Application of the GHZ formalism in a puncture scheme

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Self-force

Metric expanded as

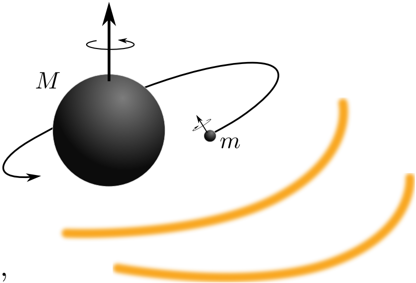
$$g_{\mu\nu} = g^0_{\mu\nu} + \frac{m}{M} h^1_{\mu\nu} + \left(\frac{m}{M}\right)^2 h^2_{\mu\nu} + \cdots$$

$$\Big\|$$

$$G_{\mu\nu}(g) = 8\pi T_{\mu\nu}$$

$$\Big\Downarrow$$

$$\begin{aligned} \delta G_{\mu\nu}[h^1] &=: \mathcal{E}_{\mu\nu}(h^1) = 8\pi T_{\mu\nu}, \\ \delta G_{\mu\nu}[h^2] &= -\delta^2 G_{\mu\nu}[h^1, h^1], \\ &\vdots \end{aligned}$$



CCK metric reconstruction

Avoid $\mathcal{E}_{ab}(h) = T_{ab} \longrightarrow$ use metric reconstruction scheme.

- Step 1: Solve Teukolsky equation in both vacuum regions

$$\mathcal{O}\psi_0^{\pm} = S^{ab} T_{ab}.$$

- Step 2: Compute the Hertz potential (radial inversion)

$$\text{þ}^4 \bar{\Phi}^{\pm} = -2\psi_0^{\pm}.$$

- Step 3: Finally, reconstruct metric

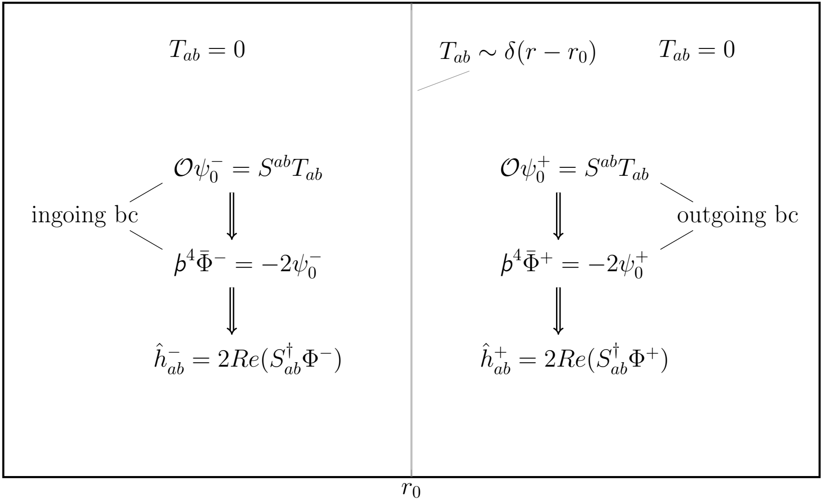
$$\hat{h}^{\pm}_{ab} = 2Re(S^{\dagger}_{ab}\Phi^{\pm}).$$

- Finally, add appropriate completion terms

$$h_{ab} = \hat{h}^{-}_{ab}\Theta^{-} + \hat{h}^{+}_{ab}\Theta^{+} + \dot{g}_{ab}\Theta^{+} - \mathcal{L}_{\xi} g_{ab}\Theta^{-}.$$

