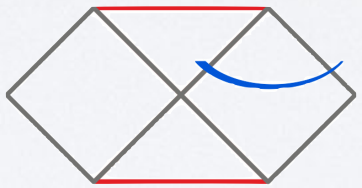
MULTI-DOMAIN SPECTRAL METHOD FOR SELF-FORCE CALCULATIONS



Rodrigo Panosso Macedo

Patrick Bourg, Adam Pound

HYPERBOLOIDAL METHODS IN SELF-FORCE

- **Goal:** develop new codes in perturbation theory, offering an alternative framework for self-force calculation

HYPERBOLOIDAL METHODS IN SELF-FORCE

- **Goal:** develop new codes in perturbation theory, offering an alternative framework for self-force calculation
- **Current status:** Benchmark test cases - Scalar field on Schwarzschild background, first order self-force, particle on circular orbits*

HYPERBOLOIDAL METHODS IN SELF-FORCE

- **Goal:** develop new codes in perturbation theory, offering an alternative framework for self-force calculation
- **Current status:** Benchmark test cases - Scalar field on Schwarzschild background, first order self-force, particle on circular orbits*

$$\Box_o \chi^{(1)} = \mathcal{S}^{(1)}$$

HYPERBOLOIDAL METHODS IN SELF-FORCE

- **Goal:** develop new codes in perturbation theory, offering an alternative framework for self-force calculation
- **Current status:** Benchmark test cases - Scalar field on Schwarzschild background, first order self-force, particle on circular orbits*

$$\Box_o \chi^{(1)} = \mathcal{S}^{(1)}$$

- **Motivation:** second order calculations*

$$\Box_o \chi^{(2)} = \mathcal{S}^{(2)}(\chi^{(1)}, \partial\chi^{(1)})$$

HYPERBOLOIDAL METHODS IN SELF-FORCE

- **Goal:** develop new codes in perturbation theory, offering an alternative framework for self-force calculation

- **Current status:** Benchmark test cases - Scalar field on Schwarzschild background, first order self-force, particle on circular orbits*

$$\Box_o \chi^{(1)} = \mathcal{S}^{(1)}$$

- **Motivation:** second order calculations*

$$\Box_o \chi^{(2)} = \mathcal{S}^{(2)}(\chi^{(1)}, \partial \chi^{(1)})$$

- Any source of systematic error at 1st order impacts performance of 2nd order calculation

HYPERBOLOIDAL METHODS IN SELF-FORCE

- **Goal:** develop new codes in perturbation theory, offering an alternative framework for self-force calculation

Numerical systematic errors

Boundary conditions: impose asymptotic conditions ensuring the energy propagates into the black hole and out to the wave zone

PROBLEM: physical space has unbounded domain $r_* \in (-\infty, +\infty)$
USUAL SOLUTION: approximate BC at finite radius $r_* = \pm r_*^{\mathrm{BC}}$

ALTERNATIVE: Compact hyperboloidal slides $\sigma \in [0, 1]$

HYPERBOLOIDAL METHODS IN SELF-FORCE

- **Goal:** develop new codes in perturbation theory, offering an alternative framework for self-force calculation

Numerical systematic errors

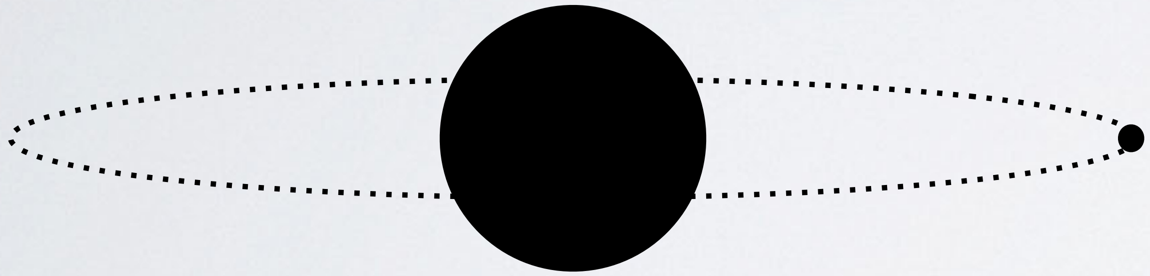
<u>Numerical discretisation</u>: *any* numerical scheme introduces errors

PROBLEM: *any* numerical scheme introduce errors
'FASTER' SOLVERS+'SLOW' CONVERGENCE: Finite difference methods, Explicit time integrators

'SLOWER' SOLVERS+'FAST' CONVERGENCE: Multidomain spectral methods, Implicit time integrators

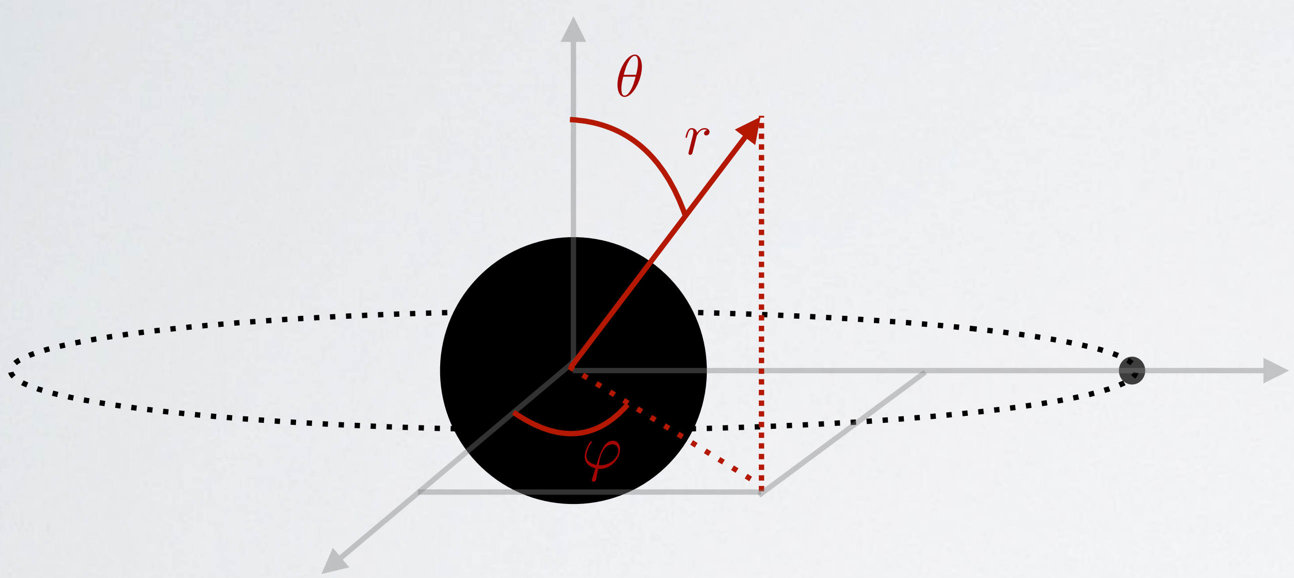
EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011



EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011



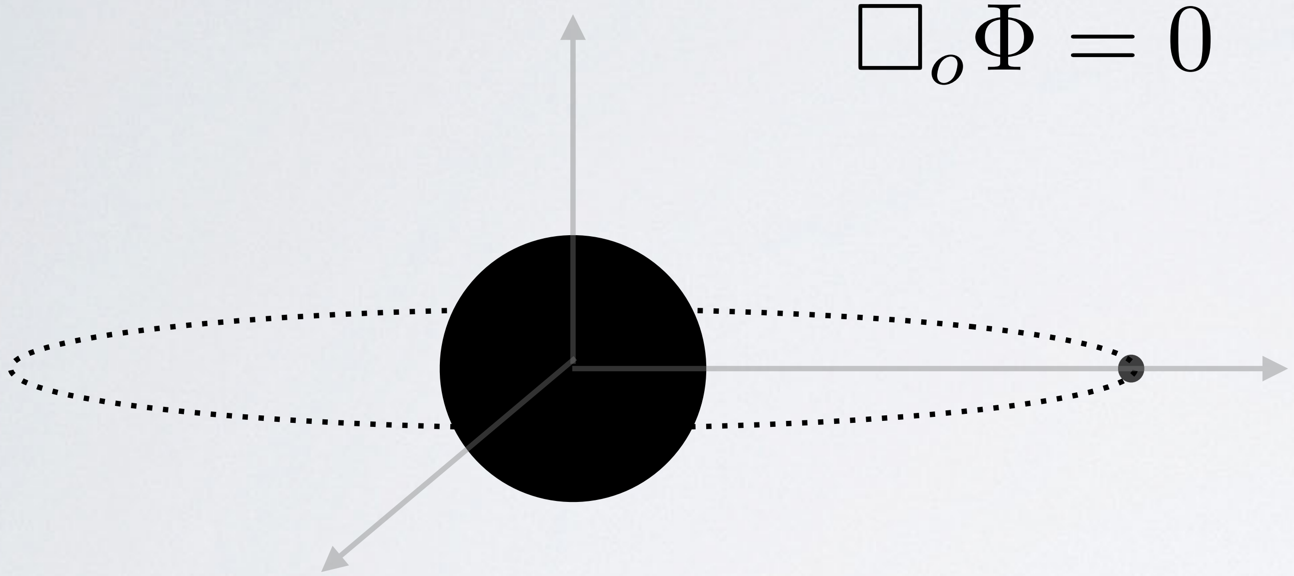
Coordinate System

(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011

$$\Box_o \Phi = 0$$



Coordinate System

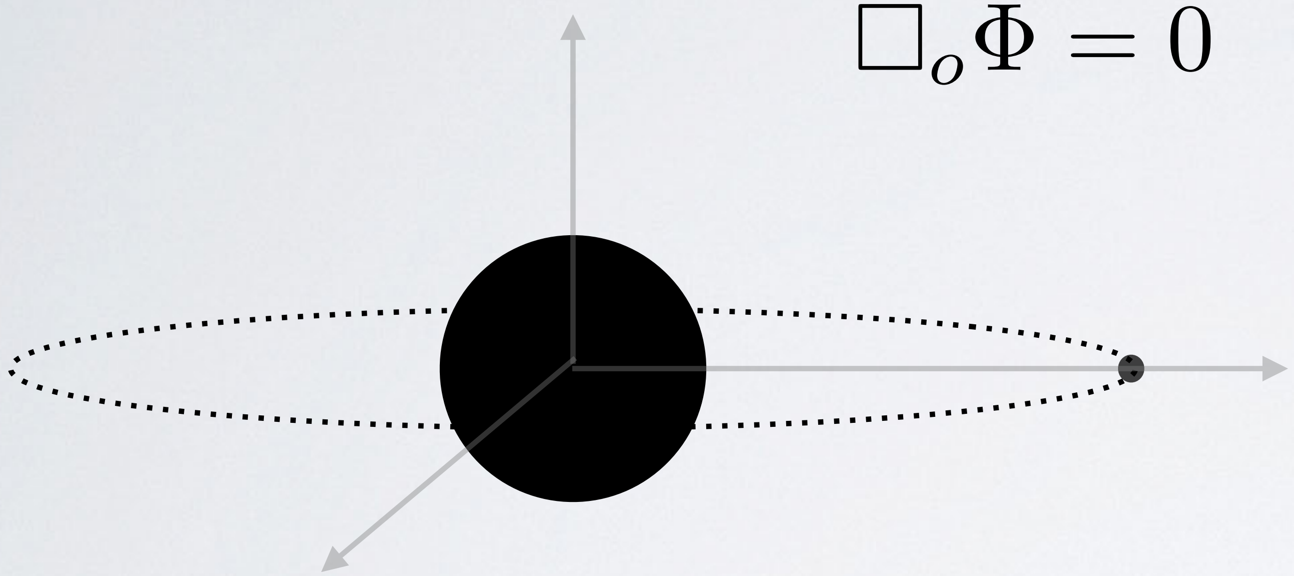
$$(t, r, \theta, \varphi)$$

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

Impose BC: Energy flow to wave zone and into black hole

$$\Box_o \Phi = 0$$



Coordinate System

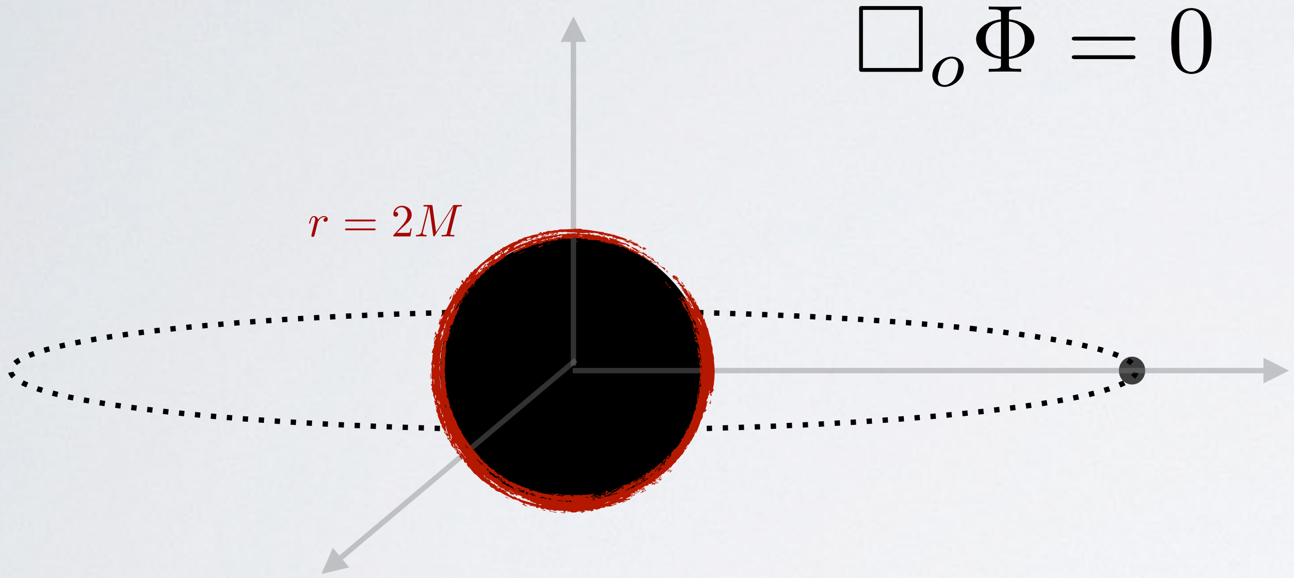
$$(t, r, \theta, \varphi)$$

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

Impose BC: Energy flow to wave zone and into black hole

$$\Box_o \Phi = 0$$



Coordinate System

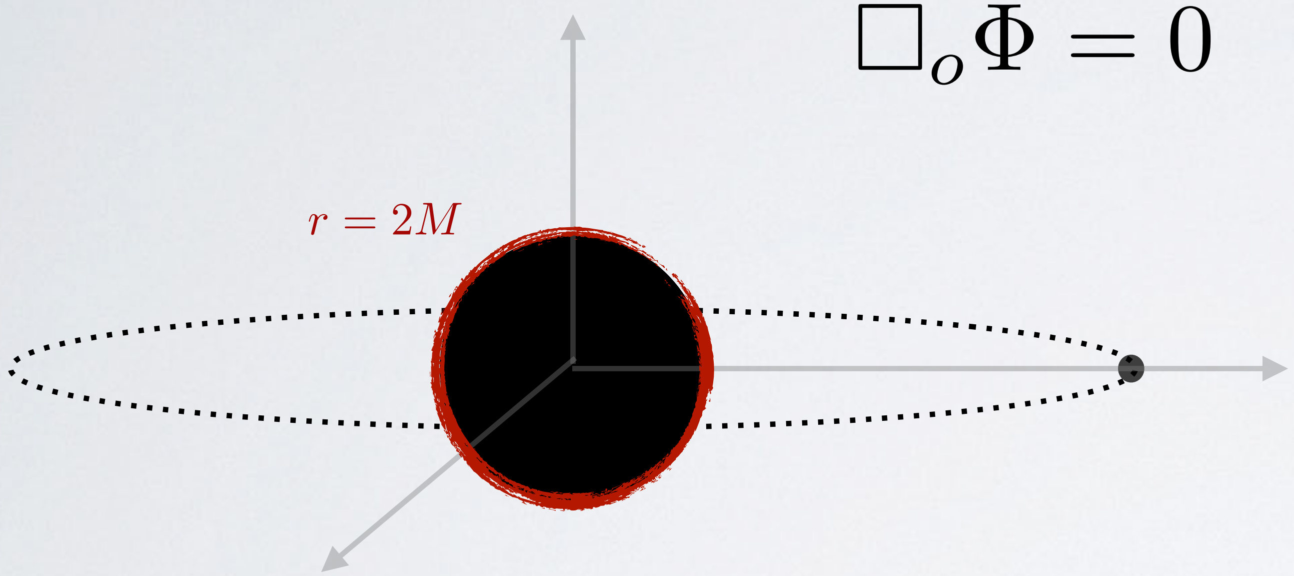
$$(t, r, \theta, \varphi)$$

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011

Impose BC: Energy flow to wave zone and into black hole

$$\Box_o \Phi = 0$$



Coordinate System

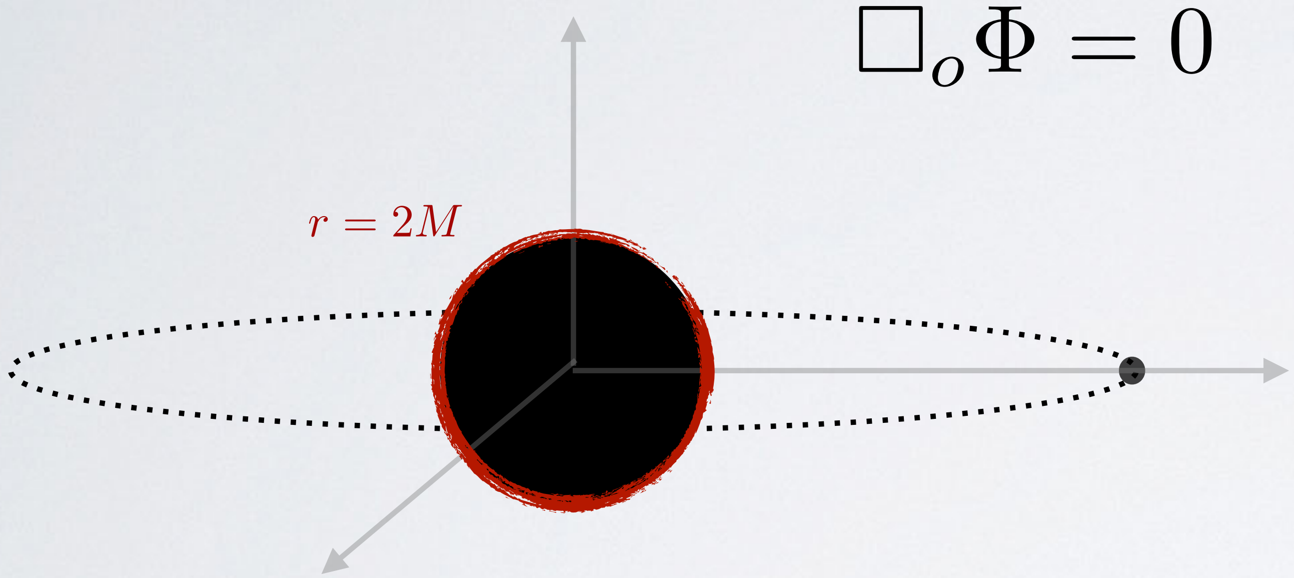
(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

Impose BC: Energy flow to wave zone and into black hole

$$\Box_o \Phi = 0$$



$r \to \infty$

Solution: retarded field

Φ^{ret}

Coordinate System

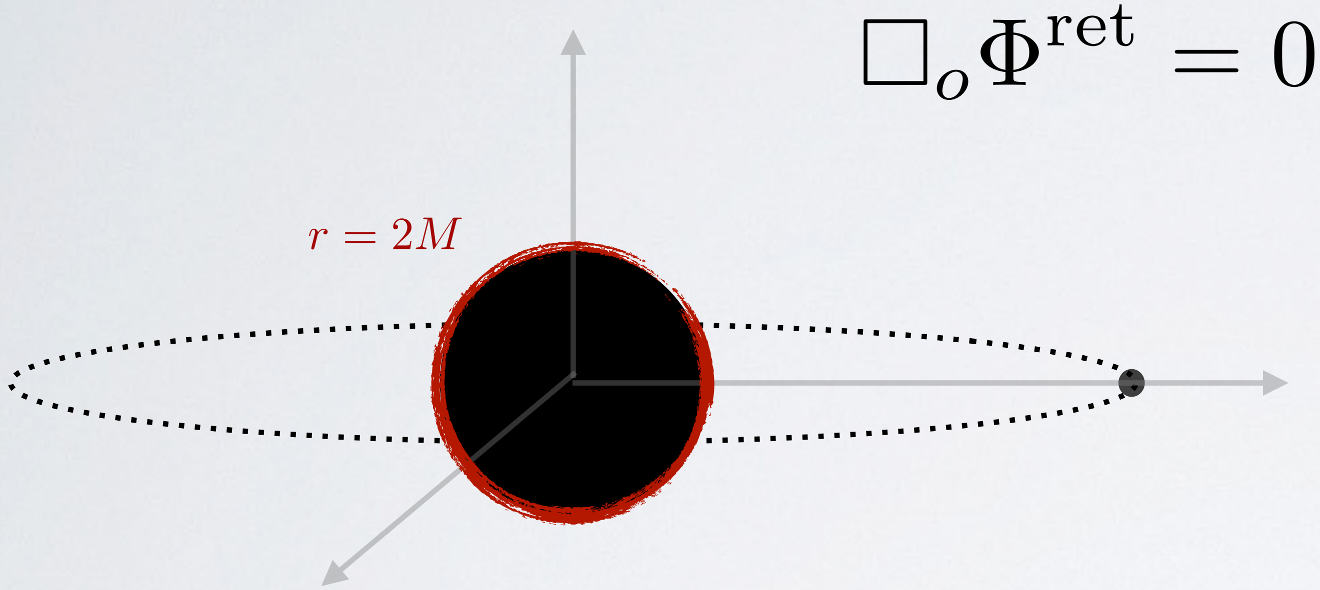
(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

