Tidal Heating in eccentric orbits

Sayak Datta

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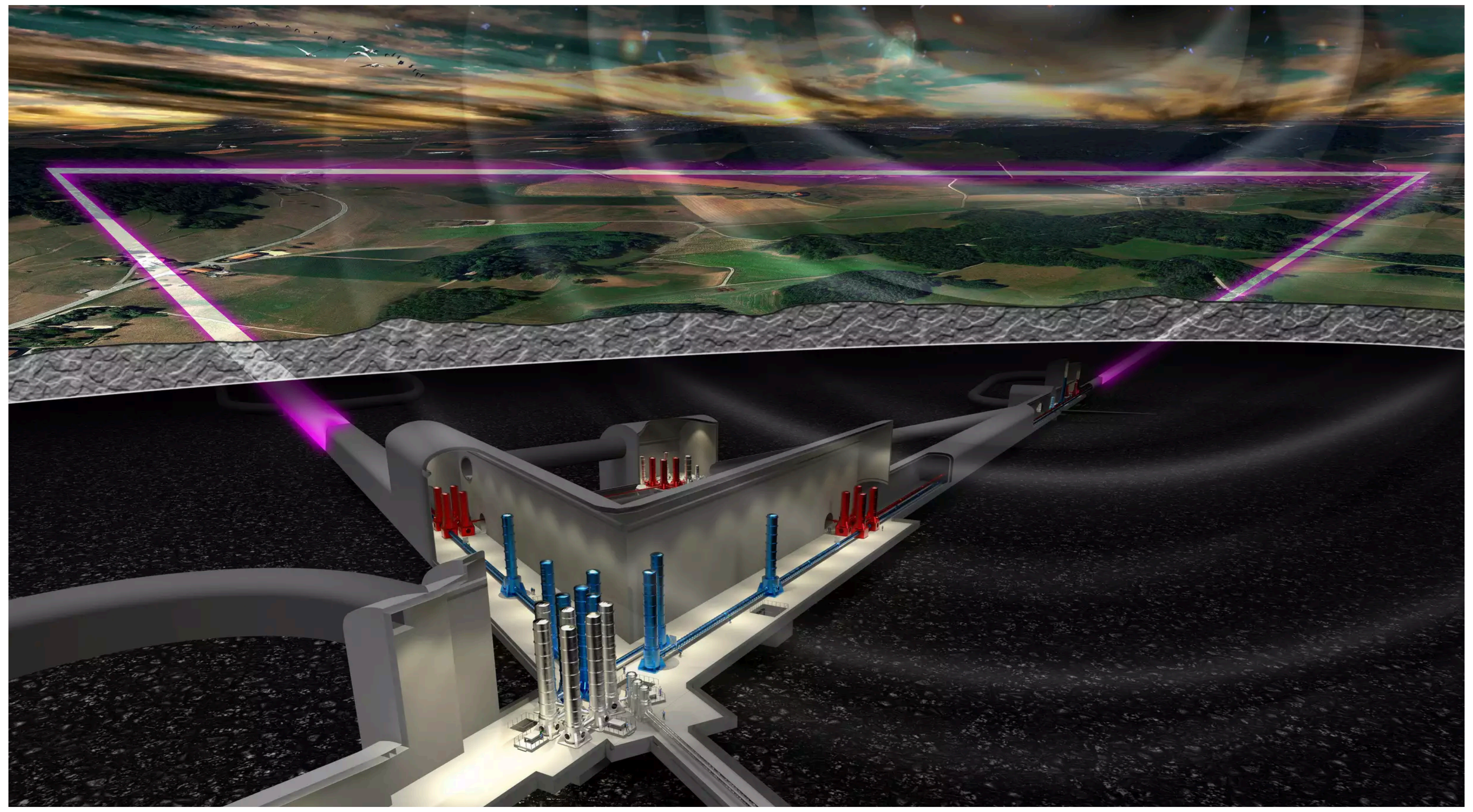
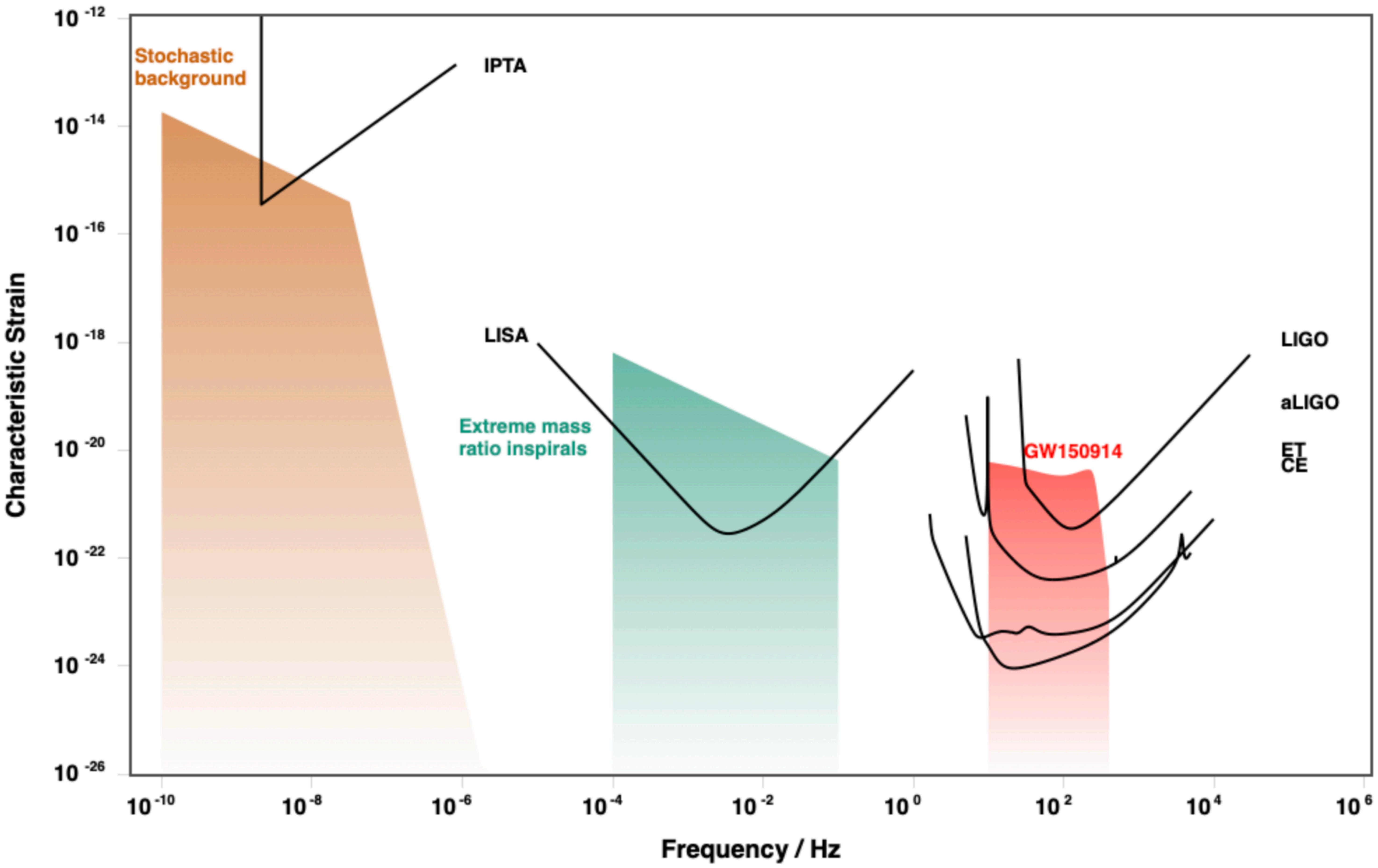
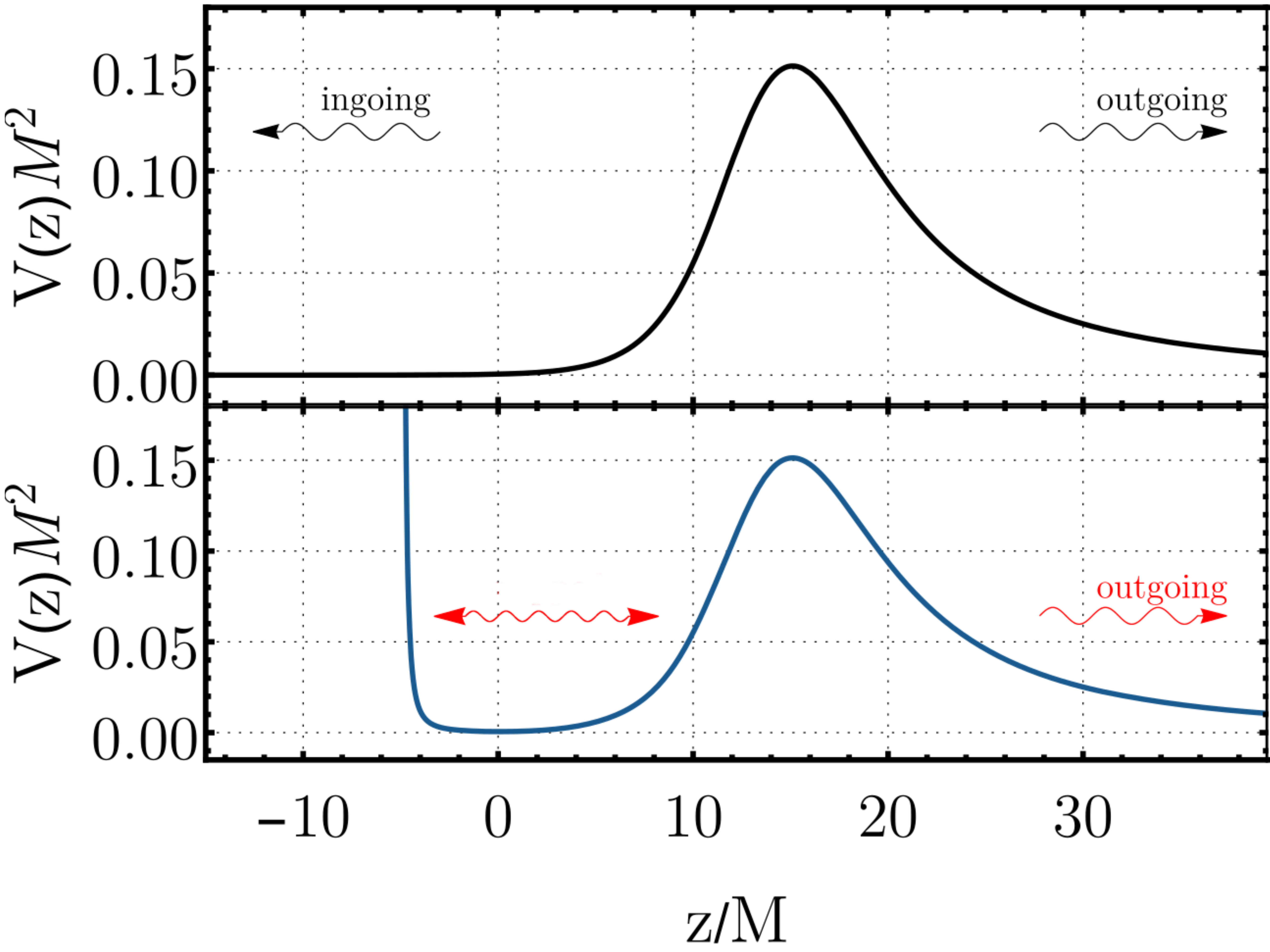
- Current GBDs are being upgraded.
- New detectors Cosmic Explorer, Einstein telescope, and space based LISA is also coming.
- These will be more sensitive detectors.
- This opportunity can be used to test GR.
- Also the nature of the compact objects.
- Exotic compact object (ECO), quantum effects near BH.



