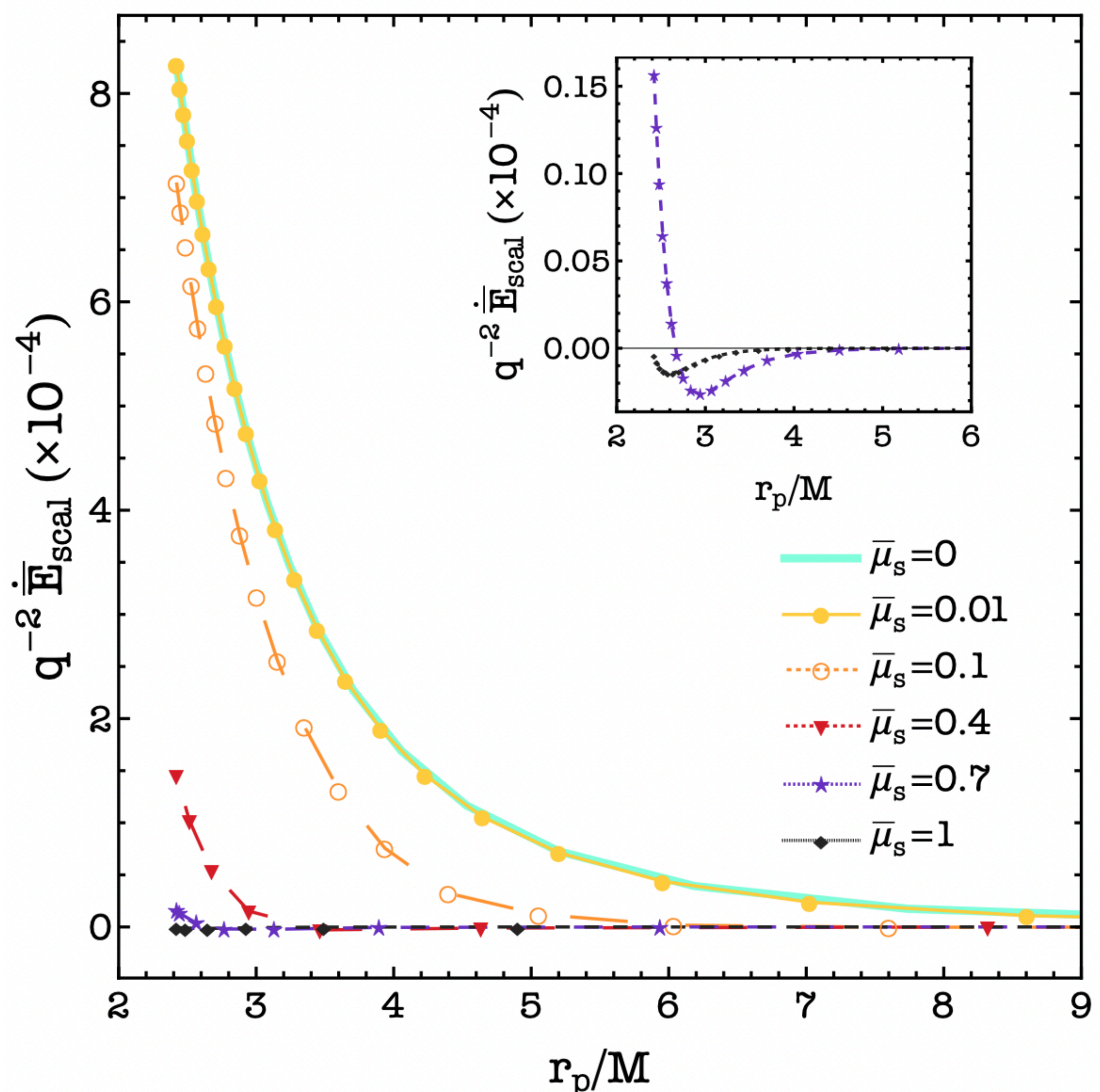
Energy emission:

- $\dot{E}_{scal} = d^2\dot{\bar{E}}_{scal}$
- $\bar{\mu}_s = \mu_s M$
- $\chi = a/M = 0.9$