Impose BC: Energy flow to wave zone and into black hole

$$\Box_o \Phi^{\mathrm{ret}} = 0$$



Solution: retarded field

$$\Phi^{\mathrm{ret}}$$

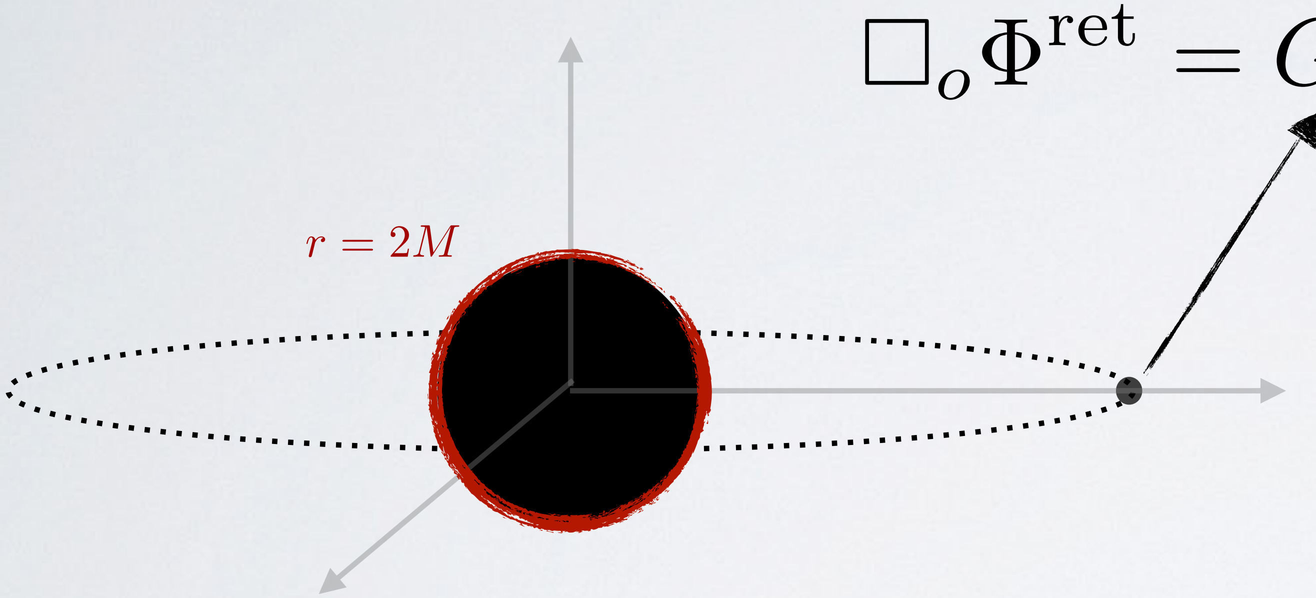
Coordinate System

(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

$$\Box_o \Phi^{\rm ret} = G\delta(x^a - x_p^a)$$



$r \to \infty$

Solution: retarded field

$\Phi^{\rm ret}$

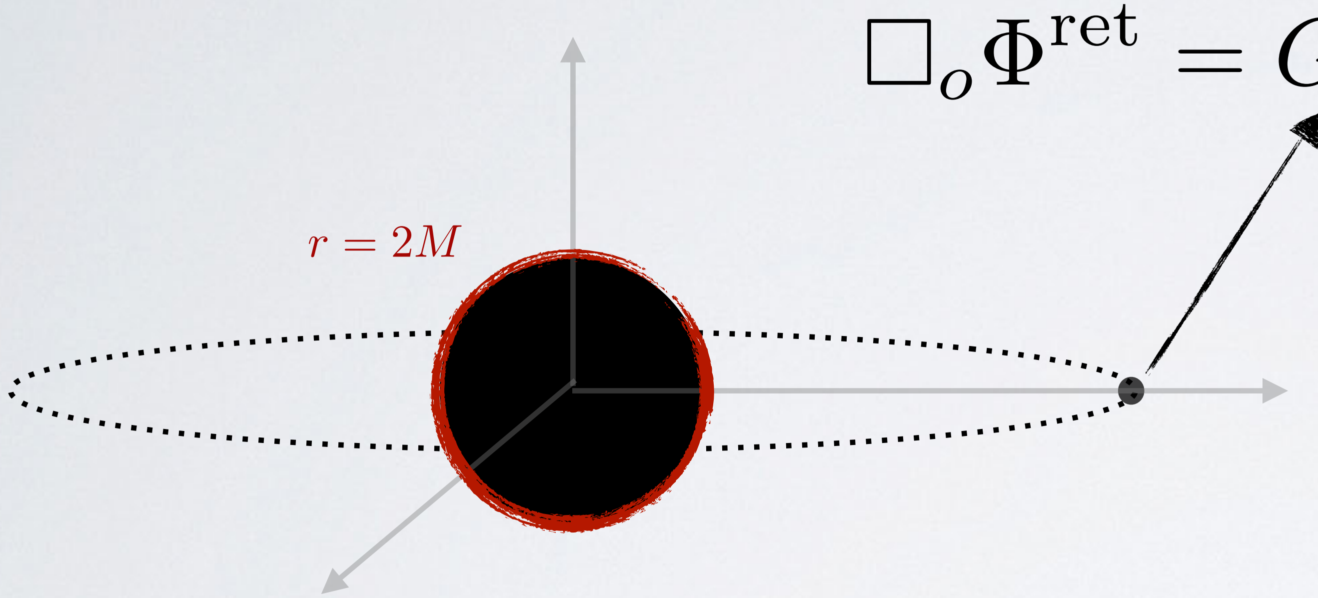
Coordinate System

(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

$$\Box_o \Phi^{\mathrm{ret}} = G\delta(x^a - x_p^a)$$



$r \to \infty$

Solution: retarded field

$$\Phi^{\mathrm{ret}} = \Phi^{\mathrm{sing}} + \Phi^{\mathrm{reg}}$$

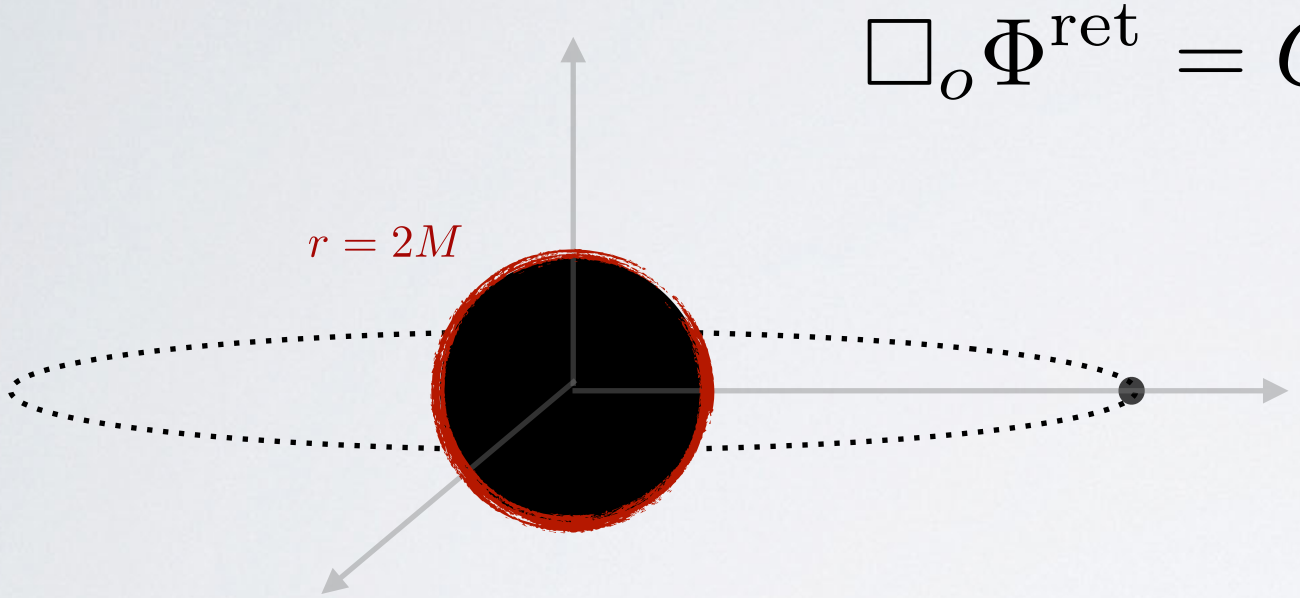
Coordinate System

(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011

$$\Box_o \Phi^{\text{ret}} = G\delta(x^a - x_p^a)$$



Solution: retarded field

$$\Phi^{\text{ret}} = \Phi^{\text{sing}} + \Phi^{\text{reg}}$$

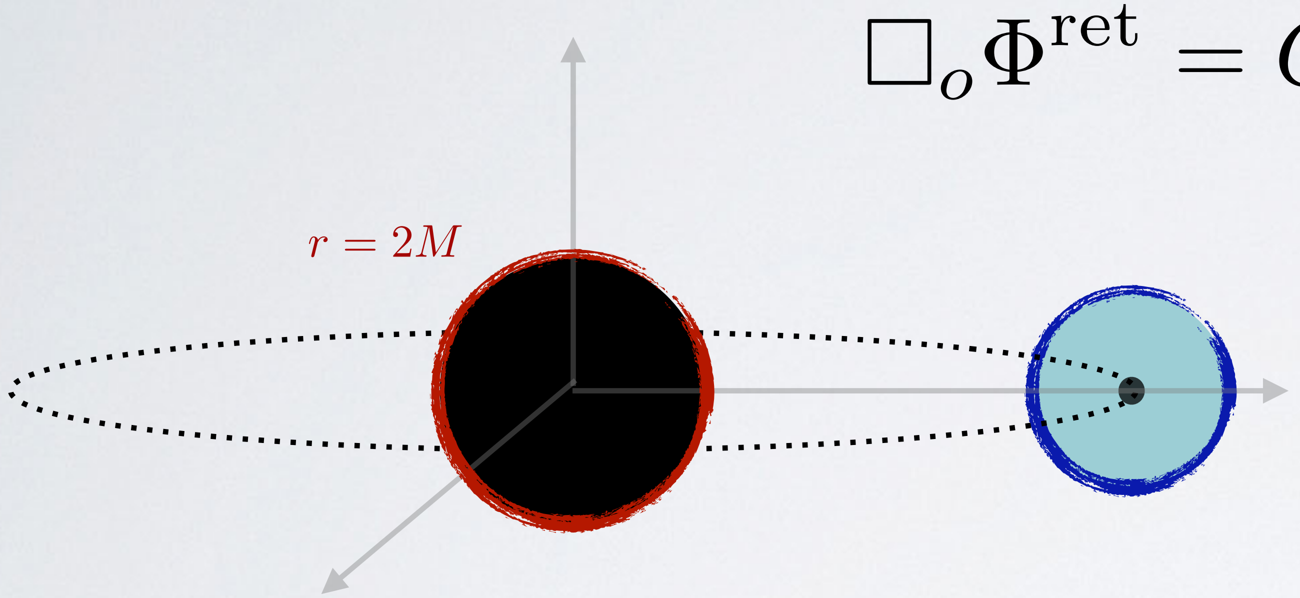
Coordinate System

(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

$$\Box_o \Phi^{\rm ret} = G\delta(x^a - x_p^a)$$



$r \to \infty$

Solution: retarded field

$$\Phi^{\rm ret} = \Phi^{\rm sing} + \Phi^{\rm reg}$$

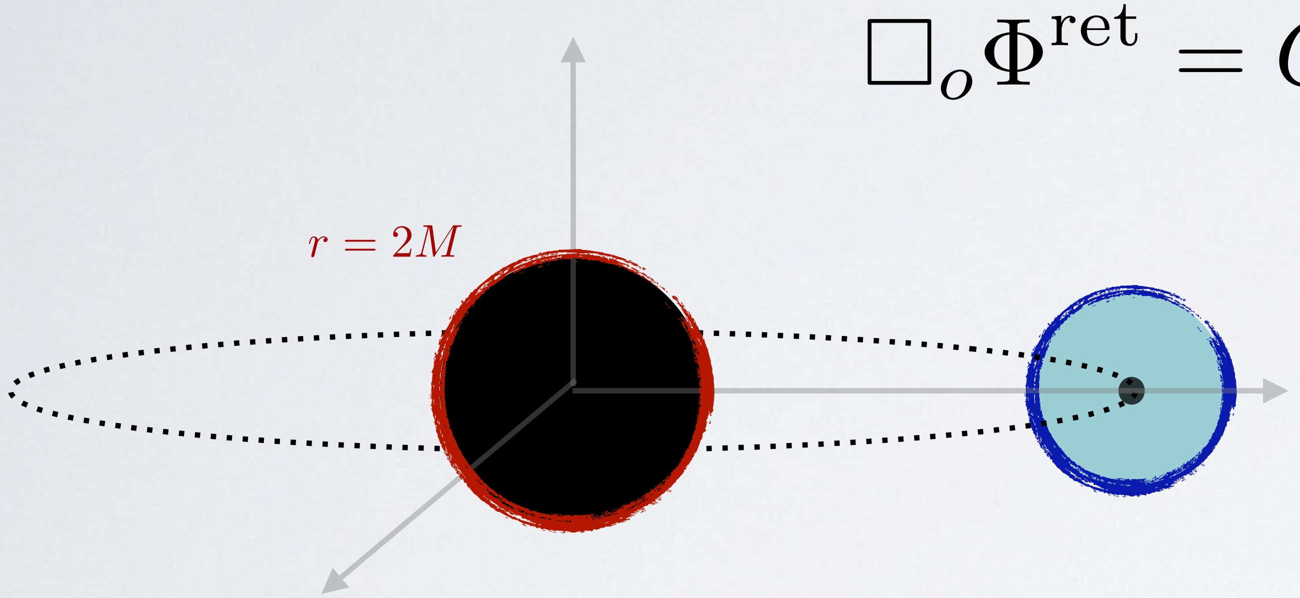
Coordinate System

(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011

$$\Box_o \Phi^{\text{ret}} = G\delta(x^a - x_p^a)$$



$r \to \infty$

Solution: retarded field

$$\Phi^{\text{ret}} = \Phi^{\text{sing}} + \Phi^{\text{reg}}$$

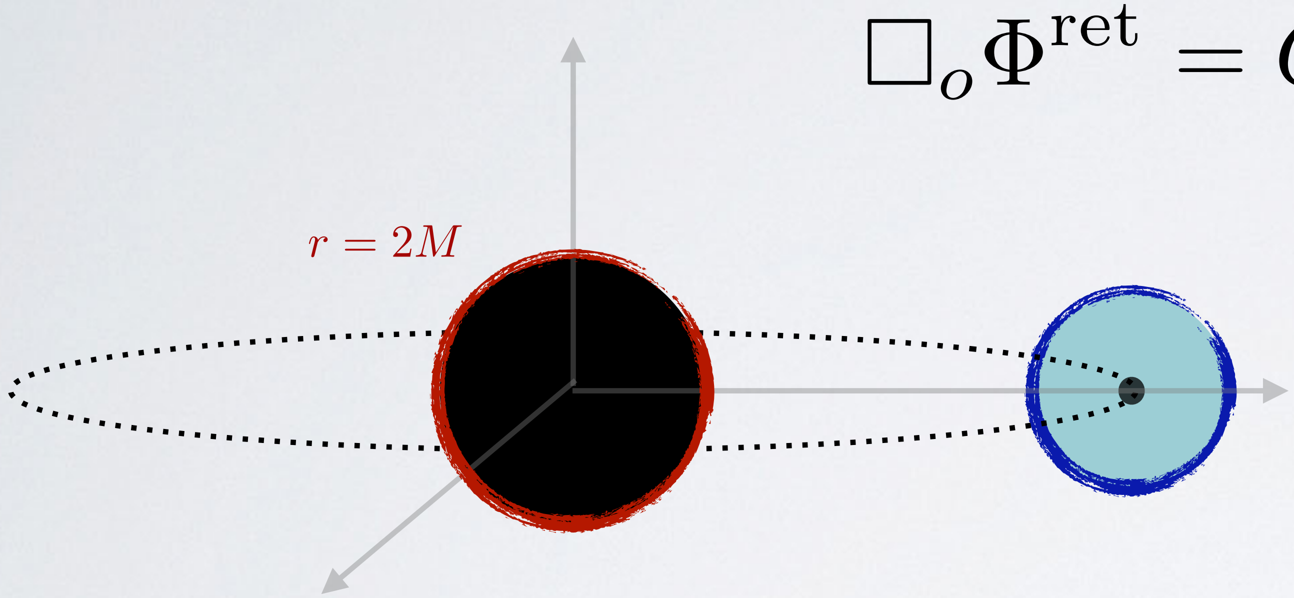
Coordinate System

(t, r, θ, φ)

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

$$\Box_o \Phi^{\rm ret} = G\delta(x^a - x^a_p)$$



Solution: retarded field

~~$$\Phi^{\rm ret} = \Phi^{\rm sing} + \Phi^{\rm reg}$$~~

$$\approx$$

$$\Phi^{\rm ret} = \Phi^{\rm punc} + \Phi^{\rm res}$$

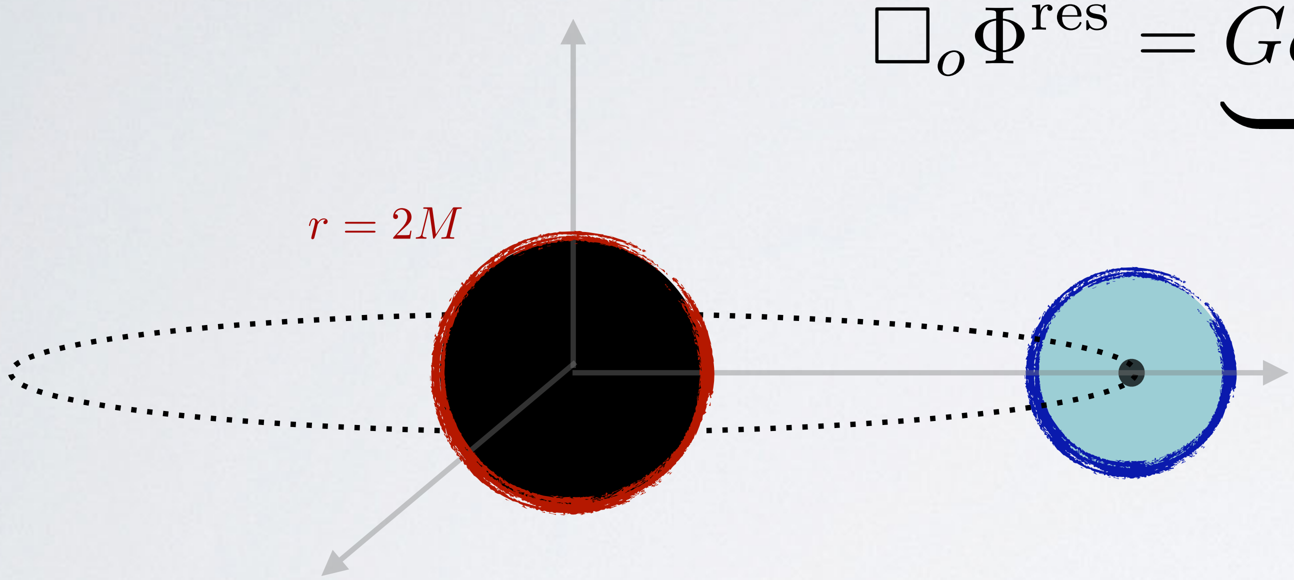
Coordinate System

$$(t, r, \theta, \varphi)$$

EFFECTIVE SOURCE APPROACH

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

$$\Box_o \Phi^{\rm res} = \underbrace{G\delta(x^a - x_p^a) - \Box_o \Phi^{\rm punc}}_{S_{\rm eff}}$$



$r \to \infty$

Solution: retarded field

~~$\Phi^{\rm ret} = \Phi^{\rm sing} + \Phi^{\rm reg}$~~

$$\wr\wr$$

$$\Phi^{\rm ret} = \Phi^{\rm punc} + \Phi^{\rm res}$$

Coordinate System

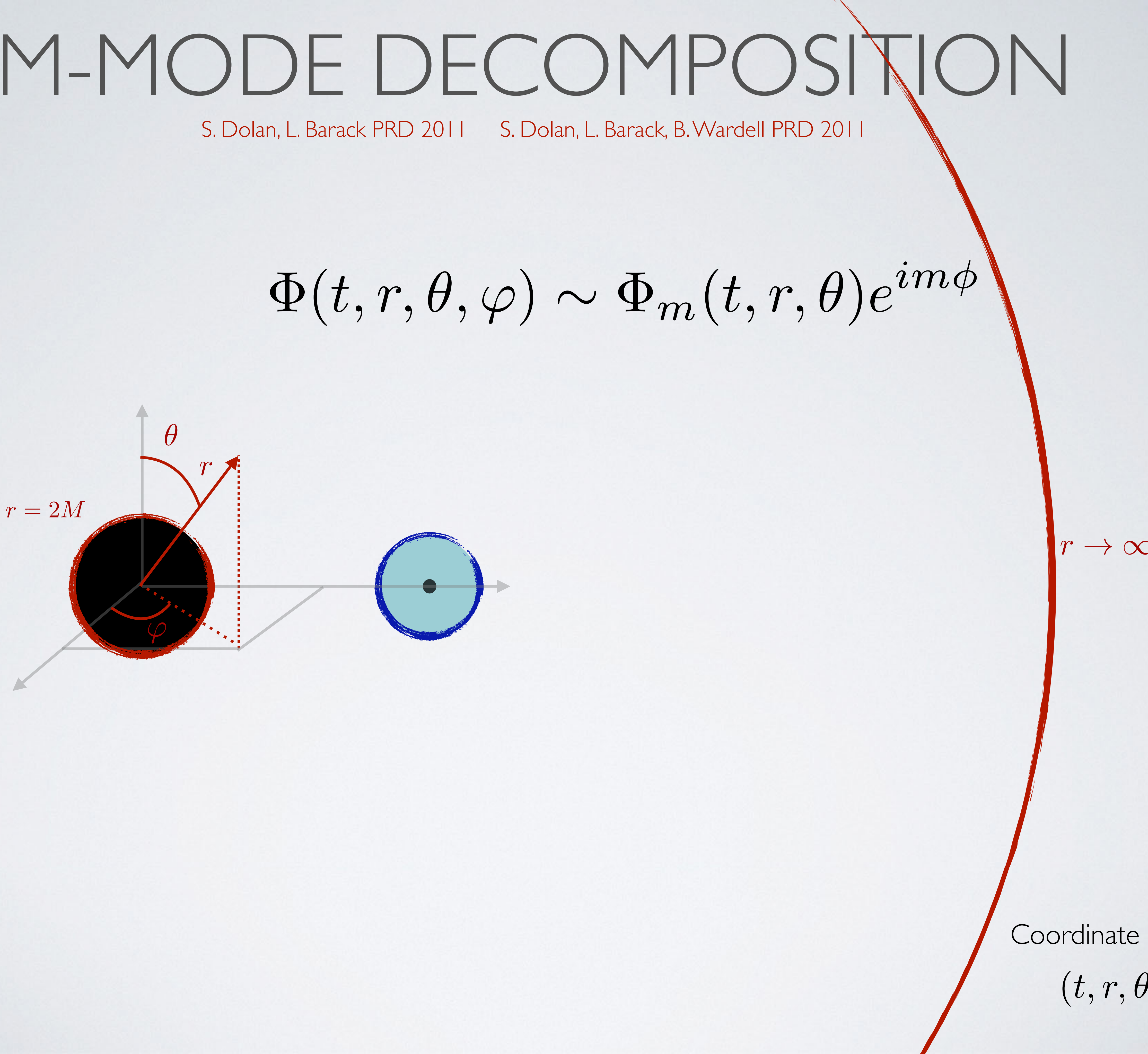
(t, r, θ, φ)