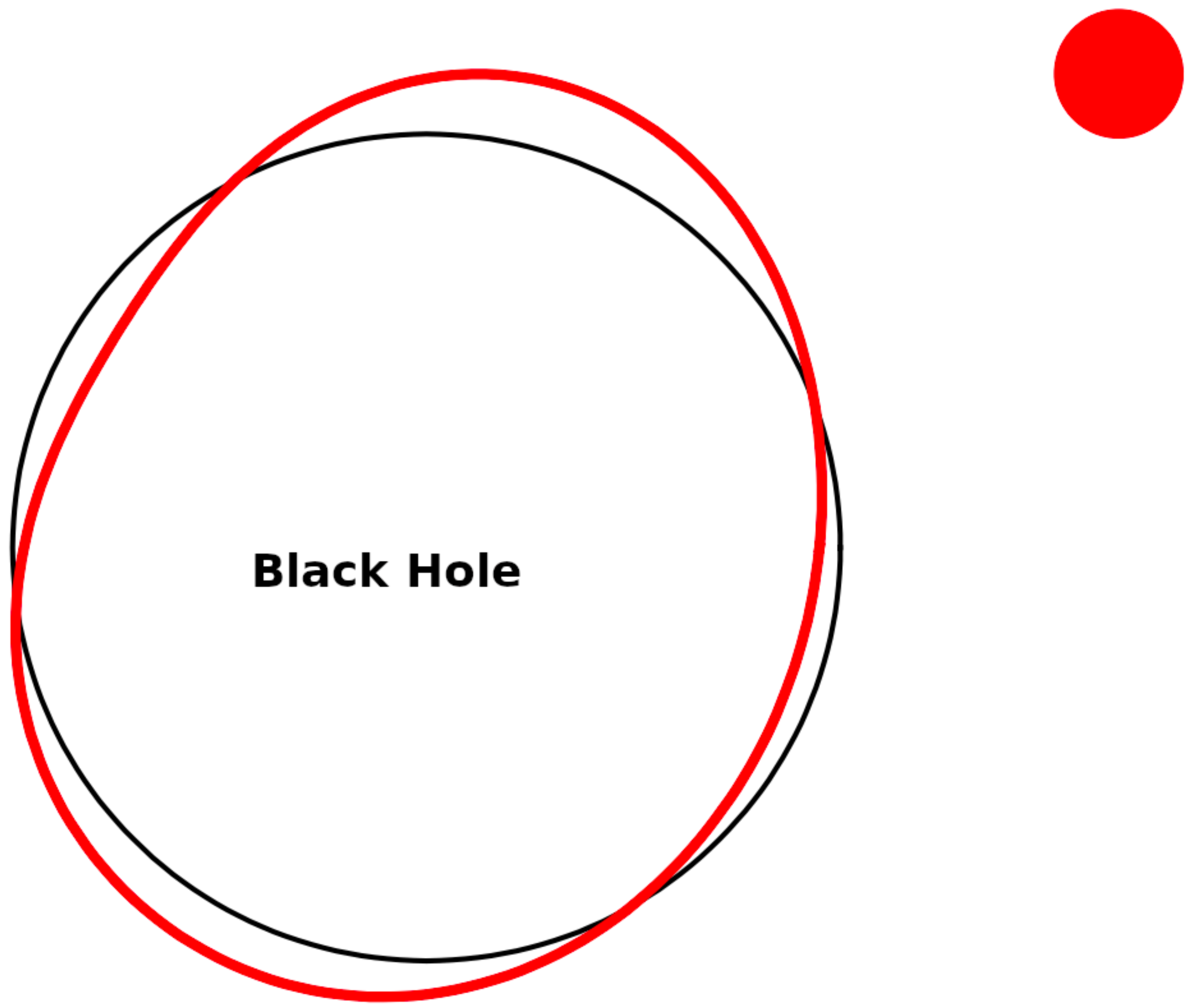


- Classical BH's horizon is perfect absorber due to causality.
- Absence (modification) of this implies imperfect absorption.
- Measuring nonzero reflectivity of compact object surface will be signature of deviation.
- Tidal heating is one such effects.



- Living Rev.Rel. 22 (2019) 1, 4

- Components in a binary feel each others' tidal fields (strongly in the late inspiral).
- If the bodies are(at least partially) absorbing, these backreact on the orbit, exchanging energy and angular momentum with the orbit.
- This effect is called tidal heating J. B. Hartle, PRD8, 1010 (1973), S. A. Hughes, PRD64,064004 (2001).



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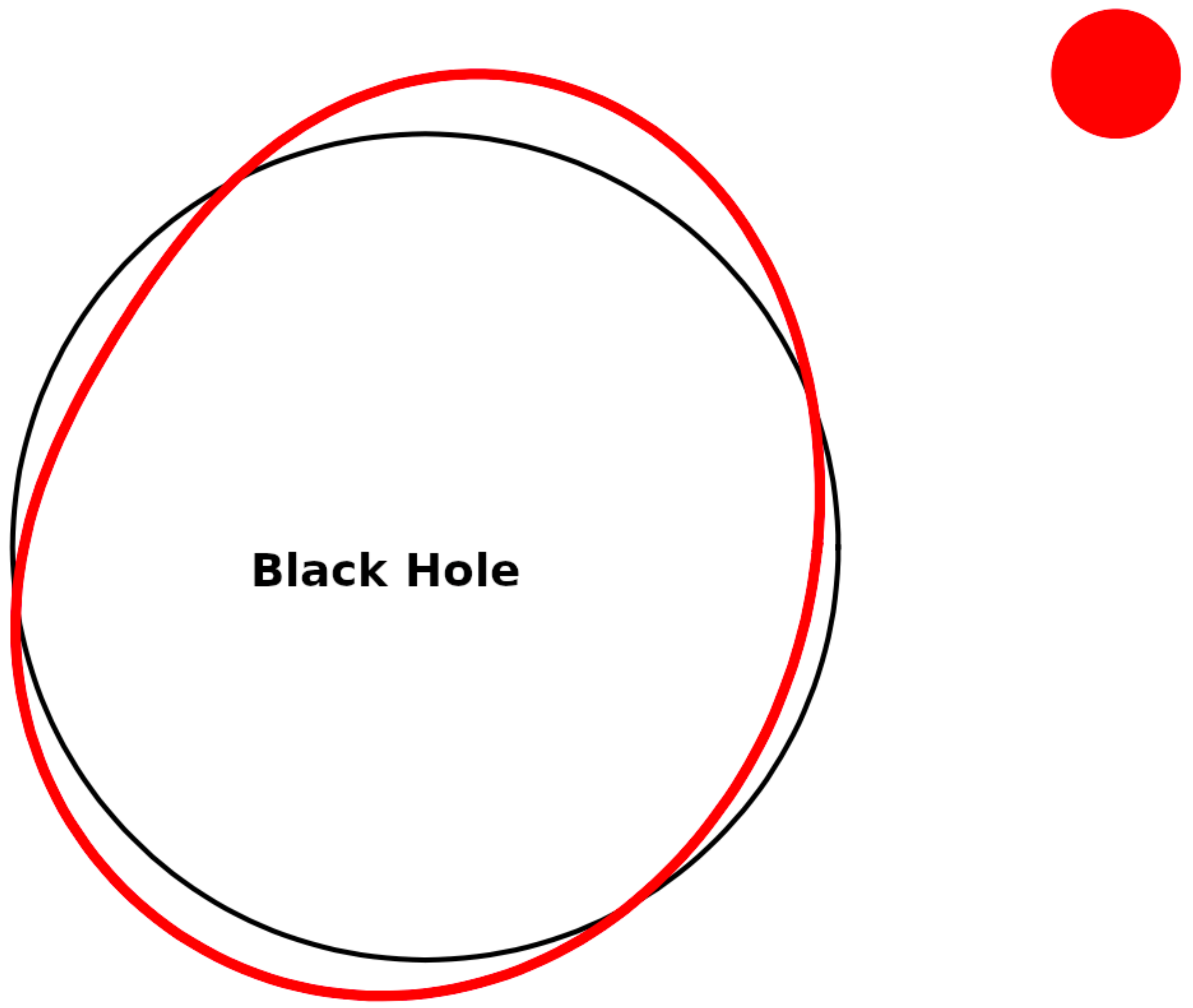
- In stars this absorption comes due to viscous heating in the material.
- In BHs it caused by the change in the BH mass.



