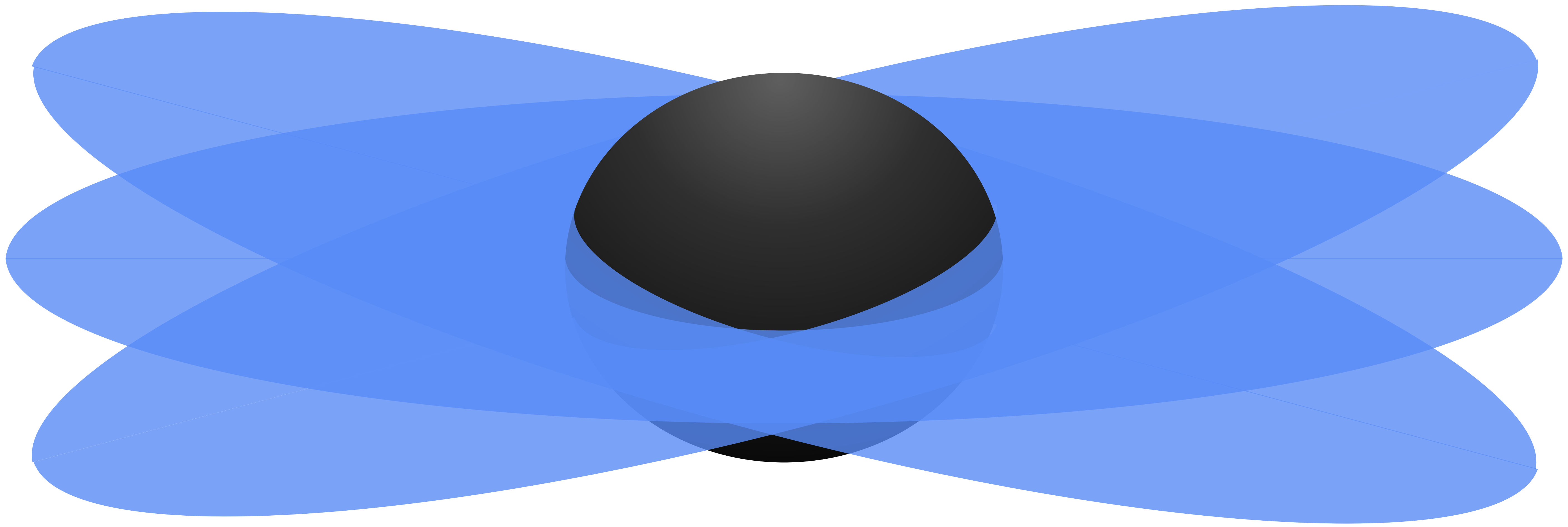
New results on black-hole quasibound states



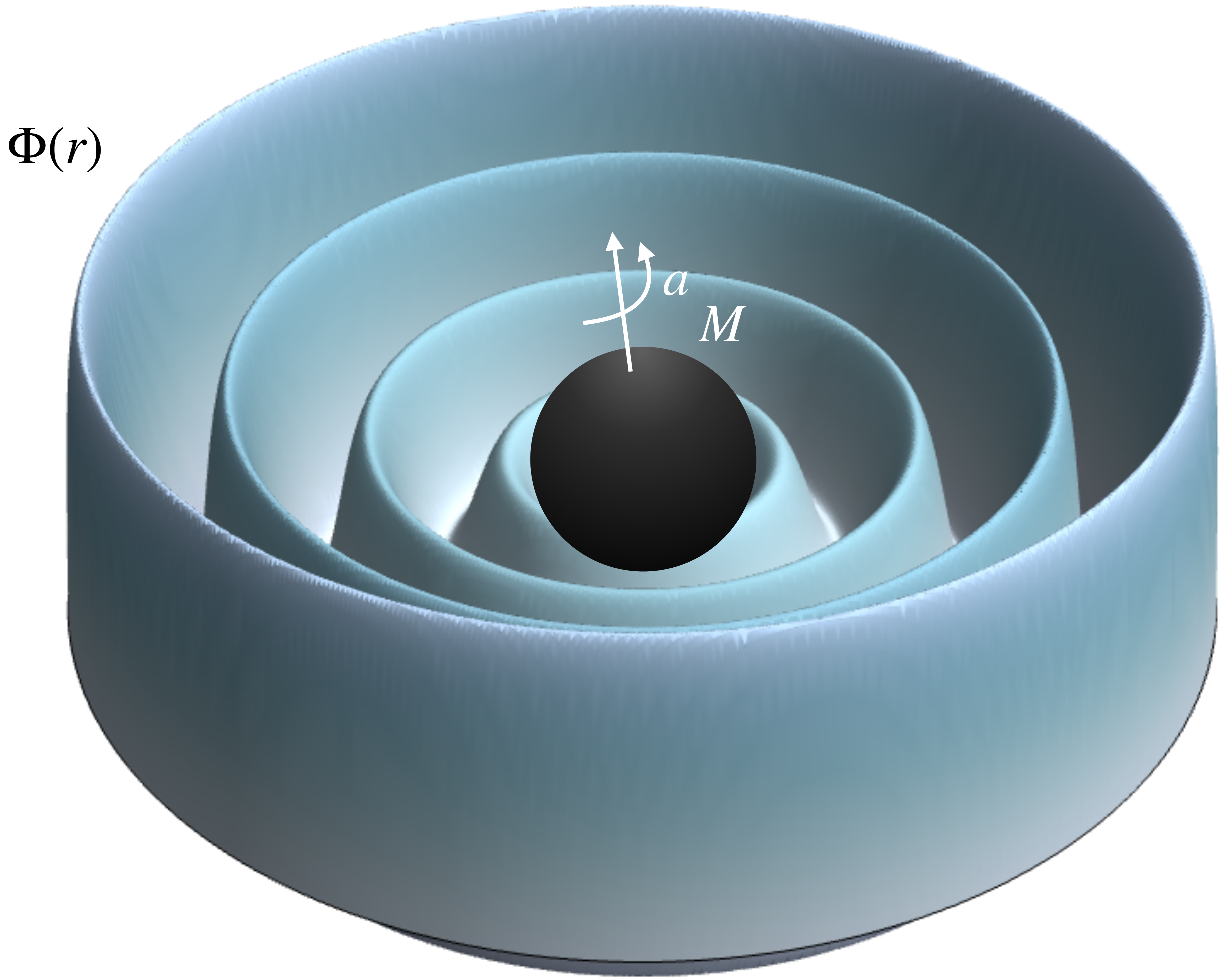
Laura Sberna (Max Planck Institute for Gravitational Physics, Potsdam)

with Enrico Cannizzaro, Stephen Green, Stefan Hollands

Capra Meeting
Copenhagen, July 2023

Work in progress

$$\Box_{g_{\mu\nu}} \Phi + \mu^2 \Phi = 0$$



QUASINORMAL

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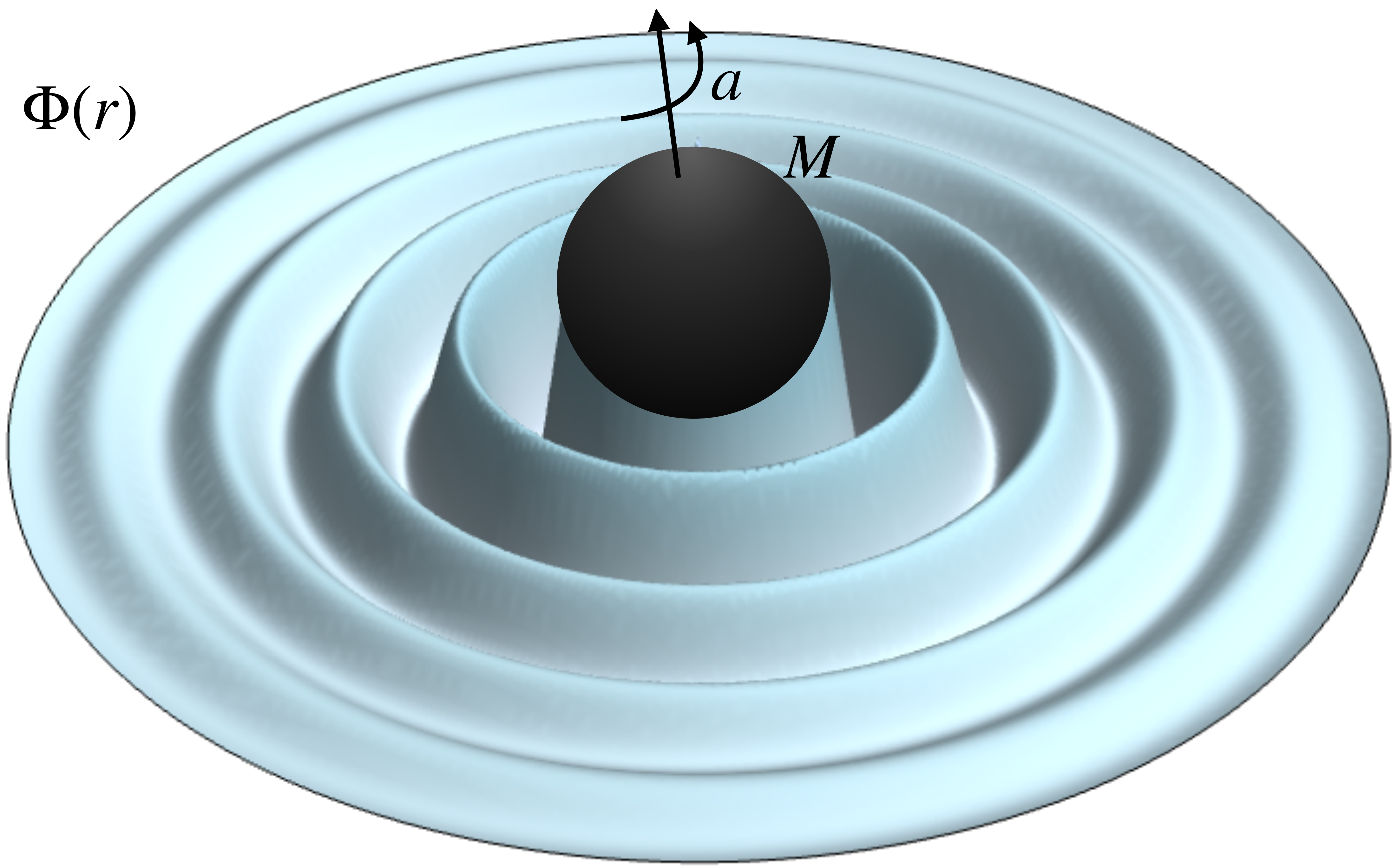

QUASIBOUND

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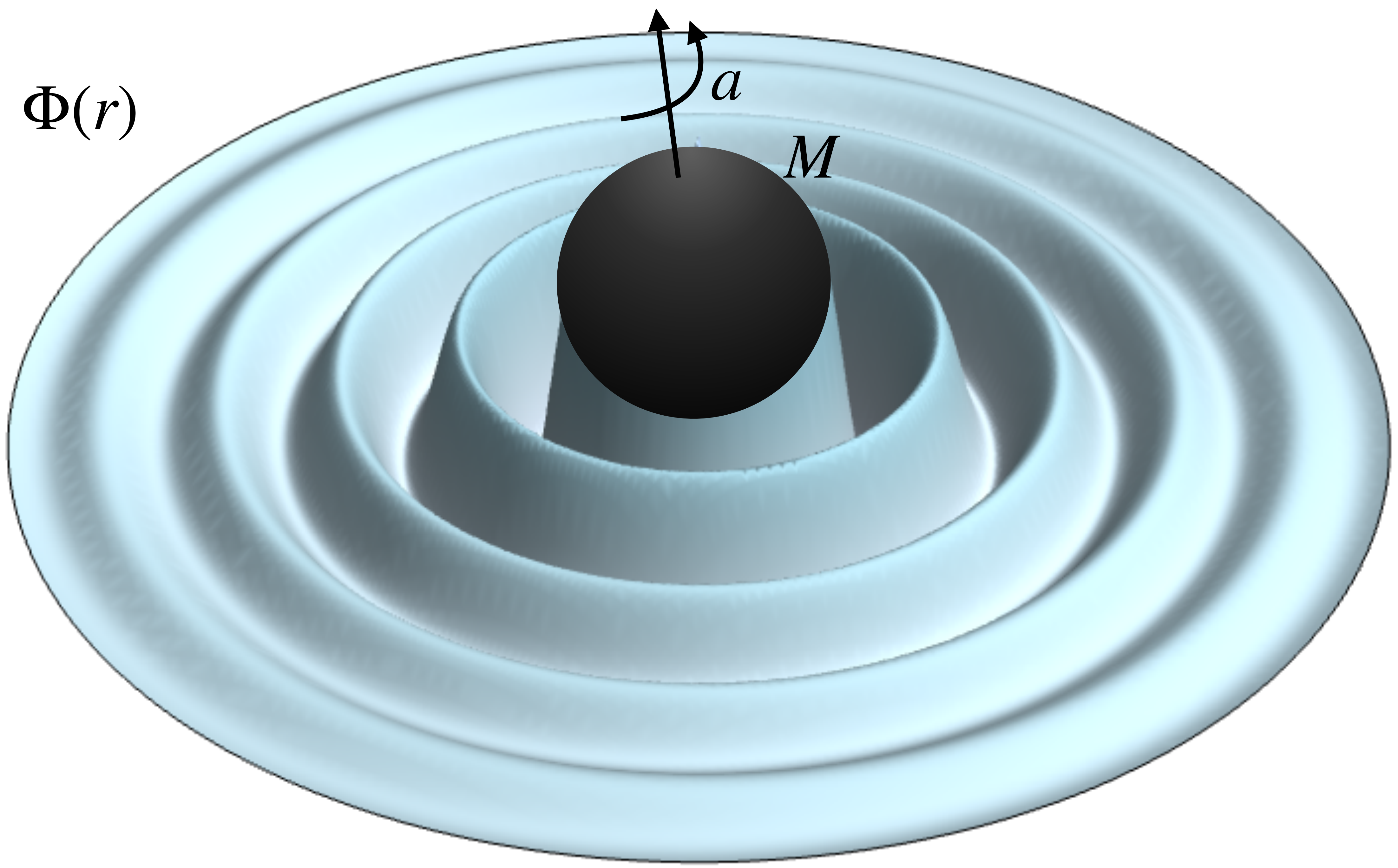
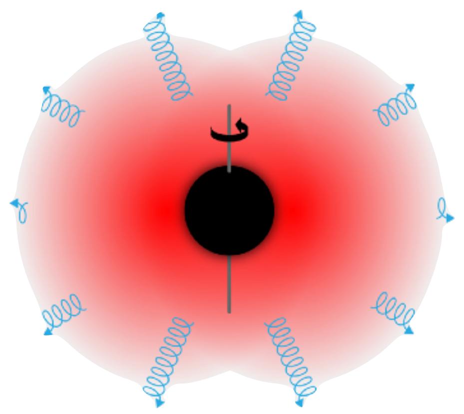


Can be unstable
(***superradiance***)