M-MODE DECOMPOSITION

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011

t

$$\Phi(t, r, \theta, \varphi) \sim \Phi_m(t, r, \theta)e^{im\phi}$$



Coordinate System

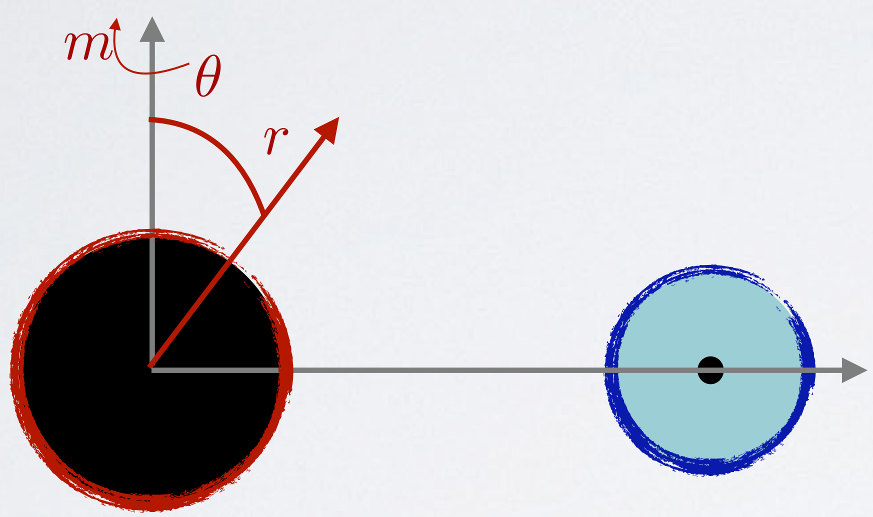
(t, r, θ)

M-MODE DECOMPOSITION

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011

t

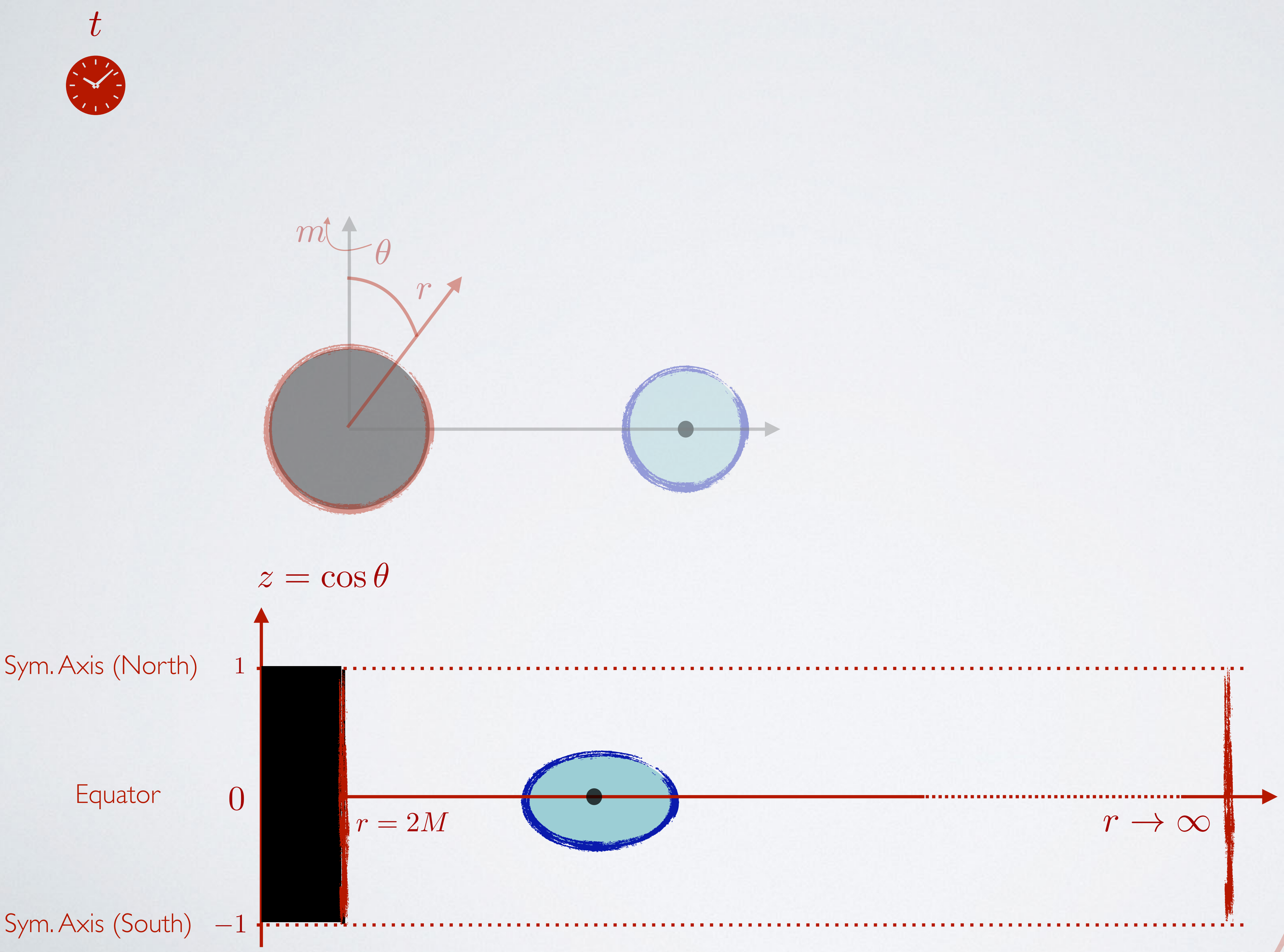
$$\Phi(t, r, \theta, \varphi) \sim \Phi_m(t, r, \theta) e^{im\phi}$$



Coordinate System
(t, r, θ)

M-MODE DECOMPOSITION

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011



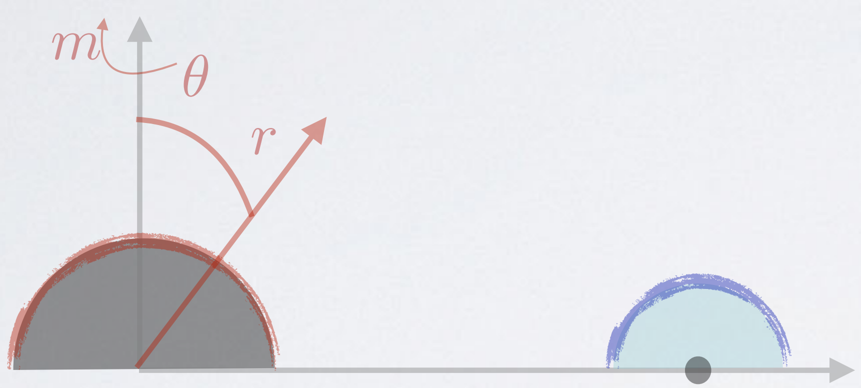
Coordinate System
(t, r, z)

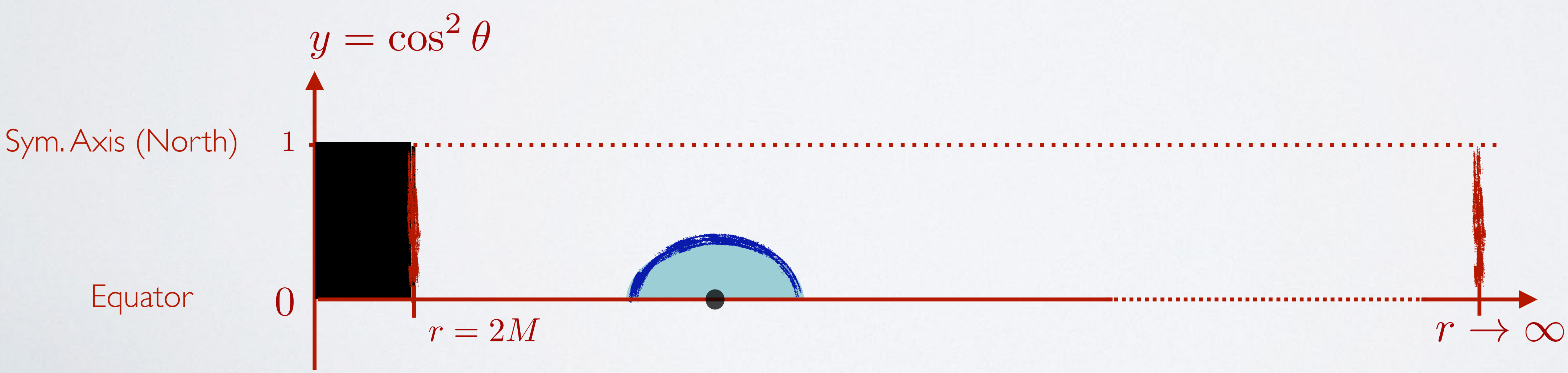
M-MODE DECOMPOSITION

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011

Exploit: Equatorial Symmetry

t





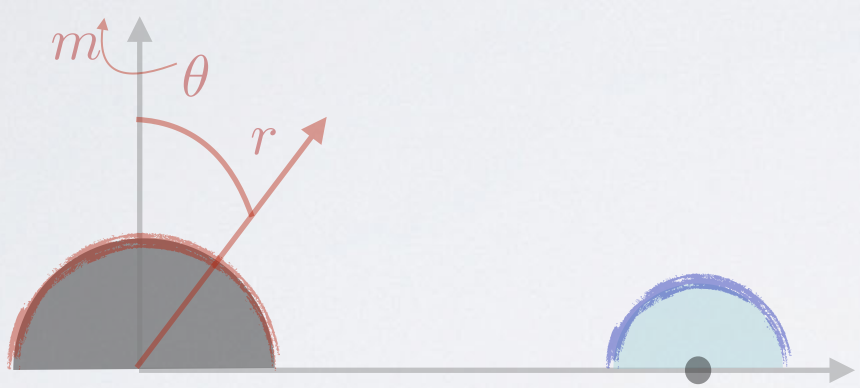
Coordinate System
(t, r, y)

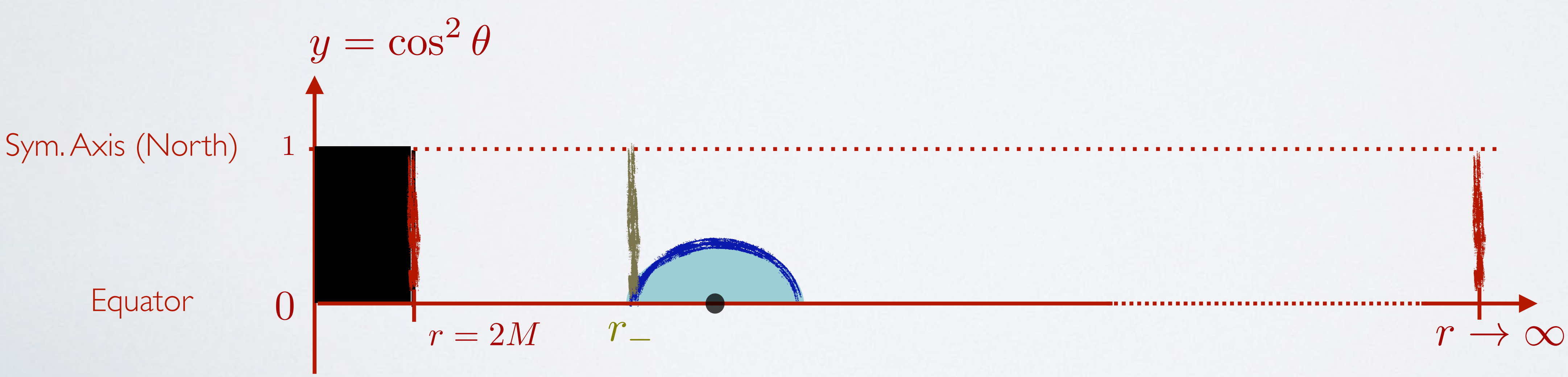
M-MODE DECOMPOSITION

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

Exploit: Equatorial Symmetry

t





Coordinate System

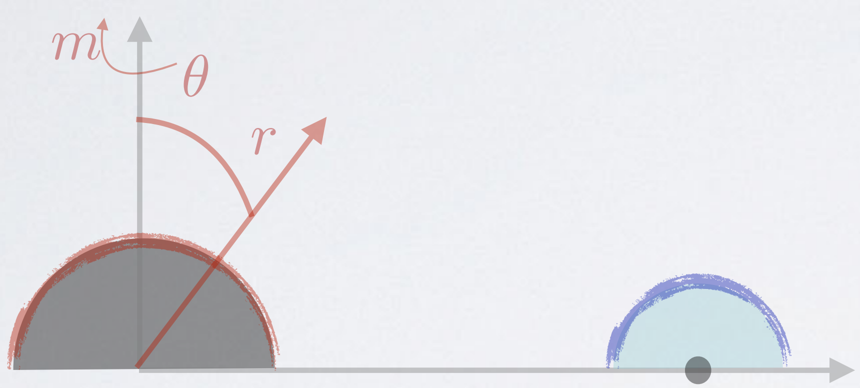
(t, r, y)

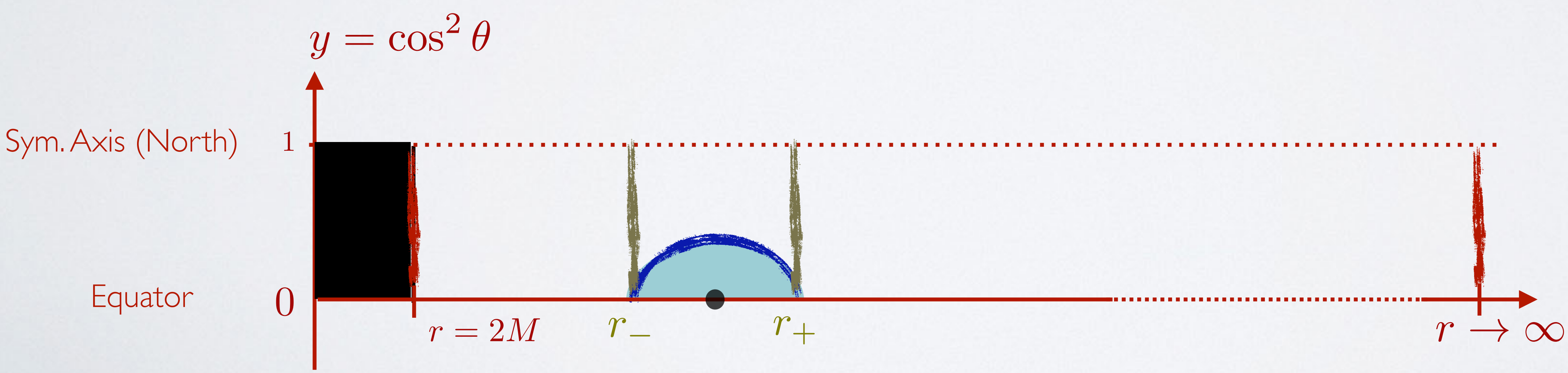
M-MODE DECOMPOSITION

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

Exploit: Equatorial Symmetry

t





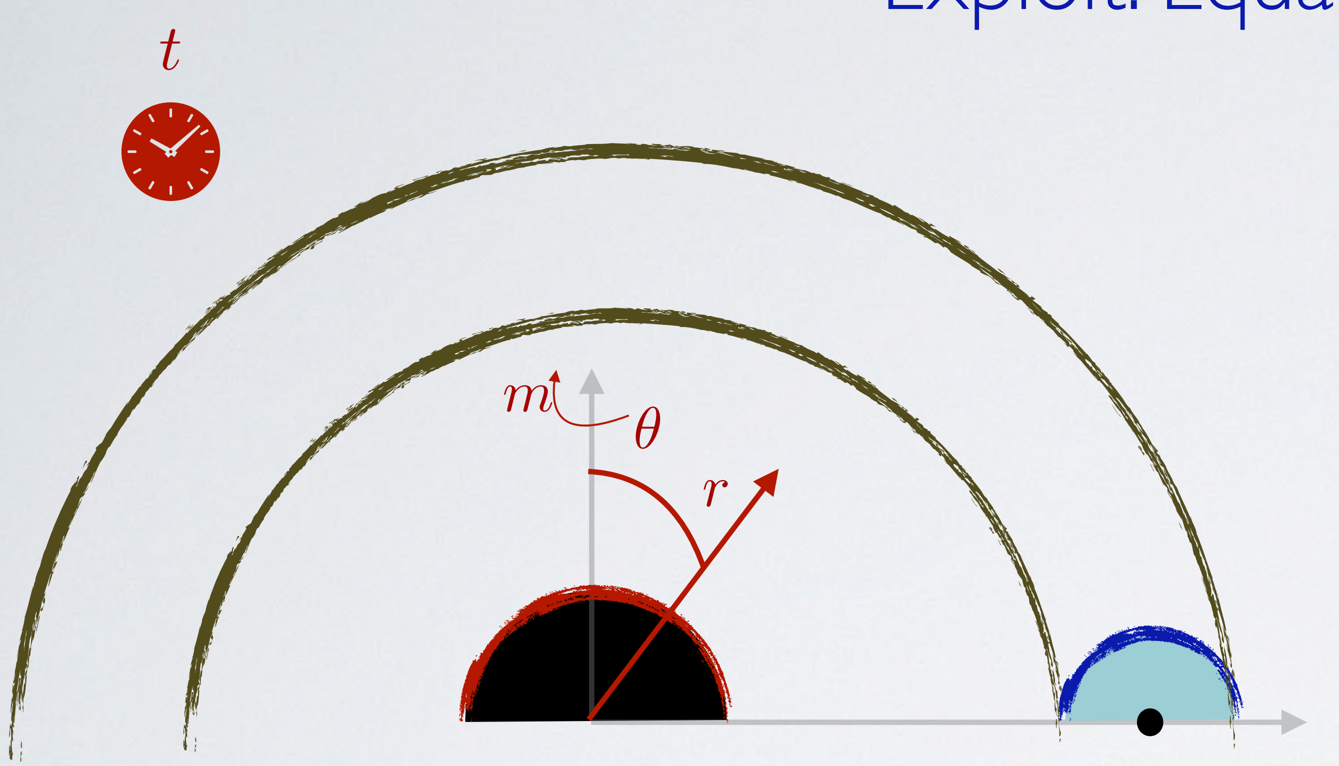
Coordinate System

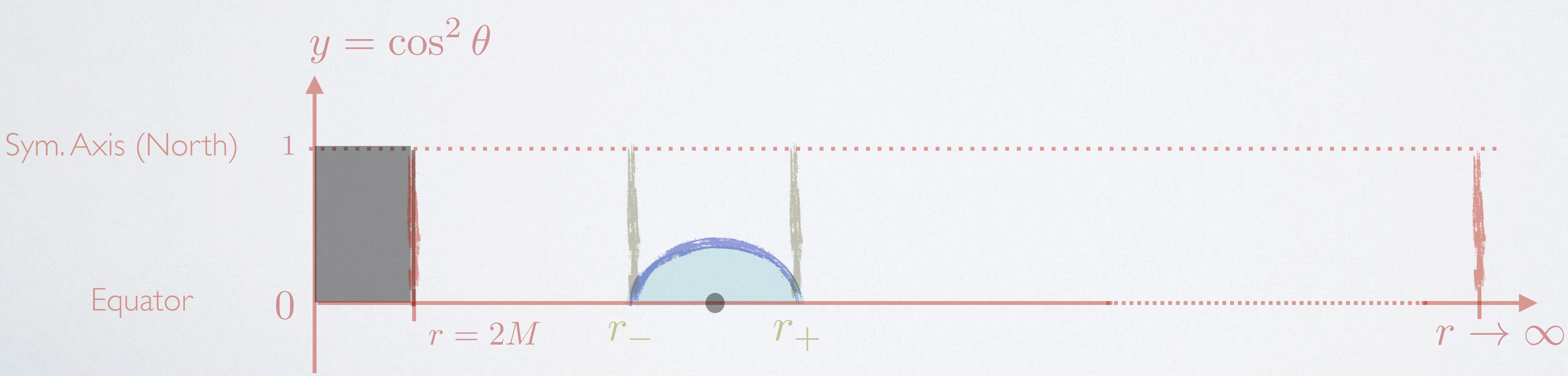
(t, r, y)