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- Expression for TH of a star and BH can be brought into same footing with viscosity coefficient ($\nu_{BH} \sim M$). K. Glampedakis+ PRD89,024007(2014)
- For NS, $$\nu_{NS} = 10^4 \left(\frac{\rho}{10^{14} gm cm^{-3}}\right)^{\frac{5}{4}} \left(\frac{10^8 K}{T}\right)^2 cm^2 s^{-1}$$
- $$\nu_{BH} = 8.6 \times 10^{14} \left(\frac{M}{M_\odot}\right) cm^2 s^{-1}$$
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- Distinguish BH and NS in this range can change NS mass upperbound and BH mass lower bound.

TH in eccentric orbits

- TH depends on the tidal environment, $\mathscr{E}_{ab}(r, \dot{r}, \phi, \dot{\phi})$ and $\mathscr{B}_{ab}(r, \dot{r}, \phi, \dot{\phi})$. Taylor+ PRD 78, 084016 (2008)

- Orbital quantities depend on, $\xi(f, e_t), u, e_t$. Moore+ PRD93, 124061 (2016)

- $\dot{m} = \dot{m}(\bar{\mathscr{E}}_{ab}, \bar{\mathscr{B}}_{ab})$ and similarly $\dot{J}$. Taylor+ PRD 78, 084016 (2008) , Poisson, PRD70, 084044 (2004)

- Upon averaging over the orbit,
  $\langle \dot{m} \rangle = \dot{m}_{circ}\mathscr{M}(e_t), \ \langle \dot{m} \rangle_\chi = \dot{m}_{circ,\chi}\mathscr{M}_\chi(e_t)\ , \ \langle \dot{J} \rangle_\chi = \dot{J}_{circ,\chi}\mathscr{J}_\chi(e_t)$
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SD, 2305.03771, Munna+ 2306.12481, Jezereel talk

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- Inspiral gives, $\dfrac{de_t}{dt}$ and $\dfrac{dx}{dt}$, $x \sim f^{2/3}$. Arun+ PRD80, 124018, Klein+ PRD81, 124001
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- $e_t = e_0\, \epsilon_1(x, x_0) + e_0^3\, \epsilon_3(x, x_0) + \mathcal{O}(e_0^5)$. SD, 2305.03771, SD 2306.12522

  $\langle \dot{m} \rangle = \mathscr{M}_0 + e_0^2 \mathscr{M}_2 + e_0^4 \mathscr{M}_4 + \mathcal{O}(e_0^6)$. SD, 2305.03771

- $$\langle \dot{J}_1 \rangle = -\frac{8 m_1^5 m_2^2 \xi^4 \chi \left(3\chi^2+1\right)}{5M^6} \frac{\left(3e_t^4+24e_t^2+8\right)}{8\left(1-e_t^2\right)^{9/2}}$$
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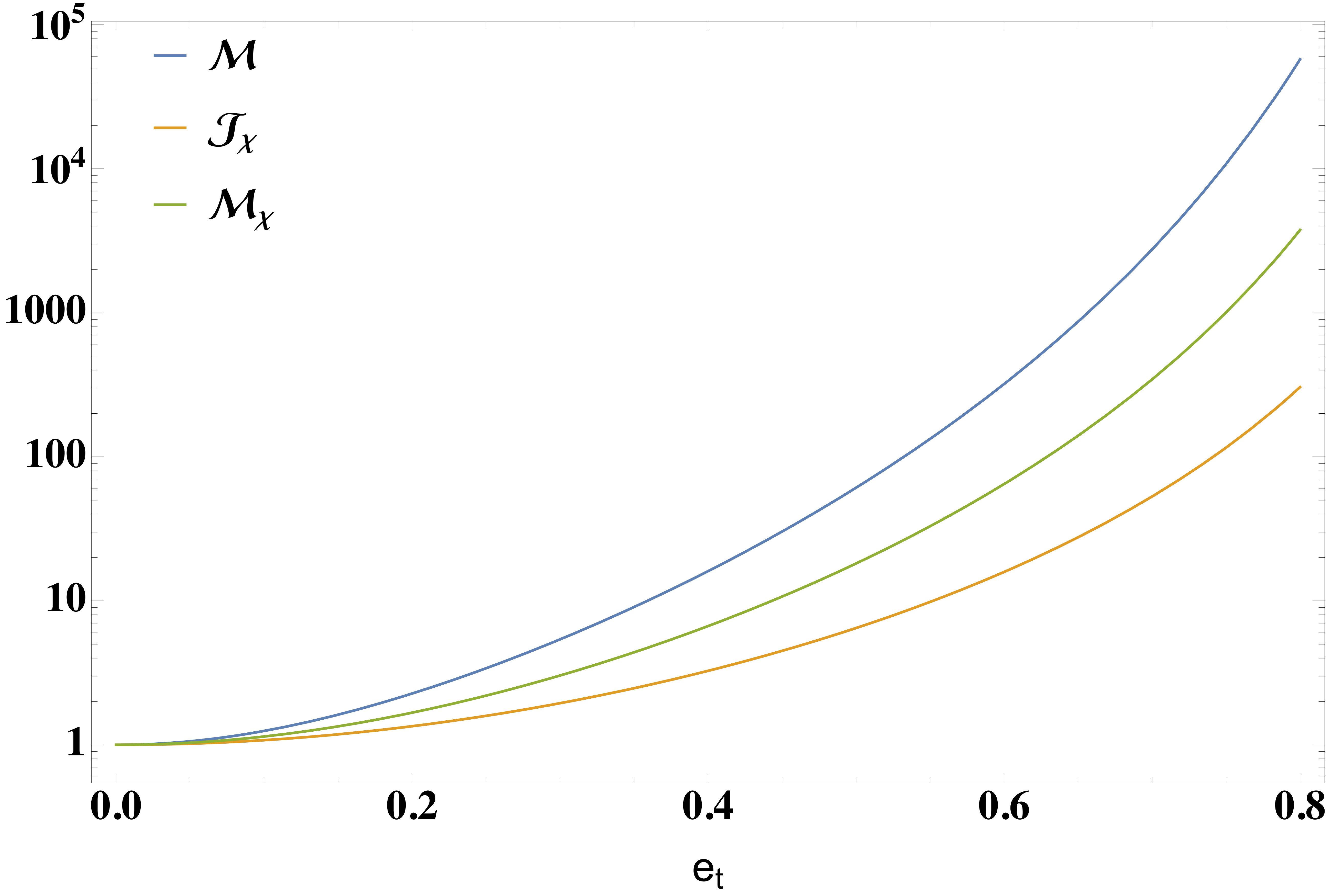

