@BHPToolkit

The Black Hole Perturbation Toolkit

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Outline

- Overview of BHPToolkit
- Major components
- Latest release
- In the pipeline

http://bhptoolkit.org

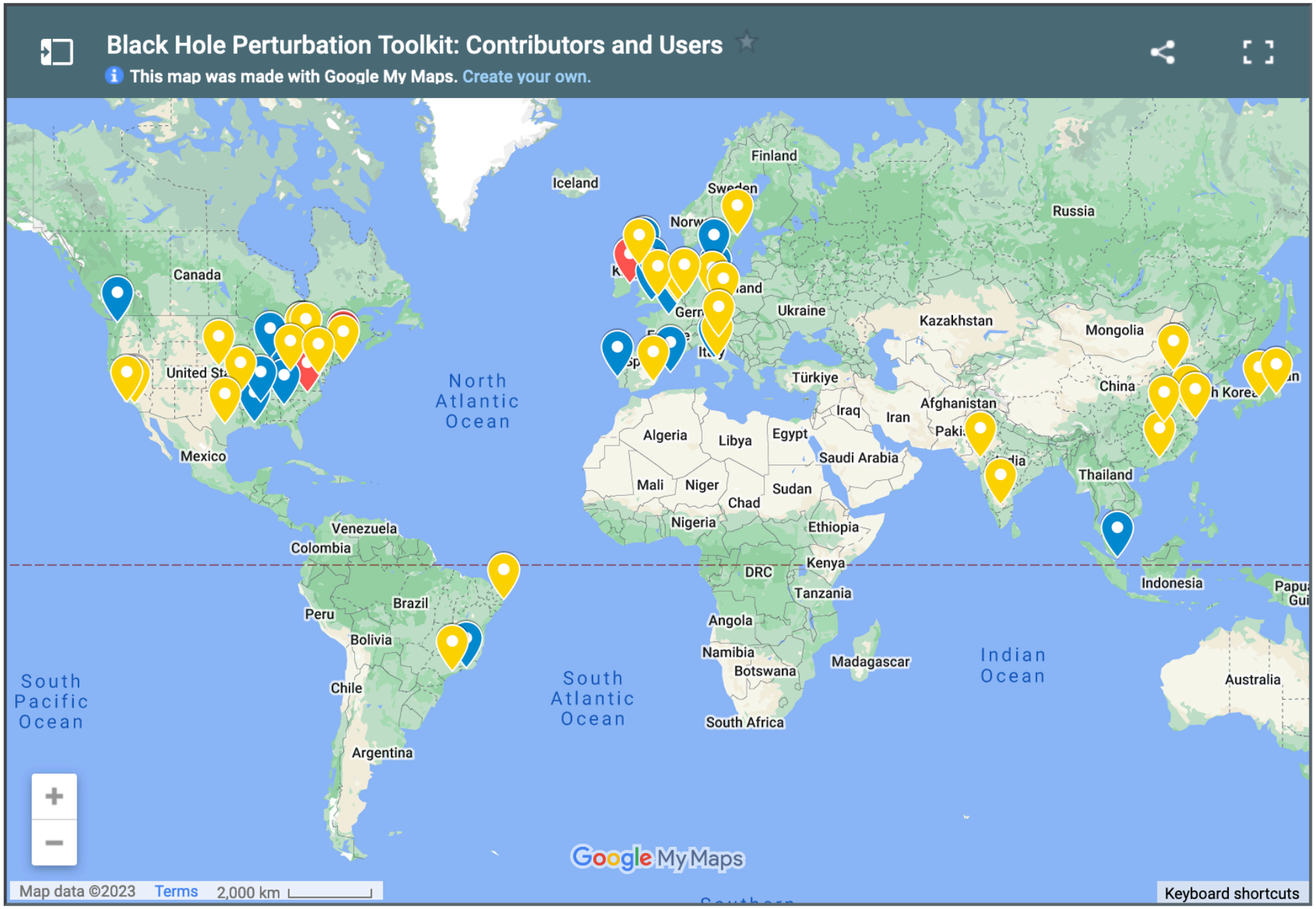
Overview of the BHPToolkit

The BHPToolkit is an open-source and community developed collection of analytic and numerical software and datasets for black hole perturbation theory and self-force calculations.