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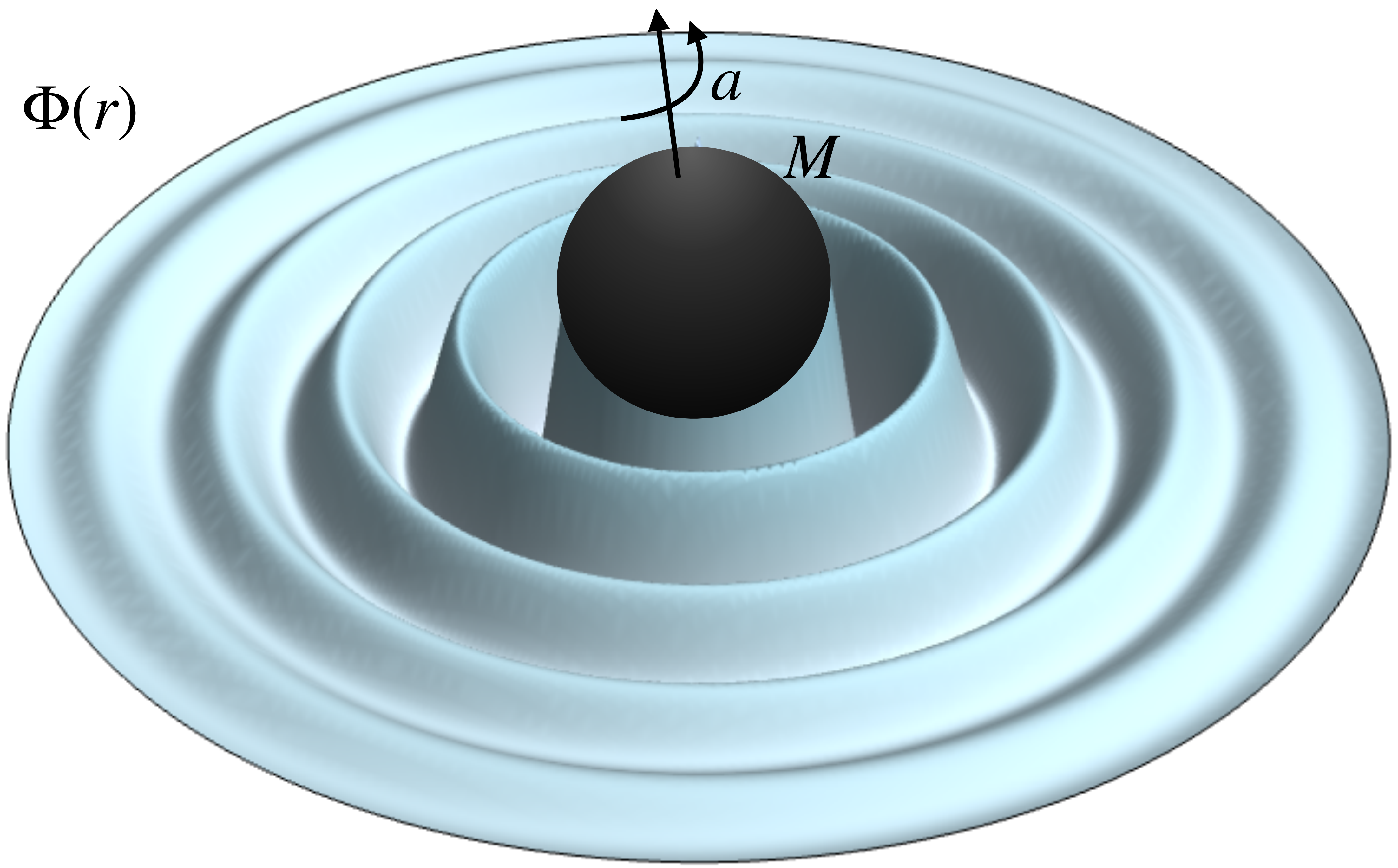

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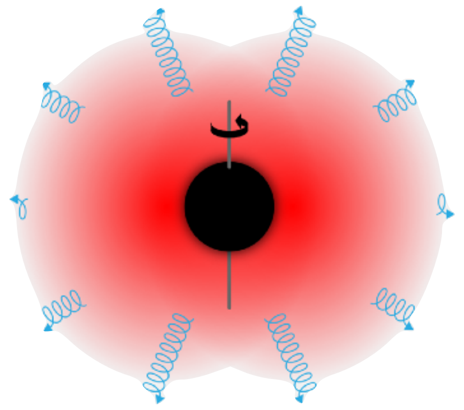
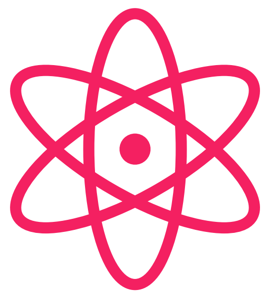


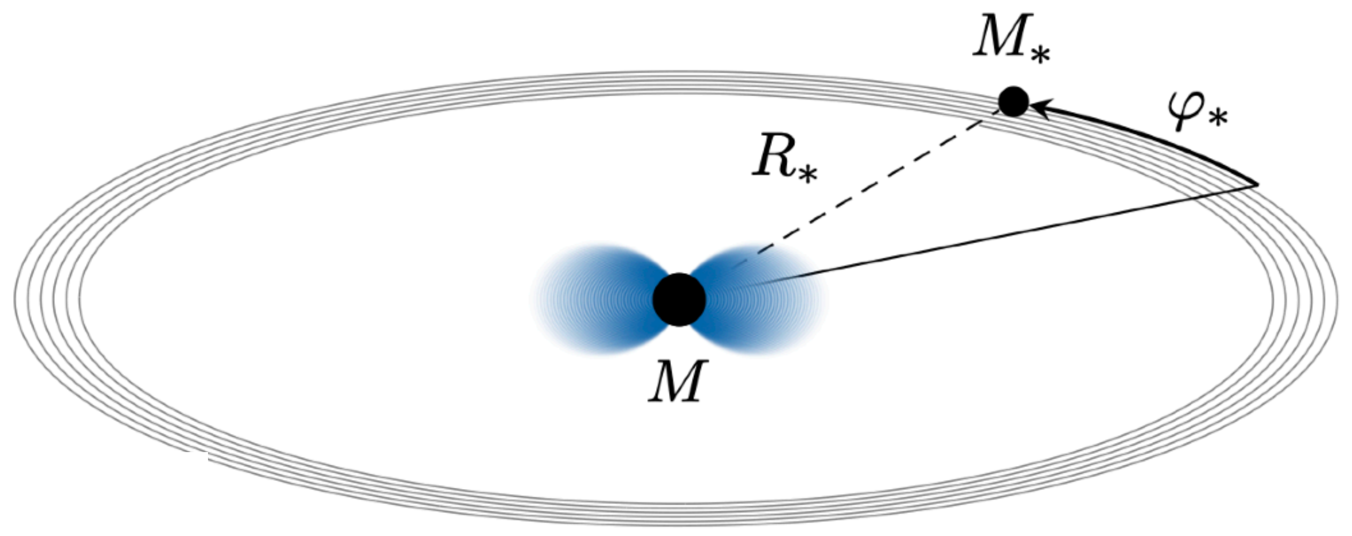
Non-relativistic limit $M\mu \ll 1$:
hydrogen atom spectrum



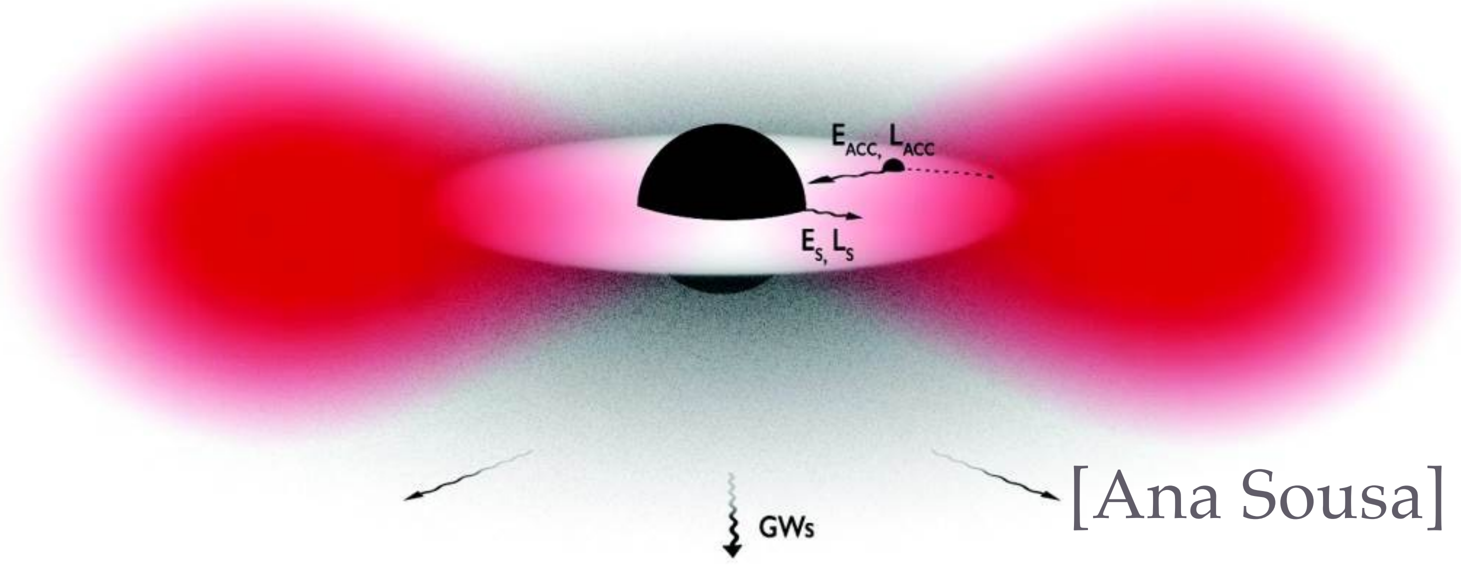
MOTIVATIONS

1) Purely gravitational probes of new physics:

- black hole **spin down**
- **continuous** gravitational **waves**
- **environmental** effects



[Baumann+ 2022]



[Ana Sousa]

MOTIVATIONS

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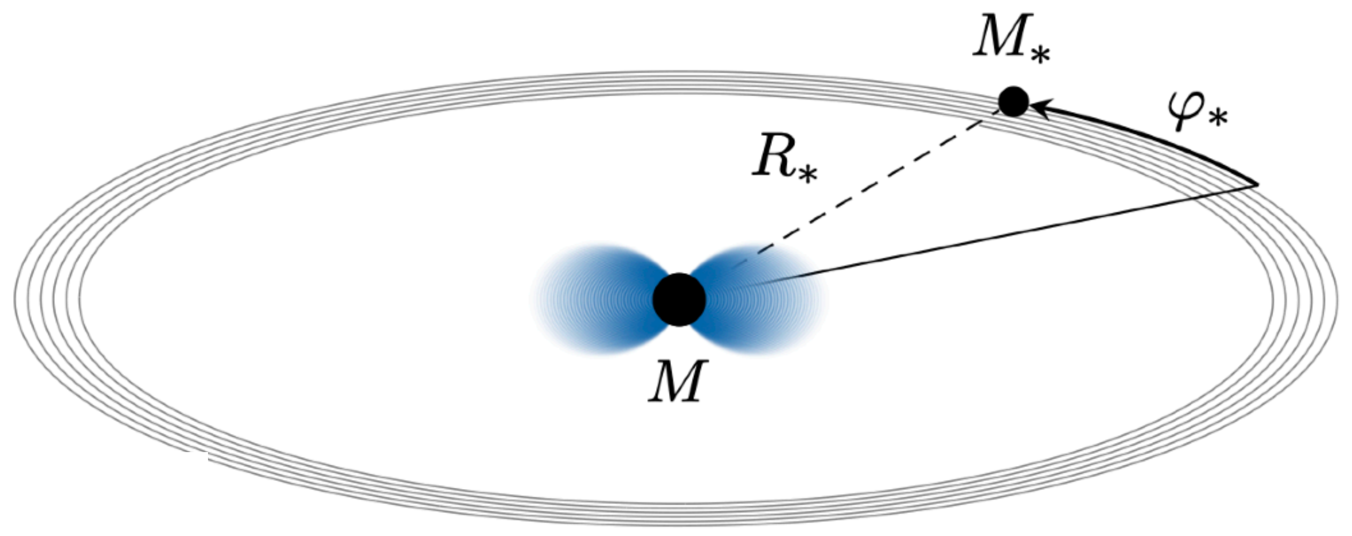