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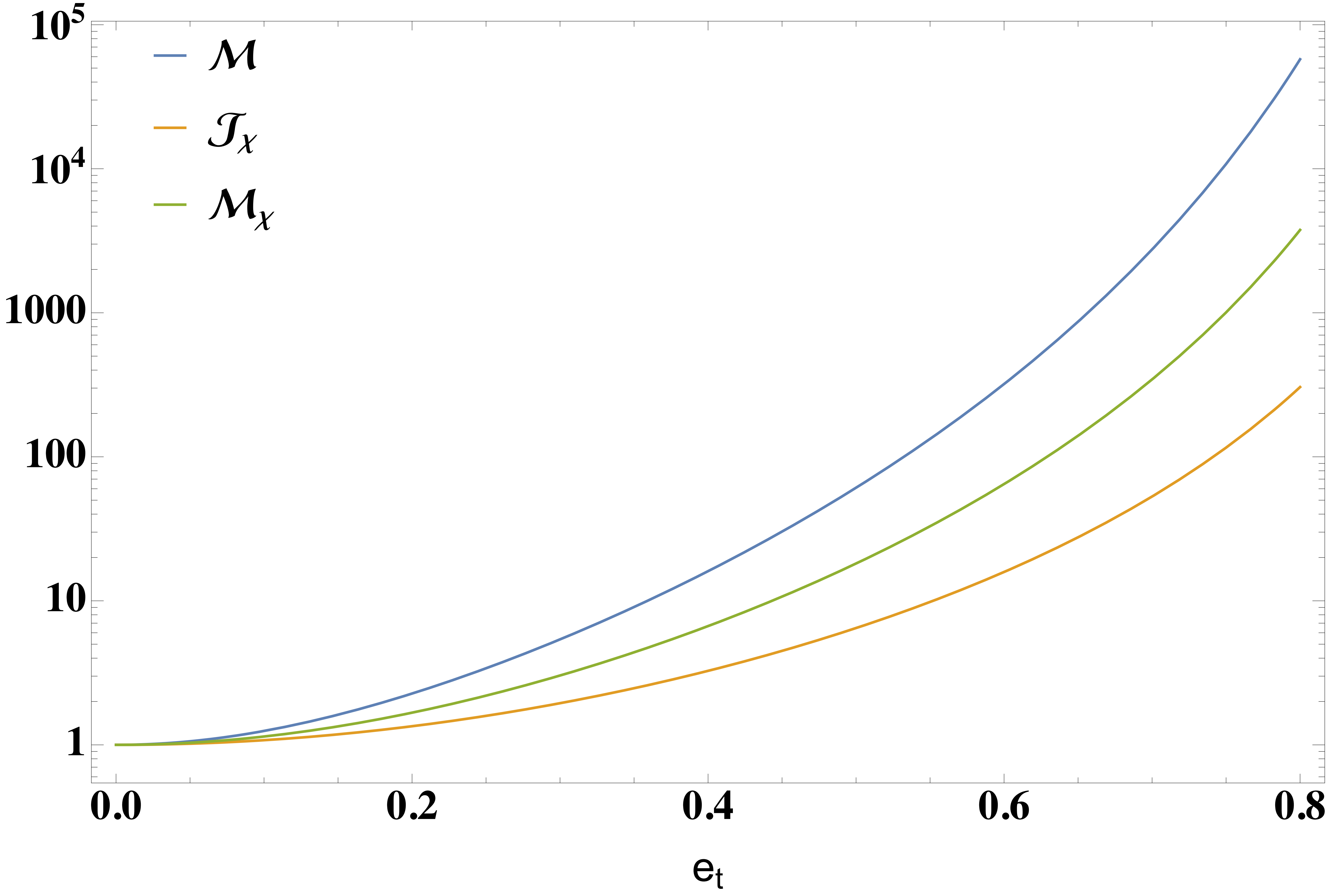


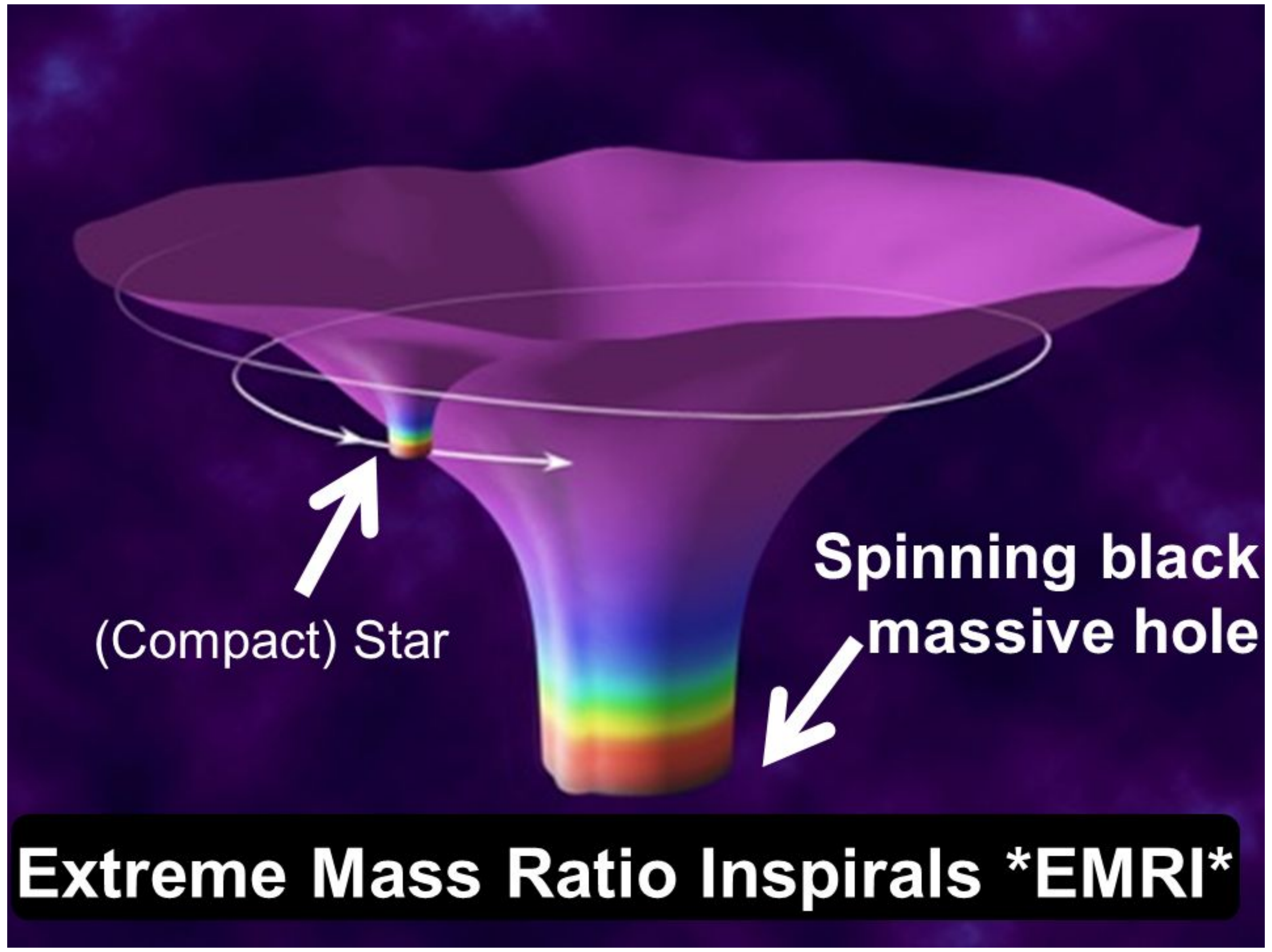
$$F^H \propto \Omega(\Omega - \Omega_H)$$

$$\Omega \sim v^3$$

Strength in EMRI

SD, Scott Hughes, Richard Brito, Paolo Pani

- We will focus on EMRI, where a stellar mass $\sim 10 - 100 M_\odot$ Inspirals around SMBH of $\sim 10^5 - 10^7 M_\odot$, observable in LISA.
- Hence we calculate perturbation around Kerr BH by a small particle.