M-MODE DECOMPOSITION

S. Dolan, L. Barack PRD 2011 S. Dolan, L. Barack, B. Wardell PRD 2011

Exploit: Equatorial Symmetry



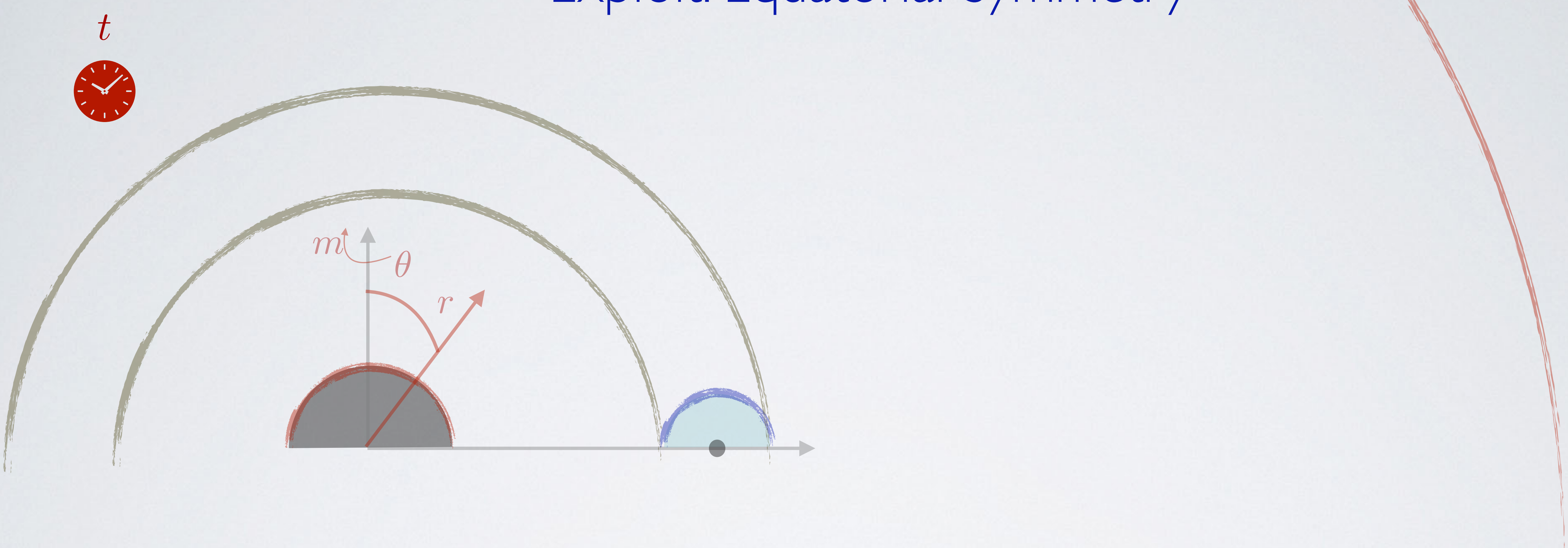


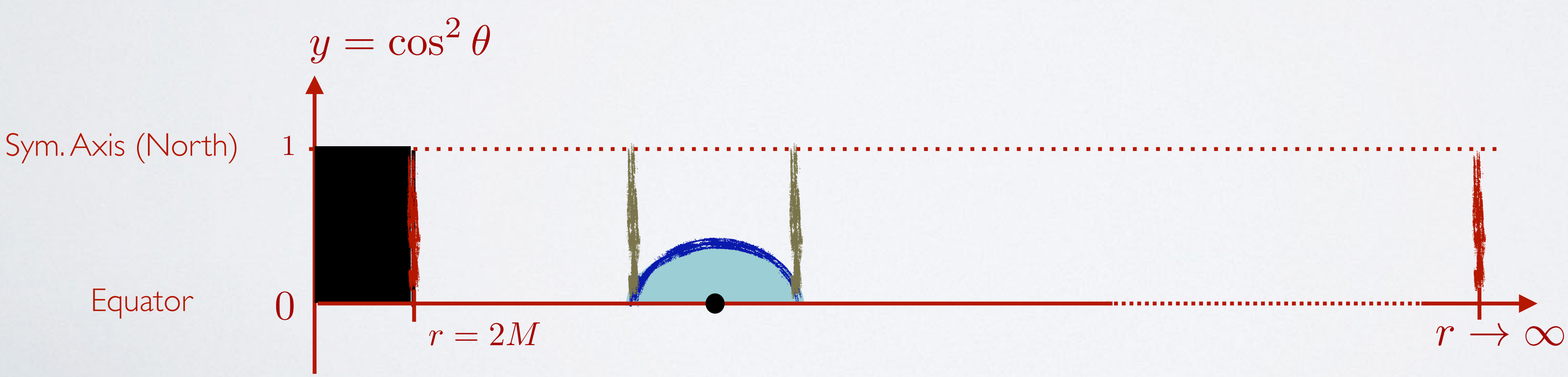
Coordinate System
(t, r, y)

M-MODE DECOMPOSITION

S. Dolan, L. Barack PRD 2011   S. Dolan, L. Barack, B. Wardell PRD 2011

Exploit: Equatorial Symmetry





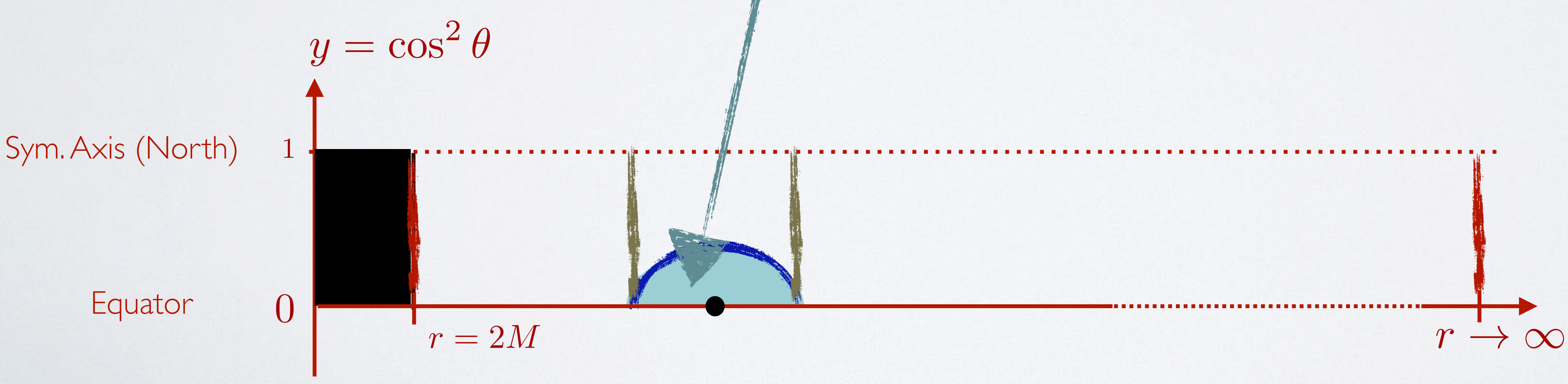
Coordinate System

(t, r, y)

M-MODE DECOMPOSITION

PARTICLE REGION

HOW TO OBTAIN THE PUNCTURE AND THE EFFECTIVE SOURCE?



Coordinate System
(t, r, y)

M-MODE DECOMPOSITION

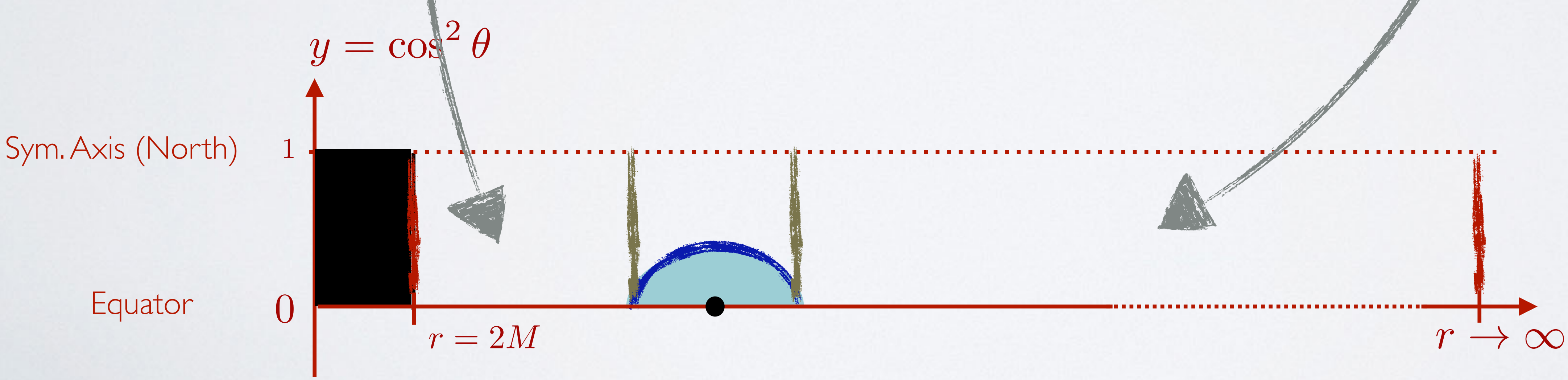
PARTICLE REGION

HOW TO OBTAIN THE PUNCTURE AND THE EFFECTIVE SOURCE?

BLACK HOLE + WAVE ZONE REGION

HOW TO IMPOSE BOUNDARY CONDITIONS?

HOW TO OBTAIN GLOBAL SOLUTIONS?



Coordinate System
(t, r, y)

M-MODE DECOMPOSITION

PARTICLE REGION

HOW TO OBTAIN THE PUNCTURE AND THE EFFECTIVE SOURCE?

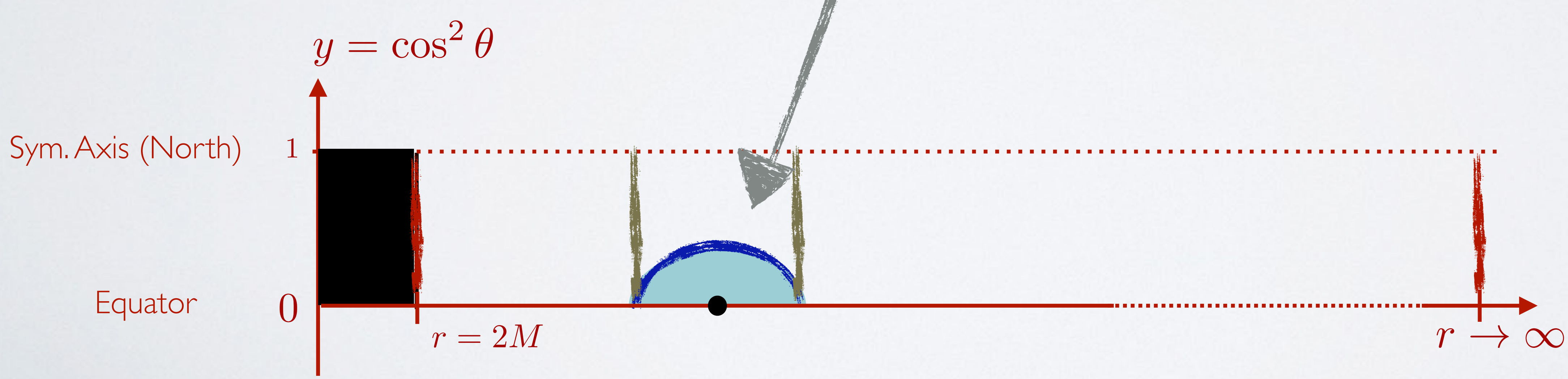
BLACK HOLE + WAVE ZONE REGION

HOW TO IMPOSE BOUNDARY CONDITIONS?

HOW TO OBTAIN GLOBAL SOLUTIONS?

VACCUM REGION

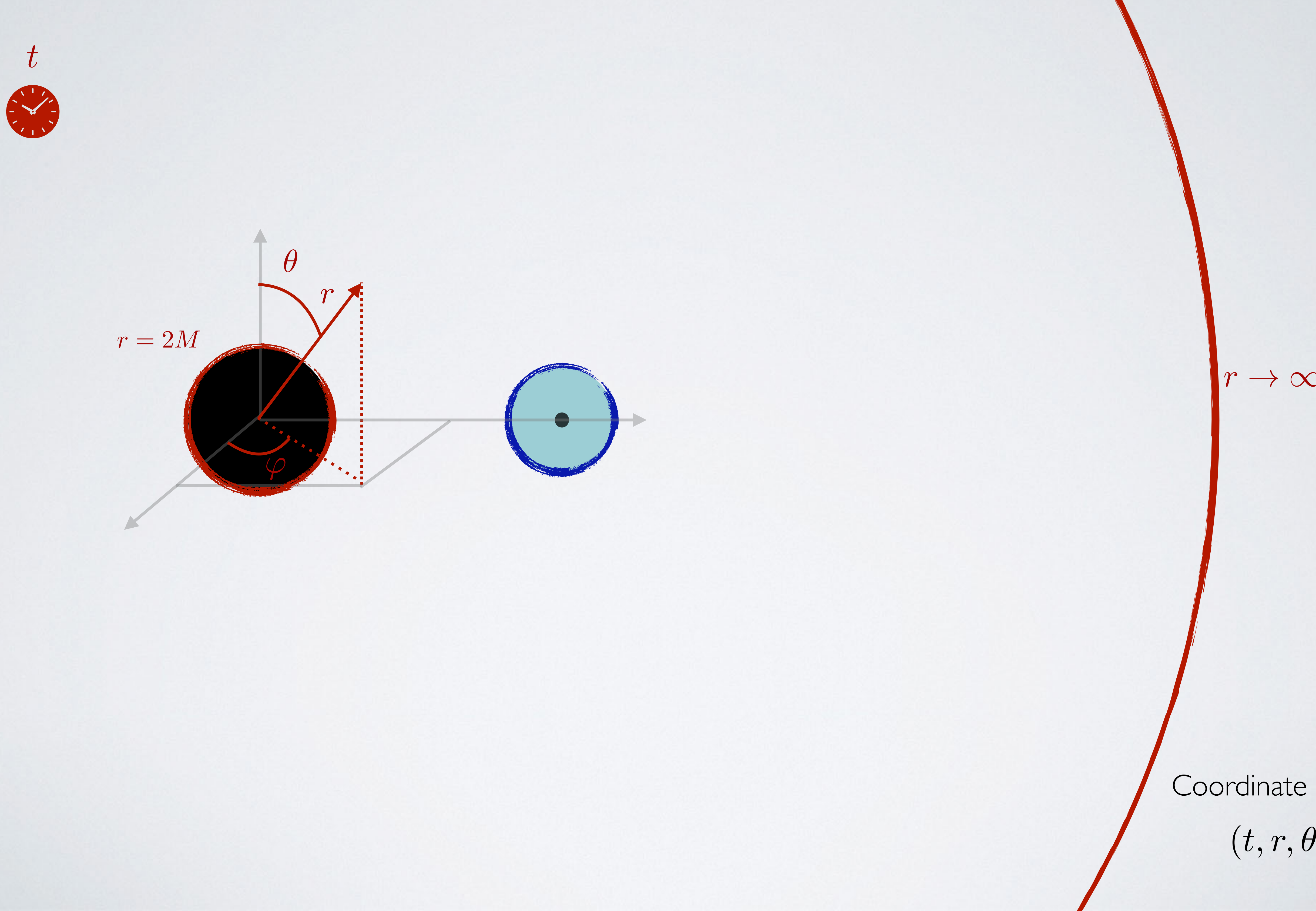
HOW TO CONNECT PARTICLE AND ASYMPTOTIC REGIONS?



Coordinate System
(t, r, y)

PUNCTURE FIELD

A.Pound PRD (2012), P. Bourg, A. Pound, S. Upton 2022 (23?)

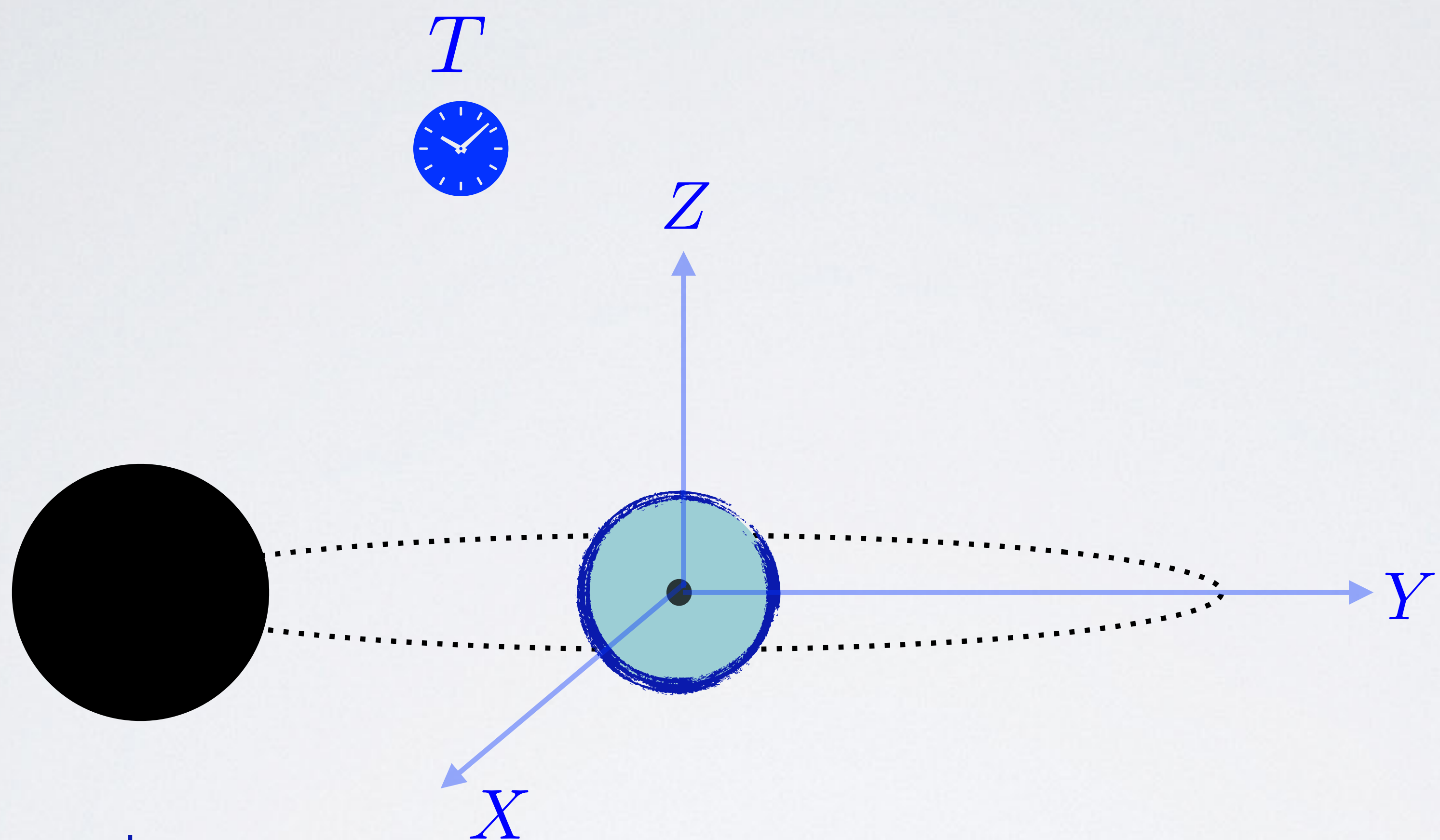


Coordinate System

(t, r, θ, φ)

PUNCTURE FIELD

A.Pound PRD (2012), P. Bourg, A. Pound, S. Upton 2022 (23?)