Motivation:

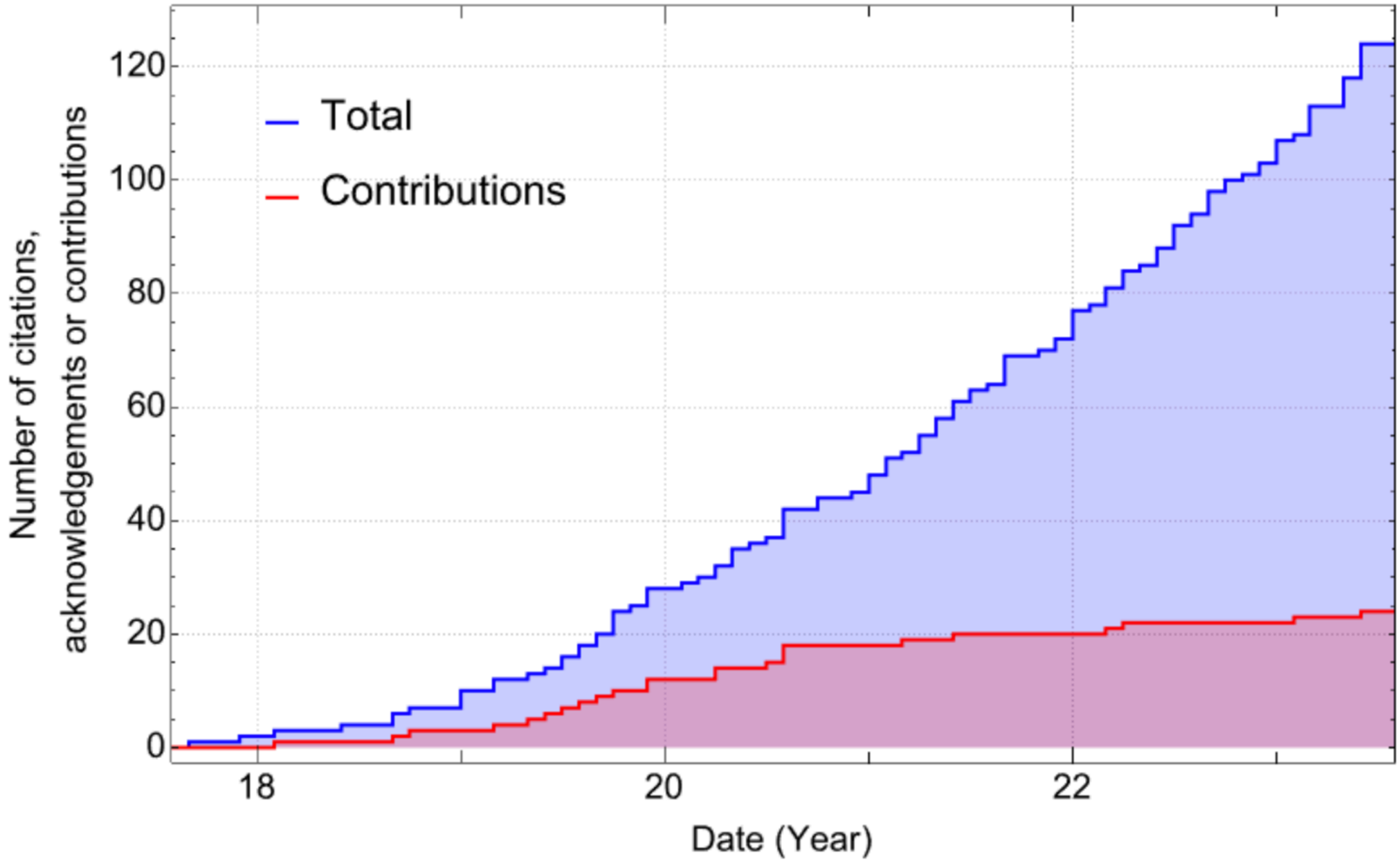
- Stop community reinventing the wheel
- Reduce barrier to entry to help grow the field
- Lots of work to be done before LISA launches
- Plenty of other interesting applications

Contributors and users



http://bhptoolkit.org/users.html

We know of 125+ papers that make use of or extend the Toolkit



http://bhptoolkit.org/users.html

Citation guideline

If you make use of any of the Toolkit in your research please acknowledge using:

> This work makes use of the Black Hole Perturbation Toolkit.