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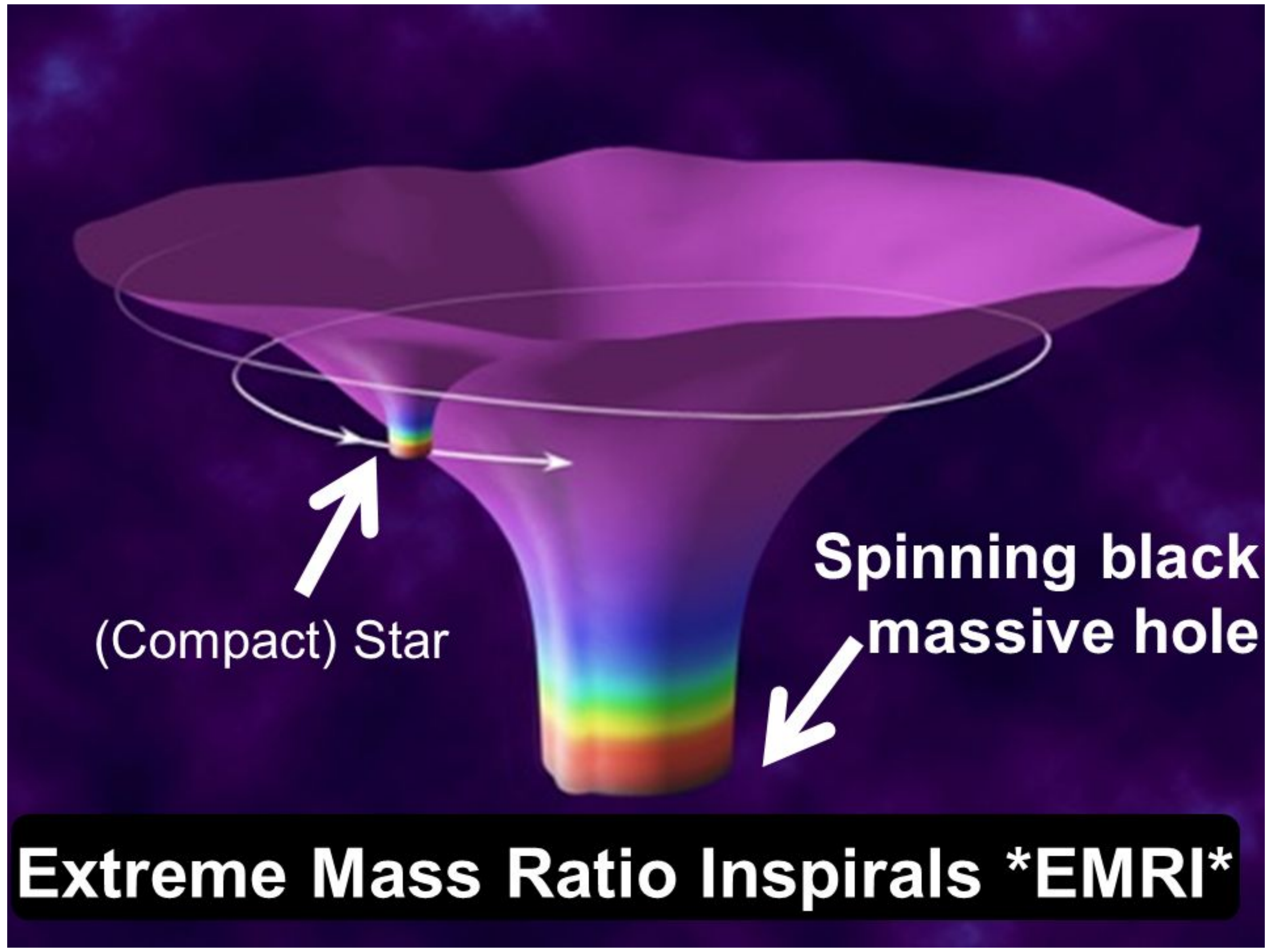
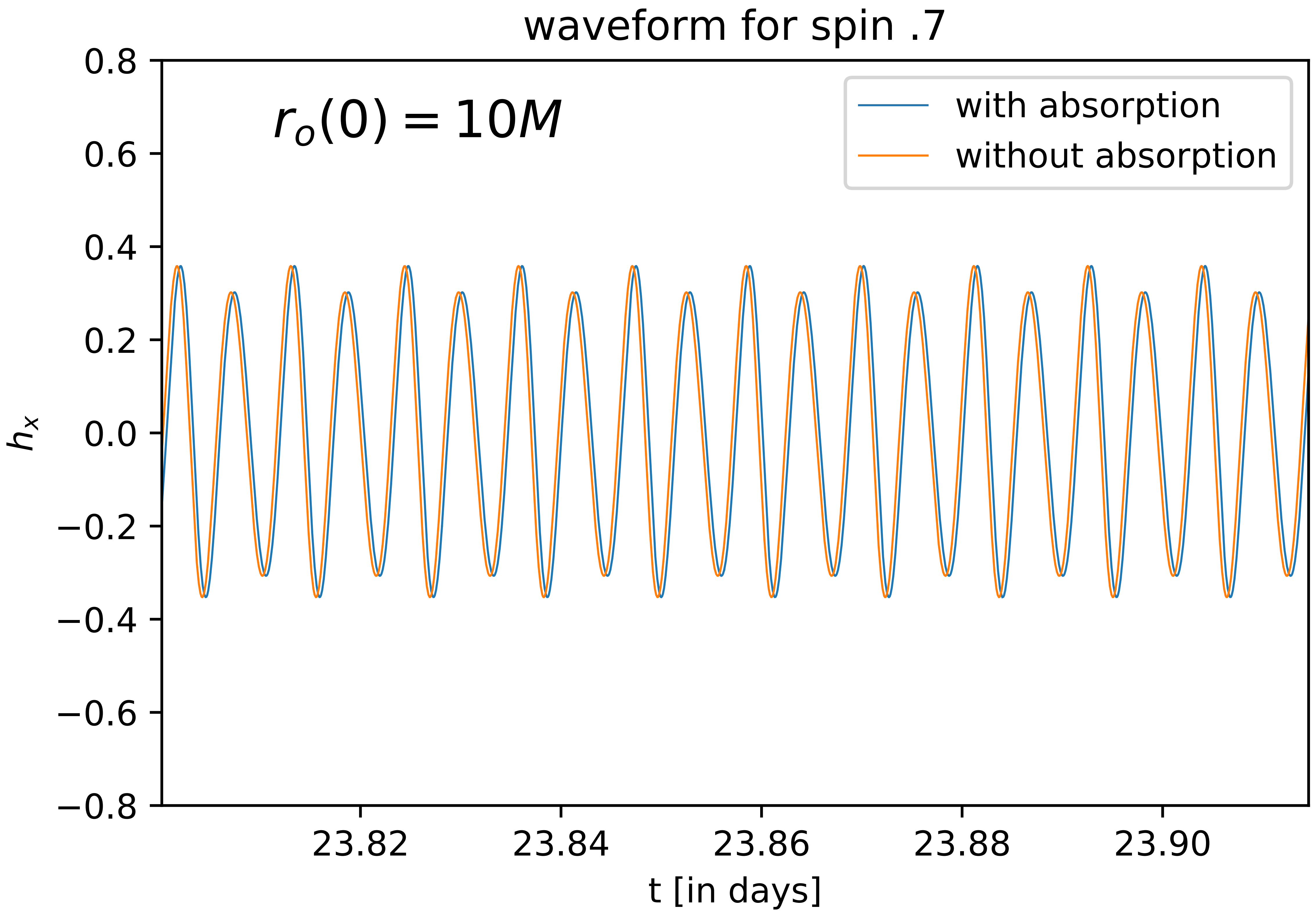
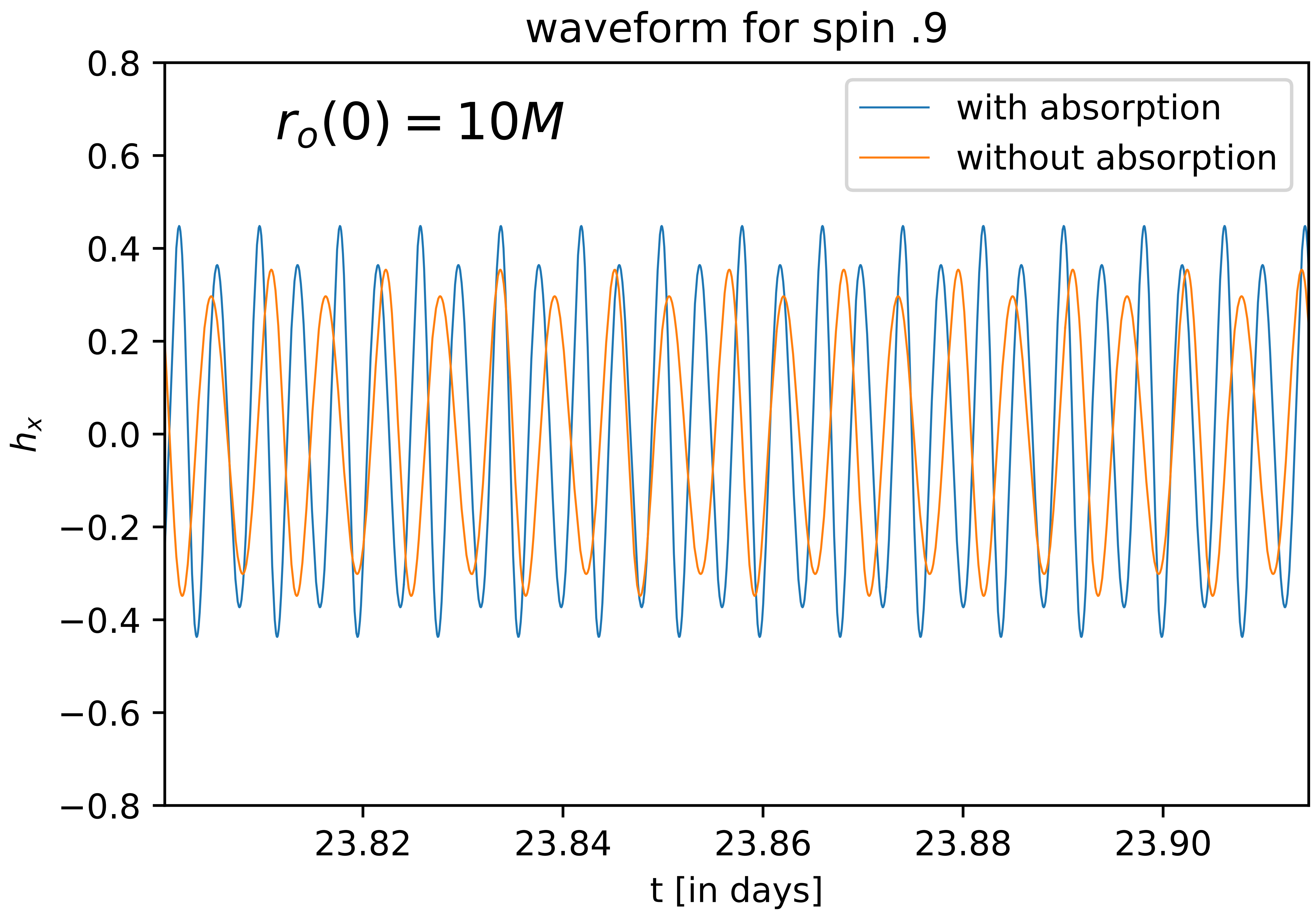