- The flux at infinity *vanishes* for $\omega < \mu_s$

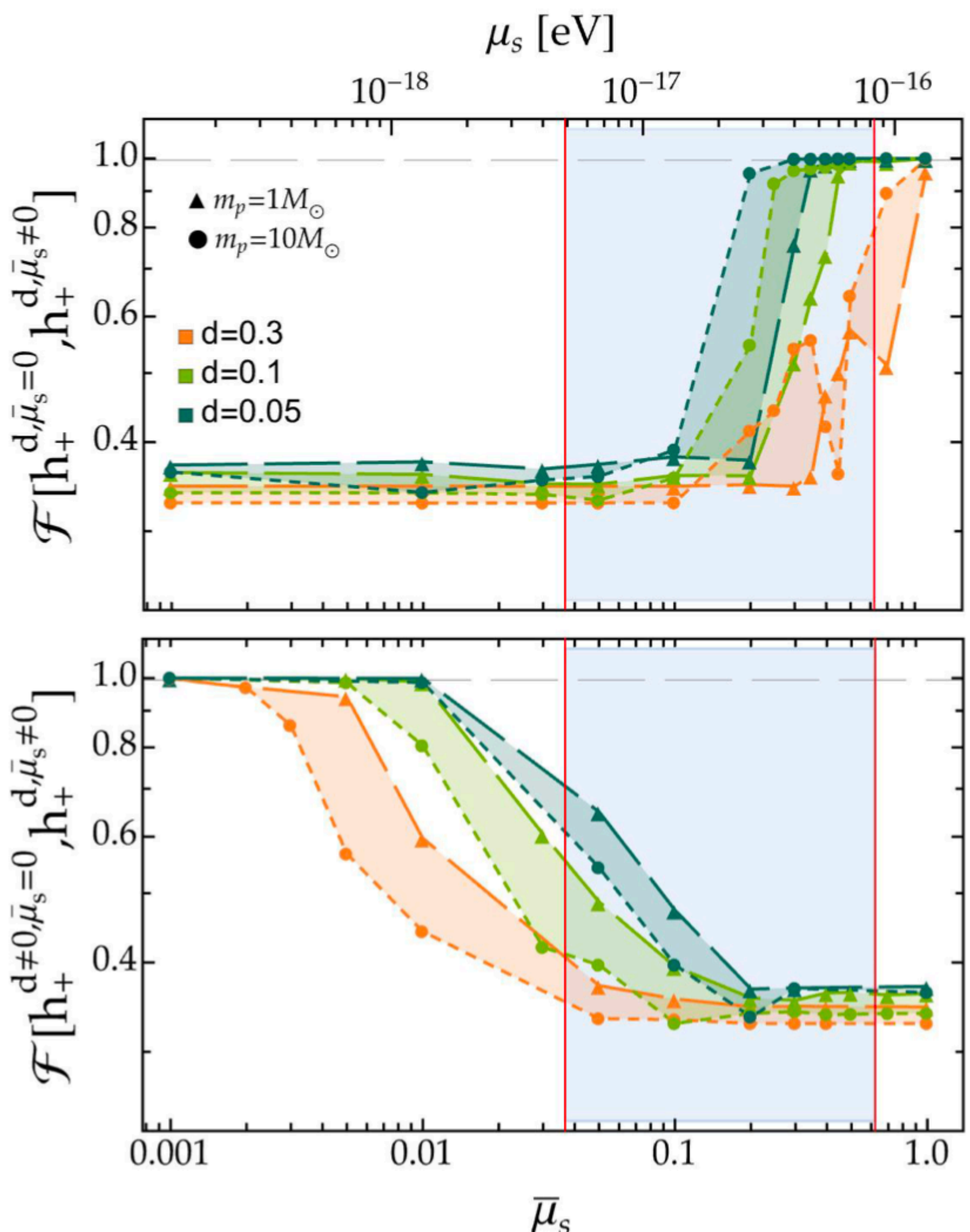
— For each (ℓ, m) exist r_s such that $\dot{E}^{\infty}_{scal}(r > r_s) = 0$

- The flux at the horizon is active during all the inspiral



The emitted GW flux drives the adiabatic orbital evolution

Massive scalar fields: faithfulness



[S.B +, PRL accepted]

- $a = 0.9M$ - $d = 0.1$
- $\mathscr{F}[h^+_{d=0}, h^+_{d\neq 0}]$: between a GR template and one with massive scalar fields
- $\mathscr{F}[h^+_{\mu_s=0}, h^+_{\mu_s\neq 0}]$: between templates with massive/massless scalar fields

- Shaded band: superradiance instability
  - $\chi = 0.9$ [Brito+, Lect.Notes Phys. 971 (2020) pp.1-293]
  - $M = 10^6 M_\odot$

To eV :

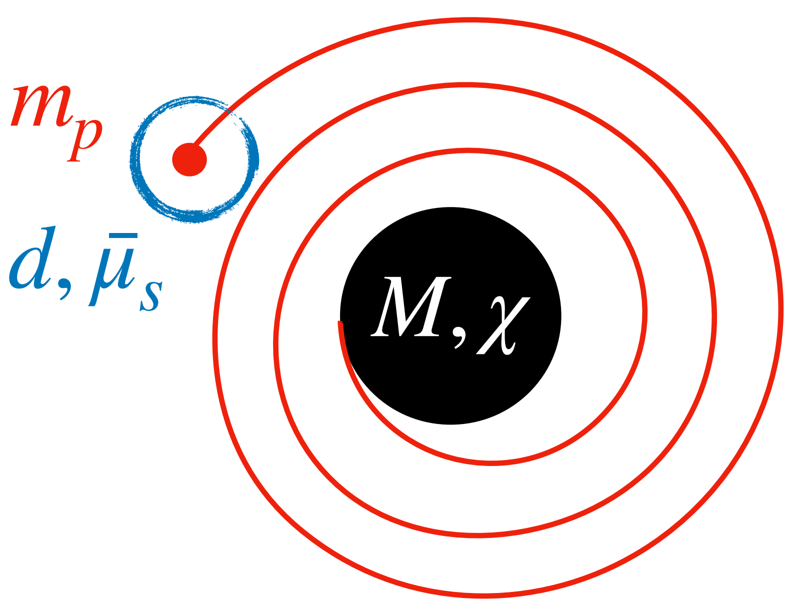
$$\left(\frac{\mu_s M}{0.75}\right) \cdot \left(\frac{10^6 M_\odot}{M}\right) 10^{-16}\ eV$$

FIM: Fisher Information Matrix analysis

- Inject parameters to generate the waveform $\vec{\theta} = \Big(\ln M, \ln m_p, \chi, \ln D, \theta_s, \phi_s, \theta_1, \phi_1, r_0, \Phi_0, d, \bar{\mu}_s \Big)$
- Fisher Information Matrix analysis

$$\Gamma_{ij} = \left\langle \frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right\rangle_{\theta=\hat{\theta}} \longrightarrow \mathbf{\Sigma} = \mathbf{\Gamma}^{-1} \longrightarrow \sigma_i = \Sigma_{ii}^{1/2} \quad , \quad c_{\theta_i \theta_j} = \Sigma_{ij}^{1/2} / (\sigma_{\theta_i} \sigma_{\theta_j})$$