To cite the Toolkit please use this BibTeX entry (or similar). Some modules also request additional citations. Please check the documentation for individual modules.

Citing

In addition to acknowledging the Black Hole Perturbation Toolkit as suggested on the front page we also recommend citing the specific package version you use via the citation information on the package's Zenodo page linked from the above DOI.

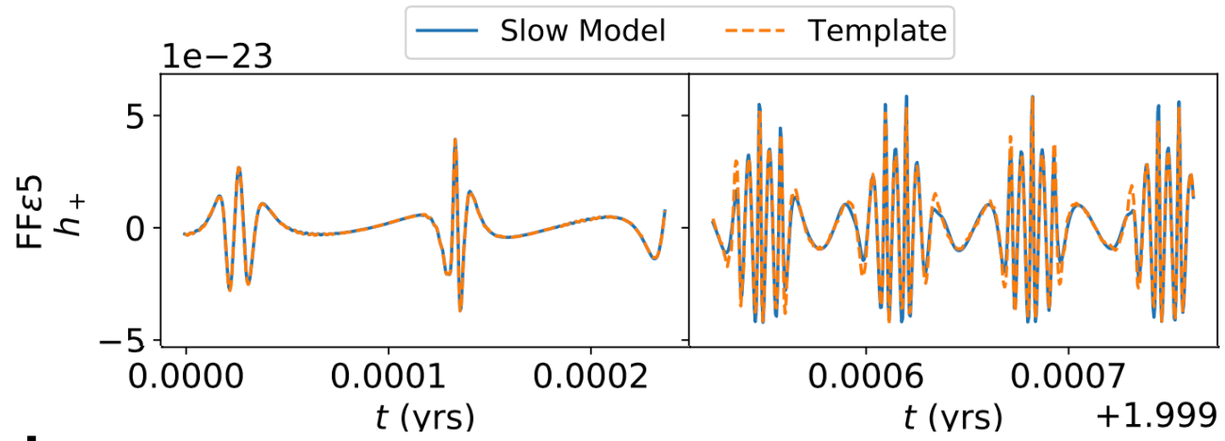
Why cite?

A lot of researcher time and effort goes into developing the Toolkit. Acknowledging and citing this work demonstrates that it is being used which helps us secure additional funding to support further development.

What can I do with the BHPToolkit?

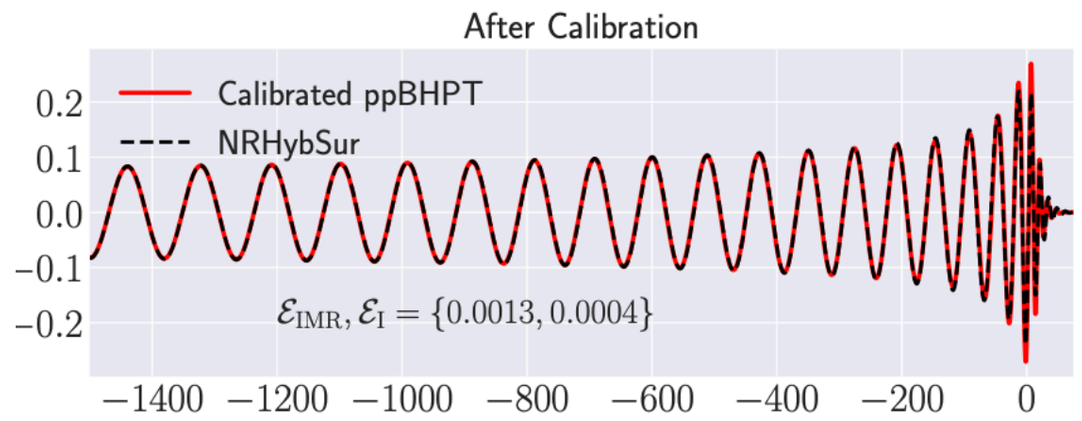
Compute waveforms

- Fast EMRI Waveforms (FEW)
- BHPTNRSurrogate



Solve radial and angular Teukolsky and Regge-Wheeler equations

- SpinWeightedSpheroidalHarmonics
- Teukolsky
- ReggeWheeler
- Gremlin



Work with high-order PN series

- PostNewtonianSelfForce

```
Flux∞["Series"] + O[y]^7
```

$$\frac{32\,y^5}{5} - \frac{2494\,y^6}{105} + \frac{128}{5}\,\pi\,y^{13/2} + O[y]^7$$