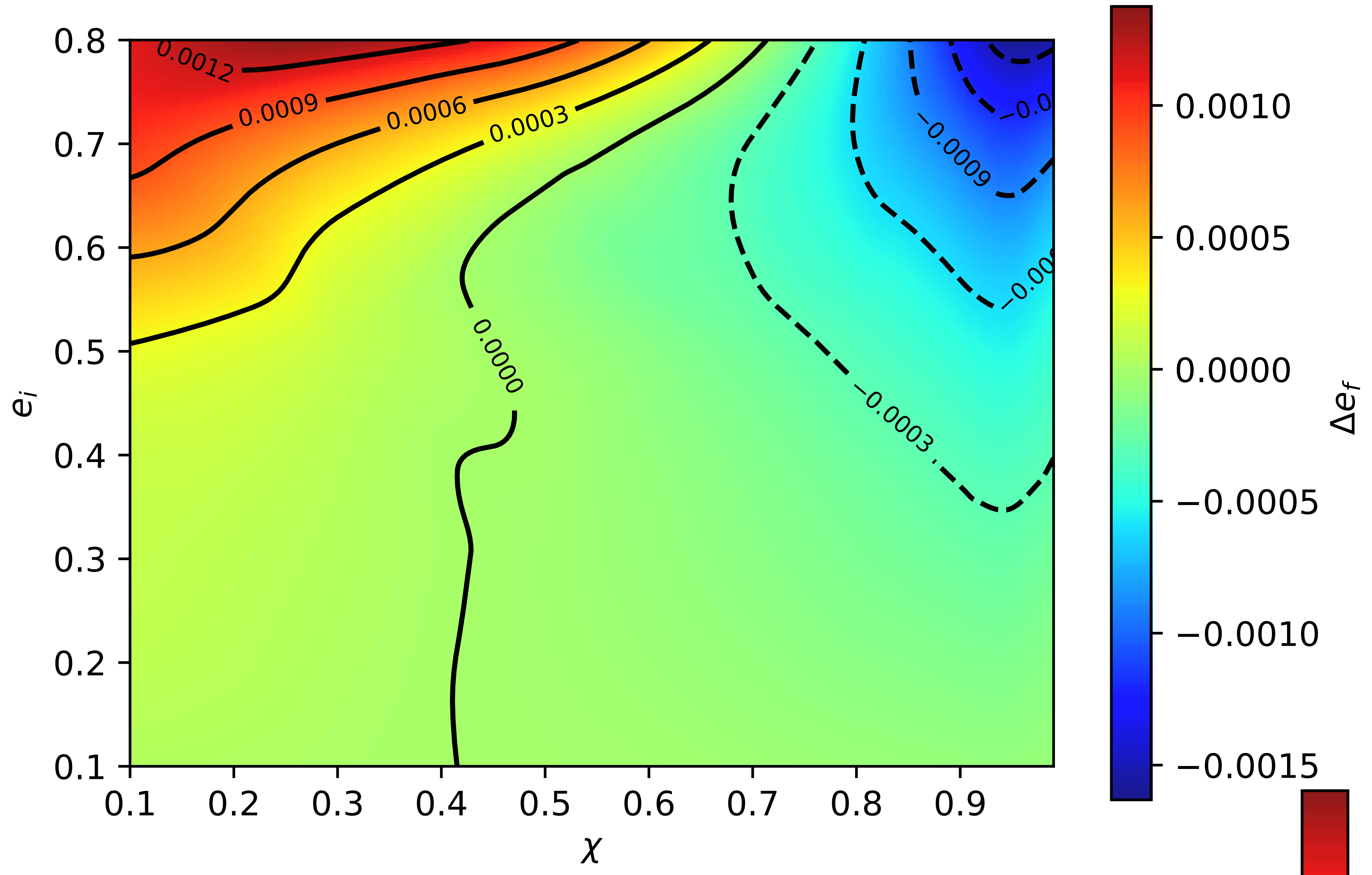
- We solve BHP equation.
- Energy fluxes at infinity and the horizon can be calculated from the perturbation GW waveform.
- $\dot{E} = \dot{E}_\infty + \dot{E}_H$
- $\dot{E}_{\rm TH} = (1 - |\mathscr{R}|^2)\dot{E}_H$