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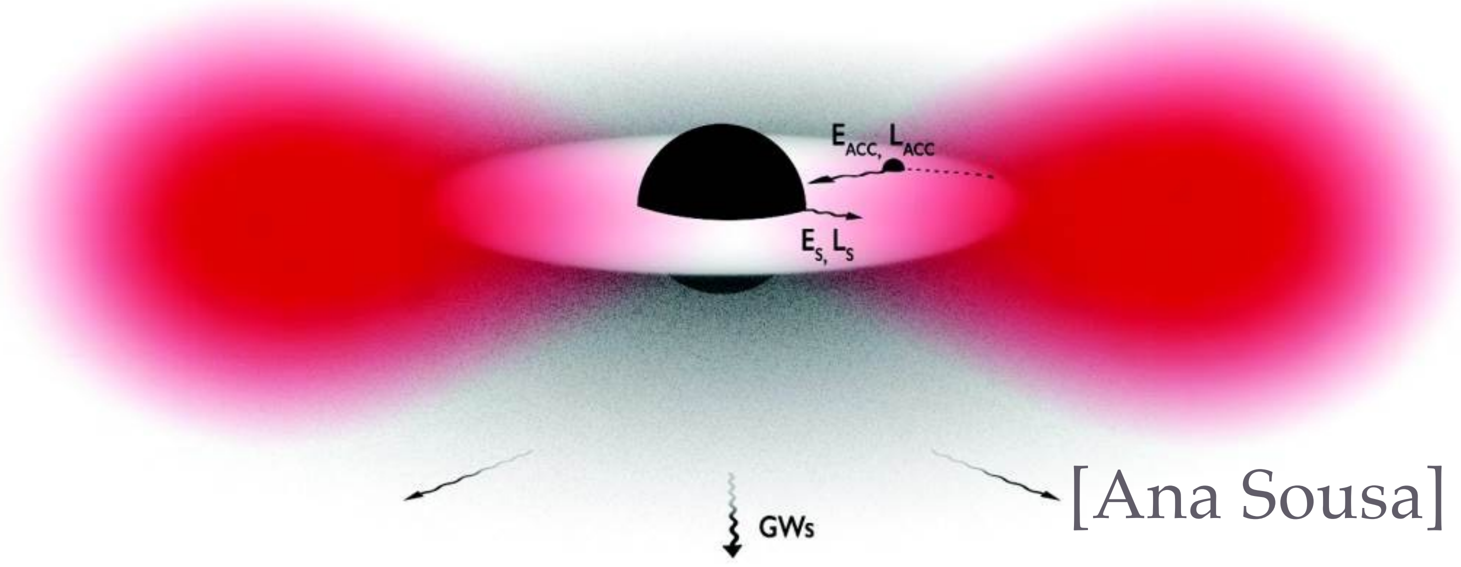
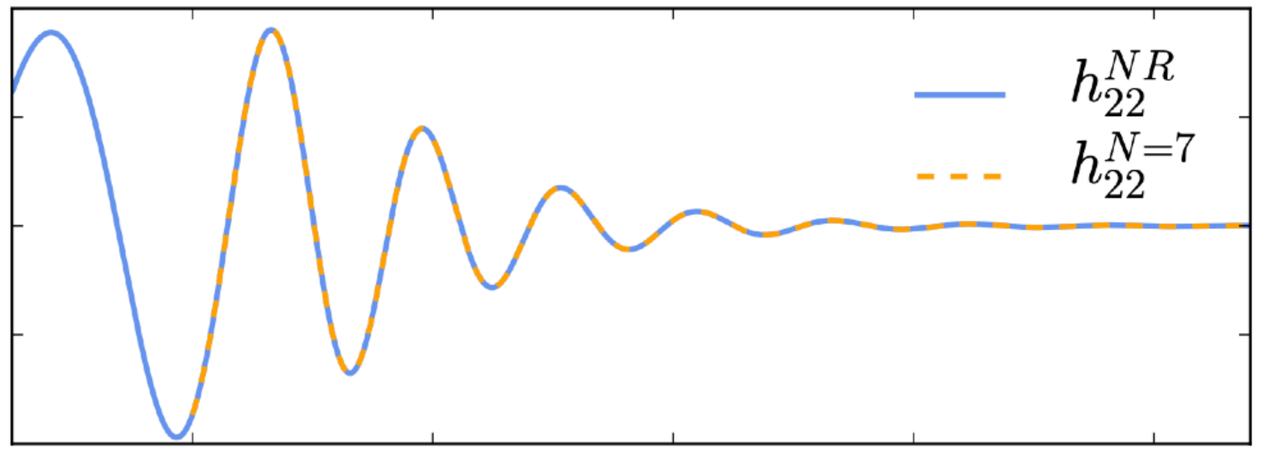
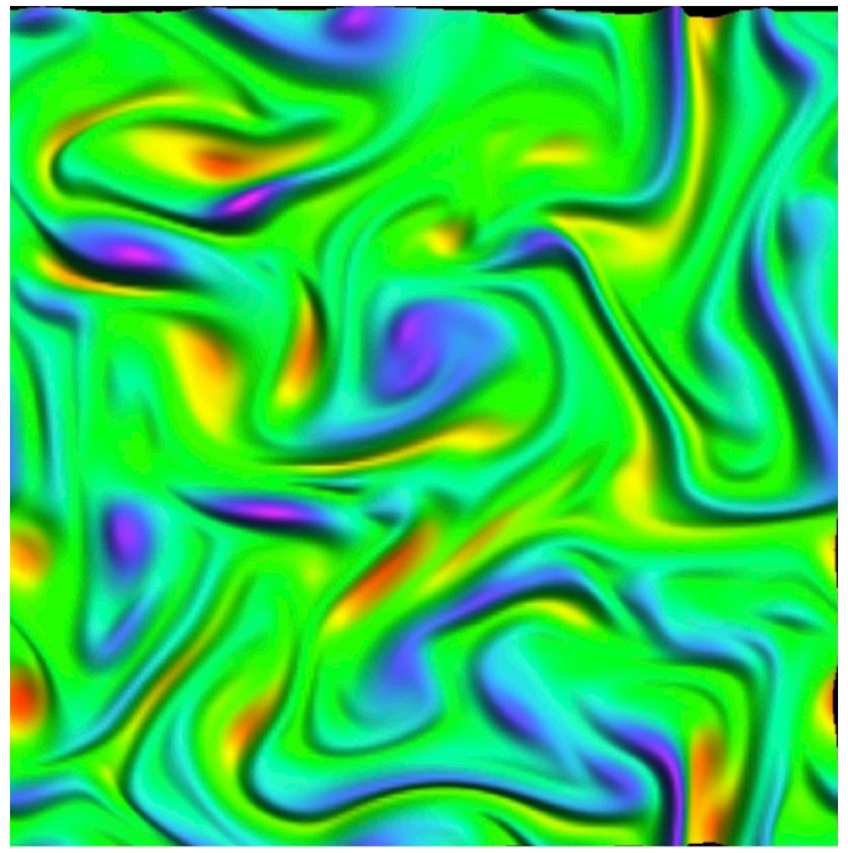


[Ana Sousa]

2) A toy model for **higher order/nonlinear** effects in *gravitational* perturbations



[Giesler+ 2019]



[Green+ 2014]

MODE SOLUTIONS: THE GREEN'S FUNCTION

$$\Box_{g_{\mu\nu}}\Phi + \mu^2\Phi = 0$$

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$$r \to +\infty$$

- **quasinormal** (exponential in *r*)
- **quasibound** (exponential decay in *r*)

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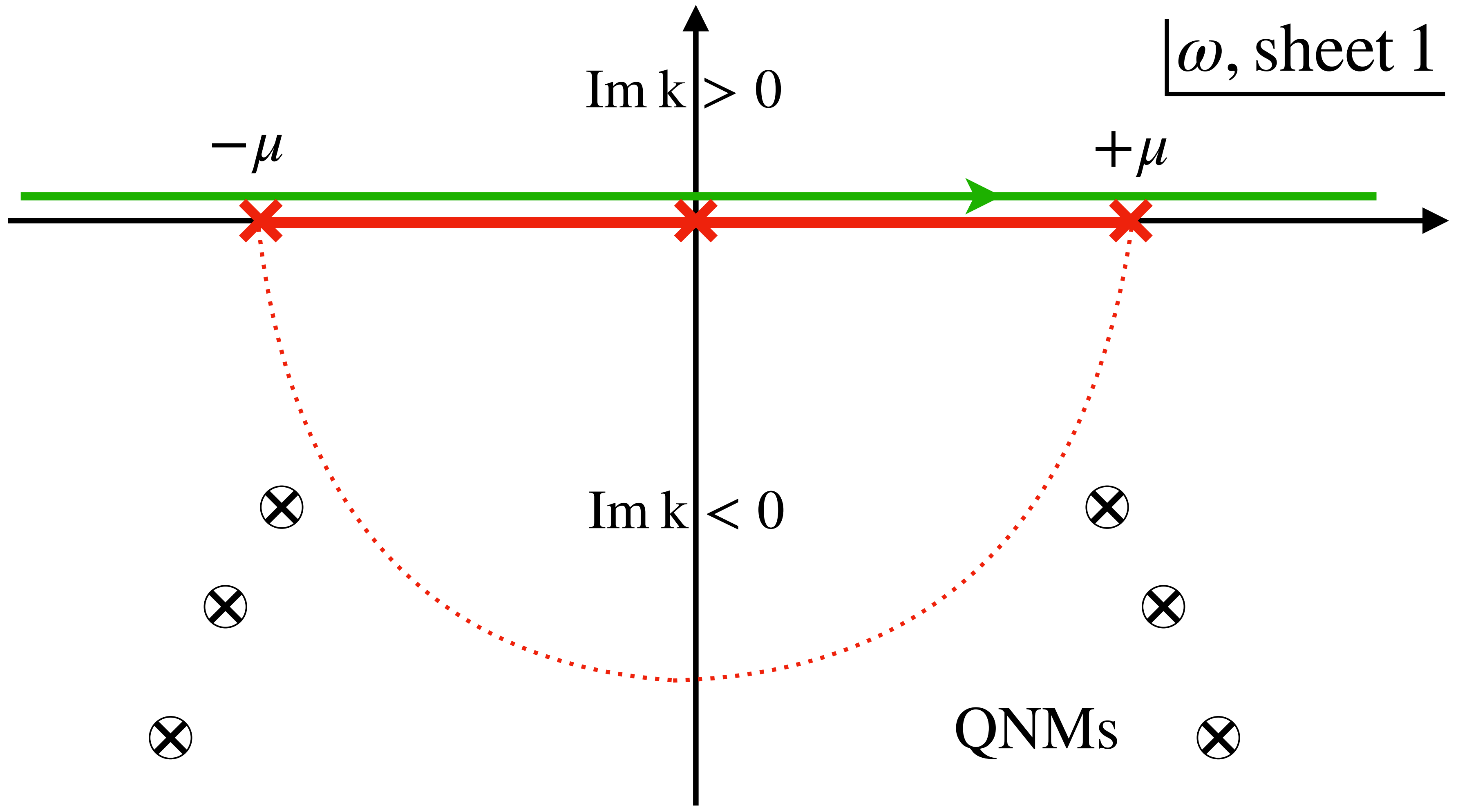

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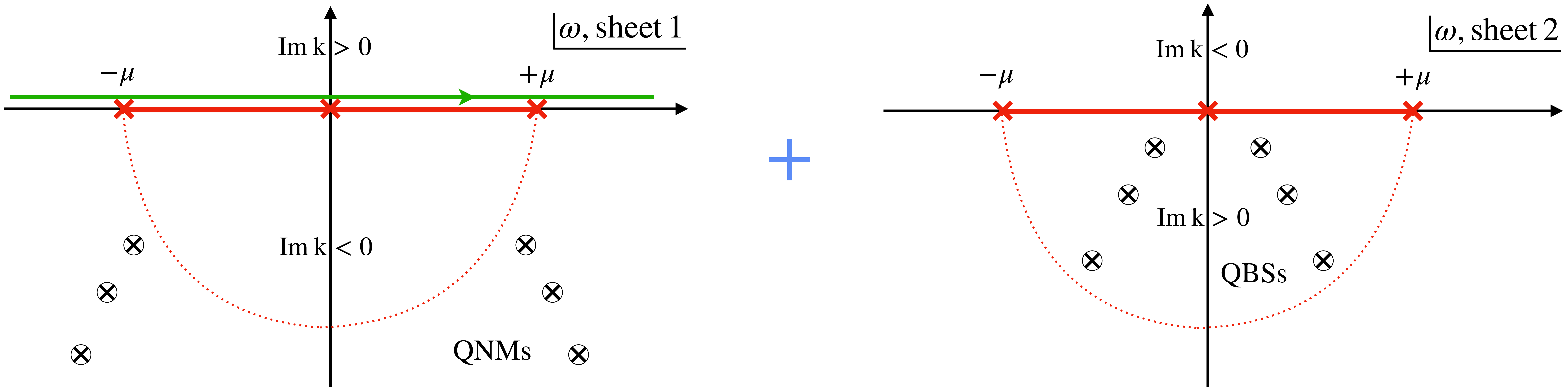
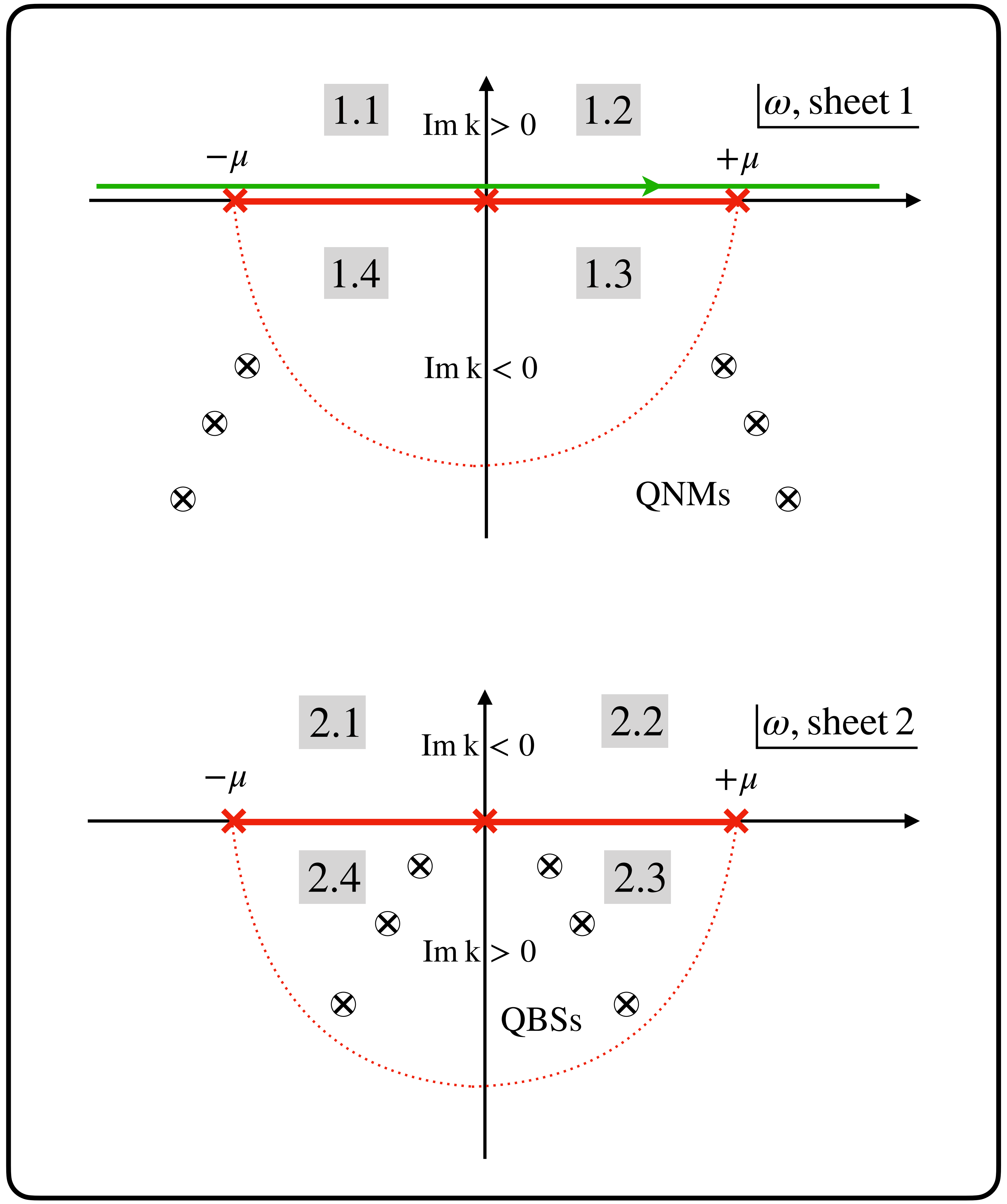


[see e.g. Décanini+ 2015]

MODE SOLUTIONS, HAPPIER TOGETHER

Work in progress



[inspired by optics literature! Tamir and Oliver 1963 and more]