Compute quasi-normal modes

- qnm

Plus a lot more: dG time-domain solver, snapshot waveforms, numerical data,…

First second-order codes appearing

PerturbationEquations v0.0.1

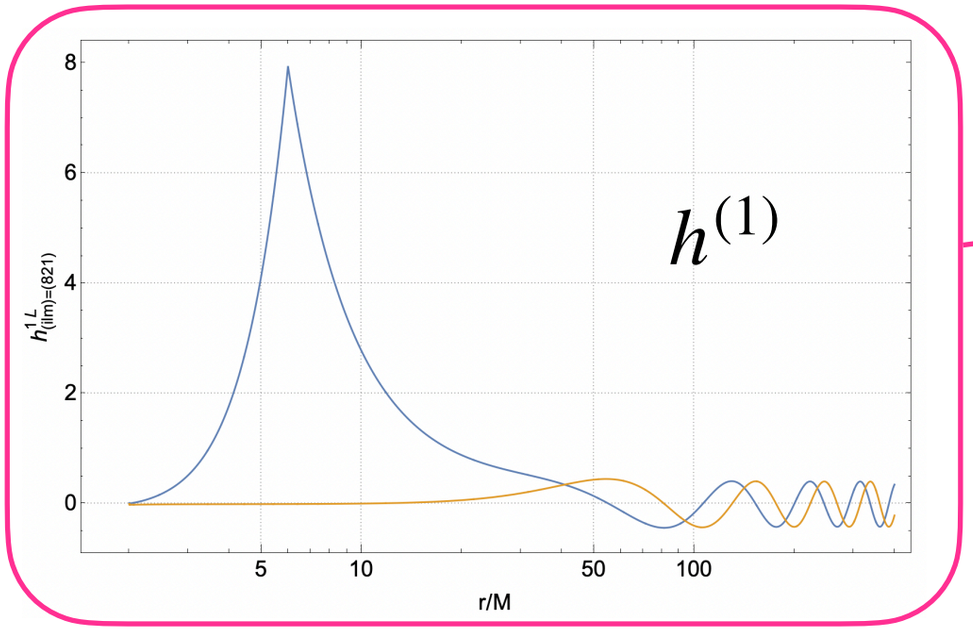
Perturbation equations, including formula for second-order Einstein tensor

$d^2 R_{ll}$ in the Carter tetrad basis for ingoing radiation gauge:

```
In[1]:= SchwarzschildQuadraticOperator["d2G", "IngoingRadiationGauge", "Carter", "Carter"]["ll"]
```