- We considered just the dipole for the scalar emission $(\ell = 1)$
- 1 year of observation before the plunge

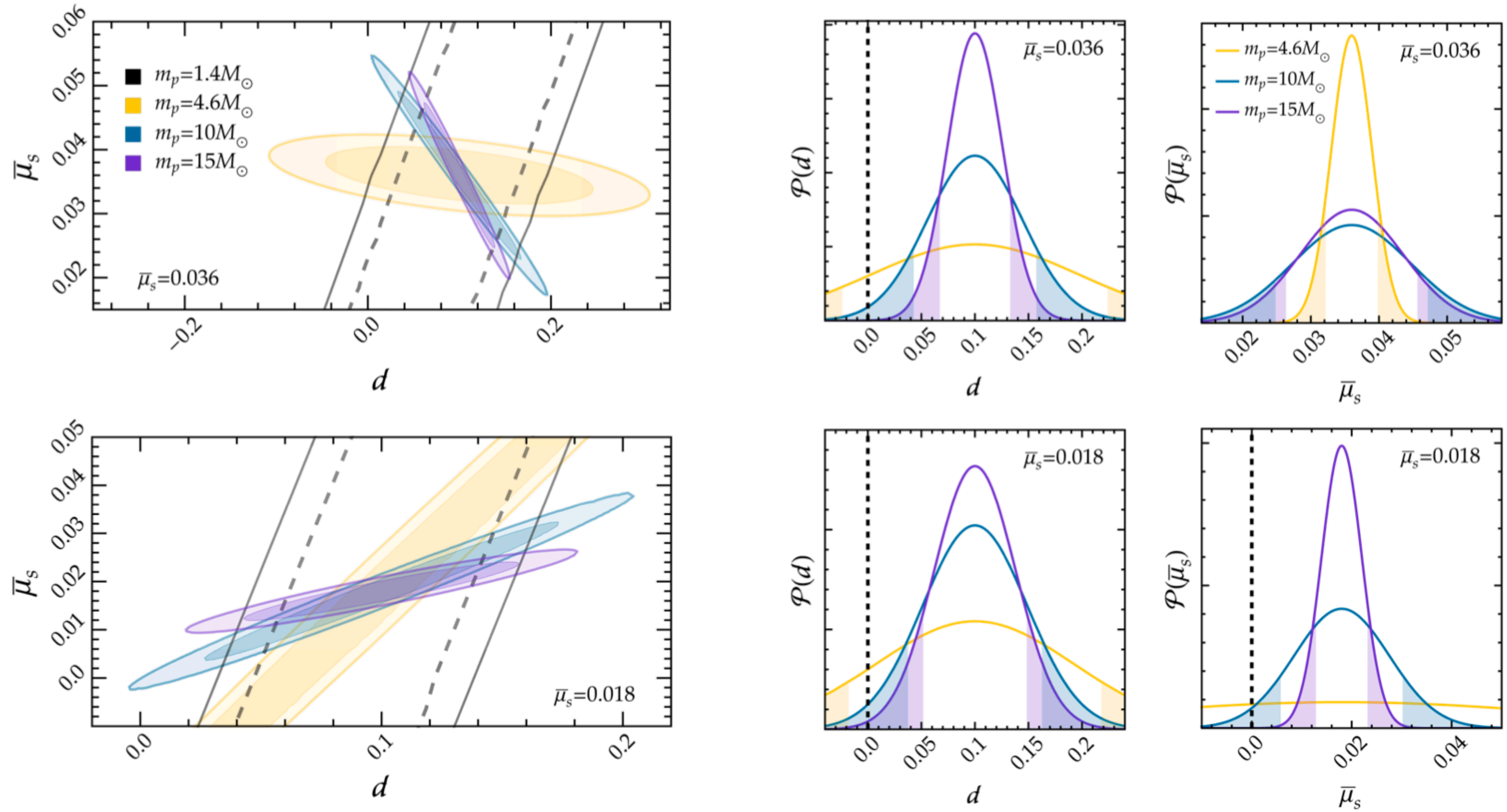


— Primary :
- $M/M_\odot = 10^6$
- $\chi = 0.9$

— Secondary :
- $m_p/M_\odot = 1.4,\ 4.6,\ 10,\ 15$
- $d = 0.1$
- $\bar{\mu}_s = 0.018,\ 0.036$

- The scalar flux at infinity is significant throughout the entire inspiral

FIM: scalar charge and mass detectability



Credible intervals at 68 % and 90 % for the joint posterior distribution of d, $\bar{\mu}_s$