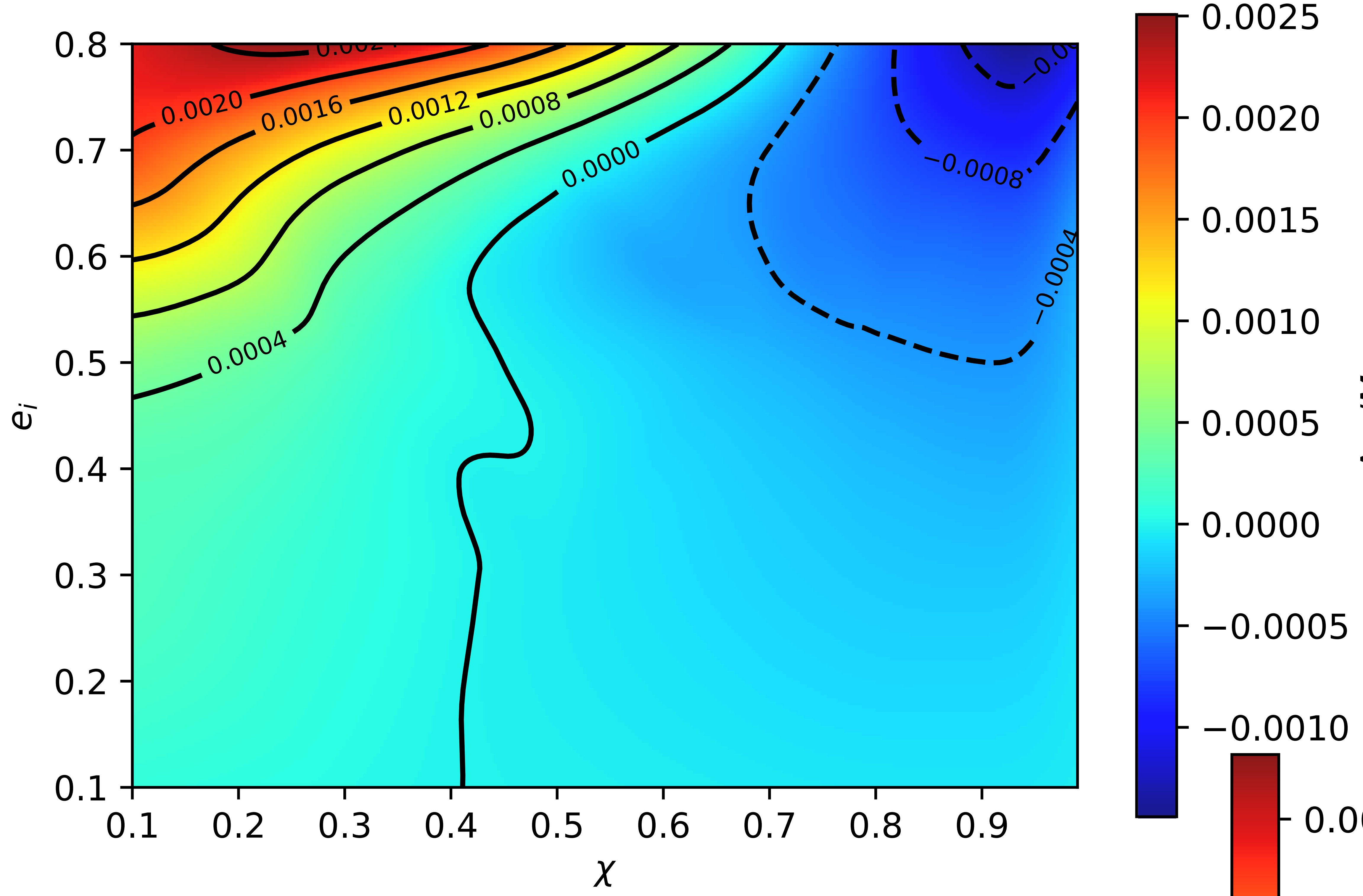
- $M = 10^6 M_\odot, M = 30000\mu$

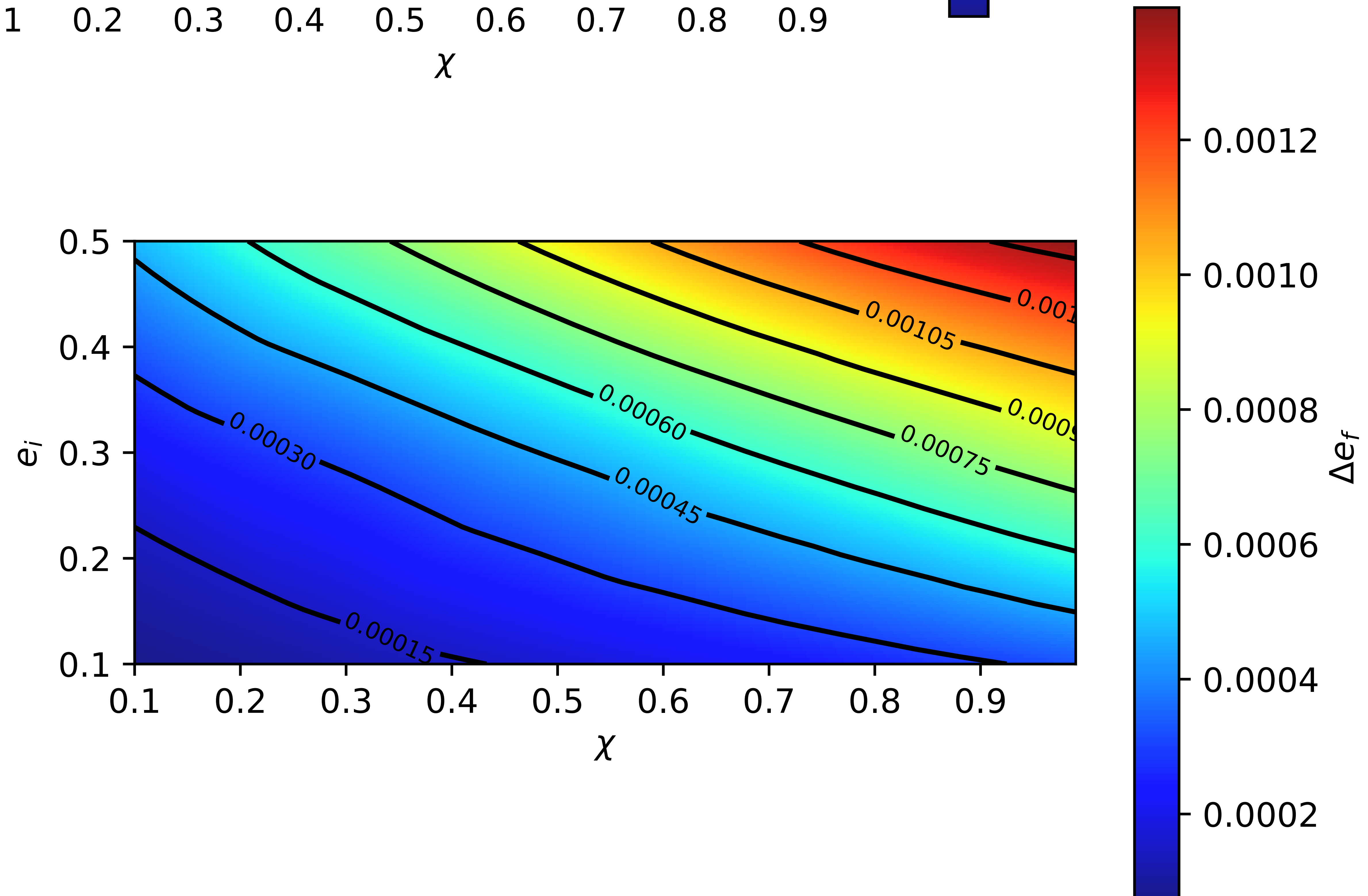


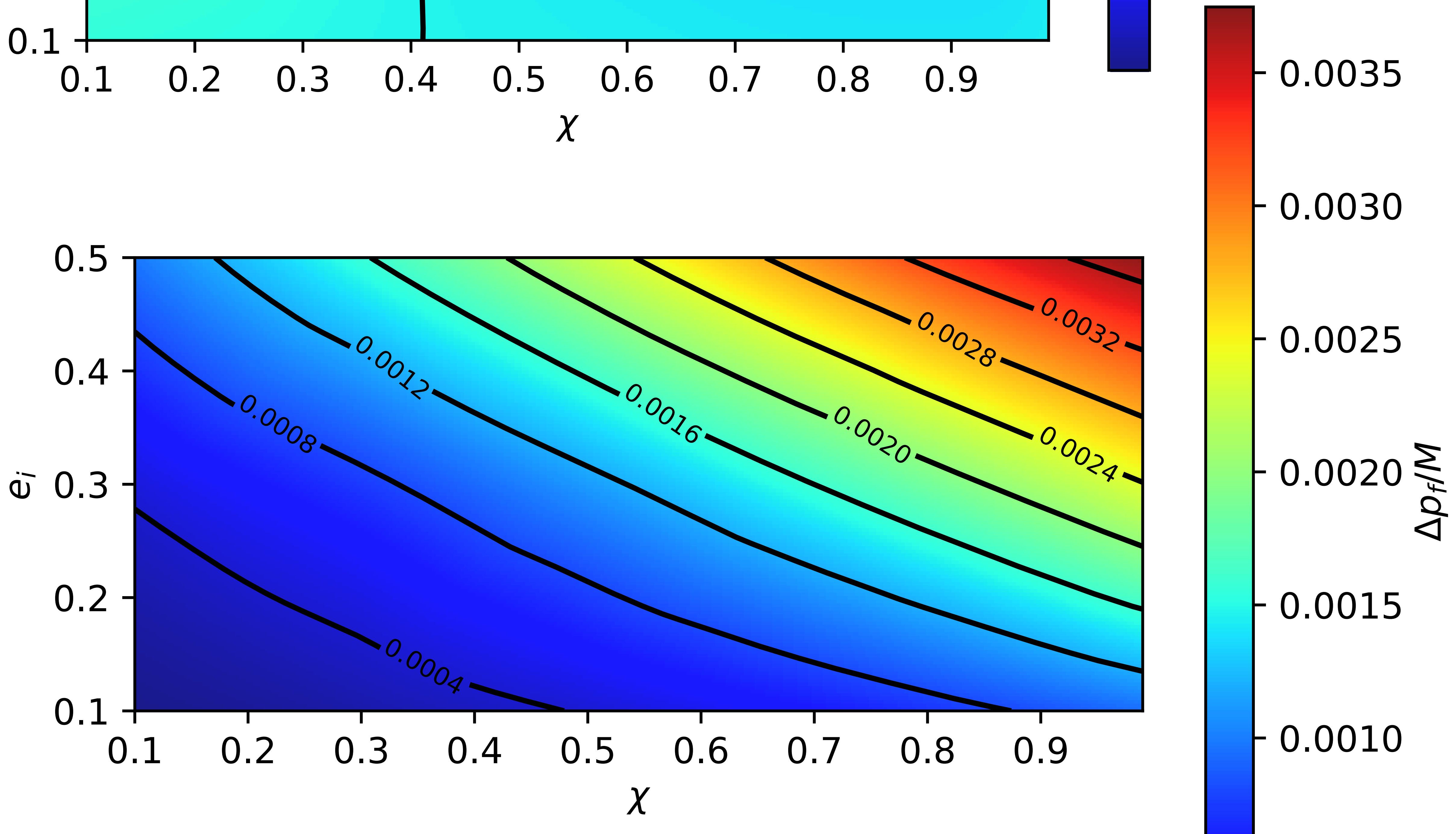


- Inspiral of binary is driven by energy loss at infinity and at horizon.
- Switching off energy exchange due to heating modifies inspiral rate, resulting in change in GW.
- Calculate GW with and without TH and calculate dephasing PRD.101.044004 SD, Brito, Bose, Pani, Hughes.