$$\frac{1}{f[r]\, r^4}\Big(C^{lm0}{}_{l_2 m_2 - 1 l_1 m_1 1}\Big(\sqrt{f[r]}\; r\Big(-h_{11}{}^{0 l_2 m_2}\;\mu\;{l_2}_1\; r\Big(-\sqrt{f[r]}\;h_{33}{}^{2 l_1 m_1}\;\mu\;{l_1}_2 - r\,\partial_0 h_{13}{}^{1 l_1 m_1} + f[r]\, r\,\partial_1 h_{13}{}^{1 l_1 m_1}\Big) + h_{11}{}^{0 l_1 m_1}\;\mu\;{l_1}_1$$
$$r\Big(\sqrt{f[r]}\;h_{44}{}^{-2 l_2 m_2}\;\mu\;{l_2}_2 - r\,\partial_0 h_{14}{}^{-1 l_2 m_2} + f[r]\, r\,\partial_1 h_{14}{}^{-1 l_2 m_2}\Big) + 2\, r^2\Big(h_{44}{}^{-2 l_2 m_2}\;\mu\;{l_2}_2\,\partial_0 h_{13}{}^{1 l_1 m_1} -$$
$$h_{33}{}^{2 l_1 m_1}\;\mu\;{l_1}_2\,\partial_0 h_{14}{}^{-1 l_2 m_2} + f[r]\Big(h_{44}{}^{-2 l_2 m_2}\;\mu\;{l_2}_2\,\partial_1 h_{13}{}^{1 l_1 m_1} - h_{33}{}^{2 l_1 m_1}\;\mu\;{l_1}_2\,\partial_1 h_{14}{}^{-1 l_2 m_2}\Big)\Big)\Big) -$$
$$h_{14}{}^{-1 l_2 m_2}\; r\Big(\sqrt{f[r]}\;\Big(3\,M\,h_{11}{}^{0 l_1 m_1}\;\mu\;{l_1}_1 - 2\Big(M\,h_{33}{}^{2 l_1 m_1}\;\mu\;{l_1}_2 + \mu\;{l_1}_1\, r^2\,\partial_0 h_{11}{}^{0 l_1 m_1}\Big)\Big) + 4\,M\, r\,\partial_0 h_{13}{}^{1 l_1 m_1} -$$
$$2\, r^3\,\partial_0\partial_0 h_{13}{}^{1 l_1 m_1} + f[r]^{3/2}\, r\Big(-h_{11}{}^{0 l_1 m_1}\;\mu\;{l_1}_1 + 2\,h_{33}{}^{2 l_1 m_1}\;\mu\;{l_1}_2 + 2\;\mu\;{l_1}_1\, r\,\partial_1 h_{11}{}^{0 l_1 m_1}\Big) +$$

h1Lorenz

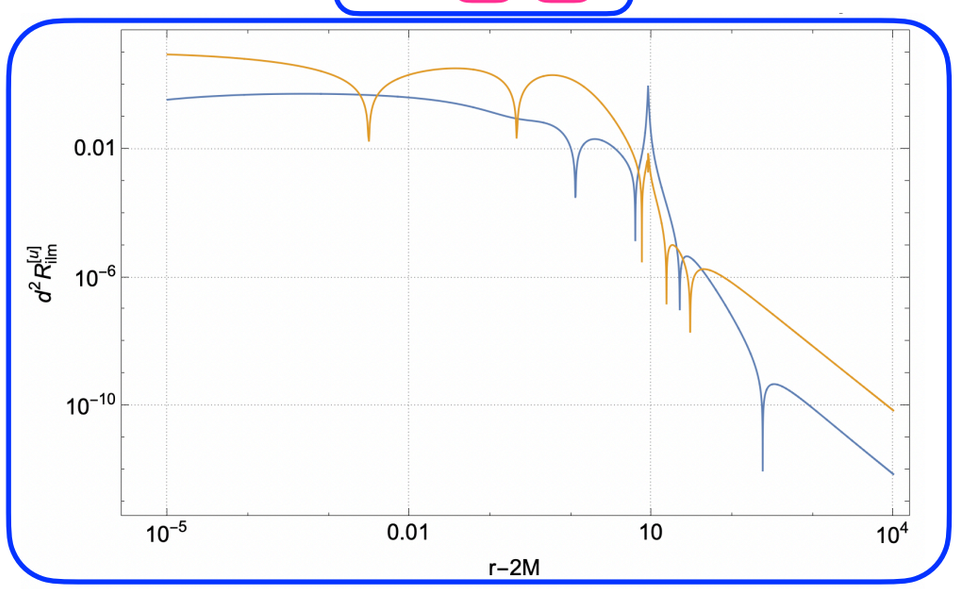


C code that computes the first-order metric perturbation in the Lorenz gauge on a grid of radial points

SecondOrderRicci

C++ code that implements formula from above package to compute the Barack-Lousto-Sago spherical-harmonic modes of the second-order Ricci tensor

$$G^{(1)}[h^{2R}] = G^{(2)}[h^1, h^1] - G^{(1)}[h^{2P}]$$



Major update to Mathematica packages

SpinWeightedSpheroidalHarmonics v1.0.0

Major update to the documentation

Expansion coefficients

$$ {}_sS_{lm}(\gamma) = \sum_{l'} b^{l'}_{lm}(\gamma)\ {}_sY_{lm} $$

- Scope (9)
  - Numerical Evaluation (4)
  - Specific Values (1)
  - Visualisation (2)
  - Function Properties (2)
- Options (1)
  - Method (1)
- Properties & Relations (1)
- Possible Issues (2)