Marginal distributions for d, $\bar{\mu}_s$

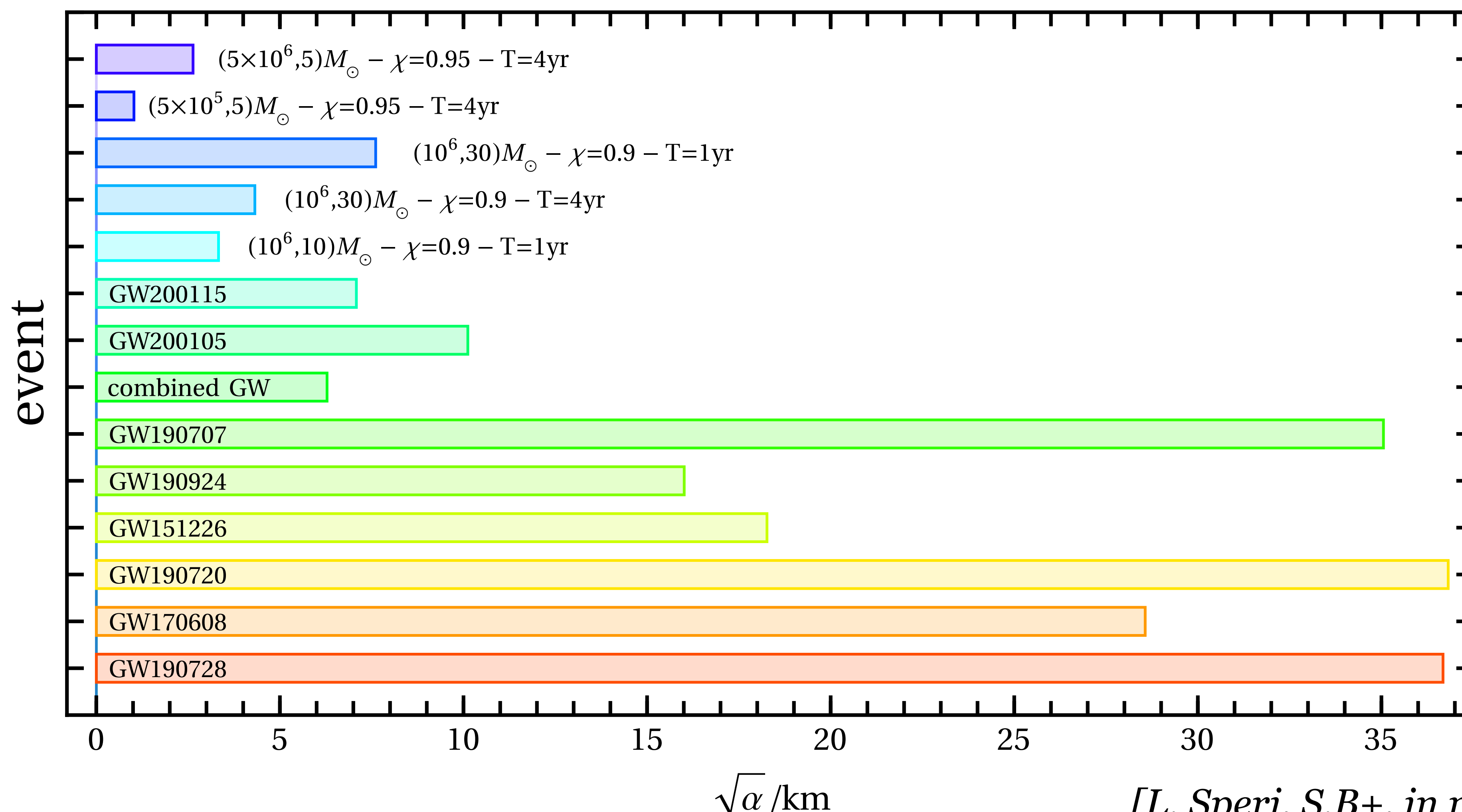
white area between shaded regions: 90 % of $\mathscr{P}$

$m_p[M_\odot]$	$\bar{\mu}_s$	σ_d/d	$\sigma_{\bar{\mu}_s}/\bar{\mu}_s$	$c_{d\bar{\mu}_s}$
4.6	0.018	92%	243%	0.995
	0.036	97%	8%	−0.485
10	0.018	49%	53%	0.984
	0.036	45%	24%	−0.990
15	0.018	38%	22%	0.938
	0.036	26%	21%	−0.986

***SIMULTANEOUS* detection of *BOTH* the scalar charge and mass with single event observations!**

Bayesian analysis: Markov Chain Monte Carlo

CIRCULAR INSPIRAL & MASSLESS FIELD



[L. Speri, S.B+, in preparation]

90% upper bound on the probability distribution of the sGB coupling constant for different EMRIs, compared against constraints currently available, inferred by nearly symmetric binaries

Conclusions

- EMRIs in a vast class of modified theories of gravity + scalar fields
- The extra energy loss modifies the binary evolution and leaves an imprint in the emitted GW
- The dephasing and the faithfulness show how scalar charges of $d \sim 0.01$ could be possibly detectable by LISA
- The Fisher analysis shows how LISA could be able to measure scalar charges with accuracy of the order of percent (massless) and to simultaneously detect both the scalar charge and mass of the new ultra-light scalar field (massive)

To look forward ..

→ Easy extensions to multiple fields and couplings

→ Self force corrections …. **stay tuned with Andrew Spiers talk!**

Thank you for attention

Back up

Field equations

$$S\left[\mathbf{g},\varphi,\Psi\right] = S_0\left[\mathbf{g},\varphi\right] + \alpha S_c\left[\mathbf{g},\varphi\right] + S_m\left[\mathbf{g},\varphi,\Psi\right]$$

$\boxed{\zeta \ll 1}$

$S_c \sim M^{-n} S_0$

$\dfrac{\delta S}{\delta g^{\mu\nu}}$

$$G_{\mu\nu} = \frac{1}{2}\partial_\mu\varphi_1\partial_\nu\varphi_1 - \cancel{\frac{1}{4}g_{\mu\nu}\left(\partial\varphi_1\right)^2} - \frac{1}{4}g_{\mu\nu}\mu_s^2\varphi_1^2 \qquad \cancel{-\frac{16\pi\alpha}{\sqrt{-g}}\frac{\delta S_c}{\delta g^{\mu\nu}} \sim \zeta G_{\mu\nu}} \qquad +8\pi\int m\left(\varphi\right)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}\frac{dy_p^\alpha}{d\lambda}\frac{dy_p^\beta}{d\lambda}d\lambda$$