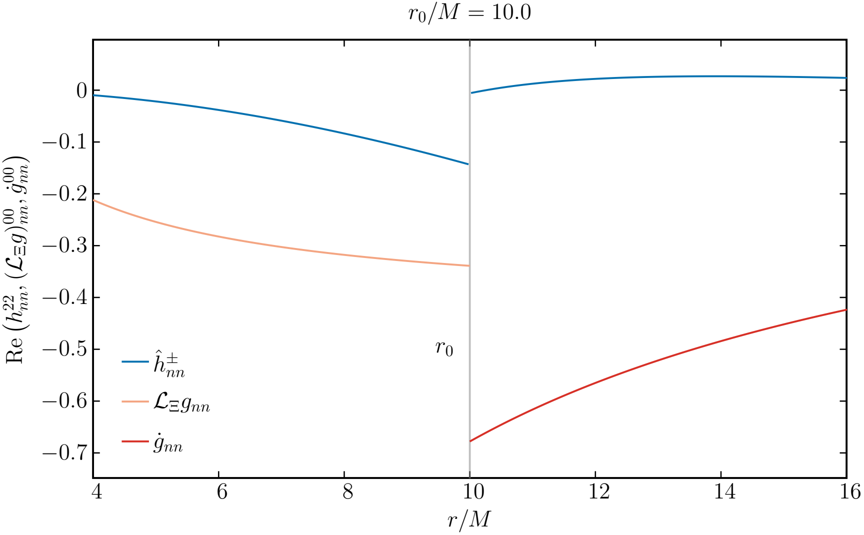
CCK no-string reconstruction

CCK reconstruction scheme

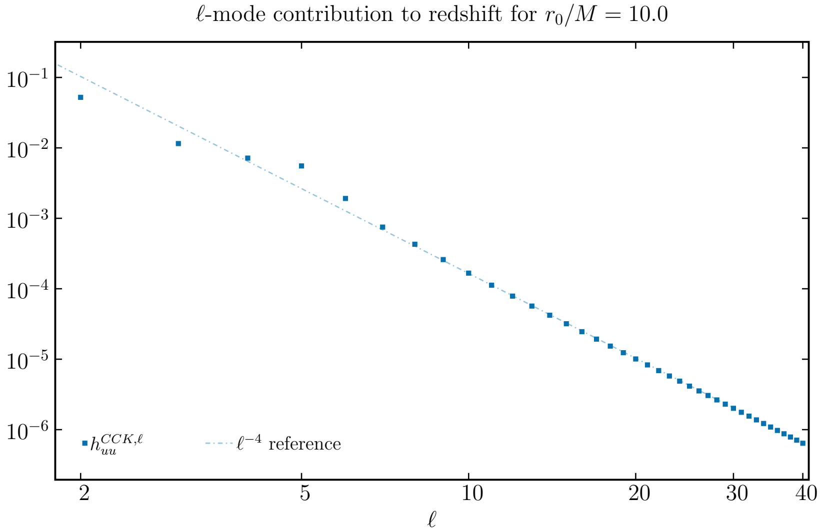


CCK no-string fields

$$h_{ab} = \hat{h}^{-}_{ab}\Theta^{-} + \hat{h}^{+}_{ab}\Theta^{+} + \dot{g}_{ab}\Theta^{+} - \mathcal{L}_{\xi} g_{ab}\Theta^{-}.$$



Redshift: $h_{uu} := h_{ab}u^a u^b$



From CCK to GHZ reconstruction

- CCK cannot be applied to problems with extended sources $\implies$ not applicable to puncture schemes.
- Problem already shows for point particle source, where $\mathcal{E}(\hat{h}^{\pm}_{ab}) \neq 0$ everywhere.

$\implies$ New reconstruction formalism by Green, Hollands, and Zimmerman.

- Supplements the CCK procedure with a corrector tensor x_{ab}.

$$\hat{h}^{\pm}_{ab} = 2Re(S^{\dagger}_{ab}\Phi^{\pm}) + x^{\pm}_{ab}.$$

Satisfies $\mathcal{E}_{ab}(\hat{h}^{\pm}) = 8\pi T_{ab}$, by imposing

$$(8\pi T_{ab} - \mathcal{E}_{ab}(x))\ell^{a} = 0.$$

x_{ab} satisfy a triangular system of 3 ODEs.