Properties

$$g^{o}_{ab}\big|_{p} = \eta_{ab}$$

$$\Box_{o}\big|_{\mathrm{p}} = \Delta$$

High punc. order → Analytical Solution

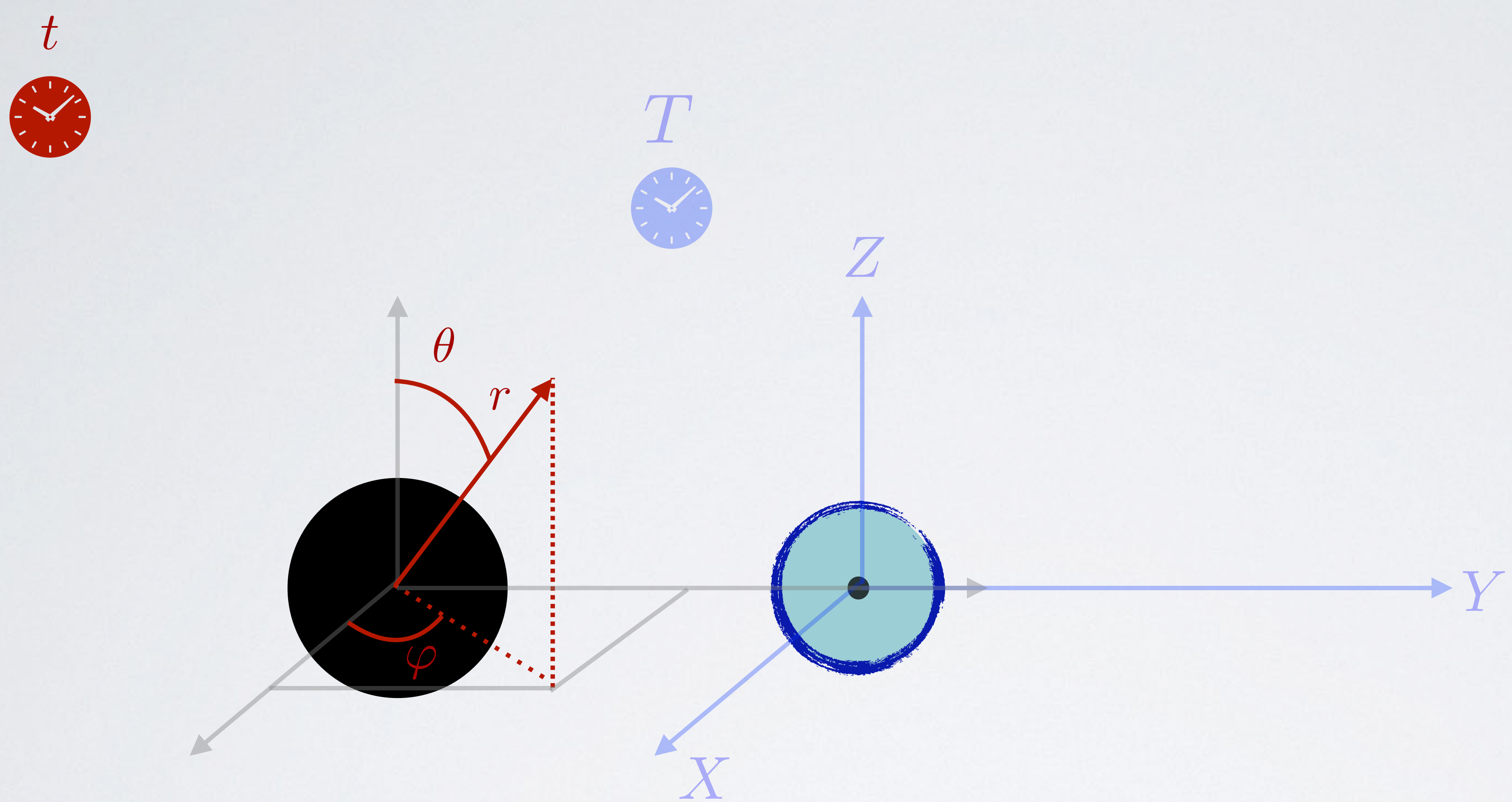
$$\hat{\Phi}^{\mathbf{punc}}_{\bar{n}_{\mathbf{max}}}(T, X, Y, Z)$$

Coordinate System

(T, X, Y, Z)

PUNCTURE FIELD

A.Pound PRD (2012), P. Bourg, A. Pound, S. Upton 2022 (23?)



$$\Phi^{\rm punc}_{\bar{n}_{\rm max}}(x) = \hat{\Phi}^{\rm punc}_{\bar{n}_{\rm max}}(\hat{x}(x))$$

Coordinate System

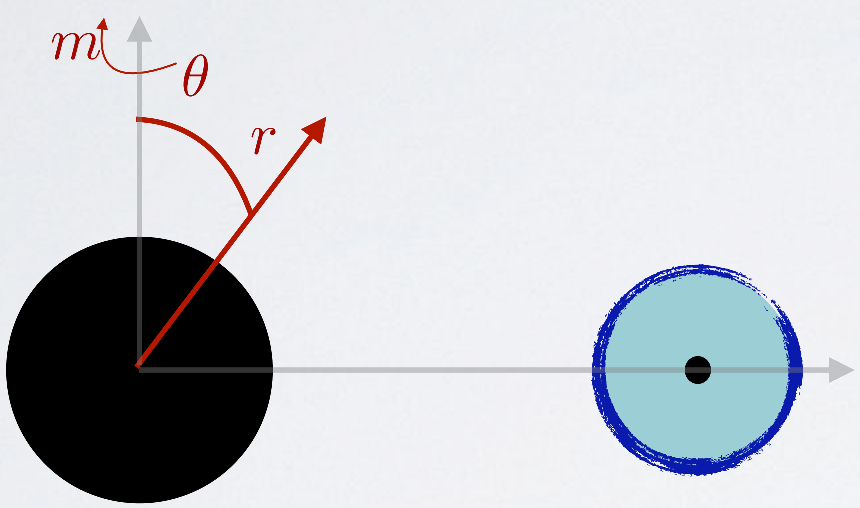
(t, r, θ, φ)

PUNCTURE FIELD

A.Pound PRD (2012), P. Bourg, A. Pound, S. Upton 2022 (23?)

t

$$\Phi(t, r, \theta, \varphi) \sim \Phi_m(t, r, \theta)e^{im\phi}$$



$$\Phi^{\text{punc}}_{\bar{n}_{\max}}(x) = \hat{\Phi}^{\text{punc}}_{\bar{n}_{\max}}(\hat{x}(x))$$

Coordinate System

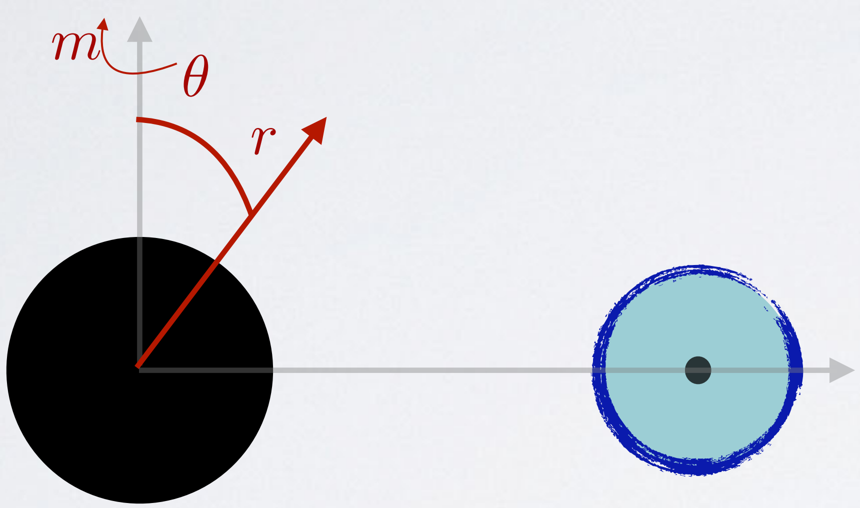
(t, r, θ)

PUNCTURE FIELD

A.Pound PRD (2012), P. Bourg, A. Pound, S. Upton 2022 (23?)

t

$$\Phi(t, r, \theta, \varphi) \sim \Phi_m(t, r, \theta) e^{im\phi}$$



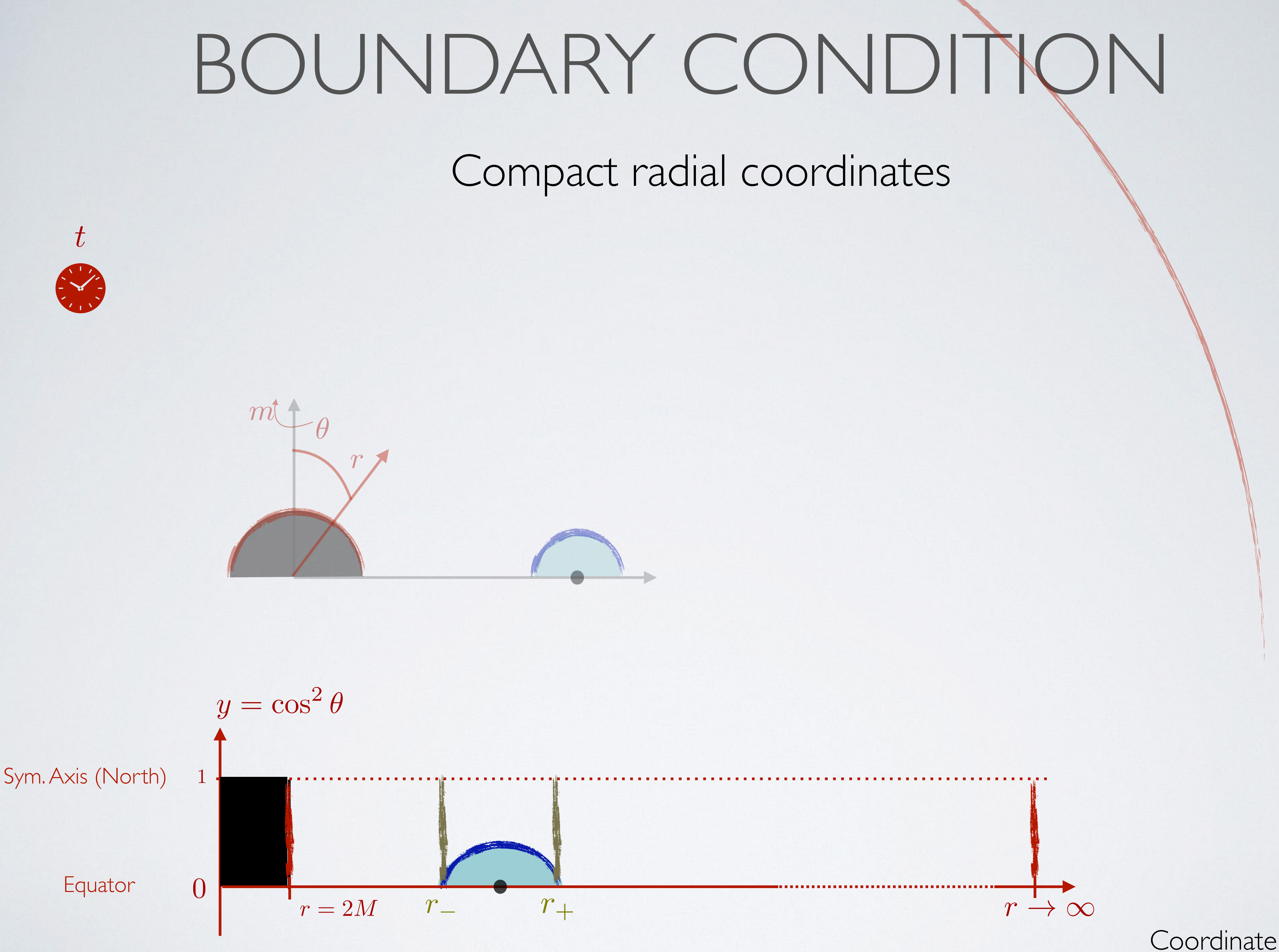
$$\Phi^{\rm punc}_{m,\bar{n}_{\rm max}}(x) = \int_0^{2\pi} d\varphi \, e^{-im\varphi} \, \hat{\Phi}^{\rm punc}_{\bar{n}_{\rm max}}(\hat{x}(x))$$

Coordinate System

(t, r, θ)

BOUNDARY CONDITION

Compact radial coordinates

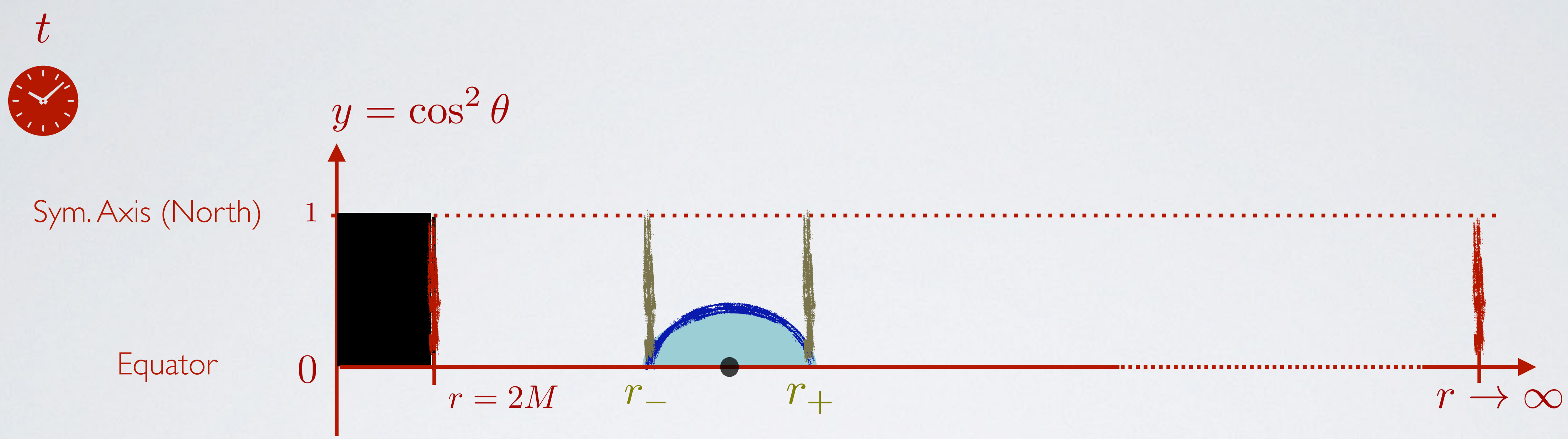


Coordinate System

(t, r, y)

BOUNDARY CONDITION

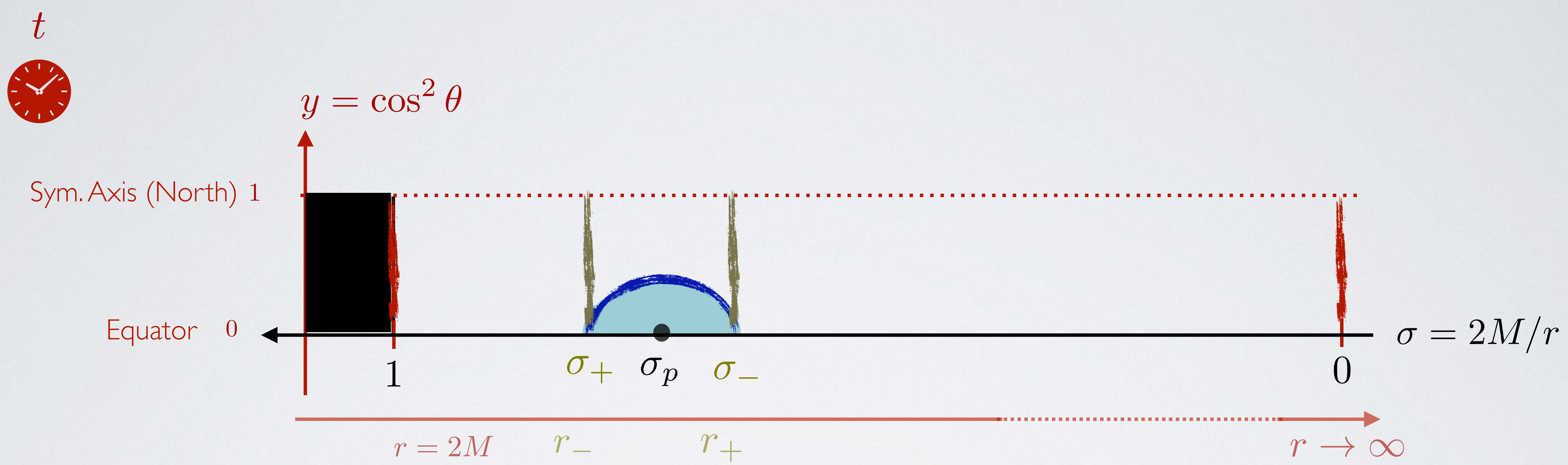
Compact radial coordinates



Coordinate System
(t, r, y)

BOUNDARY CONDITION

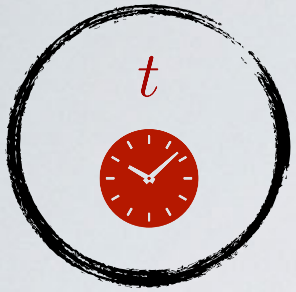
Compact radial coordinates $r = \dfrac{2M}{\sigma}$

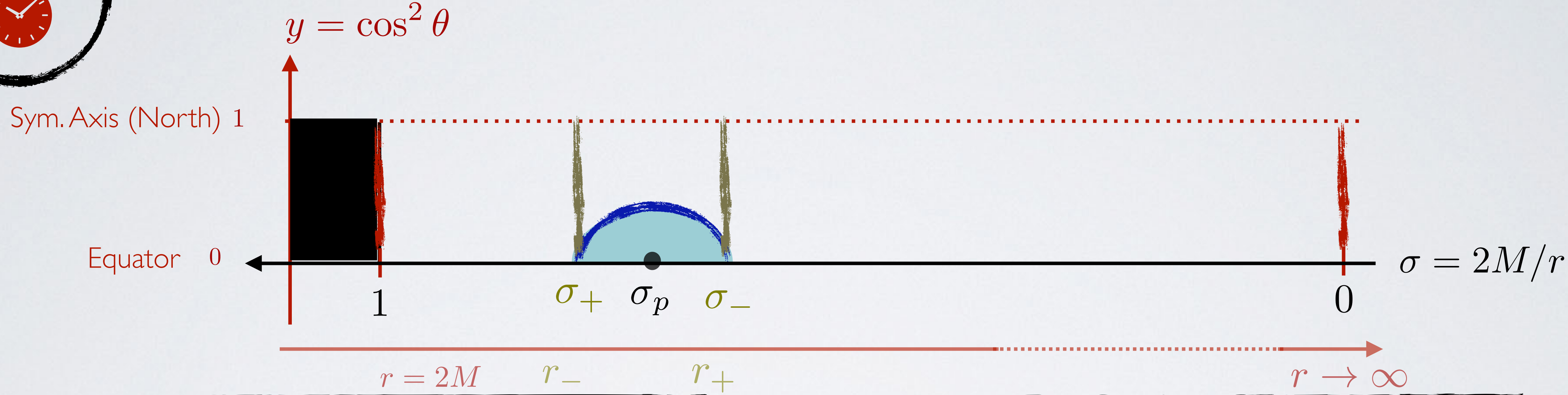


Coordinate System
(t, r, y)

BOUNDARY CONDITION

Compact radial coordinates $r = \frac{2M}{\sigma}$





GR is a geometrical theory on space+time

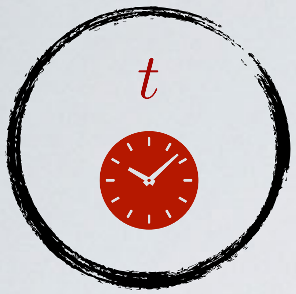
Limits on spatial coordinates are taken

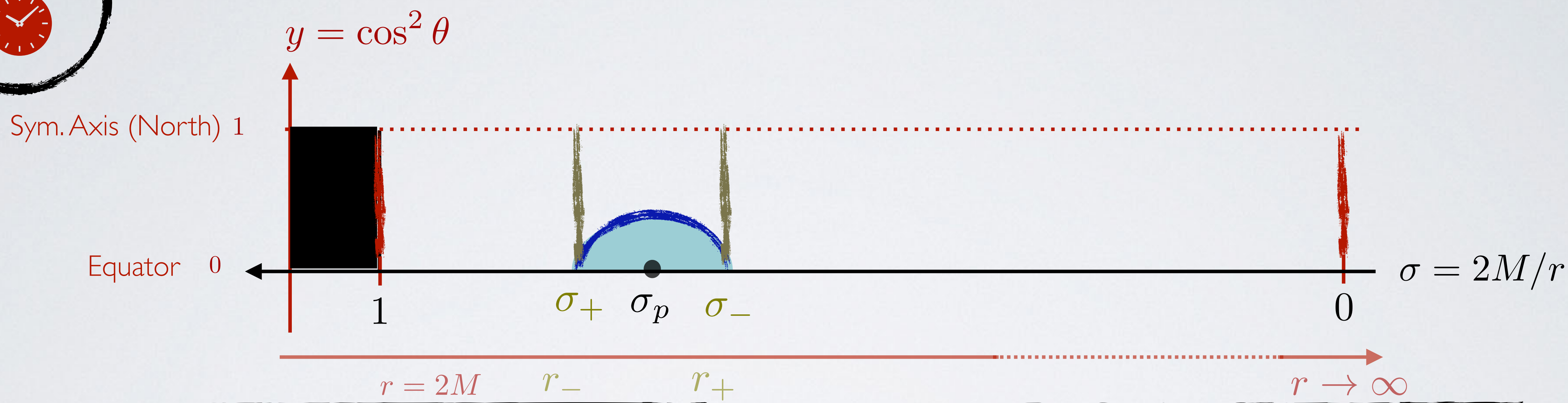
along surfaces of constant time:

Coordinate System
(t, r, y)

BOUNDARY CONDITION

Compact radial coordinates $r = \frac{2M}{\sigma}$





GR is a geometrical theory on space+time

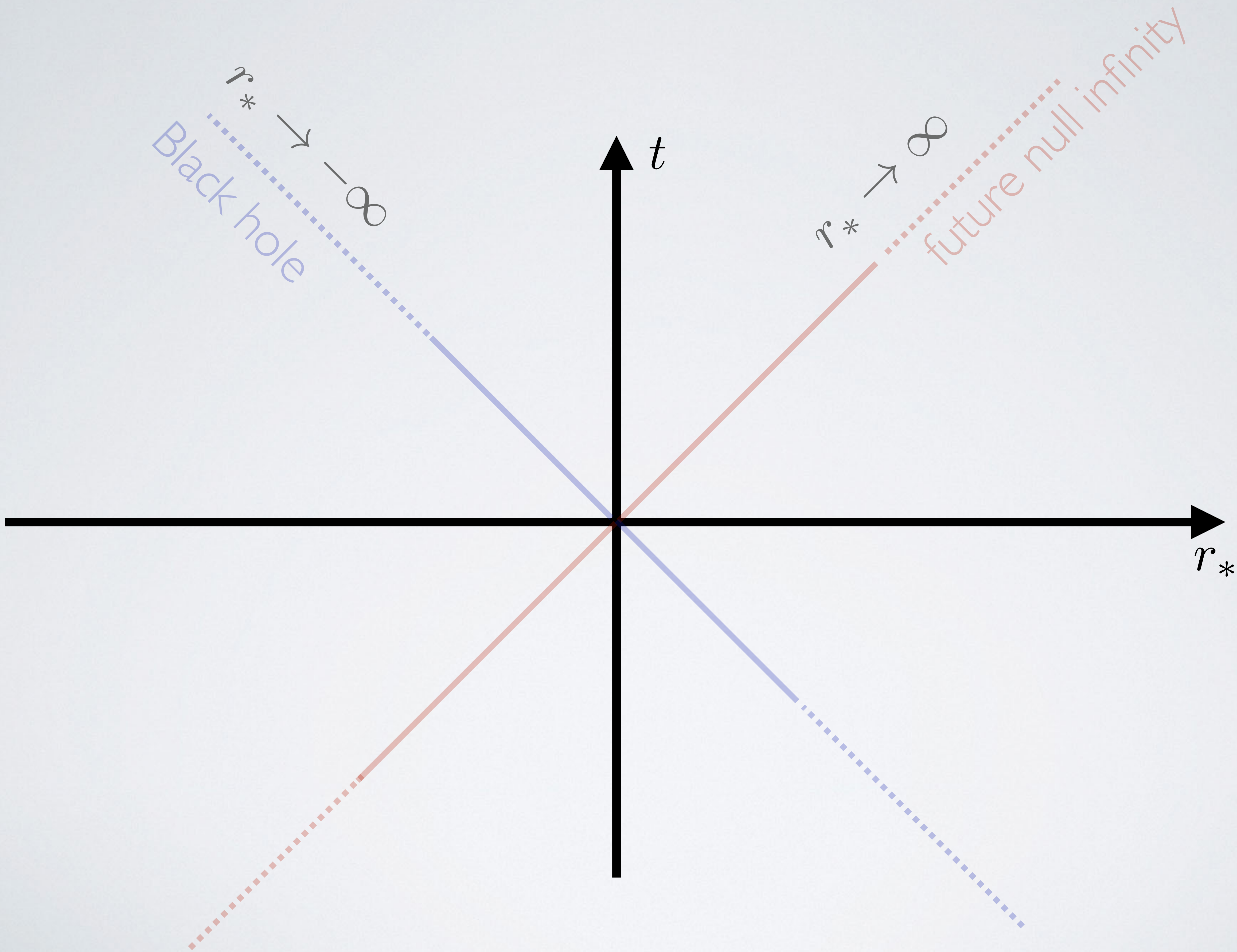
Limits on spatial coordinates are taken

along surfaces of constant time:

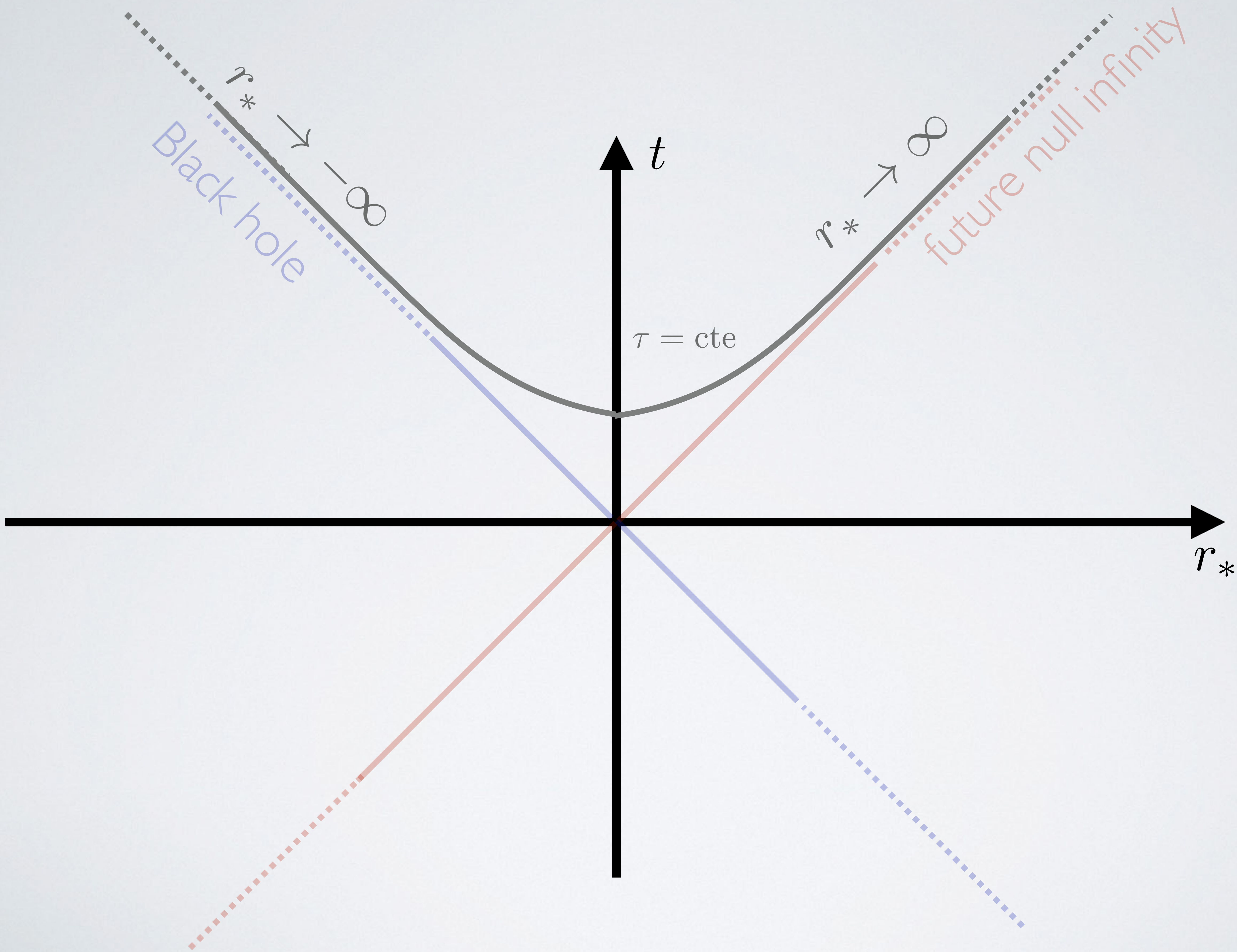
Which time coordinate?

Coordinate System
(t, r, y)

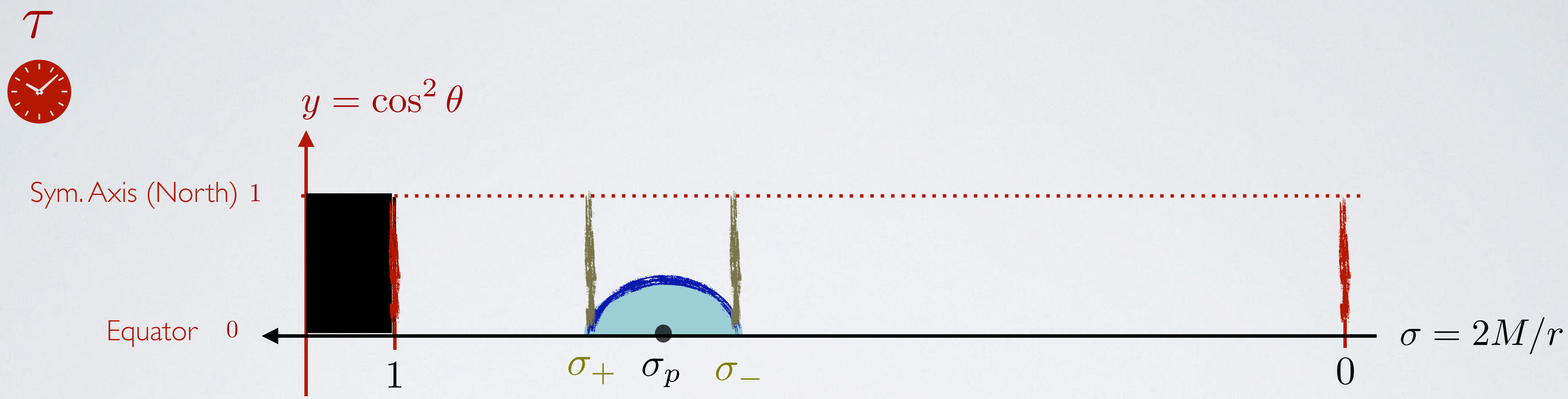
HYPERBOLOIDAL TIME



HYPERBOLOIDAL TIME



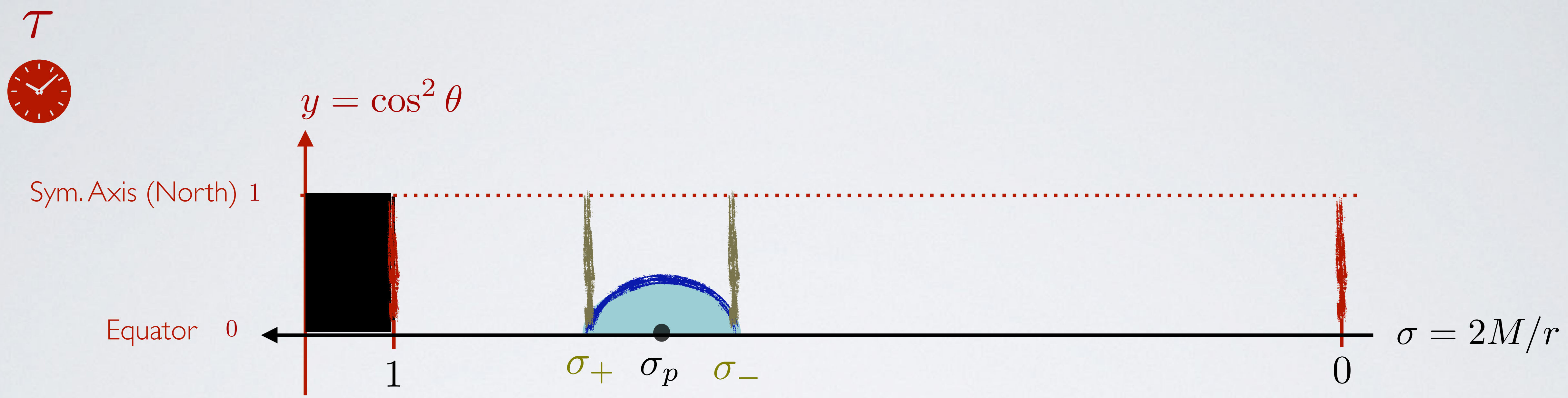
BOUNDARY CONDITION



Coordinate System

(τ, σ, y)

FREQUENCY DOMAIN



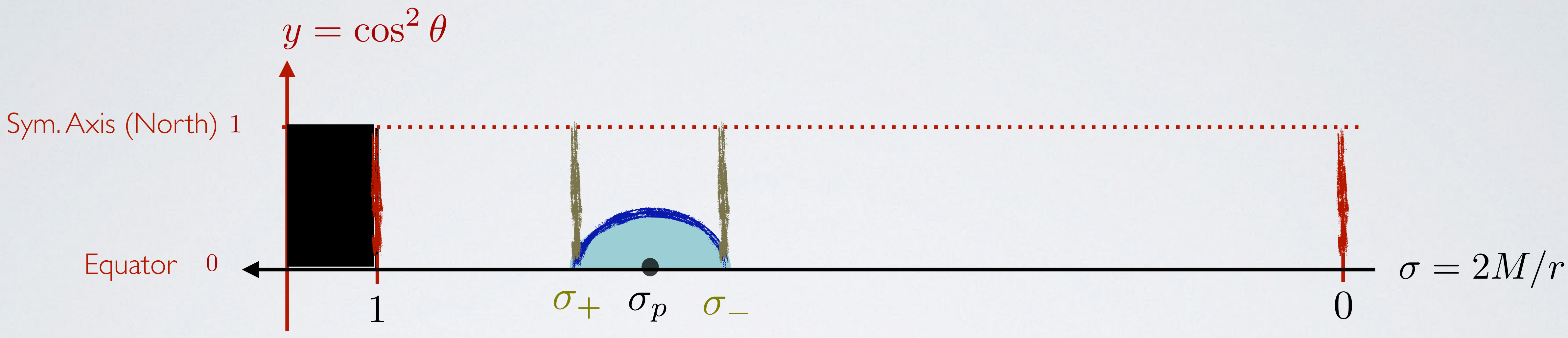
$$\Phi \sim e^{-i\omega_m \tau}\, \tilde{\phi}_m(\sigma, y)$$

Coordinate System

(τ, σ, y)

FREQUENCY DOMAIN

$$\omega_m = m\sqrt{\frac{M}{r_p^3}}$$

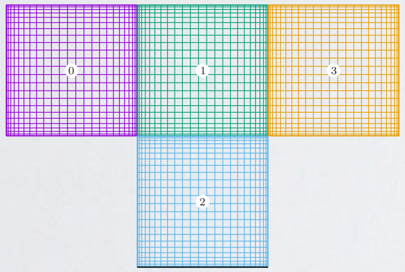


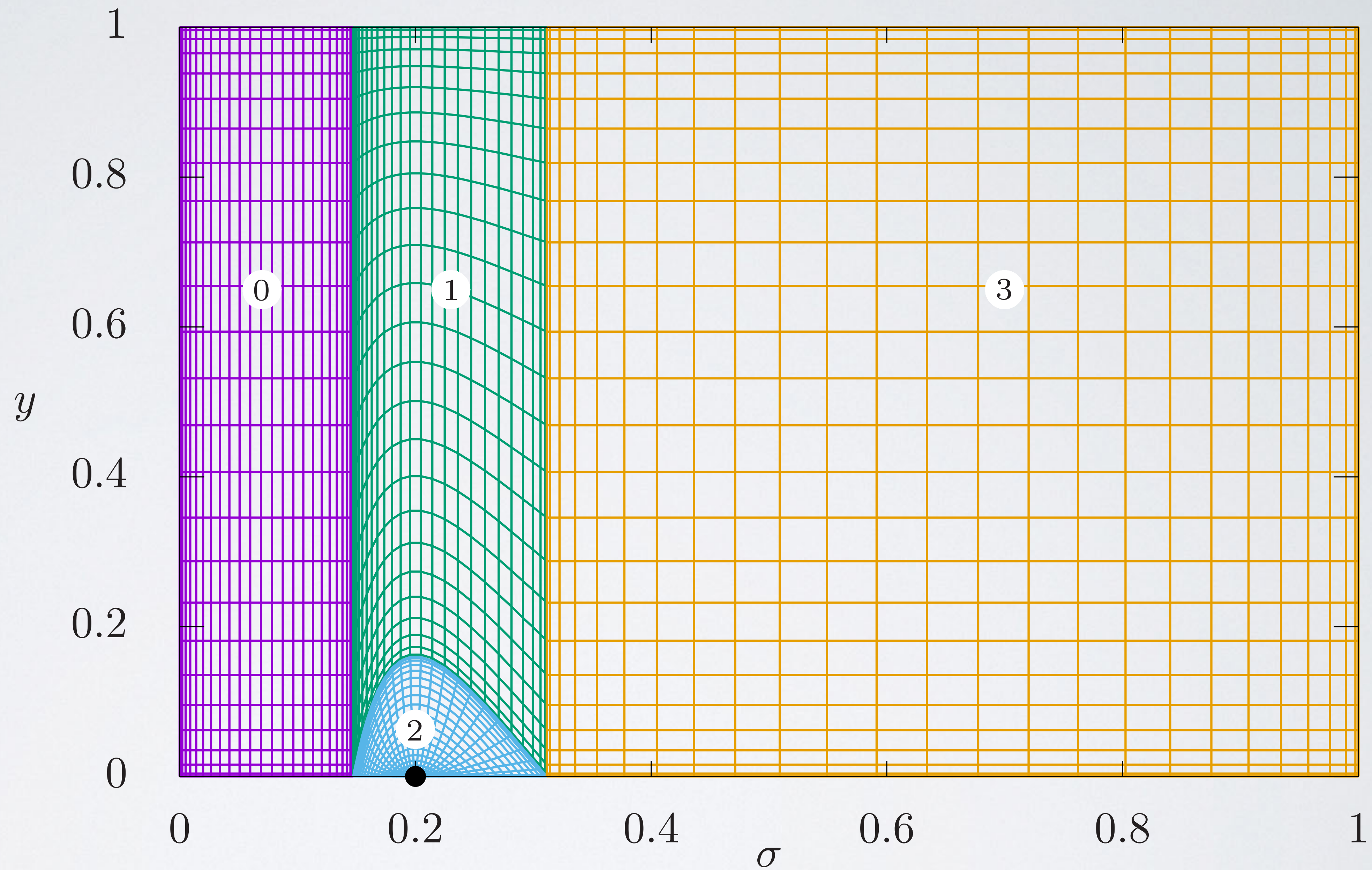
$$\Phi \sim e^{-i\omega_m \tau}\,\tilde{\phi}_m(\sigma, y)$$

Coordinate System
(σ, y)

MULTI-DOMAIN ELLIPTIC SOLVER

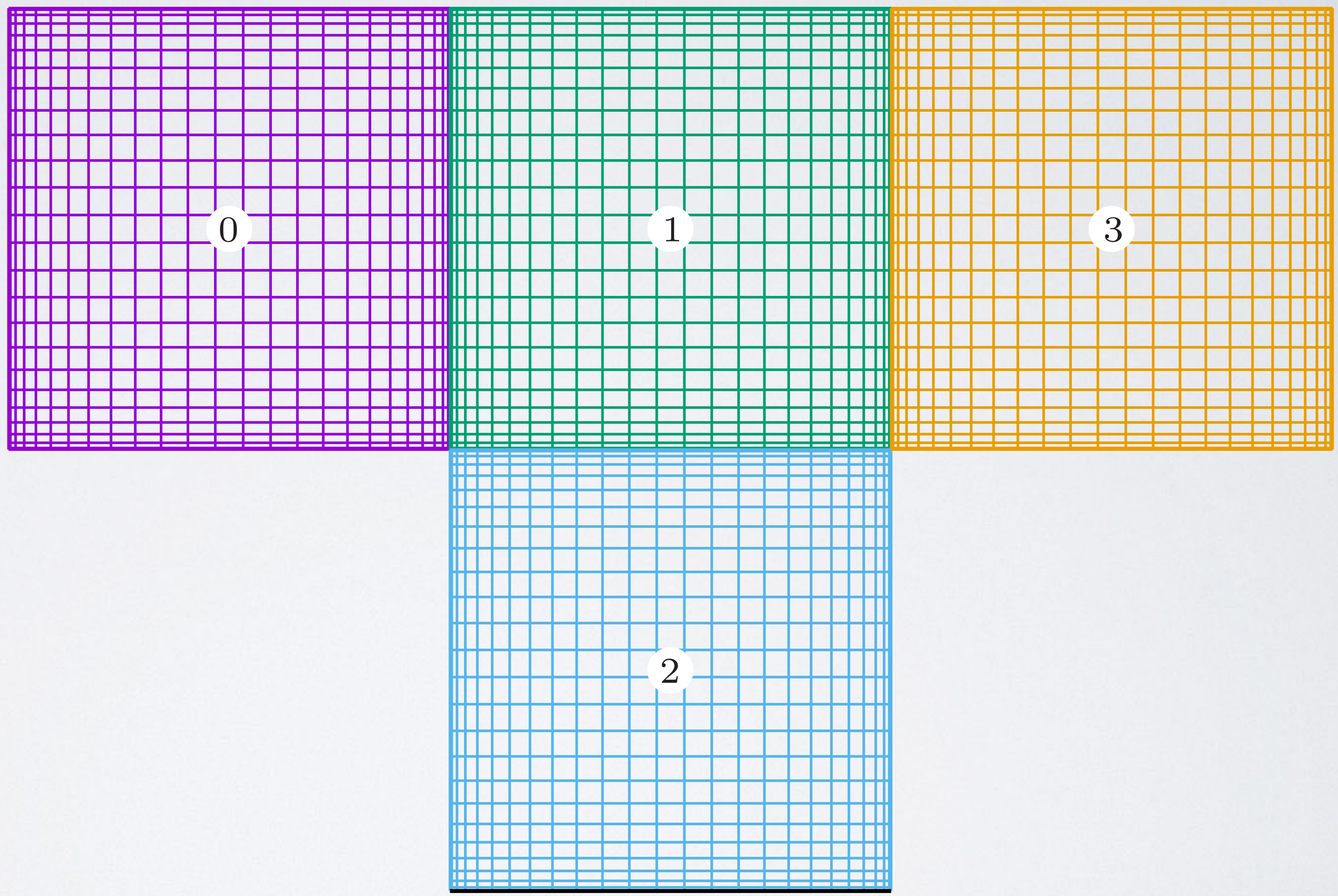
- Domain 0:
  Future null infinity
- Domain 1:
  Near Particle-Vacuum
- Domain 2:
  Near Particle-Puncture
- Domain 3:
  Black Hole

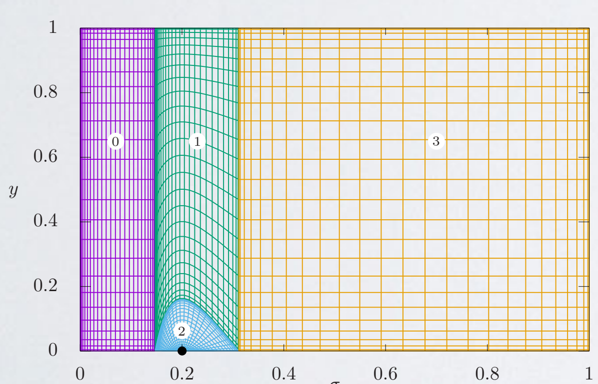




MULTI-DOMAIN ELLIPTIC SOLVER

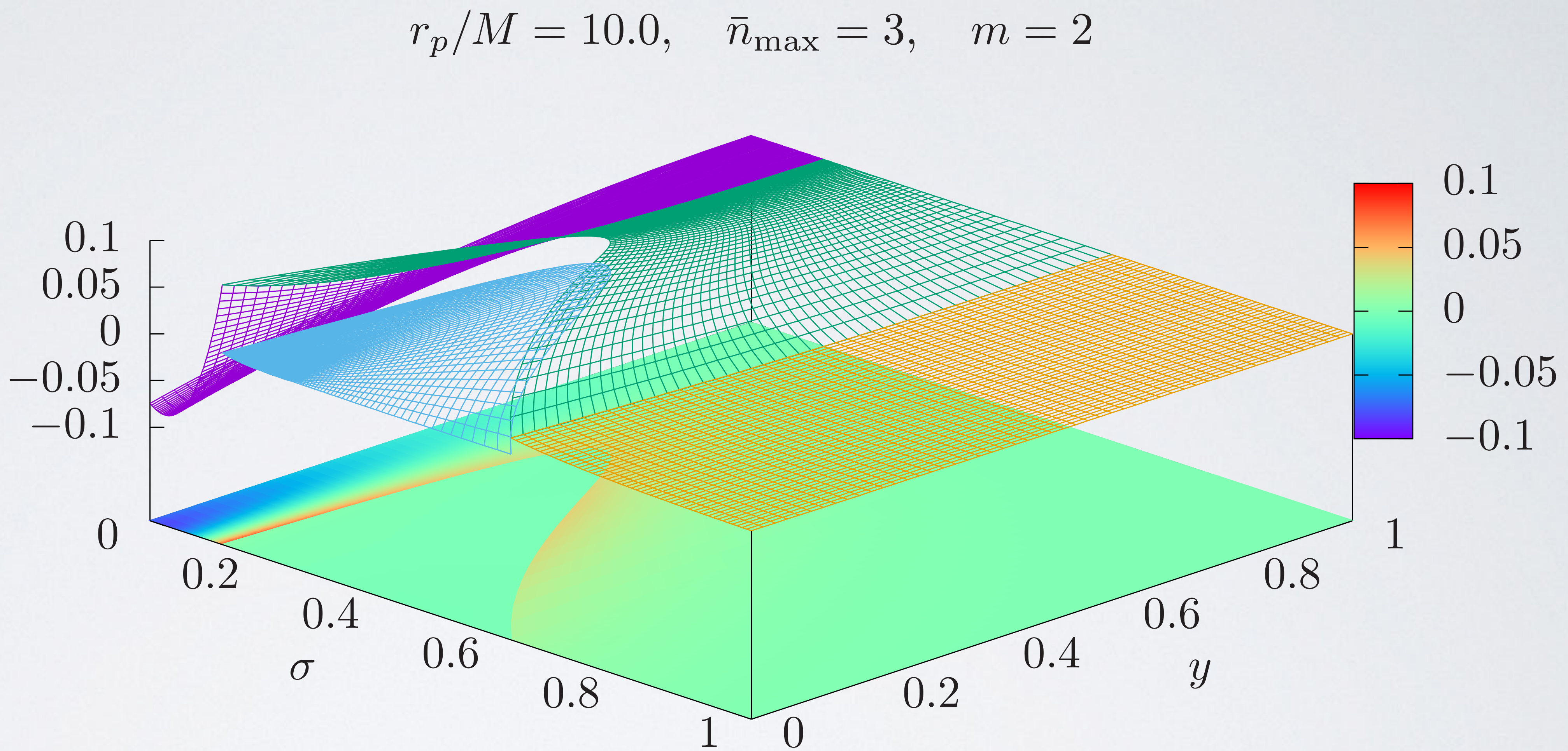
- Domain 0:
  Future null infinity
- Domain 1:
  Near Particle-Vacuum
- Domain 2:
  Near Particle-Puncture
- Domain 3:
  Black Hole