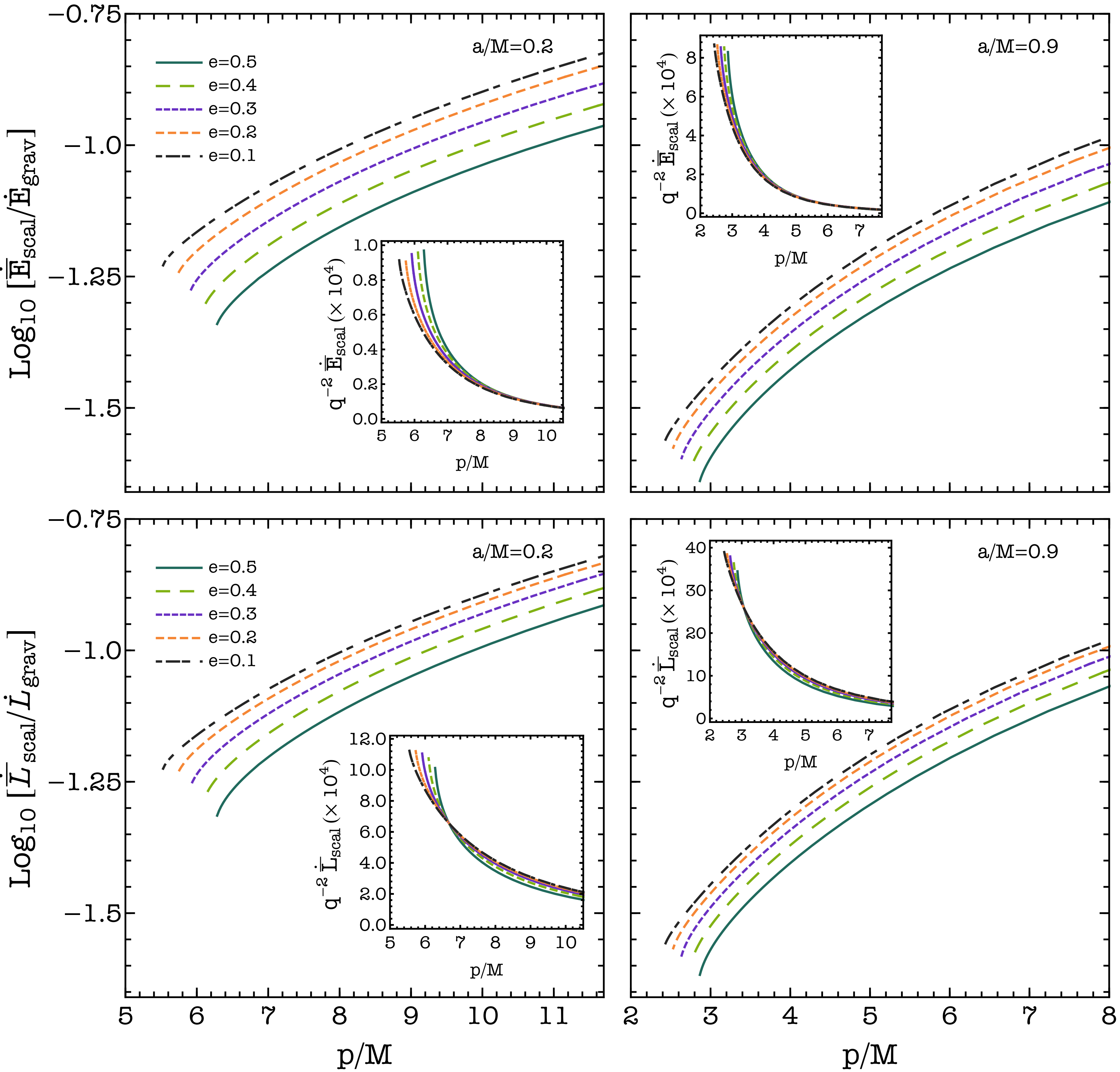
$\dfrac{\delta S}{\delta \varphi}$

$$\left(\Box - \mu_s^2\right)\varphi = \cancel{-\frac{16\pi\alpha}{\sqrt{-g}}\frac{\delta S_c}{\delta\varphi} \sim \zeta\,\Box\varphi} \qquad +16\pi\int m'\left(\varphi\right)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}d\lambda$$

- m, m' to be evaluated at φ_0
- In a reference frame centered on the particle : $\varphi = \dfrac{m_p\, d}{\tilde{r}}e^{-\mu_s\tilde{r}} + O\left(\dfrac{m_p^2}{\tilde{r}^2}e^{-\mu_s\tilde{r}}\right)$
- Matching with the scalar field eq. outside the world tube:
- (tt)-stress energy tensor in the weak field limit: matter density:

$$\boxed{\begin{aligned} m'(\varphi_0) &= -\frac{d}{4}m_p \\ m(\varphi_0) &= m_p \end{aligned}}$$

Energy flux: eccentric orbits



$$\text{Rel. Diff.} = \frac{\dot{E} - \dot{E}_{grav}}{\dot{E}_{grav}} = \frac{\dot{E}_{scal}}{\dot{E}_{grav}}$$

For a fixed e:

The Rel. Diff. decreases for smaller p, due to faster growth of $\dot{E}_{grav}$ and $\dot{L}_{grav}$w.r.t. to the scalar sector

For a fixed p:

The scalar energy flux increases with the eccentricity

The Rel. Diff. decreases with the increasing of eccentricity

Orbital Evolution

The emitted GW flux drives the adiabatic orbital evolution

- Balance law $\dot{E} = -\dot{E}_{GW}$ & $\dot{L} = -\dot{L}_{GW}$

- From the rate of change of the integrals (E, L), we obtain the time derivatives of (p, e)

$$\dot{p} = (L_{,e}\dot{E} - E_{,e}\dot{L})/H \qquad H = E_{,p}L_{,e} - E_{,e}L_{,p}$$

$$\dot{e} = (E_{,p}\dot{L} - L_{,p}\dot{E})/H$$

- And of the phases $\psi_{\phi,r}$ related to the frequencies $\Omega_{\phi,r}(e,p) = \frac{d}{dt}\Psi_{\phi,r}$