See Also

SpinWeightedSpheroidalHarmonicS

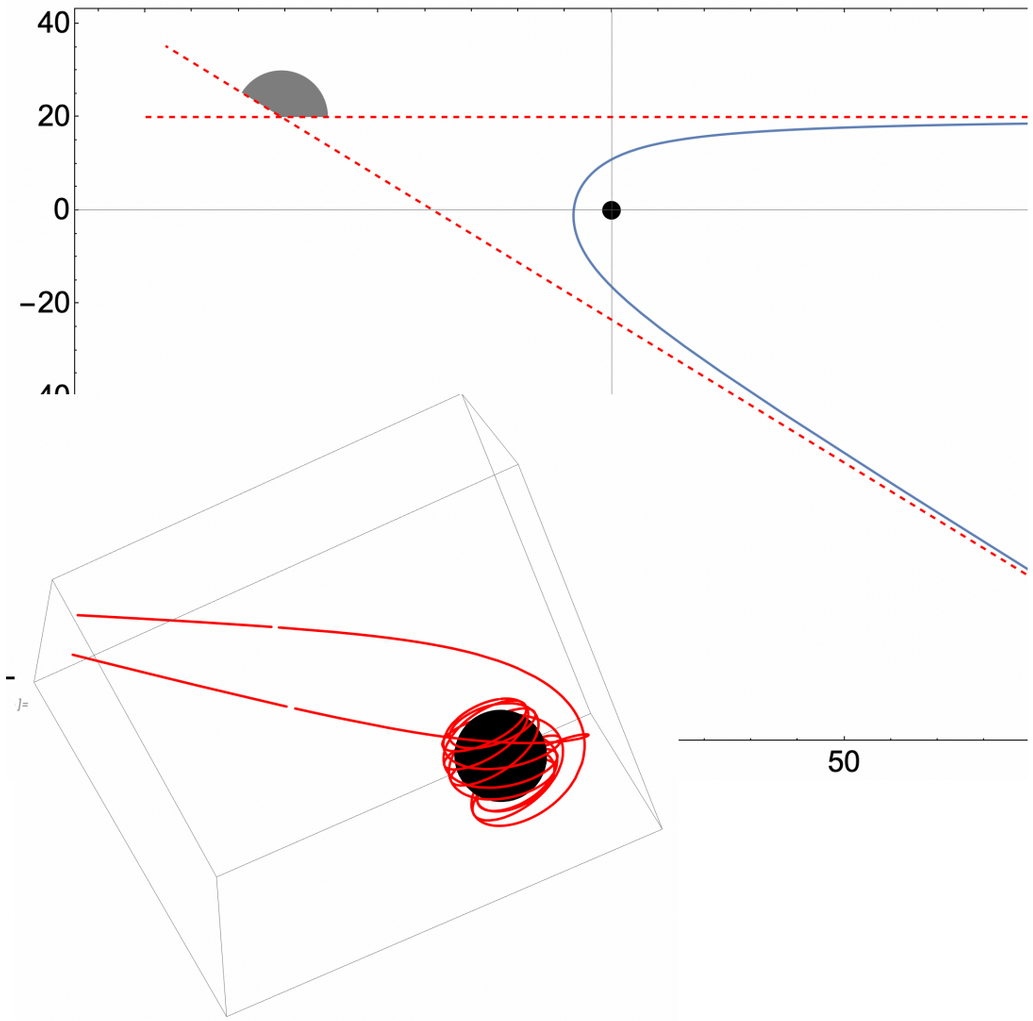
```
S = SpinWeightedSpheroidalHarmonicS[-2, 2, 2, 0.5, Method → {"SphericalExpansion", "NumTerms" → 10}];

S["ExpansionCoefficients"]
```

```
⟨| 2 → 0.995552, 3 → 0.0939677, 4 → 0.00682532, 5 → 0.000388212, 6 → 0.0000182142,
 7 → 7.23009 × 10^-7, 8 → 2.48914 × 10^-8, 9 → 7.5513 × 10^-10, 10 → 2.0481 × 10^-11, 11 → 5.01869 × 10^-13, 12 → 1.12149 × 10^-14 |⟩
```