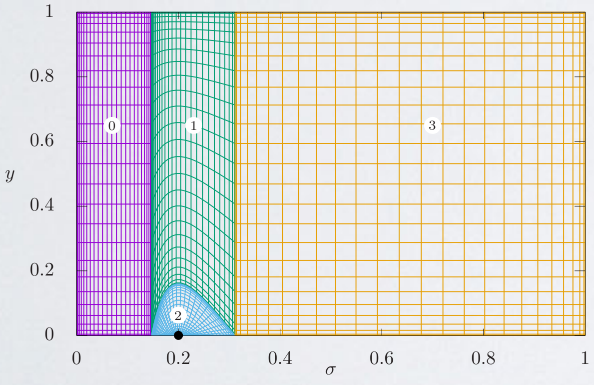




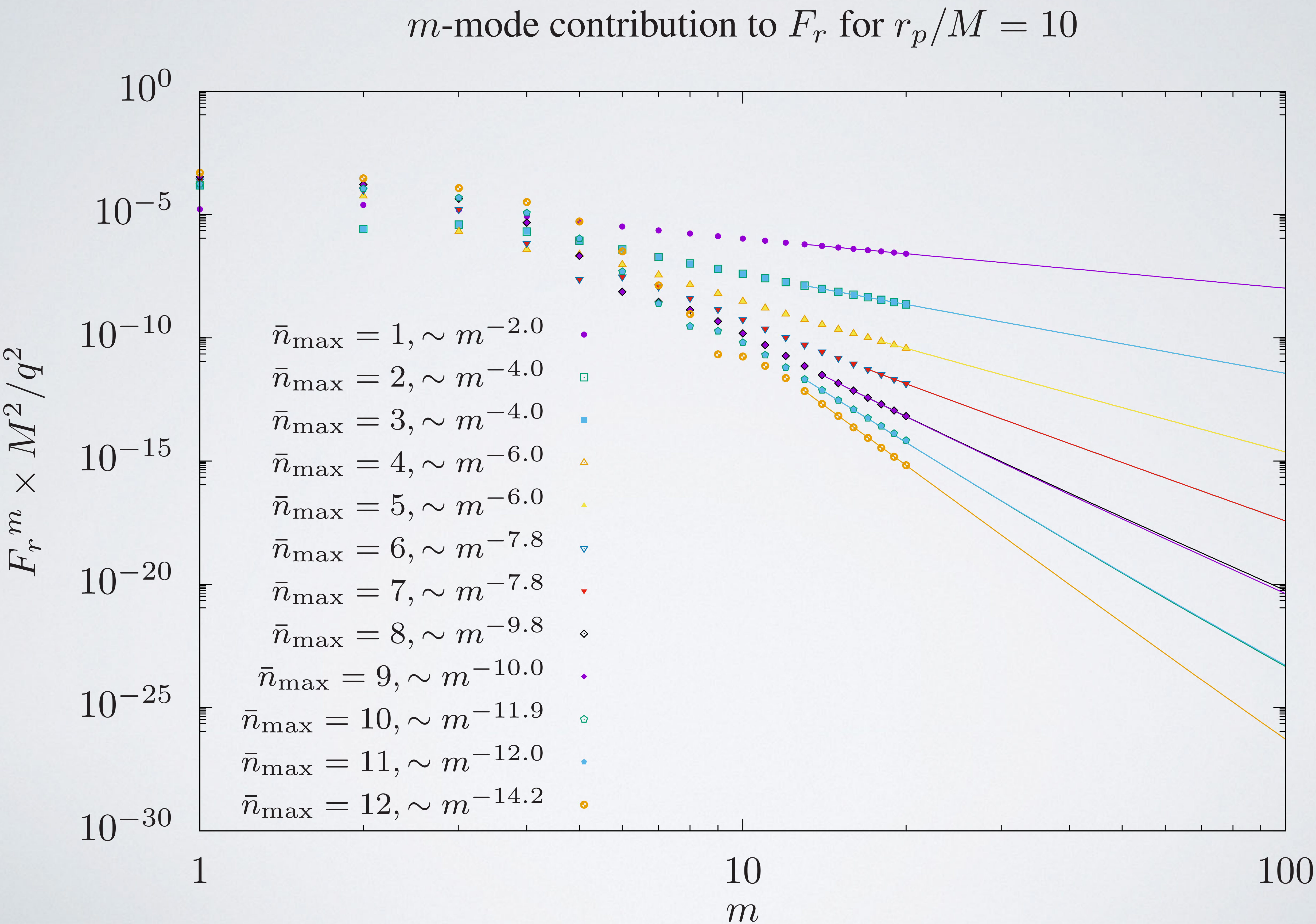
SOLUTION

- Domain 0:
  Future null infinity
- Domain 1:
  Near Particle-Vacuum
- Domain 2:
  Near Particle-Puncture
- Domain 3:
  Black Hole



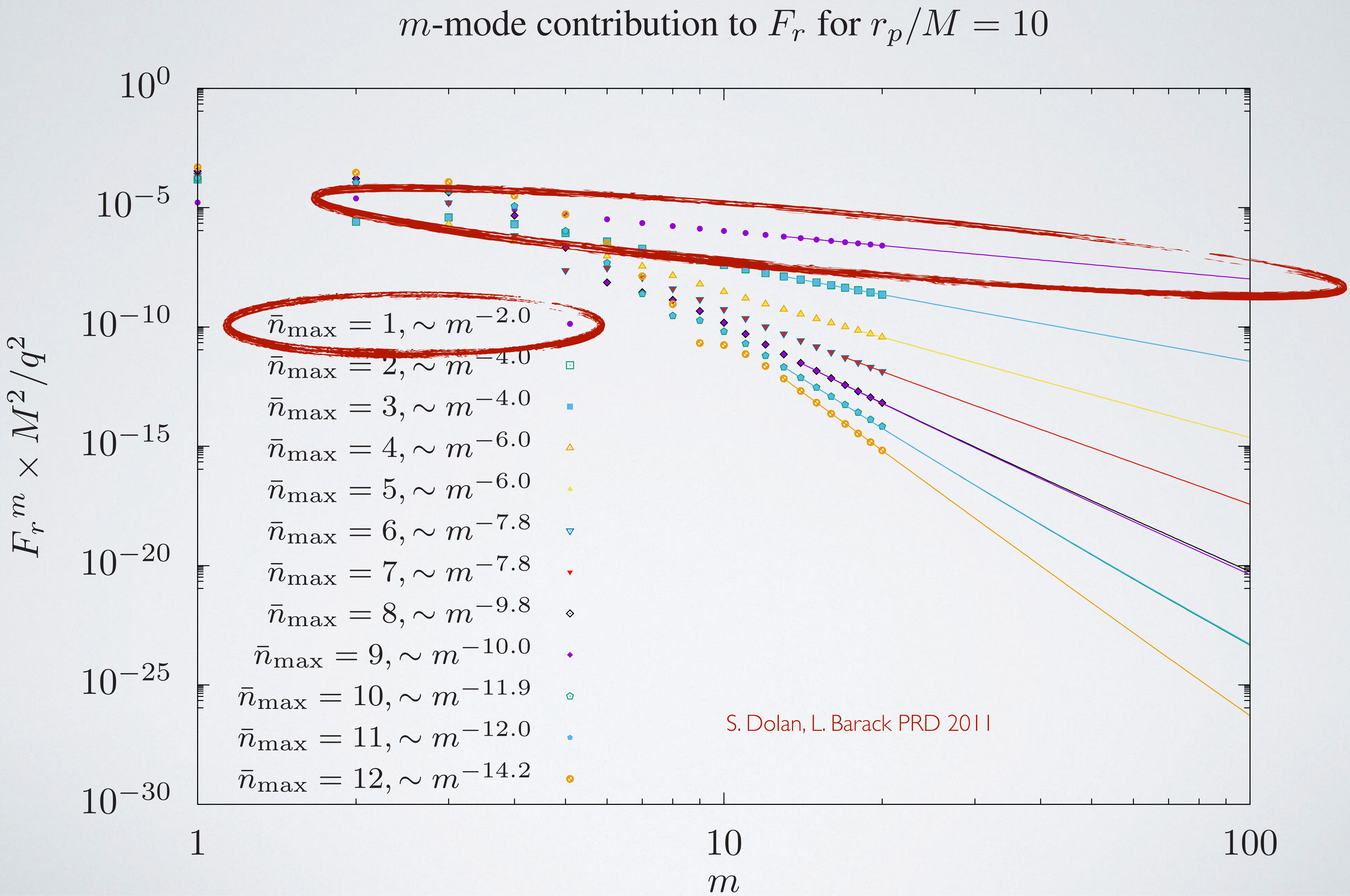


SELF-FORCE



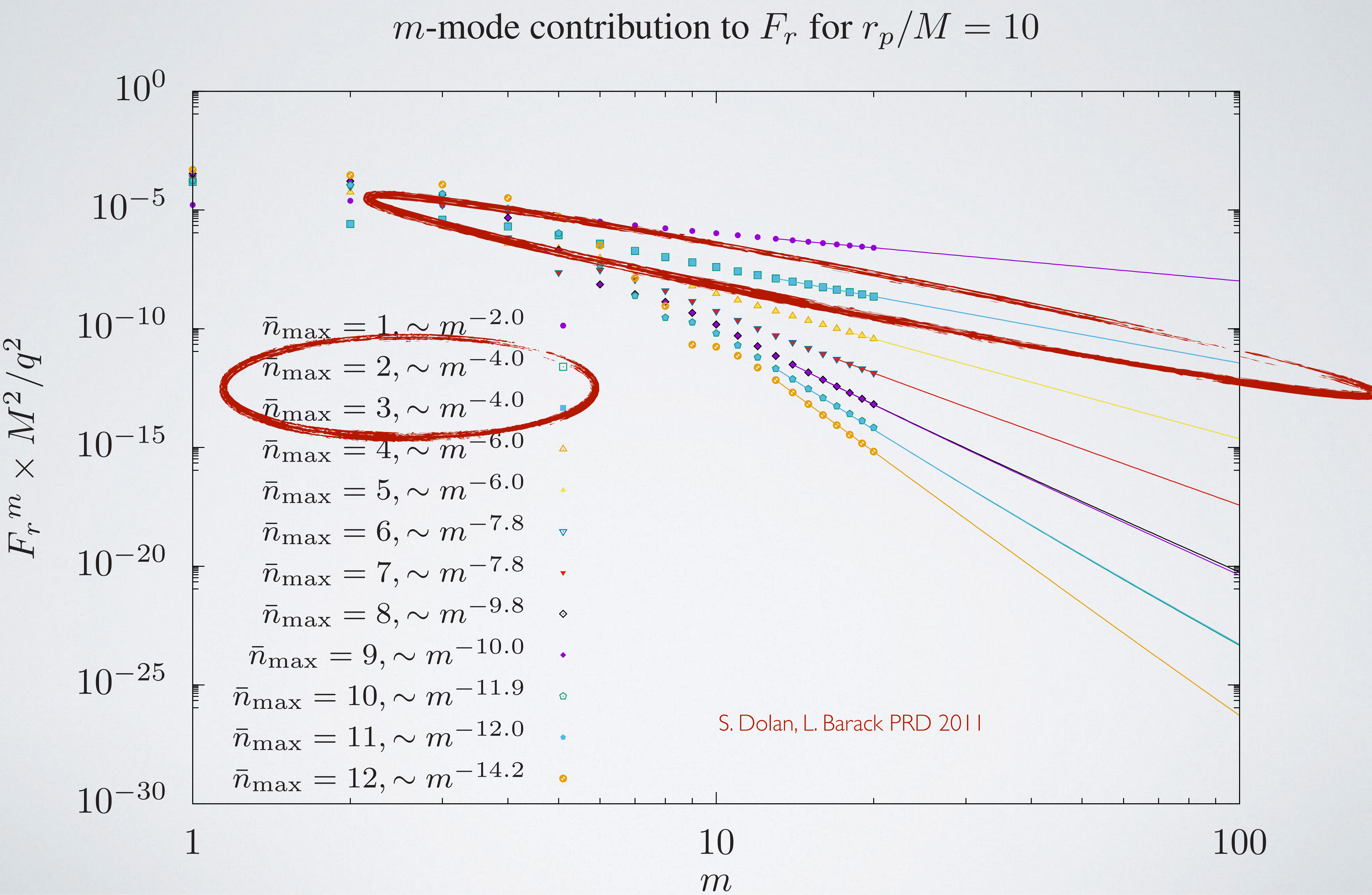
m-mode contribution to F_r for $r_p/M = 10$
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
${F_r}^m \times M^2/q^2$
1
10
100
m
$\bar{n}_{\max} = 1, \sim m^{-2.0}$
$\bar{n}_{\max} = 2, \sim m^{-4.0}$
$\bar{n}_{\max} = 3, \sim m^{-4.0}$
$\bar{n}_{\max} = 4, \sim m^{-6.0}$
$\bar{n}_{\max} = 5, \sim m^{-6.0}$
$\bar{n}_{\max} = 6, \sim m^{-7.8}$
$\bar{n}_{\max} = 7, \sim m^{-7.8}$
$\bar{n}_{\max} = 8, \sim m^{-9.8}$
$\bar{n}_{\max} = 9, \sim m^{-10.0}$
$\bar{n}_{\max} = 10, \sim m^{-11.9}$
$\bar{n}_{\max} = 11, \sim m^{-12.0}$
$\bar{n}_{\max} = 12, \sim m^{-14.2}$

SELF-FORCE



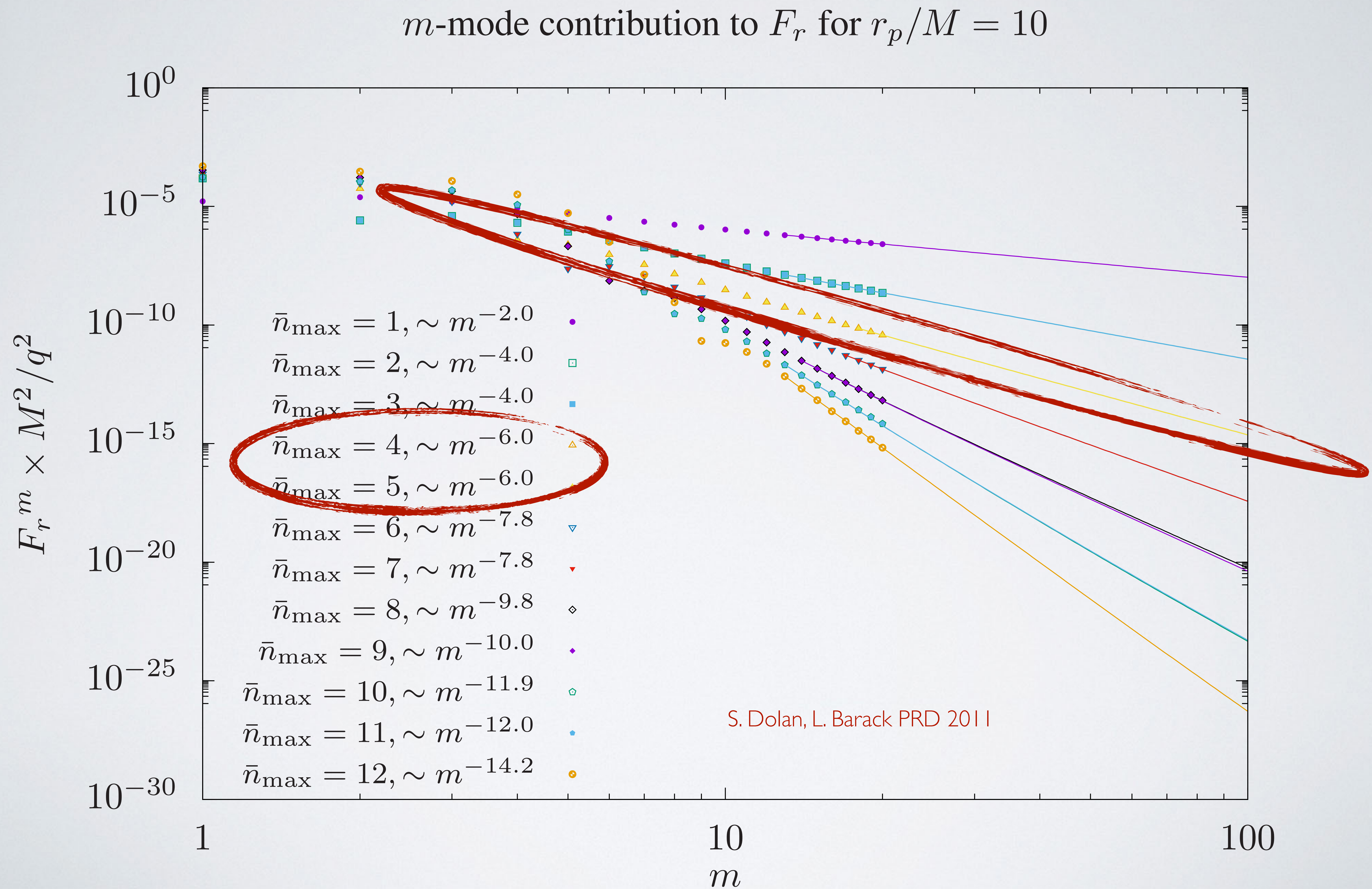
m-mode contribution to F_r for $r_p/M = 10$
10^0
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
$F_r{}^m \times M^2/q^2$
1
10
100
m
$\bar{n}_{\max} = 1, \sim m^{-2.0}$
$\bar{n}_{\max} = 2, \sim m^{-4.0}$
$\bar{n}_{\max} = 3, \sim m^{-4.0}$
$\bar{n}_{\max} = 4, \sim m^{-6.0}$
$\bar{n}_{\max} = 5, \sim m^{-6.0}$
$\bar{n}_{\max} = 6, \sim m^{-7.8}$
$\bar{n}_{\max} = 7, \sim m^{-7.8}$
$\bar{n}_{\max} = 8, \sim m^{-9.8}$
$\bar{n}_{\max} = 9, \sim m^{-10.0}$
$\bar{n}_{\max} = 10, \sim m^{-11.9}$
$\bar{n}_{\max} = 11, \sim m^{-12.0}$
$\bar{n}_{\max} = 12, \sim m^{-14.2}$
S. Dolan, L. Barack PRD 2011

SELF-FORCE

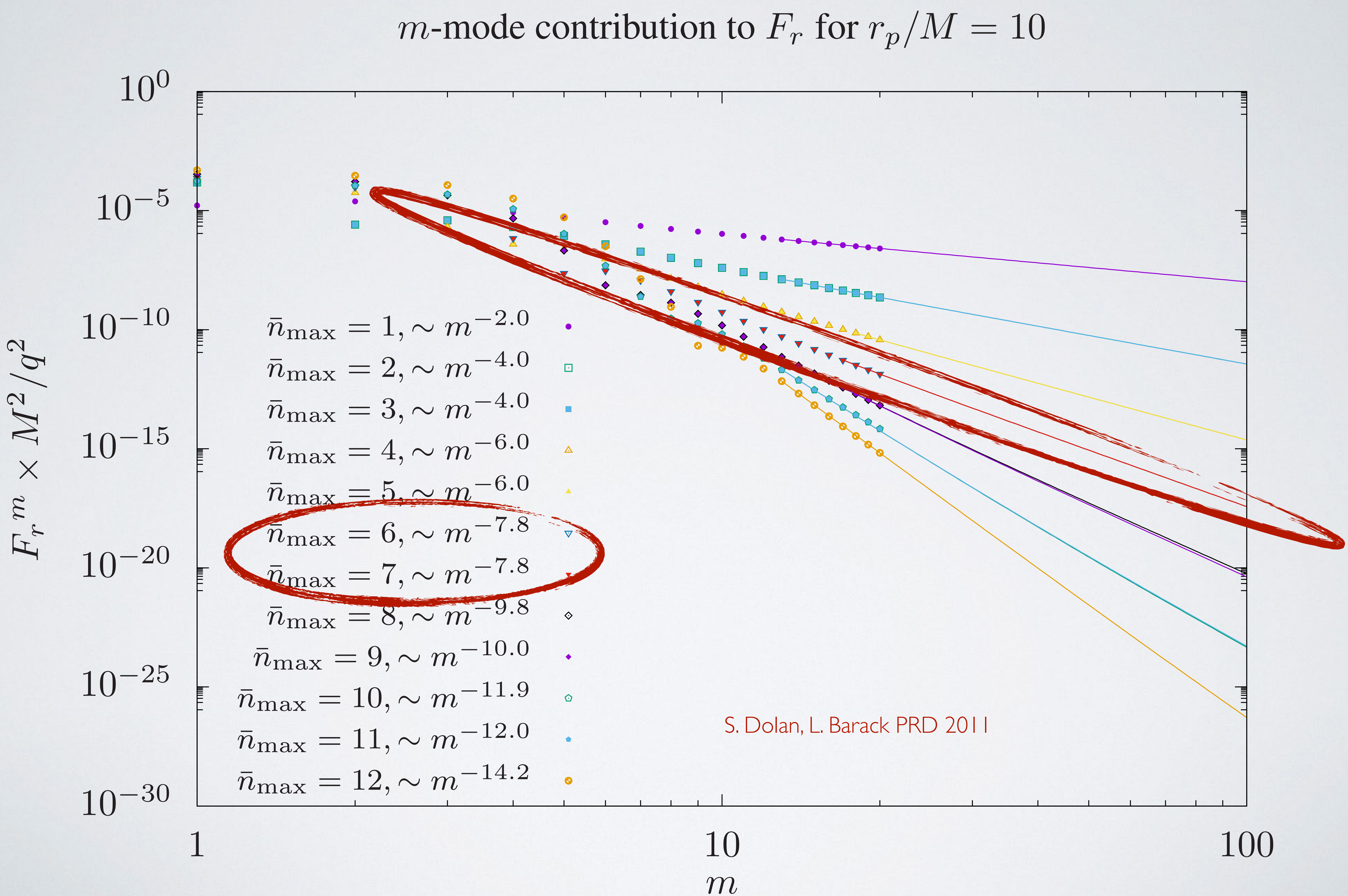


m-mode contribution to F_r for $r_p/M = 10$
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
$F_r{}^m \times M^2/q^2$
1
10
100
m
$\bar{n}_{\max} = 1, \sim m^{-2.0}$
$\bar{n}_{\max} = 2, \sim m^{-4.0}$
$\bar{n}_{\max} = 3, \sim m^{-4.0}$
$\bar{n}_{\max} = 4, \sim m^{-6.0}$
$\bar{n}_{\max} = 5, \sim m^{-6.0}$
$\bar{n}_{\max} = 6, \sim m^{-7.8}$
$\bar{n}_{\max} = 7, \sim m^{-7.8}$
$\bar{n}_{\max} = 8, \sim m^{-9.8}$
$\bar{n}_{\max} = 9, \sim m^{-10.0}$
$\bar{n}_{\max} = 10, \sim m^{-11.9}$
$\bar{n}_{\max} = 11, \sim m^{-12.0}$
$\bar{n}_{\max} = 12, \sim m^{-14.2}$
S. Dolan, L. Barack PRD 2011