- The extra emission accelerates the binary coalescence and affects the GW phase, causing a **dephasing** w.r.t the case $d = 0$

- Compute the dephasing

$$\Delta\Psi_i = 2\int_0^{T_{obs}} \Delta\Omega_i dt \qquad i = \phi, r$$

$$\Delta\Omega_i = \Omega_i^d - \Omega_i^{d=0}$$

GW Signal

- Quadrupolar approximation

$$h_{ij}^{TT} = \frac{2}{D}\left(P_{i\ell}P_{jm} - \frac{1}{2}P_{ij}P_{\ell m}\right)\ddot{I}_{\ell m}$$

[L. Barack and C. Cutler, Phys. Rev. D 69 (2004) 082005]

$$I_{ij} = \int d^3x T^{tt}(t, x^i)x^i x^j = m_p x^i x^j$$

- Strain measured by the detector

$$h(t) = \sum_n h_n(t) \qquad h_n(t) = \frac{\sqrt{3}}{2}\left[F^+(t)A_n^+(t) + F^\times(t)A_n^\times(t)\right]$$

LISA pattern functions

$$F_+ = \frac{1+\cos^2\theta}{2}\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi$$

$$F_\times = \frac{1+\cos^2\theta}{2}\cos 2\phi\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi$$

Amplitudes

$$A_n^+ = -[1+(\hat{L}\cdot\hat{N})^2][a_n\cos(2\gamma) - b_n\sin(2\gamma)] + [1-(\hat{L}\cdot\hat{N})^2]c_n$$

$$A_n^\times = 2(\hat{L}\cdot\hat{N})[b_n\cos(2\gamma) + a_n\sin(2\gamma)]$$

$$a_n = -n\boxed{\mathscr{A}}\left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + (2/n)J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne)\right]\cos[n\Phi(t)]$$

$$b_n = -n\boxed{\mathscr{A}}(1-e^2)^{1/2}[J_{n-2}(ne) - 2J_n(ne) + J_{n+2}(ne)]\sin[n\Phi(t)]$$

$$c_n = 2\boxed{\mathscr{A}}J_n(ne)\ \cos[n\Phi(t)]$$

$$\downarrow$$

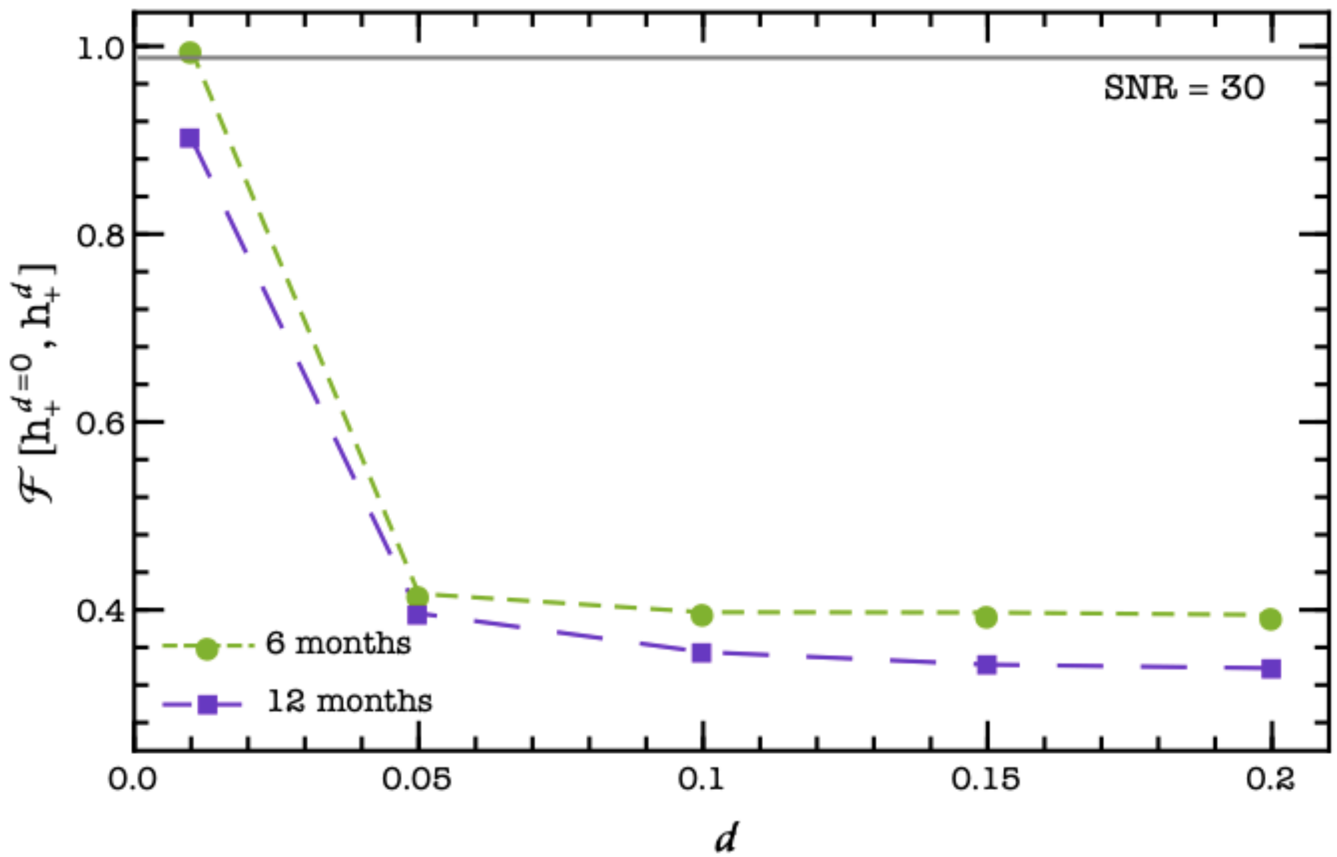
$$(2\pi\nu M)^{2/3}m_p/D$$

$$\begin{cases} 2\pi\nu = d\Phi/dt \\ \Phi = \Psi_\phi \\ \cos\gamma = \cos\Psi_R \end{cases}$$