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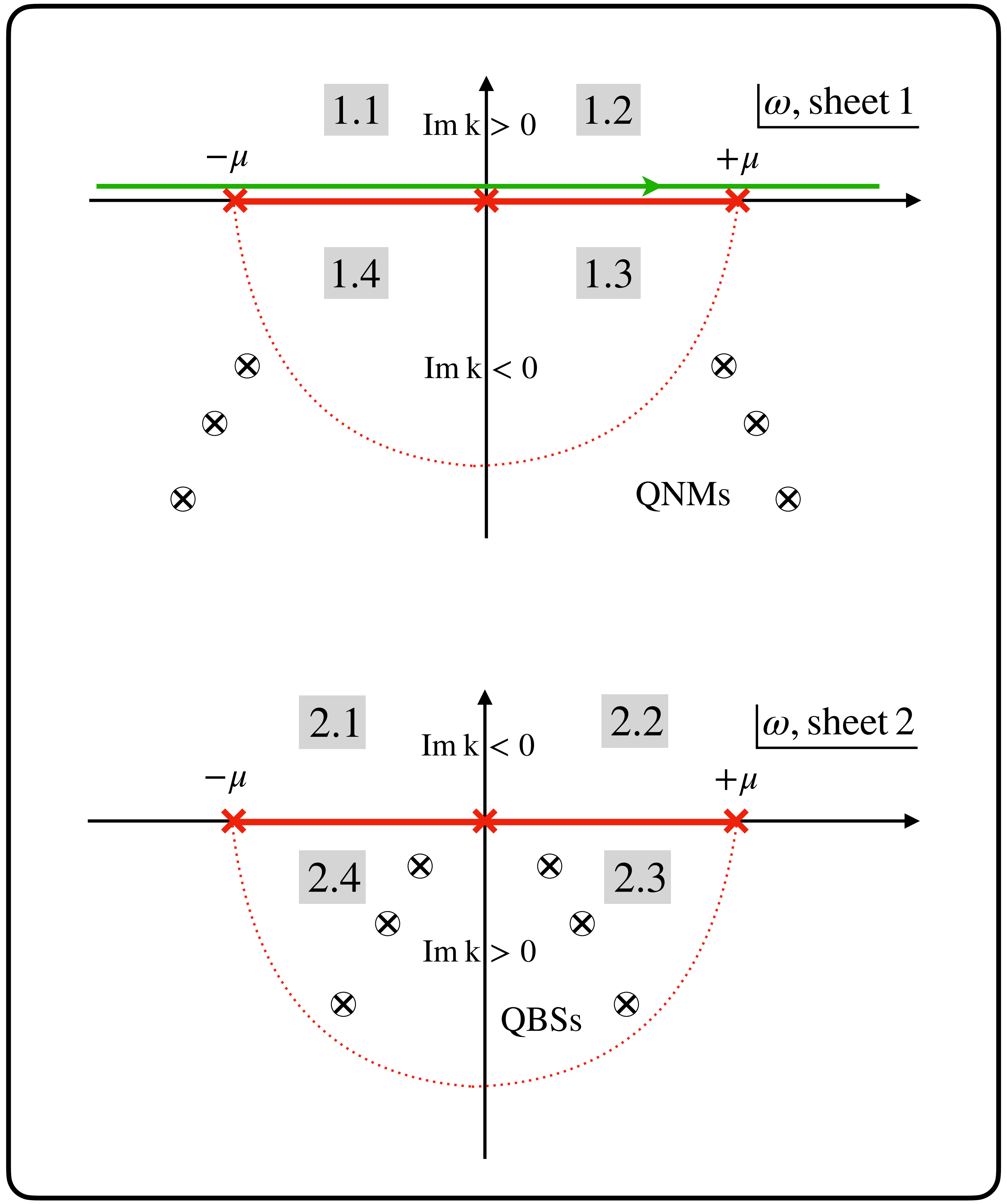
Work in progress



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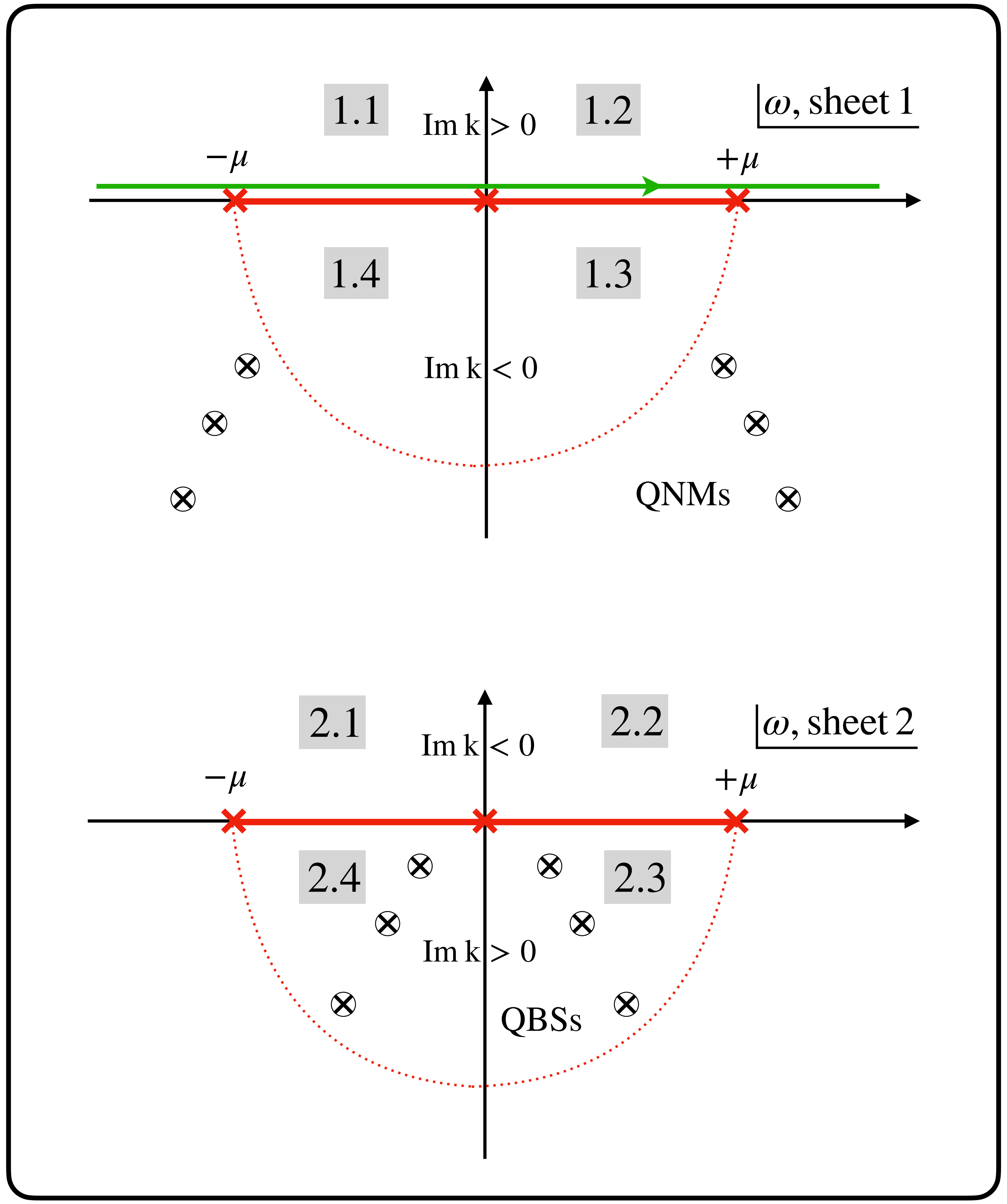

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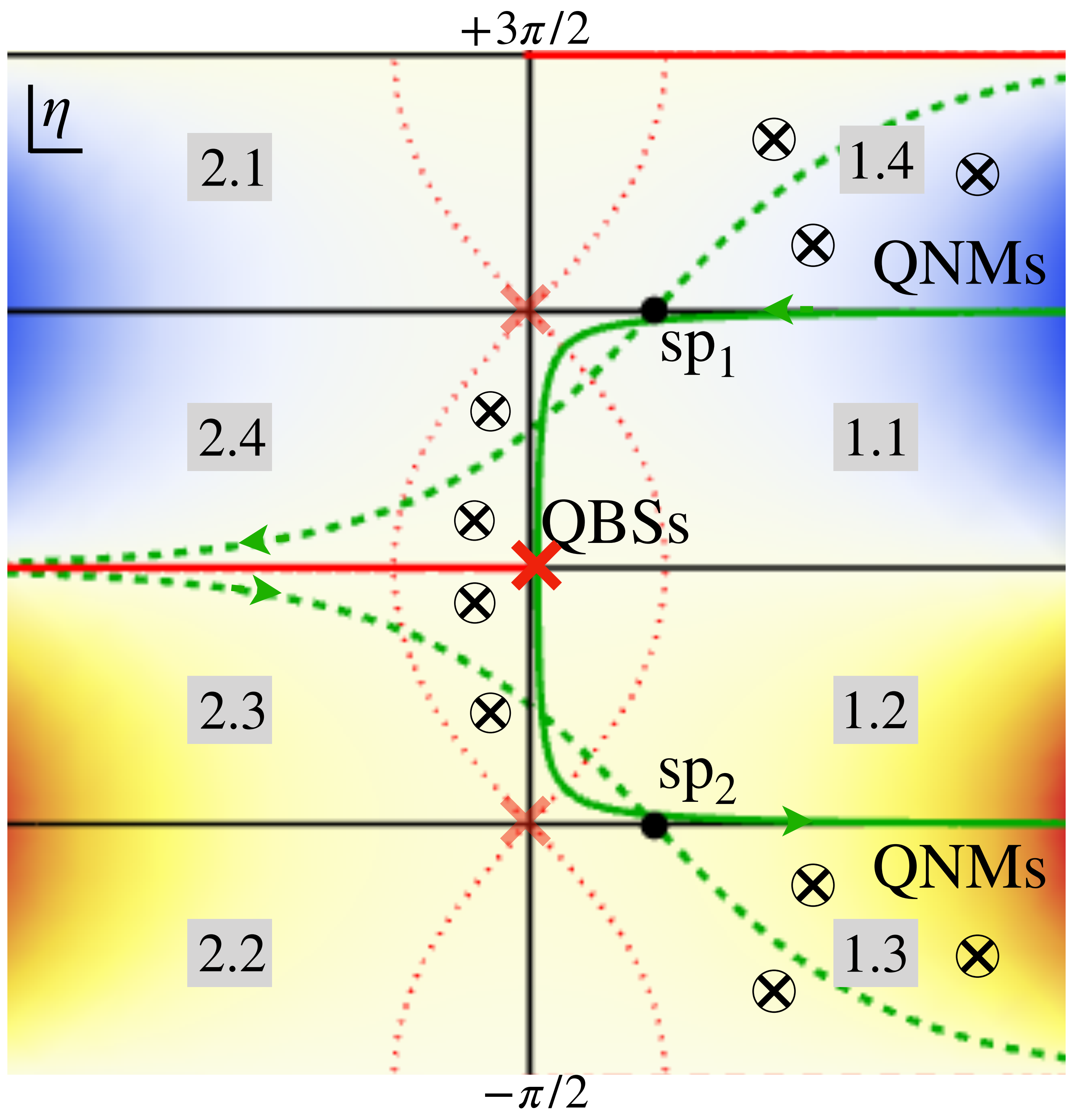

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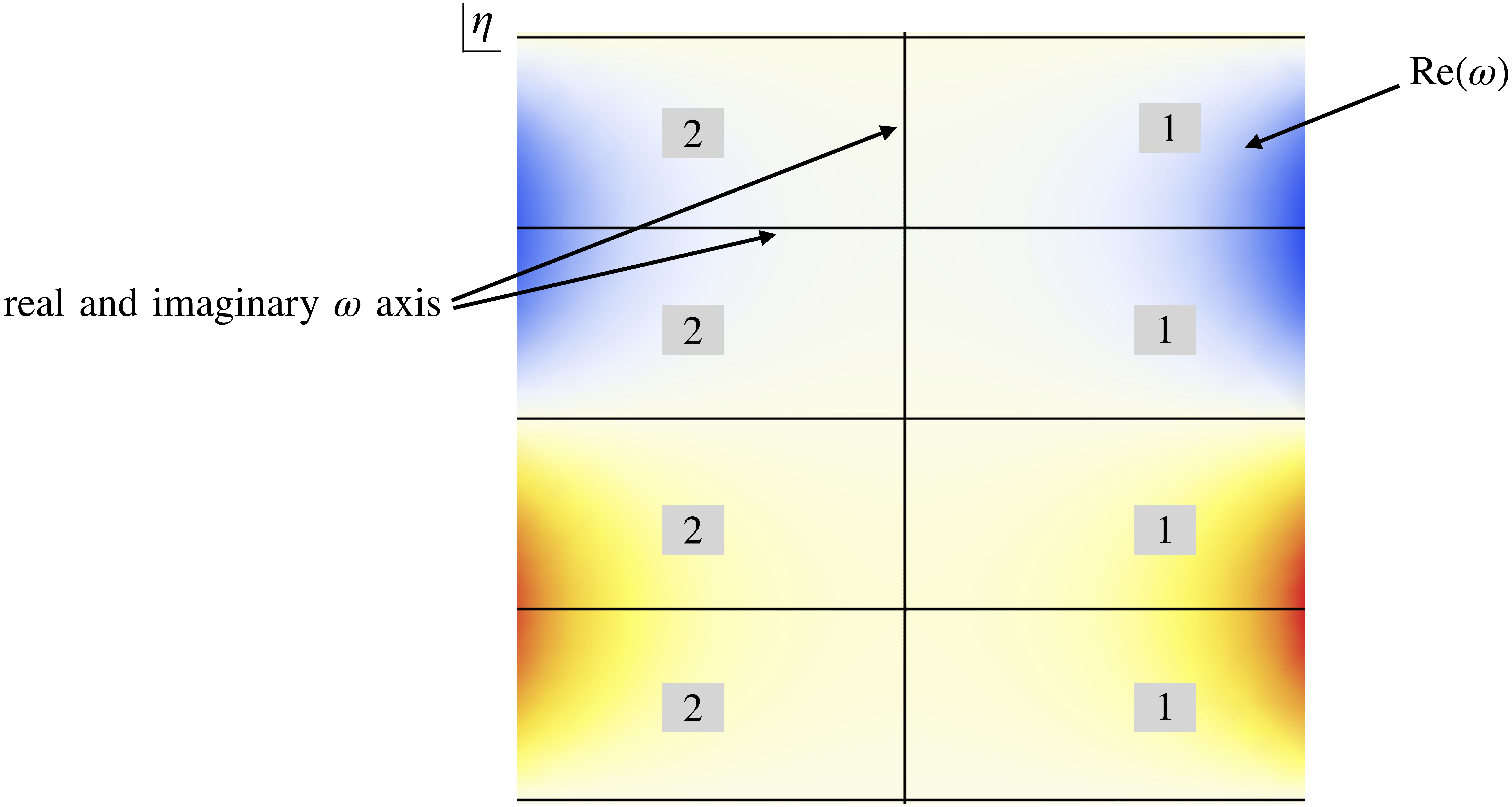
MODE SOLUTIONS: THE NEW COMPLEX PLANE

Work in progress



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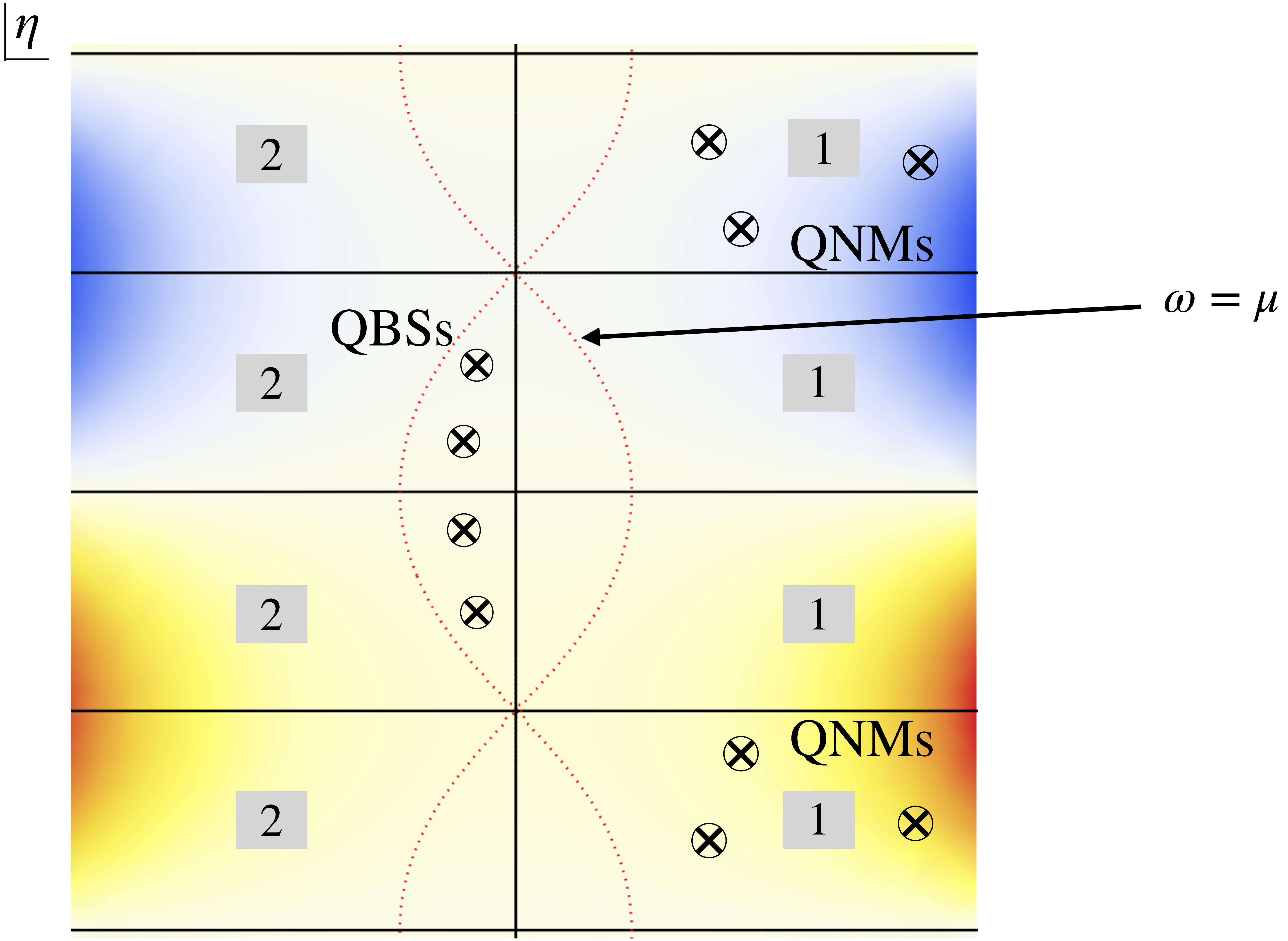