Puncture scheme

Split retarded field, $h_{ab}^{ret} = h_{ab}^{R} + h_{ab}^{P}$, and solve for residual field

$$\mathcal{E}(h^R) = 8\pi T_{ab} - \mathcal{E}(h^P) =: T_{ab}^{eff}.$$

$\implies$ associated Weyl scalar

$$\psi_0^R := \mathcal{T}^{ab} h_{ab}^R = \psi_0^{ret} - \psi_0^P.$$

$\implies$ associated Hertz potential

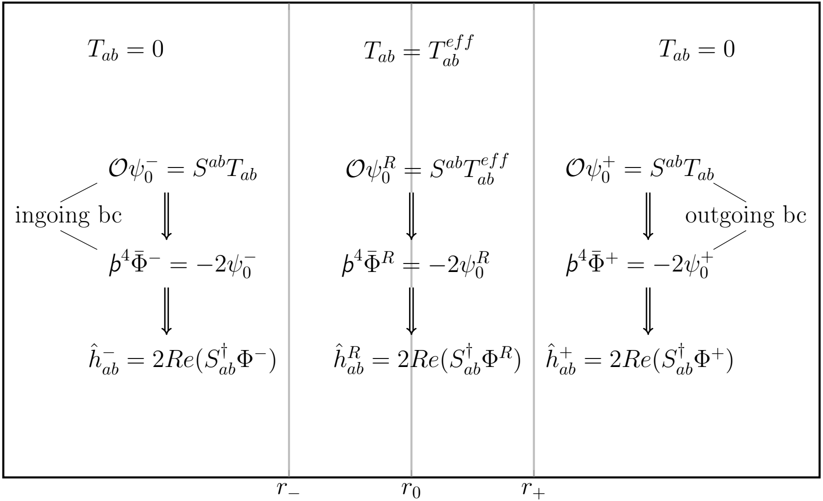
$$þ^4 \bar{\Phi}^R = -2\psi_0^R.$$

$\implies$ metric reconstruction

$$\hat{h}_{ab}^R = 2Re(S_{ab}^{\dagger}\Phi^R).$$

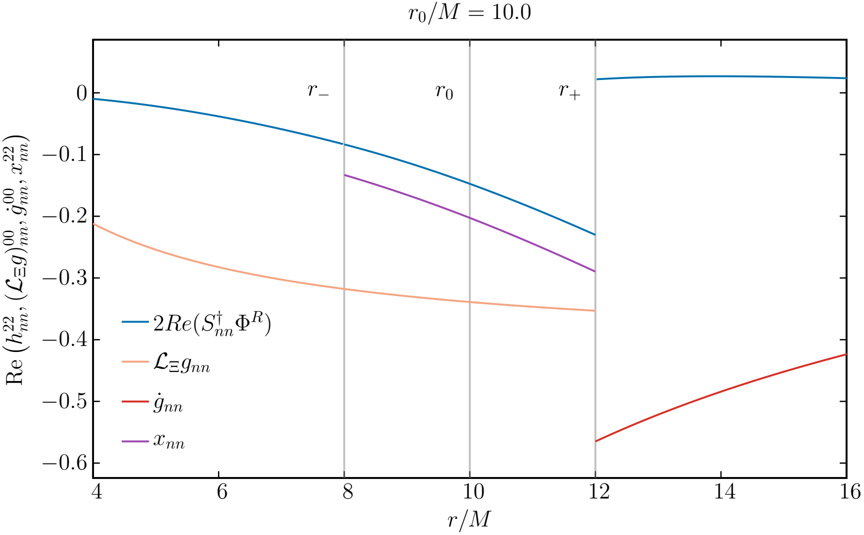
GHZ puncture scheme

GHZ puncture scheme