Faithfulness

Estimate of how much two signals differ

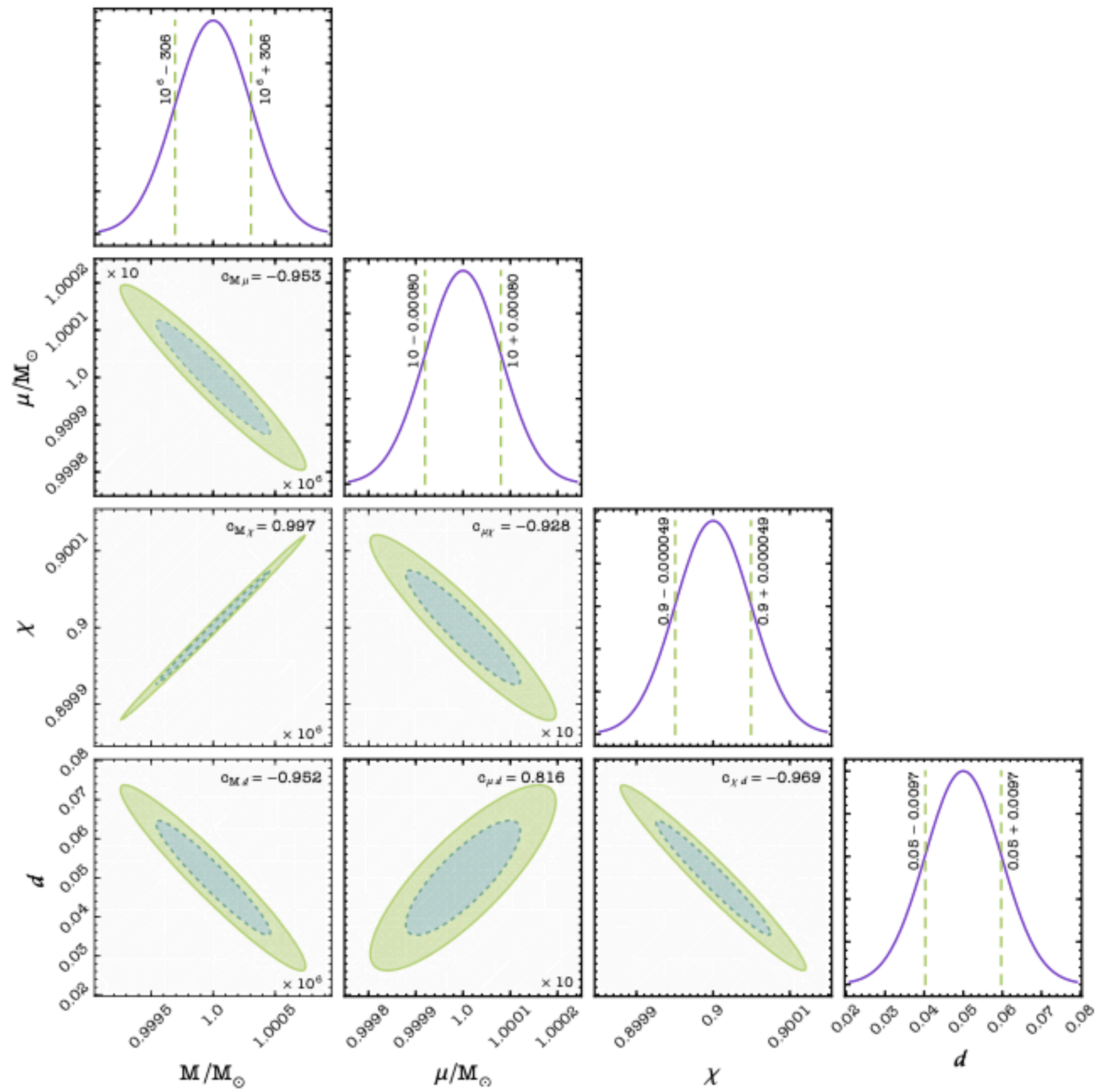
$$\mathcal{F}[h_1, h_2] = \max_{\{t_c, \phi_c\}} \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_2 | h_2 \rangle}}$$



- Grey line: threshold under which the signals are significantly different and don't provide a faithful description of one another

- After one year the faithfulness is always smaller than the threshold set by $SNR = 30$, even for scalar charges as small as $d = 0.01$

Probability distribution



- Corner plot of the probability distribution of (M, μ, χ, d), after 12 months of observation, with $d = 0.05$ and $SNR = 150$

- Vertical lines: 1-σ distribution for each waveform parameters

- Colored contours: 68 % and 95 % probability confidence intervals

- Measurement of the scalar charge with a relative error smaller than 10 % , with a probability distribution that does not have any support on $d = 0$ at more than 3-σ

- Scalar charge d highly correlated with μ and anti-correlated with M and χ

From the scalar charge to the coupling constant !