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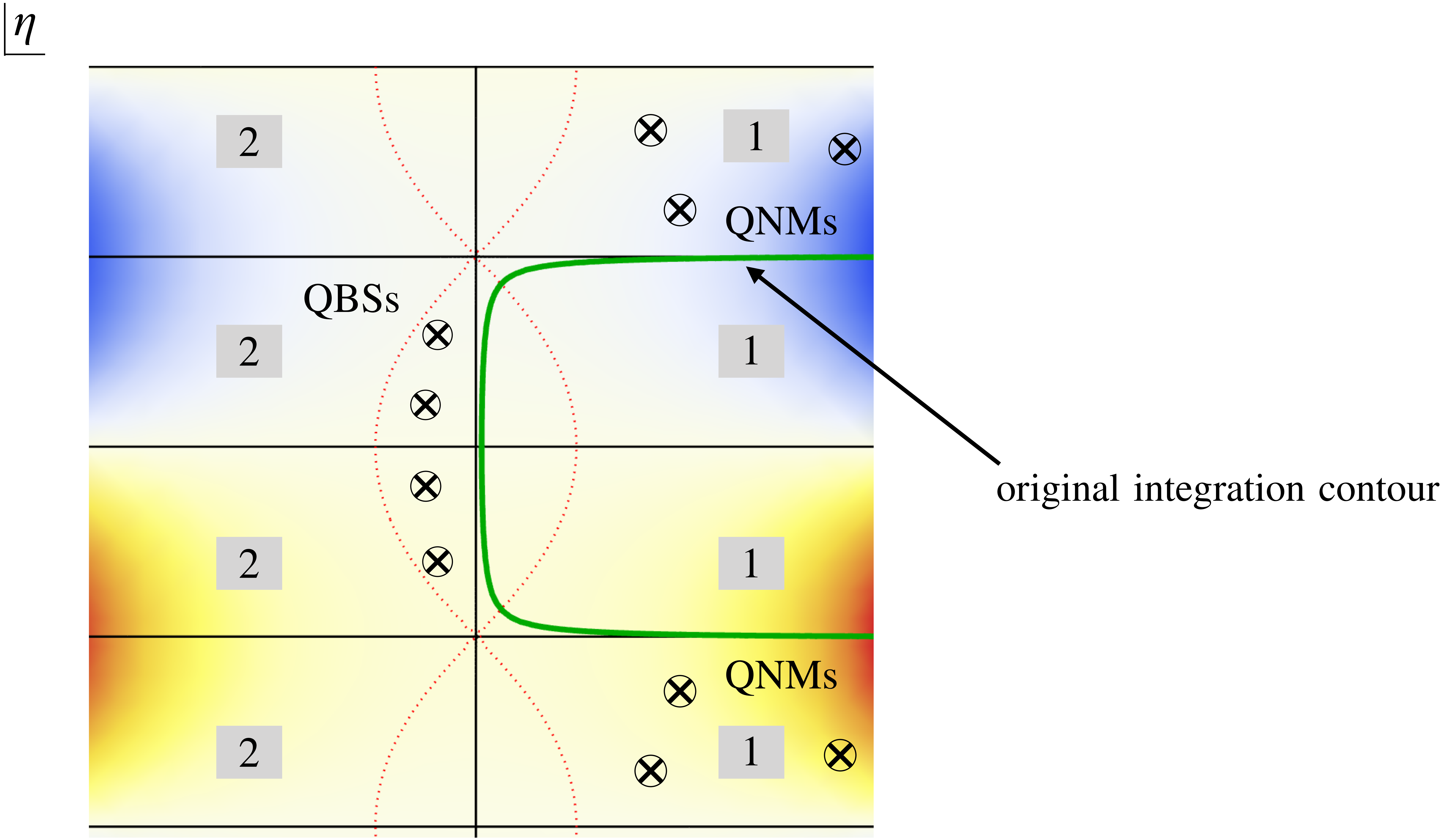

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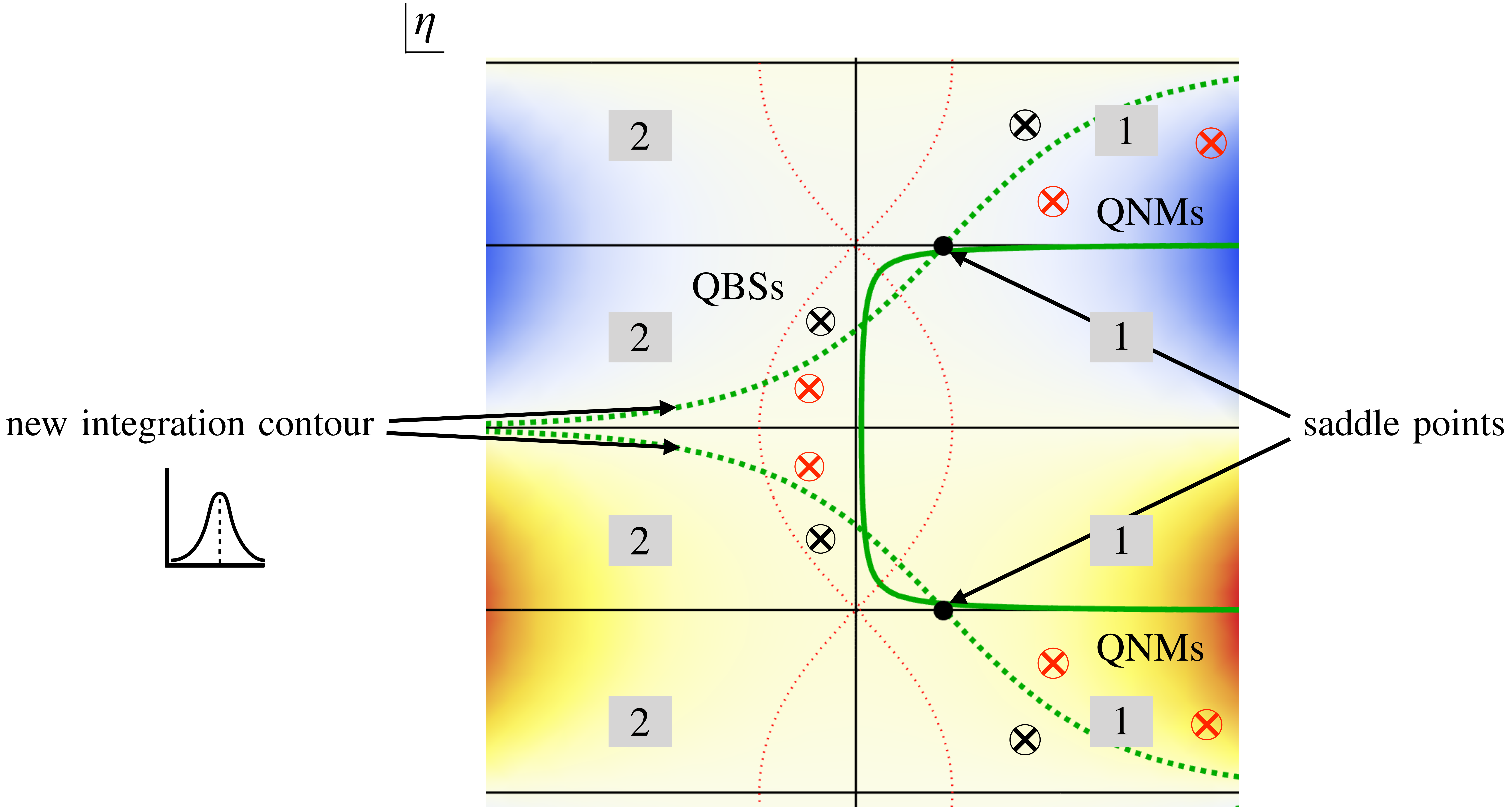
Work in progress



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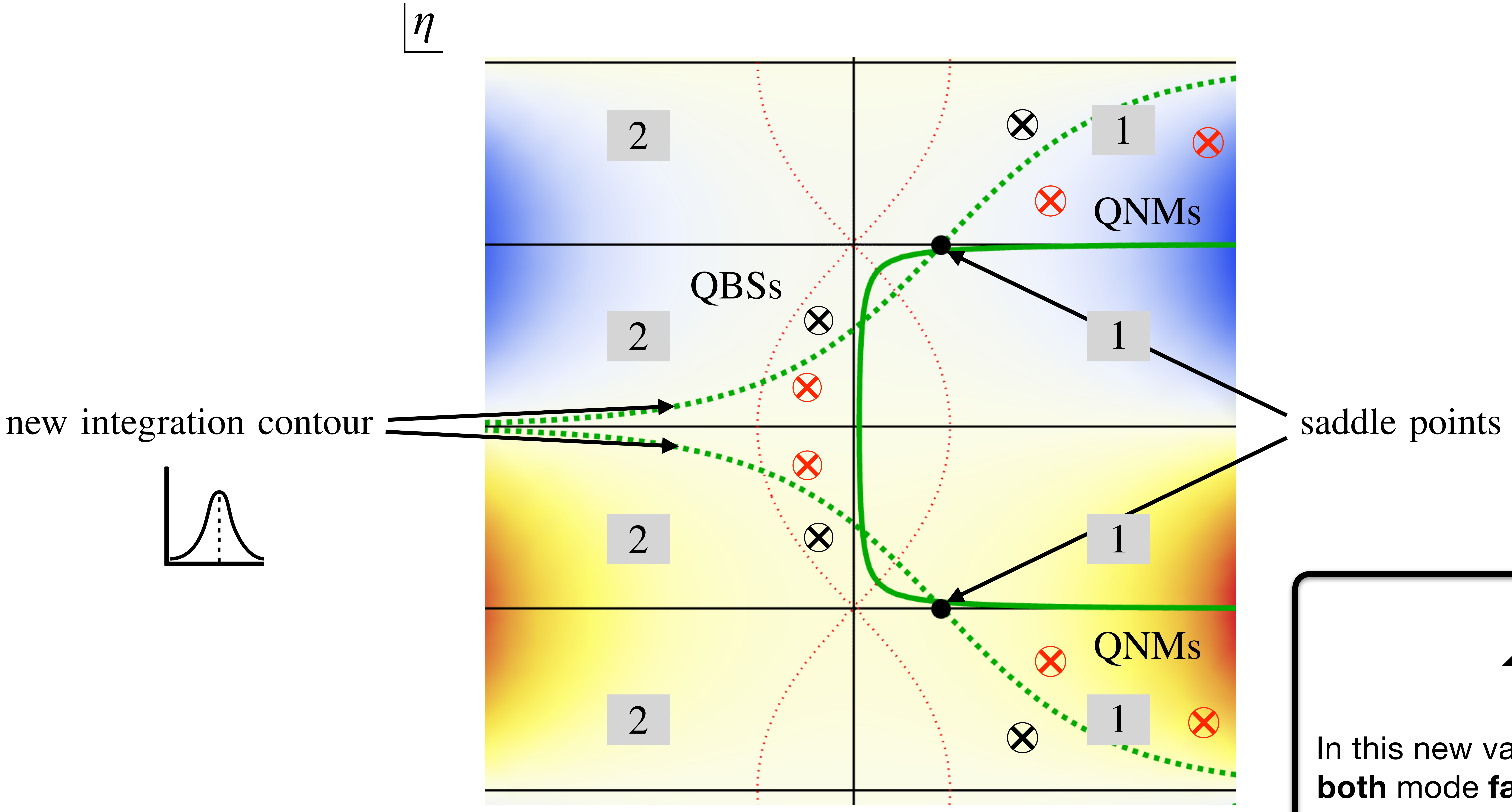
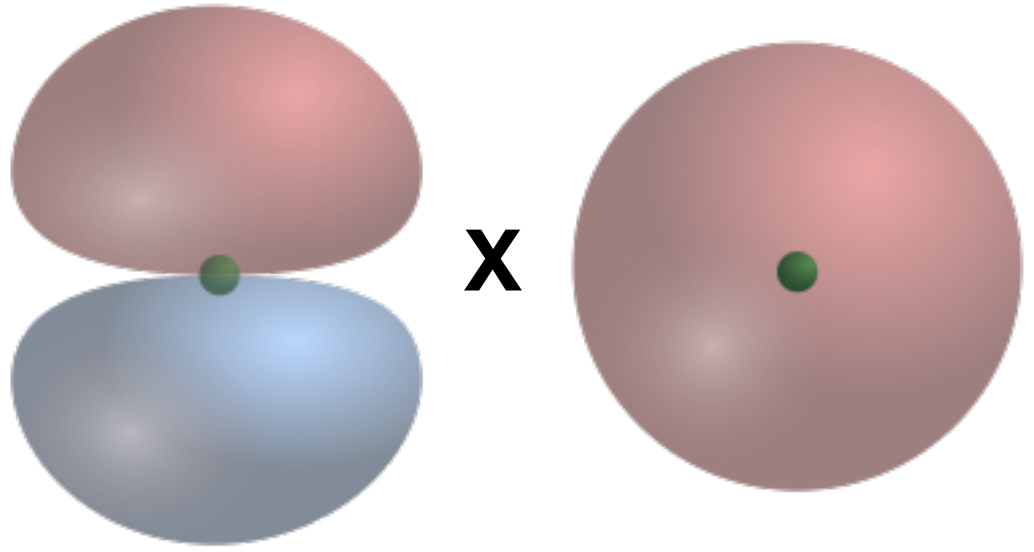


In this new variable, we can see **both** mode **families contribute** to the evolution!