- In preparation

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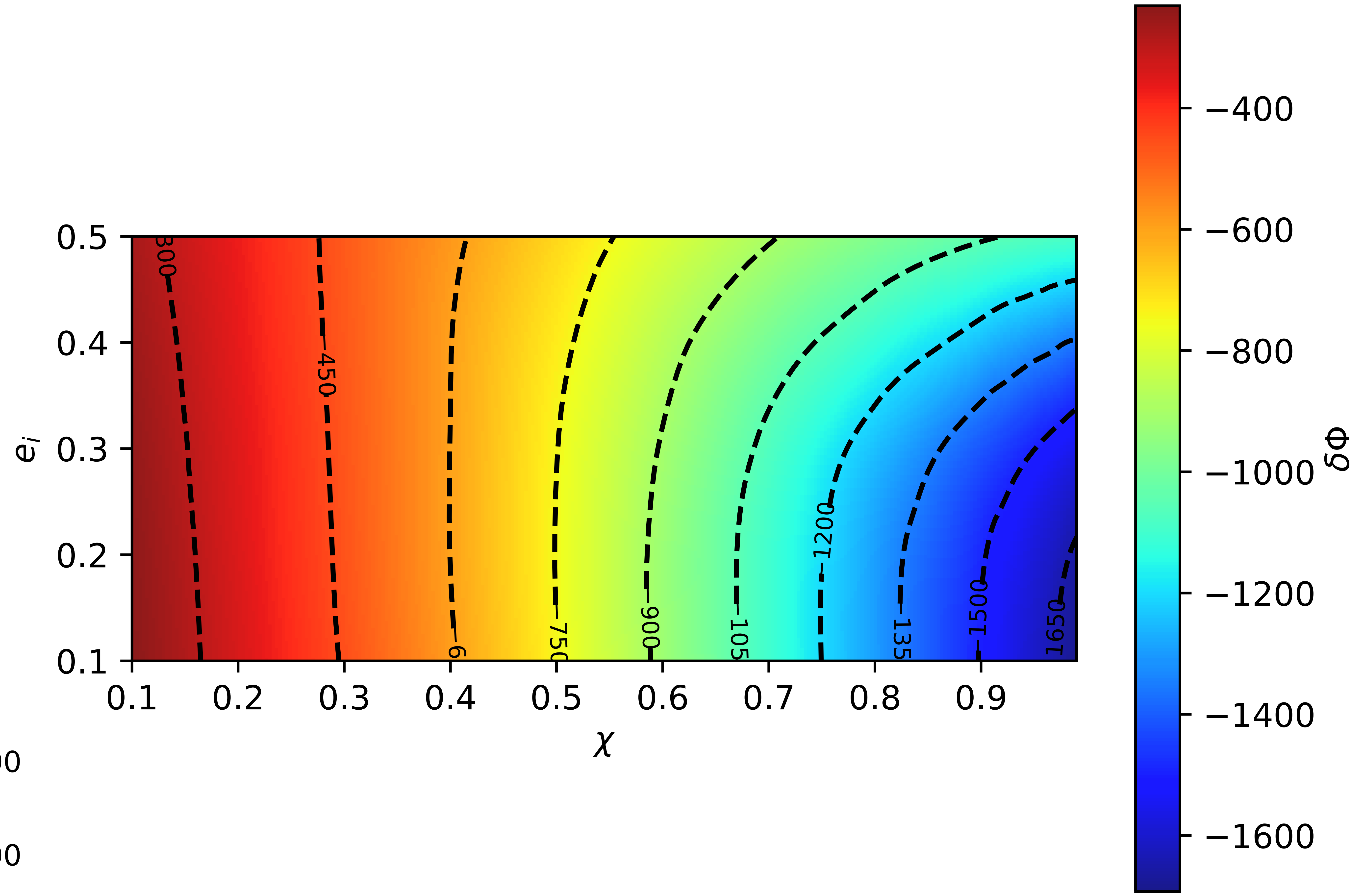
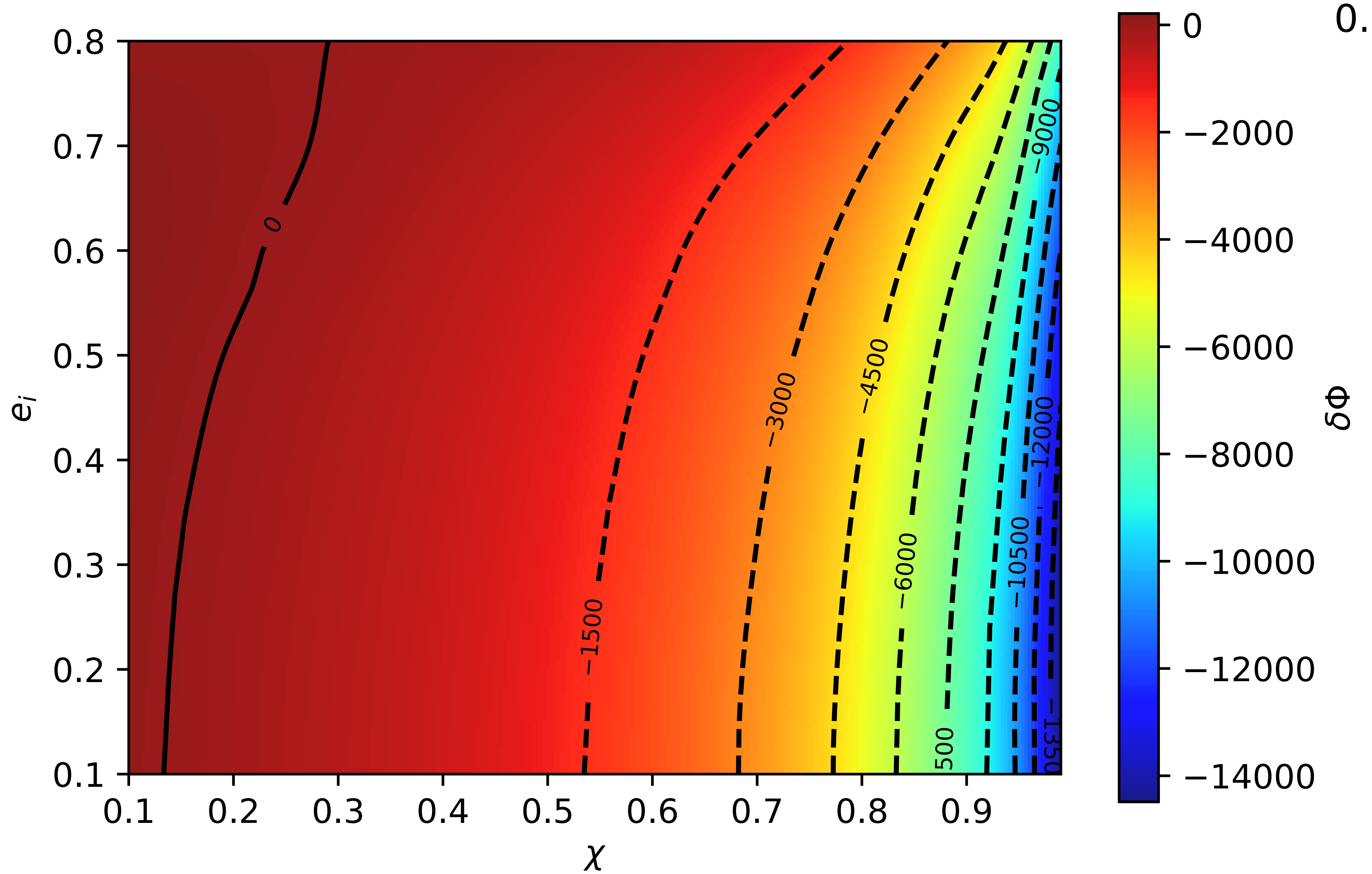
- $\delta\Phi_{m,n} = m\delta\Phi_\phi + n\delta\Phi_r$
- $\delta\Phi = 2\delta\Phi_\phi + \delta\Phi_r$

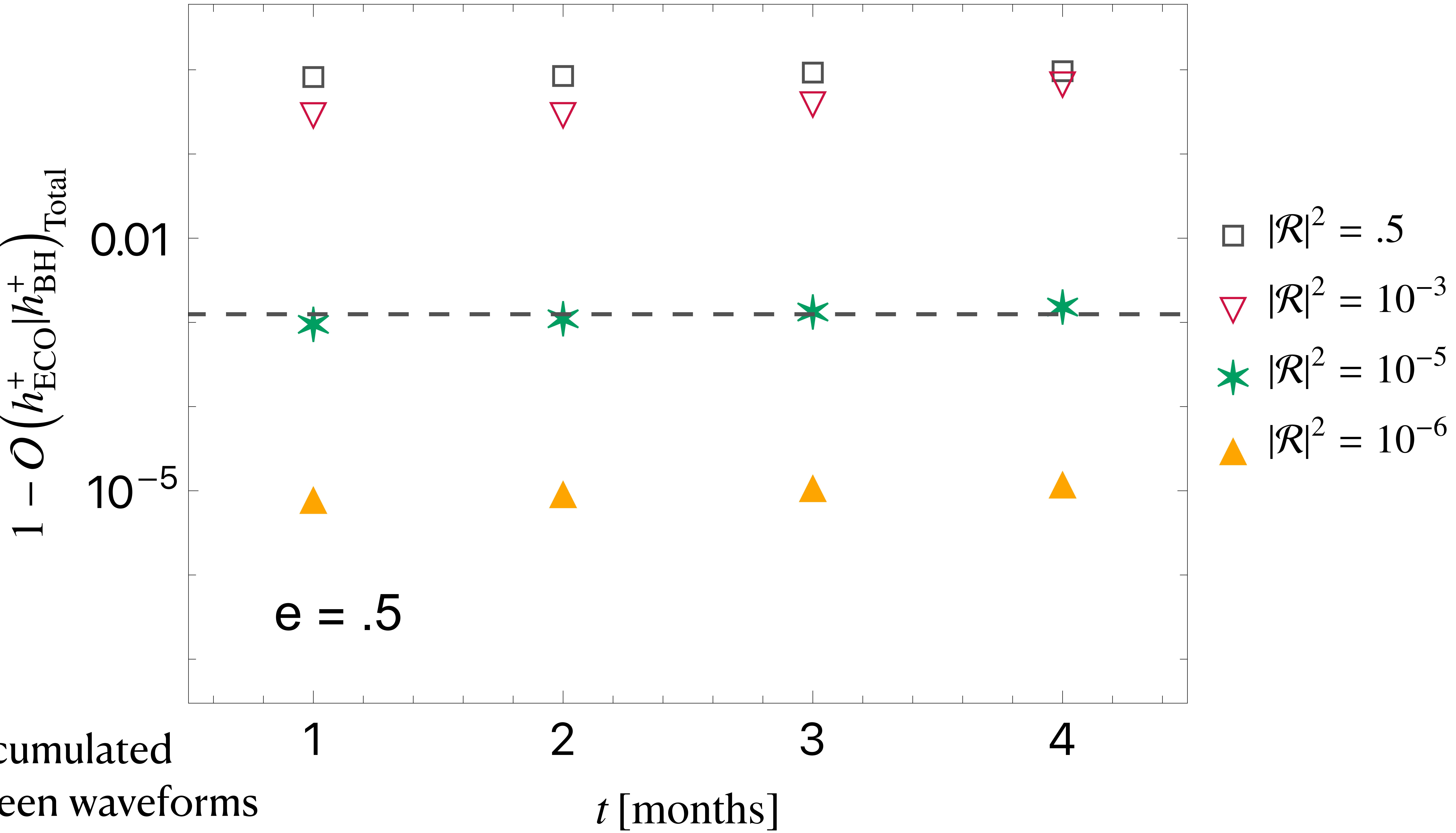




- In preparation

Usefulness of TH

- $\dot{E}_{\rm ECO} = (1 - |\mathscr{R}|^2)\dot{E}_H + \mathcal{O}(\epsilon)$ SD, PRD.102.064040 , Maggio+ PRD104 (2021) 10, 104026
- $\mathscr{R}$ is the reflectivity of the ECO (QBH).
- position of the reflective surface $r_s = r_H(1 + \epsilon)$.
- Measuring $|\mathscr{R}|^2$ tests the "Blackness" of the hole and ϵ tests the horizon position. SD, S. Bose, PRD99,084001 (2019), Maselli+, PRL120,081101(2018)



- Image is the accumulated mismatch between waveforms with $|\mathscr{R}|^2 \neq 0$ and BH. with time. In preparation

Take Home

- TH in eccentric case computed analytically.
- Using eccentricity evolution frequency evolution is computed.
- In EMRI, final orbital quantities can change due to modified TH, implying changed inspiral.
- TH induces large dephasing.
- In EMRI TH can lead to significant dephasing, resulting in constraining $|\mathscr{R}|^2 \sim 10^{-5} - 10^{-6}$.