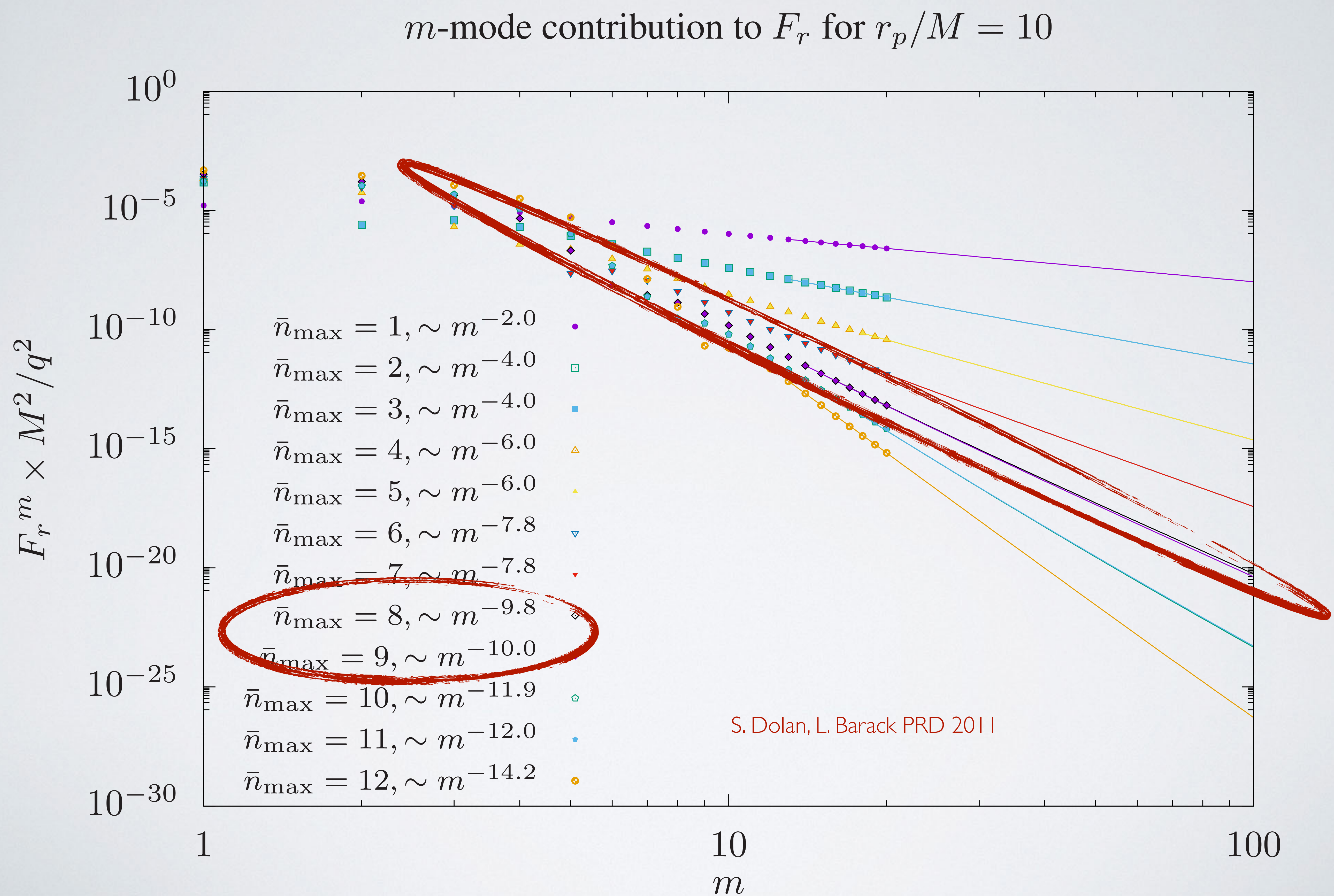
SELF-FORCE


m-mode contribution to F_r for $r_p/M = 10$
$F_r{}^m \times M^2/q^2$
m
10^0
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
1
10
100
$\bar{n}_{\max} = 1, \sim m^{-2.0}$
$\bar{n}_{\max} = 2, \sim m^{-4.0}$
$\bar{n}_{\max} = 3, \sim m^{-4.0}$
$\bar{n}_{\max} = 4, \sim m^{-6.0}$
$\bar{n}_{\max} = 5, \sim m^{-6.0}$
$\bar{n}_{\max} = 6, \sim m^{-7.8}$
$\bar{n}_{\max} = 7, \sim m^{-7.8}$
$\bar{n}_{\max} = 8, \sim m^{-9.8}$
$\bar{n}_{\max} = 9, \sim m^{-10.0}$
$\bar{n}_{\max} = 10, \sim m^{-11.9}$
$\bar{n}_{\max} = 11, \sim m^{-12.0}$
$\bar{n}_{\max} = 12, \sim m^{-14.2}$
S. Dolan, L. Barack PRD 2011

SELF-FORCE

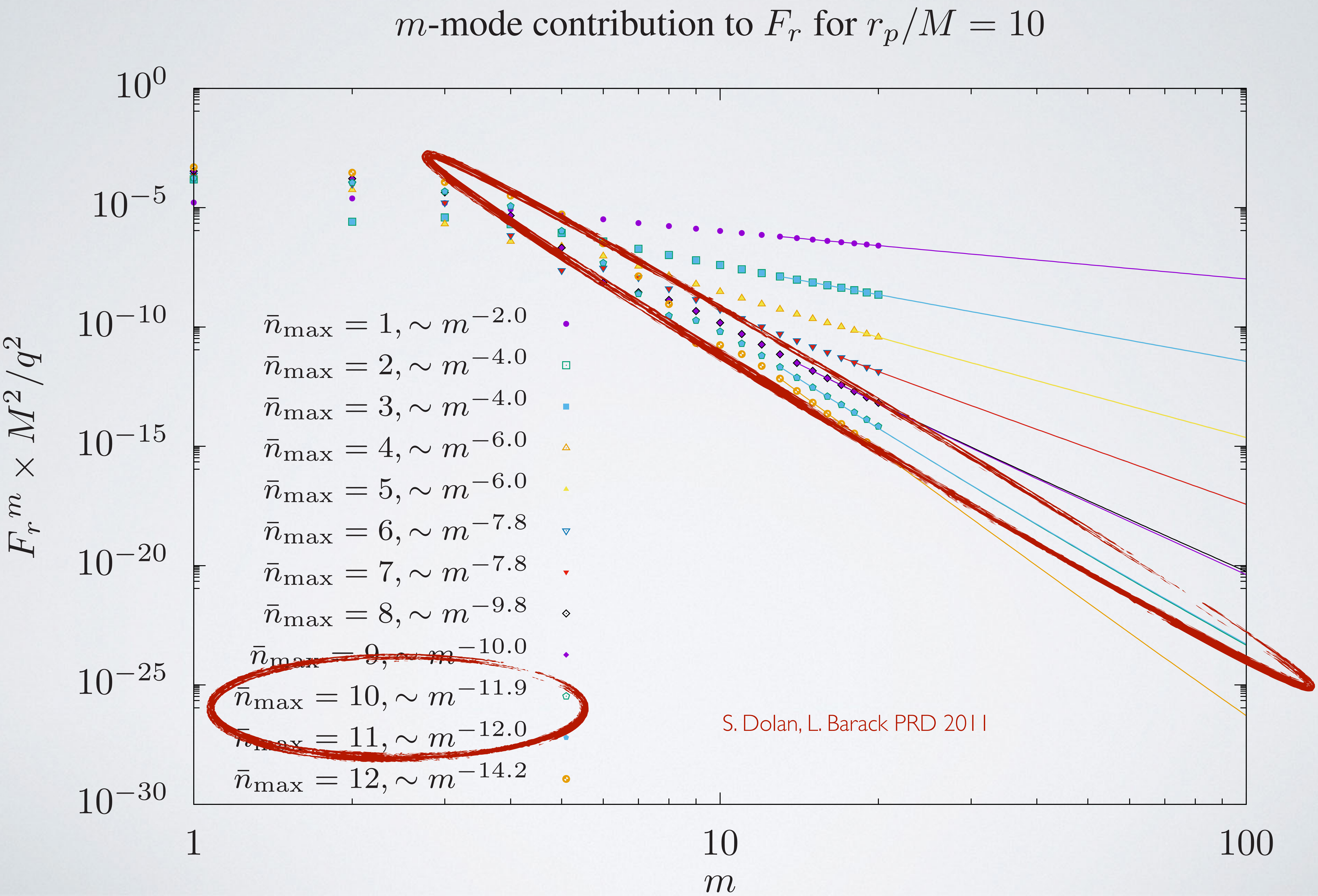

m-mode contribution to F_r for $r_p/M = 10$
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
$F_r{}^m \times M^2/q^2$
1
10
100
m
$\bar{n}_{\max} = 1, \sim m^{-2.0}$
$\bar{n}_{\max} = 2, \sim m^{-4.0}$
$\bar{n}_{\max} = 3, \sim m^{-4.0}$
$\bar{n}_{\max} = 4, \sim m^{-6.0}$
$\bar{n}_{\max} = 5, \sim m^{-6.0}$
$\bar{n}_{\max} = 6, \sim m^{-7.8}$
$\bar{n}_{\max} = 7, \sim m^{-7.8}$
$\bar{n}_{\max} = 8, \sim m^{-9.8}$
$\bar{n}_{\max} = 9, \sim m^{-10.0}$
$\bar{n}_{\max} = 10, \sim m^{-11.9}$
$\bar{n}_{\max} = 11, \sim m^{-12.0}$
$\bar{n}_{\max} = 12, \sim m^{-14.2}$
S. Dolan, L. Barack PRD 2011

SELF-FORCE

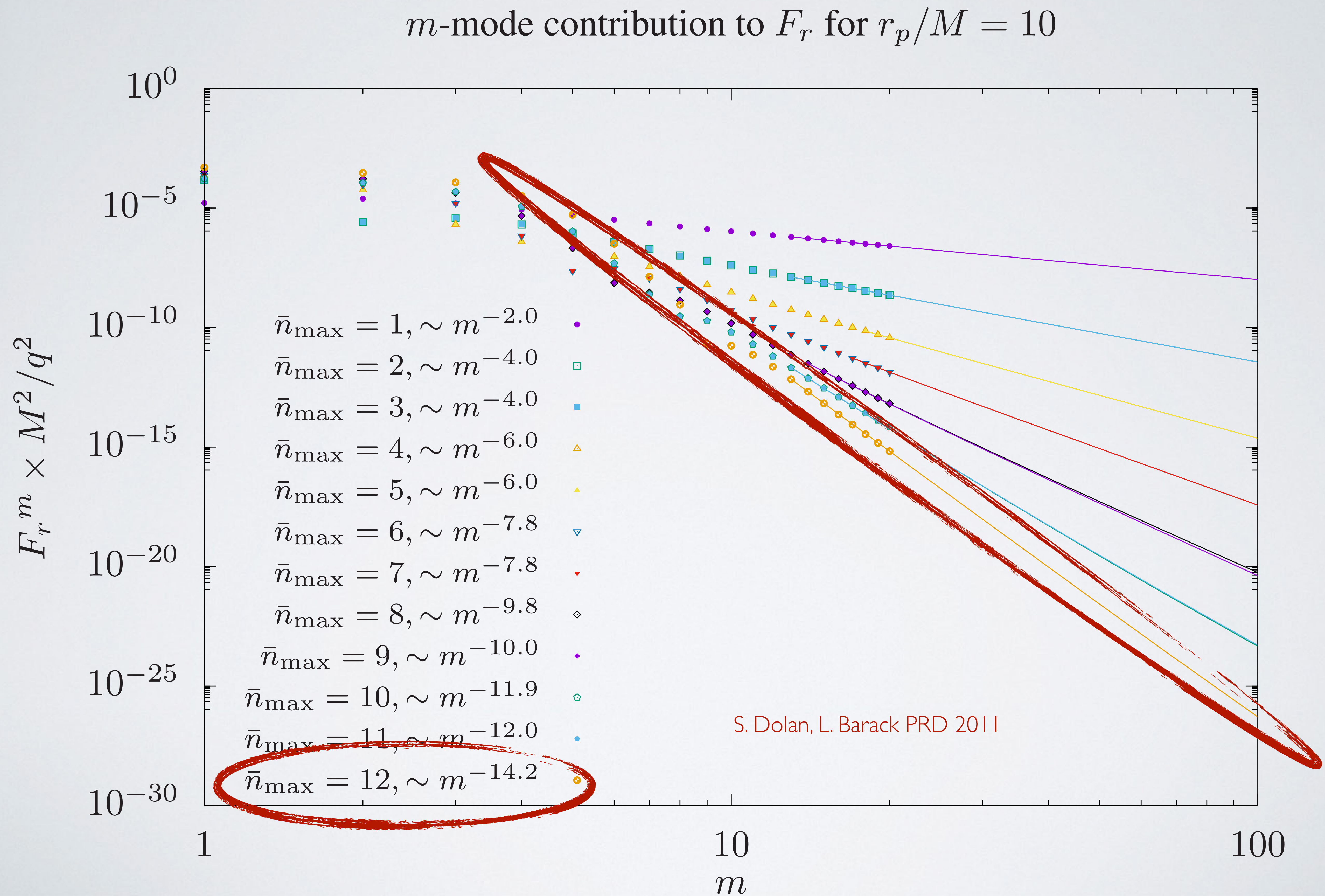


m-mode contribution to F_r for $r_p/M = 10$
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
$F_r{}^m \times M^2/q^2$
1
10
100
m
$\bar{n}_{\max} = 1, \sim m^{-2.0}$
$\bar{n}_{\max} = 2, \sim m^{-4.0}$
$\bar{n}_{\max} = 3, \sim m^{-4.0}$
$\bar{n}_{\max} = 4, \sim m^{-6.0}$
$\bar{n}_{\max} = 5, \sim m^{-6.0}$
$\bar{n}_{\max} = 6, \sim m^{-7.8}$
$\bar{n}_{\max} = 7, \sim m^{-7.8}$
$\bar{n}_{\max} = 8, \sim m^{-9.8}$
$\bar{n}_{\max} = 9, \sim m^{-10.0}$
$\bar{n}_{\max} = 10, \sim m^{-11.9}$
$\bar{n}_{\max} = 11, \sim m^{-12.0}$
$\bar{n}_{\max} = 12, \sim m^{-14.2}$
S. Dolan, L. Barack PRD 2011

SELF-FORCE


m-mode contribution to F_r for $r_p/M = 10$
$F_r{}^m \times M^2/q^2$
10^0
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
1
10
100
m
$\bar{n}_{\max} = 1, \sim m^{-2.0}$
$\bar{n}_{\max} = 2, \sim m^{-4.0}$
$\bar{n}_{\max} = 3, \sim m^{-4.0}$
$\bar{n}_{\max} = 4, \sim m^{-6.0}$
$\bar{n}_{\max} = 5, \sim m^{-6.0}$
$\bar{n}_{\max} = 6, \sim m^{-7.8}$
$\bar{n}_{\max} = 7, \sim m^{-7.8}$
$\bar{n}_{\max} = 8, \sim m^{-9.8}$
$\bar{n}_{\max} = 9, \sim m^{-10.0}$
$\bar{n}_{\max} = 10, \sim m^{-11.9}$
$\bar{n}_{\max} = 11, \sim m^{-12.0}$
$\bar{n}_{\max} = 12, \sim m^{-14.2}$
S. Dolan, L. Barack PRD 2011

SELF-FORCE



m-mode contribution to F_r for $r_p/M = 10$
$F_r{}^m \times M^2/q^2$
10^0
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
1
10
100
m
$\bar{n}_{\max} = 1, \sim m^{-2.0}$
$\bar{n}_{\max} = 2, \sim m^{-4.0}$
$\bar{n}_{\max} = 3, \sim m^{-4.0}$
$\bar{n}_{\max} = 4, \sim m^{-6.0}$
$\bar{n}_{\max} = 5, \sim m^{-6.0}$
$\bar{n}_{\max} = 6, \sim m^{-7.8}$
$\bar{n}_{\max} = 7, \sim m^{-7.8}$
$\bar{n}_{\max} = 8, \sim m^{-9.8}$
$\bar{n}_{\max} = 9, \sim m^{-10.0}$
$\bar{n}_{\max} = 10, \sim m^{-11.9}$
$\bar{n}_{\max} = 11, \sim m^{-12.0}$
$\bar{n}_{\max} = 12, \sim m^{-14.2}$
S. Dolan, L. Barack PRD 2011

SELF-FORCE

Elliptic Solver for inspiral model
(Finite Boundary. Fin. Diff.)

$$F_r = 1.3784 \times 10^{-5}\, q^2/M^2$$

T. Osburn, N. Nishimura PRD 2022

High-Order mode-sum regularisation

$$F_r = 1.37844825756679 59 \times 10^{-5}\, q^2/M^2$$

A. Heffernan, A. Ottewill, B. Wardell PRD 2012

$$F_r \times 10^5\, M^2/q^2$$

$\bar{n}_{\max}$	Numerical ($m \in [0, 20]$)	Rel. Error	High-m fit ($m \in [0, 100]$)	Rel. Error
1	0.8773923605	3×10^{-1}	1.277643667	7×10^{-2}
2, 3	1.377054775	1×10^{-3}	1.378440334	6×10^{-6}
4, 5	1.378435277	9×10^{-6}	1.378448291	2×10^{-8}
6, 7	1.378447937	2×10^{-7}	1.378448259	7×10^{-10}
8, 9	1.378448251	4×10^{-9}	1.378448261	2×10^{-9}
10, 11	1.378448266	6×10^{-9}	1.378448266	6×10^{-9}

CONCLUSION

CONCLUSION

- Multi-domain spectral code for elliptical equation arising in an m-mode decomposition in the frequency domain

CONCLUSION

- Multi-domain spectral code for elliptical equation arising in an m-mode decomposition in the frequency domain
- High order puncture scheme in the particle region (circular orbit, Schwarzschild)

CONCLUSION

- Multi-domain spectral code for elliptical equation arising in an m-mode decomposition in the frequency domain
- High order puncture scheme in the particle region (circular orbit, Schwarzschild)
- Hyperboloidal approach for the asymptotic region

CONCLUSION

- Multi-domain spectral code for elliptical equation arising in an m-mode decomposition in the frequency domain
- High order puncture scheme in the particle region (circular orbit, Schwarzschild)
- Hyperboloidal approach for the asymptotic region
- The infrastructure is generic enough to direct use in Kerr (modulo: puncture+effective source calculation)

CONCLUSION

- Multi-domain spectral code for elliptical equation arising in an m-mode decomposition in the frequency domain
- High order puncture scheme in the particle region (circular orbit, Schwarzschild)
- Hyperboloidal approach for the asymptotic region
- The infrastructure is generic enough to direct use in Kerr (modulo: puncture+effective source calculation)

Hyperboloidal approach + Spectral methods framework

CONCLUSION

- Multi-domain spectral code for elliptical equation arising in an m-mode decomposition in the frequency domain
- High order puncture scheme in the particle region (circular orbit, Schwarzschild)
- Hyperboloidal approach for the asymptotic region
- The infrastructure is generic enough to direct use in Kerr (modulo: puncture+effective source calculation)

Hyperboloidal approach + Spectral methods framework

Phase I complete:

Benchmark tests for scalar field on Schwarzschild,
particle at circular orbit, first order GSF
(ODE, Elliptic and 1+1 solvers)

CONCLUSION

- Multi-domain spectral code for elliptical equation arising in an m-mode decomposition in the frequency domain
- High order puncture scheme in the particle region (circular orbit, Schwarzschild)
- Hyperboloidal approach for the asymptotic region
- The infrastructure is generic enough to direct use in Kerr (modulo: puncture+effective source calculation)

Hyperboloidal approach + Spectral methods framework

Phase I complete:
Benchmark tests for scalar field on Schwarzschild,
particle at circular orbit, first order GSF
(ODE, Elliptic and 1+1 solvers)

Phase I (proof-of-principle) complete:
include gravity case, polish up, system of equations, add to BHPT
[I need help for that! Volunteers?]

PERSPECTIVE

Hyperboloidal approach + Spectral methods framework

Phase II:

Problems in Kerr at first order GSF
Start developing the interface with
problems at second order GSF