PRODUCT BETWEEN MODES



bilinear form adapted from *Leung+; Green, LS+ 2022:*

$$\langle\langle \Phi_1, \Phi_2 \rangle\rangle = \int dr \int d\Omega \left[\frac{2Mra}{\Delta} \left(\Phi_2^{\substack{t \to -t \\ \phi \to -\phi}} \partial_\phi \Phi_1 - \Phi_1 \partial_\phi \Phi_2^{\substack{t \to -t \\ \phi \to -\phi}} \right) + \frac{\Sigma}{\Delta} \left(r^2 + a^2 + \frac{2Mra^2}{\Sigma} \sin^2\theta \right) \left(\Phi_2^{\substack{t \to -t \\ \phi \to -\phi}} \partial_t \Phi_1 - \Phi_1 \partial_t \Phi_2^{\substack{t \to -t \\ \phi \to -\phi}} \right) \right]$$

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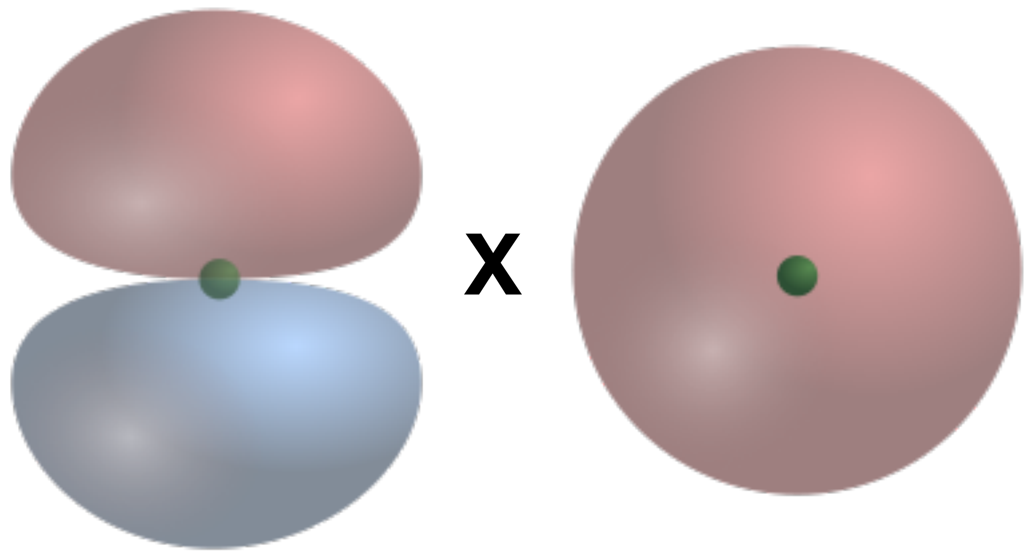

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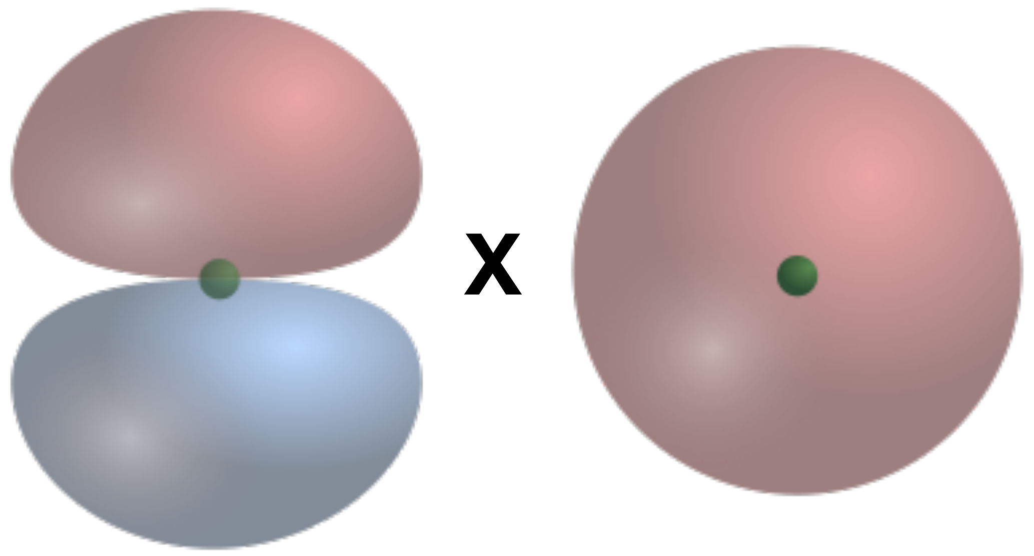

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On quasibound modes:

- $r \to +\infty$: no divergence
- $r \to r_+$: mild divergence   boundary term subtraction

QUASIBOUND STATES ARE ORTHOGONAL

Work in progress

From the properties of the product [*Green, LS+ 2022*]:

$$\langle\langle \Phi_1, \Phi_2 \rangle\rangle = \delta_{\omega_1 \omega_2}$$

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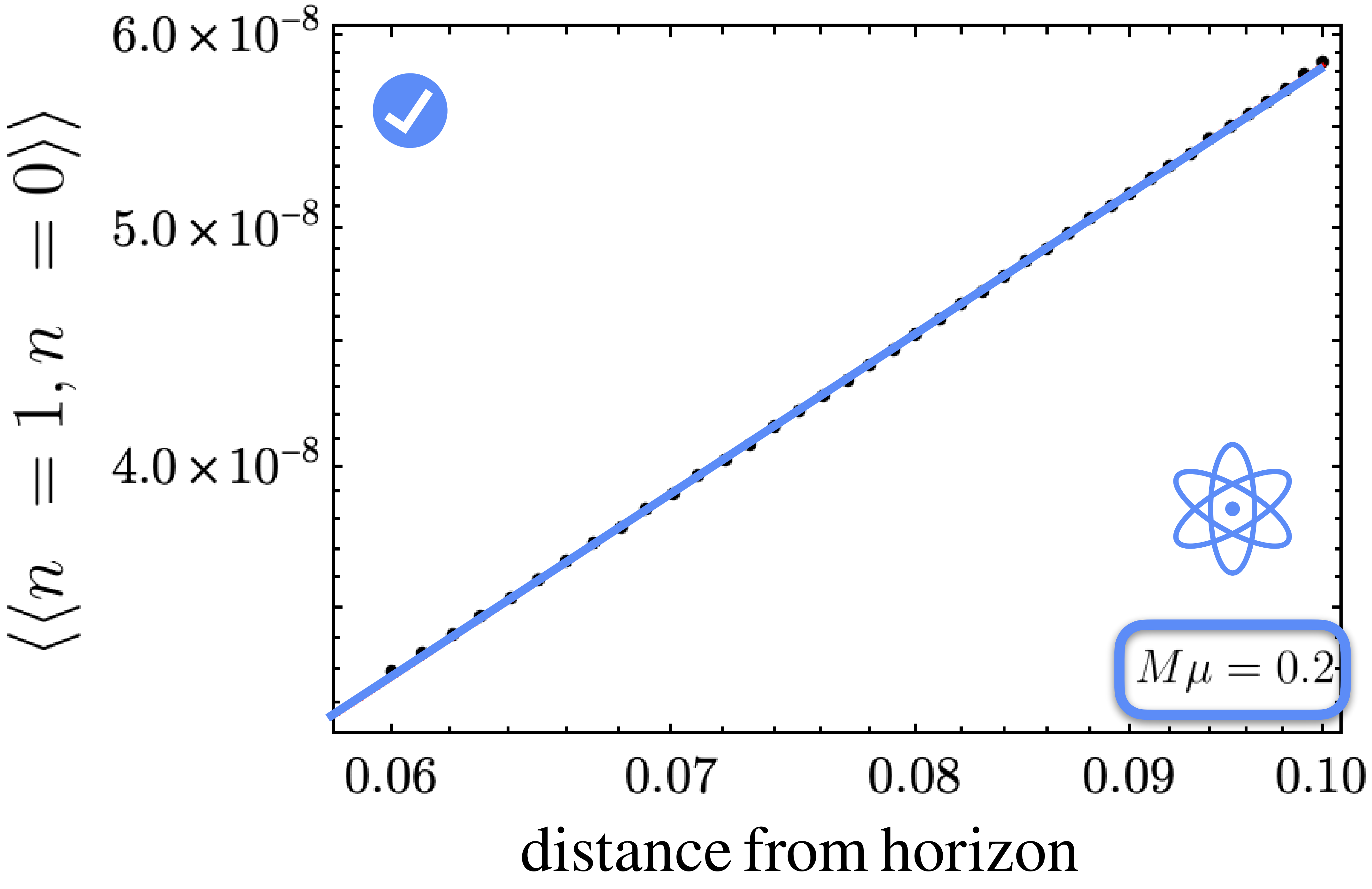

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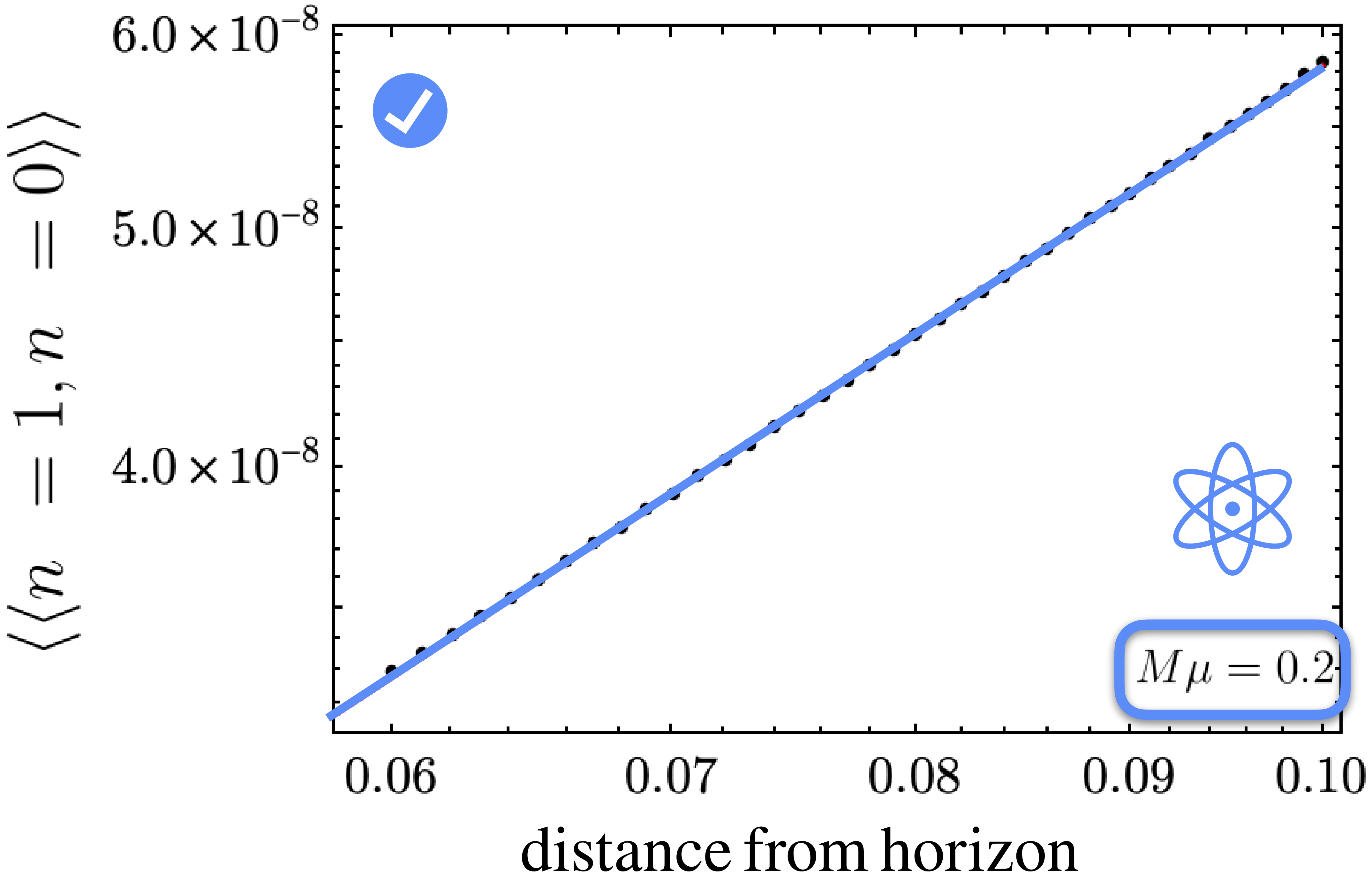
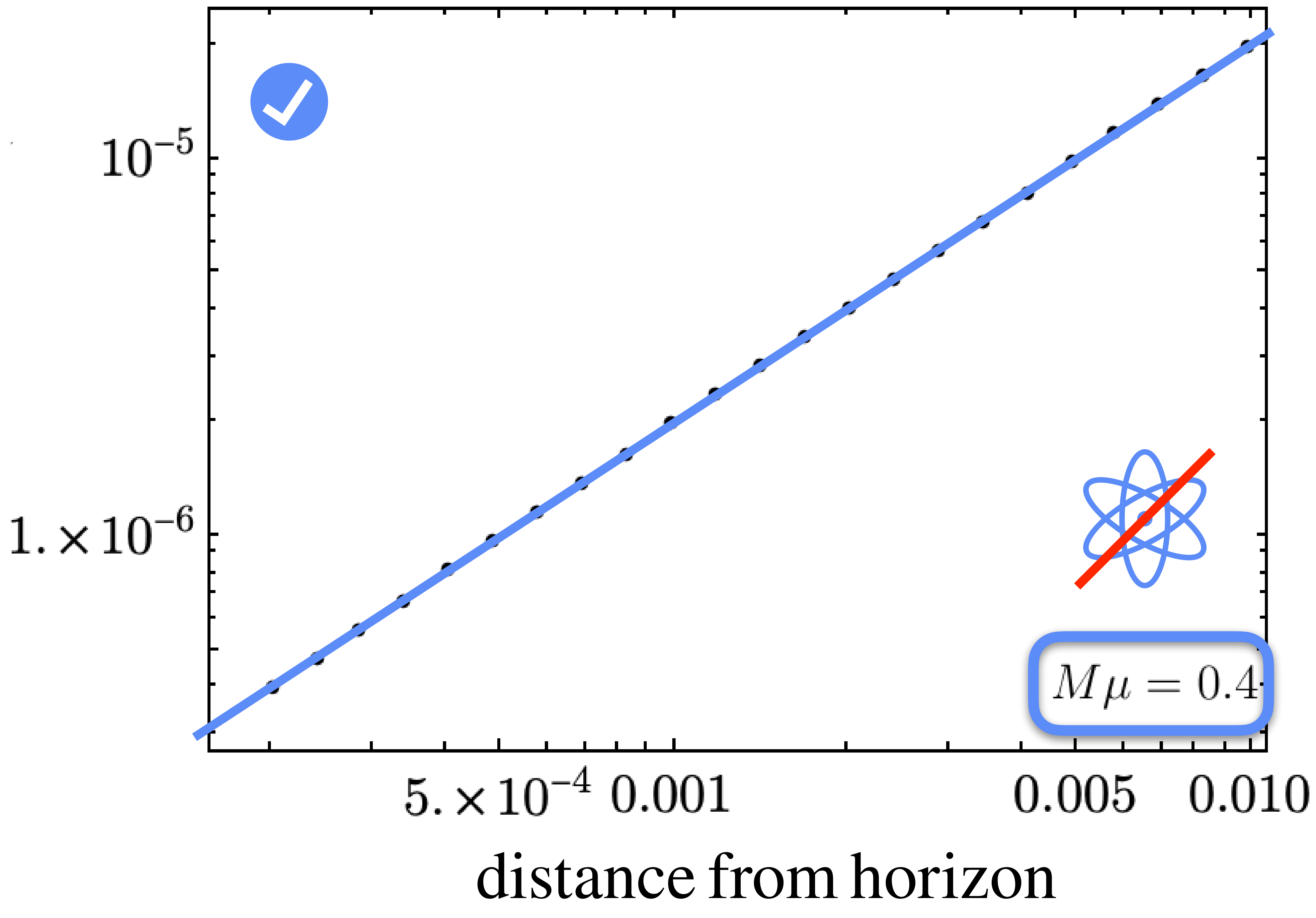
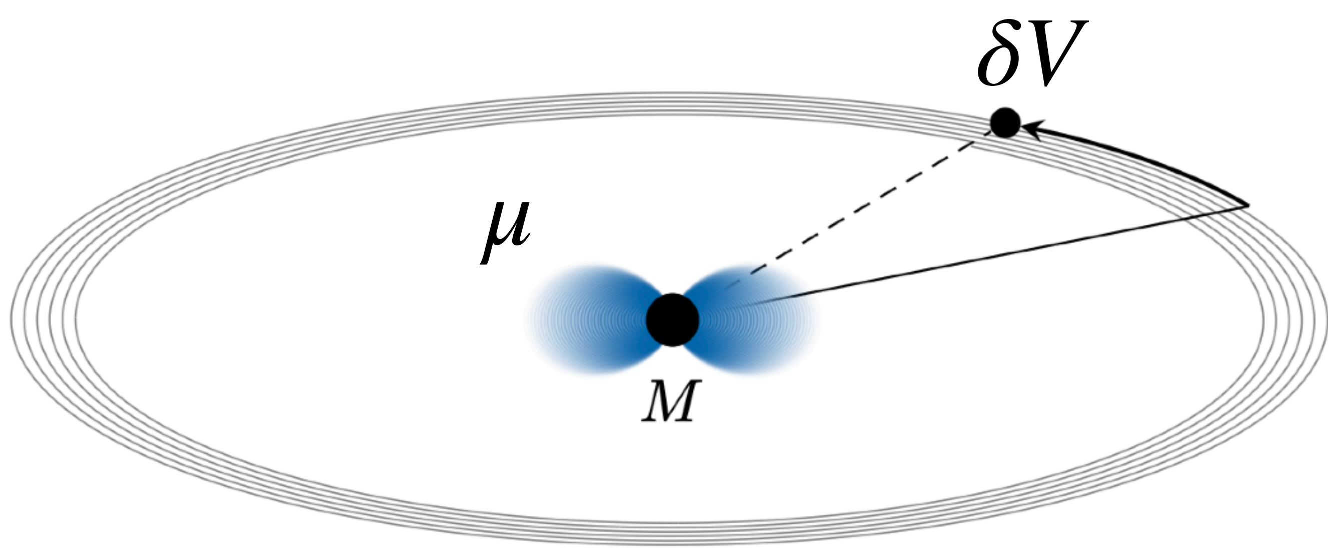