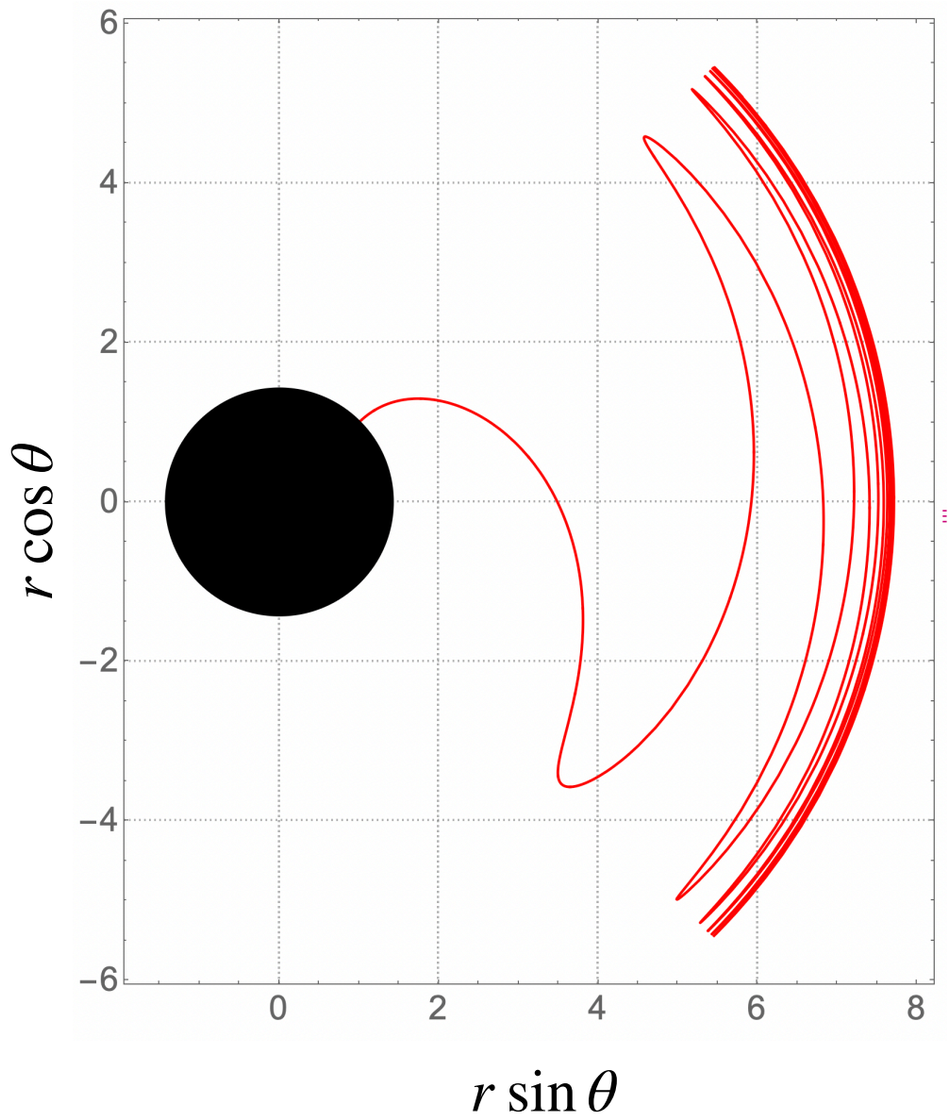
Major update to Mathematica packages

KerrGeodesics v0.9.0

Hyperbolic orbits added



Ready to accept code for plunging orbits (currently in pull request)