For theories with hairy BHs, it is possible to find a relation $\mathbf{d}(\alpha)$

Example of theories: scalar Gauss-Bonnet gravity (sGB)

$$\alpha S_c = \frac{\alpha}{4} \int d^4x \frac{\sqrt{-g}}{16\pi} f(\varphi)\, \mathscr{G}$$

$[\alpha] = (\mathrm{mass})^{\mathrm{n}}$

- n=2
- Dimensionless coupling constant $\beta \equiv \alpha / m_p^2$
- Gauss-Bonnet invariant $\mathscr{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$

$\longrightarrow \quad f(\varphi) = e^{\varphi} \quad \longrightarrow \quad d = 2\beta + \dfrac{73}{30}\beta^2 + O(\beta^3)$

$\longrightarrow \quad f(\varphi) = \varphi \quad \longrightarrow \quad d = 2\beta + \dfrac{73}{60}\beta^3 + O(\beta^4)$

bounds on d can be translated to bounds on β

Coupling constant

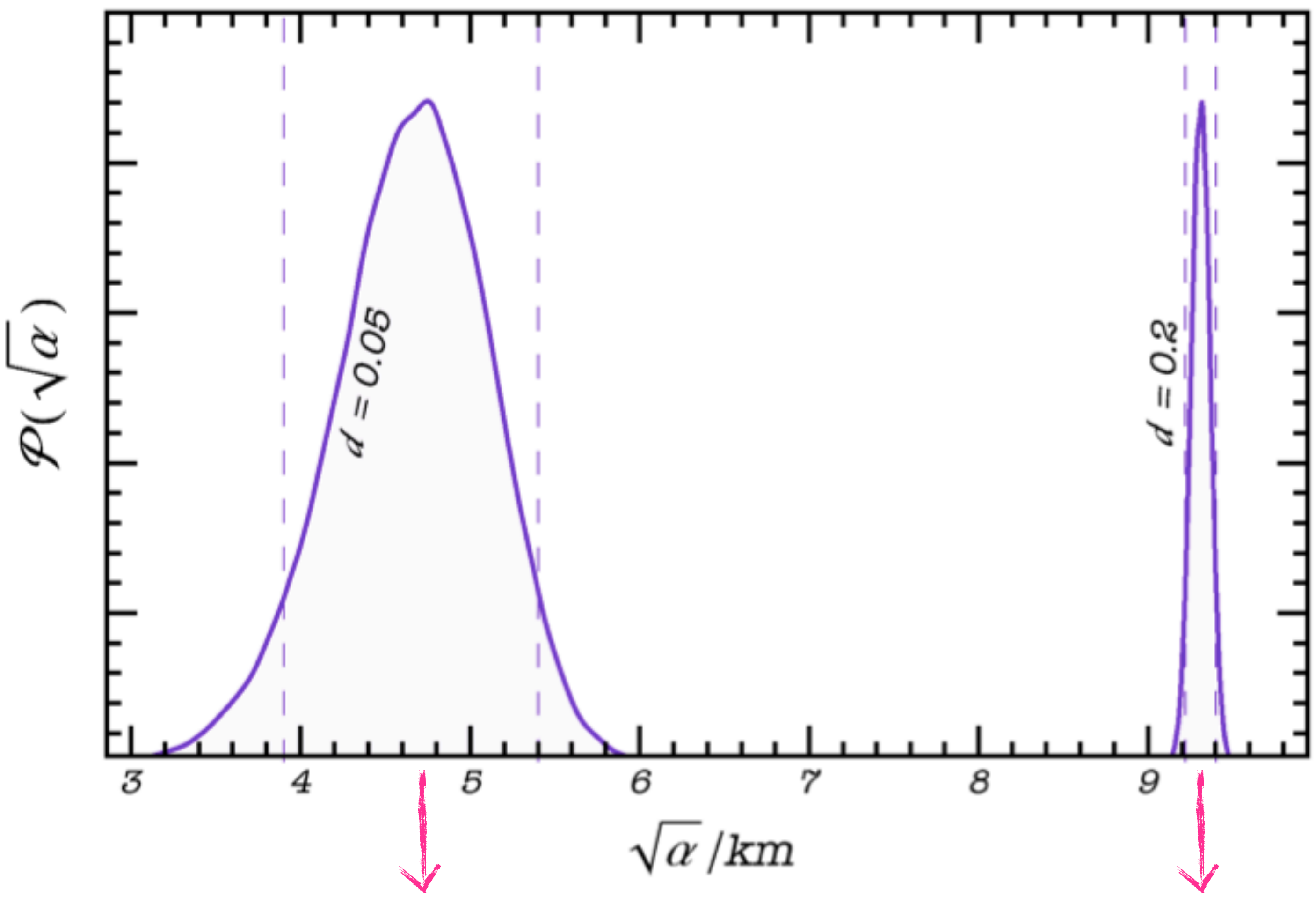
For hairy BHs, if the little body is a BH, we find a relation $d(\alpha)$

Shift-symmetric Gauss Bonnet gravity

$$S_c = \frac{\alpha}{4}\int d^4x \frac{\sqrt{-g}}{16\pi} f(\varphi)\mathcal{G}$$

$$f(\varphi) = \varphi$$

$$\alpha \simeq 2d\mu^2 - \frac{73}{240}d^3\mu^2$$



$\sqrt{\alpha}/km = 4.67^{+0.73}_{-0.77}$ $\sqrt{\alpha}/km = 9.312^{+0.092}_{-0.092}$

- Probability density function of $\sqrt{\alpha}$ obtained from the joint probability distribution of μ and d obtained from the Fisher analysis (SNR=150)
- Vertical lines: 90 % confidence interval
- Even for $d = 0.05$, the probability density functions do not have support with $\alpha = 0$