APPLICATION: A NEW FRAMEWORK

Work in progress

$$\mathcal{O}\Phi + \delta V \Phi = 0$$



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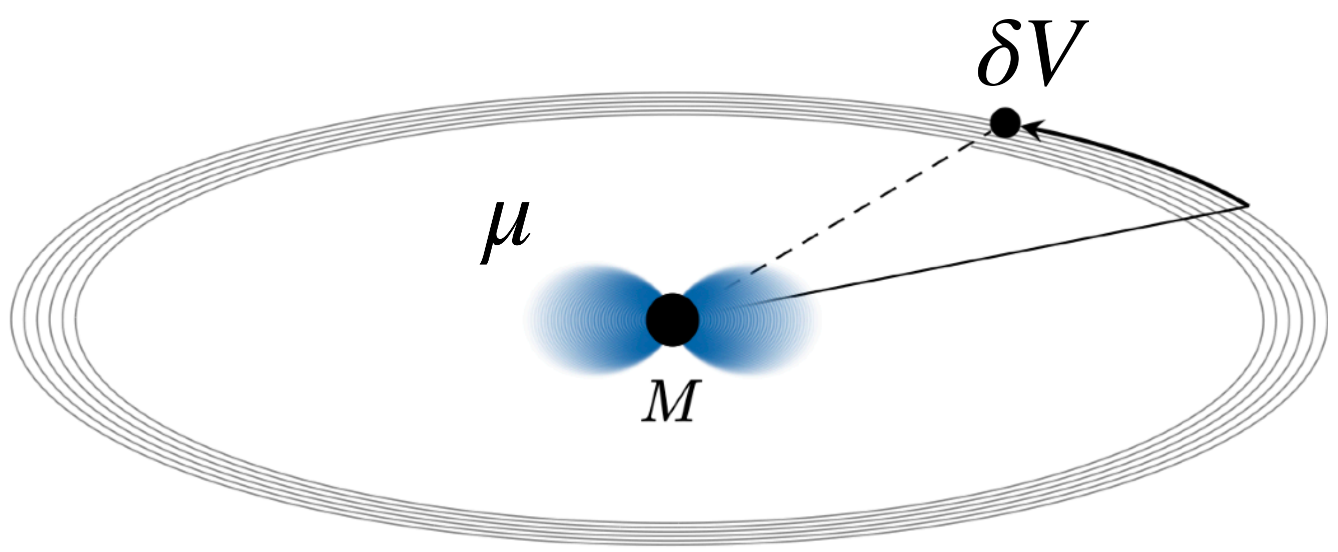

Mode **ansatz**: $\Phi = \sum_q c_q(t) \Phi_q$

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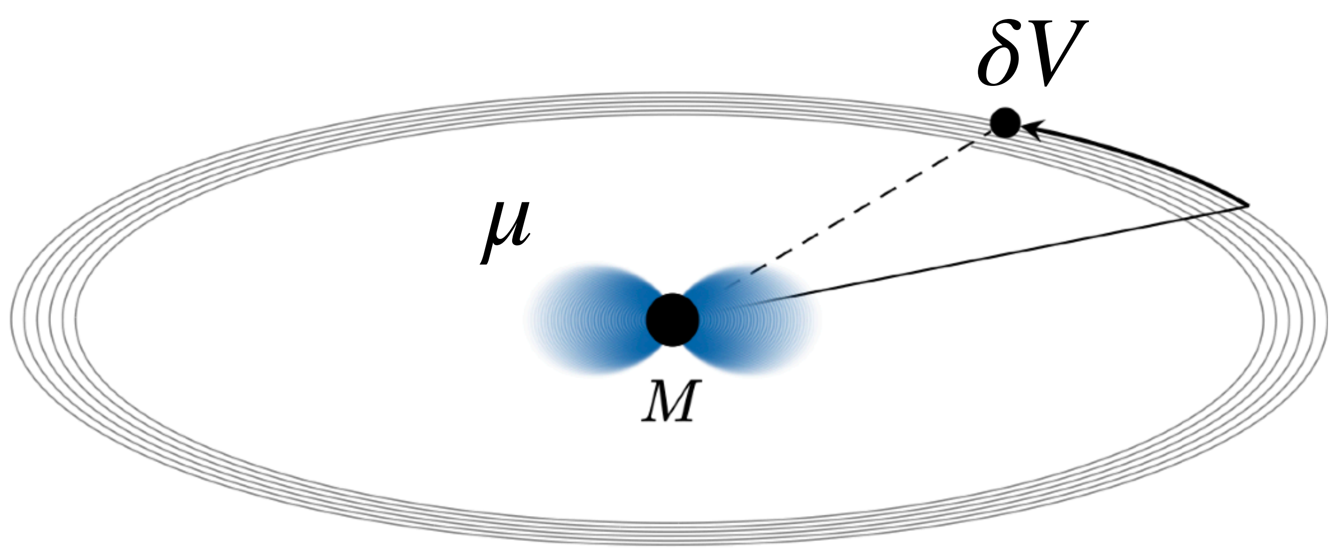

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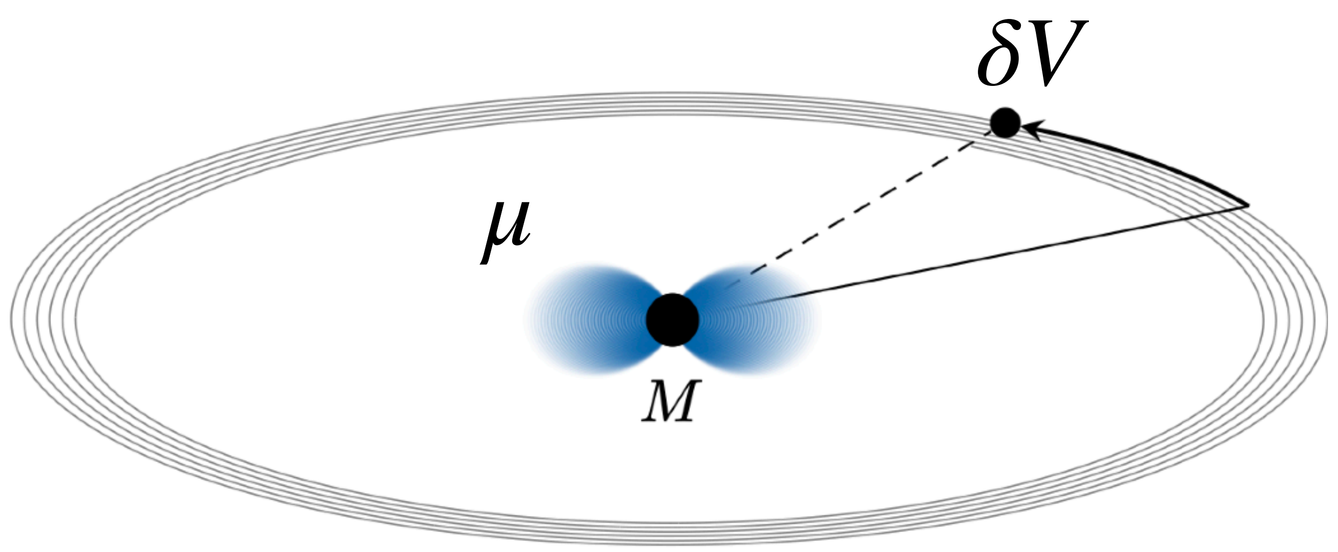

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Departure from quantum mechanics:

$\langle\langle , \rangle\rangle \sim \partial_t$

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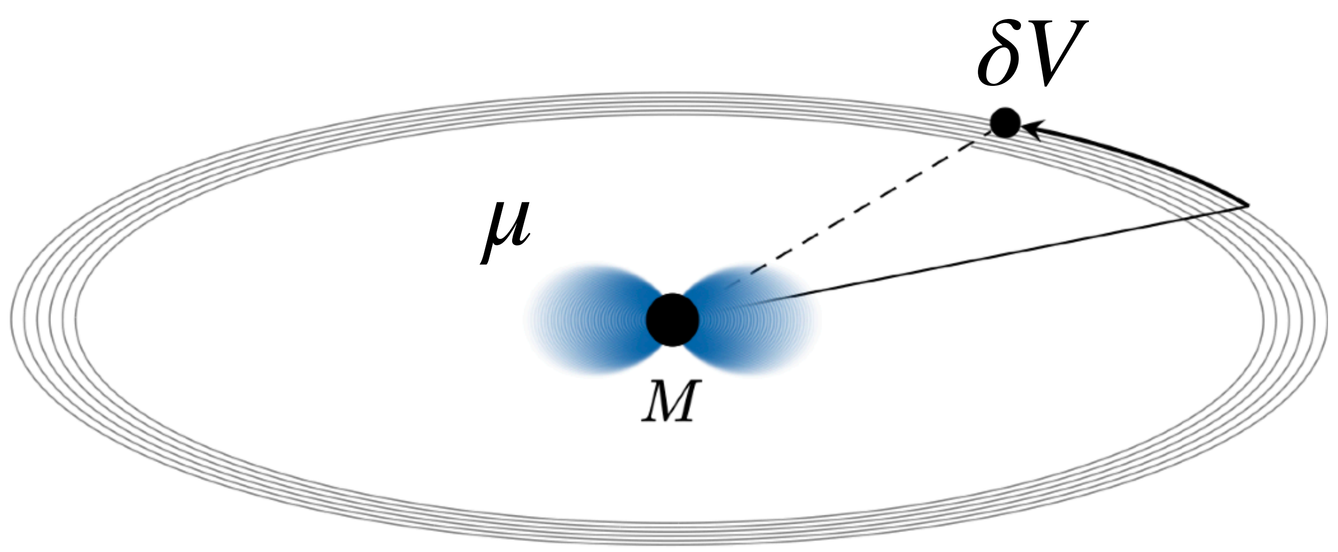

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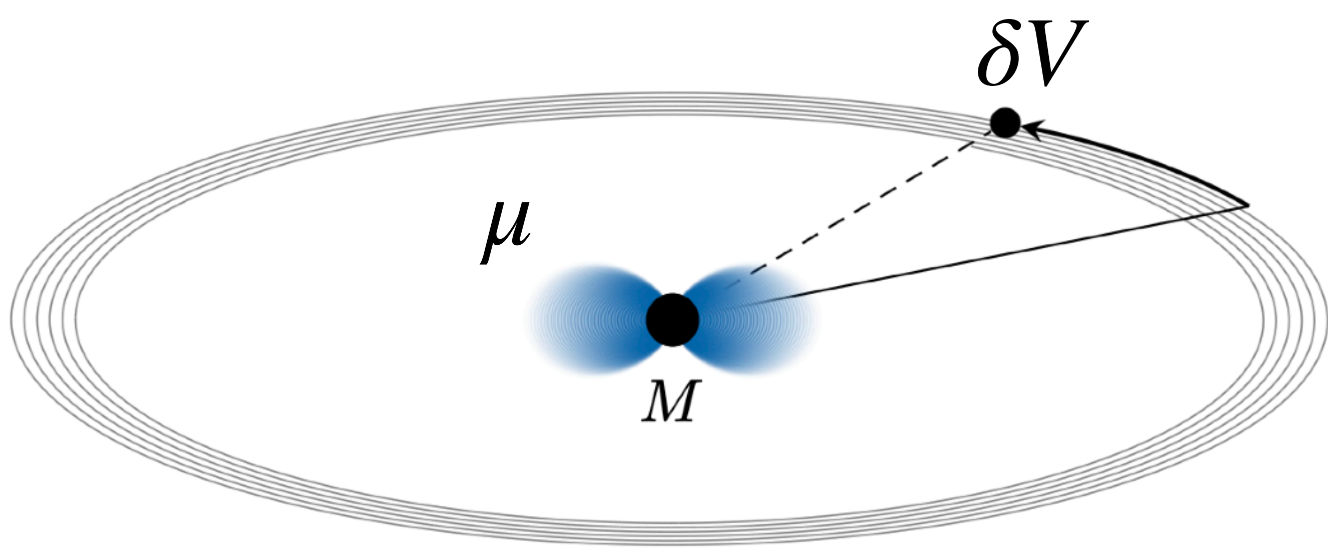
Assuming $c(t)$ evolve slowly and using orthogonality:

$$-2i\omega_n \dot{c}_n \langle\langle \Phi_n, \Phi_n \rangle\rangle = -\sum_q c_q(t) \langle\langle \Phi_n, \delta V \Phi_q \rangle\rangle$$

! Departure from quantum mechanics: $\langle\langle , \rangle\rangle \sim \partial_t$