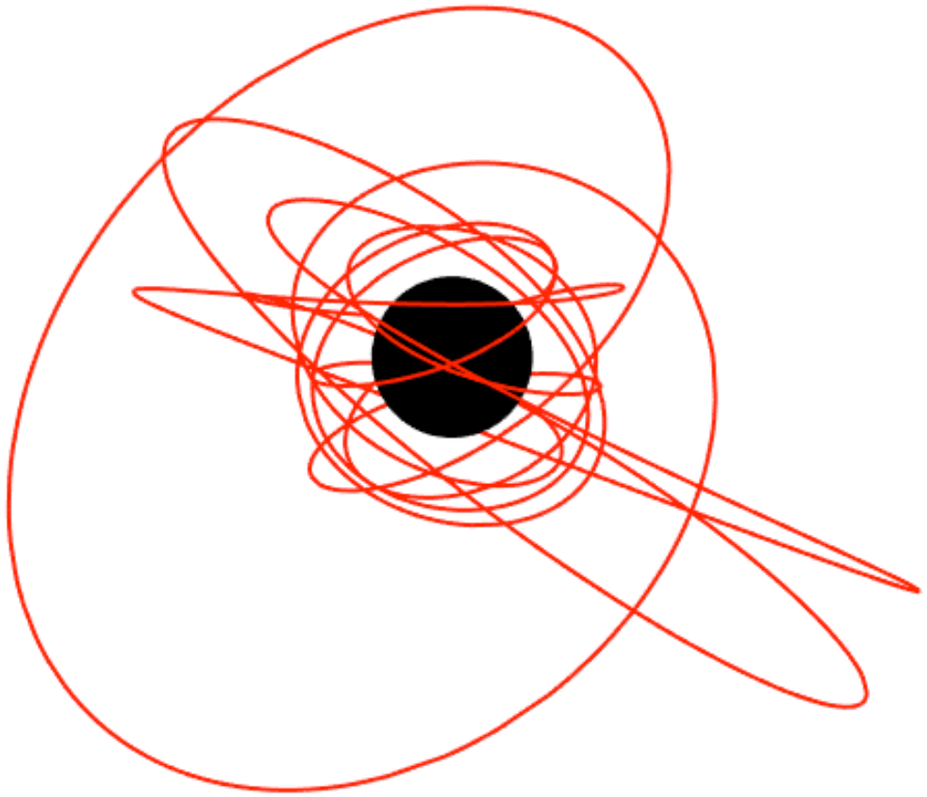
See Conor Dyson's talk

Major update to Mathematica packages

Teukolsky v1.0.0

TeukolskyPointParticleMode now works for generic bound orbits with spin-weights

$s = \{0, \pm 1, \pm 2\}$



```
<< Teukolsky`

orbit = KerrGeoOrbit[0.9`32, 5, 0.4`32, 0.5`32];

mode = Quiet@TeukolskyPointParticleMode[-2, 2, 2, -3, 4, orbit];

mode["Fluxes"]["Energy"]
```

$\langle | \mathcal{I} \to 1.1976874637309 \times 10^{-12}, \mathcal{H} \to -7.562260856327 \times 10^{-15} | \rangle$

What's in the pipeline?

KerrSpinningSecondary v0.1.0

- Extends KerrGeodesics to include spin on the secondary
- Allows for linear and non-linear spin corrections
- Allows for difference spin supplementary conditions

```
KerrSpinEnergy[0, p, 0, 1, σ]
```

$$\frac{1 - \frac{2}{p} - \frac{\sigma}{2\left(1-\frac{3}{p}\right)p^{5/2}}}{\sqrt{1-\frac{3}{p}}}$$

MetricPerturbation v0.1.0

- Depends the Teukolsky and Regge-Wheeler packages
- Will allow construction of metric in RW, radiation, and Lorenz gauges

```
orbit = KerrGeoOrbit[0, 8.1`32, 0, 1];

habL = MetricPerturbation[2, 1, orbit, "Gauge" → "Lorenz"]
```

```
MetricPerturbationMode[ l: 2  m: 1  Gauge: Lorenz
                        Type: PointParticleCircular ]
```

Future directions

- Please download and use the code
- If you find a problem file an issue on GitHub (or better yet, create a pull request to fix it!)
- New packages? pybhpt (Zach Nasipak's talk). Interest in Julia language is growing — see recent GeneralizeSazakiNakamura.jl from arXiv:2306.16469
- We ran an intensive developer workshop last year. It was very successful so want to do something similar again.
- Training workshops (online?)
- Don't mention the paper…