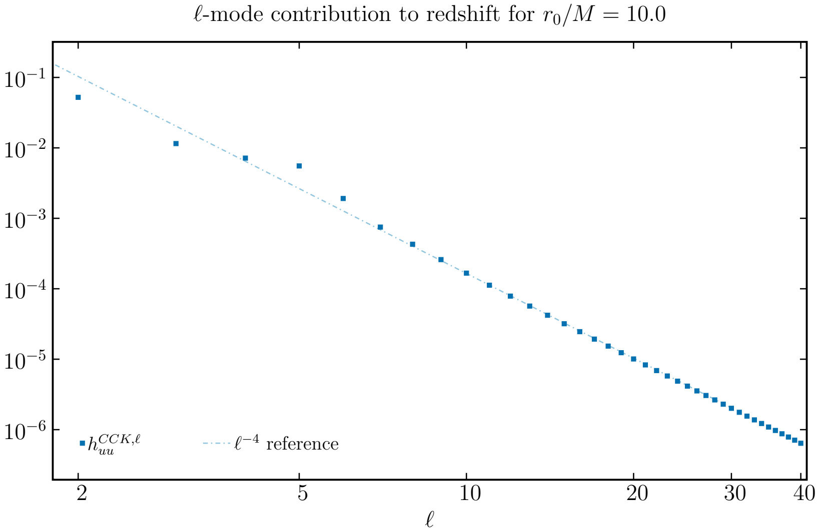


GHZ puncture scheme fields

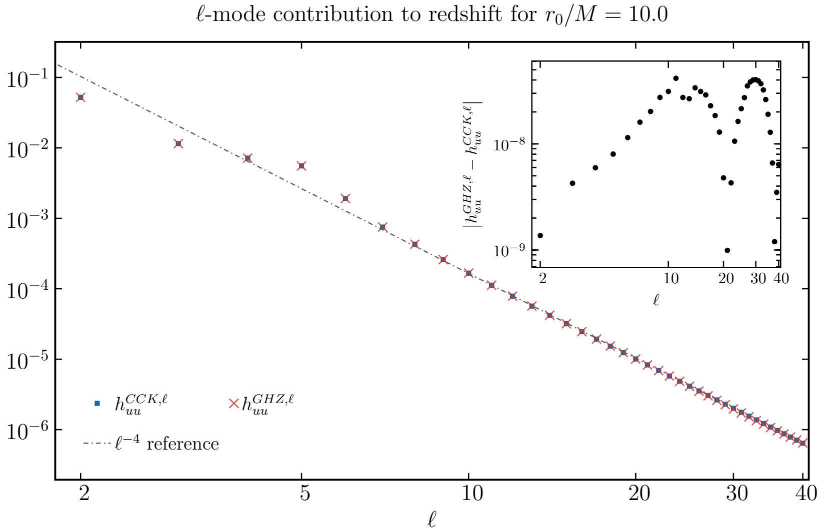
$$h_{ab} = 2Re\left(S^{\dagger}_{ab}\Phi^{R}\right) + x_{ab}\Theta^{M} + \dot{g}_{ab}\Theta^{+} - \mathcal{L}_{\xi}g_{ab}\Theta^{-}.$$



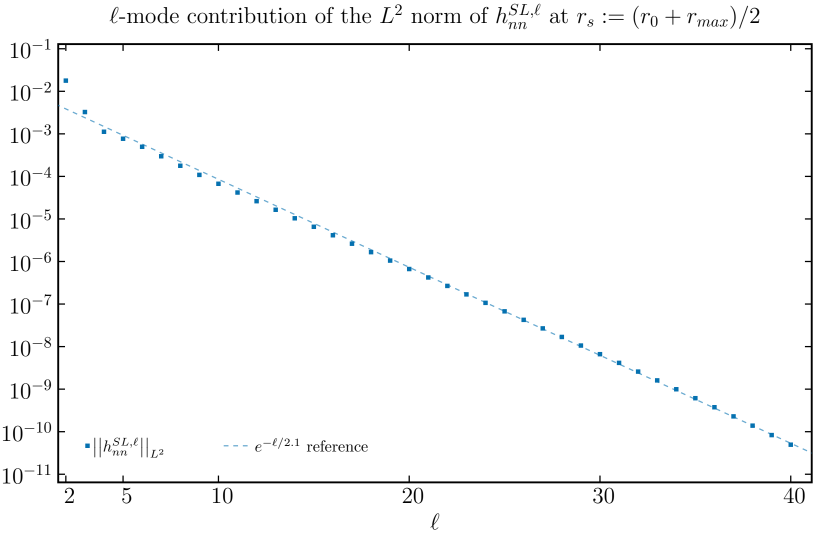
Redshift $h_{uu} = h_{ab}u^a u^b$



Redshift $h_{uu} = h_{ab} u^a u^b$



Thank you!



ℓ-mode contribution of the L^2 norm of $h_{nn}^{SL,\ell}$ at $r_s := (r_0 + r_{max})/2$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
2
5
10
20
30
40
ℓ
$||h_{nn}^{SL,\ell}||_{L^2}$
$e^{-\ell/2.1}$ reference