MATRIX ELEMENTS FOR MODE MIXING

Work in progress



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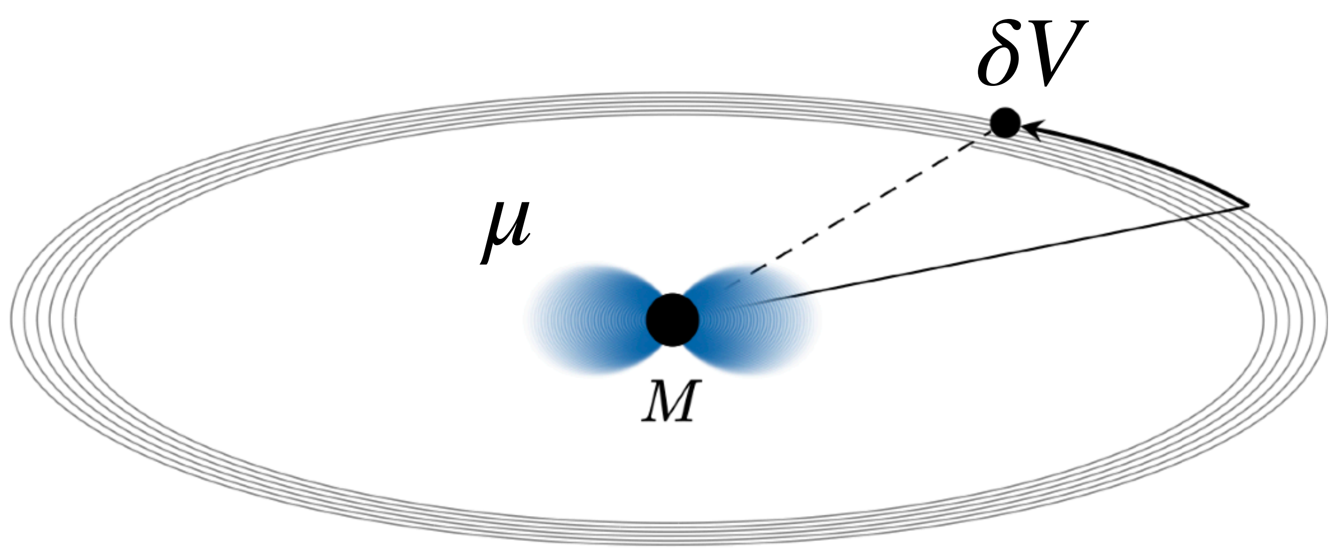

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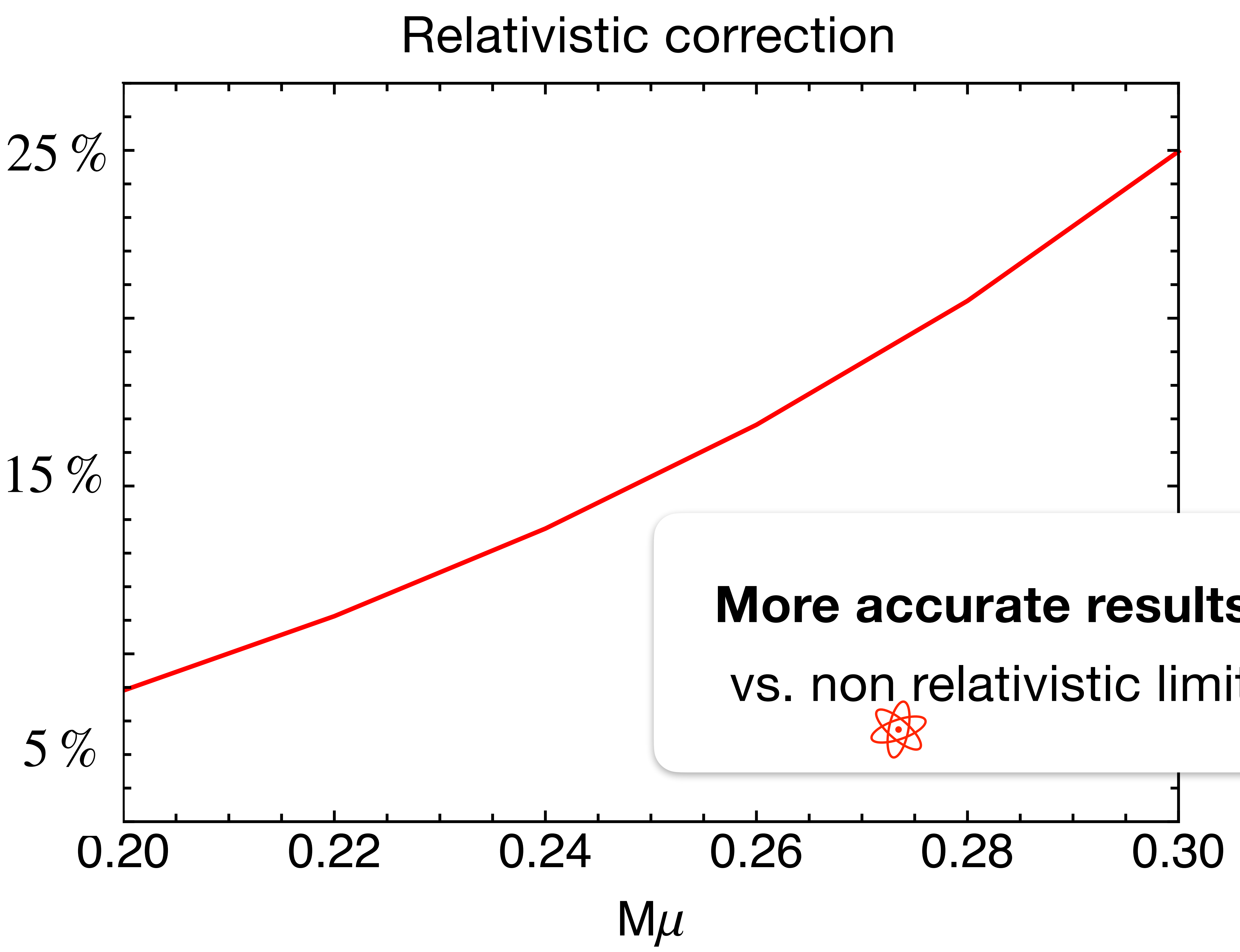
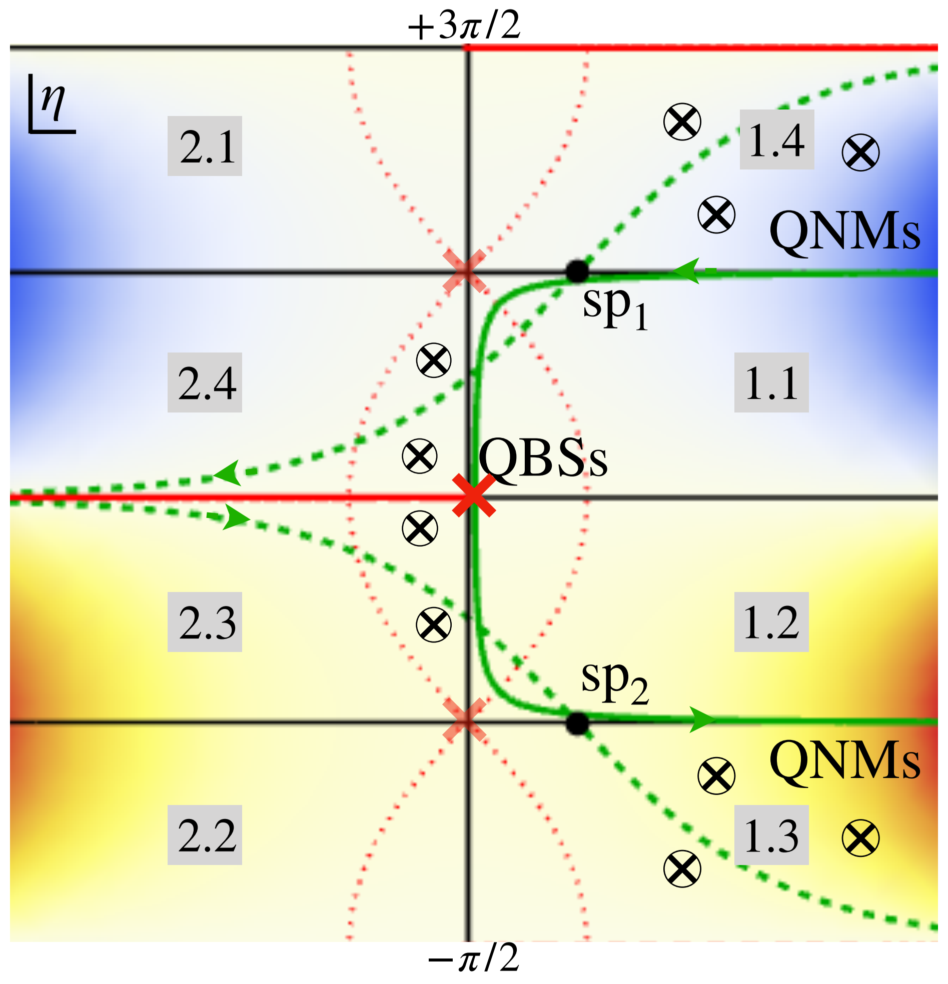


More accurate results

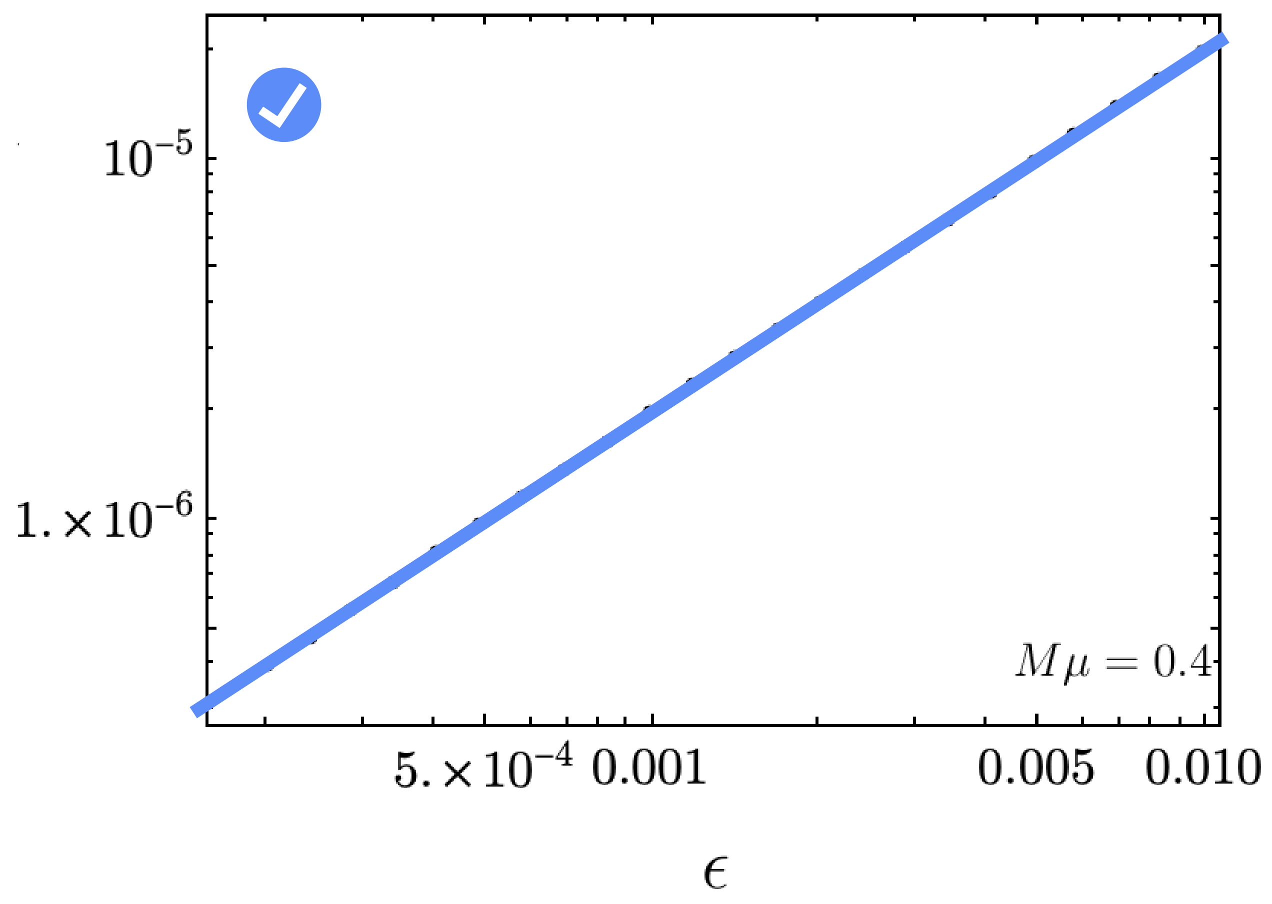
vs. non relativistic limit

NEW RESULTS ON BLACK-HOLE QUASIBOUND STATES

1) Green's function structure



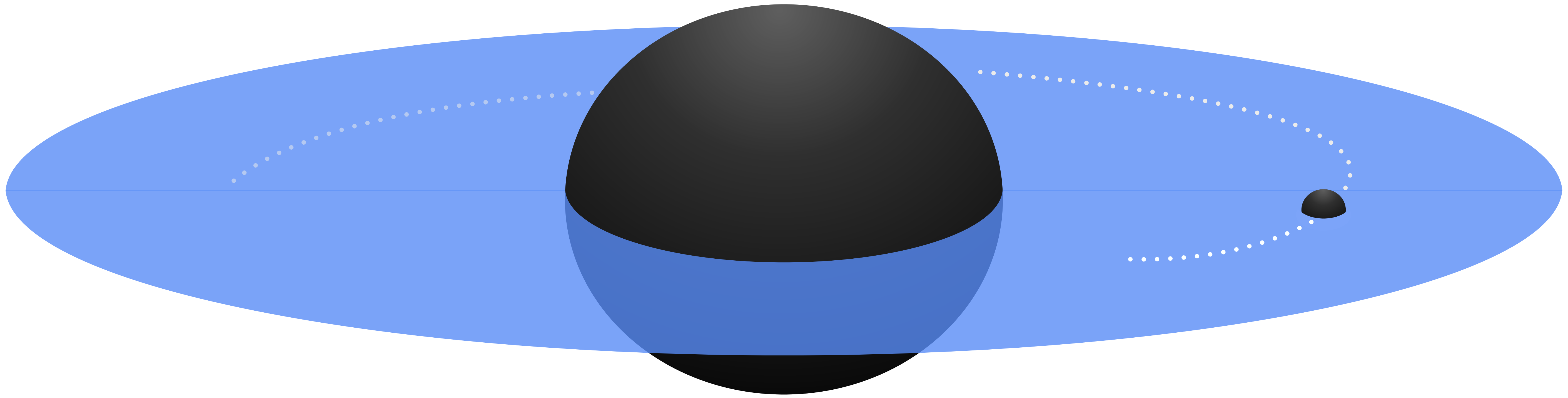
2) quasi bound states are **orthogonal**



3) time-dependent excitation

$$2i\omega_n \dot{c}_n \langle\langle \Phi_n, \Phi_n \rangle\rangle = \sum_q c_q(t) \langle\langle \Phi_n, \delta V \Phi_q \rangle\rangle$$

Thank you!



Laura Sberna (Max Planck Institute for Gravitational Physics, Potsdam)

Capra Meeting
Copenhagen, July 2023